ELECTRIC CHARGE AND ELECTRIC FIELD

4 FORCES IN NATURE : GRAVITY
ELECTROMAGNETISM
STRONG (NUCLEAR)
WEAK (RADIOACTIVE DECAY)

ELECTROMAGNETISM = ELECTRICITY + MAGNETISM.

NEED CHARGE (OR CURRENT) FOR ELECTROMAGNETISM.

CHARGE IS PROPERTY OF OBJECT (PARTICLES).

CHARGED OBJECT CAN BE ACCELERATED BY ELECTRIC FORCE.

ELECTROMAGNETISM IS THE FORCE OF OUR EVERY DAY WORLD: CHEMISTRY
BIOLOGY
TECHNOLOGY (PHYSICS)
EVEN LIGHT

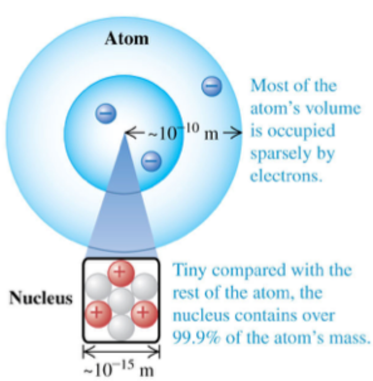
ELECTRIC CHARGE

2 KINDS OF ELECTRIC CHARGE : POSITIVE
NEGATIVE

2 CHARGES : $\left.\begin{matrix} + & + \\ - & - \end{matrix}\right\}$ REPEL, $\left.\begin{matrix} + & - \\ - & + \end{matrix}\right\}$ ATTRACT } INDEPENDENT OF THE VALUE OF THE CHARGE

OFTEN IT IS THE NET ELECTRIC CHARGE THAT IS IMPORTANT.

STRUCTURE OF MATTER



ELECTRONS, −'VE
PROTONS, +'VE } CHARGE MAGNITUDES EQUAL

NEUTRONS, NO (ELECTRIC) CHARGE

QUARKS $\pm 1/3$, $\pm 2/3$ ELECTRIC CHARGE

ATOMS ARE CHARGE NEUTRAL (ZERO NET CHARGE)
IONIZATION : ADD AN ELECTRON (-'VE ION)
REMOVE AN ELECTRON (+'VE ION)

ELECTRIC CHARGE CONSERVATION

CONSERVATION OF CHARGE : ALGEBRAIC SUM OF ALL ELECTRIC CHARGES IN CLOSED SYSTEM IS CONSERVED.

UNIT OF CHARGE : MAGNITUDE OF ELECTRON OR PROTON CHARGE.

CHARGE IS QUANTIZED.

CONDUCTORS, INSULATORS, AND INDUCED CHARGE

2 CLASSES OF MATERIALS FOR THIS COURSE.

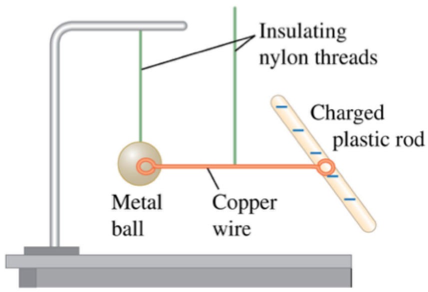
CONDUCTOR - PERMITS EASY MOVEMENT OF CHARGE.
- ELECTRONS CAN MOVE FREELY.
- METALS ARE GOOD CONDUCTORS.

INSULATORS - CHARGE CAN BUILD UP OR SEPARATE A BIT, BUT NOT FLOW.

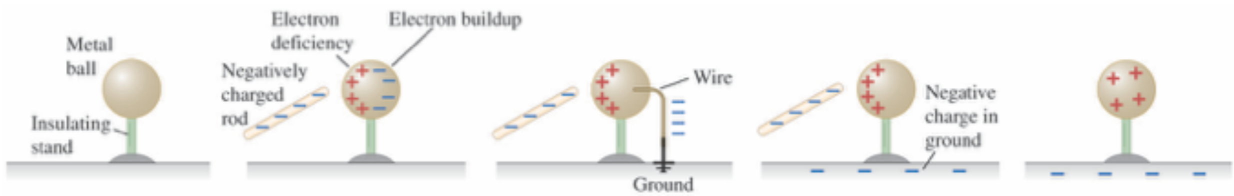
ALSO, SEMICONDUCTORS - INTERMEDIATE BETWEEN CONDUCTOR AND INSULATOR

SUPERCONDUCTOR - NO RESISTANCE.

CHARGE BY INDUCTION



NEED CONDUTOR



(a) Uncharged metal ball

(b) Negative charge on rod repels electrons, creating zones of negative and positive **induced charge**.

(c) Wire lets electron buildup (induced negative charge) flow into ground.

(d) Wire removed; ball now has only an electron-deficient region of positive charge.

(e) Rod removed; electrons rearrange themselves, ball has overall electron deficiency (net positive charge).

ELECTRIC FORCE ON UNCHARGED OBJECTS

INDUCED CHARGE EFFECT. (ATTRACTIVE FORCE)

NETCHARGE IS ZERO.

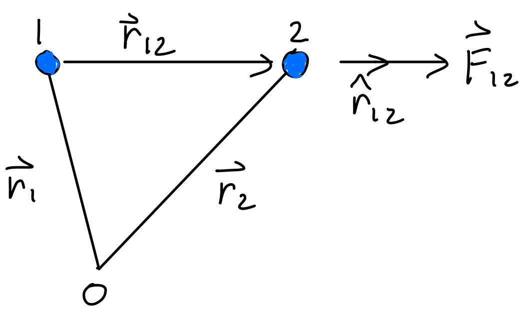
POLARIZATION - SEPARATION OF CHARGE IN OBJECT

COULOMB'S LAW

FOR STATIC POINT CHARGES IN VACUUM.

SIZE OF OBJECT SMALL COMPARED TO DISTANCE BETWEEN THEM.

FORCE ON 2 DUE TO 1



$$\boxed{\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}}$$

$$\vec{F}_{12} = F_{12}\hat{r}_{12}$$

$k \equiv$ PROPORTIONALITY CONSTANT (COULOMB CONSTANT)
(VALUE DEPENDS ON UNITS)

q_1, q_2 ARE SIGNED CHARGES

$\vec{r}_{12} = -\vec{r}_{21}$ AND $r_{12} = r_{21}$; $\hat{r}_{12} = \dfrac{\vec{r}_{12}}{r_{12}} \Rightarrow \hat{r}_{12} = -\hat{r}_{21}$

CONSIDER FORCE ON 1 DUE TO 2

$$\vec{F}_{21} = k\,\frac{q_2 q_1}{r_{21}^2}\,\hat{r}_{21} = -k\,\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12} = -\vec{F}_{12}$$

SATISFIES NEWTON'S 3RD LAW

MAGNITUDE $F = k\dfrac{|q_1 q_2|}{r^2}$

ANALOGOUS TO GRAVITY $F = G\dfrac{m_1 m_2}{r^2}$

MASS HAS ONLY "POSITIVE CHARGE" $\Rightarrow$ FORCE ONLY ATTRACTIVE

CHARGE HAS 2 SIGNS $\Rightarrow$ ATTRACTIVE OR REPULSIVE.

FUNDAMENTAL ELECTRIC CONSTANT

[C] IS COULOMB OF CHARGE

$q \equiv e = 1.6 \times 10^{-19}$ C FOR ELECTRON (OR PROTON)

$k = 8.988 \times 10^{9}\ \mathrm{N\,m^2/C^2} \approx 8.99 \times 10^{9} \sim 9 \times 10^{9}$

WILL WRITE $k = \dfrac{1}{4\pi\epsilon_0}$

$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N\,m^2} \approx 8.9 \times 10^{-12} \sim 9 \times 10^{-12}$

$\epsilon_0 \equiv$ PERMITIVITY OF FREE SPACE (ELECTRIC CONSTANT)

(EQUATE GRAVITY TO COULOMB AT SAME DISTANCE $\Rightarrow \dfrac{F_e}{F_g}$ HUGE)
(AT ATOMIC SCALES)

SUPERPOSITION OF FORCES

IF MULTIPLE CHARGES, USE VECTOR SUM OF FORCES.

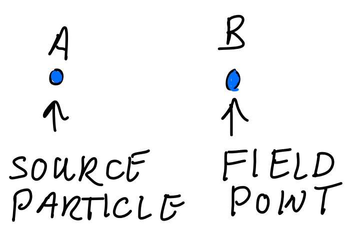
AIR $\approx$ VACUUM (DOES NOT WORK IN MATRIALS)

(3 EXAMPLES IN BOOK ; DO 21.72)

ELECTRIC FIELD AND ELECTRIC FORCES

ELECTRIC FIELD

CONSIDER 2 POINT PARTICLES LABEL AS SUCH



B FEELS THE ELECTRIC FIELD OF A.

A PRODUCES ELECTRIC FIELD INDEPENDENT OF B

REMOVE B.

B CAN REPLACE A IN THIS PICTURE

BUT, A CHARGE CAN NOT PRODUCE A FORCE ON ITSELF.

TO DETECT A FIELD, PUT TEST CHARGE IN SPACE AND SEE IF FORCE ACTS ON IT.

$$\vec{E}(\vec{x}) = \frac{\vec{F}_0(\vec{x})}{|q_0|}$$ ELECTRIC FIELD AT POINT $(\vec{x})$

$|q_0| \leftarrow$ TEST CHARGE

$\vec{x} \equiv$ FIELD POINT (NOT SOURCE POINT)

$\vec{F}_0(\vec{x}) \equiv$ ELECTRIC FORCE ON TEST CHARGE q_0 LOCATED AT $\vec{x}$.

$\vec{E}(\vec{x}) \equiv$ ELECTRIC FORCE PER UNIT CHARGE AT POINT $\vec{x}$.

$\vec{E}(\vec{x})$ DOES NOT DEPEND ON THE TEST CHARGE

THE TEST CHARGE SHOULD BE A POINT CHARGE.
(BECAUSE FIELD DEFINED AT POINT).

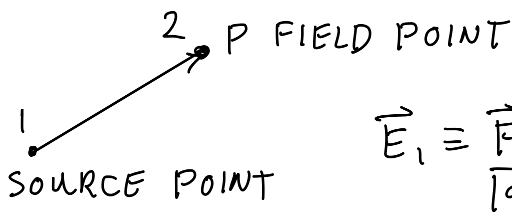
ANY CHARGE CONFIGURATION CAN PRODUCE THE $\vec{E}$ FIELD.

UNITS OF ELECTRIC FIELD [N/C].

$\vec{E}$ PARALLEL TO $\vec{F}$ AT $\vec{x}$ IF q_0 POSITIVE. $\quad \begin{matrix} \longrightarrow \vec{E} \\ \longrightarrow \vec{F} \end{matrix}$

$\vec{E}$ ANTI-PARALLEL TO $\vec{F}$ AT $\vec{x}$ IF q_0 NEGATIVE. $\quad \begin{matrix} \longrightarrow \vec{E} \\ \longleftarrow \vec{F} \end{matrix}$

ELECTRIC FIELD OF A POINT CHARGE



$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$\vec{E}_1 \equiv \frac{\vec{F}_{12}}{|q_2|} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}^2} \hat{r}_{12}$$

FOR ARBITRARY q AND r, $\vec{E} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q}{r^2} \hat{r}$

SOURCE TO FIELD

AWAY FOR POSITIVE
TOWARDS FOR NEGATIVE

THIS IS ELECTRIC FIELD AT POINT P DUE TO ELECTRIC CHARGE q

MAGNITUDE $E = \dfrac{1}{4\pi\epsilon_0} \dfrac{|q|}{r^2}$

THE FIELD AT P IS INDEPENDENT IF THERE IS CHARGE AT POINT P, OR NOT.

$\vec{E}(\vec{x})$ IS A VECTOR FIELD $\vec{E}(x, y, z)$, $\vec{E}(r, \theta, \phi)$, $\vec{E}(\rho, z, \phi)$

$E_x(x, y, z)$, $E_y(x, y, z)$, $E_z(x, y, z)$

UNIFORM FIELD – SAME VALUE (AND DIRECTION) IN SOME REGION OF SPACE

IN ELECTROSTATICS, THE ELECTRIC FIELD AT EVERY POINT WITHIN THE MATERIAL OF A CONDUCTOR MUST BE ZERO

CALCULATIONS ARE OFTEN 2 STEPS:
1) CALCULATE FIELD
2) CALCULATE MOTION DUE TO FIELD

(3 EXAMPLE IN BOOK WITH NO INTEGRATION)

ELECTRIC FIELD CALCULATIONS

SUPERPOSITION OF ELECTRIC FIELDS

TOTAL FORCE $\vec{F}_0 = \sum_i \vec{F}_i = q_0 \sum_i \vec{E}_i = q_0 \vec{E}$

$\therefore \vec{E} = \dfrac{\vec{F}_0}{q_0}$

CONSIDER FIELDS DUE TO CHARGE DISTRIBUTIONS.

ρ VOLUME CHARGE DENSITY $[C/m^3]$, $\rho = dq/dV$

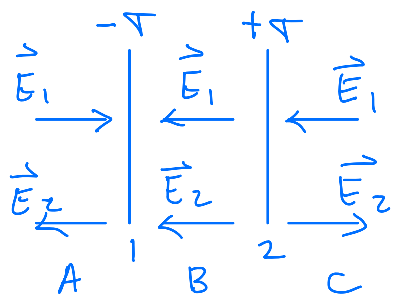
σ SURFACE CHARGE DENSITY $[C/m^2]$, $\sigma = dq/dA$

λ LINEAR CHARGE DENSITY $[C/m]$, $\lambda = dq/dl$

(5 EXAMPLES IN BOOK) DO 21.68, 21.84

INFINITE SHEET EXAMPLE $E = \dfrac{\sigma}{2\epsilon_0}$, $E_1 = E_2$

$\vec{E}_1 = -\vec{E}_2$



$A: \vec{E} = \vec{E}_1 + \vec{E}_2 = 0$

$B: \vec{E} = \vec{E}_1 + \vec{E}_2 = -\dfrac{\sigma}{\epsilon_0}\hat{i}$

$C: \vec{E} = \vec{E}_1 + \vec{E}_2 = 0$

ELECTRIC FIELD LINES

IMAGINARY LINES OR CURVES DRAWN THROUGH A REGION OF SPACE SO THAT THE TANGENT AT ANY POINT IS IN THE DIRECTION OF THE ELECTRIC FIELD VECTOR AT THAT POINT.

OFTEN CALLED FIELD MAPS.

FIELD LINES GIVE THE DIRECTION OF $\vec{E}$

NO INFORMATION ABOUT MAGNITUDE AT ANY POINT.

SPACING OF FIELD LINE GIVE QUALITATIVE IDEA OF MAGNITUDE.

FIELD LINE ARE STRAIGHT AND PARALLEL FOR UNIFORM FIELD.

ELECTRIC FIELD LINES ARE NOT TRAJECTORIES.

FIELD LINES NEVER INTERSECT.

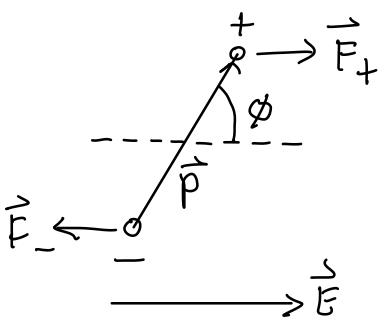
SHOW: $\pm$ CHARGES, DIPOLE, QUADRAPOLE, 2 CHARGES

ELECTRIC DIPOES

NOT RESPONSIBLE FOR THIS TOPIC

2 CHARGES OF EQUAL AND OPPOSITE SIGN SEPARATED BY A DISTANCE

FORCE AND TORQUE ON AN ELECTRIC DIPOLE



UNIFORM EXTERNAL FIELD

NET FORCE $\vec{F}_+ + \vec{F}_-$

$= q_+\vec{E} + q_-\vec{E}$

$= (q_+ + q_-)\vec{E}$

FOR DIPOLE (DEFINITION) $q_+ = -q_-$

$d \equiv$ SEPARATION $\therefore$ NET FORCE $= 0$

IF THE FIELD IS NON UNIFORM A FORCE MAY BE ON THE DIPOLE.

TORQUE WRT CENTRE $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = F_+ r_+ \sin\phi_+ + F_- r_- \sin\phi_- = qE\frac{d}{2}\sin\phi + qE\frac{d}{2}\sin\phi$

$\tau = qEd\sin\phi$

ELECTRIC DIPOLE MOMENT $\equiv \vec{p} = q\vec{d}$, $p = qd$

$\vec{p}$ IS NEGATIVE TO POSITIVE

$\tau = pE\sin\phi$; $\vec{\tau} = \vec{p} \times \vec{E}$

POTENTIAL ENERGY OF ELECTRIC DIPOLE

ELECTRIC FIELD TORQUE DOES WORK dW ON THE DIPOLE.

$\Rightarrow$ CHANGE IN POTENTIAL ENERGY

$$dW = \tau \, d\phi = -pE \sin\phi \, d\phi$$

τ IN DIRECTION OF DECREASING ϕ

$$W = \int_{\phi_1}^{\phi_2} d\phi \, (-pE\sin\phi) = pE\cos\phi_2 - pE\cos\phi_1$$

$$W = -\Delta U = -(U_2 - U_1) = U_1 - U_2 \Rightarrow U(\phi) = -pE\cos\phi = -\vec{p}\cdot\vec{E}$$

QUADRUPOLE $\equiv$ 2 EQUAL DIPOLES WITH OPPOSITE ORIENTATION.

GAUSS'S LAW

CHARGE AND ELECTRIC FLUX

IF THE ELECTRIC FIELD IS KNOWN IN A REGION, WHAT CAN WE DETERMINE ABOUT THE CHARGE IN THAT REGION?

Q CHARGE DISTRIBUTION

ENCLOSED SURFACE

WITH TEST CHARGE MAP OUT $\vec{E}$ OUTSIDE SURFACE.

ONLY NEED TO KNOW $\vec{E}$ ON ENCLOSED SURFACE.

ELECTRIC FLUX AND ENCLOSED CHARGE

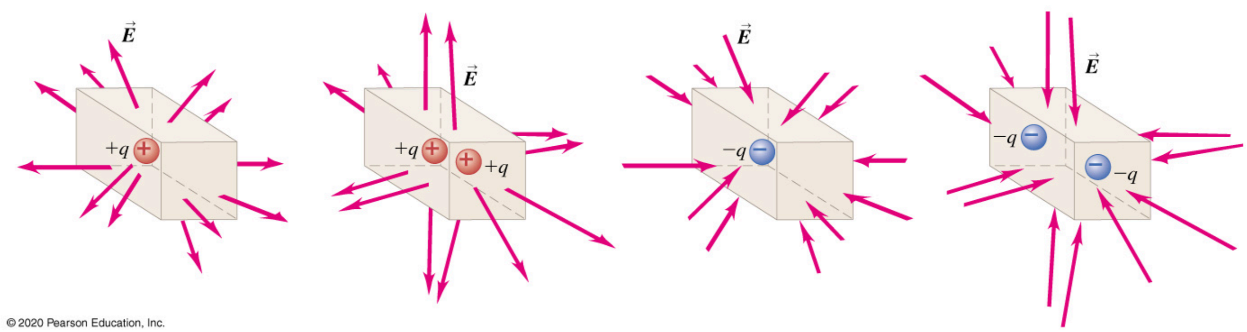
OUTWARD ELECTRIC FLUX ≡ FIELD LINES POINT OUT OF SURFACE.

INWARD ELECTRIC FLUX ≡ FIELD LINES POINT INTO SURFACE.

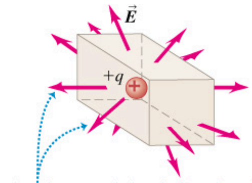
OUTWARD FLUX ⇒ POSITIVE CHARGE INSIDE

INWARD FLUX ⇒ NEGATIVE CHARGE INSIDE

(a) Positive charge inside box, outward flux
(b) Positive charges inside box, outward flux
(c) Negative charge inside box, inward flux
(d) Negative charges inside box, inward flux

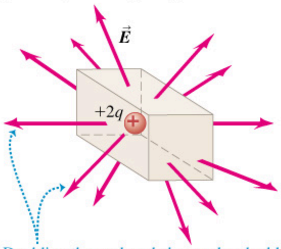


(a) A box containing a positive point charge $+q$



There is outward electric flux through the surface.

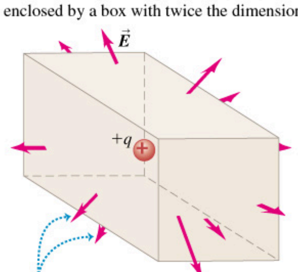
(b) The same box as in (a), but containing a positive point charge $+2q$



Doubling the enclosed charge also doubles the magnitude of the electric field on the surface, so the electric flux through the surface is twice as great as in (a).

THE MAGNITUDE OF FLUX PROPORTIONAL TO AMOUNT OF NET CHARGE.

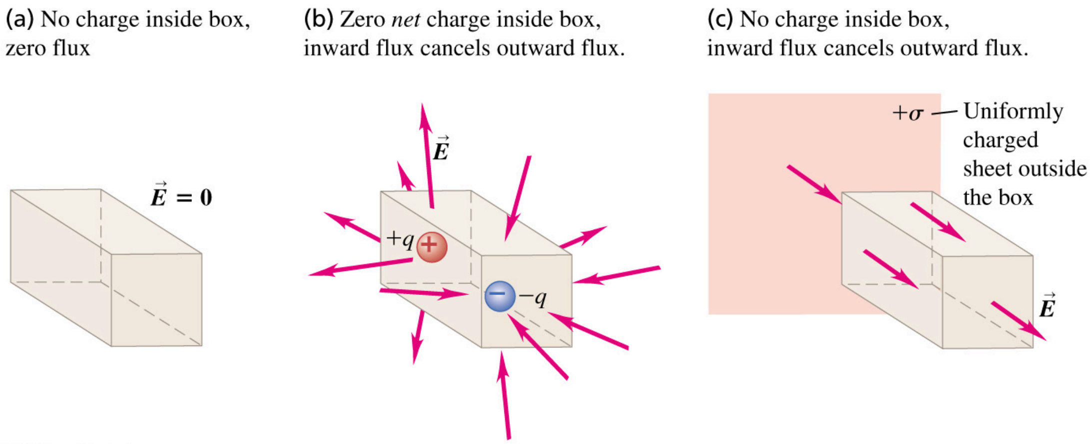
(c) The same positive point charge $+q$, but enclosed by a box with twice the dimensions



The electric flux is the same as in (a): The magnitude of the electric field on the surface is $\frac{1}{4}$ as great as in (a), but the area through which the field "flows" is 4 times greater.

FLUX IS INDEPENDENT OF THE SURFACE SIZE (OR SHAPE)

$E \sim \frac{1}{r^2}$, $A \sim r^2 \Rightarrow EA$ CONSTANT



IF NO CHARGE INSIDE ENCLOSED SURFACE, $\vec{E}=0$

$\Rightarrow$ ELECTRIC FLUX 0

FOR 0 NET CHARGE INSIDE ENCLOSE SURFACE, $\vec{E} \neq 0$

$\Rightarrow$ NET ELECTRIC FLUX 0

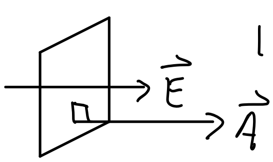
IF CHARGE IS OUTSIDE ENCLOSED SURFACE, $\vec{E} \neq 0$

$\Rightarrow$ NET ELECTRIC FLUX 0 (FLOWS IN AND OUT)

ALL THE ABOVE APPLIES TO ANY CHARGE DISTRIBUTION AND ANY ENCLOSED SURFACE.

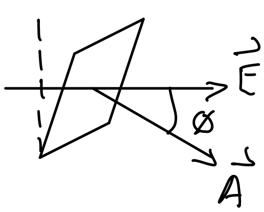
CALCULATING ELECTRIC FLUX

FLUX OF A UNIFORM ELECTRIC FIELD



IF $\vec{E}$ IS PERPENDICULAR TO A SURFACE $\vec{A}$

FLUX $\Phi_E = EA$



ϕ IS ANGLE BETWEEN $\vec{E}$ AND PERPENDICULAR TO $\vec{A}$

$$\Phi_E = EA\cos\phi = E_\perp A = EA_\perp$$

$\vec{A} = A\hat{n}$, $\hat{n} \equiv$ NORMAL UNIT VECTOR (OUTWARD)

IN GENERAL

$\Phi_E = \vec{E} \circ \vec{A}$

$\vec{E}, \vec{A}$ CAN BE FINITE BUT Φ_E CAN BE 0.

Φ_E UNITS $\left[\frac{N}{C} m^2\right]$

FLUX OF A NONUNIFORM ELECTRIC FIELD

IF $\vec{E}$ IS NOT UNIFORM OR A IS PART OF A CURVED SURFACE

$$\Phi_E = \int \vec{E} \circ d\vec{A} = \int E \cos\phi \, dA = \int E_\perp dA$$ SURFACE INTEGRAL

[3 EXAMPLE IN BOOK] DO 22.6

GAUSS'S LAW

EQUIVALENT, OR ALTERNATIVE TO COULOMB'S LAW.

POINT CHARGE INSIDE A SPHERICAL SURFACE

$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \hat{r}$; $\vec{A} = A\hat{r}, 4\pi R^2 \hat{r}$

$$\Phi_E = \vec{E} \circ \vec{A} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} 4\pi R^2 \hat{r} \circ \hat{r} = \frac{q}{\epsilon_0}$$

POINT CHARGE INSIDE A NONSPHERICAL SURFACE

$$\Phi_E = \oint \vec{E} \circ d\vec{A} = \frac{q}{\epsilon_0}$$ ANY ENCLOSED SURFACE

CLOSED

ELECTRIC FIELD LINES CAN BEGIN OR END INSIDE A REGION OF SPACE ONLY WHEN THERE IS A CHARGE IN THAT REGION.

GENERAL FORM OF GAUSS'S LAW

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \frac{Q_{ENCL}}{\epsilon_0} \leftarrow \text{TOTAL ENCLOSED CHARGE}$$

↑ TOTAL FIELD

THERE MIGHT BE CHARGES INSIDE AND OUTSIDE THE SURFACE. GAUSSIAN SURFACES ARE IMAGINARY.

APPLICATION OF GAUSS'S LAW

WHEN AN EXCESS CHARGE IS PLACED ON A SOLID CONDUCTOR, IT RESIDES ENTIRELY ON THE SURFACE.

THE ELECTRIC FIELD IN THE CONDUCTOR IS 0.

[6 EXAMPLES IN BOOK] DO 22.18

CHARGES ON CONDUCTORS

FIELD AT THE SURFACE OF A CONDUCTOR

$\vec{E}$ IS ALWAYS PERPENDICULAR TO SURFACE

$$E_\perp = \frac{\sigma}{\epsilon_0}$$

σ LARGEST WHERE CURVATURE LARGEST

SHOW FIG 22.27 FARADAY CAGE

ELECTRIC POTENTIAL

WORK, ENERGY, ELECTRIC POTENTIAL, VOLTAGE

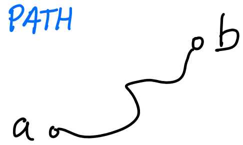
ELECTRIC POTENTIAL ENERGY

WORK DONE BY A FORCE

ANGLE AT EVERY POINT ALONG PATH

$$W_{a \Rightarrow b} = \int_a^b \vec{F} \cdot d\vec{\ell} = \int_a^b F \cos \phi \, d\ell$$

LENGTH ELEMENT ALONG PATH



IF THE WORK IS CONSERVATIVE, WORK DONE BY $\vec{F}$ CAN BE EXPRESSED IN TERMS OF POTENTIAL ENERGY.

$$W_{a \Rightarrow b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

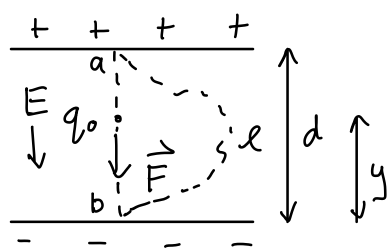
WORK ENERGY THEORM $\quad -\Delta U = \Delta K$

$\Delta K = K_b - K_a$ CHANGE IN KINETIC ENERGY

$\Delta K = -\Delta U$ OR $\Delta K + \Delta U = 0$

$K_a + U_a = K_b + U_b$

ELECTRIC POTENTIAL IN A UNIFORM FIELD



$\vec{E}$ CONSTANT

$\vec{F} = q_0 \vec{E}$ $\quad F_y = -q_0 E$

$\vec{E}$ DOES WORK ON q_0

$W_{a \Rightarrow b} = Fd = q_0 E d = q_0 \vec{E} \cdot \vec{d}$

$q_0, \vec{E}, \vec{d}$ ALL HAVE SIGNS

IF THE WORK DOES NOT DEPEND ON THE PATH THE FORCE IS CONSERVATIVE. WORK DONE BY FIELD IS INDEPENDENT OF PATH.

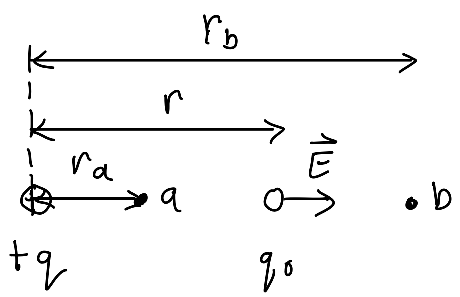
POTEENTIAL ENERGY FOR ELECTRIC FORCE $F = -qE_0$ IS
$U = q_0 E y$

WORK DONE IN MOVING FROM HEIGHT y_a TO y_b

$$W_{a \to b} = -\Delta U = -(U_b - U_a) = -(q_0 E y_b - q_0 E y_a)$$
$$= q_0 E (y_a - y_b) = q_0 E \Delta y$$

OPPOSITE FOR NEGATIVE CHARGE

POTENTIAL ENERGY OF TWO POINT PARTICLES



$$F_r = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}$$

$\vec{F}$ IS NOT CONSTANT

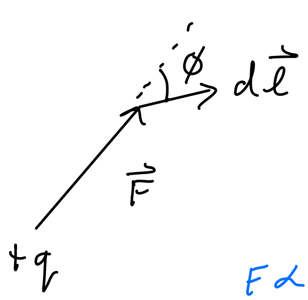
IF THE DISPLACEMENT IS RADIAL

$$W_{a \to b} = \int_{r_a}^{r_b} F_r\, dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2} dr = \frac{q q_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

DEPENDS ONLY ON END POINTS

FOR A MORE GENERAL DISPLACEMENT

$$W_{a \to b} = \int_{r_a}^{r_b} F \cos\phi\, dl = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\cos\phi\, dl$$



$\cos\phi\, dl = dr$

THE FORCE ON q_0 IS CONSERVATIVE

POTENTIAL ENERGY $U = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r}$

$F \propto \frac{1}{r^2}$, $U \propto \frac{1}{r}$

POTENTIAL ENERGY IS DEFINED RELATIVE TO A REFERENCE POINT WHERE $U=0 \Rightarrow r \to \infty$, $U \to 0$ IS REFERENCE POINT.

POTENTIAL IS NOT A PROPERTY OF THE CHARGE ALONE.

(1 EXAMPLE IN BOOK) DO 23.4

ELECTRIC POTENTIAL ENERGY WITH SEVERAL POINT CHARGES

$$U = \frac{q_0}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

WE CAN REPRESENT ANY CHARGE DISTRIBUTION AS A COLLECTION OF POINT CHARGES.

FOR EVERY ELECTRIC FIELD DUE TO A STATIC CHARGE DISTRIBUTION, THE FORCE EXERTED BY THAT FIELD IS CONSERVATIVE.

CHOOSE $U=0$ AT INFINITY.

POTENTIAL ENERGY TO ASSEMBLE ALL THE CHARGES

$$U = \frac{1}{4\pi\epsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

INTERPRETING ELECTRIC POTENTIAL ENERGY

TWO VIEWS

$U_a - U_b$ WORK DONE BY ELECTRIC FIELD WHEN PARTICLE MOVES FROM a TO b.

$U_a - U_b$ WORK DONE BY EXTERNAL FORCE WHEN MOVING A PARTICLE FROM b TO a IN AN ELECTRIC FIELD.

(1 EXAMPLE IN BOOK)

ELECTRIC POTENTIAL

POTENTIAL IS THE POTENTIAL ENERGY PER UNIT CHARGE.

$V(x) = \frac{U(x)}{q_0}$ OR $U(x) = q_0 V(x)$; UNITS $\left[\frac{J}{C}\right] = [V]$ VOLT

WORK PER UNIT CHARGE

$$\frac{W_{a \to b}}{q_0} = \frac{-\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a) = V_a - V_b$$

$V_a - V_b = V_{ab}$ POTENTIAL OF a WRT b (VOLTAGE)

V_{ab} EQUALS WORK DONE BY ELECTRIC FORCE WHEN <u>UNIT</u> CHARGE MOVES FROM a TO b.

V_{ab} EQUALS WORK THAT MUST BE DONE TO MOVE <u>UNIT</u> CHARGE SLOWLY FROM b TO a AGAINTS THE ELECTRIC FORCE.

CALCULATING ELECTRIC POTENTIAL

POINT CHARGE $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ INDEPENDENT OF TEST CHARGE

COLLECTION OF POINT CHARGES $V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

CONTINOUS CHARGE DISTRIBUTION $V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$

IF THE CHARGE DISTRIBUTION EXTENDS TO INFINTY CAN <u>NOT</u> TAKE $V=0$ AT INFINITY.

FINDING ELECTRIC POTENTIAL FROM ELECTRIC FIELD

$$W_{a \to b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b q_0 \vec{E} \cdot d\vec{l}$$

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E \cos\phi \, dl$$

$$\begin{tikzcd} \vec{F} \arrow[r, "1/q_0"] \arrow[d, "\int d\vec{l}"'] & \vec{E} \arrow[d, "\int d\vec{l}"] \\ U \arrow[r, "1/q_0"'] & V \end{tikzcd}$$

POSITIVE CHARGE TENDS TO MOVE FROM A HIGH POTENTIAL TO A LOW POTENTIAL.

ELECTRIC FIELD UNITS [N/C] OR USUALLY [V/m]

ELECTRON VOLT

DIFFERENT UNIT OF ENERGY.

ELECTRONS ARE COMMON.

$e = 1.602 \times 10^{-19}$ C CHARGE MAGNITUDE

$U_a - U_b = q(V_a - V_b) = q V_{ab}$

FOR $V_{ab} = 1V = 1J/C$

$U_a - U_b = (1.602 \times 10^{-19} C)(1 J/C) = 1.602 \times 10^{-19} J \equiv 1 eV$

USEFUL UNIT FOR OTHER FORMS OF ENERGY, SUCH AS KINETIC ENERGY.

(5 EXAMPLES IN BOOK)

CALCULATING ELECTRIC POTENTIAL

CALCULATE POTENTIAL DUE TO CHARGE DISTRIBUTIONS

IONIZATION AND CORONA DISCHARGE

ONCE AIR MOLECLES BECOME IONIZED, AIR BECOMES A CONDUCTOR.

DIELECTRIC STRENGTH OF AIR 3×10^6 V/m

CORONA DISCHARGE – RESULTING CURRENT AND ASSOCIATED GLOW

SMALLER RADIUS OF CURVATURE – DECREASES THE POTENTIAL FOR CORONA DISCHARGE

$V = R E$ ← LIMITED

(4 EXAMPLES IN BOOK)

EQUAL POTENTIAL SURFACES

A 3D SURFACE ON WHICH THE ELECTRIC POTENTIAL V IS THE SAME AT EVERY POINT.

EQUIPOTENTIAL SURFACES FOR DIFFERENT POTENTIALS CAN NEVER TOUCH OR INTERSECT

EQUIPOTENTIAL SURFACES AND FIELD LINES

FIELD LINES AND EQUIPOTENTIAL SURFACES ARE ALWAYS PERPENDICULAR.

(SHOW MY FIELD MAPS)

EQUIPOTENTIALS AND CONDUCTORS

WHEN ALL CHARGES ARE AT REST, THE SURFACE OF A CONDUCTOR IS ALWAYS AN EQUIPOTENTIAL SURFACE.

WHEN ALL CHARGES ARE AT REST, THE ELECTRIC FIELD JUST OUTSIDE A CONDUCTOR MUST BE PERPENDICULAR TO

THE SURFACE AT EVERY POINT.

$\vec{E}$ IS PERPENDICULAR TO SURFACE AT EACH POINT

WHEN ALL CHARGES ARE AT REST, THE ENTIRE VOLUME OF A CONDUCTOR IS AT THE SAME POTENTIAL.

$\rightarrow$ EQUIPOTENTIAL VOLUME $\quad$ $\vec{E} = 0$ INSIDE $\Rightarrow$ NO FORCE $\Rightarrow$ NO WORK $\Rightarrow$ POTENTIAL CONSTANT

POTENTIAL GRADIANT

CALCULATE $\vec{E}$ GIVEN V. $V_a - V_b = \int_b^a dV = -\int_a^b dV$

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{\ell}$$

TURN INTEGRAL EQUATION INTO DIFFERENTIAL EQUATION

$\boxed{\vec{E} = -\vec{\nabla} V}$ $\quad$ $\vec{\nabla} \equiv$ GRADIANT ; $\vec{\nabla} V$ POTENTIAL GRADIANT

$\vec{E} = \vec{E}(x, y, z)$; $V = V(x, y, z)$ IN CARTESIAN COORDINATES

$\vec{E} = E_x \hat{\imath} + E_y \hat{\jmath} + E_z \hat{k}$; V IS SCALAR FUNCTION

$\vec{\nabla} = \left(\hat{\imath} \frac{\partial}{\partial x} + \hat{\jmath} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right)$ OPERATOR

$\therefore \vec{\nabla} V = \hat{\imath} \frac{\partial V}{\partial x} + \hat{\jmath} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z}$ IS VECTOR

AND $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$ 3-EQUATIONS

RECALL PARTIAL DERIVATIVE

$$\frac{\partial}{\partial x} = \frac{d}{dx} \Big|_{y, z \text{ COSTANT}}$$

FOR RADIAL ELECTRIC FIELD $E_r = -\frac{\partial V}{\partial r}$; $\vec{E} = -\frac{\partial V}{\partial r} \hat{r}$

DERIVE V FROM $\vec{E}$ $\Rightarrow$ INTEGRATION

DERIVE $\vec{E}$ FROM V $\Rightarrow$ DIFFERENTIATION

(2 EXAMPLES IN BOOK) DO 23.76

CAPACITANCE AND DIELECTRICS

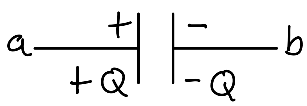
A CAPACITOR STORES ELECTRIC POTENTIAL ENERGY AND CHARGE.

CAPACITORS AND CAPACITANCE

CAPACITOR $\equiv$ ANY 2 CONDUCTORS SEPARATED BY AN INSULATOR (OR VACUUM).

THE NET CHARGE ON A CAPACITOR IS ZERO.

ONE SIDE CARRIES CHARGE $+Q$, AND THE OTHER $-Q$.



$C \equiv \dfrac{Q}{V_{ab}}$ UNITS $\left[\dfrac{C}{V}\right] = [F]$ FARAD

CALCULATING CAPACITANCE: CAPACITORS IN VACUUM

PARALLEL-PLATE CAPACITOR.

AREA A, SEPARATION d.

CHARGE IS DISTRIBUTED UNIFORMLY.

ELECTRIC FIELD BETWEEN PLATES CONSTANT.

$E = \dfrac{\sigma}{\epsilon_0} = \dfrac{1}{\epsilon_0}\dfrac{Q}{A}$; $V_{ab} = Ed = \dfrac{1}{\epsilon_0}\dfrac{Q}{A}d$

$\therefore\ C = \dfrac{Q}{V_{ab}} = \epsilon_0 \dfrac{A}{d}$

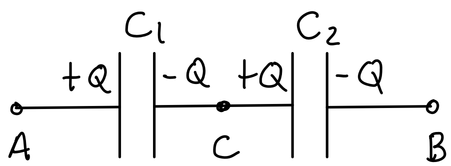
CAPACITANCE DEPENDS ONLY ON GEOMETRY.

CAPACITANCE IN AIR ABOUT THE SAME AS VACUUM.

(4 EXAMPLES IN BOOK) DO 24.10

CAPACITORS IN SERIES AND PARALLEL

CAPACITORS IN SERIES



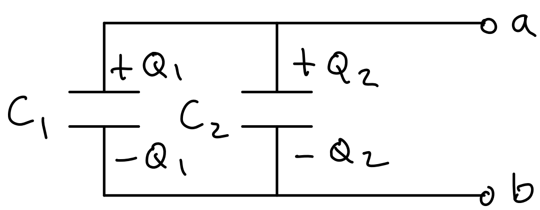
THE MAGNITUDE OF CHARGE ON ALL PLATES IS THE SAME

$$V_{ac} = V_1 = \frac{Q}{C_1}, \quad V_{cb} = V_2 = \frac{Q}{C_2}, \quad V_{ab} = V = V_1 + V_2$$
$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{eq}}$$ EQUIVALENT CAPACITANCE

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i} \quad ; \quad C_{eq} < C_i$$

CAPACITORS IN PARALLEL



THE POTENTIAL DIFFERENCE ACROSS ALL INDIVIDUAL CAPACITORS IS THE SAME

$$Q_1 = C_1 V, \quad Q_2 = C_2 V, \quad Q = Q_1 + Q_2 = (C_1 + C_2) V$$

$$\frac{Q}{V} = C_1 + C_2 = C_{eq}$$ EQUIVALENT CAPACITANCE

$$C_{eq} = \sum_i C_i \quad ; \quad C_{eq} > C_i$$

(2 EXAMPLES IN BOOK) DO 24.20

ENERGY STORAGE IN CAPACITORS AND ELECTRIC-FIELD ENERGY

CONSIDER CHARGING A CAPACITOR

$V = \frac{Q}{C}$ FINAL VALUE ; INTERMEDIATE VALUES $v = \frac{q}{C}$

WORK REQUIRE TO CHARGE CAPACITOR = ENERGY STORED

$$dW = v\,dq = \frac{q}{C}\,dq$$

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$ ANY TWO OF (Q, V, C)

ALSO WORK DONE BY ELECTRIC FIELD ON CHARGE WHEN CAPACITOR DISCHARGES.

APPLICATIONS OF CAPACITORS : ENERGY STORED

ELECTRIC-FIELD ENERGY

THINK OF THE ENERGY AS STORED IN THE FIELD BETWEEN THE PLATES

ENERGY DENSITY $u = \frac{1/2\,CV^2}{Ad}$

FOR PARALLEL PLATE CAPACITOR $C = \epsilon_0 \frac{A}{d}$, $V = Ed$

$$\therefore u = \frac{\epsilon_0 A}{2Ad^2} E^2 d^2 = \frac{1}{2}\epsilon_0 E^2$$

TRUE FOR ANY CAPACITOR IN VACUUM AND ANY FIELD CONFIGURATION IN VACUUM.

EMPTY SPACE CAN HAVE ENERGY.

ELECTRIC FIELD ENERGY = ELECTRIC POTENTIAL ENERGY.
VIEW AS ENERGY DUE TO A SYSTEM OF CHARGES
= ENERGY OF ELECTRIC FIELD THE CHARGES CREATE.

DIELECTRICS

DIELECTRIC ≡ NON CONDUCTING MATERIAL (CAN BE POLARIZED)

1) PROVIDES MECHANICAL SEPARATION BETWEEN CONDUCTING PLATES.
2) ALLOWS HIGHER POTENTIALS BEFORE DIELECTRIC BREAKDOWN (IONIZATION THAT PERMITS CONDUCTION).
3) CAPACITANCE IS GREATER (V CAN BE MADE SMALLER FOR SAME CHARGE Q.

CONSIDER A CAPACITOR WITH AND WITHOUT DIELECTRIC.
Q SAME ON BOTH CAPACITORS.
$C_0 = Q/V_0$ NO DIELECTRIC , $C = Q/V$ WITH DIELECTRIC

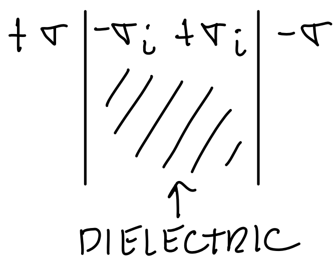
$K = \dfrac{C}{C_0}$ ≡ DIELECTRIC CONSTANT (PURE NUMBER, UNITLESS)
$K = 1$ FOR VACUUM (AND ≈ FOR AIR)
$K > 1$ FOR MOST DIELECTRICS

AND $V = \dfrac{V_0}{K}$ REDUCED

ALWAYS SOME LEAKAGE CURRENT.

INDUCED CHARGE AND POLARIZATION

ALSO $E = \frac{E_0}{K}$ FOR CONSTANT Q



IF E DECREASE σ DECREASES,
BUT Q IS CONSTANT.

$\Rightarrow$ INDUCED CHARGE IN DIELECTRIC SURFACE REDUCES σ

CHARGES IN DIELECTRIC ARE REDISTRIBUTED $\Rightarrow$ POLARIZATION

$\sigma_i \equiv$ INDUCED SURFACE CHARGE DENSITY.

$$E_0 = \frac{\sigma}{\epsilon_0} \quad ; \quad E = \frac{\sigma - \sigma_i}{\epsilon_0} \quad \Rightarrow \quad \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

$\epsilon \equiv K\epsilon_0$ PERMITTIVITY OF DIELECTRIC

$$\therefore \quad E = \frac{\sigma}{\epsilon}$$

IT FOLLOWS $\quad C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$

$$u = \frac{1}{2} K\epsilon_0 E^2 = \frac{1}{2} \epsilon E^2$$

(2 EXAMPLES IN BOOK) DO 24.34

DIELECTRIC BREAKDOWN

MOLECULAR MODEL OF INDUCED CHARGE

GAUSS'S LAW IN DIELECTRICS

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q(\text{ENCL-FREE})}{\epsilon}$$

CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

WILL DISCUSS CHARGES IN MOTION

CHARGES MOVING IN A CLOSED LOOP IS AN ELECTRIC CIRCUIT.

CURRENTS TRANSFER ENERGY.

CURRENT

CURRENT $\equiv$ MOTION OF CHARGE.

IN ELECTROSTATICS $\vec{E} = 0 \Rightarrow I = 0$

BUT ELECTRONS ARE IN RANDOM MOTION, $v_R \sim 10^6 \, m/s$.

FOR STEADY $\vec{E}$, $v_d \sim 10^{-4} \, m/s$ DRIFT VELOCITY.

THE DIRECTION OF CURRENT FLOW

$\vec{E}$ DOES WORK ON MOVING THE CHARGES.

KINETIC ENERGY TRANSFERED TO VIBRATIONAL ENERGY OF MATERIAL $\Rightarrow$ MATERIAL HEATS UP.

$I = \frac{dQ}{dt}$ NET CHARGE FLOW THRU AREA A (CONVECTION CURRENT)

I POSITIVE FOR POSITIVE dQ THRU AREA.

CURRENT IS NOT A VECTOR, BUT HAS A SIGN.

UNITS $\left[\frac{C}{s}\right] = [A]$ AMPERE

CURRENT, DRIFT VELOCITY, AND CURRENT DENSITY

$I = \frac{dQ}{dt} = n|q|v_d A$ $\quad [m^{-3}][C][m/s][m^2] = [C/s]$

$n \equiv$ NUMBER OF CHARGED PARTICLES PER UNIT VOLUME. (PARTICLE CONCENTRATION) $[m^{-3}]$

$J \equiv \frac{I}{A}$ CURRENT DENSITY

VECTOR CURRENT DENSITY $\vec{J} = nq\vec{v}_d$

IF $q > 0$, $\hat{v}_d = \hat{E}$
IF $q < 0$, $\hat{v}_d = -\hat{E}$
} $\vec{J}$ IS IN THE SAME DIRECTION AS $\vec{E}$

$\vec{J}$ IS A VECTOR AT EACH POINT.

I HAS SAME VALUE EVERYWHERE IN WIRE

(1 EXAMPLE IN BOOK)

RESISTIVITY

$\vec{J} \propto \vec{E}$ OHM'S "LAW". IDEALIZED MODEL

RESITIVITY $\rho = \frac{E}{J}$

UNITS $\left[\frac{V/m}{A/m^2}\right] = \left[\frac{V}{A} m\right] = [\Omega m]$ $\Omega \equiv$ OHM

CONDUCTIVITY $\equiv \frac{1}{\rho}$

$\rho \to 0$ GOOD CONDUCTOR

$\rho \to \infty$ GOOD INSULATOR

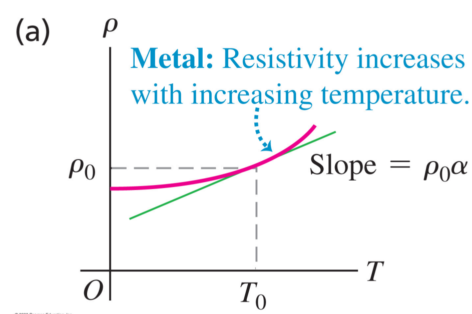
GOOD ELECTRICAL CONDUCTORS ARE USUALLY GOOD HEAT CONDUCTORS.

RESISTIVITY AND TEMPERATURE

$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$

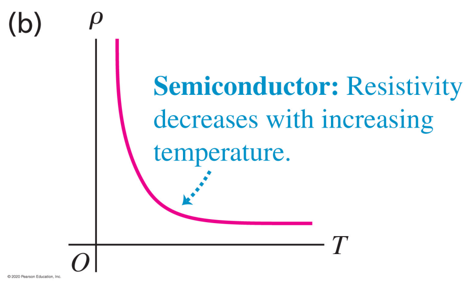
$T_0 \equiv$ REFERENCE TEMPERATURE

$T > T_0$ OR $T < T_0$ $[T_0 = 0°$ OR $20°C]$



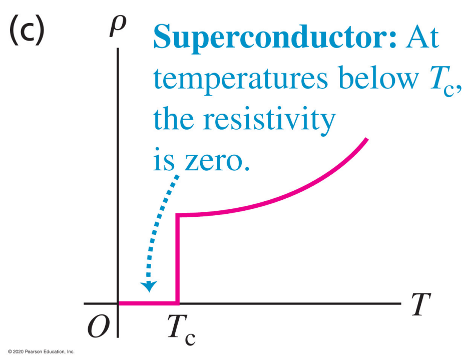
$\rho_0 = \rho(T)$

$\alpha \equiv$ TEMPERATURE COEFFICIENT OF RESISTIVITY (PROPERTY OF MATERIAL)



SEMICONDUCTOR

HIGHER TEMPERATURE CREATES MORE FREE CHARGE CARRIERS.



SUPERCONDUCTOR

RESISTIVITY BECOMES ZERO BELOW SOME CRITICAL TEMPERATURE.

RESISTANCE

$\vec{E} = \rho \vec{J}$, FOR CONDUCTOR

ρ IS CONSTANT FOR OHM'S LAW.

DEPENDS ON PROPERTY OF MATERIAL.

I, V ARE MORE USEFUL

DEFINE RESISTANCE $R = \frac{V}{I}$ ALWAYS TRUE BUT R NEED NOT BE CONSTANT.

FOR WIRE, $V = EL$ AND $I = JA$ $\Rightarrow R = \frac{EL}{JA} = \frac{\rho L}{A}$

DEPENDS ON MATERIAL PROPERTY AND GEOMETRY.

$\therefore$ $V = IR$ IS OHM'S LAW.

INTERPRETING RESISTANCE

$R(T) = R_0 [1 + \alpha(T - T_0)]$

$\alpha \equiv$ TEMPERATURE COEFFICIENT OF RESTANCE

$=$ TEMPERATURE COEFFICIENT OF RESISTIVITY.

(2 EXAMPLES FROM BOOK) DO 25.18

ELECTROMOTIVE FORCE AND CIRCUITS

FOR A STEADY CURRENT, A CIRCUIT LOOP IS NEEDED. RESISTANCE CAUSES POTENTIAL TO DECREASE. NEED SOURCE TO INCREASE POTENTIAL BACK UP SO ITS ZERO CHANGE AROUND THE LOOP.

ELECTROMOTIVE FORCE

ELECTROMOTIVE FORCE (emf) $\mathcal{E}$ MAKES CURRENT FLOW. FROM A LOWER POTENTIAL TO A HIGHER POTENTIAL. EVERY CIRCUIT NEEDS A SOURCE FOR CURRENT TO FLOW.

NOT FORCE (ENERGY PER UNIT CHARGE)

$V_{ab} = \mathcal{E}$ OPEN CIRCUIT (FOR IDEAL emf)

$\mathcal{E} = V_{ab} = IR$ CLOSED CIRCUIT (FOR IDEAL emf)

INTERNAL RESISTANCE

$V_{ab} = \mathcal{E} - rI$, $r \equiv$ INTERNAL RESISTANCE OF emf

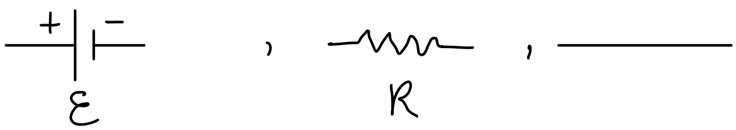
$V_{ab} \equiv$ TERMINAL VOLTAGE

$V_{ab} = \mathcal{E}$ FOR $I = 0$ (OPEN CIRCUIT)

FOR SOURCE WITH INTERNAL RESTANCE

$$\mathcal{E} - rI = IR \Rightarrow I = \frac{\mathcal{E}}{R+r}$$

SYMBOLS FOR CIRCUIT DIAGRAMS



CONDUCTER WITH NEGLIGIBLE RESTANCE

AMMETER (ZERO RESISTANCE)

VOLTMETER (INFINITE RESISTANCE)

IDEALIZED METERS DO NOT DISTURB CIRCUIT THEY ARE MEASURING

(4 EXAMPLES IN BOOK) DO 25.28

POTENTIAL CHANGES AROUND A CIRCUIT

THE NET CHANGE IN POTENTIAL AROUND A CIRCUIT IS ZERO.

$\Delta V_{NET} = 0$, $\mathcal{E}$ = POTENTIAL GAIN, IR = POTENTIAL DROP

$V_{ab} = V_a - V_b$

ENERGY AND POWER IN ELECTRIC CIRCUITS

POTENTIAL ENERGY CHANGE $V_{ab}\,dQ = V_{ab} I\,dt$

TIME RATE OF ENERGY TRANSFER = POWER

$$P = \frac{V_{ab}\,dQ}{dt} = V_{ab} I$$

POWER DELIVERED TO OR EXTRACTED FROM ELEMENT.

V_{ab} = VOLTAGE ACROSS ELEMENT.

I = CURRENT THRU ELEMENT.

UNITS $\left[\frac{J}{C}\right]\left[\frac{C}{S}\right] = \left[\frac{J}{S}\right] = [W]$ WATT

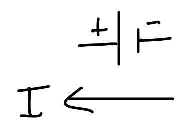
POWER INPUT TO A PURE RESISTANCE

$$P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R}$$

I^2R GOES INTO RESISTOR HEAT

POWER OUTPUT OF A SOURCE

$P = V_{ab} I$; $V_{ab} = \mathcal{E} - Ir \Rightarrow P = \mathcal{E}I - I^2 r$



$I^2 r$ GOES TO HEAT

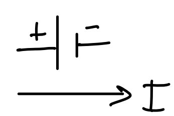
$\mathcal{E} I$ GOES TO REST OF CIRCUIT

$P = \mathcal{E}I - I^2 r$ QUADRATIC IN I.
P ALSO QUADRATIC IN R

$\}$ MAXIMUM POWER CAN BE CALCULATED

POWER INPUT TO A SOURCE

$V_{ab} = \mathcal{E} + Ir$ (REVERSE CURRENT)



CHARGING

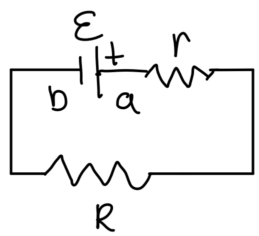
$P = V_{ab} I = \mathcal{E}I + I^2 r$

$I^2 r$ GOES TO HEAT

$\mathcal{E} I$ GOES TO emf

(3 EXAMPLES IN BOOK) DO 25.34

THEORY OF METALLIC CONDUCTION



$$\varepsilon - Ir = IR = V_{ab}$$
$$\varepsilon = I(R+r)$$
$$I = \frac{\varepsilon}{R+r}$$

$P = V_{ab} I = (\varepsilon - Ir) I \Rightarrow rI^2 - \varepsilon I + P = 0$ TWO CURRENTS.

DIFFERENTIATE WRT $I \Rightarrow I = \frac{\varepsilon}{2r}$ $(\Rightarrow r = R)$

$P_{MAX} = \frac{\varepsilon^2}{4r}$ INDEPENDENT OF R

$P = I^2 R = \frac{\varepsilon^2 R}{(R+r)^2}$ TWO RESISTANCES

DIFFERENTIATE WRT R $\Rightarrow 0 = \varepsilon^2 \left[\frac{1}{(R+r)^2} - \frac{2R}{(R+r)^3} \right]$

$\therefore R = r$, $P_{MAX} = \frac{\varepsilon^2}{4r} = \frac{\varepsilon^2}{4R}$

DIRECT-CURRENT CIRCUITS

DIRECT CURRENT (DC) VS. ALTERNATING CURRENT (AC)

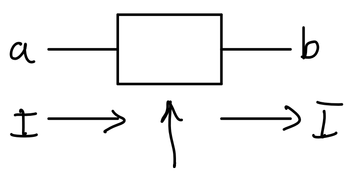
DC ≡ CURRENT DOES NOT CHANGE DIRECTION WITH TIME.

RESISTORS IN SERIES AND PARALLEL

SERIES ≡ SINGLE CURRENT PATH THRU BOTH COMPONENTS.

PARALLEL ≡ CURRENT CAN SPLIT BETWEEN TWO PATHS.

RESISTOR NETWORK CAN BE REPLACED BY EQUIVALENT RESISTANCE GIVING SAME I AND V.



$$V_{ab} = I R_{eq}$$

$$R_{eq} = \frac{V_{ab}}{I}$$

RESISTOR NETWORK

RESISTORS IN SERIES

$$V_{i,i+1} = I R_i \;;\; V_{TOT} = \sum_i^1 V_{i,i+1} = \sum_i^1 I R_i = I \sum_i^1 R_i = I R_{eq}$$

$$\therefore R_{eq} = \sum_i^1 R_i \qquad (R_{eq} > R_i)$$

RESISTORS IN PARALLEL

$$I_i = \frac{V_{ab}}{R_i} \;;\; I_{TOT} = \sum_i^1 I_i = \sum_i^1 \frac{V_{ab}}{R_i} = V_{ab} \sum_i^1 \frac{1}{R_i} = \frac{V_{ab}}{R_{eq}}$$

$$\therefore \frac{1}{R_{eq}} = \sum_i^1 \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \qquad (R_{eq} < R_i)$$

MORE CURRENT FLOWS THRU THE PATH OF LEAST RESISTANCE.

(2 EXAMPLES IN BOOK)

KIRCHHOFF'S RULES

JUNCTION ≡ POINT IN CIRCUIT WHERE THREE OR MORE CONDUCTORS MEET

LOOP ≡ CLOSED CONDUCTING PATH.

JUNCTION RULE: $\sum_i I = 0$ SUM OF CURRENT INTO A JUNCTION.

⇒ CONSERVATION OF ELECTRIC CHARGE.

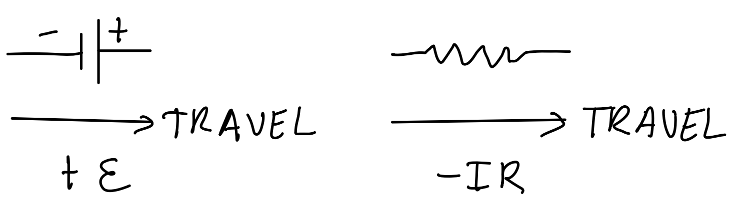
LOOP RULE: $\sum_i V = 0$ SUM OF POTENTIAL DIFFERENCES AROUND LOOP.

⇒ ELECTROSTATIC FORCE IS CONSERVATIVE.

SIGN CONVENTION FOR THE LOOP RULE

CURRENT FLOWS FROM HIGHER POTENTIAL TO LOWER POTENTIAL.

ASSUME DIRECTION OF CURRENT



(5 EXAMPLES IN BOOK)

ELECTRICAL MEASURING INSTRUMENTS

AMMETER

CURRENT FLOWS THRU AMMETER ⇒ CONNECT IN SERIES

IDEAL AMMETER HAS ZERO RESISTANCE

VOLTMETER

VOLTAGE ACROSS CIRCUIT $\Rightarrow$ CONNECT IN PARALLEL.

IDEAL VOLTMETER HAS INFINITE RESISTANCE

AMMETERS AND VOLTMETERS IN COMBINATION

OHMMETERS

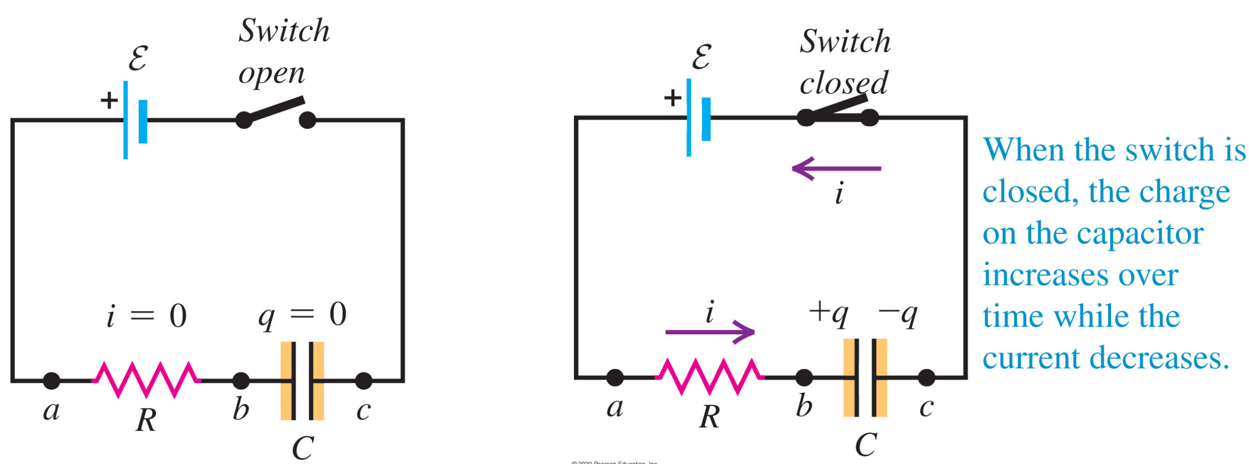
THE POTENTIOMETER

R-C CIRCUITS

CONSIDER A TIME DEPENDENCE IN A CIRCUIT.

CHARGING A CAPACITOR

(a) Capacitor initially uncharged (b) Charging the capacitor



$t < 0,\ i = 0,\ q = 0$

$t = 0,\ i \neq 0 = \dfrac{\mathcal{E}}{R}$ (DISCONTINUOUS) $I_0 = \dfrac{\mathcal{E}}{R}$

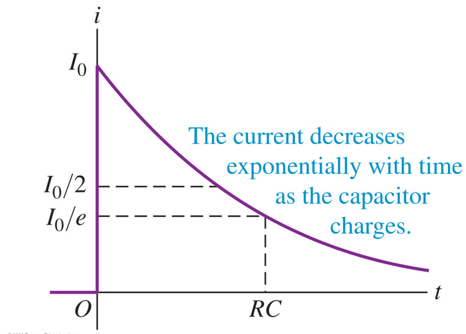
$v_{ab} = \mathcal{E},\ v_{bc} = 0$

t ARBITARY, $v_{ab} = iR,\ v_{bc} = \dfrac{q}{C}$

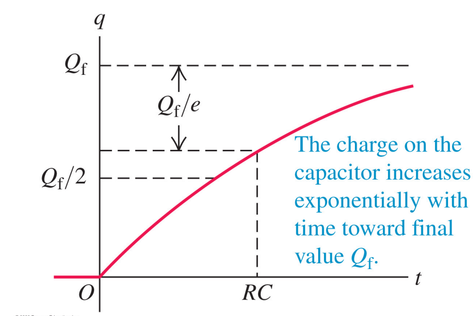
$v_{ab} + v_{bc} = \mathcal{E}$

$t \to \infty, \; i = 0, \; v_{ab} = 0, \; v_{bc} = \mathcal{E} = \frac{Q_f}{C}$

(a) Graph of current versus time for a charging capacitor



(b) Graph of capacitor charge versus time for a charging capacitor



$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

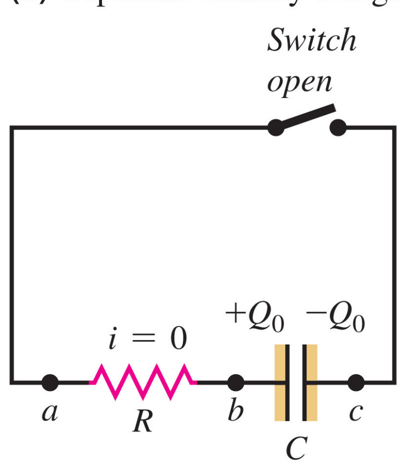
$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC})$$

TIME CONSTANT

$\tau = RC$ CHARACTERISTIC TIME OF CIRCUIT

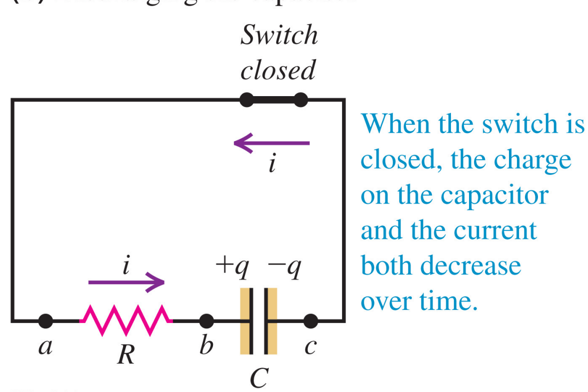
DISCHARGING A CAPACITOR

(a) Capacitor initially charged



No emf

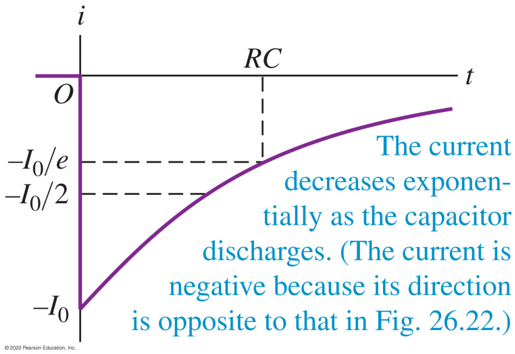
(b) Discharging the capacitor



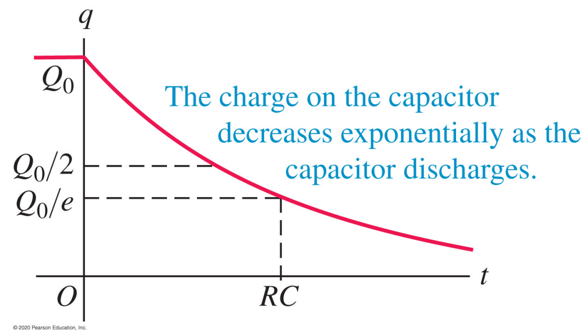
$t < 0, \; q = Q_0, \; i = 0$

$t = 0, \; q = Q_0, \; i = I_0 = -Q_0/RC$ DISCONTINOUS

(a) Graph of current versus time for a discharging capacitor



(b) Graph of capacitor charge versus time for a discharging capacitor



$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC}$$
$$= -I_0 e^{-t/RC}$$

$$q = Q_0 e^{-t/RC}$$

POWER CONSIDERATIONS : $\mathcal{E} i = \underset{\uparrow}{i^2 R} + \frac{iq}{C}$ ← STORED
(WHEN CHARGING)

DISSIPATED

BATTARY ENERGY ⇒ 1/2 STORED IN CAPACITOR
+ 1/2 DISSIPATED IN RESISTOR

PROOF : INTEGRATE OVER TIME TO GET ENERGY

emf : $\mathcal{E} \int_0^\infty i\, dt = \mathcal{E} \int_0^\infty \frac{dq}{dt} dt = \mathcal{E} \int_0^{Q_f} dq = \mathcal{E} Q_f$

OR $\mathcal{E} I_0 \int_0^\infty e^{-t/RC} dt = \mathcal{E} I_0 RC = \mathcal{E} \frac{\mathcal{E}}{R} R \frac{Q_f}{\mathcal{E}} = \mathcal{E} Q_f$ ✓

CAPACITOR : $\frac{1}{C} \int_0^\infty q\, i\, dt = \frac{1}{C} \int_0^\infty q \frac{dq}{dt} dt = \frac{1}{C} \int_0^{Q_f} q\, dq$

$$= \frac{1}{C} \frac{Q_f^2}{2} = \frac{\mathcal{E}}{Q_f} \frac{Q_f^2}{2} = \frac{\mathcal{E} Q_f}{2}$$

OR $\frac{1}{C} Q_f I_0 \int_0^\infty (1 - e^{-t/RC}) e^{-t/RC} dt = \frac{Q_f I_0}{C} \left(RC - \frac{RC}{2} \right)$

$$= \frac{Q_f}{C} \frac{\mathcal{E}}{R} \frac{RC}{2} = \frac{Q_f \mathcal{E}}{2} \checkmark$$

RESISTOR : $R \int_0^\infty i^2 dt = R I_0^2 \int_0^\infty e^{-2t/RC} = R I_0^2 \frac{RC}{2}$

$$= R \frac{\mathcal{E}^2}{R^2} \frac{R}{2} \frac{Q_f}{\mathcal{E}} = \frac{\mathcal{E} Q_f}{2}$$

I CAN NOT SOLVE IT IN GENERAL ✗

FINALLY $\mathcal{E} Q_f = \frac{\mathcal{E} Q_f}{2} + \frac{\mathcal{E} Q_f}{2}$ INDEPENDENT OF R, C.

POWER DISTRIBUTION SYSTEMS

CIRCUIT OVERLOADS AND SHORT CIRCUITS

HOUSEHOLD AND AUTOMOTIVE WIRING

MAGNETIC FIELD AND MAGNETIC FORCES

NEED MOVING CHARGES (CURRENTS).
FIRST STUDY RESPONSE TO MAGNETIC FIELD.
(NOT WHAT PRODUCES THE FIELI

MAGNETISM

PERMANENT MAGNETS (MAGNETIC POLES).
MAGNETS INTERACT WITH EACH OTHER.
OPPOSITE POLES ATTRACT. SAME POLES REPEL.
MAGNETS INTERACT WITH NON-MAGNETS (METALS).
MAGNET ATTRACTS NON-MAGNET REGARDLESS OF POLE.
THE EARTH IS A MAGNET
BAR MAGNET NORTH POLE POINTS TO GEOGRAPHIC NORTH.
FOR EARTH, MAGNETIC NORTH IS CURRENTLY GEOGRAPHIC SOUTH.

MAGNETIC POLES VERSUS ELECTRIC CHARGE

NO MAGNETIC MONOPOLES.
POINT CHARGES ARE ELECTRIC MONOPOLES.
MAGNETIC DIPOLES ARE MOST FUNDAMENTAL.

COMPASS IS EFFECTED BY NEARBY CURRENT.
CURRENT GENERATED BY MOVING MAGNET.
THIS LED TO UNIFICATION ELECTRIC AND MAGNETIC INTERACTIONS → FIELD THEORY.

MAGNETIC FIELD

STATIC CHARGES CREATE $\vec{E}$ FIELD.

MOVING CHARGES CREATE $\vec{E}$ AND $\vec{B}$ FIELDS.

$\vec{E}$ FIELD CREATES FORCE ON CHARGED PARTICLE.

$\vec{B}$ FIELD CREATES FORCE ON MOVING CHARGED PARTICLE.

$\vec{E} = \vec{E}(x,y,z)$, $\vec{B} = \vec{B}(x,y,z)$ ARE VECTOR FIELDS.

$\vec{B}$ DIRECTION IN WHICH NORTH POLE OF COMPASS NEEDLE POINTS.

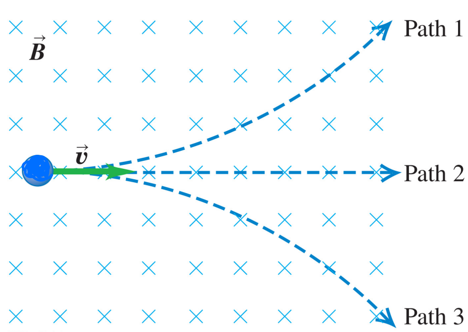
THIS CLASS STUDIES EFFECT OF A "GIVEN" FIELD.

MAGNETIC FORCES ON MOVING CHARGES

$\vec{F} = q\vec{v} \times \vec{B}$ $\qquad$ $\vec{F}, \vec{v}, \vec{B}$ ALL PERPENDICULAR

$\vec{B}$ UNITS $\dfrac{[N]}{[C][m/s]} = \left[\dfrac{N}{A\,m}\right] \equiv [T]$ TESLA

OR [G] GAUSS $= 10^{-4}$ T



POSITIVE CHARGED PARTICLE

0 CHARGED PARTICLE

NEGATIVE CHARGED PARTICLE

MEASURING MAGNETIC FIELDS WITH TEST CHARGES

IN GENERAL

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ LORENTZ FORCE

BASED ON EXPERIMENT.

EXAMPLE IN BOOK, 27.8

MAGNETIC FIELD LINES AND MAGNETIC FLUX

MAGNETIC FIELD LINES ALWAYS FORM CLOSED LOOPS.

MAGNETIC FIELD LINES HAVE NO ENDS.

$\vec{B}$ FIELD LINES IN DIRECTION OF COMPASS NEEDLE.

$\vec{B}$ FIELD LINES NOT IN DIRECTION OF $\vec{F}$.

FIELD LINES NEVER CROSS.

$\times$ FIELD INTO PAGE

$\circ$ FIELD OUT OF PAGE

$\vec{B}$ UNIFORM IN MAGNET

$\vec{B}$ AROUND CURRENT CARRYING WIRE.

MAGNETIC FLUX AND GAUSS'S LAW FOR MAGNETISM

MAGNETIC FLUX

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos\phi \, dA = \int B_\perp \, dA$$

UNITS $\left[\frac{N}{Am}\right][m^2] = \left[\frac{N\,m}{A}\right] = [Wb]$ WEBER $(= [T][m^2])$

GAUSS'S LAW FOR MAGNETISM

$$\oint \vec{B} \cdot d\vec{A} = 0$$ NO MAGNETIC MONOPOLES

MAGNETIC FIELD LINES ALWAYS FORM CLOSED LOOPS.
BOTH NORTH AND SOUTH POLES INSIDE THE SURFACE
$\Rightarrow$ NO NET MAGNETIC "CHARGE" IN SURFACE.

$\Phi_B = \int B \, dA_\perp \Rightarrow B = \dfrac{d\Phi_B}{dA_\perp}$ FLUX PER UNIT AREA

SOMETIMES $\vec{B}$ CALLED MAGNETIC FLUX DENSITY.

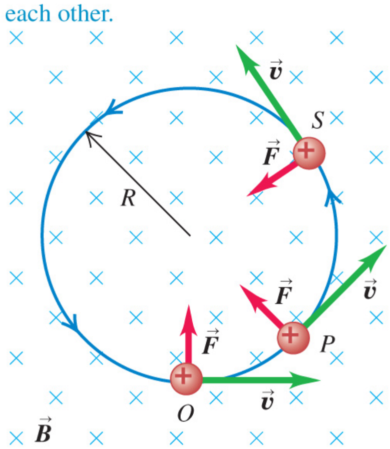
1 EXAMPLE IN BOOK

MOTION OF CHARGED PARTICLES IN A MAGNETIC FIELD

MAGNETIC FORCE CAN NEVER DO WORK ON A PARTICLE.
FOR A MAGNETIC FIELD ONLY, PARTICLE SPEED IS CONSTANT.
$\vec{F} \perp \vec{v} \Rightarrow$ CAN ONLY CHANGE DIRECTION.

(a) The orbit of a charged particle in a uniform magnetic field

A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.



CIRCULAR MOTION

$F = |q| v B = m \dfrac{v^2}{R}$

SIGN OF q GIVES DIRECTION OF ORBIT.

$R = \dfrac{mv}{|q|B}$ ← MOMENTUM

ANGULAR SPEED

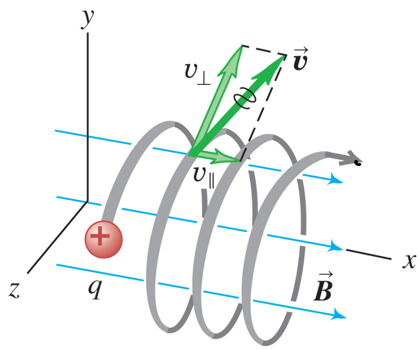
$\omega = \dfrac{v}{R} = \dfrac{v|q|B}{mv} = \dfrac{|q|B}{m}$

INDEPENDENT OF v

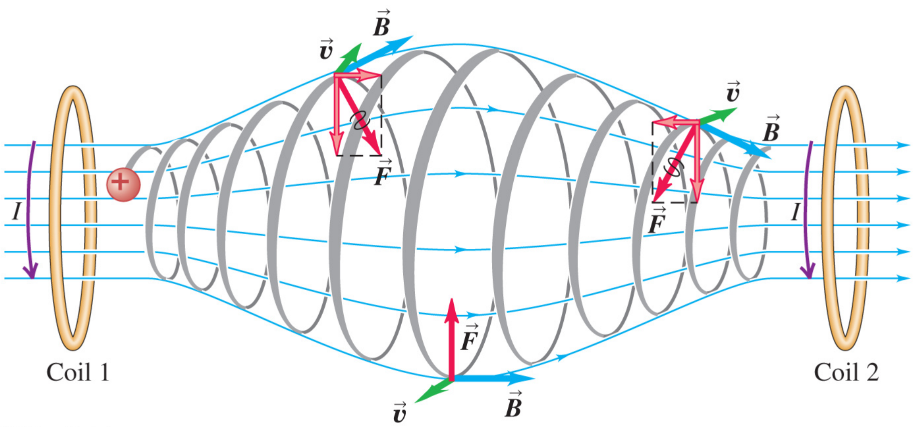
$f = \dfrac{\omega}{2\pi} = \dfrac{|q|B}{2\pi m}$ CYCLOTRON FREQUENCY
INDEPENDENT OF R (OR v)

IF v LARGE RELATIVISTIC CORRECTIONS NEEDED.
(INDEPENDENCE BREAKS DOWN)

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.

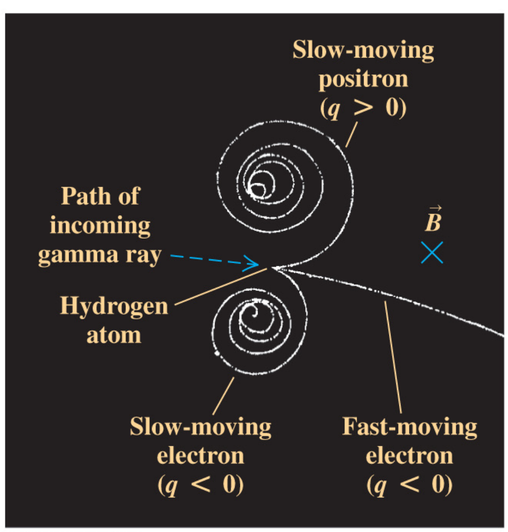


IF $\vec{v}$ SOME COMPONENT PARALLEL TO $\vec{B}$



VARING $\vec{B}$ FIELD

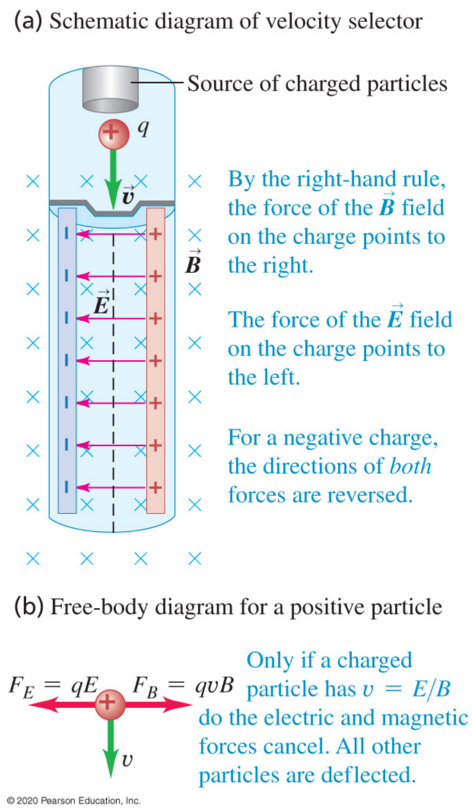
VAN ALLEN RADIATION BELTS



CREATION OF ANTI MATTER

2 EXAMPLES IN BOOK

APPLICATIONS OF MOTION OF CHARGED PARTICLES



VELOCITY SELECTOR

$\longleftarrow \vec{F}_E = q\vec{E}$

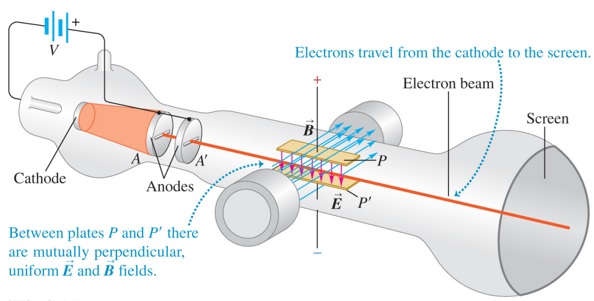
$\longrightarrow \vec{F}_B = q\vec{v} \times \vec{B}$

$\vec{F}_E + \vec{F}_B = 0$ FOR CHARGED PARTICLES TO ESCAPE

NO DEFLECTION

$qE = qvB \Rightarrow v = \frac{E}{B}$

ANY CHARGE



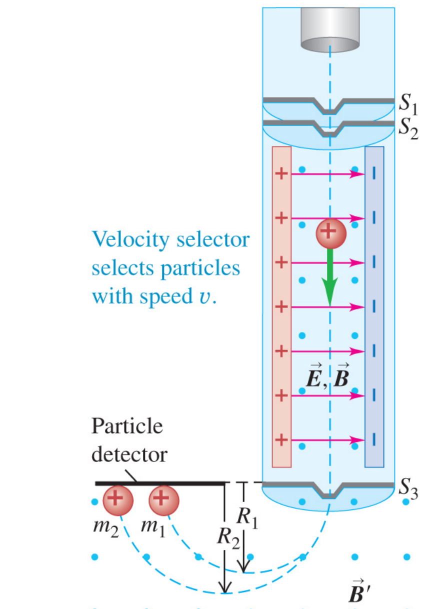
THOMSON'S e/m EXPERIMENT

$$\frac{1}{2} m v^2 = eV$$

$$v = \left(\frac{2eV}{m}\right)^{1/2}$$

$$\frac{E}{B} = v = \left(\frac{2eV}{m}\right)^{1/2} \Rightarrow \frac{e}{m} = \frac{E^2}{2VB^2}$$

$\frac{e}{m}$ ONE VALUE INDEPENDENT OF CATHOD MATERIAL OR ANYTHING ELSE $\rightarrow$ FUNDAMENTAL PARTICLE $\rightarrow$ ELECTRON.



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

MASS SPECTROMETER

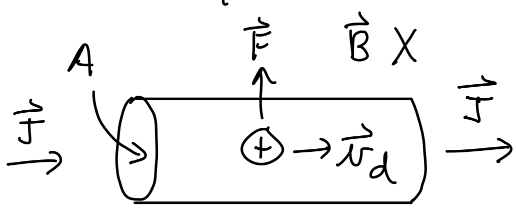
$v = \frac{E}{B}$, $R = \frac{mv}{qB'}$ ← MOMENTUM

ASSUME $q = +e$

$R \propto m$

MEASURE MASS OF ISOTOPES (DIFFERENT NUMBER OF NEUTRONS)

MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR



$\vec{v}_d \equiv$ DRIFT VELOCITY OF POSITIVE CHARGE

$\vec{F} = (nA\ell)(q\vec{v}_d \times \vec{B})$

$= (nq\vec{v}_d A) \times (\ell \vec{B}) = \vec{J} A \ell \times \vec{B} = I \vec{\ell} \times \vec{B}$

$\vec{F} = I\vec{\ell} \times \vec{B}$ STRAIGHT CURRENT CARRYING WIRE.

I DOES NOT DEPEND ON SIGN OF q. $(q \rightarrow -q, \vec{v}_d \rightarrow \vec{v}_d)$

$\vec{\ell}$ IS DIRECTION OF CURRENT.

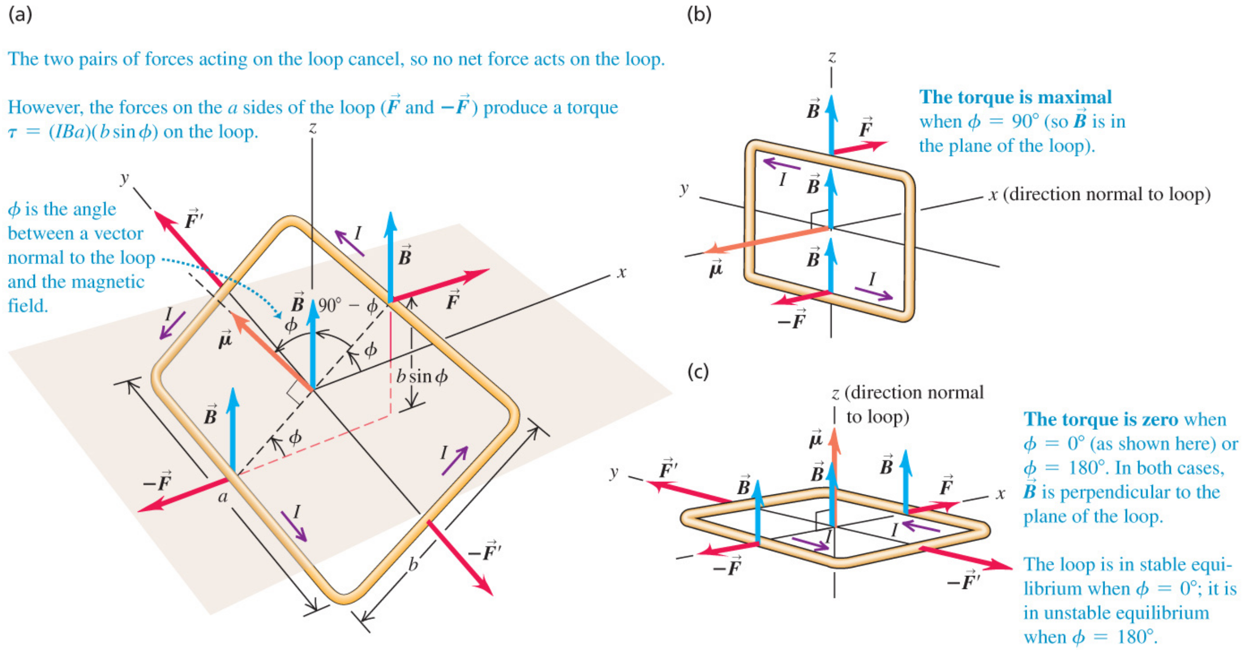
$d\vec{F} = I\, d\vec{\ell} \times \vec{B}$ NOT STRAIGHT WIRE SEGMENT.

CURRENT NOT VECTOR. I SAME AT ALL POINTS IN WIRE.

$d\vec{\ell}$ TANGENT TO CONDUCTOR

2 EXAMPLES IN BOOK.

FORCE AND TORQUE ON A CURRENT LOOP



THE NET FORCE ON A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD IS ZERO.

FORCE ON OPPOSITE SIDES CANCEL.

HOWEVER, THE NET TORQUE IS NOT IN GENERAL ZERO.

$\tau = 2F(b/2)\sin\phi = (IBa)(b\sin\theta) = IBA\sin\phi$

A = AREA OF RECTANGULAR LOOP.

IA ≡ MAGNETIC DIPOLE MOMENT (OR MAGNETIC MOMENT)

$\mu = IA \Rightarrow \tau = \mu B\sin\phi$

A MAGNETIC DIPOLE IS AN OBJECT THAT EXPERIENCES MAGNETIC TORQUE.

$\phi = 0 \Rightarrow \tau = 0$

$\phi = \pi \Rightarrow \tau = 0$ (UNSTABLE)

$\phi = \pi/2 \Rightarrow \tau = IBA$ (MAXIMUM)

TORQUE TENDS TO ROTATE LOOP IN DIRECTION OF DECREASING ϕ. $\rightarrow$ TOWARDS STABILITY.
THE MOTION WILL OSCILLATE.

MAGNETIC TORQUE : VECTOR FORM

$$\vec{\mu} = I\vec{A} \Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$$

POTENTIAL ENERGY FOR A MAGNETIC DIPOLE

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\phi$$

MAGNETIC TORQUE : LOOPS AND COILS

$\mu_{TOT} = NIA$, $N \equiv$ NUMBER OF COIL WINDINGS (SOLENOID)
WORKS FOR ANY PLANE LOOP OF ANY SHAPE.
2 EXAMPLES IN BOOK

MAGNETIC DIPOLE IN A NONUNIFORM MAGNETIC FIELD

NET FORCE IS NOT ZERO

MAGNETIC DIPOLE AND HOW MAGNETS WORK

NORTH/SOUTH POLES REPRESENT HEAD/TAIL OF $\vec{\mu}$ OF MAGNETIC MATERIAL.
$\vec{B}$ ALIGNS $\vec{\mu}$ IN MATERIAL MAKING IT MAGNETIC.

THE DIRECT-CURRENT MOTOR

THE HALL EFFECT

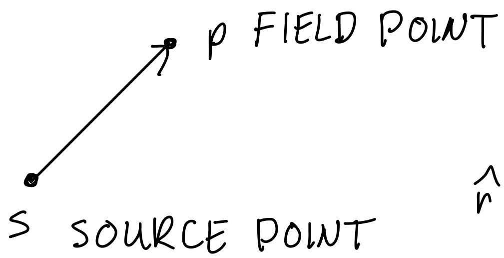
SOURCES OF MAGNETIC FIELD

PREVIOUSLY STUDIED FORCES GIVE THE FIELD.
NEED MOVING CHARGES TO CREATE THE FIELD.

MAGNETIC FIELD OF A MOVING CHARGE

MOVING CHARGE : VECTOR MAGNETIC FIELD

MOVING CHARGE : MAGNETIC FIELD LINES



$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}$$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ FROM SOURCE TO FIELD POINT

$\vec{v}$ CONSTANT

$\mu_0 \equiv$ MAGNETIC CONSTANT
(MAGNETIC PERMEABILITY OF FREE SPACE)

$\mu_0 = 4\pi \times 10^{-7}\ Tm/A$

ASIDE: $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ SPEED OF LIGHT

$\vec{E}$ FIELD RADIATES FROM PARTICLE, EVEN IF MOVING.
$\vec{B}$ FIELD LINES ARE CIRCLES AROUND $\vec{v}$

1 EXAMPLE IN BOOK

MAGNETIC FIELD OF A CURRENT ELEMENT

CURRENT ELEMENT : VECTOR MAGNETIC FIELD

CURRENT ELEMENT : MAGNETIC FIELD LINES

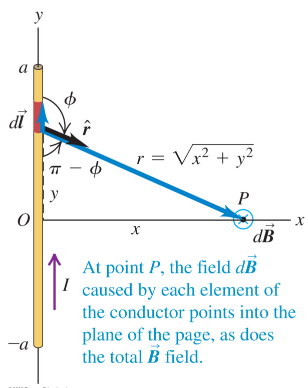
USE PRINCIPLE OF SUPERPOSITION OF MAGNETIC FIELDS.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$ BIOT-SAVART LAW

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

1 EXAMPLE IN BOOK 28.12

MAGNETIC FIELD OF A STRAIGHT CURRENT-CARRYING CONDUCTOR



WIRE LENGTH $2a$; $d\vec{l} \times \hat{r} = y \sin(\pi - \phi)$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x\,dy}{(x^2+y^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2+a^2}}$$

$a \to \infty$, $B = \frac{\mu_0 I}{2\pi x}$ $\vec{B}$ INTO PAGE

IN GENERAL $B = \frac{\mu_0}{2\pi} \frac{I}{r}$

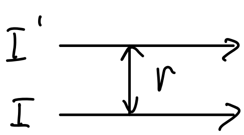
r IS PERPENDICULAR DISTANCE FROM CONDUCTOR TO FIELD POINT.

CIRCULAR MAGNETIC FIELD (NO END POINTS)

$\oint \vec{B} \cdot d\vec{A} = 0$ MAGNETIC FLUX THRU ANY CLOSED SURFACE

2 EXAMPLES IN BOOK.

FORCE BETWEEN PARALLEL CONDUCTORS



$B = \frac{\mu_0 I}{2\pi r}$ FIELD DUE TO WIRE I (OUT OF PAGE AT WIRE I')

FORCE ON WIRE I', $\vec{F} = I'\vec{L} \times \vec{B}$ (TOWARD WIRE I)

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r}$$

FORCE PER UNIT LENGTH $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$

CURRENT IN SAME DIRECTION ATTRACT.

28.60

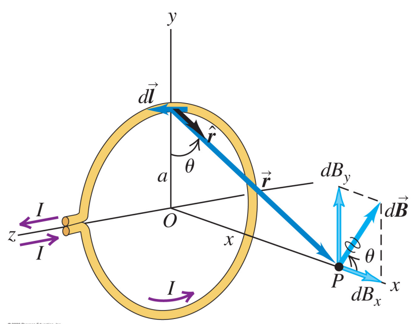
CURRENT IN OPPOSITE DIRECTIONS REPEL.

CURRENT DOES NOT NEED TO BE IN WIRE.

IN WIRE CURRENT CAN CONCENTRATE IN CENTRE (PINCH EFFECT)

MAGNETIC FORCES AND THE VALUE OF μ_0

MAGNETIC FIELD OF A CIRCULAR CURRENT LOOP



P ON AXIS OF LOOP

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2+a^2)}$$

$$dB_x = dB\cos\theta$$

$$dB_y = dB\sin\theta$$

dB_y ALL CANCEL WHEN INTEGRATING AROUND LOOP.

$$dB_x = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2+a^2)} \frac{a}{(x^2+a^2)^{1/2}}$$

$\cos\theta$ FACTOR

$$B_x = \frac{\mu_0 I}{4\pi} \frac{a}{(x^2+a^2)^{3/2}} \int dl \quad ; \quad \int dl = 2\pi a$$

$$= \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

DIRECTION OF $\vec{B}$ GIVEN BY RIGHT-HAND RULE.

MAGNETIC FIELD ON THE AXIS OF A COIL

FOR N CIRCULAR LOOPS (COIL)

$$B_x = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$$ CLOSELY SPACED SO FIELD POINT x SAME DISTANCE TO LOOPS.

AT CENTRE ($x=0$) OF LOOP $B_x = \frac{\mu_0 N I}{2a}$

$\mu = IA = I\pi a^2$ FOR ONE LOOP; $\mu = N I \pi a^2$ FOR N LOOPS.

$$B_x = \frac{\mu_0 \mu}{2\pi (x^2+a^2)^{3/2}}$$

AT CENTRE ($x=0$) OF COIL $B_x = \frac{\mu_0 \mu}{2\pi a^3}$

1 EXAMPLE IN BOOK 28.34 $\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi a^3}$

AMPERE'S LAW

$\oint \vec{B} \cdot d\vec{l}$ LINE INTEGRAL OF $\vec{B}$ AROUND A CLOSED PATH.

AMPERE'S LAW FOR LONG, STRAIGHT CONDUCTORS

$B = \frac{\mu_0 I}{2\pi r}$ FOR WIRE

$\oint \vec{B} \cdot d\vec{l} = \frac{\mu_0 I}{2\pi R} \oint dl = \left(\frac{\mu_0 I}{2\pi R}\right)(2\pi R) = \mu_0 I$ INDEPENDENT OF RADIUS

INTEGRATE IN DIRECTION OF $\vec{B}$

(-'VE IF CURRENT REVERSED; -'VE DIRECTION REVERSED)

AMPERE'S LAW: GENERAL STATEMENT

$\oint \vec{B} \cdot dl = \mu_0 I_{ENCL}$ USE SIGN RULE FOR MULTIPLE CURRENTS

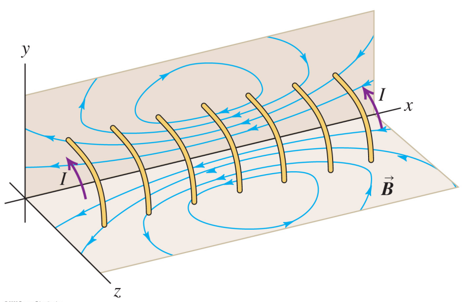
ONLY VALID FOR STEADY CURRENTS

APPLITATION OF AMPERE'S LAW

4 EXAMPLES IN BOOK

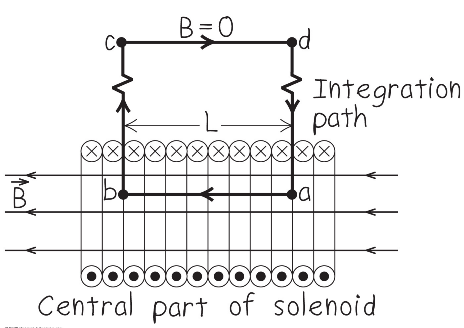
FIELD OF A SOLENOID

SOLENOIL ≡ HELICAL WOUND WIRE ON A CYLINDER.
USUALLY CIRCULAR (SOMETIME RECTANGULAR) CROSS SECTION.
EACH TURN IS A LOOP.
ALL TURNS CARRY SAME CURRENT
TOTAL $\vec{B}$ IS VECTOR SUM DUE TO EACH LOOP.
SOME FIELD



SOME FIELD LINES OUT ENDS
SOME FIELD LINES BETWEEN WINDINGS

IF SOLENOID IS LONG COMPARED TO CROSS SECTION AND COILES WOUND TIGHT.
$\vec{B}$ NEAR MIDPOINT APPROXIMATELY UNIFORM AND PARALLEL TO AXIS. EXTERNAL $\vec{B}$ VERY SMALL.



n TURNS PER UNIT LENGHT

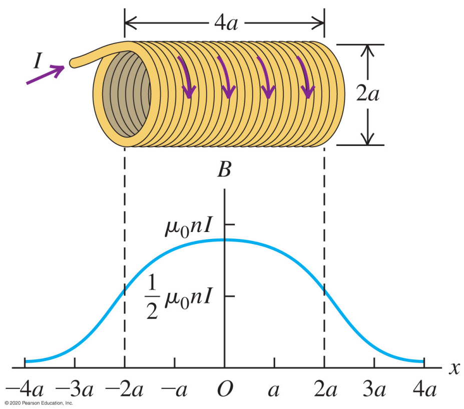
$I_{ENCL} = nLI$

$b \to c$ AND $d \to a$, $\vec{B} \cdot d\vec{\ell} = 0$

$c \to d$, $\vec{B} = 0$

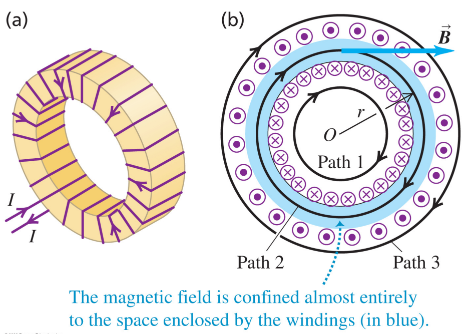
$$\oint \vec{B} \cdot d\vec{\ell} = \int_a^b \vec{B} \cdot d\vec{\ell} + \int_b^c \vec{B} \cdot d\vec{\ell} + \int_c^d \vec{B} \cdot d\vec{\ell} + \int_d^a \vec{B} \cdot d\ell = \mu_0 I_{ENCL}$$

$$\int_a^b \vec{B}\cdot d\vec{\ell} = BL = \mu_0 I_{ENCL} = nLI \Rightarrow \boxed{B = \mu_0 n I}$$



ab NEED NOT BE ON AXIS.
FIELD APPROXIMATELY UNIFORM OF ENTIRE CROSS SECTION

FIELD OF A TOROIDAL SOLENOID



EACH LOOP IS IN PLANE PEPENDICULAR TO CIRCULAR AXIS OF TORIOD.
FIELD LINES CIRCLES CENTRED ON TORIOD AXIS.
$\Rightarrow$ CIRCULAR INTEGRATION PATHS.

$$\oint \vec{B}\cdot d\vec{\ell} = B(2\pi r)$$

PATH 1 : $I_{ENC} = 0 \Rightarrow B = 0$

PATH 3 : $I_{ENC} = I - I = 0 \Rightarrow B = 0$ (EACH WINDING TWICE)

PATH 2 : $I_{ENC} = NI \Rightarrow B = \dfrac{\mu_0 NI}{2\pi r}$

TOPOLOGICALLY STRAIGHT SOLENOID BENT IN CIRCLE.

$n \approx 2\pi r N \Rightarrow B \approx \mu_0 n I$

MANY IDEALIZATIONS ARE ASSUMED.

MAGNETIC MATERIALS

THE BOHR MAGNETON

PARAMAGNETISM

DIAMAGNETISM

FARROMAGNETISM

ELECTROMAGNETIC INDUCTION

HOW TO CONVERT BETWEEN ELECTRIC ENERGY AND OTHER FORMS OF ENERGY.

TIME VARYING MEGNETIC FIELD $\Rightarrow$ ELECTRIC FIELD

TIME VARYING ELECTRIC FIELD $\Rightarrow$ MAGNETIC FIELD

$\Rightarrow$ MAXWELL'S EQUATIONS

INDUCTION EXPERIMENTS

MOVE MAGNET RELATIVE TO COIL.

MOVE TWO COILS RELATIVE TO EACH OTHER.

$\Rightarrow$ INDUCED CURRENT AND INDUCED emf

FARADAY'S LAW

$\phi_B = \int \vec{B} \circ d\vec{A}$ MAGNETIC FLUX THRU SURFACE dA

$\mathcal{E} = -\dfrac{d\phi_B}{dt}$ FARADAY'S LAW OF INDUCTION

$\mathcal{E} \equiv$ INDUCED emf IN CLOSED LOOP

EXAMPLE IN BOOK

DIRECTION OF INDUCED emf

INDUCED emf IS IN DIRECTION OF CURRENT.

DIRECTION IS GIVEN BY HOW COMBINATION $\vec{B} \circ d\vec{A}$ IS CHANGING WITH TIME

$$\mathcal{E} = -\int \frac{d}{dt}(\vec{B} \circ d\vec{A}) = -\int \left(\frac{dB}{dt}\right) \circ d\vec{A} - \int \vec{B} \circ \frac{d\vec{A}}{dt}$$

FOR COIL OF N TURNS $\mathcal{E} = -N \dfrac{d\Phi_B}{dt}$

5 EXAMPLES IN BOOK

GENERATORS AS ENERGY CONVERTERS

CONVERT MECHANICAL ENERGY TO ELECTRICAL ENERGY.

LENZ'S LAW

THE DIRECTION OF ANY MAGNETIC INDUCTION EFFECT IS SUCH AS TO OPPOSE THE CAUSE OF THE EFFECT

LENZ'S LAW MAINTAINS CONSERVATION OF ENERGY.

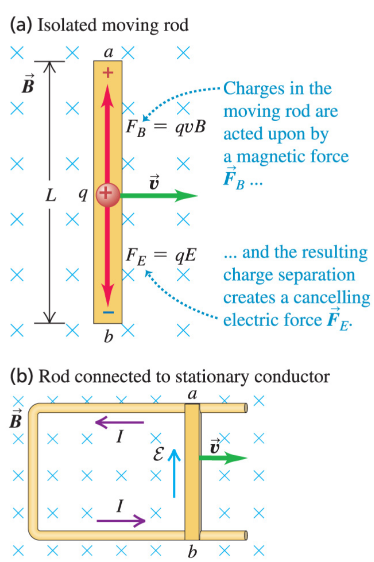
2 EXAMPLES IN BOOK

LENZ'S LAW AND THE RESPONSE TO FLUX CHANGES

LENZ'S LAW ONLY GIVES THE DIRECTION

THE MAGNITUDE OF CHANGE DEPEND ON RESISTANCE.

MOTIONAL EMF



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

$\vec{F}_m = q\vec{v} \times \vec{B}$; $\vec{F}_E = q\vec{E}$

$E = vB$

$V_{ab} = EL = vBL$

$\mathcal{E} = vBL$ MOTIONAL emf

FOR RESTANCE R

$IR = \mathcal{E} = vBL$

MOTIONAL emf: GENERAL FORM

FOR TIME INDEPENDENT MAGNETIC FIELD

$$d\mathcal{E} = \vec{E} \cdot d\vec{l} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

AROUND A CLOSED CONDUCTING LOOP

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

USE THIS FOR MOVING CONDUCTORS.

USE FARADAY'S LAW FOR STATIONARY CONDUCTORS.

2 EXAMPLES IN BOOK

INDUCED ELECTRIC FIELDS

THE INDUCED ELECTRIC FIELD IS NOT CONSERVATIVE

$$\oint \vec{E} \cdot d\vec{l} = \mathcal{E} \quad \Rightarrow \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

THIS IS ONLY TRUE FOR STATIONARY PATH

NONELECTROSTATIC ELECTRIC FIELDS

NONELECTROSTATIC $\equiv$ ELECTRIC FIELD NOT CONSERVATIVE.

POTENTIAL HAS NO MEANING

$\vec{F} = q\vec{E}$ ALWAY VALID

1 EXAMPLE IN BOOK

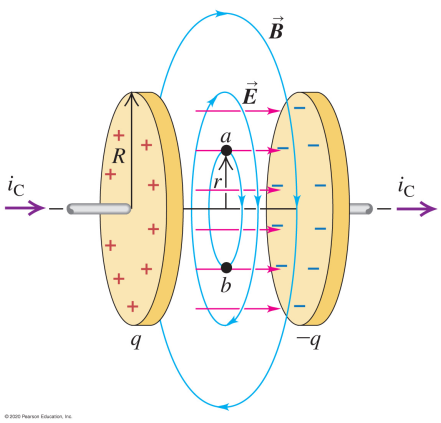
EDDY CURRENTS

DISPLACEMENT CURRENT AND MAXWELL'S EQUATIONS

VARYING ELECTRIC FIELDS ALSO GIVE RISE TO MAGNETIC FIELDS.

GENERLIZING AMPERE'S LAW

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{ENCL}$ PREVIOUS VERSION



$i_c \equiv$ NORMAL CONDUCTION CURRENT

DISPLACEMENT CURRENT

$i_D = \epsilon \dfrac{d\Phi_E}{dt}$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_c + i_D)$

i_c OR i_D WILL BE ZERO DEPENDING ON SURFACE

WE CAN CONSIDER CURRENT THRU CAPACITOR.

THE REALITY OF DISPLACEMENT CURRENT

A $\vec{B}$ FIELD CAN BE MEASURED BETWEEN THE PLATES.

MAXWELL'S EQUATIONS OF ELECTROMAGNETISM

CHARGES AND CURRENTS IN EMPTY SPACE

$\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{ENCL}}{\epsilon_0}$ GAUSS'S LAW FOR $\vec{E}$

$\oint \vec{B} \cdot dA = 0$ GAUSS'S LAW FOR $\vec{B}$

$\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ FARADAY'S LAW

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left[i_c + \epsilon_0 \dfrac{d\Phi_E}{dt} \right]_{ENCL}$ AMPERE'S LAW

$\vec{E} = \vec{E}_c + \vec{E}_N$

$\vec{E}_c \equiv$ ELECTROSTATIC (DUE TO CHARGES)

$\vec{E}_N \equiv$ NONELECTROSTATIC

$\oint \vec{E}_c \cdot d\vec{\ell} = 0$ CONSERVATIVE (NOT IN FARADAY'S LAW)

$\oint \vec{E}_N \cdot d\vec{A} = 0$ NOT DUE TO CHARGES (GAUSS'S LAW)

SYMMETRY IN MAXWELL'S EQUATIONS

IN EMPTY SPACE $q = 0$ $(i_c = 0)$

$\oint \vec{E} \cdot d\vec{A} = 0$; $\oint \vec{B} \cdot dA = 0$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \; ; \; \oint \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

REMOVING THE FLUXES $\Phi_B = \int \vec{B} \cdot d\vec{A}$; $\Phi_E = \int \vec{E} \cdot d\vec{A}$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot dA \; ; \; \oint \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

ALSO $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

SUPERCONDUCTIVITY

THE MEISSNER EFFECT

SUPERCONDUCTOR LEVITATION AND OTHER APPLICATIONS

INDUCTANCE

COILS

MUTUAL INDUCTANCE

CONSIDER TWO NEIGHBORING COILS

I_1 IN COIL 1 PRODUCES $\vec{B}_1$, AND HENCE ϕ_2 IN COIL 2.

IF i_1 IS CHANGING, ϕ_2 IS CHANGING.

FARADAY'S LAW SAYS emf INDUCED IN COIL 2.

$\Rightarrow$ CURRENT i_2 IN COIL 2.

$$\mathcal{E}_2 = -N_2 \frac{d\phi_2}{dt}$$

$\phi_2 \propto i_1 \Rightarrow N_2 \phi_2 = M_{21} i_1$; $M_{21} \equiv$ MUTUAL INDUCTANCE

$$N_2 \frac{d\phi_2}{dt} = M_{21} \frac{di_1}{dt} \Rightarrow \mathcal{E}_2 = -M_{21} \frac{di}{dt}$$

WHERE $M_{21} = \dfrac{N_2 \phi_2}{i_1}$

CONVERSLY WE COULD CONSIDER A CURRENT IN COIL 2, i_2, CAUSING A MAGNTIC FIELD $\vec{B}_2$

BUT $M_{12} = M_{21} \equiv M$ FUNCTION OF GEOMETRY AND MATERIAL PROPERTIES

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \; ; \; \mathcal{E}_1 = -M \frac{di_2}{dt}$$

THE -'VE SIGN IS A RESULT OF LENZ'S LAW

THE MUTUAL INDUCTANCE IS $M = \dfrac{N_2 \phi_2}{i_1} = \dfrac{N_1 \phi_1}{i_2}$

M UNITS [Wb/A] ≡ [H] HENRY

SELF-INDUCTANCE AND INDUCTORS

THE CHANGING CURRENT THAT CAUSES A MAGNETIC FIELD CAN CAUSE A CHANGING FLUX IN THE SAME CIRCUIT $\Rightarrow$ SELF-INDUCED emf.

$N\Phi_B \propto i$; $N\Phi_B = Li$

FOR A COIL, SELF-INDUCTANCE (INDUCTANCE) $L = \dfrac{N\Phi_B}{i}$

FUNCTION OF GEOMETRY AND MATERIAL PROPERTIES

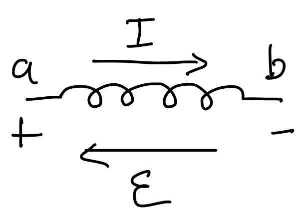
FOR CHANGING CURRENT, $N\dfrac{d\Phi_B}{dt} = L\dfrac{di}{dt}$

FROM FARADAY'S LAW SELF-INDUCED emf $\varepsilon = -L\dfrac{di}{dt}$

-'VE SIGN DUE TO LENZ'S LAW

INDUCTORS AS CIRCUIT ELEMENTS

$$V_{ab} = L\frac{di}{dt} > 0$$



OPPOSE ANY VARIATION OF CURRENT IN CIRCUIT

APPLICATION OF INDUCTORS

MAGNETIC FIELD ENERGY

ENERGY STORED IN AN INDUCTOR

$$P = V_{ab}\, i = Li\frac{di}{dt} \quad ; \quad P = \frac{dU}{dt} \Rightarrow dU = Li\,di$$

$$U = L\int_0^I i\,di = \frac{1}{2}LI^2$$

NO ENERGY IN OR OUT FOR STEADY CURRENT.

IF CURRENT INCREASES ENERGY IS STORED IN INDUCTOR.
IF CURRENT DECREASES INDUCTOR ACTS AS SOURCE OF ENERGY.

MAGNETIC ENERGY DENSITY

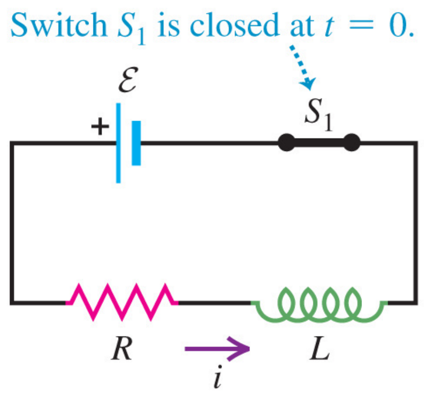
$u = \frac{B^2}{2\mu_0}$ OR $u = \frac{B^2}{2\mu}$ cf. $u_E = \frac{1}{2}\epsilon_0 E^2$

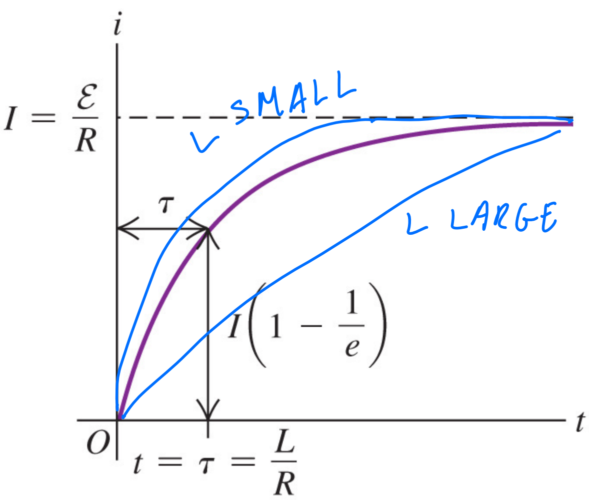
THE R-L CIRCUIT

CURRENT GROWTH IN AN R-L CIRCUIT

FARADAY'S LAW

$\mathcal{E}_{IND} = -L\frac{di}{dt} = iR - \mathcal{E}$





$\mathcal{E} - iR - L\frac{di}{dt} = 0$

$i = \frac{\mathcal{E}}{R}\left[1 - \exp\left(-\frac{R}{L}t\right)\right]$

$t \to \infty, \quad i = \frac{\mathcal{E}}{R}$

$\frac{di}{dt} = \frac{\mathcal{E}}{L}\exp\left(-\frac{R}{L}t\right)$

$t = 0, \quad \frac{di}{dt} = \frac{\mathcal{E}}{L}$

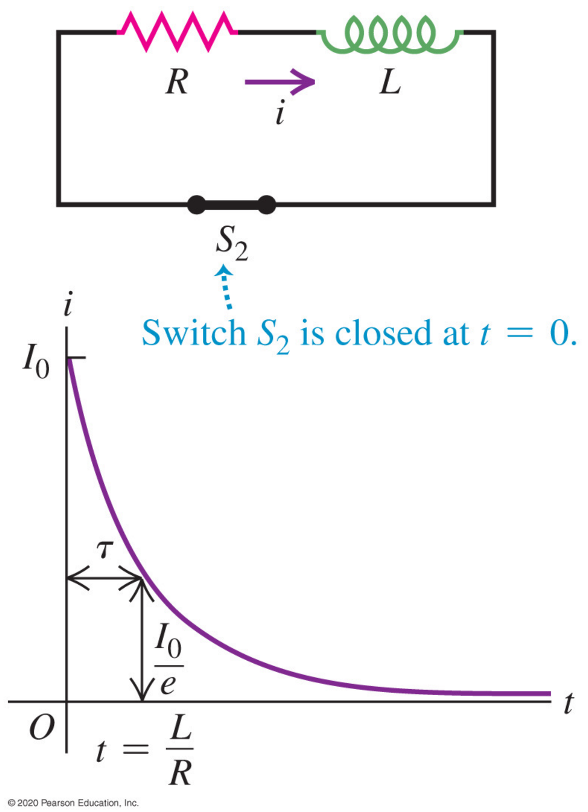
$t \to \infty, \quad \frac{di}{dt} \to 0$

TIME CONSTANT $\tau = \frac{L}{R}$

POWER (ENERGY CHANGE)

$\mathcal{E}i = i^2R + Li\frac{di}{dt} = i^2R + \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = i^2R + \frac{du_L}{dt}$

CURRENT DECAY IN AN R-L CIRCUIT



$$0 + iR + L\frac{di}{dt} = 0$$

$$i = I_0 \exp\left[-\frac{R}{L}t\right]$$

POWER

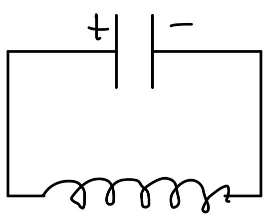
$$0 = i^2 R + Li\frac{di}{dt}$$

$Li\frac{di}{dt} < 0 \Rightarrow$ ENERGY DECREASES

$$Li\frac{di}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2}\right) = \frac{dU_L}{dt}$$

R-C AND R-L GIVE TRANSIENT BEHAVIOUR

THE L-C CIRCUIT (GIVES OSCILLATING BEHAVIOUR)



$t = 0$, $q = Q$, $i = 0$

ENERGY OSCILLATES BETWEEN COMPONENTS

ELECTRICAL OSCILLATIONS IN AN L-C CIRCUIT

$$0 - L\frac{di}{dt} - \frac{q}{C} = 0 \; ; \; i = \frac{dq}{dt}$$

$$f = \frac{\omega}{2\pi}$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \quad \text{2ND ORDER ODE}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

SOLUTION

$$\left.\begin{aligned} q &= Q\cos(\omega t + \phi) \\ i &= -\omega Q \sin(\omega t + \phi) \end{aligned}\right\} \omega = \sqrt{\frac{1}{LC}}$$

Q, ϕ GIVEN BY INITIAL CONDITIONS

IF $Q(t=0) = Q$; $i(t=0) = 0 \Rightarrow \phi = 0$

ENERGY IN AN L-C CIRCUIT

INITIALY TOTAL ENERGY $\frac{Q^2}{2C}$

$\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$ CONSERVATION OF ENERGY (FOR NO RESISTANCE)

THE L-R-C SERIES CIRCUIT

THE RESISTOR ABSORBS SOME OF THE ENERGY IN i^2R LOSSES $\rightarrow$ DAMPED OSCILLATIONS

$\left.\begin{array}{l}\text{UNDER DAMPED}\\ \text{CRITICALLY DAMPED}\\ \text{OVER DAMPED}\end{array}\right\}$ DEPENDING ON R, L, C

ANALYZING AN L-R-C SERIES CIRCUIT

$t=0,\ q=Q_0,\ i=0$

$0 - iR - L\frac{di}{dt} - \frac{q}{C} = 0$; $i = \frac{dq}{dt}$ $\leftarrow$ FOR +'VE PLATE

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

UNDER DAMPED

$R^2 < \frac{4L}{C}$; $q = A\exp\left[-\frac{R}{2L}t\right]\cos(\omega' t + \phi)$

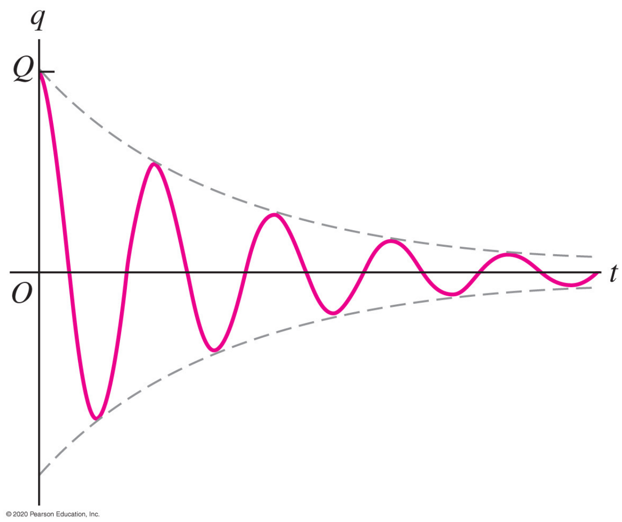
$\omega' = \left[\frac{1}{LC} - \frac{R^2}{4L^2}\right]^{1/2}$; A, ϕ GIVEN BY INITIAL CONDITIONS

$R=0$ RECOVERS OSCILLATING CASE

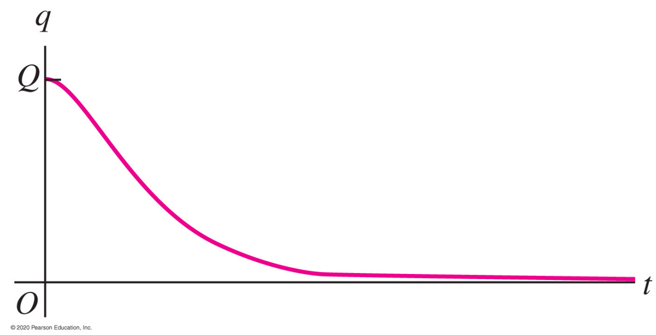
CRITICALLY DAMPED

$R^2 = \frac{4L}{C}$; TWO DECREASING EXPONENTIALS

(a) Underdamped circuit (small resistance R)



(c) Overdamped circuit (very large resistance R)



INDUCTORS SMOOTH TRANSIANTS IN CIRCUITS.

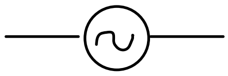
INDUCTORS SLOW DOWN RISE TIMES.

DIGITAL SWITCHING CIRCUITS REQUIRE FAST RISE TIMES.

ALTERNATING CURRENT

DC, AC, TRANSIENT

PHASORS AND ALTERNATING CURRENTS



ac SOURCE

$v = V\cos\omega t$; $V \equiv$ VOLTAGE AMPLITUDE

DO NOT CARE ABOUT HOW PRODUCED

$\omega \equiv$ ANGULAR FREQUENCY

$f = \frac{\omega}{2\pi} = 60\,Hz$ (CANADA & USA) ; $f = 50\,Hz$ ELSEWHERE

SIMILARLY, $i = I\cos\omega t$ (USE THIS CONVENTION)

PHASOR DIAGRAMS

I WILL USE COMPLEX NOTATION TO KEEP TRACK OF PHASE INFORMATION

$$c = Ce^{j\phi} = A + jB \ , \ c^* = Ce^{-j\phi} = A - jB$$

$$C = |c| = \sqrt{cc^*} = \sqrt{A^2+B^2} \ ; \ \phi = \tan^{-1}\left(\frac{B}{A}\right)$$

RECTIFIED ALTERNATING CURRENT

ROOT-MEAN-SQUARED (rms) VALUES

SQUARE THE VALUE, TAKE THE MEAN, TAKE THE SQUARE ROOT.

FOR CURRENT $i^2 = I^2\cos^2\omega t = \frac{I^2}{2}(1+\cos 2\omega t)$

$$\langle i^2 \rangle = \left\langle \frac{I^2}{2} \right\rangle + \left\langle \frac{I^2}{2}\cos 2\omega t \right\rangle = \frac{I^2}{2}$$

$I_{RMS} = \sqrt{\langle i^2 \rangle} = \frac{I}{\sqrt{2}}$; ALSO $V_{RMS} = \frac{V}{\sqrt{2}}$

$V_{RMS} = 120\,V \Rightarrow V = \pm 170\,V$ OSCILLATING 60 TIMES PER SECOND

V, I USUALLY DESCRIBED IN TERMS OF RMS

EXCEPT PRODUCTS WHICH BENIFIT FROM HIGHER RATINGS

RESISTANCE AND REACTANCE

R, C, L

RESISTOR IN AN ac CIRCUIT

$i = I\cos\omega t \Rightarrow v_R = iR = IR\cos\omega t = V_R\cos\omega t$

$V_R = IR \equiv I X_R$ AND $v_R = i z_R$ WITH $z_R = X_R = R$

VOLTAGE IN PHASE WITH CURRENT.

INDUCTOR IN AN ac CIRCUIT

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}(I\cos\omega t) = -I\omega L\sin\omega t$$

$$= I\omega L\cos(\omega t + \pi/2) = V_L\cos(\omega t + \pi/2)$$

WE USUALLY DESCRIBE PHASE OF VOLTAGE RELATIVE TO CURRENT

$i = I\cos\omega t$; $v = V\cos(\omega t + \phi)$, $\phi \equiv$ PHASE ANGLE

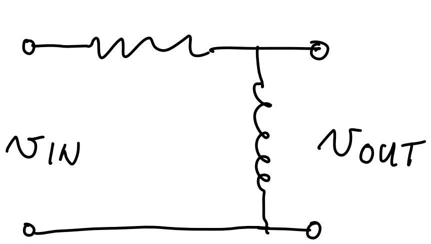
FOR INDUCTOR, VOLTAGE <u>LEADS</u> THE CURRENT BY $\phi = \pi/2$

FREQUENCY DEPENDENT

$V_L = I\omega L \equiv I X_L$ AND $v_L = i z_L$ WITH $z_L = j\omega L$

$X_L \equiv \omega L$ (INDUCTIVE REACTANCE)

THE MEANING OF INDUCTIVE REACTANCE



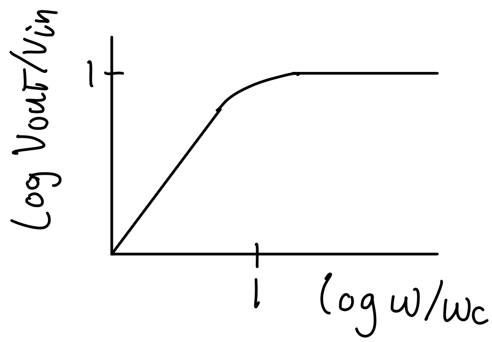
$v_{IN} = iz = i[R + j\omega L]$

$v_{OUT} = i j\omega L$

$$\frac{v_{out}}{v_{IN}} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega L/R}{1 + j\omega L/R}$$

$\omega \to 0$, $\frac{V_{out}}{V_{in}} \to 0$, $\phi \to 90°$

$\omega \to \infty$, $\frac{V_{out}}{V_{in}} \to 1$, $\phi \to 0°$ (NO PHASE SHIFT)



$$\omega_c \equiv \frac{R}{L}$$

HIGH-PASS FILTER

CAPACITOR IN AN ac CIRCUIT

$$i = \frac{dq}{dt} = I \cos \omega t \Rightarrow q = \frac{I}{\omega} \sin \omega t$$

$$q = C v_c \Rightarrow v_c = \frac{I}{\omega C} \sin \omega t = \frac{I}{\omega C} \cos(\omega t - \pi/2)$$

$$v_c = V_c \cos(\omega t - \pi/2)$$

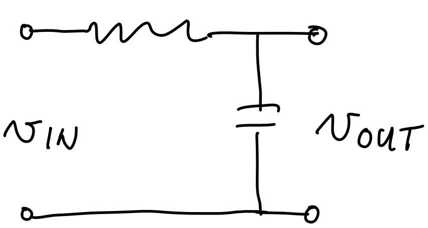
$V_c = \frac{I}{\omega C} \equiv I X_c$ AND $v_c = i z_c$ WITH $z_c = \frac{1}{j\omega C}$

FREQUENCY DEPENDENT

$X_c \equiv 1/\omega C$ (CAPACITIVE REACTANCE)

FOR CAPACITOR, VOLTAGE LAGS THE CURRENT BY $\phi = \pi/2$

THE MEANING OF CAPACITIVE REACTANCE



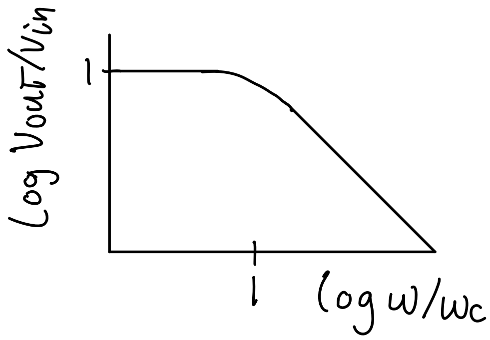
$$v_{IN} = i z = i\left[R + \frac{1}{j\omega C}\right]$$

$$v_{OUT} = i \frac{1}{j\omega C}$$

$$\frac{v_{out}}{v_{IN}} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

$\omega \to 0$, $\frac{V_{out}}{V_{in}} \to 1$, $\phi \to 0°$ (NO PHASE SHIFT)

$\omega \to \infty$, $\frac{V_{out}}{V_{in}} \to 0$, $\phi \to -90°$



$$\omega_c \equiv \frac{1}{RC}$$

LOW-PASS FILTER

COMPARING ac CIRCUIT ELEMENTS

$V_R = IR$, $v_R = i z_R$, $z_R = X_R$, $X_R = R$, v_R IN PHASE WITH i

$V_L = I\omega L$, $v_L = i z_L$, $z_L = j X_L$, $X_L = \omega L$, v_L LEADS i BY $\pi/2$

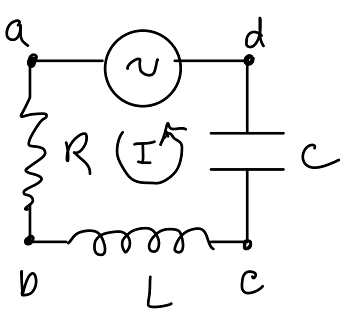
$V_C = \frac{I}{\omega C}$, $v_C = i z_C$, $X_C = -j X_C$, $X_C = \frac{1}{\omega C}$, v_C LAGS i BY $\pi/2$

NOTE: $z_C = -\frac{j}{\omega C} = \frac{1}{j\omega C}$

31.42

THE L-R-C SERIES CIRCUIT

NOW ADD ac SOURCE TO L-R-C CIRCUIT



$i = I\cos\omega t$

$v = v_{ad}$, $v_R = v_{ab}$, $v_L = v_{bc}$, $v_C = v_{cd}$

$$\begin{aligned} v &= v_R + v_L + v_C \\ &= i z_R + i z_L + i z_C \\ &= i\,(z_R + z_L + z_C) \\ &= i\,[R + j\,(X_L - X_C)] \end{aligned}$$

$$|v| = |i|\,[R^2 + (X_L - X_C)]^{1/2} = |i|\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

IMPEDANCE $Z = \frac{V}{I} = \frac{|v|}{|i|} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]$

$V = IZ$. ALSO $z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R + jX$

THE MEANING OF IMPEDANCE AND PHASE ANGLE

$i = I\cos(\omega t)$

$v = V\cos(\omega t + \phi)$

$v = iZ$; $V = IZ$

$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

CAN USE SAME EQUATIONS IF ONE COMPONENT MISSING

$R \to 0$; $L \to 0$; $C \to \infty$

$X_L > X_C \Rightarrow 0 < \phi < 90°$ OR $X_C < X_L \Rightarrow -90° < \phi < 0$

ALSO $V = IZ \Rightarrow \frac{V}{\sqrt{2}} = \frac{I}{\sqrt{2}}Z \Rightarrow V_{RMS} = I_{RMS}Z$

POWER IN ALTERNATING-CURRENT CIRCUITS

INSTANTANEOUS POWER $p = vi$

WE WILL TAKE AVERAGES OVER A PERIOD

$\langle\cos^2\omega t\rangle = 1/2$, $\langle\cos\omega t \sin\omega t\rangle = 0$

POWER IN A RESISTOR

$p_R = V\cos\omega t \, I\cos\omega t = VI\cos^2\omega t$

$P_{AVE} = \langle p_R\rangle = VI\langle\cos^2\omega t\rangle = \frac{1}{2}VI = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}} = V_{RMS}I_{RMS}$

SINCE $V_{RMS} = I_{RMS}R$, $P_{AVE} = I^2_{RMS}R = \frac{V^2_{RMS}}{R}$

POWER IN AN INDUCTOR

$$p_L = V_L \cos(\omega t + \pi/2)\, I \cos\omega t = -V_L I \sin\omega t \cos\omega t$$
$$P_{AVE} = \langle p_L \rangle = -V_L I \langle \sin\omega t \cos\omega t \rangle = 0$$

NO ENERGY STORED IN INDUCTOR
AVERAGE IS 0; NET OVER A CYCLE IS 0
INSTANTANEOUS $\neq 0$ AT ALL TIMES

POWER IN A CAPACITOR

$$p_C = V_C \cos(\omega t - \pi/2)\, I \cos\omega t = V_C I \sin\omega t \cos\omega t$$
$$P_{AVE} = \langle p_C \rangle = V_C I \langle \sin\omega t \cos\omega t \rangle = 0$$

NO ENERGY STORED IN CAPACITOR

POWER IN GENERAL ac CIRCUIT

$$p = vi = V\cos(\omega t + \phi)\, I \cos\omega t$$
$$= VI[\cos\omega t \cos\phi - \sin\omega t \sin\phi]\cos\omega t$$
$$= VI\cos\phi \cos^2\omega t - VI \sin\phi \sin\omega t \cos\omega t$$

$$P_{AVE} = VI\cos\phi \langle \cos^2\omega t \rangle - VI\sin\phi \langle \sin\omega t \cos\omega t \rangle$$
$$= \tfrac{1}{2} VI\cos\phi = V_{RMS} I_{RMS} \underbrace{\cos\phi}_{\text{POWER FACTOR}}$$

FOR RESISTOR, $\phi = 0$, $P_{AVE} = V_{RMS} I_{RMS}$; $P_{MAX} = VI = 2P_{AVE}$
FOR INDUCTOR OR CAPACITOR, $\phi = \pm \pi/2$, $P_{AVE} = 0$
FOR L-R-C SERIES CIRCUIT $\cos\phi = \dfrac{R}{Z}$

$\cos\phi \to 1$ FOR $Z \to R \Rightarrow \omega L = 1/\omega C$

RESONANCE IN ALTERNATING-CURRENT CIRCUITS

WHEN $X_L - X_C = 0$, $Z = R$ (MINIMUM)

FREQUENCY AT WHICH THE CURRENT IS A MAXIMUM FOR A GIVEN VOLTAGE

$I = \frac{V}{Z}$, $I_{MAX} \Rightarrow Z_{MIN} \Rightarrow Z = R \Rightarrow X_L = X_C$

AT RESONANCE $V_L - V_C = 0$

CIRCUIT BEHAVIOR AT RESONANCE

AT RESONANCE, $\omega = \omega_0$; $\omega_0 L = \frac{1}{\omega_0 C}$

$\omega_0 = \frac{1}{\sqrt{LC}}$ RESONANCE ANGULAR FREQUENCY

RESONANCE FREQUENCY $f_0 = \frac{\omega_0}{2\pi}$

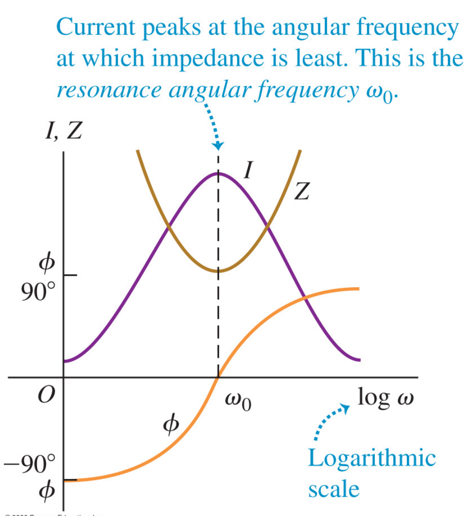
AT ω_0 IT'S LIKE THE CAPACITOR AND INDUCTOR CANCEL EACH OTHER AND DO NOT EXIST.

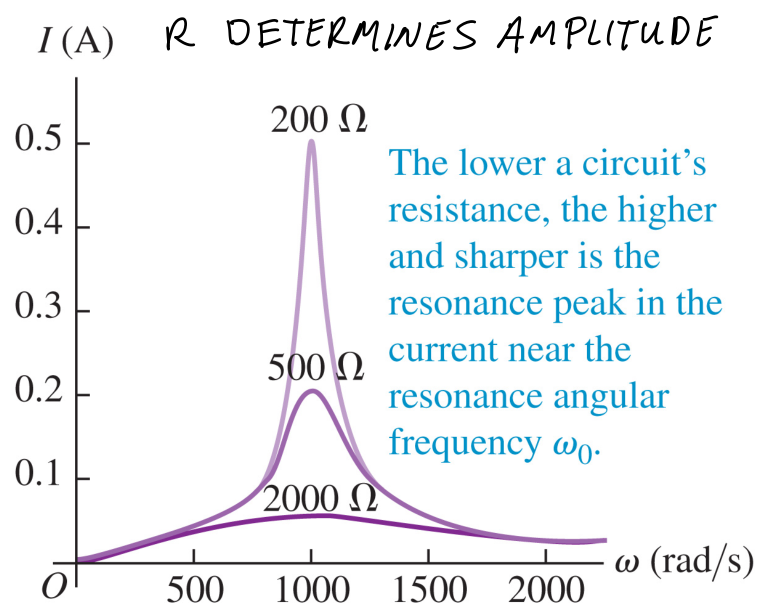
TAILORING AN ac CIRCUIT

L, C DETERMINE ω_0

R DETERMINES AMPLITUDE

(b) Impedance, current, and phase angle as functions of angular frequency





TRANSFORMERS

CONVERT ac VOLTAGES

LONG DISTANCE TRANSMISSION, V HIGH, I LOW

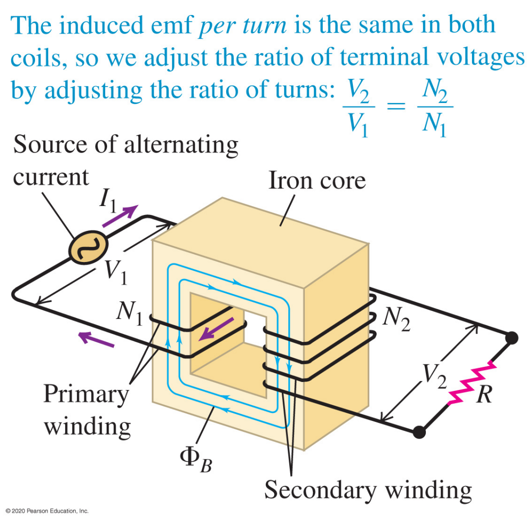
REDUCES I^2R LOSSES

$\left.\begin{array}{l} V_{RMS} = 500\text{ kV ON LINE} \\ V_{RMS} = 120\text{ V AT WALL} \end{array}\right\}$ STEP DOWN

CELL PHONE 120V → 5V USB + AC ⇒ DC CONVERSION.

→ 0.6 V ON CHIP (DC VOLTAGE DIVIDER)

HOW TRANSFORMES WORK



IRON CORE TO KEEP FIELD INSIDE ⇒ MAXIMIZE MUTUAL INDUCTANCE

ALL $\vec{B}$ IN IRON CORE, $\Phi_1 = \Phi_2 \equiv \Phi$

$\mathcal{E}_1 = -N_1 \frac{d\Phi}{dt}$, $\mathcal{E}_2 = -N_2 \frac{d\Phi}{dt}$

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$$

FOR NO RESISTANCE AND SAME ω AS SOURCE

TERMINAL VOLTAGES $\frac{V_2}{V_1} = \frac{N_2}{N_1}$ STEP UP OR STEP DOWN

ENERGY CONSIDERATIONS FOR TRANSFORMERS

FOR NO RESISTANCE, POWER NOT CHANGED

$V_1 I_1 = V_2 I_2$; $\frac{V_1}{V_2} = \frac{I_2}{I_1}$

FOR A LOAD R ON THE SECONDARY

$$\frac{V_1}{I_1} = \frac{R}{(N_2/N_1)^2}$$

TRANSFORMS RESISTANCE

IMPEDANCE MATCHING MAXIMIZES POWER TRANSFER.

ELECTROMAGONTIC WAVES

WHAT IS THE NATURE OF LIGHT?

EXPLAIN BY UNIFICATION OF ELECTRICITY AND MAGNATISM.

→ ELECTROMAGNETISM

MAXWELL'S EQUATIONS AND ELECTROBAGNETIC WAVES

WHEN THE FIELDS ARE TIME VARYING THEY ARE NOT INDEPENDENT. (FARADAY'S LAW AND AMPERE LAW WITH MAXWELL)

$\vec{E}$ AND $\vec{B}$ SUSTAIN EACH OTHER.

→ ELECTROMAGNETIC WAVES

ELECTRICITY, MAGNETISM, AND LIGHT

MAXWELL'S EQUATIONS

GAUSS'S LAW
$$\begin{cases} \oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{encl}}{\epsilon_0} \\ \oint \vec{B} \cdot d\vec{A} = 0 \end{cases}$$

$;\quad Q_{ENCL} = \int \rho \, dV$

DIVERGENCE THEOREM $\int_S \vec{E} \cdot d\vec{A} = \int_V \vec{\nabla} \cdot \vec{E} \, dV$

$$\int \vec{\nabla} \cdot \vec{E} \, dV = \frac{1}{\epsilon_0} \int \rho \, dV \Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\int \vec{\nabla} \cdot \vec{B} \, dV = 0 \Rightarrow \vec{\nabla} \cdot \vec{B} = 0$$

FARADAY'S LAW $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_M}{dt} = -\dfrac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$

STOKES'S THEOREM $\oint \vec{E} \cdot d\vec{l} = \int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{A}$

$$\int (\vec{\nabla} \times \vec{E}) \cdot d\vec{A} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A} \Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

AMPERE'S LAW $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

$i_c = \int \vec{J} \cdot d\vec{A}$ $\qquad = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d}{dt}\int_s \vec{E} \cdot d\vec{A}$

$$\int (\vec{\nabla} \times \vec{B}) \cdot d\vec{A} = \mu_0 \int \vec{J} \cdot d\vec{A} + \mu_0 \epsilon_0 \frac{d}{dt}\int \vec{E} \cdot d\vec{A}$$

$$\Rightarrow \vec{\nabla} \times B = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

THESE ARE ELECTROMAGNETIC FIELDS IN VACUUM.

FOR NO SOURCES, $Q_{ENCL} = 0$, $i_c = 0$

IN MATERIALS $\epsilon_0 \to \epsilon$, $\mu_0 \to \mu$

(SOMETIMES $\epsilon = \epsilon(t, \vec{x})$, $\mu = \mu(t, \vec{x})$)

GENERATING ELECTROMAGNETIC RADIATION

STATIC CHARGES PRODUCE $\vec{E}$ ONLY.

CONSTANT CURRENT PRODUCE $\vec{E}$ AND $\vec{B}$.

ACCELERATING CHARGES PRODUCE WAVES

ALL RADIATION DUE TO ACCELERATING CHARGE

EXAMPLE, SIMPLE HARMONIC MOTION

ELECTROMAGNETIC RADIATION (WAVES)

THE ELECTROMAGNETIC SPECTRUM

$c = 3 \times 10^8$ m/s FIXED, $c = \lambda f$

RADIO (SMALL f, LARGE λ)

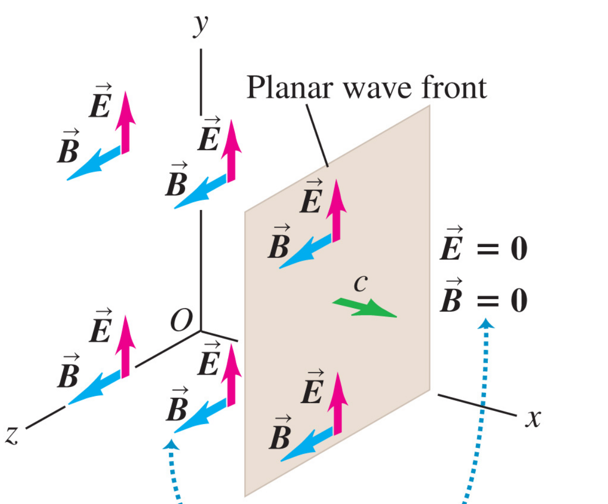
GAMMA RAYS (LARGE f, SMALL λ)

VISIBLE LIGHT SMALL REGION OF SPECTRUM

PLANE ELECTROMAGNETIC WAVES AND THE SPEED OF LIGHT

A SIMPLE PLANE ELECTROMAGNETIC WAVE

WE WILL SHOW PLANE WAVES SATISFY MAXWELL'S EQUATIONS



The electric and magnetic fields are uniform behind the advancing wave front and zero in front of it.

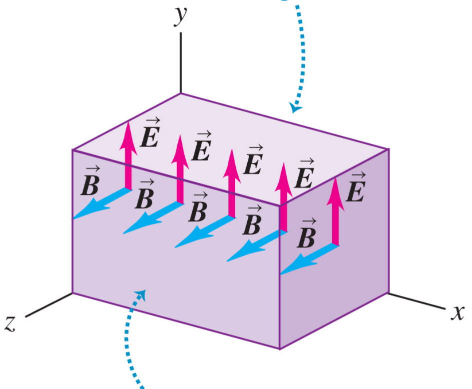
$\vec{E} = E\hat{y}$ UNIFORM

$\vec{B} = B\hat{z}$ UNIFORM

$\vec{c} = c\hat{x}$ VELOCITY OF WAVE FRONT

$\Rightarrow$ PLANE WAVE

The electric field is the same on the top and bottom sides of the Gaussian surface, so the total electric flux through the surface is zero.



The magnetic field is the same on the left and right sides of the Gaussian surface, so the total magnetic flux through the surface is zero.

GAUSS'S LAWS $\Phi_E = \Phi_M = 0$

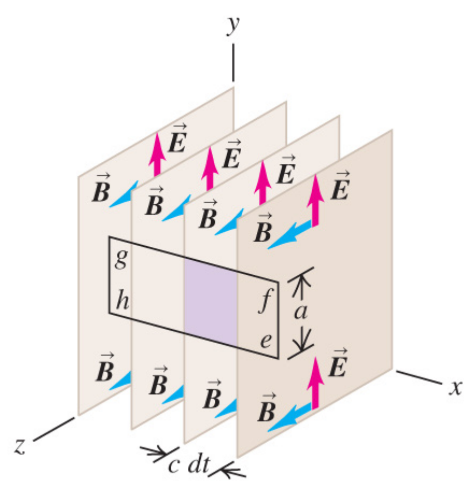
IF $\vec{E} \perp \vec{c}$ AND $\vec{B} \perp \vec{c}$

$\Rightarrow$ WAVE IS TRANSVERSE

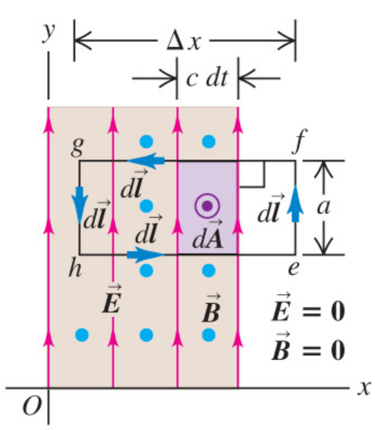
IF $\vec{E}$ OR $\vec{B}$ HAD x-COMPONENTS

Φ_E OR $\Phi_M \neq 0$

(a) In time dt, the wave front moves a distance $c\,dt$ in the $+x$-direction.



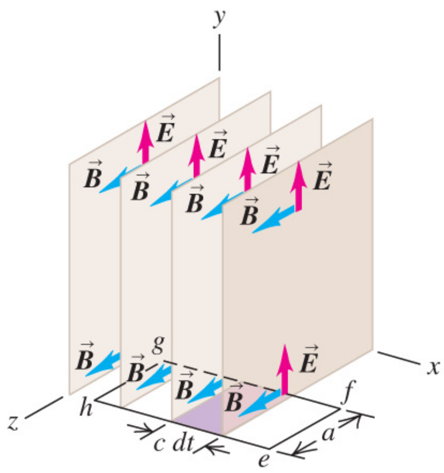
(b) Side view of situation in (a)



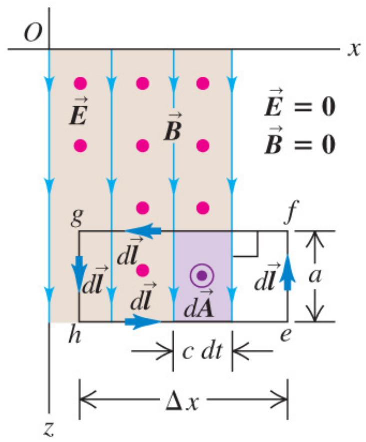
FARADAY'S LAW

$$\left.\begin{aligned} \oint \vec{E} \cdot d\vec{l} &= -Ea \\ \frac{d\Phi_B}{dt} &= Bac \end{aligned}\right\} \quad \begin{aligned} E &= cB \\ \vec{E} &\perp \vec{B} \end{aligned}$$

(a) In time dt, the wave front moves a distance $c\,dt$ in the $+x$-direction.



(b) Top view of situation in (a)



AMPERE'S LAW $(i_c = 0)$

$$\left.\begin{aligned} \oint \vec{B} \cdot d\vec{\ell} &= Ba \\ \frac{d\Phi_E}{dt} &= Eac \end{aligned}\right\} \quad \begin{aligned} &B = \epsilon_0 \mu_0 c E \\ &\vec{E} \perp \vec{B} \end{aligned}$$

$$\therefore\; c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

KEY PROPERTIES OF ELECTROMAGNETIC WAVES

1) $\vec{E} \perp c$, $\vec{B} \perp c$, $\vec{E} \perp \vec{B}$, $\vec{E} \times \vec{B} \parallel \vec{c}$ (TRANSVERSE)

2) $E = cB$

3) $v = c$ (SPEED OF LIGHT)

4) NO PROPAGATION MEDIUM REQUIRED (VACUUM)

THE FIELDS NEED NOT BE UNIFORM
e.g. SINUSOIDAL FUNCTION OF x.

$E = cB \Rightarrow \vec{E}, \vec{B}$ HAVE SAME PHASE

IF $\vec{E} = E\hat{y}$, LINEAR POLARIZED ALONG y-AXIS

DERIVATION OF THE ELECTROMAGNTIC WAVE EQUATION

IN VACUUM

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$ FARADAY'S LAW

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}\underbrace{(\vec{\nabla} \cdot \vec{E})}_{\rho/\epsilon_0} - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\underbrace{\vec{\nabla} \times \vec{B}}_{\mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

GAUSS' LAW

AMPERE'S LAW

$$\nabla^2 \vec{E} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\epsilon_0}\vec{\nabla}\rho + \mu_0 \frac{\partial \vec{J}}{\partial t}$$ WAVE EQUATION

FOR NO SOURCES, $\rho = 0$, $\vec{J} = 0$

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$ WAVE EQUATION

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$ AMPERE'S LAW

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla}\underbrace{(\vec{\nabla} \cdot \vec{B})}_{0} - \nabla^2 \vec{B} = \mu_0 \vec{\nabla} \times \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\underbrace{\vec{\nabla} \times \vec{E}}_{-\frac{\partial \vec{B}}{\partial t}}$$

GAUSS'S LAW

FARADAY'S LAW

$$\nabla^2 \vec{B} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t} = -\mu_0 \vec{\nabla} \times \vec{J}$$ WAVE EQUATION

FOR NO SOURCES, $\vec{J} = 0$ $$\frac{\partial^2 B}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$$

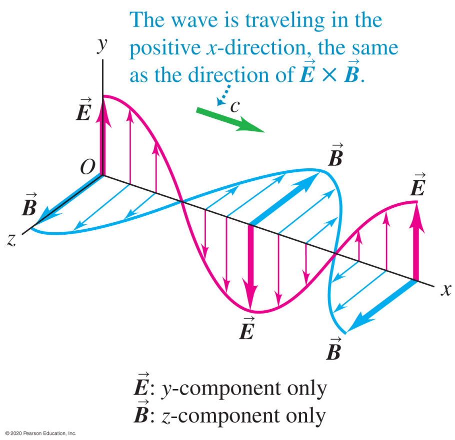
WAVE EQUATION

SINUSOIDAL ELECTROMAGNETIC WAVES

NOT ALL EM WAVES ARE PLANE WAVES.

WAVES FROM A POINT CHARGE ARE SPERICAL WAVES.

FIELDS OF A SINUSOIDAL WAVE



SINUSOIDAL IN SPACE AND TIME

$\vec{E}$ AND $\vec{B}$ ARE IN PHASE

$c = \lambda f$; $\omega = ck$

$\omega = 2\pi f$; $k = 2\pi/\lambda$

PROPOGATION DIRECTION $\vec{E} \times \vec{B}$

$$\left.\begin{aligned} \vec{E}(\vec{x},t) &= \hat{\jmath}\, E_{MAX} \cos(\vec{k}\circ\vec{x} - \omega t) \\ \vec{B}(\vec{x},t) &= \hat{k}\, B_{MAX} \cos(\vec{k}\circ\vec{x} - \omega t) \end{aligned}\right\} \text{IN PHASE}$$

$E_{MAX} = c\, B_{MAX}$

FOR PROPOGATION IN -'VE $\vec{x}$ DIRECTION $(\vec{k}\circ\vec{x} + \omega t)$

PLANE WAVES VS SPHERICAL WAVES ($\approx$ PLANE WAVES)

ELECTROMAGNETIC WAVES IN MATTER

CONSIDER DIELECTRICS

$E = vB$ AND $B = \epsilon\mu v E$; $v = \dfrac{1}{\sqrt{\mu\epsilon}}$

ENERGY AND MOMENTUM IN ELECTROMAGNETIC WAVES

$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$ ENERGY DENSITY IN VACUUM

$B = \frac{E}{c} \Rightarrow u = \epsilon_0 E^2 \Rightarrow u_E = u_B = \frac{1}{2} u$

ALSO $u = B^2/\mu_0$

ELECTROMAGNETIC ENERGY FLOW AND THE POYNTING VECTOR

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ POYNTING VECTOR (IN VACUUM)

ENERGY FLOW PER UNIT TIME PER UNIT AREA [J/s-m²]
OR POWER PER UNIT AREA [W/m²]

$\vec{S}$ IN THE DIRECTION OF PROPAGATION

$P = \oint \vec{S} \cdot d\vec{A}$ POWER OUT OF A CLOSED SURFACE

$$S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \epsilon_0 c E^2 = \frac{c}{\mu_0} B^2$$

FOR SINUSODIAL WAVE $\vec{S}(\vec{x},t) = \frac{1}{\mu_0} \vec{E}(\vec{x},t) \times \vec{B}(\vec{x},t)$

$$= \frac{1}{\mu_0} [\hat{j}\, E_{MAX} \cos(\vec{k} \cdot \vec{x} - \omega t)] \times [\hat{k}\, B_{MAX} \cos(\vec{k} \cdot \vec{x} - \omega t)]$$

$$= \frac{E_{MAX} B_{MAX}}{\mu_0} \cos^2(\vec{k} \cdot \vec{x} - \omega t)\, \hat{i} \qquad (\vec{S} = S\hat{i})$$

INTENSITY IS TIME AVERAGE OF S $(\vec{S}_{AV} = S_{AV}\hat{i})$

$$I = S_{AV} = \frac{E_{MAX} B_{MAX}}{\mu_0} \langle \cos^2(\vec{k} \cdot \vec{x} - \omega t) \rangle = \frac{E_{MAX} B_{MAX}}{2\mu_0}$$

ELECTROMAGNETIC MOMENTUM FLOW AND RADIATION PRESSURE

S IS ENERGY / TIME / AREA.

$\frac{S}{c}$ IS ENERGY / VOLUME

$\frac{S}{c^2}$ MOMENTUM / VOLUME $(E = pc)$

MOMENTUM DENSITY $\frac{dp}{dV} = \frac{S}{c^2}$

MOMENTUM FLOW RATE $\frac{1}{A}\frac{dp}{dt} = \frac{S}{c}$

AVERAGE MOMENTUM FLOW RATE $= \frac{S_{AV}}{c} = \frac{I}{c}$

$I \equiv$ INTENSITY

$\frac{dp}{dt} = F$; $\frac{1}{A}\frac{dp}{dt} = \frac{F}{A} = p_{RAD}$ (PRESSURE)

MOMENTUM $\Rightarrow$ RADIATION PRESSURE p_{RAD}

$p_{RAD} = \frac{S_{AV}}{c} = \frac{I}{c}$ WAVE TOTALLY ABSORBED

$p_{RAD} = 2I/c$ WAVE TOTALLY REFLECTED

STANDING ELECTROMAGNETIC WAVES

STANDING WAVE $\equiv$ SUPERPOSITION OF INCIDENT WAVE AND REFLECTED WAVE

FOR CONDUCTOR $\vec{E} = 0$ ON SURFACE $\Rightarrow$ STANDING WAVE