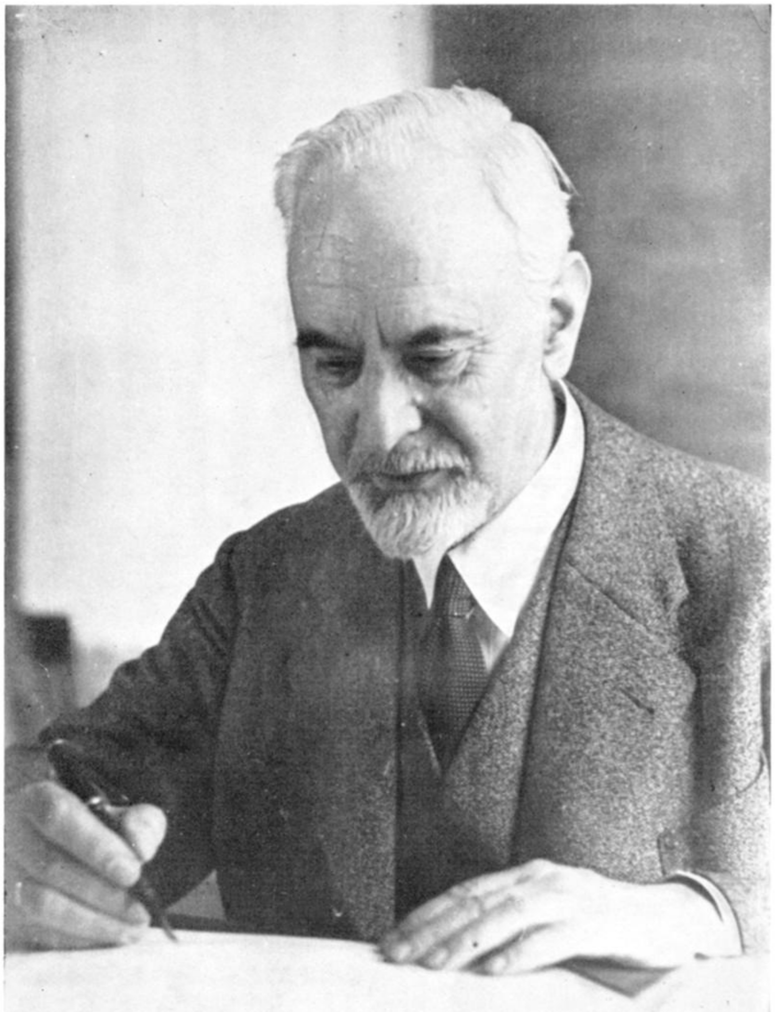
LUDWIG PRANDTL

1875-1953

IT IS A TRAGEDY in our age of technological evolution that those who initiate progress in important fields so often live to see their results applied to quicker and more thorough destruction. Certainly aviation is one of those dubious blessings. And the father of aerodynamics, LUDWIG PRANDTL, familiar since his youth with treasures of art and architecture in the lovely old city of Munich, doubtless could not help being haunted by the thought of why the day, on which mankind learns how to fly, must be the last day for many of these creations.

In heavier-than-air aviation, which Prandtl advanced by advocacy of wide-span monoplanes, he was not himself a pilot, while in the lighter-than-air department, in which he fathered the streamline shape of dirigibles, he was at least a licensed practising balloonist. But he was not even a passenger in an airplane until about 1930, and his aerodynamical laboratory flourished without any connexion to an airfield. Prandtl preferred a small staff of rational thinkers and was almost afraid of the emotional interference which comes with the touch of the stick and with real flying. In no way does that mean that all play and fun were taboo at work. On the contrary, Prandtl's own way to tackle the Navier–Stokes equations was to toy with flows far beyond the range of practical application, showing the pleasure and also the marvelling observation of a child, till a phenomenon crystallized that asked for explanation. Spheres and cylinders were used as models—there would be no 'Karman vortex street' without that—sharp-edged corners were preferred to complicated shapes. He liked to reduce the number of arbitrary shape parameters, etc., so the brain would have no excuse for misinterpretation. The growth in time of starting vortices or of the curling ends of discontinuity layers, or the steady supersonic flow around the corner, are famous examples without any geometrical length, as indeed is the development of the boundary layer from the edge of a flat plate.

Prandtl was born on 4 February 1875 in the Bavarian town of Freising, the only remaining child of three born to Magdalene Ostermann and Alexander Prandtl, professor of surveying and engineering at the Weihenstephan agricultural college. He learned Latin and Greek at a high school in Munich before he entered college to become a mechanical engineer. During his college years he lost both parents. In Munich at that time was the famous professor of engineering mechanics, August Foeppl, who was a revolutionary spirit engaged in rationalizing continuum mechanics, and a rival to the school of C. Bach, the more conservative group dominating the German engineering



society. There was no doubt that Prandtl felt at home in Foeppl's laboratory, when in his doctor's thesis, appearing in 1900, he tackled the torsional instability of beams with an extreme depth-width ratio. This paper and his soap-film analogy for the torsion problem, with its immediate recognition, could have been the start of a career in elasticity, but this was not his fate, though, as a matter of record, he never abandoned this subject nor the subject of plasticity.

A freak assignment, while Prandtl was employed at the firm, M.A.N., at Nuernberg, was to become the turning point in his life. He was told to investigate the failure of the pneumatic chip-removal installation in the carpentry shops. With his background in rational mechanics, the pneumatic installation, designed by empirical hydraulics, was bound to be a challenge for a young engineer who had not yet felt his limitations. His investigation was so successful that the firm not only reconstructed its installation accordingly but offered similar installations as a new line of products.

Soon he left this job for a start in his father's beloved academic career, when called as professor of mechanics to Hannover in the autumn of 1901. The 26-year-old Prandtl, however, did not give up his studies aimed at placing fluid mechanics on a rational basis. In a manually-driven portable water channel of a do-it-yourself type he observed flows and took pictures with one of the early photographic cameras and within four years he had elucidated one of the great riddles of flow with vanishing viscosity—the boundary layer, or, in Prandtl's original terms, the friction layer on the walls. He presented this result at the Third International Congress of Mathematicians at Heidelberg in 1904.

While Prandtl's entry to his lifetime task was admittedly accidental, his further climb on the ladder of success was comparatively logical, but there were still unusual circumstances which brought him to the place of his lifetime work. The mathematician and editor of the *Encyclopedia of Mathematical Sciences*, Felix Klein, famous for his unifying mathematical concepts, for influencing the teaching of mathematics in schools, and for tracking down the sources of mathematics wherever they may be, had the idea that the University of Göttingen needed a cross-breeding of its pure mathematical tendencies with applied sciences in order to keep up its traditions. So he brought four strangers, men of chemistry and engineering, to a place of worship of the humanities, which meant the breaking down of quite a few fences. Time proved all of these men to be well chosen, and Prandtl was one of them. In the autumn of 1904, he was made head of the applied physics—later to be called applied mechanics—laboratory. Prandtl's own students were from then on majoring in mathematics, men like H. Blenk, O. Tietjens, W. Tollmien, and H. Schlichting. The vital construction engineers for his wind tunnels, like G. Fuhrmann, A. Betz, M. Munk, C. Wieselsberger, J. Ackeret and many more, were usually hired on recommendation or earlier acquaintance.

The Wright brothers having already succeeded in their first flight, and others soon afterwards, the era of flight had begun and the necessity to develop the aeronautical sciences became apparent. In 1906 a study committee was set up by proponents of dirigibles and, with Felix Klein as spokesman, plans were presented to create a model testing laboratory at Göttingen. The tool for such tests, a 35 h.p. wind tunnel, was built in 1908. This was the first offspring of the do-it-yourself water channel; it retained the turning vanes in the four corners of its closed circuit, which is one of the family traits of the Göttingen-type tunnels. The velocity distribution in the test chamber was subsequently improved by the incorporation of a large contraction, following a settling chamber with honeycomb, so familiar in conventional tunnel design. In 1915, during World War I, another wind tunnel project was started and completed in 1917, with 300 horse-power. In 1926, a third wind tunnel for propeller research, with increased velocities at reduced pressures, was built on the same location, while in 1933, a full-scale wind tunnel followed.

Wind tunnels of larger size are beyond the budget of a university laboratory and require sponsors for their building and an organization for their operation. Prandtl was lucky to find such sponsors and vice versa. After the study committee for dirigibles had completed its task and dissolved, the Kaiser Wilhelm Society for the Advancement of Science was prevented from creating a large aerodynamical centre only by the outbreak of the first world war. Soon after, however, by a joint effort of army and navy during the war, the laboratory was organized and the staff recalled from the armed forces. After the war a private organization of sponsors of the Aerodynamical Laboratory took responsibility and found support from the Ministry of Transport while, after 1933, more substantial support came from the research department of the Air Ministry. Prandtl, whose endeavour it was to put aerodynamics on a rational basis, wondered whether theory would do the main job and wind tunnels be for spot checks, or whether the tunnels would still remain his most successful children. Timetables of the total horse power installed in wind tunnels around the earth were sometimes drawn in Göttingen for the past, and extrapolated with a daring exponential curve into the future; but it seems that during Prandtl's lifetime the extrapolations into the future proved always below the actual growth.

Prandtl's favourite type of research was more closely realized in 1925 when after the years of inflation the recovered Kaiser Wilhelm (later named Max Planck) Society for the Advancement of Science succeeded in surrounding Prandtl with the proper equipment. The new laboratory for flow research adjacent to the wind tunnels contained on a larger scale facilities for supersonic flow in air and for cavitation tests in water. They consisted mainly of a couple of tanks, a gasometer for dried air with the drying equipment, and a compressor, a vacuum pump and a water pump, to continue the study of diversified flow phenomena so successfully started in 1904 to 1908 in the applied mechanics laboratory of the University. His meteorological interest was supported by a rotating chamber for flow observations. When, therefore, at about the time of the Volta Conference in Rome, 1935, the compressibility effects at high-speed flight became a matter of concern, Göttingen was again well prepared with results, like the Prandtl–Glauert rule for subsonic flow and the characteristics method for supersonic flow, which Prandtl had used in a primitive form (only valid for perfect gases at $\sqrt{2}$ times the sonic velocity) as early as 1906 to shape the exit of his Laval nozzles for parallel supersonic jets—work which was laying the foundation of compressible aerodynamics as surely as his work on boundary layers was building up viscous flow theory.

Prandtl's scientific achievements in aerodynamics start with the proof that the Navier–Stokes equation for small viscosity leads to solutions in which the viscosity is relevant only in a thin layer surrounding solid walls, the so-called boundary layer. Separation of this layer from the body forms wakes and causes large pressure drag. Proper streamlining of the body reduces the drag to a mere shear drag if the boundary layer can be kept on the body till it unites from all sides at the rear. This insight into the causes of drag makes streamlining a rational operation and explains the stalling at larger angles of attack of wing profiles. Two other phenomena became thereafter ready for a full explanation. Streamlined wings of finite span showed a drag due to lift, which Prandtl called induced drag after being able to determine its numerical value and the lift distribution to minimize this drag. His basic concepts are the same as in Lanchester's vortex theory of lift. But Prandtl's trick, to start with small lift values on a lifting line normal to the flight direction, avoids most of the difficulties encountered in the general problem. While the separation of boundary layer has generally only a minor dependence on the Reynolds number, discontinuous changes were observed by Eiffel and others for the drag of spheres, cylinders, and for the stalling angles of wing profiles at a Reynolds number of the order of half a million. Prandtl was able to explain this critical point by the alternative possibilities of laminar or turbulent boundary layer flow, similar to Reynolds's laminar or turbulent pipe flow. Since the boundary layer thickness grows with the square root of the body length, the order of a million reduces to the order of a thousand for a comparable pipe flow, which is the familiar order of Reynolds numbers for flows becoming turbulent in channels and pipes.

The relation to turbulent pipe flow, discovered as early as 1913 and used qualitatively to explain the similarity between shear and heat transfer along the surface, induced Prandtl later to investigations about the origin of turbulence. Though a self-supporting turbulence is strictly a three-dimensional phenomenon, the instability for small disturbances, if it exists, can be investigated by superposition of two-dimensional particular solutions. Prandtl, whose own doctor's thesis was about elastic instabilities, had a number of his Göttingen students, O. Tietjens, W. Tollmien and H. Schlichting. and also A. Sommerfeld's pupil W. Heisenberg, working on the hydrodynamic instability of boundary layer profiles for their theses. H.B. Squire, a guest from England, completed the three-dimensional aspect by adding oblique disturbances. The work itself proved successful though it is on the border line where those two necessities, permissible simplifications and convincing rigor, might not readily be agreed to by a sceptical audience. Thus it was not until after the second world war that this chapter of smooth wall turbulence found international recognition along with the other possibility of starting turbulence at finite amplitudes with the help of roughness or other imperfections.

During this time Prandtl worked simultaneously on the problems of fully developed turbulence in wakes by introducing the concept of mixing length. He also generalized the knowledge of criteria for separation, by comparing a compressible flow to an incompressible flow with similiar pressure distribution. The equivalence of a thin compressible and a thicker incompressible flow of reduced scale in the crosswise direction is the so-called Prandtl–Glauert rule.

Looking at Prandtl's aerodynamic achievements, which he presented in his steadily expanding book, starting as two handbook articles in 1913 and ending as *Essentials of fluid dynamics* in 1952, and considering the many other subjects of his publications, one feels small wonder that his greatest enjoyment came from his work and the discussion of problems with pupils, whose number and variety of background increased with the growth and fame of the laboratory. Special studies at the laboratory were performed by foreign guests; some English guests were J. W. Maccoll, S. Goldstein, H. B. Squire, and Miss L. M. Swain. Nevertheless his tour around the world in 1929 on the occasion of the World Engineering Congress at Tokyo and the reception by his friends in foreign countries were highlights in his career.

Prandtl as a person was a modest, almost shy, man who regarded his gift, of such amazing clarity in dissecting and composing mechanical phenomena and finding ways to reduce their complexity, as a grace not as a gold mine. The influence, in developing this gift, of his famous teachers, August Foeppl and Felix Klein, resulted in everlasting gratitude. He asked for the hand of Foeppl's eldest daughter, Gertrude, in 1909; he was then well prepared to support a wife, and the two daughters which came in 1914 and 1917. His refusal of a call to Foeppl's chair in Munich, 1923, in favour of a new laboratory in Göttingen, is an example of the traditional method of trading in academic life, and is the reason for many excellent laboratories in Germany. Prandtl loved his native state, Bavaria, and hiking in the hills around Göttingen was only a weekend substitute for the mountains of his summer vacations in Bavaria. He was musical, and calibrated his absolute pitch while a member of an orchestra in his youth. His private life and his family life were inconspicuous except for the annual party which he enjoyed giving in later years for his laboratory staff. His pupils found Prandtl's work and life such a complete unity that they instinctively oversimplified his personality, as is revealed by the few anecdotes which claim his reactions to be predictable. Of course, he could hardly pass a toy without touching it, and to figure out how a magician's trick works was to him just as serious a problem as an unbelievable result in a test. He was a passionate teacher, but in spite of all his admiration for witty remarks of others, he preferred to make the ideas shine, not his words.

Prandtl's publications share the authorship with his pupils. He would be more tempted to write for them than to have their contribution incorporated in his papers, even if it were one as personal as an anniversary contribution. He observed rigid rules of personal integrity which were not open to anybody's advice. Thus he could surprise his friends in small things, like refusing his signature on an application for extra food rations for his laboratory, as long as his name was part of the list. And he could surprise them in serious things. When in the laboratory, one professional party speaker finished his standard speech by saying that nothing positive was done in the past fourteen years except the marching of the brown columns, Prandtl could jump up and summarize in an eloquent little address for the members of his staff just those few important positive things done in aerodynamics within the last fourteen years by people who also love their country. Such talk and the following applause could ruin a man. It took some quick thinking of his friends to make Goering allocate money for a new wind tunnel, to get Field Marshall Milch to present it as an anniversary gift to Prandtl, and to assure the irreversibility of his honour by a torch parade of the students to his residence.

Commonly Prandtl's talk was not quite so eloquent as in times of emotion. According to his aim to make his sentences foolproof, they required a re-wording, a re-phrasing, an explanation of the reasons for re-phrasing and perhaps the discussion of some exception to the stated rule. It is quite obvious how different the result of such lectures must have been for beginners and for listeners with background. Seeing the details as clearly as Prandtl did, and never trying to circumvent difficulties by omitting them, made his lectures, seminars, colloquia a rich source of information for all his pupils, besides those fortunate few who walked with him home from work. Spotting at the gate the group bringing each other home did not mean a thing, as the wives had to learn: it still was not sure at which house the discussion would stop, to be carried on by a smaller group with fewer residences as points for further decrystallization. This is the secret behind handbook articles offered to Prandtl but written by his pupils. Prandtl's own work was done from supper to midnight in the study of his rented flat. Late in the morning after classes, or direct from home, he came with new results or with ideas not quite ready, which he would like to discuss.

The second world war took much of his time for the many obligations which he had to accept because of his knowledge, or because his reputation might shield others, when the war machine faltered. Among all those creations, the Academy of Aircraft Research with its monthly meetings was one institution in which Prandtl was again receptive and productive to the delight of his former pupils. Giving from his large store of experience thoughts and details worthy of preservation for posterity, made him seem to enjoy these meetings just as much as the other members.

When his first grandchild died, when his first son-in-law was killed during the war, and when his wife died immediately after the war, these events and the general situation of his creations diminished his vitality to a dangerous degree and friends in Switzerland invited him to a successful recovery. His work, the nerve of his life, was continued with new editions of his book and a number of papers on different subjects. His own favourite is the explanation of 'Weather phenomena in the upper troposphere' of 1949. But his body lost its strength and, his light being already dimmed during his last year, death came on 15 August 1953.

Prandtl received many honours and medals (some British ones are the honorary doctor's degree from the University of Cambridge in 1936, foreign membership of the Royal Society of London from 1928, and honorary membership of the Royal Aeronautical Society and receipt of its gold medal). They made him happy when they arrived, though the recording of this happiness he left to his wife. His gift, combined with his modest personality and his pleasure in helping others, made him many real friends. Only a very few were able to return him a favour, and nobody as much as Albert Betz, his associate for about forty years. The rest will be fortunate if they succeed in giving to others as much as they received from him.

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