Relativistic Aspects of Modern Physics

PHYS 200

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Historical Perspective

- Near end of 19'th century it was thought that all important laws of physics had been discovered.
  - All that was left was to work out remaining details.
- We had
  - Newton's laws of motion and gravity.
  - Maxwell's equations of electricity and magnetic phenomena.
- Discovery of radioactivity by Becquerel and work by Planck, Einstein, Rutherford, Millikan, Bohr, de Broglie, Schrodinger, Heisenberg and others in early 20'th century lead to two completely new theories: relativity and quantum mechanics.
- These theories revolutionized the world of science and became the foundation for new technologies that have changed the face of civilization.

Module 1

Classical Physics

Classical Physics

- The state of any mechanical system at some time t_0 can be specified by constructing a set of coordinate axes and giving the coordinates and moment of the various parts of the system at that time.
- If forces are acting between the various parts, Newton's laws make it possible to calculate the state of the system at any future time t in terms of its state at t_0.

Newton's Laws

- Free particles move with constant velocity.
  - With zero acceleration, or in other words, with constant speed along straight lines.
- The vector-force on a particle equals the product of its mass and its vector-acceleration:

$$\vec{F} = m\vec{a}$$

- The forces of action and reaction are equal and opposite: e.g., if a particle A exerts a force F on a particle B, then B exerts a force $-F$ on A.

Relativity

- It is often desirable to specify the state of a system in terms of a new set of coordinate axes, which is moving relative to the first set.
  - How do we transform our description of the system from the old to the new coordinates?
  - What happens to the equations which govern the behaviour of the system when we make the transformation.
- The same questions arise when treating electromagnetic systems.

Newton's Principle of Relativity

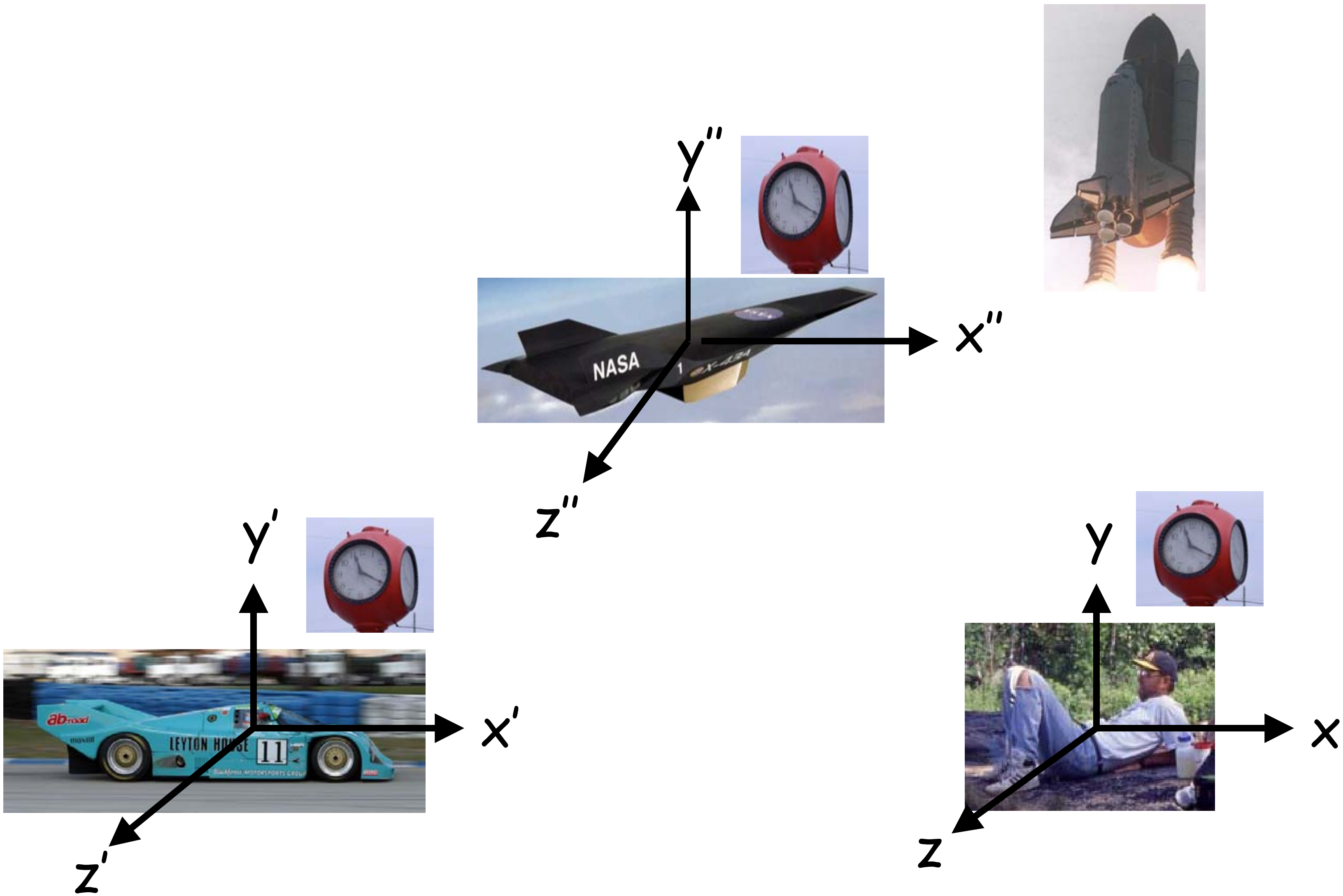
- Newton's first law does not distinguish between a particle at rest and one moving with constant velocity.

  "The motions of bodies included in a give space are the same among themselves, whether that space is at rest or moves uniformly forward in a straight line".
- The laws of physics appear the same in many different reference frames.
- Newton's force equation does not change its mathematical form under a change of reference frame.

Some Definitions

- **Event** is a physical "happening" that occurs at a certain place and at a certain time.
- **Reference frame** (3D) – consists of a set of x, y, z-axes (called a coordinate system).
- **Measurements** are made relative to a chosen reference system.
- **Record event** with reference frame (where event occurs) and clock (when event occurs).
- An **observer** is at rest relative to his own reference frame.

Reference Frames



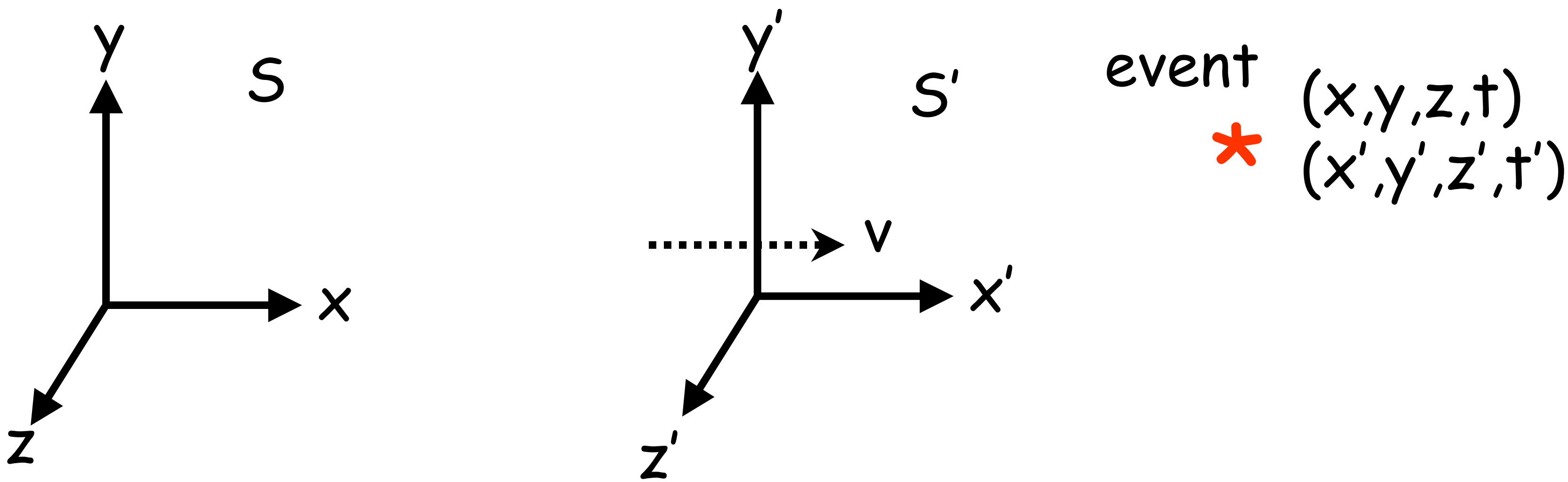
Inertial Reference Frame

- A reference frame in which Newton's law of inertia is valid.
- A body at rest in an inertial reference frame remains at rest.
- The acceleration of a body is zero when measured in an inertial reference frame.
- Rotating and accelerating reference frames are **not** inertial reference frames.
- The earth-based reference frame is not an inertial frame but it is a good approximation.

Galilean Transformation

Two Cartesian frames S and S′ in "standard configuration".

$t = t' = 0$



Coordinates: $x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$

Velocities: $u_x' = u_x - v, \quad u_y' = u_y, \quad u_z' = u_z$

More on Galilean Transformation

Galilean transformation

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

Inverse transformation

$$x = x' + vt'$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

Velocity transformation

$$u_x^{'} = u_x - v$$

$$u_y^{'} = u_y$$

$$u_z^{'} = u_z$$

Acceleration transformation

$$a_x^{'} = a_x$$

$$a_y^{'} = a_y$$

$$a_z^{'} = a_z$$

Same in both frames

Newtonian's laws hold in all frames

Relative Motion

- Consider a particle at rest relative to you with no forces acting on it.
  - According to Newton's first law, it will remain at rest.
- Now consider the same particle from the point of view of a second observer who is moving with constant velocity relative to you.
  - From the observer's frame of reference both you and the particle are moving with constant velocity.
  - Newton's first law holds also for him.
- How might we distinguish whether you and the particle are at rest and the second observer is moving with constant velocity, or the second observer is at rest and you and the particle are moving?

Absolute Motion

- All reference frames moving at constant velocity relative to an inertial reference frame are also inertial reference frames.
- There is no mechanics experiment that can tell us which is at rest and which is moving or if they are both moving.

Absolute motion cannot be detected

- By the late 19'th century it was generally thought that absolute motion could be detected in principle by a measurement of the speed of light.

Newton's Absolute Space

- Acceleration is absolute. You know you are accelerating because you fell a force.
- A particle does not resist uniform motion, but it does resist any change in velocity.
- The coefficient m is a measure of the particle's inertia, i.e. its resistance to acceleration.
- But, acceleration with respect to what?
- With respect to any one of the inertial frames.
- What in the world singles out the class of inertial frames from all the others as standards of nonacceleration?
- Newton postulated the existence of absolute space.
- This is suppose to interact with every particle so as to resist its acceleration.
- Identify absolute space with the frame of the fixed stars.

Objections to Absolute Space

- It is purely ad hoc and explains nothing.
- There is no unique way of locating Newton's absolute space within the infinite class of inertial frames.
- "It conflicts with one's scientific understanding to conceive of a thing which acts but cannot be acted upon".
- Newton's theory offers no satisfactory explanation for the existence of inertial frames.

Electromagnetic Waves

- Maxwell's equations of the electromagnetic field describe electricity, magnetism, and light in one uniform system.
- They lead to the conjecture that light must be an electromagnetic wave phenomenon.
- They predict electromagnetic disturbances which propagate in a vacuum in the manner of wave motion.
- The propagation of waves requires the existence of a propagation medium.
- The medium was given the name *ether* (assumed to fill all space).

Maxwell's Equations

- Perhaps Maxwell's ether could be identified with Newton's absolute space?
- Maxwell's equations do not obey Newton's principle of relativity.
- Their mathematical form does not remain the same under a Galilean transformation.
- In a moving ship electrical and optical phenomena will be different from those in a stationary ship.

Luminiferous Ether

- Ether must be massless since electromagnetic waves such as light can travel in a vacuum.
- It must have elastic properties in order to sustain the vibrations which are inherent to wave motion.
- Disturbance propagates with a fixed velocity with respect to the ether.
- Assumed Maxwell's equations valid for the frame of reference which was at rest with respect to the either – the ether frame.

Predictions of the Speed of Light

- A solution of Maxwell's equations leads to the propagation velocity of electromagnetic waves.
- The propagation velocity predicted by the equations depends on the form of the equations.
- In a new frame of reference in uniform motion with respect to the ether, the equations would have a different mathematical form.
- Thus a Galilean transformation of Maxwell's equations predict different values for the speed of light.
- Light traveled at the speed c only when measured with respect to the ether.

Speed of Light

Light waves travel from a source in any direction with speed c.

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Coulomb's Law

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

ε_0 permittivity of free space
($8.85 \times 10^{-12}\ \mathrm{C^2/(Nm^2)}$)

Biot-Savat Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

μ_0 permeability of free space
($4\pi \times 10^{-7}$ Tm/A)

$$c = \frac{1}{\sqrt{(8.85 \times 10^{-12})(4\pi \times 10^{-7})}} = 3.00 \times 10^{8}\ \mathrm{m/s}$$

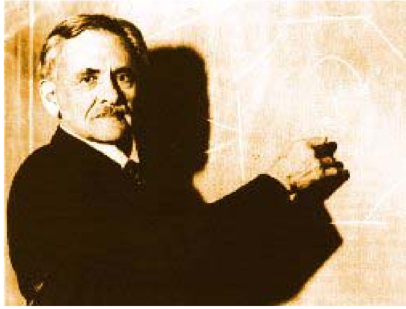
Detection of the Ether

- If one assumed the existence of a unique ether frame, it seemed that as the earth orbits around the sun, it must be moving relative to the ether frame.
- Detect ether by measure the speed (relative to the earth) of light traveling in various directions.
- Hard experiment:

$$\begin{aligned} v_{\text{earth}} &\approx 3\times10^4\,\text{m/s} \\ c &= 3\times10^8\,\text{m/s} \end{aligned} \Rightarrow \frac{v_{\text{earth}}}{c} \approx 0.0001$$

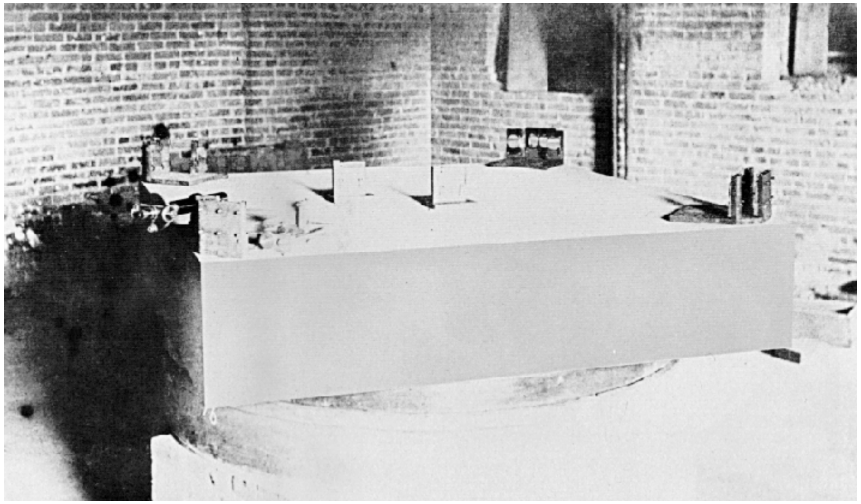
1 part in 10^4

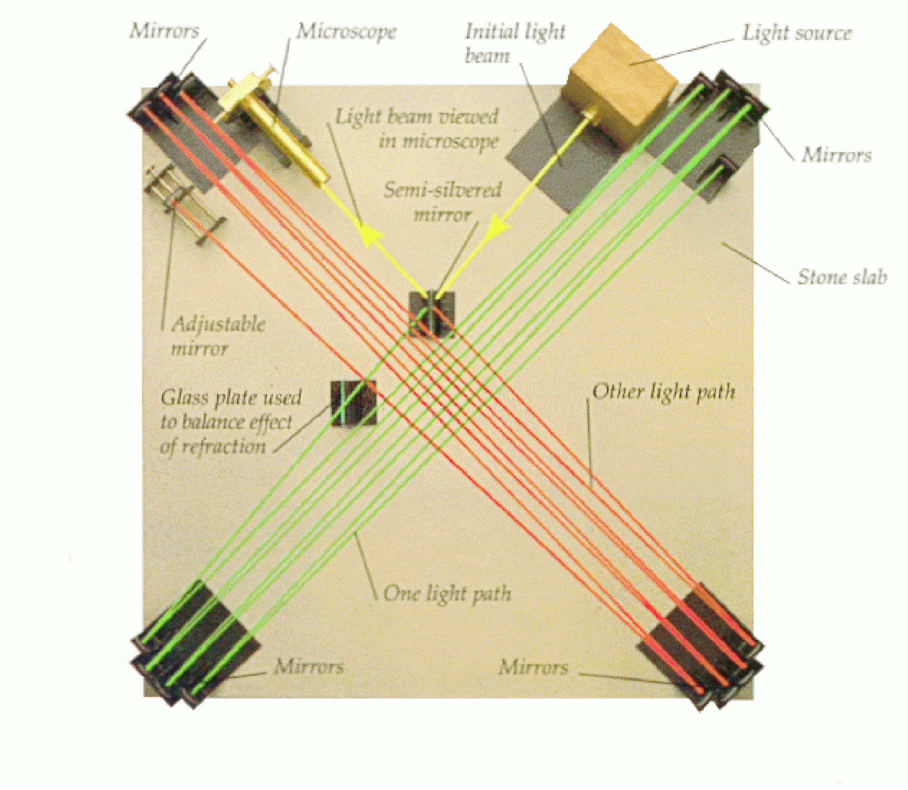
Michelson-Morley Experiment



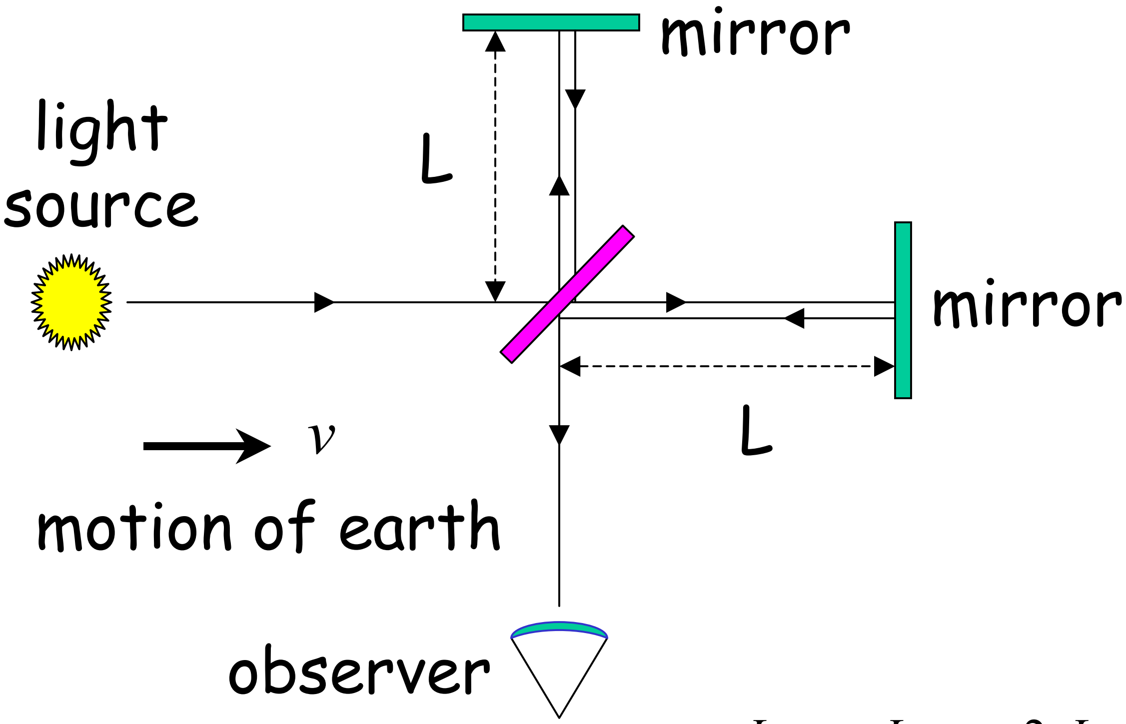
Conducted experiments from 1883 to 1887.

Result not consistent with the ether theory.





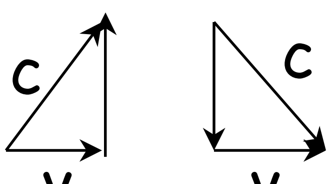
Simplified Description



t_1 time light travels parallel to motion of Earth

t_2 time light travels perpendicular to motion of Earth

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2cL}{c^2 - v^2} = \frac{2L}{c}\left[1 - \left(\frac{v}{c}\right)^2\right]^{-1} \approx \frac{2L}{c}\left[1 + \left(\frac{v}{c}\right)^2 + \cdots\right]$$



$$t_2 = \frac{2L}{\left(c^2 - v^2\right)^{1/2}} = \frac{2L}{c}\left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} \approx \frac{2L}{c}\left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \cdots\right]$$

Simplified Analysis

time difference $\Delta t = t_1 - t_2 \approx \frac{2L}{c}\left[1+\left(\frac{v}{c}\right)^2\right] - \frac{2L}{c}\left[1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right]$

$$= \frac{L}{c}\left(\frac{v}{c}\right)^2$$

Rotate apparatus by 90 degrees and observe shift in interference fringes.

$$\Delta t = -\frac{L}{c}\left(\frac{v}{c}\right)^2$$

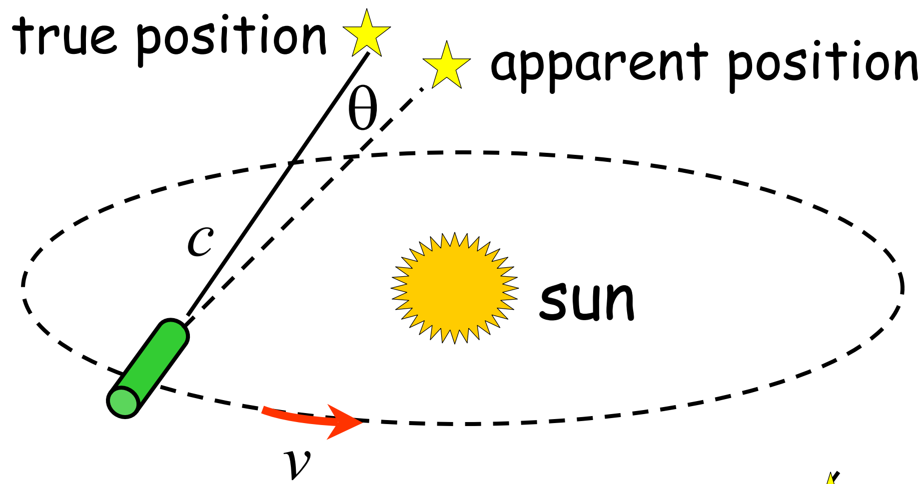
total shift $\Delta N = \frac{2c\Delta t}{\lambda} = \frac{2L}{\lambda}\left(\frac{v}{c}\right)^2$

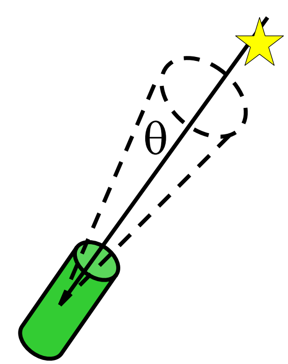
Expected shift was about 30 times the minimum observable shift.

No shift was observed.

Ether Drag Non-Explanation

- Could it be that the Earth drags the ether with it just as it does the atmosphere?
- Ruled out by aberration of light from distant stars.
  - If the light comes straight down while the telescope is moving with speed v, the telescope must be inclined at the vary small angle $\theta \sim \tan\theta = v/c$ so that the light will not hit the sides of the telescope.
  - If the ether is dragged with the telescope, the telescope should point vertically
  - Experiment shows that the telescope must be inclined at this small angle.





Module 2

Einstein's Special Theory

Einstein's Special Theory

- 26-year old paten clerk.
- Published in 1905.
- Special Relativity primary focus is restricted to un-accelerated frames of reference and excludes gravity.
- General Relativity includes accelerated frames of reference and gravity.

Theories of Relativity

- The special theory of relativity is special in the sense that it restricts our considerations to frames with no acceleration.
- The more general case, in which accelerated motion is considered, is the subject of the general theory of relativity.

Postulates of Special Relativity

1) **The Relativity Postulate**: the laws of physics are the same in every inertial reference frame.
2) **The Speed Of Light Postulate**: the speed of light in a vacuum, c, is the same in all inertial reference frames, independent of the motion of the source or the receiver.

The Postulates

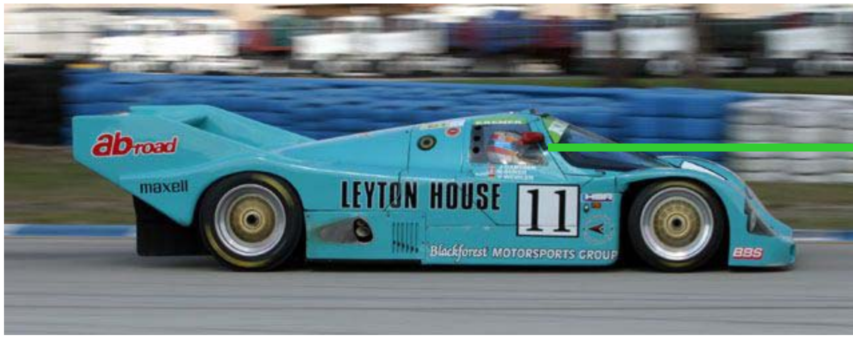
- Postulate 1: This is an extension of the Newtonian principle of relativity to include all types of physical measurements (not just those that are mechanical).
- Postulate 2: Common property of all waves.

  For example, the speed of sound does not depend on the motion of the sound source. The frequency does however (Doppler effect). The speed of the wave depends only on the properties of the air, such as its temperature.

Consequences of Relativity Postulate

- Any inertial reference frame is as good as any other for expressing the laws of physics.
- There is no experiment that can distinguish between an inertial frame that is at rest and one that is moving at a constant velocity.
- It is not possible to single out one particular inertial reference frame as being at "absolute rest".
- Only the relative velocity between objects, not their absolute velocity, can be measure and is physically meaningful.

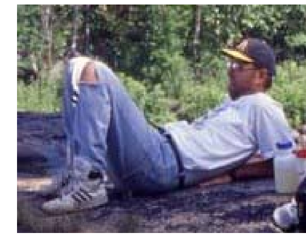
Speed of Light Postulate



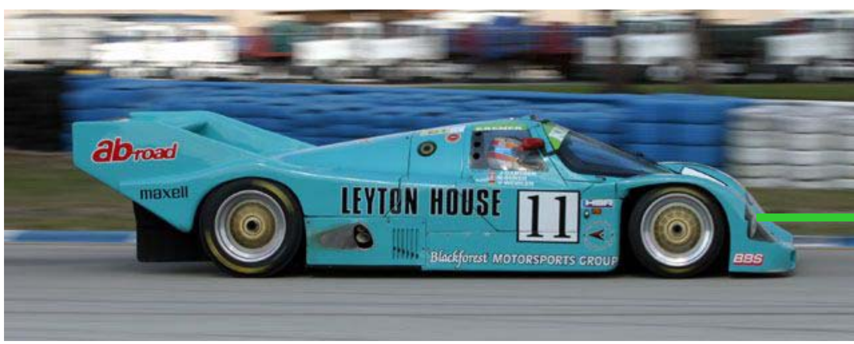
sound wave

343 m/s

15 m/s speed of car



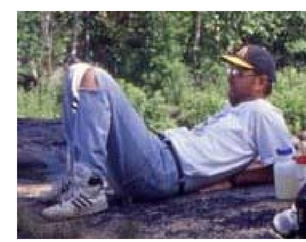
sound wave
343 m/s



light wave

3×10^8 m/s

15 m/s speed of car



light wave
3×10^8 m/s

Speed of Light and Everyday Life

- Each observer measures the same value c for the speed of light.
- The behaviour of space and time must differ from our everyday experience when speeds approach the speed of light.
- In everyday circumstances, however, the physics described by Newton's laws are perfectly adequate.
- The greatest speed a human might reasonably attain today is the speed of the space shuttle in orbit. The speed is about 7,700 m/s (28,000 km/hr). Although this is a rather large speed, it is still only 1/39,000th the speed of light.

Significance of the Postulates

- Each postulate seems quite reasonable.
- But many implications of the two together are quite surprising and contradict what is often called common sense.
- Rather than look for special explanations of the problems with absolute space, no ether, etc. Einstein used the empirical evidence to postulate a new fundamental principle of nature. He then used this principle to make predictions and thus modify all physics laws (expect for Maxwell's theory which needed no modification.)

Origins of General Relativity

- Special relativity does not provide an explanation nor a substitute for absolute space.
- General relativity sheds a little light on this.

Mach's Principle

- Space is not a "thing" in its own right; it is merely an abstraction from the totality of distance-relations between matter.
- A particle's inertia is due to some (unfortunately unspecified) interaction of that particle with all the other masses in the universe
- The local standards of nonacceleration are determined by some average of the motions of all the masses in the universe
- All that matters in mechanics is the relative motion of all the masses.

Consequences of Mach's Principle

- "… it does not mater if we think of the earth as turning around on its axis, or at rest while the fixed stars revolve around it…"
- A spinning elastic sphere bulges at its equator. How does the sphere "know" that it is spinning and must bulge?
  - Newton: it "felt" the action of absolute space.
  - Mach: it "felt" the action of the cosmic masses rotating around it.
- Centrifugal (inertial forces)
  - Newton: rotation with respect to absolute space (separate from gravity).
  - Mach: centrifugal forces are gravity, i.e. caused by action of mass upon mass.

Classical Newton Theory

- We must examine all aspects of classical Newton theory to check if it is consistent with the special relativity postulates.
- We will find that classical Newton theory for time, length, addition of velocities, momentum, and kinetic energy are only true at small speeds compared to that of light.
- Relativistic view of these concepts apply at all speeds between zero and the speed of light.

Modifications to Classical Concepts

- Kinematics
  - Time interval
  - Length
  - Addition of velocities
- Dynamics
  - Momentum
  - Energy
  - Optical
- Electromagnetic
- Hydrodynamic

Working Assumptions

- In each reference frame, we will assume that there are as many observers as are needed who are equipped with measuring devices, such as clocks and rulers, that are identical when compared at rest.
- We need many observers, for example, to determine the times of events. If one observer is distant from an event, then his time observations can be thrown off by the time it takes for the information about the event to travel to his location (such as the transit time of the light pulse).
- The observer can avoid such problems by recording only events *local* to him, and leaving others events to other observers at those locations.

Module 3

Clock Synchronization and Simultaneity

Synchronization

- The proper time is the time interval between two events that occur at the same point in some reference frame.
  - It can therefore be measured on a single clock.
- In another reference frame moving relative to the first, the same two events occur at different places, so two clocks are needed to record the times.
- This procedure requires that the clock be synchronized.

Synchronous Clocks

- In order to have a time scale valid for an entire frame of reference, we must have a number of clocks distributed throughout the frame so that there will everywhere be a nearby clock which can be used to measure time in its vicinity.
- These clocks my be synchronized.

Asynchronous Clocks

- If two clocks are synchronized in the frame in which they are at rest, they will be out of synchronization in another frame.
- In the frame in which they are moving, the chasing clock leads (shows a later time) by an amount

$$\Delta t_S = L_P \frac{v}{c^2}$$

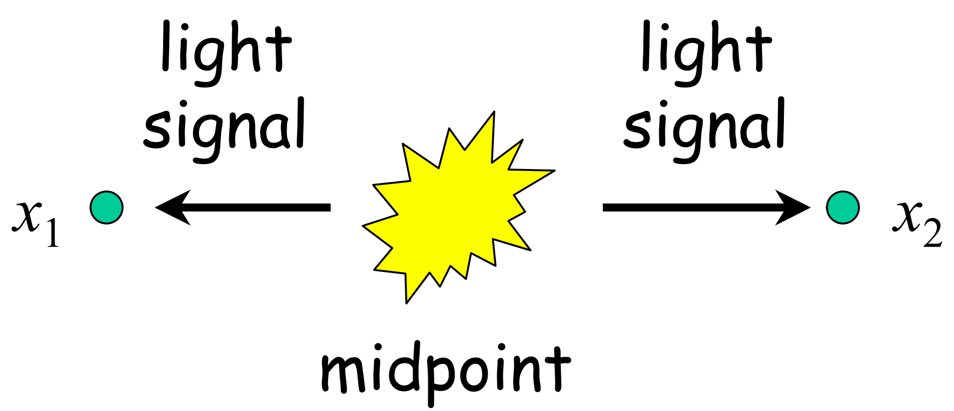
where L_P is the proper distance between the clocks.

Simultaneity

- Two events that take place at the same instant of time, though not necessarily at the same point in space, are said to occur simultaneously.
- *Common sense* tells us that if two events occur simultaneously according to one person, these events must occur simultaneously from the point of view of all observers. **Wrong.**
- In special relativity, simultaneity does not have an absolute meaning independent of the spatial coordinates, as it does in the classical theory.

Definition of Simultaneity

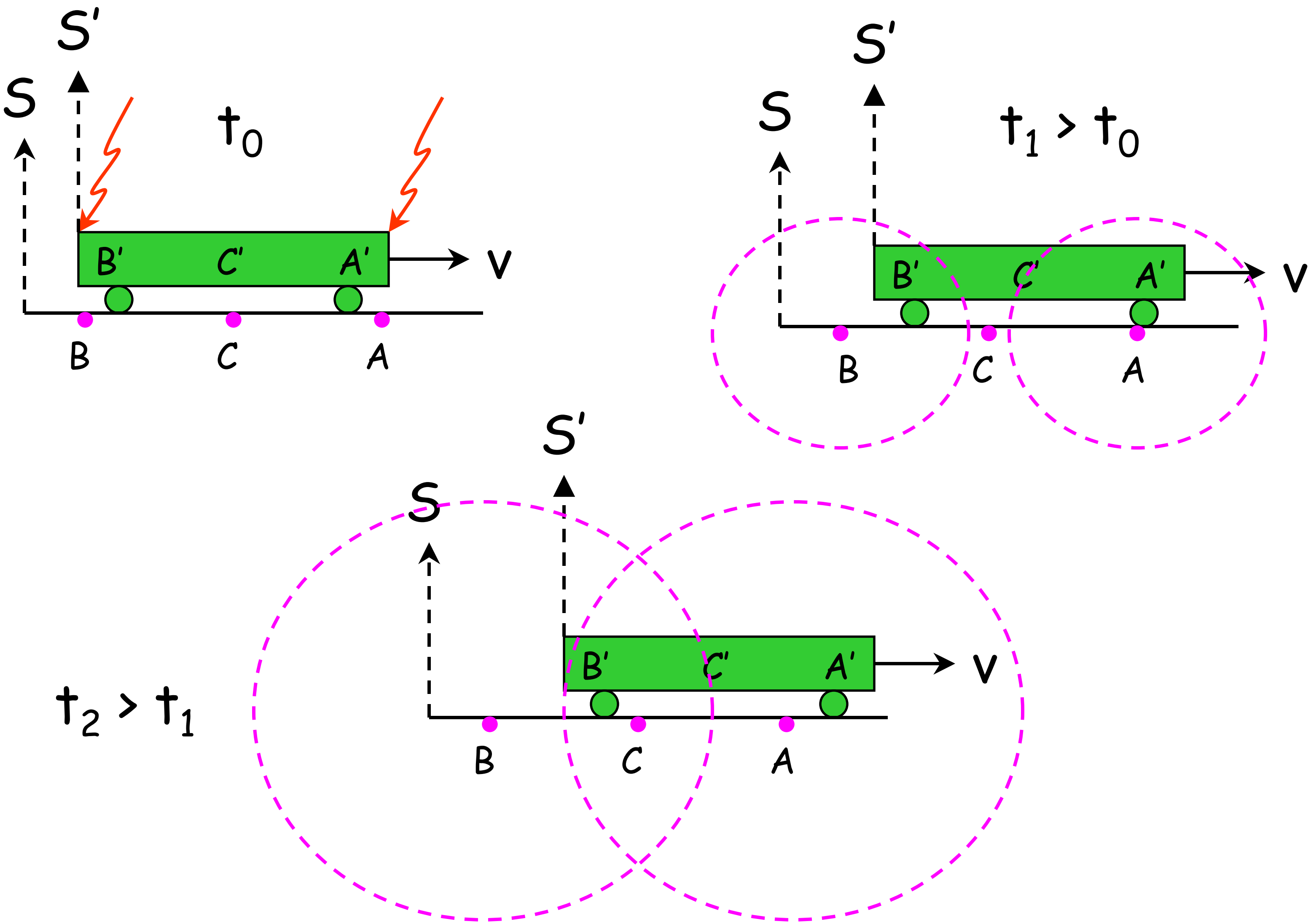
- Two instants of time t_1 and t_2, observed at two points x_1 and x_2 in a particular frame, are simultaneous if light signals simultaneously emitted from the geometrically measured midpoint between x_1 and x_2 arrive at x_1 at t_1 and at x_2 at t_2.



Simultaneity in Relativity

- t_1 and t_2 are simultaneous if light signals emitted at t_1 from x_1 and at t_2 from x_2 arrive at the midpoint simultaneously.
- Two events which are simultaneous when viewed from one frame of reference are in general not simultaneous when viewed from a second frame which is moving relative to the first.

Lightning Bolt Train Example



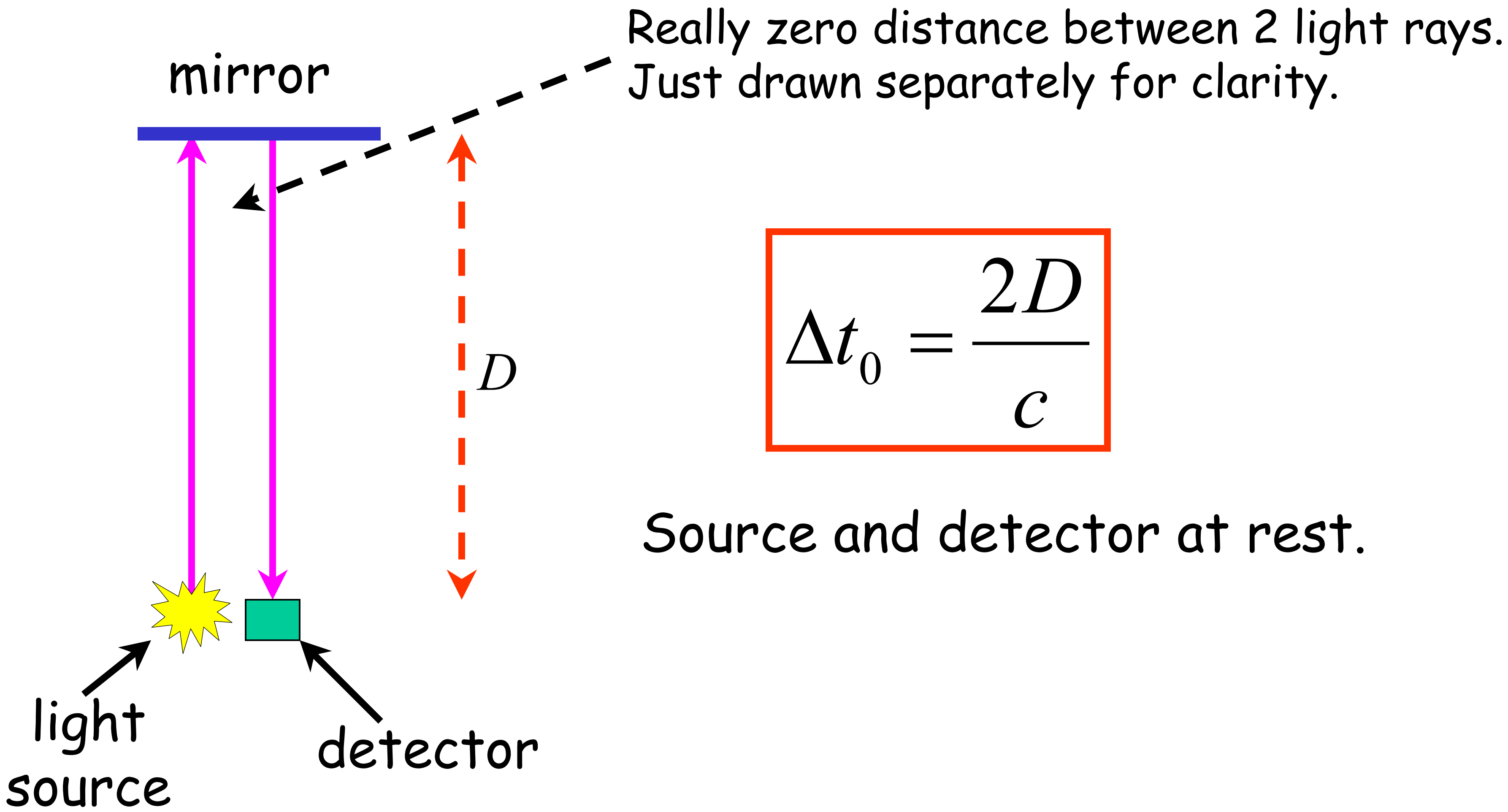
Module 4

Time Dilation

Relativity of Time: Time Dilation

- To dilate means to expand.
- We experience time to be the same regardless of how we move.
- Time dilation arguments apply to any temporal change or process: subatomic particle decay or even human lifetimes.

Proper Time

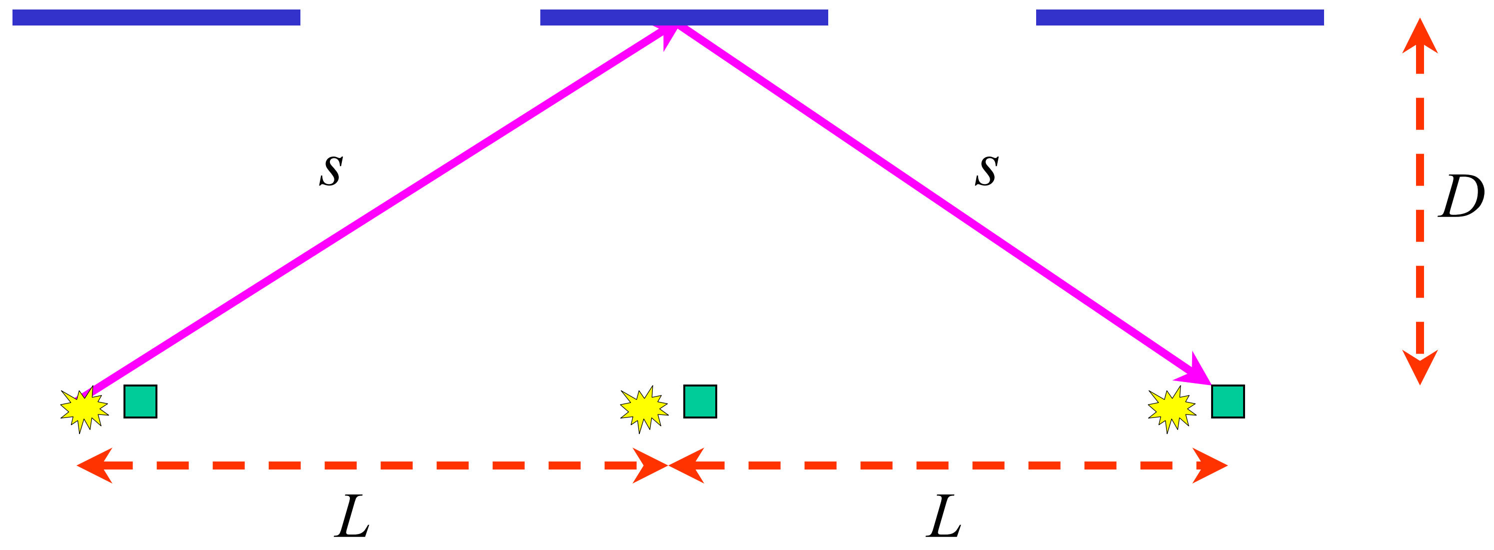


$$\Delta t_0 = \frac{2D}{c}$$

Source and detector at rest.

Time Measured by Observer on Earth

Source and detector moving to right with speed v.



$$2s = 2\sqrt{D^2 + L^2} = 2\sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2} = c\Delta t$$

$$\therefore \quad c\Delta t = 2\sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2} \quad \Rightarrow \quad \boxed{\Delta t = \frac{2D}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Time Dilation

$$\Delta t_0 = \frac{2D}{c}$$

$$\Delta t = \frac{2D}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Δt_0 = proper time interval between two events, as measured by an observer who is at rest with respect to the events and who views them as occurring at the same place.

Δt = time interval measured be an observer who is in motion with respect to the events and who views them as occurring at different places.

v = relative speed between the two observers.

c = speed of light in vacuum.

Time Longer in Moving Frame

$$v < c$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} < 1$$

$$\boxed{\Delta t > \Delta t_0}$$

The proper time interval is always shorter than the dilated time interval.

A moving clock is observed to run slow.

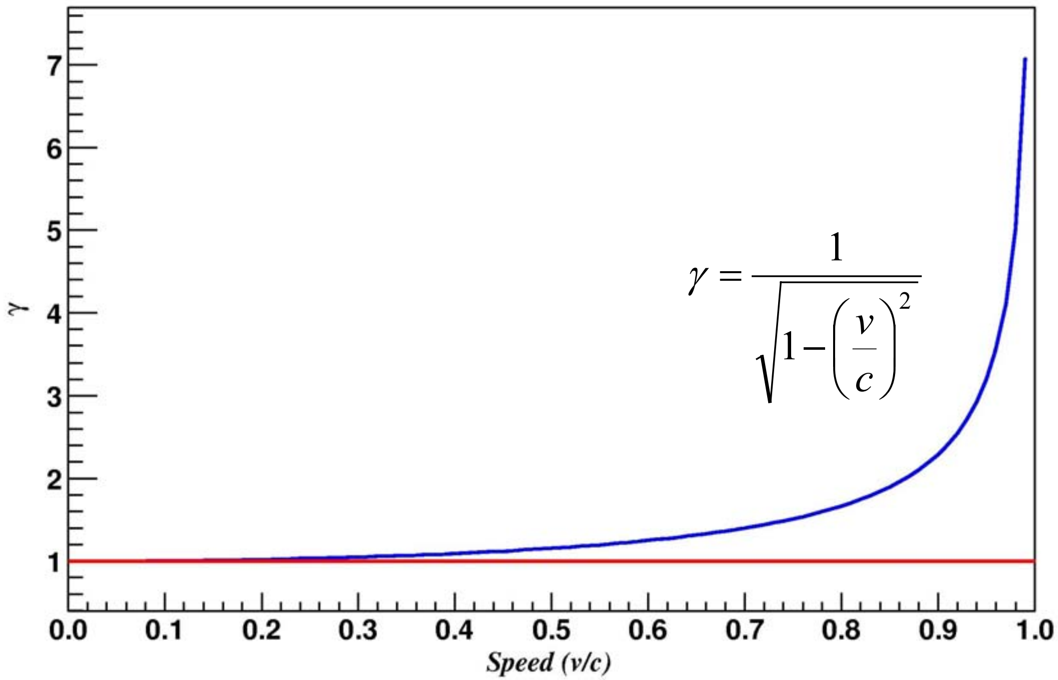
Some Useful Definitions

$$\boxed{\beta \equiv \frac{v}{c}} \quad \Rightarrow \quad 0 \le \beta \le 1 \ \text{since} \ 0 \le v \le c$$

$$\boxed{\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\beta^2}} = \left(1-\beta^2\right)^{-1/2} \quad \Rightarrow \quad \gamma \ge 1$$

Therefore we can write $\Delta t = \gamma \Delta t_0$

Classical Approximation



Binomial expansion

$$(1-x)^n = 1 - nx + n(n-1)\frac{x^2}{2} + \cdots$$

In our case $x = \frac{v^2}{c^2}$

and $n = -\frac{1}{2}$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\left(\frac{v^2}{c^2}\right)^2 + \cdots$$

Often a typical calculator does not have enough decimal places to give the correct answer. To find the correct answer, we must use the binomial expansion.

Example: Time Dilation

Spacecraft moving past the earth at a constant speed v that is 0.92 times the speed of light.

$$v = (0.92)\times(3.0\times10^{8}\,\text{m/s}) = 0.92c$$

The astronaut measures a time interval to be $\Delta t_0 = 1.0\ \text{s}$

What is the time interval Δt on earth?

$$\Delta t = \gamma\Delta t_0 = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.0\ \text{s}}{\sqrt{1-\left(\frac{0.92c}{c}\right)^2}} = 2.6\ \text{s}$$

Proper Time Interval

- The proper time interval Δt_0 between two events is the time interval measured by an observer who is at rest relative to the events and sees them at the same location in space.
- Working situation:
  - First identify the two events that define the time interval.
  - Then determine the reference frame in which the two events occur at the same place
  - An observer at rest in this reference frame measures the proper time interval Δt_0.

Example: Space Travel

Alpha Centauri is 4.3 light-years away.
A rocket leave for Alpha Centauri at a speed $v = 0.95c$ relative to the earth.

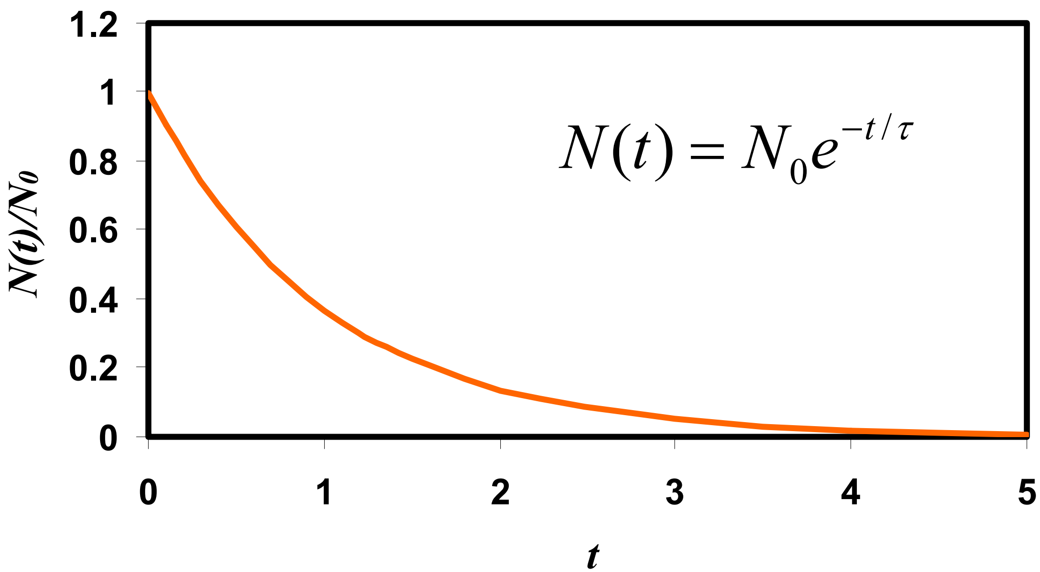
By how much will the passengers have aged, according to their own clock, when they reach their destination?

Time on earth $$\Delta t = \frac{4.3\ \text{ly}}{0.95c} = \frac{4.3c\ \text{yr}}{0.95\text{c}} = \frac{4.3\ \text{yr}}{0.95} = 4.5\ \text{yr}$$

Time according to passengers $$\Delta t_0 = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1-\beta^2} = (4.5\ \text{yr}) \times \sqrt{1-0.95^2} = 1.4\ \text{yr}$$

Muons and Cosmic Rays

- Muons are secondary radiation from cosmic rays.
- Muons are created from the decay of pions high in the atmosphere.
- It is easy to distinguish experimentally between the classical and relativistic predictions of the observation of muons at sea level.
- Muons decay according to the statistical law of radioactivity



$$N(t) = N_0 e^{-t/\tau}$$

where N_0 is the original number of muons at time $t = 0$, $N(t)$ is the number remaining at time t, and τ is the mean lifetime.

Example: Lifetime of a Muon

Average lifetime of a muon at rest is 2.2×10^{-6} s.
A muon is created in the upper atmosphere, thousands of meters above sea level.
A muon travels towards the earth at a speed of $v = 0.998c$.

a) How long does a muon live according to an observer on earth?

a) Lifetime according to observer on earth

$$\Delta t = \gamma \Delta t_0 = \frac{\Delta t_0}{\sqrt{1-\beta^2}} = \frac{2.2 \times 10^{-6}\,\mathrm{s}}{\sqrt{1-0.998^2}} = 3.5 \times 10^{-6}\,\mathrm{s}$$

Distance Traveled by Muon

b) How far does the muon travels before decaying?

b) Distance according to observer on earth

$$l = v\Delta t = (0.998)\times(3.00\times10^{8}\,\text{m/s})\times(3.5\times10^{-6}\,\text{s}) = 10{,}000\,\text{m}$$

If its lifetime were only 2.2×10^{-6} s on earth, a muon would travel only 660 m before disintegrating and could never reach the earth.

Experimental Evidence

- Hafele and Keating conducted a test in 1971 by placing an atomic clock on board a jet airplane and leaving an identical clock at rest in the laboratory. They found it to have run slower than the clock left in the lab. The discrepancy in time agreed with the predictions of relativity.

Module 5

Length Contraction

Relativistic Length: Length Contraction

- Since time is different for two observers, is distance different for two observers?
- The answer is yes, since distance is speed divided by time.
- Since the speed is a constant, both the time and distances must be different.

Definition of Velocity

$$v = \frac{L}{\Delta t_0}$$

Observer in rocket

$$v = \frac{L_0}{\Delta t}$$

Observer on earth

$$v = \frac{L}{\Delta t_0} = \frac{L_0}{\Delta t} = \frac{L_0}{\gamma \Delta t_0} \quad \Rightarrow \quad L = \frac{L_0}{\gamma}$$

Length contraction occurs only along the direction of the motion.
Those dimensions that are perpendicular to the motion are not shortened.

Length Contraction

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

L_0 = proper length (distance between two points that are at rest relative to the observers).

L = time interval measured by an observer who is in motion with respect to the events and who view them as occurring at different places.

v = relative speed between the two observers.

c = speed of light in vacuum.

Length Short in Moving Frame

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

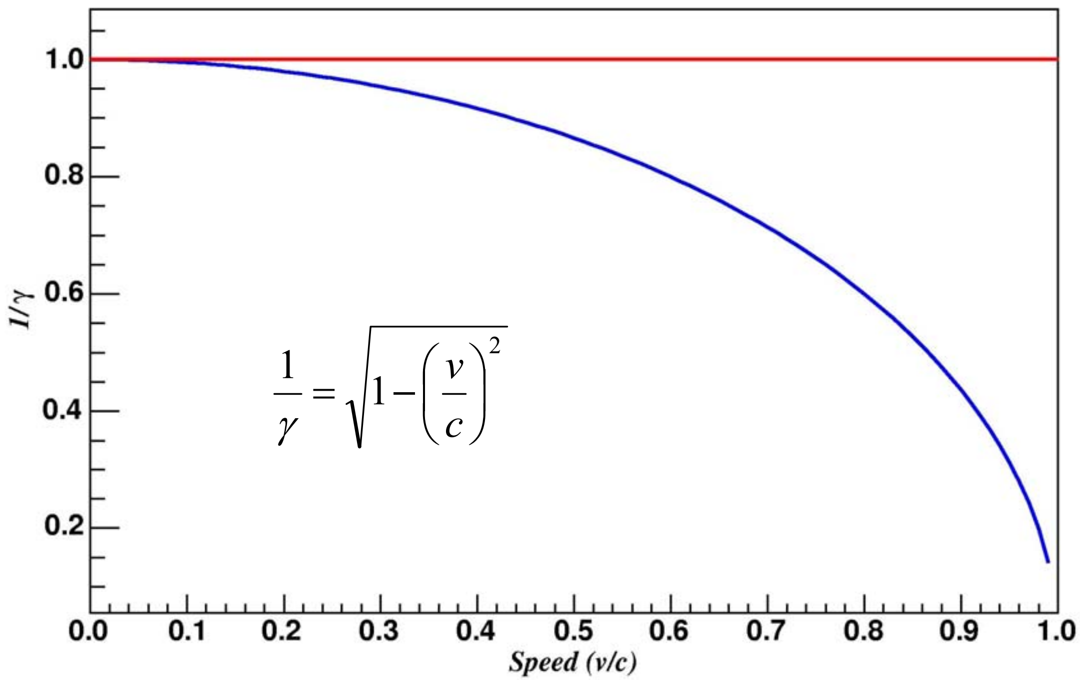
$$v < c$$

$$\sqrt{1 - \frac{v^2}{c^2}} < 1$$

$$L < L_0$$

The proper length L_0 is always larger than the contracted length L.

Classical Approximation



Binomial expansion

$$(1-x)^n = 1 - nx + n(n-1)\frac{x^2}{2} + \cdots$$

In our case $x = \frac{v^2}{c^2}$

and $n = \frac{1}{2}$

$$\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}} = 1 - \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{8}\left(\frac{v^2}{c^2}\right)^2 - \cdots$$

Often a typical calculator does not have enough decimal places to give the correct answer. To find the correct answer, we must use the binomial expansion.

Example: Contraction of a Spacecraft

Astronaut measures the length of the spacecraft to be 82 m. The spacecraft moves with a constant speed of $v = 0.95c$ relative to the earth.

What is the length of the spacecraft as measured by an observer on earth?

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1-\frac{v^2}{c^2}} = (82\text{ m})\times\sqrt{1-\left(\frac{0.95c}{c}\right)^2} = 26\text{ m}$$

Example: The Half Meter Stick

Find the speed for which the length of a meterstick is 0.500 m?

$L = 0.500$ m

The meterstick at rest has its proper length $L_0 = 1.00$ m

$$L = \frac{L_0}{\gamma} \Rightarrow \frac{L}{L_0} = \frac{1}{\gamma} = \frac{1}{2}$$

$$\gamma = 2 \Rightarrow \beta = \frac{\sqrt{3}}{2} \Rightarrow v = \frac{\sqrt{3}}{2} c \approx 0.866c$$

Proper Length

- The proper length L_0 of an object is the length measured by an observer who is at rest with respect to the object.
- "Proper" does not mean correct or preferred.

Distance Traveled by Muon Revisited

- The lifetime of a muon at rest is 2.2×10^{-6}s.
- A muon travels towards the earth at a speed of $v = 0.998c$.
- From previous time dilation example:
  - Lifetime of muon according to observer on earth 3.5×10^{-6} s.
  - Distance traveled according to observer on earth 10,000 m.
- According to a muon it travels a distance

$$l = v\Delta t_0 = 0.998 \times (3.0 \times 10^{8}) \times (2.2 \times 10^{-6}) = 660 \text{ m}$$

This is the length-contracted distance.

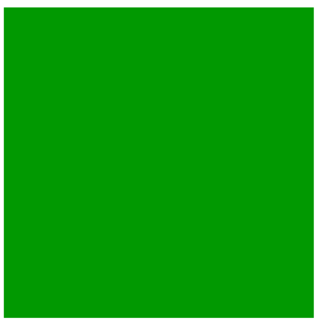
The distance in the rest frame (earth) is

$$l_0 = \gamma l = \frac{l}{\sqrt{1-\beta^2}} = \frac{660}{\sqrt{1-0.998^2}} = 10{,}000 \text{ m}$$

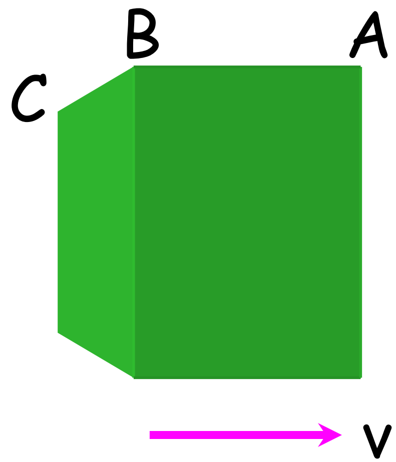
Assorted Observations

- Lengths perpendicular to the relative motion are unchanged.
- The length contraction formula applies only to the length parallel to the relative motion.

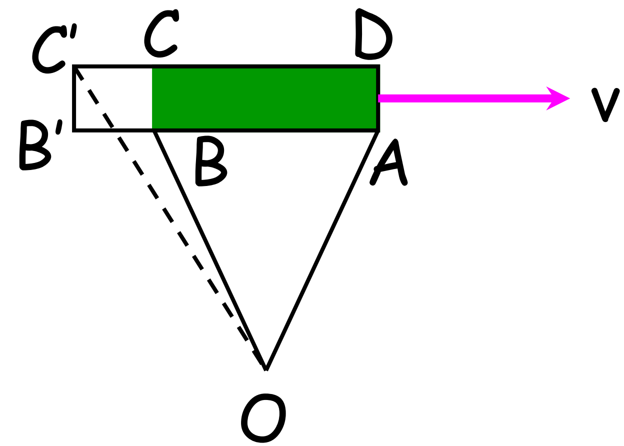
Appearance of Objects



seen at rest



seen moving at high speed



top view

The building is moving at a high speed and gets out of the way so that light at C can reach O.

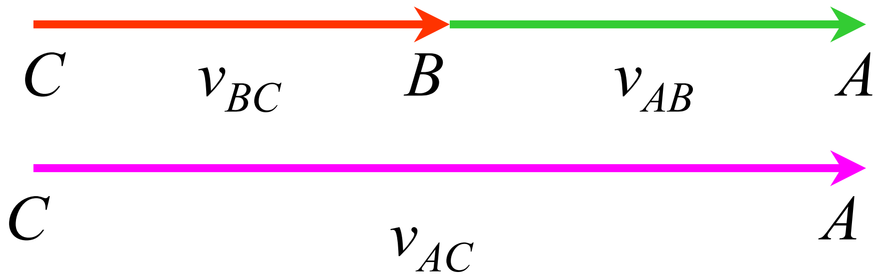
Also, the sides would appear curved because of differing distances from the observer's eye of the various points from top to bottom along a vertical side.

Spherical objects are not distorted.

Module 6

Velocity Addition

Classical Addition of Velocities



$\vec{v}_{AB}$ velocity of object A with respect to B

$\vec{v}_{BC}$ velocity of object B with respect to C

$\vec{v}_{AC}$ velocity of object A with respect to C

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

Relativistic Addition of Velocities

- If the velocity is sufficiently close to the speed of light, the classical addition of velocities would predict a velocity faster than the speed of light.
- The velocity-addition formula can be used only when the velocities are measured relative to different inertial reference frames.

relativistic velocity addition

$$\vec{v}_{AC} = \frac{\vec{v}_{AB} + \vec{v}_{BC}}{1 + \frac{\vec{v}_{AB} \cdot \vec{v}_{BC}}{c^2}}$$

Nonrelativistic Limit

$$\vec{v}_{AB} << c \text{ and } \vec{v}_{BC} << c$$

$$\vec{v}_{AC} \rightarrow \vec{v}_{AB} + \vec{v}_{BC}$$

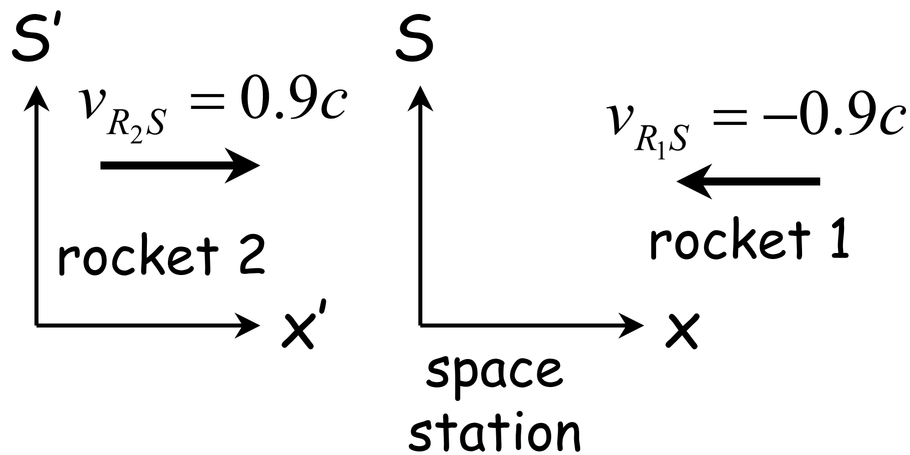
Example: Galilean and Lorentz

Two rocket ships are approaching a space station, each at a speed of $0.9c$, with respect to the stations, as shown.

What is their relative speed according to

A) The Galilean transformation?

B) The Lorentz transformation?



Galilean

$$v_{R_1R_2} = v_{R_1S} + v_{SR_2} = v_{R_1S} - v_{R_2S} = -0.9c - 0.9c = -1.8c$$

Lorentz

$$v_{R_1R_2} = \frac{v_{R_1S} + v_{SR_2}}{1 + v_{R_1S} v_{SR_2} / c^2} = \frac{v_{R_1S} - v_{R_2S}}{1 - v_{R_1S} v_{R_2S} / c^2} = \frac{-0.9c - 0.9c}{1 - (-0.9)(0.9)} = -0.994c$$

Addition with the Speed of Light

If $v_{AB} = c$

$$v_{AC} = \frac{c + v_{BC}}{1 + \frac{c v_{BC}}{c^2}} = \frac{c + v_{BC}}{1 + \frac{v_{BC}}{c}} + \frac{c(c + v_{BC})}{(c + v_{BC})} = c$$

Both observers measure the same velocity for the light.

Consistent with the speed of light postulate.

Nothing can travel faster than the speed of light.

Global Positioning System (GPS)

- GPS relies on relativistic corrections for proper functioning and accuracy.
- With a GPS receiver you can determine your location on earth (latitude, longitude, and elevation) to within a meter and the exact time.
- GPS consists of 24 satellites each containing an atomic clock accurate to a few nanoseconds.
- Each satellite continuously broadcasts its position and time.
- The GPS receiver computes its position from this information.

GPS Required Corrections

- Corrections required to obtain 1 m accuracy.
  - The speed of light is a constant.
  - Index of refraction of the earth's atmosphere.
  - Use relativistic velocity addition formula, else position error would be about 100 m.
  - Correct for time dilation of moving satellite clocks.
  - Correct for gravitational redshift, else error of about 100 m.

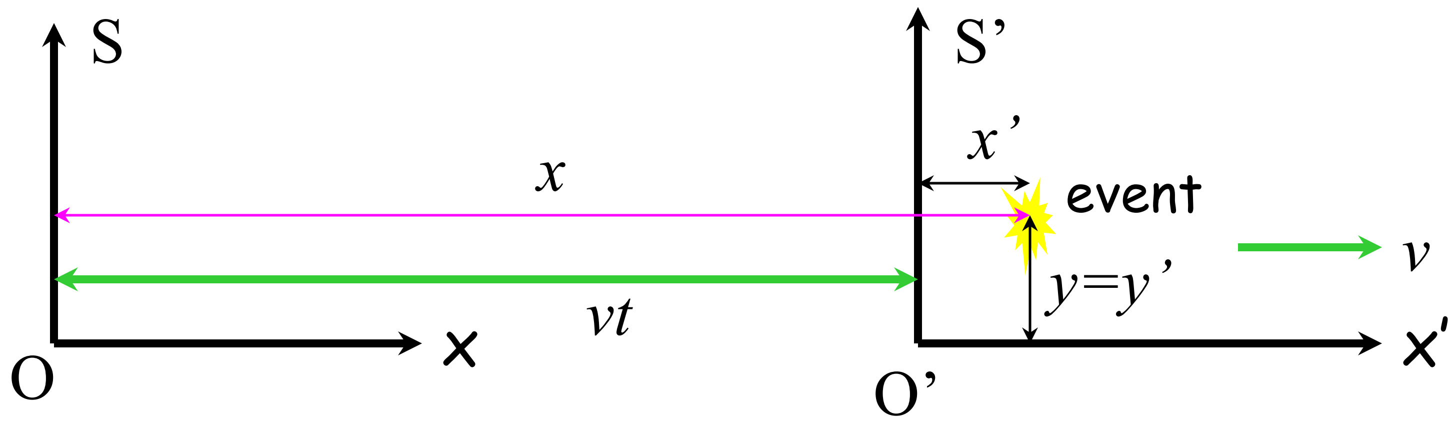
Module 7

Lorentz Transformation

Galilean Transformation Breakdown

- The Galilean transformation is not consistent with Einstein's second postulate of special relativity.
- If light moves along the x-axis with speed c in S, these equations imply that the speed in S' is $u_x' = c - v$; rather than $u_x' = c$.

Lorentz Transformation



This is the standard configuration.

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Galilean transformation

$$x' = \gamma (x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma \left(t - \frac{vx}{c^2} \right)$$

Lorentz transformation

Example: Transformation of Coordinates

A man on earth measures an event at a point 5.00 m from him at a time of 3.00 s.

If a rocket ship flies over the man at a speed of $0.800c$, what coordinates does the astronaut in the rocket ship attribute to this event?

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} = \frac{5.00\,\text{m} - (0.800)(3.00\times10^8\,\text{m/s})(3.00\,\text{s})}{\sqrt{1-(0.800c)^2/c^2}} = -1.20\times10^9\,\text{m}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}} = \frac{3.00\,\text{s} - (0.800)(3\times10^8\,\text{m/s})(5.00\,\text{m})/(3\times10^8\,\text{m/s})^2}{\sqrt{1-(0.800c)^2/c^2}} = 5.00\,\text{s}$$

More on the Lorentz Transformation

Lorentz transformation

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Inverse Lorentz transformation

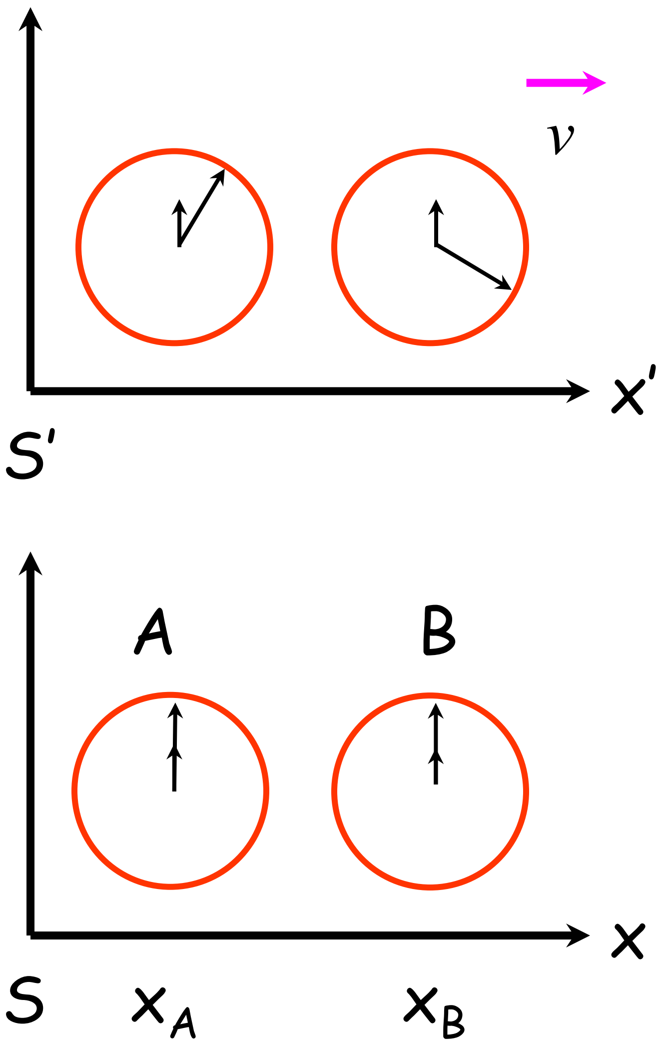
$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

limit of Lorentz transformation for small velocity

$$\frac{v}{c} \to 0 \Rightarrow \gamma \to 1 \Rightarrow \begin{cases} x' = x - vt \\ t' = t \end{cases}$$

Galilean transformation

Clock Synchronization Revisited



What is t_A on clock A and t_B on clock B as observed by S′ at time t'_0?

$$t'_0 = \gamma\left(t_A - \frac{vx_A}{c^2}\right)$$

$$t'_0 = \gamma\left(t_B - \frac{vx_B}{c^2}\right)$$

$$t_B - t_A = \frac{v}{c^2}(x_B - x_A) = L_0\frac{v}{c^2}$$

$$\text{where } L_0 = x_B - x_A$$

Lorentz transformation reproduces earlier result.

Time Dilation Revisited

Consider two events that occur at x'_0 at times t'_1 and t'_2 in frame S′. We can find the times t_1 and t_2 for these events in S using the Lorentz transformation

$$t_1 = \gamma\left(t_1' + \frac{vx_0'}{c^2}\right)$$

$$t_2 = \gamma\left(t_2' + \frac{vx_0'}{c^2}\right)$$

$$t_2 - t_1 = \gamma\left(t_2' - t_1'\right)$$

$$\Delta t = \gamma \Delta t_p$$

Δt_p is the proper time interval between events happening at the same place in a reference frame.

Length Contraction Revisited

Consider a rod at rest in frame S′ with one end at x'_1 and the other at x'_2. The length of the rod in S is defined as $L = x_2 - x_1$, where x_1 is the position at one end at some time t_1, and x_2 is the position at the same time $t_1 = t_2$ as measured in frame S.

$$x'_2 = \gamma(x_2 - vt_2)$$
$$x'_1 = \gamma(x_1 - vt_1)$$

Since $t_2 = t_1$ we obtain

$$x_2 - x_1 = \frac{1}{\gamma}(x'_2 - x'_1)$$

$$L = \frac{1}{\gamma} L_p$$

L_p is the proper length.

Matrix Representation

Lorentz transformation

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\frac{v\gamma}{c^2} & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

Hyperbolic Representation

Recall the definitions

$$\cosh\theta = \cos i\theta$$
$$i\sinh\theta = \sin i\theta$$

If we define θ such that

$$\cosh\theta = \gamma$$
$$\sinh\theta = \beta\gamma$$
$$\tanh\theta = \beta$$

The Lorentz transformation can be written as

$$x' = x\cosh\theta - ct\sinh\theta$$
$$ct' = -x\sinh\theta + ct\cosh\theta$$

We recall

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

The Lorentz transformation is reminiscent of a "rotation", but in x and ct.

It is formally a rotation in x and ict, by an angle $i\theta$.

Module 8

Spacetime

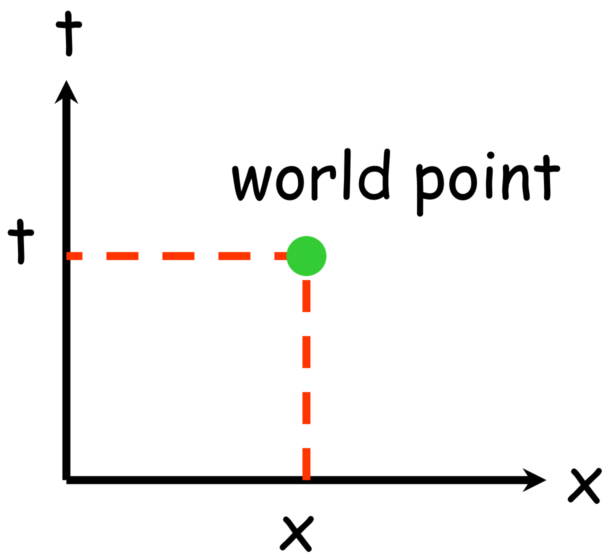
Spacetime

- Time and space are inseparable.
- "Nobody has ever noticed a place except at a time, or a time except at a place."
- A point in space at a point in time (x,y,z,t) is called a world point.
- All (x,y,z,t) systems of values is the world.
- Minkowski taught us to think of the totality of events in the world as the "points" of an absolute four-dimensional manifold called spacetime.
- Any occurrence in spacetime will be called an event. The location of this event is a world point.

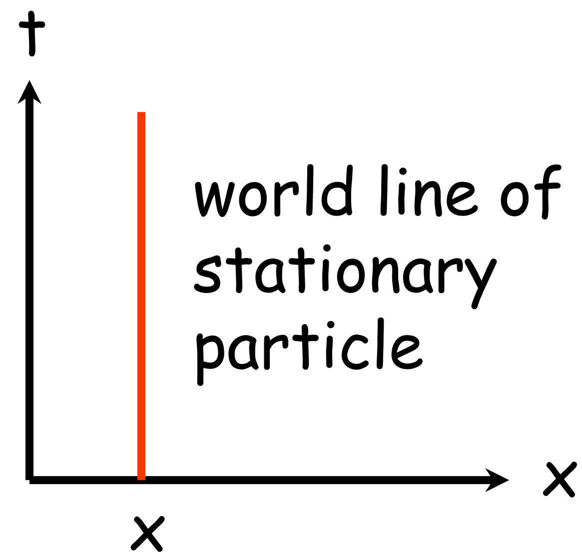
Spacetime Diagrams

- Certain lines in spacetime correspond to the history of material particles and are called particle world lines.
- To simplify the diagrams we consider only one space dimension, the x-coordinate.
- Usual to draw x-axis and t-axis (corresponding to frame S) orthogonal.
- History (or a world line) of each *fixed* point on the spatial x-axis of S corresponds to a vertical line, $x =$ constant.
- "Moments" in S have $t =$ constant (horizontal lines).

Some Spacetime Diagrams 1



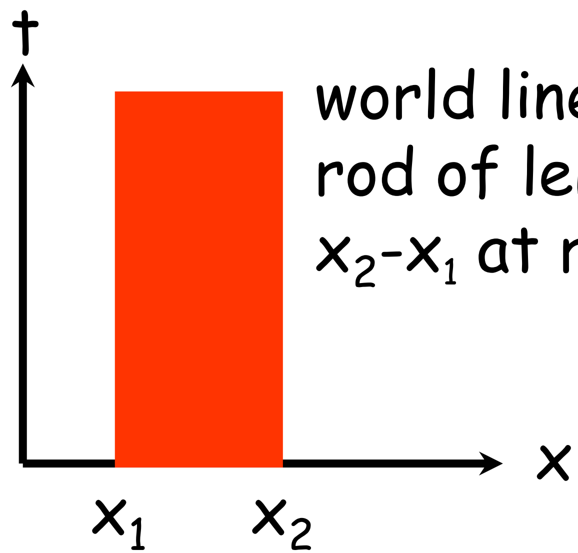
event in spacetime



Even though the particle is at rest in space, it is still moving through time.

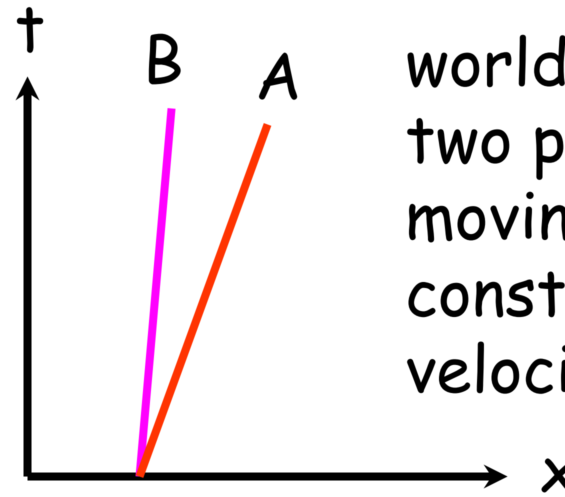
Extended bodies have "worldtubes" in spacetime.

Some Spacetime Diagrams 2



world line of rod of length $x_2 - x_1$ at rest

rod sweeps out an area

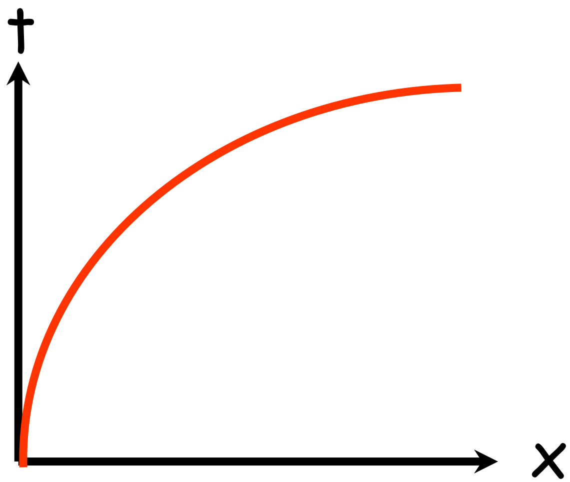


world lines of two particle moving with constant velocity

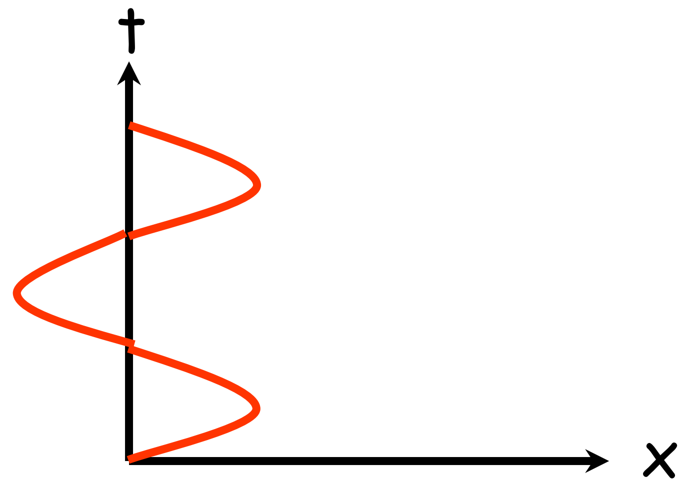
straight lines with smaller slope have greater velocity

$$v_A > v_B$$

Curved Spacetime World Lines



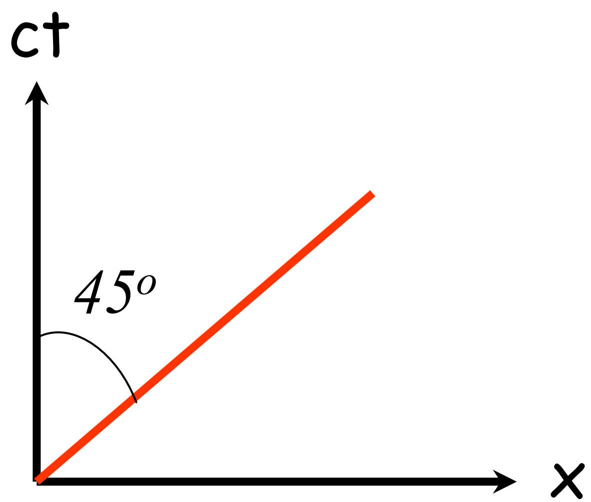
world line of accelerating particle is curved



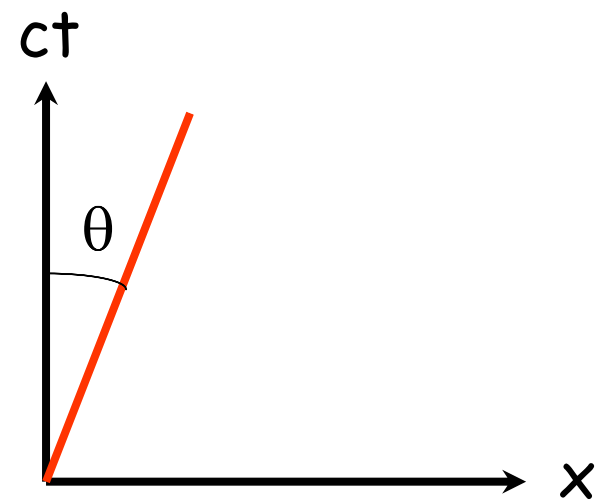
world line of particle executing simple harmonic motion

Straight Spacetime World Lines

It is convenient to make both axis have the same dimension by changing the time axis to ct, rather than t.



world line of a particle (photon) moving with speed c

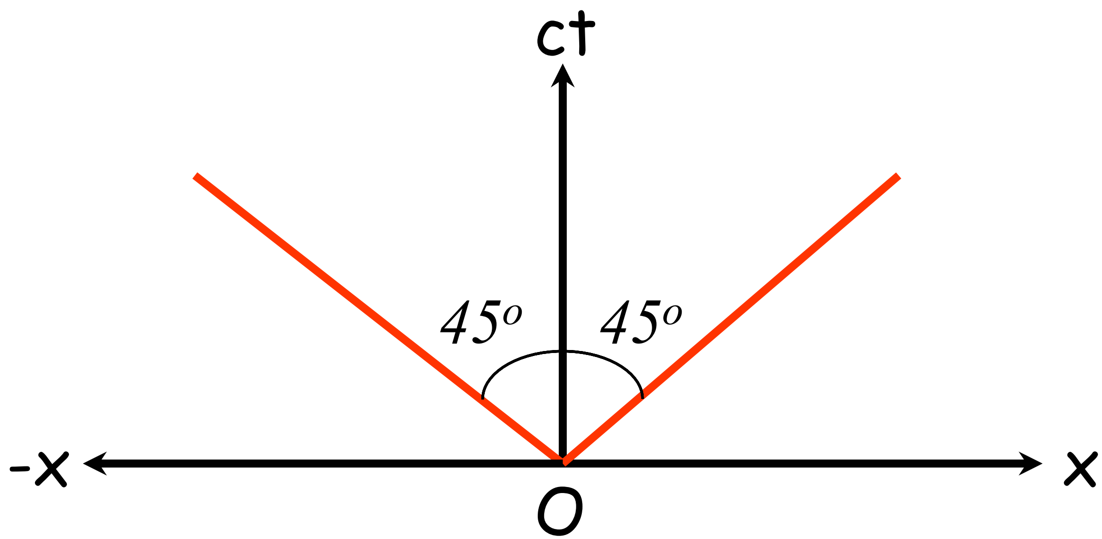


word line of a particle moving with speed $v < c$

$$\tan\theta = \frac{v}{c}$$

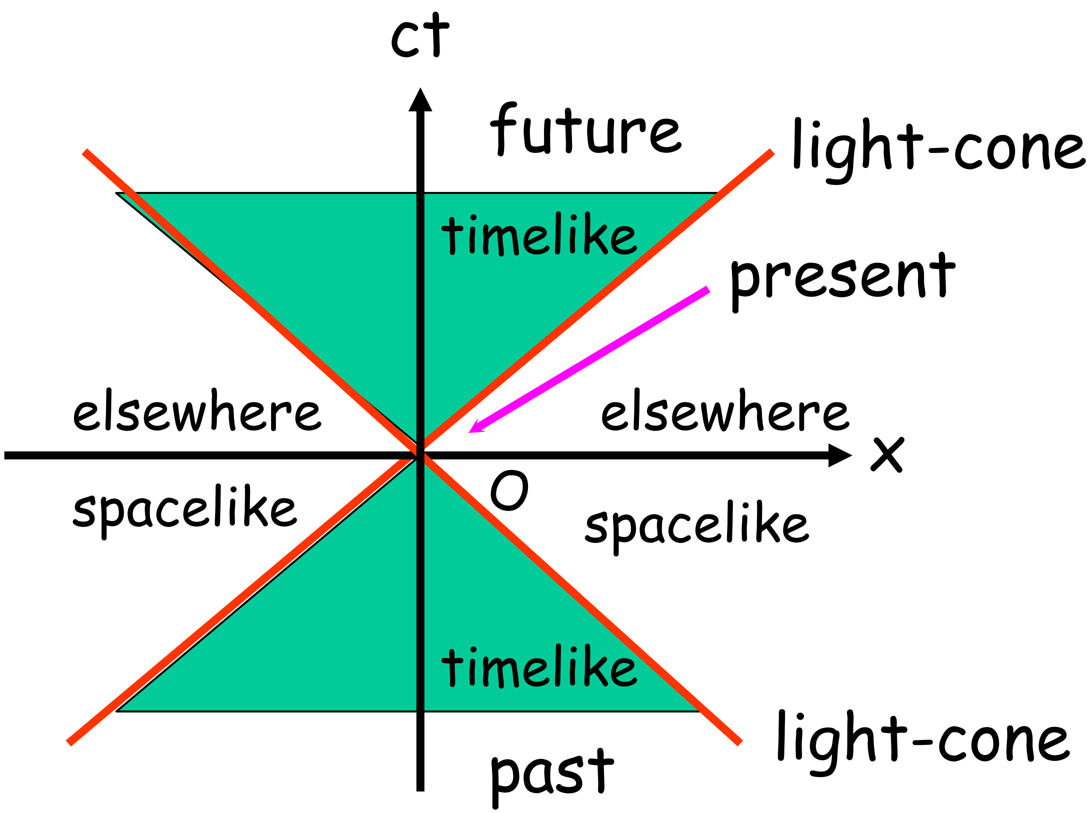
World Lines of Light

Light source at origin emits rays of light simultaneously towards the right and left.



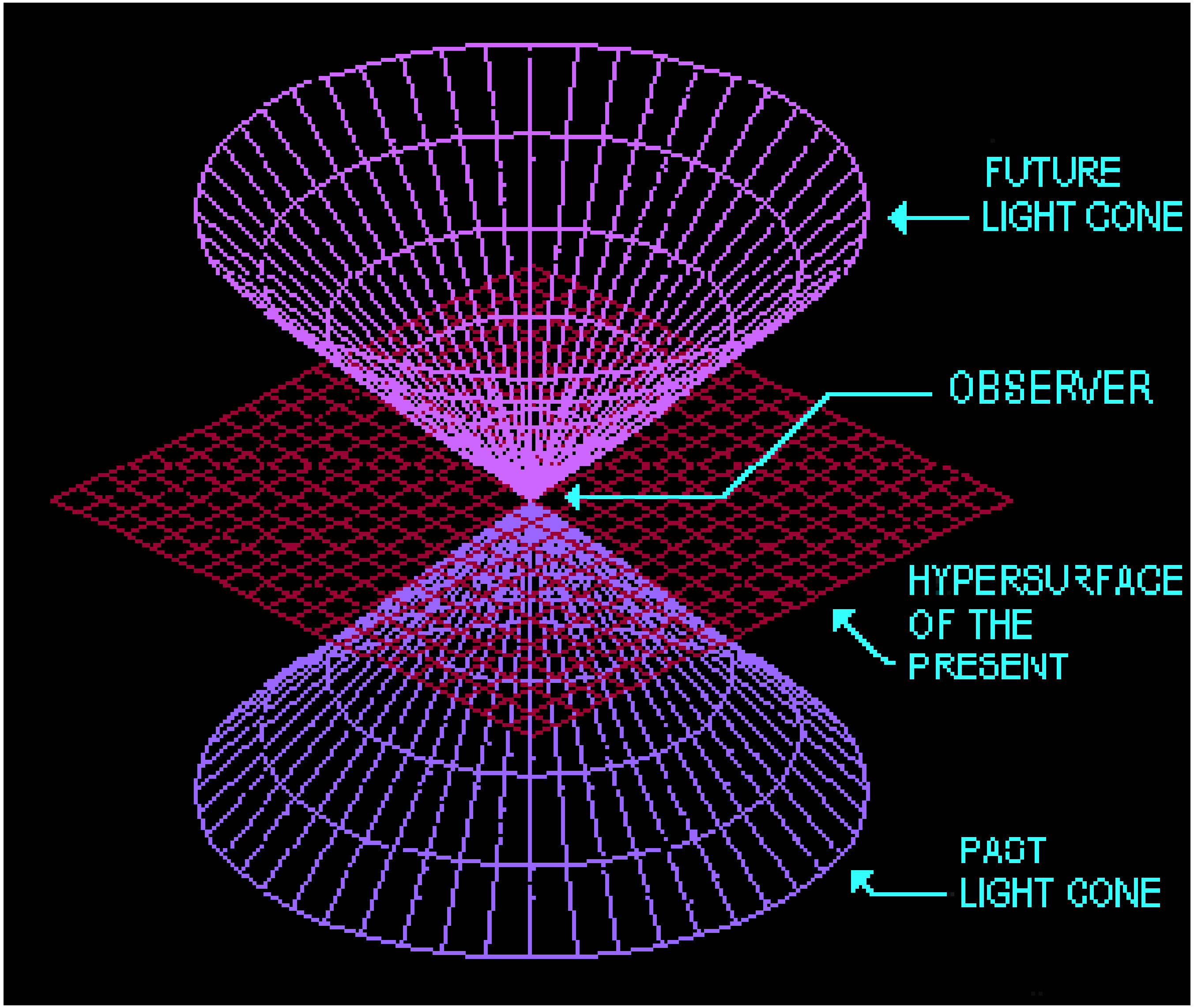
The world line of any particle situated at O must have a slope more than 45^o and be contained within the two light world lines.

Past, Present and Future



- Only world lines within the cone can have a cause and effect relationship on the particle or observer at O.
- World lines within the cone are called timelike because they are accessible to us in time.
- Events outside the cone are called spacelike because they occur in another part of space that is not accessible to use and hence is called elsewhere.

Light Cone in 3 Dimensions



Null Cone (Light Cone)

- For a fixed event O, the set of all events P such that $v = c$ is said to form the null cone (or light cone) at O.
- The half of the null cone generated by light signals emitted at O is called the future null cone at O.
- The half that is generated by light signals which reach O, is called the past null cone at O.
- The future of the event O is the set of all events on or within the future null cone, and the past of the event O is defined to be all the events lying on or inside the past null cone at O.
- O can influence any event in his future set and O can be influenced by any event in his past set.

Module 9

Invariants

Invariance

- It seem that everything is relative.
- In the varying world of spacetime is there anything that remains constant?
- Is there one single thing that all observers, regardless of their state of motion, can agree on?
- In the field of physics we always look for some characteristic constants of motion.
- In classical physics the total energy of an isolated system remains constant.
- In spacetime the constant value that all observers agree on, regardless of their state of motion, is called the invariant interval.

Space Rotation

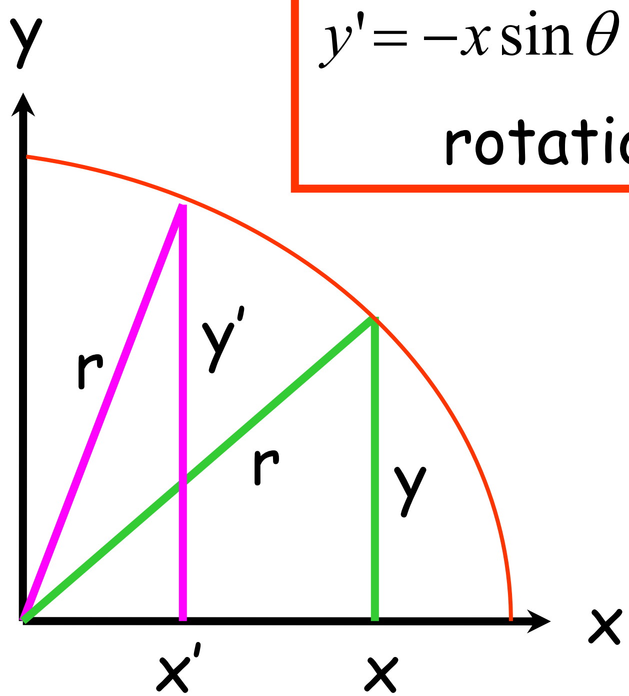
A rotation in the x-y plane leaves the distance from axis of rotation constant. The radius of the circle is fixed.

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

rotation

$$\sin^2\theta + \cos^2\theta = 1$$

tells use that



$$(x')^2 + (y')^2 = x^2 + y^2$$

is an invariant interval under a rotation in 3-space

$r^2 = x^2 + y^2$ is equation of circle.

Analogy with Rotation

$$x' = x\cos\theta + y\sin\theta$$
$$y' = y\cos\theta - x\sin\theta$$

rotation

$$x' = x\cosh\theta - ct\sinh\theta \qquad x' = \gamma(x - vt)$$
$$ct' = -x\sinh\theta + ct\cosh\theta \qquad t' = \gamma(t - vx/c^2)$$

Lorentz transformations

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cosh^2\theta - \sinh^2\theta = 1$$

The Lorentz transformation for x and ct is analogous in form to the rotation of x and y about the z-axis.

It is formally a rotation in x and ict, by an angle $i\theta$.

It preserves $x^2 + (ict)^2$ and we find $$(x')^2 - (ct')^2 = x^2 - (ct)^2$$

Lorentz transformation is a *rotation* in space and time.

Invariant Interval

In 3 space coordinates

$$(x')^2 + (y')^2 + (z')^2 - (ct')^2 = x^2 + y^2 + z^2 - (ct)^2$$

This quantity as measure by the S observer is equal to the same quantity as measured by the S′ observer.

This can only be true if each side of the equation is a constant.

Thus the quantity $s^2 = (ct)^2 - x^2 - y^2 - z^2$ is an invariant interval.

That is, it is the same in all inertial reference systems.

The invariant interval is thus a constant in spacetime.

The interval can be real or imaginary.

Not equation of circle but equation of hyperbola $s^2 = (ct)^2 - x^2$

Two-Dimensional Lorentz Geometry

Euclidean geometry

Continuous group (orthogonal group in 2 dimensions) of coordinate transformation

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

which leaves invariant the *positive definite* quadratic form

$$x^2 + y^2$$

The loci of all points Q having a fixed distance from a specified point P are circles centered on P.

Two-Dimensional Lorentz Geometry

Lorentz geometry (non-Euclidean)

Continuous group (Lorentz group in 2 dimensions) of coordinate transformation

$$x' = x\cosh\theta - ct\sinh\theta$$

$$ct' = -x\sinh\theta + ct\cosh\theta$$

which leave invariant the *indefinite* quadratic form $s^2 = (ct)^2 - x^2$

The loci of all points Q having a fixed spacetime separation from a specified event P fall into 3 classes depending on

$$s^2 < 0, \quad s^2 = 0, \quad s^2 > 0$$

We obtain a family of hyperbolae, together with their asymptotic lines (flat-hyperbolic geometry).

Spacetime Separation

- $s^2 > 0$, P ,Q have timelike separation $(ct)^2 > x^2$:
  - P, Q lie on the world line of some inertial observer.
  - All observers agree on their order in time.
  - $|s| = \tau =$ time elapsed between P, Q as measured by the inertial observer who experiences both events.
- $s^2 < 0$, P, Q have spacelike separation $x^2 > (ct)^2$:
  - There exists an inertial frame in which P, Q are simultaneous.
  - $|s| = \sigma =$ *distance between* P, Q as measured in the inertial frame. in which P, Q are simultaneous.
- $s^2 = 0$, P, Q have null (lightlike) separation:
  - P, Q lie on the world line of a light signal.

Module 10

Minkowski Diagrams

Geometrical Picture of Spacetime

- The invariant interval allows us to draw a good geometrical picture of spacetime.
- Different inertial observers draw different sections through spacetime as their "instants".

Two Views of a Transformation

- There are two ways of regarding any transformation of coordinates (x,ct) into (x',ct').

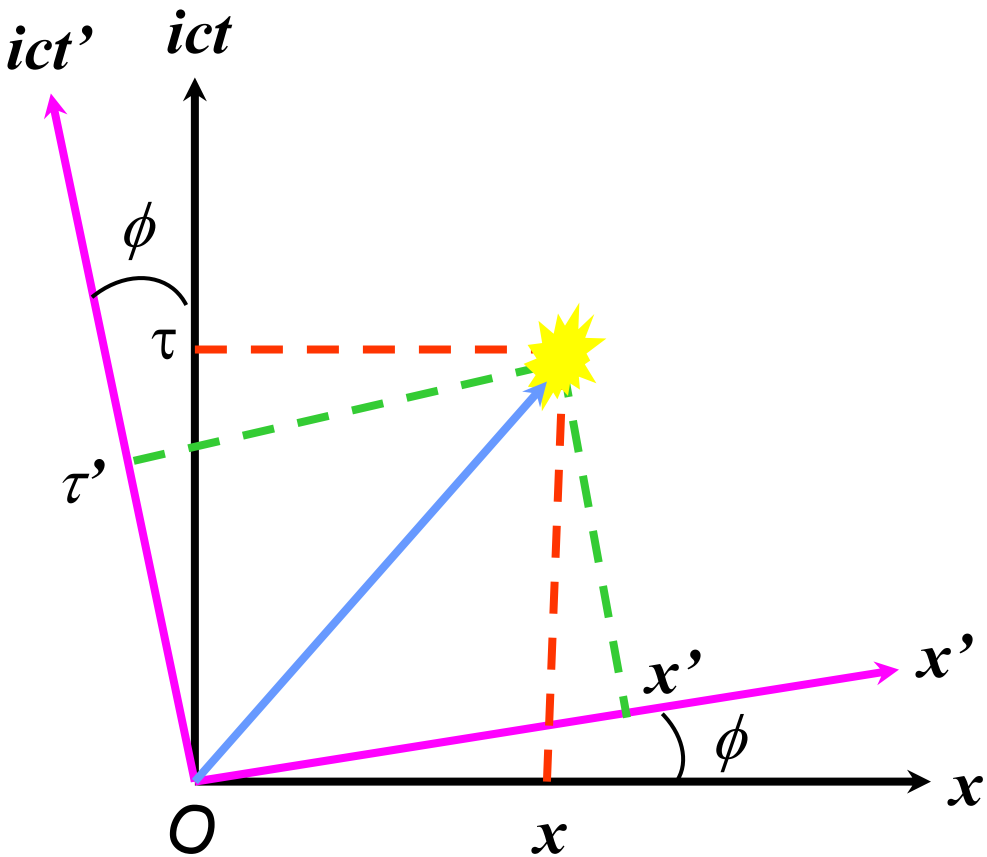
  Active view: We think of the point (x,ct) as moving to a new position (x',ct') on the same set of axes, i.e., we regard the transformation as a motion in x,ct space.

  Passive view: We regard (x',t') as merely a new label of the old point (x,t).

- In special relativity the passive view is the more relevant.

Complex Rotation Diagram

We introduce the variable $\tau = i(ct)$, where $i = \sqrt{-1}$

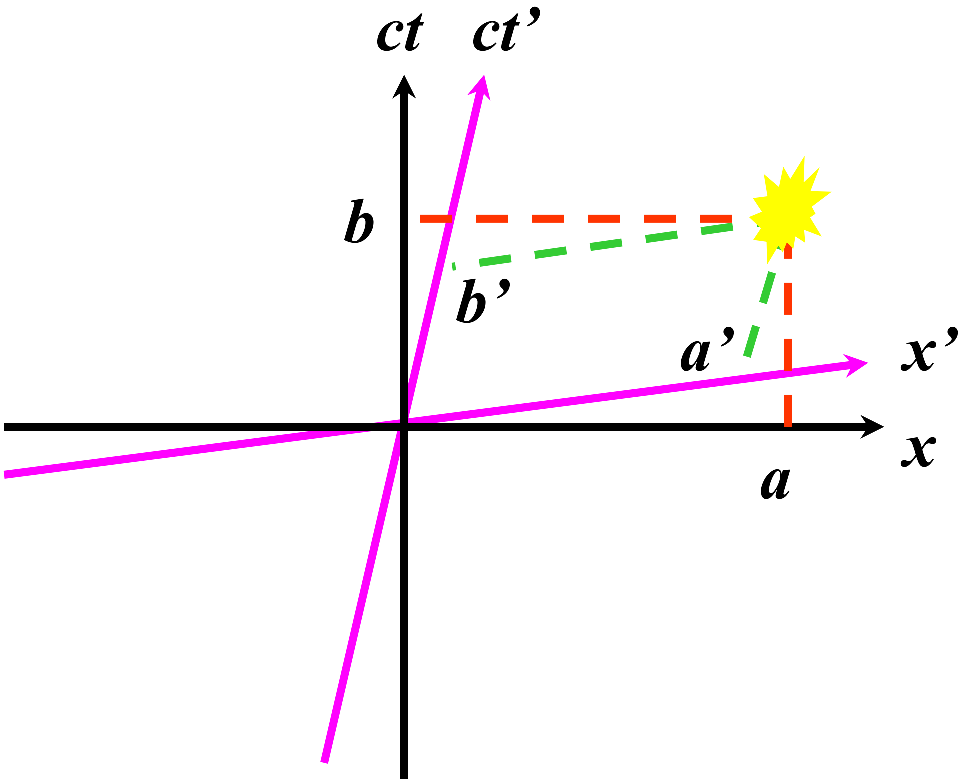


$$s = \sqrt{(ct)^2 - x^2 - y^2 - z^2}$$
$$= i\sqrt{x^2 + y^2 + z^2 + \tau^2}$$
$$\equiv ir$$

$$\tan\phi = i\frac{v}{c} = i\beta$$

Imaginary 4'th dimension

Minkowski Diagram



t'-axis is give by $x'=0$

$$x' = \gamma(x - vt) \Rightarrow x = \frac{v}{c}ct = \beta(ct)$$

straight line slope $1/\beta$

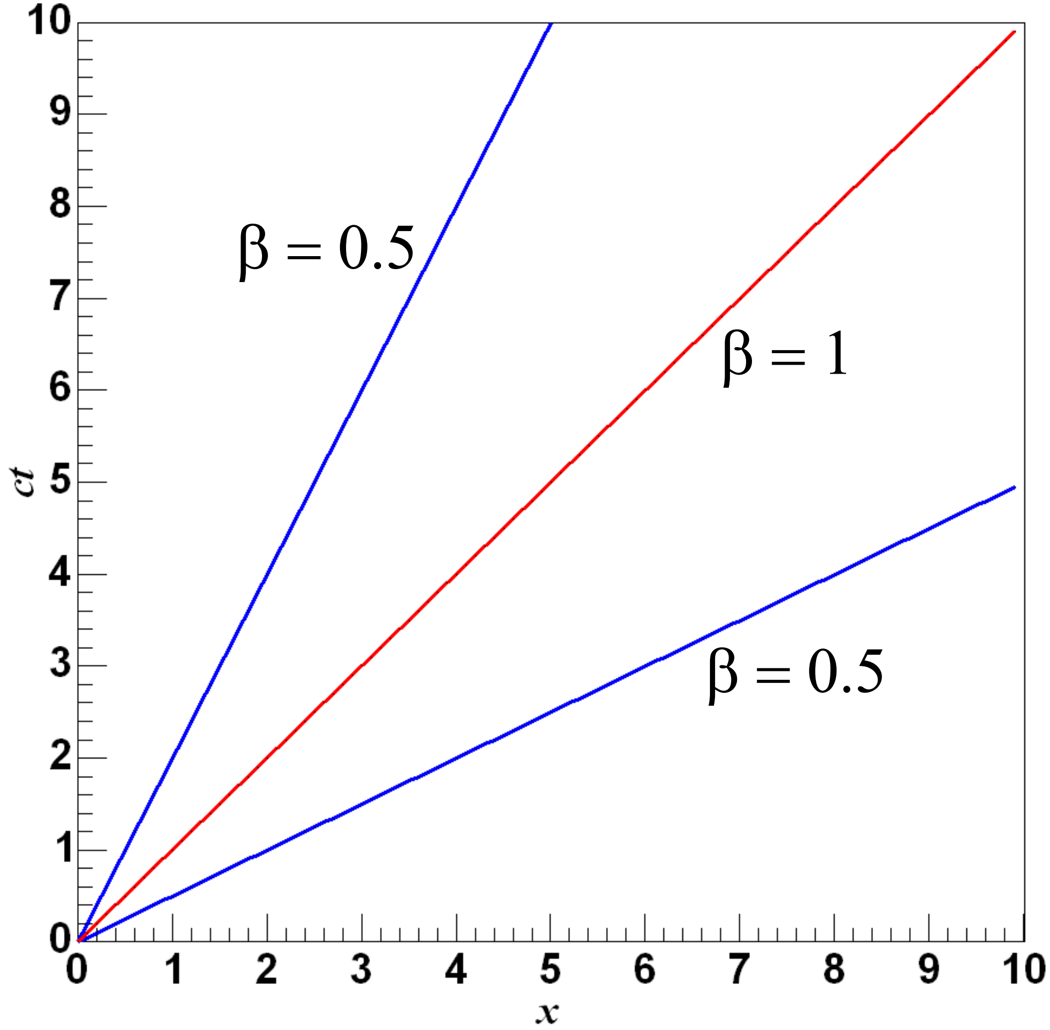
x'-axis is give by $t'=0$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \Rightarrow ct = \frac{v}{c}x = \beta x$$

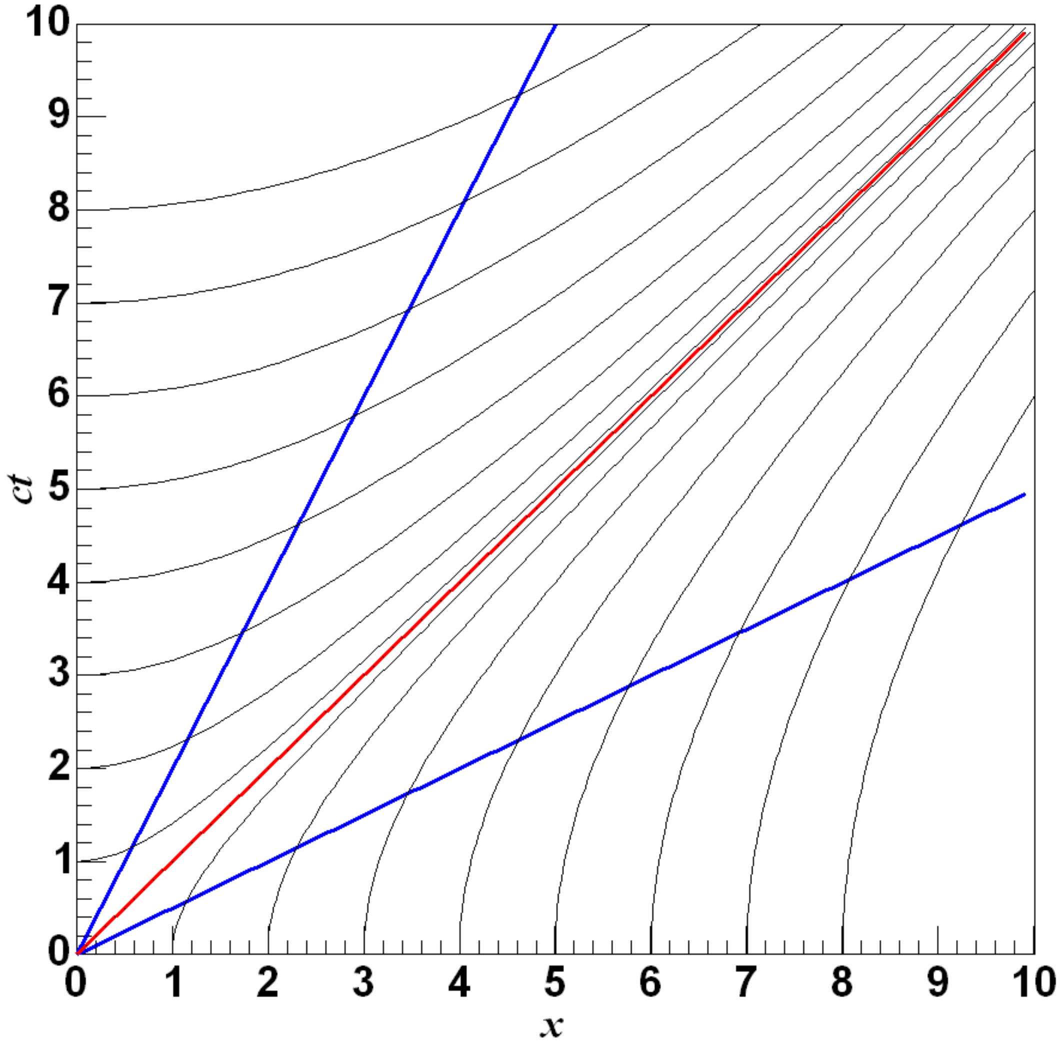
straight line slope β

The t'- and x'-axis are thus symmetrically placed with respect to the world line of the light signal from the origin, which is given by $x = t$, or $x' = t'$.

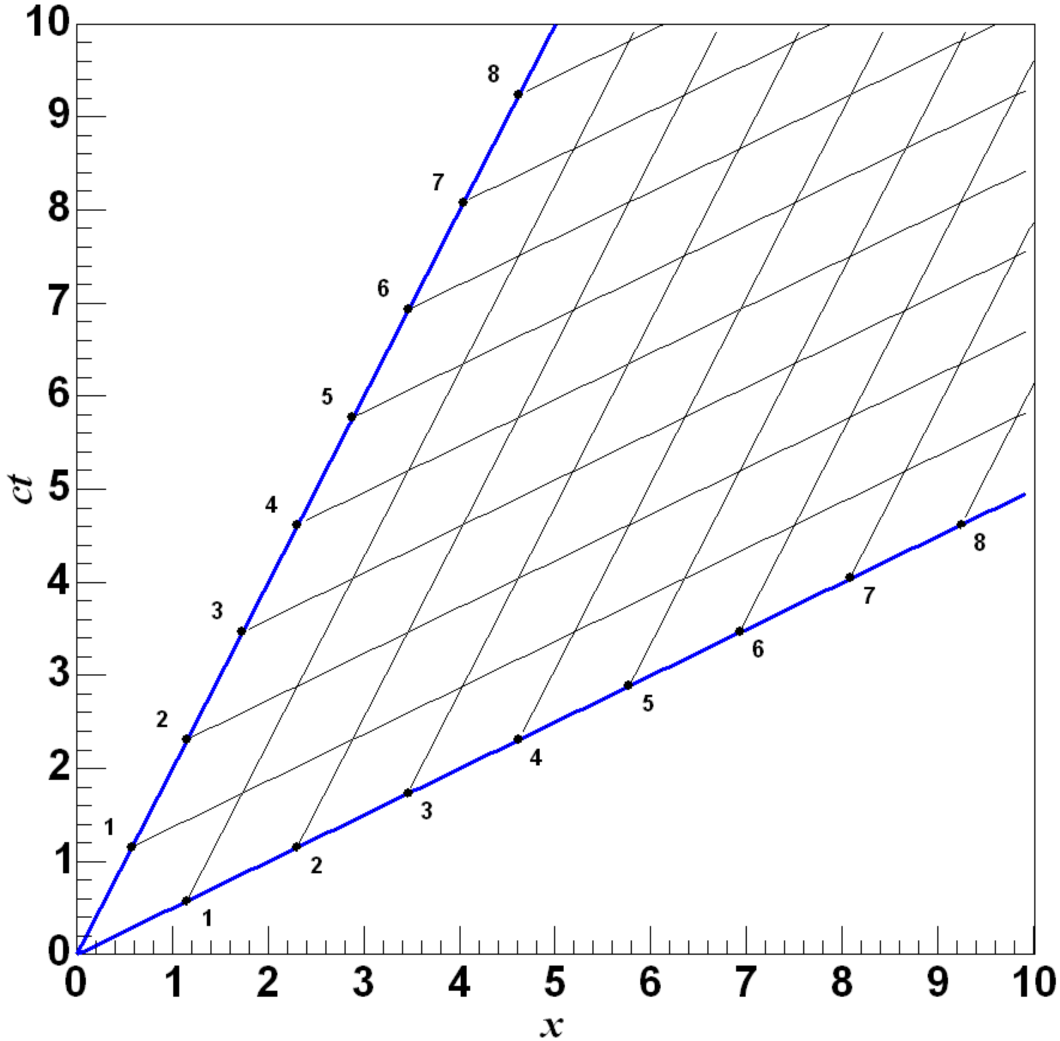
Minkowski Axis



Minkowski Scale

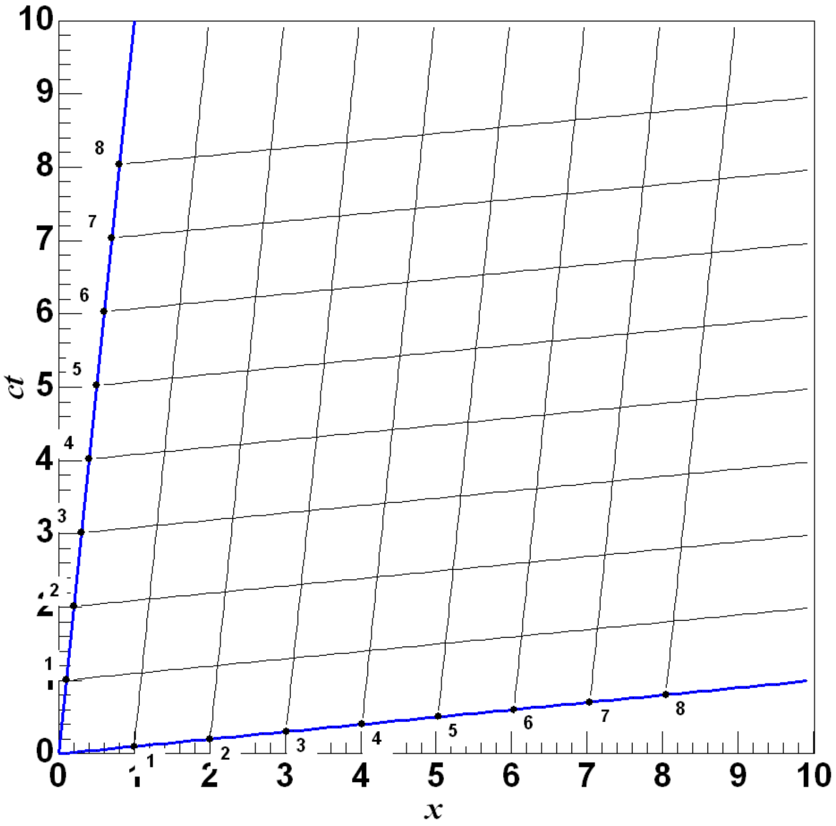


Minkowski Grid Lines

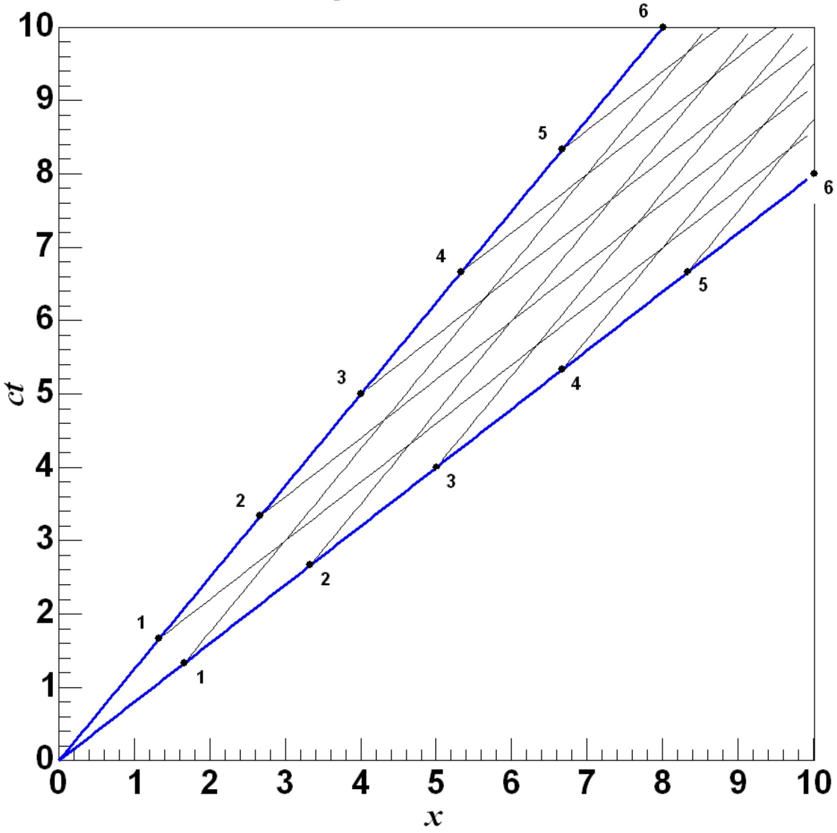


Axis Depends on Velocity

$\beta = 0.1$



$\beta = 0.8$



Features of Minkowski Diagrams

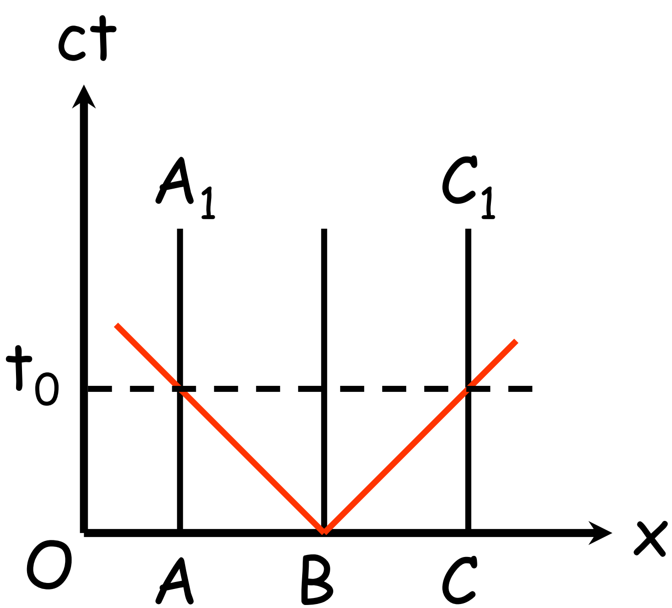
- These spacetime diagrams are based on the invariant interval.
- Because the invariant interval is based on hyperbolas, spacetime is non-Euclidean.
- The motion of the inertial observer S′ seems to warp the simple orthogonal spacetime into a skewed spacetime.
- The scales of the S′ frame are not the same as the scales on the S frame.
- Length contraction and time dilation can be explained by skewed spacetime.

Features of Minkowski Diagrams

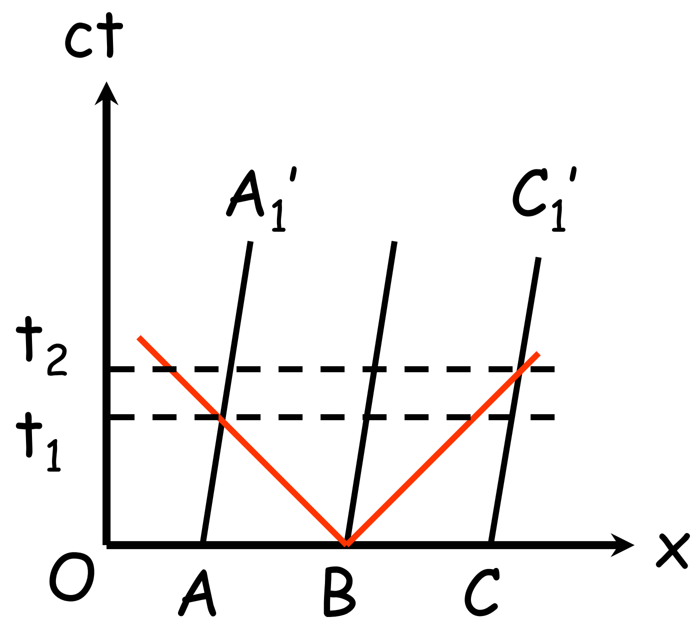
- All quantities are real.
- One set of axes is nonorthogonal and requires a different scale than the other, orthogonal set of axes.
- Observers are treated asymmetrically.
- Only the observer with the orthogonal axes has a scale which is independent of velocity.

Simultaneity Diagram

Light signals emitted from B midway between A and C.

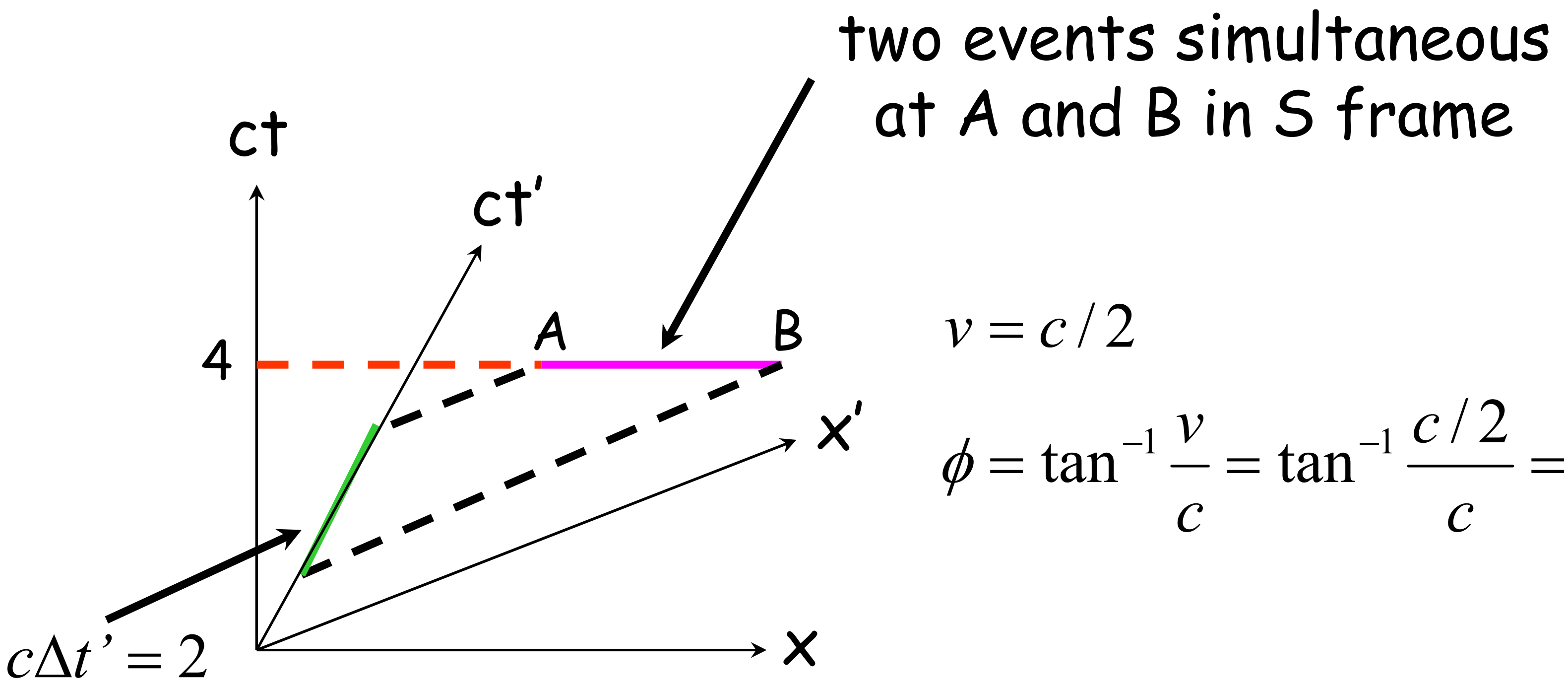


A, B, and C at rest.



A, B, and C moving.

Two Events Simultaneous in S Frame



$v = c/2$

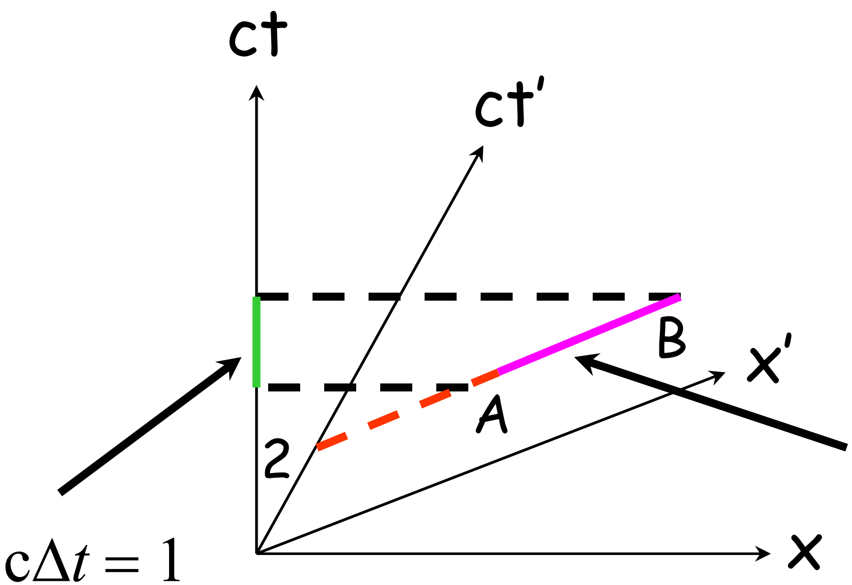
$\phi = \tan^{-1}\dfrac{v}{c} = \tan^{-1}\dfrac{c/2}{c} = 26.6^o$

B occurs before A in S' frame

Two Events Simultaneous in S' Frame

$v = c/2$

$\phi = 26.6^o$



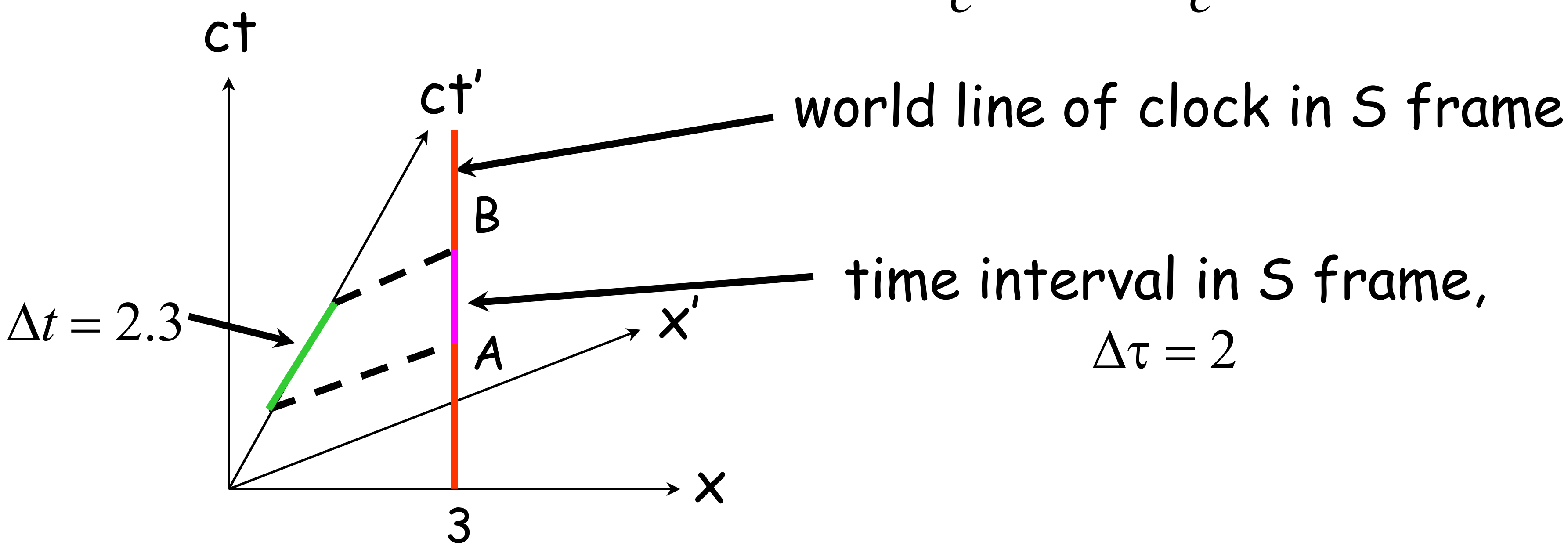
two events simultaneous at A and B in S' frame

A occurs before B in S' frame

Time Dilation in S Frame

$$v = c/2$$

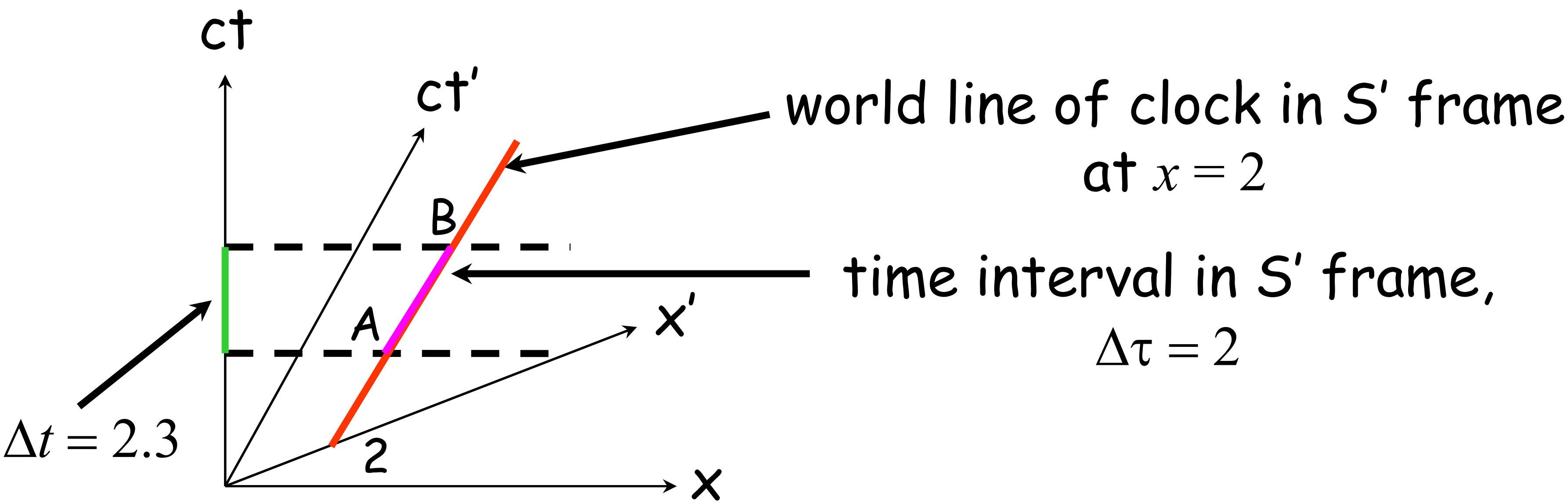
$$\phi = \tan^{-1}\frac{v}{c} = \tan^{-1}\frac{c/2}{c} = 26.6^{o}$$



Time Dilation in S' Frame

$$v = c/2$$

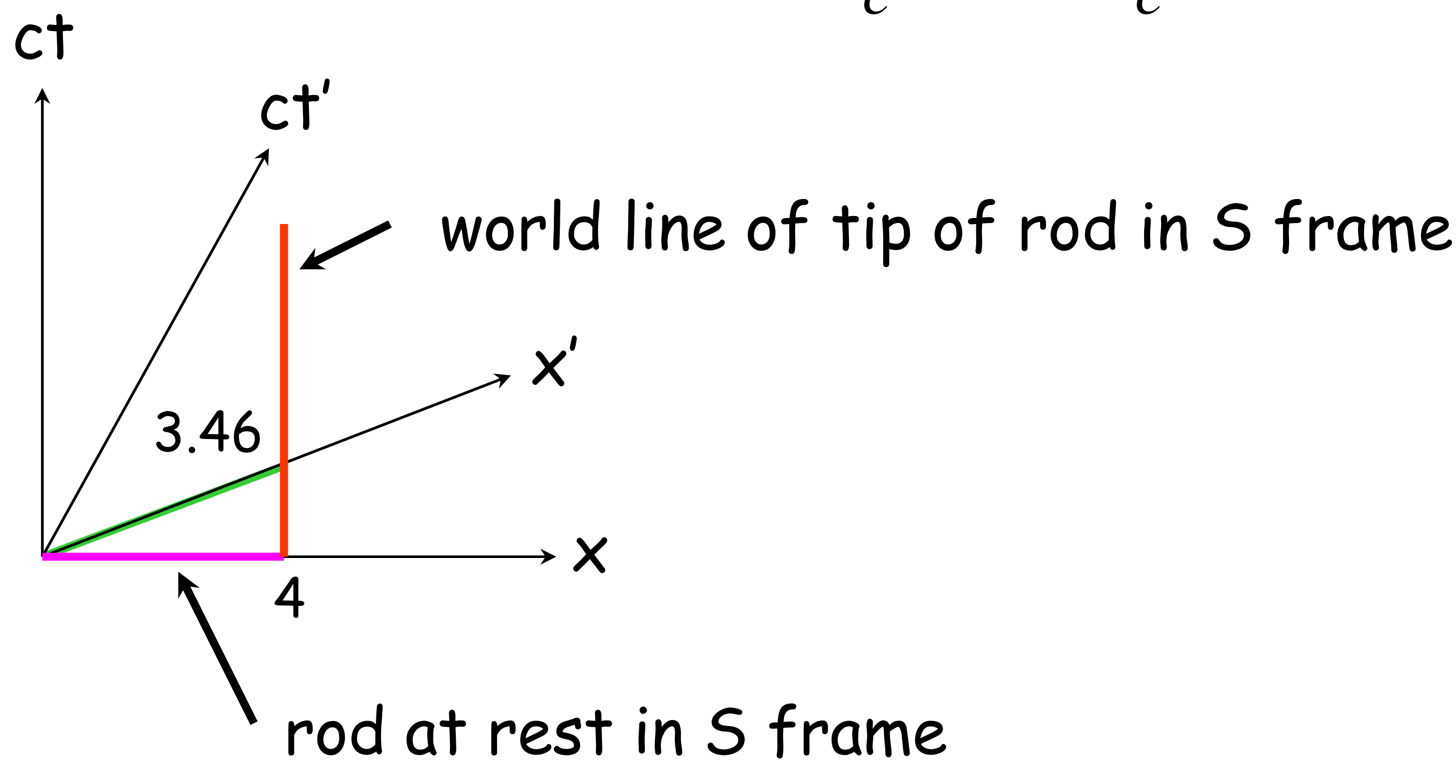
$$\phi = 26.6^o$$



Length Contraction in S Frame

$$v = c/2$$

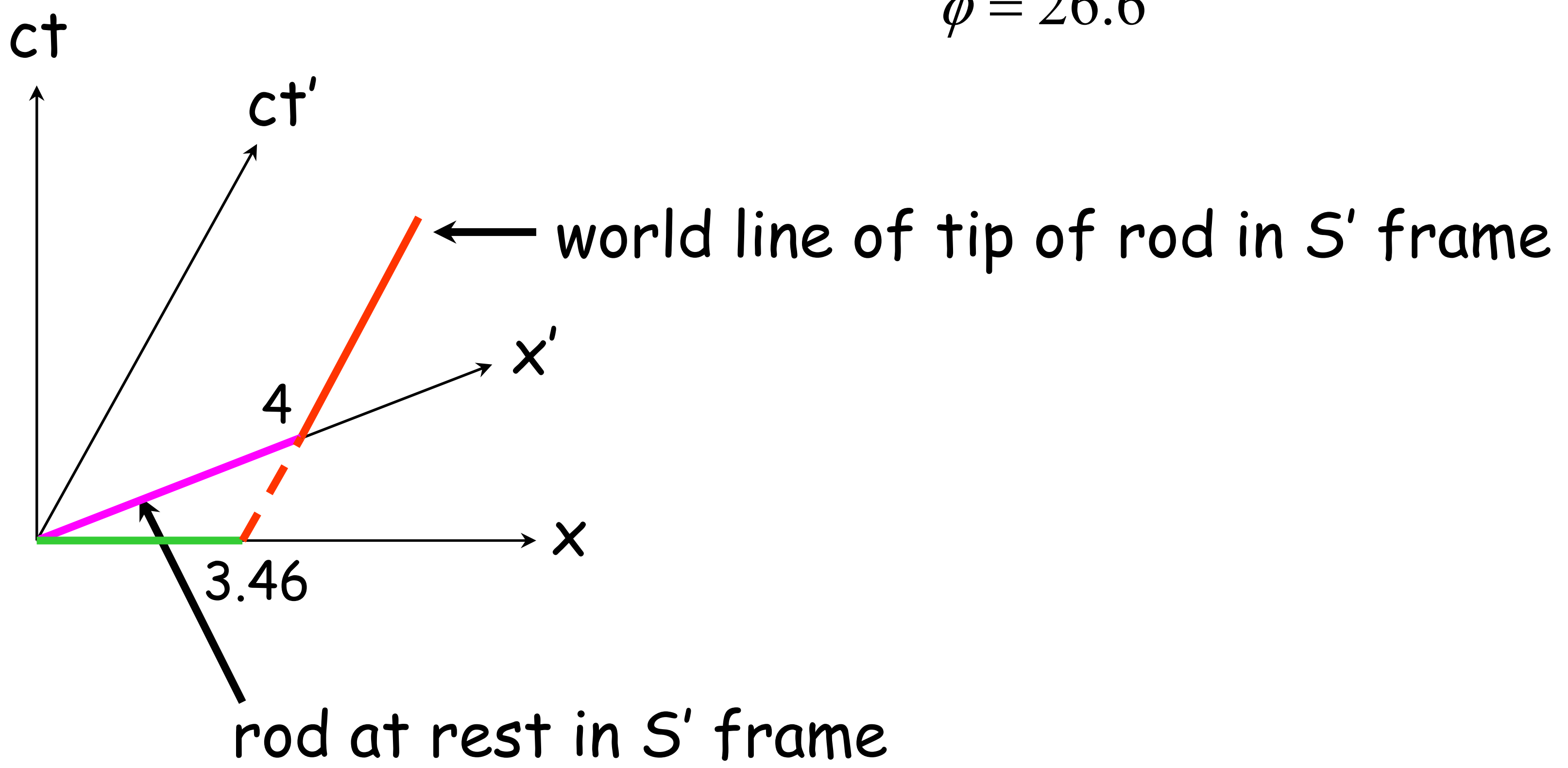
$$\phi = \tan^{-1}\frac{v}{c} = \tan^{-1}\frac{c/2}{c} = 26.6^o$$



Length Contraction in S' Frame

$$v = c/2$$

$$\phi = 26.6^o$$



Module 11

Four-Vectors

What is a 4-Vector?

- A vector is a mathematical object that has both magnitude and direction.
- The meaning of magnitude and direction differ between one geometry and another.
- Euclidean geometry defines 3-vectors in 3-dimensional space.
  - Each coordinate system has different values for the components of the 3-vector,
  - But all agree on the magnitude.
- Lorentz geometry defines 4-vectors in 4-dimensional spacetime.
  - Each inertial system has different values for the components of the 4-vector,
  - But all agree on the spacetime interval.

Four-vectors

Invariant interval

$$X^2 = (ct)^2 - x^2 - y^2 - z^2 = (ct)^2 - \vec{r}^{\,2}$$

$$x^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2 = x_0^2 - \vec{x}^{\,2}$$

Position 4-vector

$$X = (ct, x, y, z) = (ct, \vec{r})$$

$$x^\mu = (x_0, x_1, x_2, x_3) = (x_0, \vec{x})$$

Matrix Representation

Lorentz transformation

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$L = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad X = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \qquad X' = LX$$

Four-Velocity

Squared interval in differential form

$$s^2 = (ct)^2 - x^2 - y^2 - z^2$$

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

Proper time interval

$$(d\tau)^2 \equiv \frac{(ds)^2}{c^2} = (dt)^2 - \frac{(dx)^2 + (dy)^2 + (dz)^2}{c^2}$$

$d\tau$ is the relativistic analog of dt

4-velocity $\quad U = \frac{dX}{d\tau} = \left(\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right)$

notice $\quad \left(\frac{d\tau}{dt} \right)^2 = 1 - \frac{(dx)^2 + (dy)^2 + (dz)^2}{c^2 (dt)^2} = 1 - \frac{u^2}{c^2} \quad \Rightarrow \quad \frac{dt}{d\tau} = \gamma$

$$U = \gamma(1, \vec{u})$$

Four-Acceleration

4-acceleration
$$A = \frac{dU}{d\tau} = \frac{d^2 X}{d\tau^2}$$
$$= \gamma\left(\frac{d\gamma}{dt}, \frac{d\gamma}{dt}\vec{u} + \gamma\vec{a}\right)$$

In the instantaneous rest frame of the particle ($\mathrm{u} = 0$)

$$A = (0, \vec{a})$$

If the proper acceleration vanishes, A vanish.
Noticed that U can never vanish.

Module 12

Paradoxs

Twin Dilation

- Relativistic time dilation applies equal to *all physical processes*, including biological functions.
- Two twins 35 yr old. One travels with a speed 0.99c to Vega which is 25.3 light years from earth.
- Time of journey from earth.

$$v = \frac{d}{\Delta t} \quad \Rightarrow \quad \Delta t = \frac{d}{v} = \frac{25.3c(1\text{yr})}{0.99c} = 25.6\text{yr}$$

Age of twin at home 35 + 25.6 = 60.6 yr.

Twin Paradox

- Time of journey for astronaut

$$\Delta t = \gamma \Delta t_0 \quad \Rightarrow \quad \Delta t_0 = \frac{\Delta t}{\gamma} = \frac{25.6c(\text{yr})}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}} = 3.61\,\text{yr}$$

Age of astronaut 35 + 3.61 = 38.6 yr.

Compared to 60.6 yr on earth.

"Paradox"

- From the point of view of the astronaut the twin on earth was moving and should appear to age slower. *The paradox is the seemingly asymmetry.*
- But in order for them to come back together and make a comparison, the astronaut must either stop at the end of the trip and make a comparison of clocks or, more simply, he has to come back, and the one who comes back must be the twin who was moving, and he knows this, because he had to turn around.
- The twin who has felt the accelerations is the one who would be the younger.

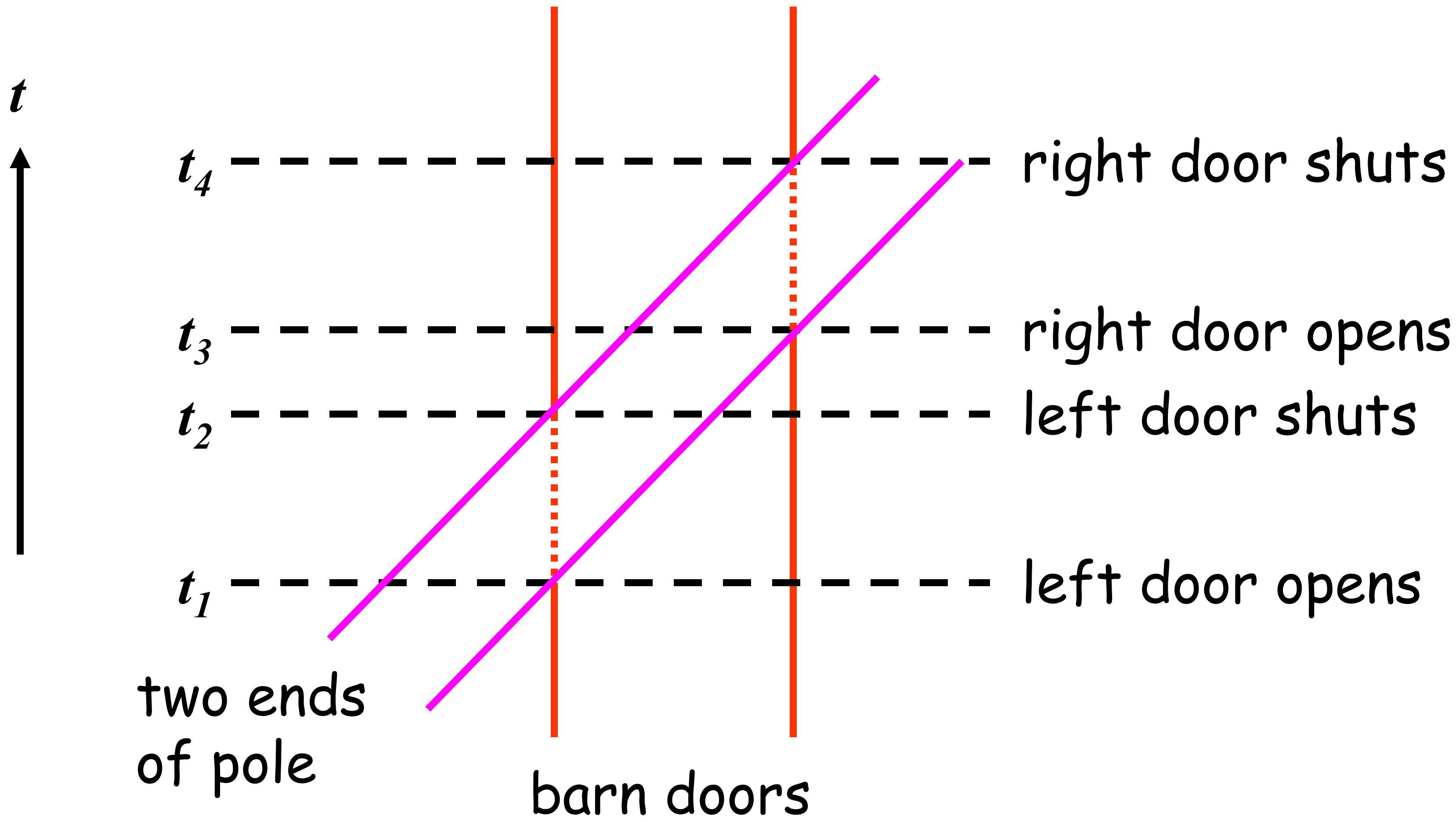
Length Contraction Paradox

- A barn 40 m long with a door at each end lies along the x-axis. Let both doors be closed.
- A man comes running down the x-axis from the left, holding a horizontal pole of proper length 50 m, parallel to the x-axis.
- The man runs at 4/5 the speed of light.
- The Lorentz contraction reduces the length of the pole in the frame of the barn to a mere 30 m.
- It appears that we can exploited Lorentz contraction to enclose completely within a contain period a moving object which, if stationary, would have been too long to fit inside the container.

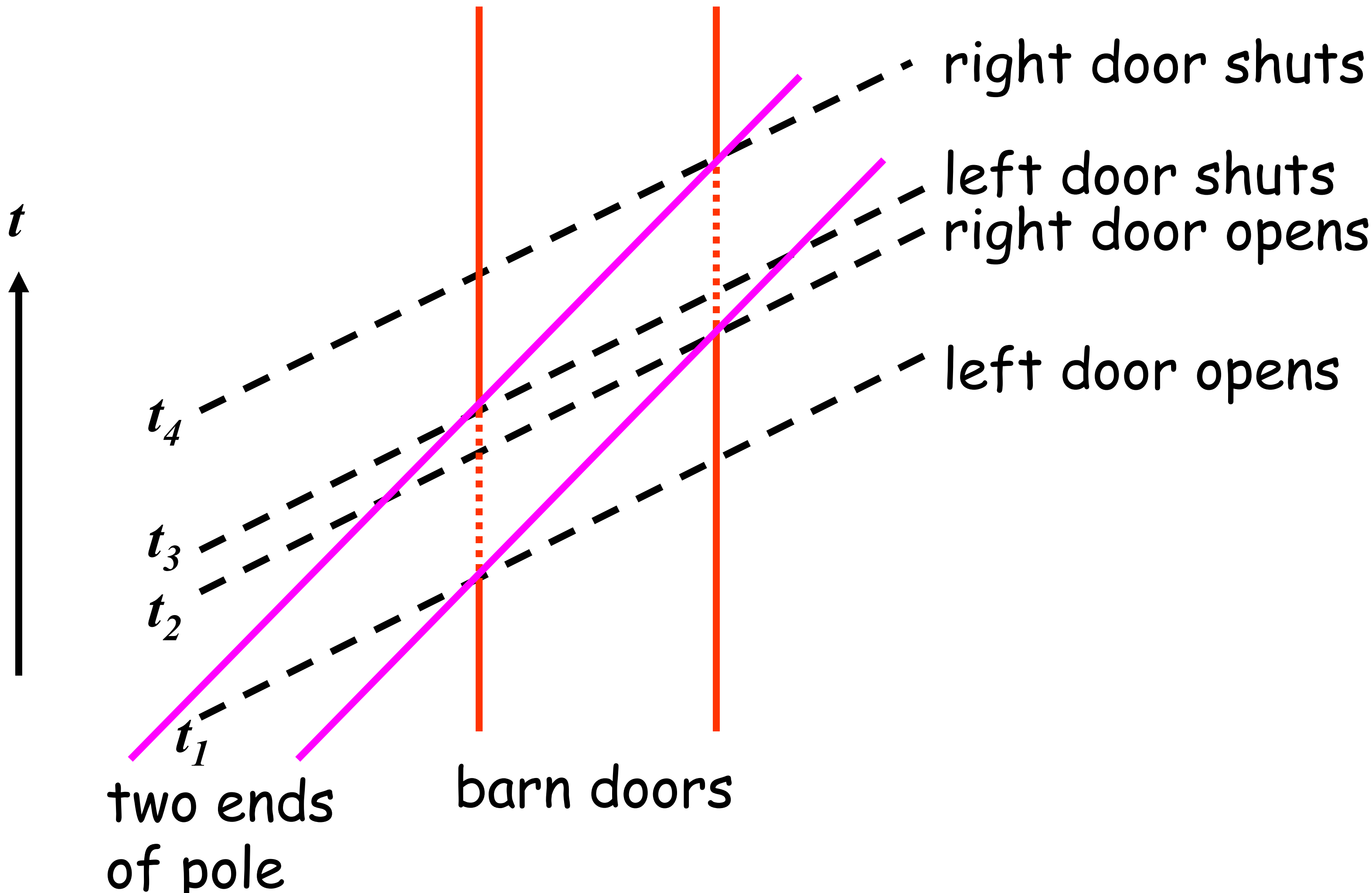
Pole in Barn Paradox

- In the runner's frame the pole is a full 50 m long, while the barn has shrunk to 24 m in length.
- How can the 50 m pole be momentarily enclosed in a barn only 24 m long?
- It is not. The statement that the pole is enclosed in the barn depends on the fact that the left door closes before the right door opens. But this fact is based on a judgment of the order in time of two spatially separated events, which sometimes can change, depending on which observer makes the judgment.

Rest Frame of the Barn



Rest Frame of Pole



"Paradox"

- The conclusions deduced in both frames of reference are correct.
- This is not contradictory, since the ordering of time of events can depend on the frame of reference used.

Module 13

Velocity Transformation

Speed of Light is the Ultimate Speed

- Objects with mass cannot reach the speed of light in a vacuum.

$$v \to c$$

$$\frac{v}{c} \to 1$$

$$1-\left(\frac{v}{c}\right)^2 \to 0$$

$$\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \to \infty$$

We will see that the kinetic energy would have to be infinite which means, according to the work-energy theorem, that an infinite amount of work would have to be done.

Since an infinite amount of work is not available, an object with mass can not attain the speed of light.

$u' = u + v$ is not correct.

Arbitrarily large velocities are possible for signals that carry no information.

Velocity Transformation

Need transformation which does not allow $v > c$.

$$u_x = \frac{u_x' + v}{1 + vu_x'/c^2}$$

$$u_y = \frac{u_y'}{\gamma\left(1 + vu_x'/c^2\right)}$$

$$u_z = \frac{u_z'}{\gamma\left(1 + vu_x'/c^2\right)}$$

For $v/c \to 0$

$$u_x \to u_x' + v$$

$$u_y \to u_y'$$

$$u_z \to u_z'$$

Replace v by $-v$ for the inverse transformation.

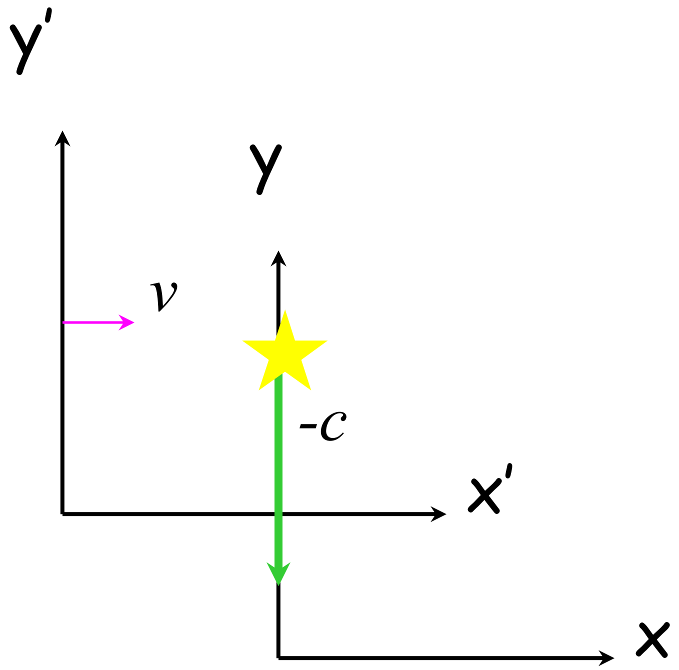
Notice u_y and u_z change since t changes.

Example: Aberration of Starlight

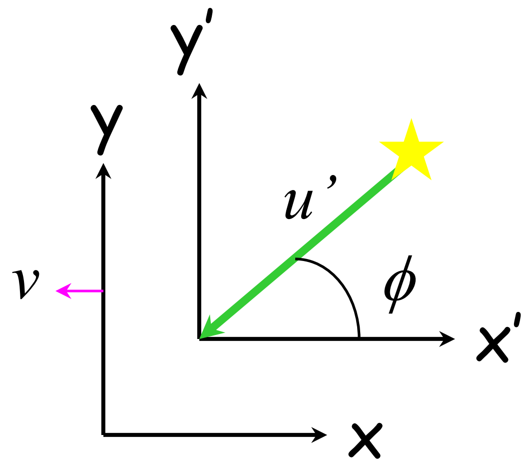
An astronomer stationary in the S-frame sees light from a distant star traveling toward him exactly down the y-axis of his coordinate system.

A second astronomer is traveling at a velocity of $0.800c$ parallel to the x-axis in the S′-frame.

At what angle does the moving astronomer see the light, and how fast is the light from that star traveling relative to her?



Example: Stellar Aberration



$$u_x = 0, \quad u_y = -c, \quad v = 0.800c$$

$$u'_x = \frac{u_x - v}{1 - vu_x / c^2} = \frac{0 - 0.800c}{1 - (0.800c)(0)/c^2} = -0.800c$$

$$u'_y = \frac{u_y}{\gamma(1 - vu_x / c^2)} = \frac{-c}{5/3[1 - (0.800c)(0)/c^2]} = \frac{3}{5}(-c) = -0.600c$$

$$u' = \sqrt{(u'_x)^2 + (u'_y)^2} = \sqrt{(-0.800c)^2 + (-0.600c)^2} = c$$ of course

$$\phi = \tan^{-1}\left(\frac{u'_y}{u'_x}\right) = \tan^{-1}\left(\frac{-0.600c}{-0.800c}\right) = 36.9^o$$

Module 14

Relativistic Momentum

Classical Momentum

Newtonian (nonrelativistic) momentum $\vec{p} = m\vec{v}$

Galilean transformation for velocity $\vec{v}' = \vec{v} - \vec{V}_0$

$$\vec{p}' = m\vec{v}' = m\vec{v} - m\vec{V}_0 = \vec{p} - m\vec{V}_0$$

constant

If the total momentum of a system of colliding particles is conserved in one reference frame, it will be conserved in the other reference frame.

Relativistic Momentum

classical momentum

$$\vec{p} = m\vec{v} = m\frac{d\vec{r}}{dt}$$

relativistic momentum

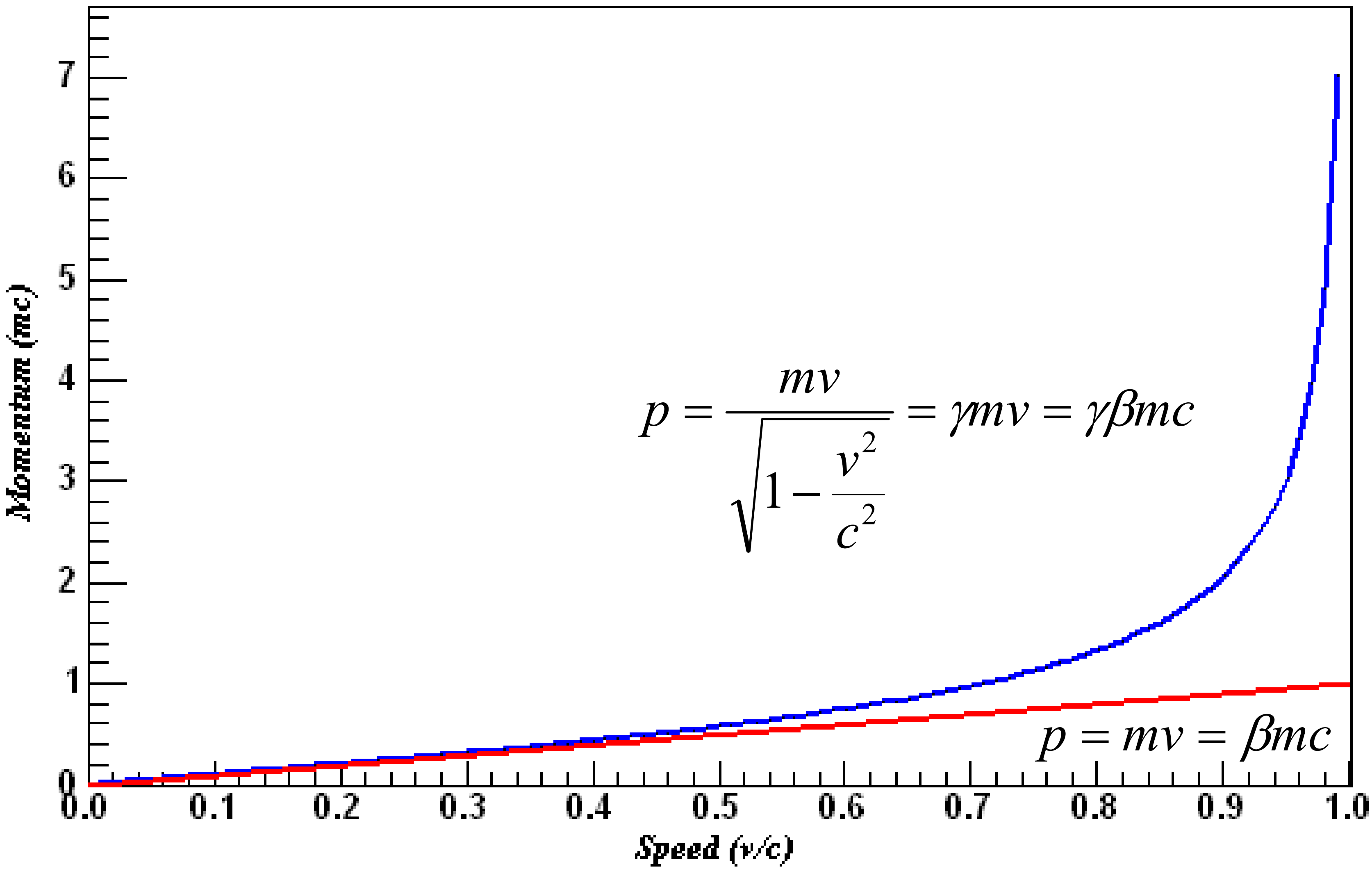
$$\vec{p} = m\frac{d\vec{r}}{dt_0} = m\gamma\frac{d\vec{r}}{dt} = \gamma m\vec{v} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

nonrelativistic limit

$$v << c \quad \Rightarrow \quad \gamma \to 1 \quad \Rightarrow \quad \vec{p} \to m\vec{v}$$

Conservation of momentum of an isolated system also holds in special relativity.

Momentum Comparison



Example: TV Tube

An electron in the beam of a TV tube has a speed of 1.0×10^{8} m/s.

What is the magnitude of the momentum of this electron?

$$p = \frac{(9.1\times10^{-31}\,\mathrm{kg})\times(1.0\times10^{8}\,\mathrm{m/s})}{\sqrt{1-\left(\dfrac{1.0\times10^{8}\,\mathrm{m/s}}{3.0\times10^{8}\,\mathrm{m/s}}\right)^{2}}} = 9.7\times10^{-23}\,\mathrm{kg\cdot m/s}$$

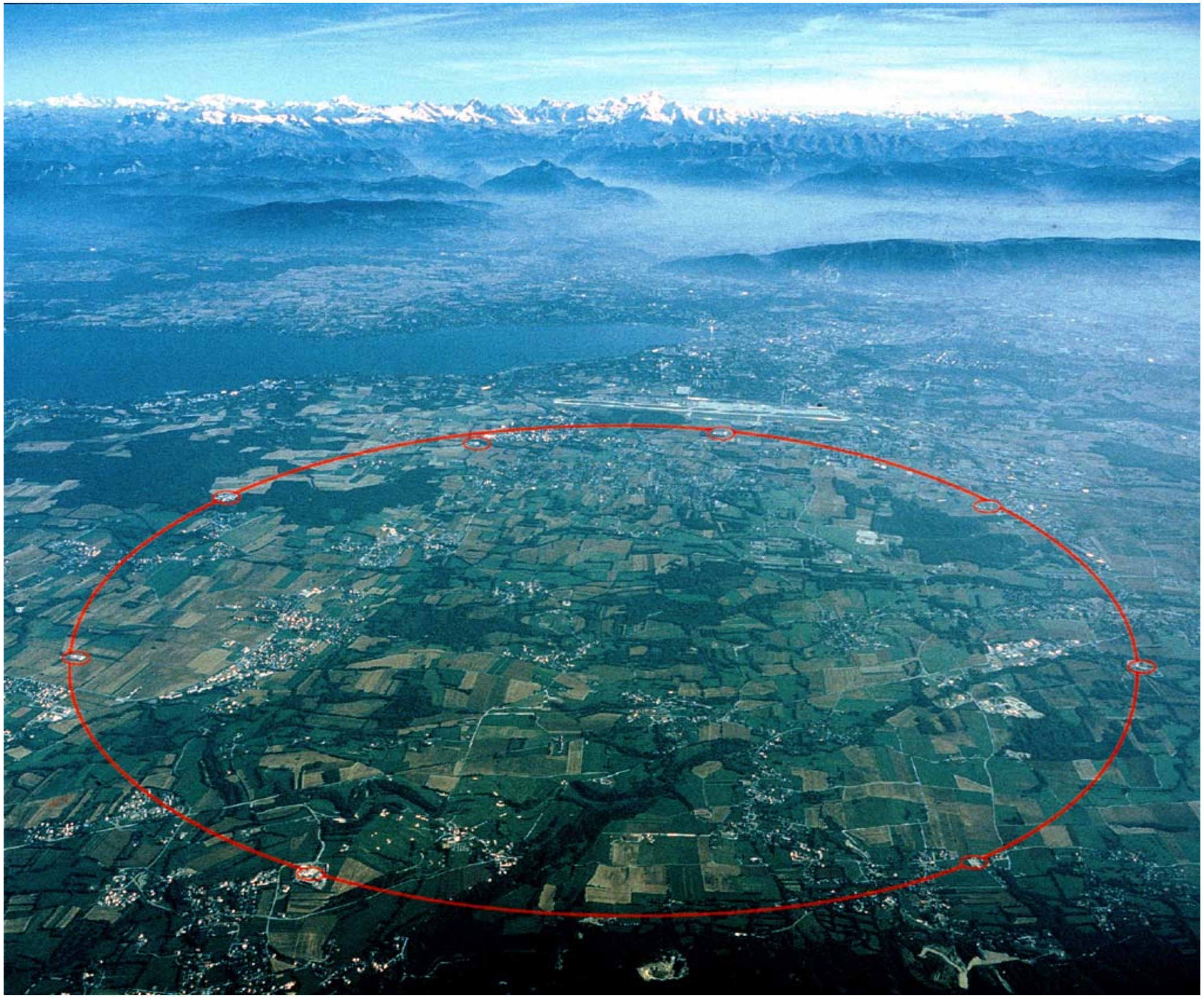
nonrelativistic momentum

$$p = (9.1\times10^{-31}\,\mathrm{kg})\times(1.0\times10^{8}\,\mathrm{m/s}) = 9.1\times10^{-23}\,\mathrm{kg\cdot m/s}$$

a 6% error

SLAC Linear Electron Accelerator

CERN LEP Accelerator



Example: High-Speed Electron

The accelerator accelerates electrons to a speed of $0.9999999997c$.

Find the magnitude of the relativistic momentum of an electron and compare it with the nonrelativistic value.

relativistic momentum

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(9.11\times10^{-31}\,\text{kg})\times(0.9999999997c)}{\sqrt{1-\frac{(0.9999999997c)^2}{c^2}}} = 1\times10^{-17}\,\text{kg}\cdot\text{m/s}$$

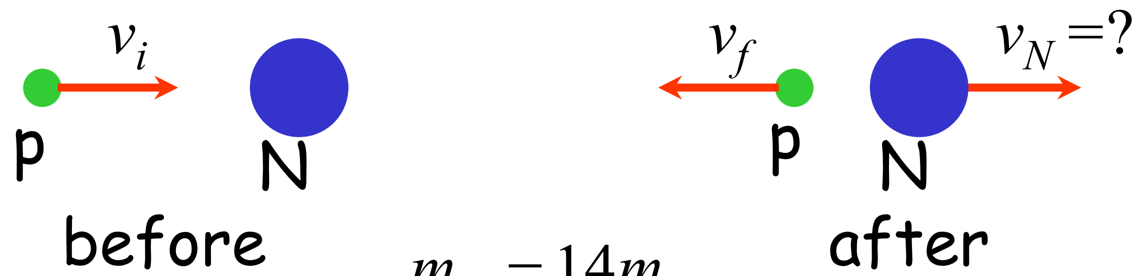
gamma factor

$$\frac{p}{mv} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.9999999997c)^2}{c^2}}} = 4\times10^4$$

Example: Collision in Atmosphere

Cosmic rays collide with atoms in the upper atmosphere.

$v_i = 0.70c$



$v_f = 0.63c$

$m_N = 14 m_p$

initial proton $\quad \gamma = \left(1 - 0.70^2\right)^{-1/2} = 1.4003$

$$p_i = 1.4003 m_p \times 0.70c = +0.9802 m_p c$$

final proton $\quad \gamma = \left(1 - 0.63^2\right)^{-1/2} = 1.288$

$$p_f = -1.288 m_p \times 0.63c = -0.8114 m_p c$$

Example: Collision in Atmosphere

change in proton momentum

$$\Delta p = -0.8114 m_p c - 0.9802 m_p c = -1.7916 m_p c$$

conservation of momentum give nitrogen momentum

$$P = +1.7916 m_p c = \gamma m_N v_N = \gamma \times 14 m_p v_N$$

$$\gamma v_N = \frac{v_N}{\sqrt{1-\left(\frac{v_N}{c}\right)^2}} = 0.1280c$$

$$\gamma \approx 1 \quad \Rightarrow \quad v_N \approx 0.13c$$

nonrelativistic calculation (26% error)

$$0.70 m_p c = -0.63 m_p c + 14 m_p v_N$$

$$v_N = \frac{1.33c}{14} = 0.095c$$

Module 15

Relativistic Force

Forces

In classical mechanics $\vec{F} = m\vec{a}$ and $\vec{F} = \frac{d\vec{p}}{dt}$

These are equivalent since m is constant.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

In relativity $\vec{p} = \gamma m\vec{u}$ and $\frac{d\vec{p}}{dt} \neq m\vec{a}$

The total F acting on a body with momentum p is defined as

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

Transformation of Forces

- A force directed along the direction of the relative velocity is the same in both frames. Surprising.
- Forces equal and opposite in one frame are not necessarily so in another frame.
- Then two forces are collinear in one frame are not necessarily collinear in another frame.

Module 16

Relativistic Energy

Relativistic Energy

Desirable Features

- When applied to slowly moving bodies, the new definition of E should reproduce as closely as possible the classical definition.
- The total energy ΣE of an isolated system of bodies should be conserved in all inertial frames.

Equivalence of Mass and Energy

total energy of an object

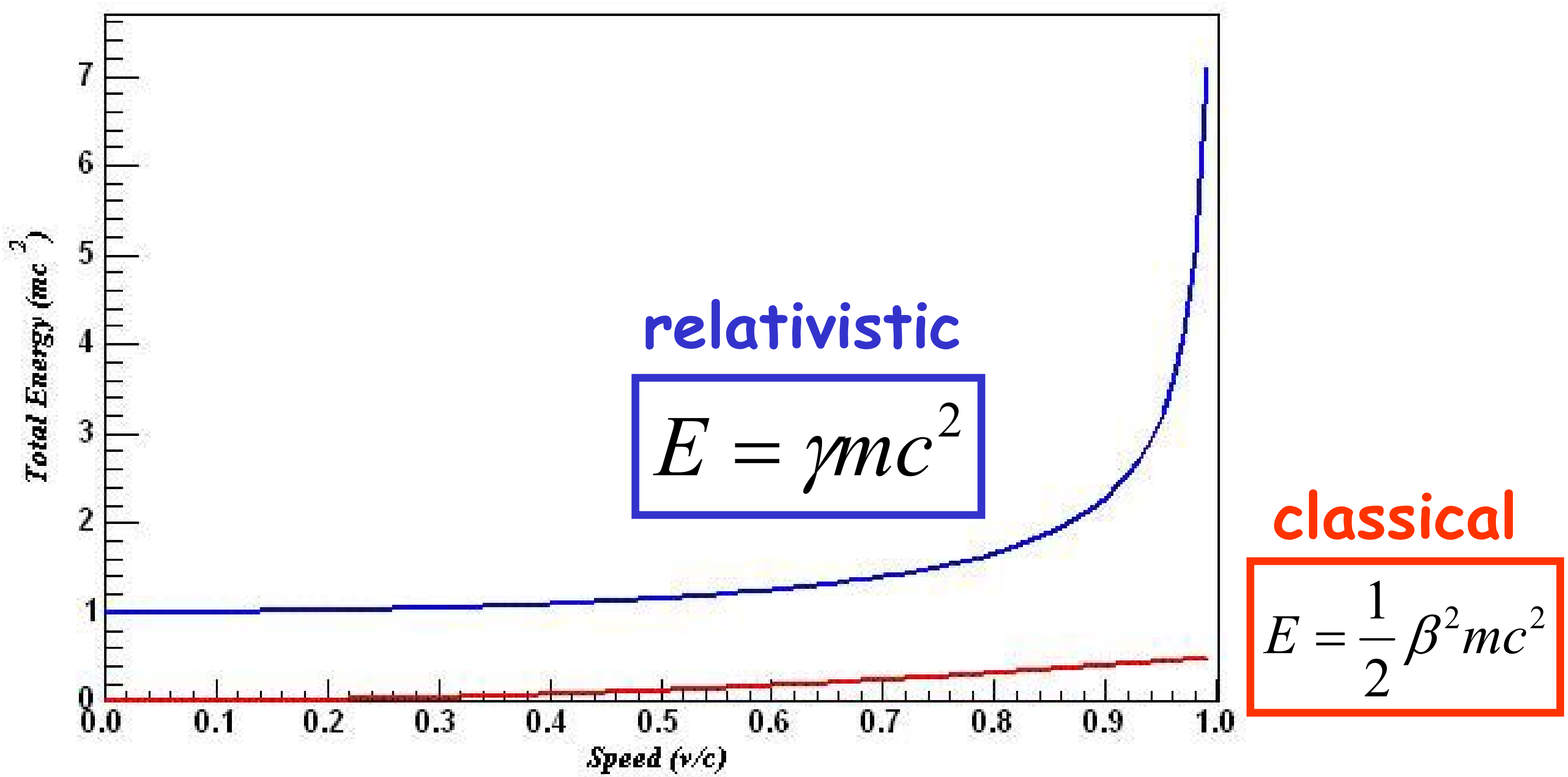
$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \equiv \gamma mc^2$$

rest energy of an object

$$E_0 = mc^2$$

Recall that one was always at liberty to add or subtract an overall constant from the energy since the zero of energy was arbitrary.

Relativistic Versus Classical Energy



No object with mass can travel at the speed of light since it would need to have an infinite total energy.

Example: Energy of a Golf Ball

A 0.046 kg golf ball lying on the green.

a) Rest energy of golf ball?
b) How long could this energy operate a 75 W light bulb?

$$E_0 = mc^2 = (0.046\text{ kg}) \times (3.0 \times 10^8\text{ m/s})^2 = 4.1 \times 10^{15}\text{ J}$$

$$t = \frac{E_0}{P} = \frac{4.1 \times 10^{15}\text{ J}}{75\text{ W}} = 5.5 \times 10^{13}\text{ s} = 1.7\text{ million years}$$

Kinetic Energy

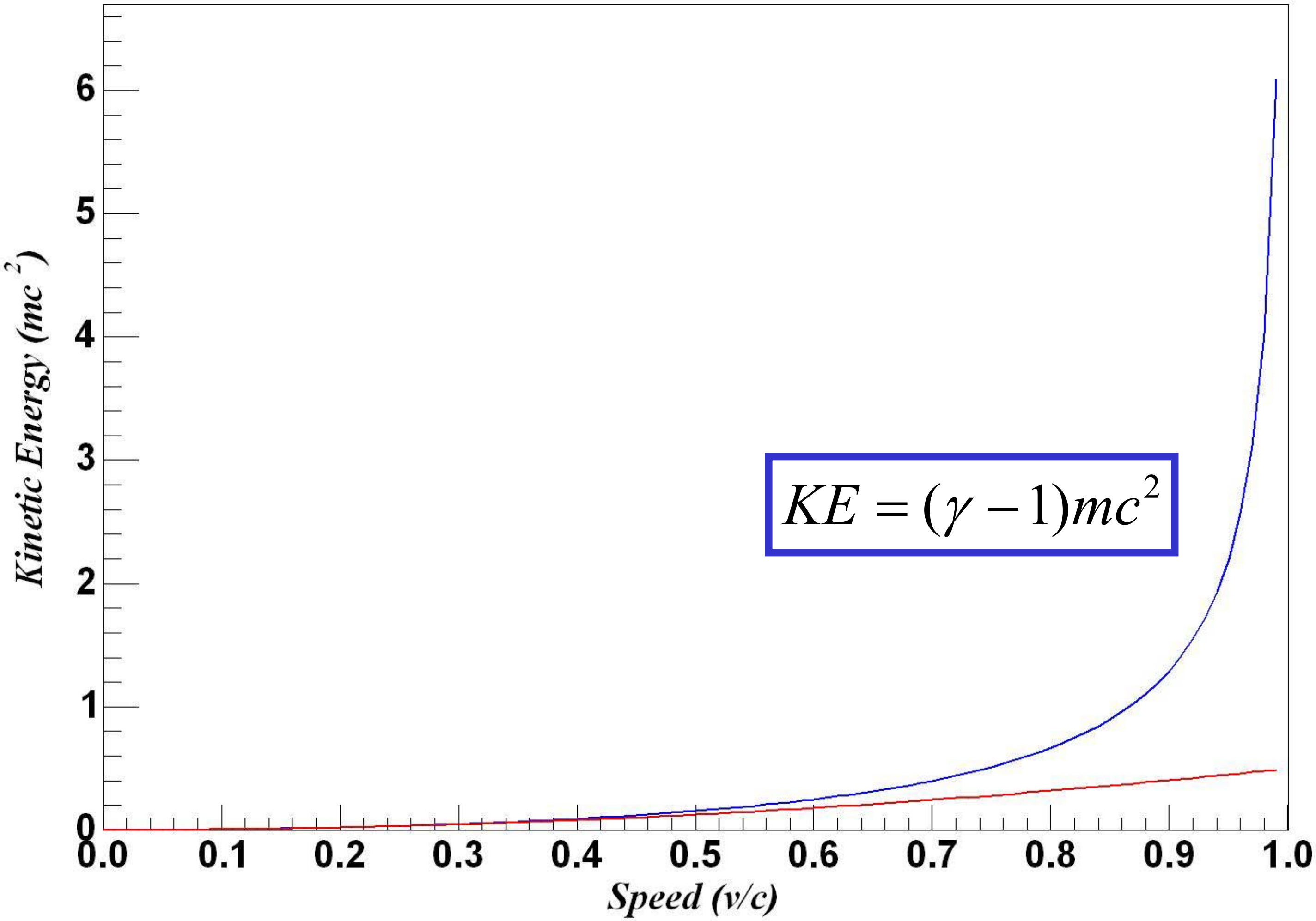
Kinetic energy is the work done by an unbalanced force in accelerating a particle from rest to some velocity.

relativistic energy $E = E_0 + KE$

kinetic energy $KE = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$

classical limit $KE = E - E_0 = \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots - 1\right)mc^2 \approx \frac{1}{2}mv^2 = \frac{1}{2}\beta^2 mc^2$

Kinetic Comparison



$$KE = (\gamma - 1)mc^2$$

$$KE = \frac{1}{2}\beta^2 mc^2$$

Example: High-Speed Electron

An electron ($m = 9.109\times10^{-31}$kg) is accelerated from rest to a speed of $v = 0.9995c$ in a particle accelerator.

Determine in millions of electron volts or MeV

a) Rest energy,
b) Total energy,
c) Kinetic energy.

$$E_0 = mc^2 = (9.109\times10^{-31}\,\text{kg})\times(2.998\times10^8\,\text{m/s})^2 = 8.187\times10^{-14}\,\text{J}$$ or 0.511 MeV

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(9.109\times10^{-31})\times(2.998\times10^8)^2}{\sqrt{1-\left(\frac{0.9995c}{c}\right)^2}} = 2.59\times10^{-12}\,\text{J}$$ or 16.2 MeV

$$KE = E - E_0 = 2.59\times10^{-12} - 8.2\times10^{-14} = 2.51\times10^{-12}\,\text{J}$$ or 15.7 MeV

Invariance Versus Conserved

- Invariance quantity: at a given moment in history, has the same numerical value in all inertial frames.
  - Example: speed of light and mass.
- Conserved quantity: maintains the same value in a given reference frame; the value may differ from one frame to another, but in any given frame its value is constant.
  - Example: momentum and energy.

Mass in Relativity

- Same as classical definition with proviso that object is at rest.
- This is the rest mass or proper mass (mass measured in frame in which the object is at rest).
- Observers in different inertial frames all agree on the rest mass of an object.
  - Invariant rest mass.
- Mass is not conserved.
- Mass is just another form of energy.
- We could talk about variable mass.

Conversion of Mass to Energy

- The conversion of rest energy with a corresponding loss in rest mass is a common occurrence in radioactive decay and nuclear reactions, including nuclear fission and nuclear fusion.
- Energy and inertia, which were formerly two distinct concepts, are related through

$$E_0 = mc^2$$

$$\Delta E = \Delta mc^2$$

Example: Sun Is Losing Mass

The Sun radiates electromagnetic energy at the rate of 3.92×10^{26}W.

The mass of the Sun is 1.99×10^{30} kg.

a) What is the change in the Sun's mass during each second?
b) What fraction of the Sun's mass is lost during a human lifetime of 75 years.

$$\Delta m = \frac{\Delta E_0}{c^2} = \frac{3.92\times10^{26}}{(3.00\times10^{8})^2} = 4.36\times10^{9}\,\text{kg}$$

$$\Delta m = (4.36\times10^{9}\,\text{kg/s})\times\left(\frac{3.16\times10^{7}\,\text{s}}{1\,\text{yr}}\right)\times 75\,\text{yr} = 1.0\times10^{19}\,\text{kg}$$

$$\frac{\Delta m}{m_{\text{Sun}}} = \frac{1.0\times10^{19}}{1.99\times10^{30}} = 5.0\times10^{-12}$$

Some Useful Relations

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\, mv \qquad \text{and} \qquad E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\, mc^2$$

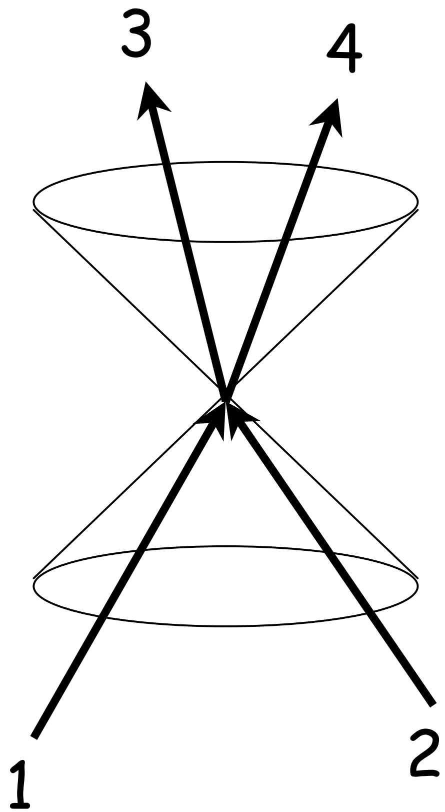
$$\frac{p}{E} = \frac{v}{c^2} \quad \Rightarrow \quad \beta = \frac{v}{c} = \frac{pc}{E} \qquad\qquad \gamma = \frac{E}{mc^2}$$

$$\boxed{E^2 = (pc)^2 + (mc^2)^2}$$

Units in Relativity

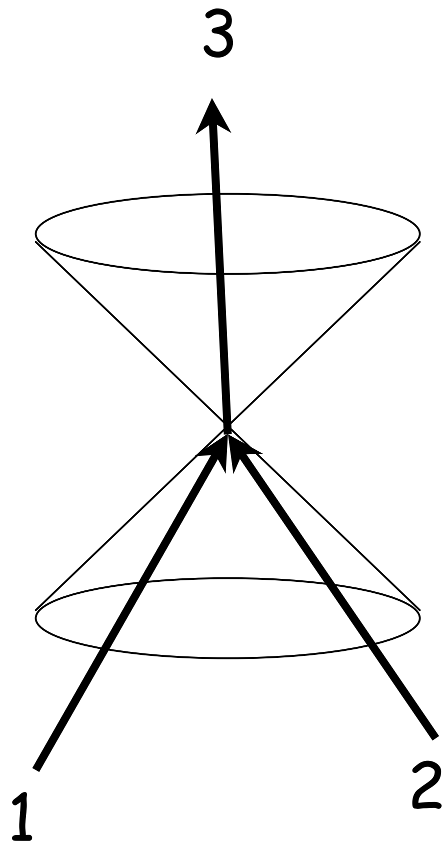
- Relativistic velocities are best expressed as fractions of c or by using the dimensionless quantity $\beta = v/c$.
- Most applications are in atomic and subatomic physics, where the joule (J) is an inconveniently large unit of energy.
- A popular unit is the electron volt or eV.
  - One electron volt is the kinetic energy that a particle of charge $\pm e$ gains when it is accelerated though a potential difference of 1 V.
  - $1 \text{ eV} = 1.60\times10^{-19}$ J.
- Typical energies in atomic physics are about 1 eV, typical energies in nuclear physics are about 1 MeV (1 MeV = 10^6 eV).
- The unit of mass is eV/c^2 and the unit of momentum is eV/c.
- Sometimes we drop the c in when expressing mass and momentum in units of eV.
- We usual speak of mc^2 when referring to mass and pc when referring to momentum.

Particle Collision Phenomena



elastic collision

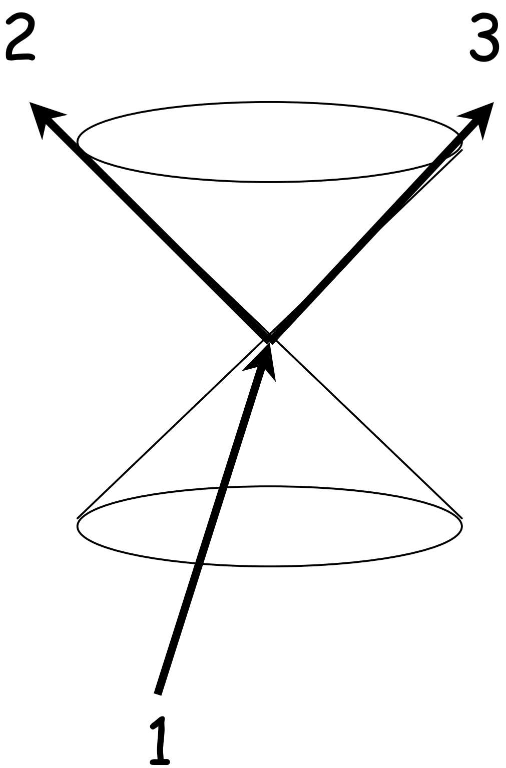
$1+2 \rightarrow 3+4$



inelastic collision

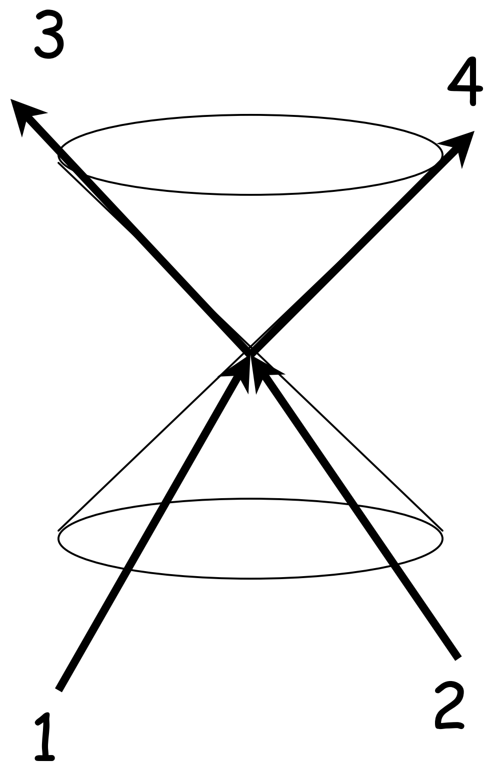
$1+2 \rightarrow 3$

Photon Collision Phenomena



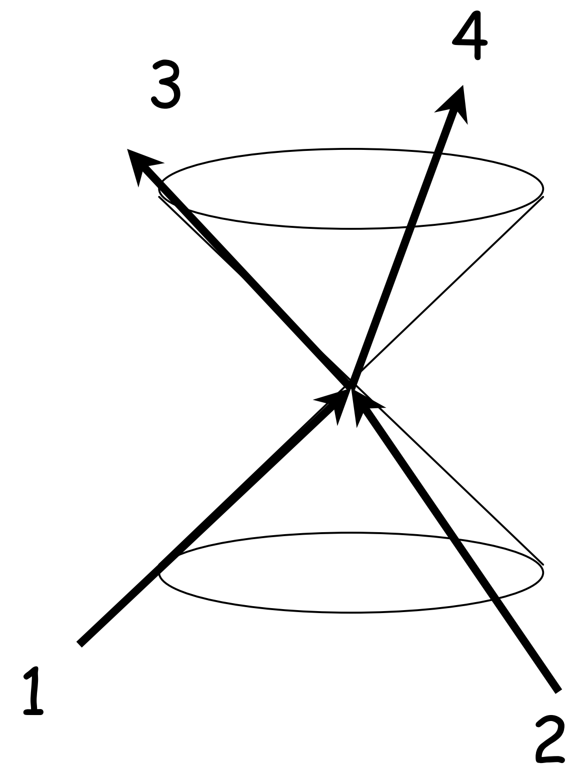
decay to 2 photons

$$1 \rightarrow \gamma + \gamma$$



annihilation

$$1 + 2 \rightarrow \gamma + \gamma$$



Compton scattering

$$\gamma + 2 \rightarrow \gamma + 4$$

Inelastic Collisions

- The rest mass of a system must change in an inelastic collision if conservation of momentum is to hold in all inertial reference frames.
- Classically, kinetic energy is lost in such a collision.
- If the total relativistic energy is conserved, the loss in kinetic energy equals the gain in rest energy of the system.
- One awkward feature of Newtonian theory is that mechanical energy is lost unless the collision is perfectly elastic. Some energy is converted into heat, and is no longer described mechanically.
- Since a hot object has greater rest energy than a cold object of the same type, this difficulty is avoided in a simple and natural way in special relativity.

Example: Inelastic Collision

conservation of energy $E_i = E_1 + E_2 = E_f$

$$KE_i = KE_1 + KE_2$$
$$= \left(E_1 - m_1c^2\right) + \left(E_2 - m_2c^2\right)$$

$$KE_f = \left(E_f - m_0c^2\right)$$

$$KE_i - KE_f = \left(E_1 + E_2 - m_1c^2 - m_2c^2\right) - \left(E_f - m_0c^2\right)$$
$$\Delta KE = \left[m_0 - \left(m_1 + m_2\right)\right]c^2 = \left(\Delta m\right)c^2$$

Loss in kinetic energy equals gain in rest-mass energy

Binding Energy

- The conversion of rest energy to kinetic energy with a corresponding loss in rest mass is a common occurrence in radioactive decay and nuclear reactions, including nuclear fission and nuclear fusion.
- If the potential energy is negative corresponding to attraction between the masses, the rest mass of the combined system is less than that of the separate masses.
- The difference in the rest energy for this case is called the binding energy of the system.
- In order to separate the masses, energy equal to the binding energy must be supplied.
- The binding energies of atoms and molecules are of the order of a few electron volts, which leads to a negligible difference in mass.
- The binding energies of nuclei are of the order of several MeV, which leads to a noticeable difference in mass.

Deutron Binding Energy

- A deutron consists of a proton and a neutron bound together.
- It is the nucleus of the deuterium atom, which is an isotope of hydrogen called heavy hydrogen and written ^2H.
- How much energy is required to separate the proton from the neutron in the deuteron?
  - 1875.63 MeV rest energy of deutron,
  - 938.28 MeV rest energy of proton,
  - 939.57 MeV rest energy of neutron,
  - 938.28 + 939.57 = 1877.75 MeV sum of rest energies of proton and neutron,
  - 1877.85 – 1875.63 = 2.22 MeV rest energy of parts greater than rest energy of deutron.
- The energy need to break up a nucleus into its constituent parts is called the binding energy of the nucleus.
- The binding energy of the deuteron is 2.22 MeV.
- This is the energy that must be added to the deutron to break it up into a proton plus a neutron.
- This can be done by bombarding deuterons with energetic particles or with electromagnetic radiation with energy of at least 2.22 MeV.

Making a Deutron

- When a deutron is formed by the combination of a neutron and proton, energy must be released
- When neutrons from a reactor collide with protons, some neutrons are captured to form deuterons.
- In the capture process, 2.22 MeV of energy is released, usually in the form of electromagnetic radiation.

Nuclear Fusion Reaction

- A tritium nucleus (^3H) and a deuterium nucleus (^2H) fuse together to form a helium nucleus (^4He) plus a neutron.
- The reaction is written $$^2H + ^3H \rightarrow ^4He + n$$
- How much energy is released in this fusion reaction?
  - 1875.628 MeV rest energy of deutron,
  - 2808.944 MeV rest energy of tritium,
  - 3727.409 MeV rest energy of helium,
  - 939.573 MeV rest energy of neutron,
  - 1875.628 + 2808.944 = 4684.572 MeV sum of rest energies of deutron and tritium,
  - 3727.409 + 939.573 = 4666.982 MeV sum of rest energies of helium and neutron,
  - 4684.572 – 4666.982 = 17.59 MeV energy released in reaction.
- This and other fusion reactions occur in the Sun and are responsible for the energy supplied to the Earth.
- As the Sun gives off energy, its rest mass continually decreases.

Energy-Momentum Four-Vector

$$P = \left(E/c, p_x, p_y, p_z\right) = \left(E/c, \vec{p}\right)$$
$$p^\mu = \left(p_0, p_1, p_2, p_3\right) = \left(p_0, \vec{p}\right)$$

$$P^2 = \frac{E^2}{c^2} - p_x^2 - p_y^2 - p_z^2 = \frac{E^2}{c^2} - \vec{p}^{\,2} = (mc)^2$$
$$p^2 = p_0^2 - p_1^2 - p_2^2 - p_3^2 = p_0^2 - \vec{p}^{\,2} = (mc)^2$$

m of an elementary particle is an invariant

$$E^2 = (\vec{p}c)^2 + (mc^2)$$

Lorentz Transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2 / c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - xv / c^2}{\sqrt{1 - v^2 / c^2}}$$

$$p'_x = \frac{p_x - vE / c^2}{\sqrt{1 - v^2 / c^2}}$$

$$p'_y = p_y$$

$$p'_z = p_z$$

$$E' = \frac{E - p_x v}{\sqrt{1 - v^2 / c^2}}$$

$(x,y,z) \rightarrow (p_x,p_y,p_z)$ and $t \rightarrow E / c^2$

Module 17

Photons and Antimatter

Massless Particles

In classical mechanics

$$\text{if } m = 0, \quad \vec{p} = m\vec{v} \quad \Rightarrow \quad \vec{p} = 0$$

$$\text{and} \quad KE = \frac{1}{2}mv^2 \quad \Rightarrow \quad KE = 0$$

If a particle has no momentum and no kinetic energy is it a particle? Does it exist?

In relativity

$$\text{if } m = 0, \quad E^2 = (pc)^2 + (mc^2)^2 \quad \Rightarrow \quad E = pc$$

$$\text{if } m = 0, \quad \beta = \frac{v}{c} = \frac{pc}{E} = 1 \quad \text{or} \quad v = c$$

A massless particle travels at the speed of light. Conversely a particle traveling at the speed of light must be massless.

Photons

$$\text{Since } v \to c \quad \Rightarrow \quad \gamma \to \infty,$$

$$m \to 0 \quad \Rightarrow \quad \vec{p} = \gamma m \vec{v} \neq 0 \quad \text{and} \quad E = \gamma mc^2 \neq 0$$

- These definitions of momentum and energy fail for massless particles, but they do not contradict the possible existence of massless particles.
- The most important example of a massless particle is the photon, or a particle of light.
- In classical mechanics, light is a wave.
- In relativity, light is a photon, having momentum and energy like other particles (but they travel only at the speed of light and have no mass).
- Other possible massless particles are the neutrinos and the graviton.

Photoionization

- Light shining on an atom can tear loose one of the atom's electrons.
- In the process the photon is absorbed and disappears.

$$\gamma + \mathrm{H} \rightarrow \mathrm{e} + \mathrm{p}$$

H, e, and p have mass, and hence a well defined momentum and energy.
If momentum and energy is conserved we can calculate the momentum and energy of the photon from the known values for the other three particles.

Matter and Antimatter

- For every elementary particle known to exist, there is a corresponding antimatter particle that has precisely the same mass and exactly the opposite charge.
- For example the electron. Since the anti-electron has a positive charge, it is often referred to as the positron.
- Anitmatter is frequently created in accelerators, where particles collide at speeds approaching the speed of light.
- It is possible to create anti-atoms in the lab made entirely of antimatter.
- An intriguing possibility is that the universe may actually contain entire anti-galaxies of antimatter.

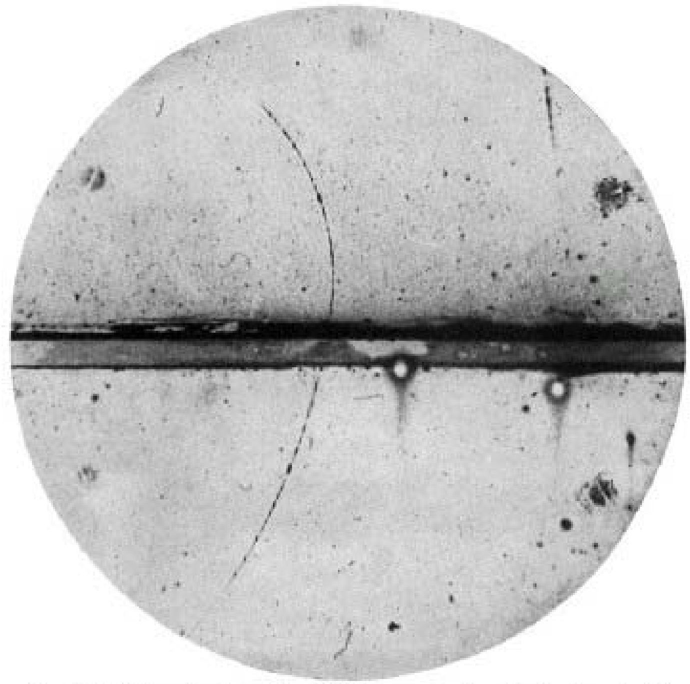
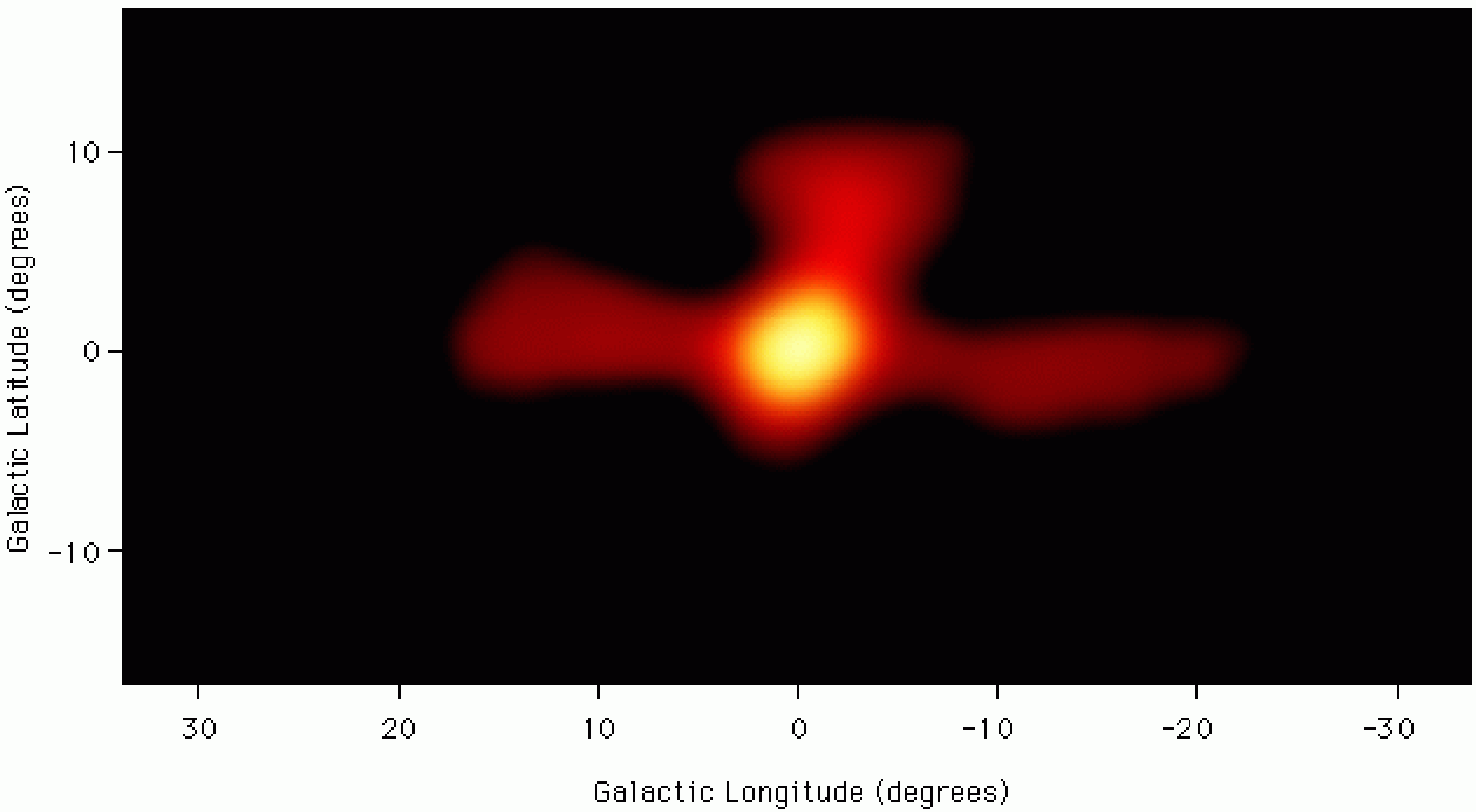


FIG. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

Antimatter Clouds

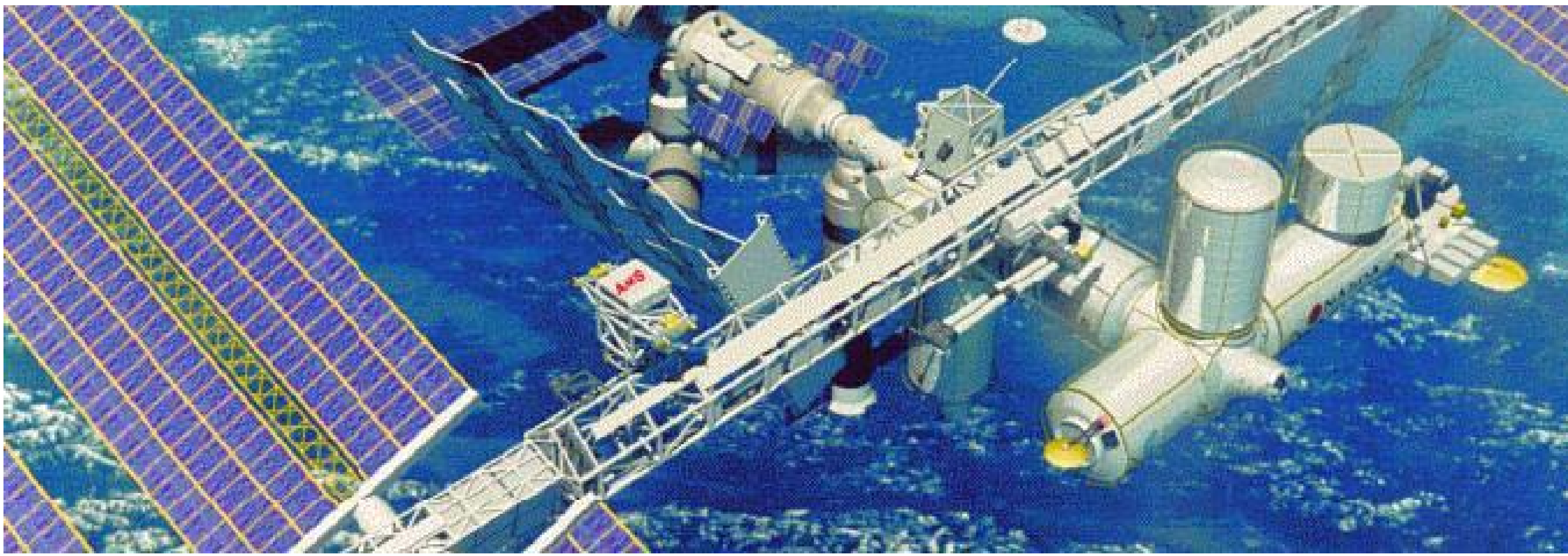


Caption: Map of the distribution of positrons towards the center of the Milky Way Galaxy, including the newly discovered antimatter "cloud". The brightest feature corresponds to the nucleus of the Galaxy. The horizontal structure lies along the plane of the Galaxy. The antimatter "cloud" is located above the Galactic center.

Courtesy of D. D. Dixon (University of California, Riverside) and W. R. Purcell (Northwestern University)

Alpha Magnetic Spectrometer

Search in space for dark matter, missing matter and antimatter on the international space station.



AMS to be launched October 2005.

Interpretation of Antimatter

Quantum mechanics (relativistic quantum mechanics) tells us that the energy can be negative as well as positive.

$$E^2 = (pc)^2 + (mc^2)^2$$

$$E = \pm\sqrt{(pc)^2 + (mc^2)^2}$$

A negative-energy particle of 4-momentum *(-E,-p)* and electric charge e appears as a positive-energy particle of 4-momentum *(E,p)* and charge $-e$.

A negative energy solutions to the quantum-mechanical wave equation traveling backwards in time is a positive energy anti-particle traveling forward in time.

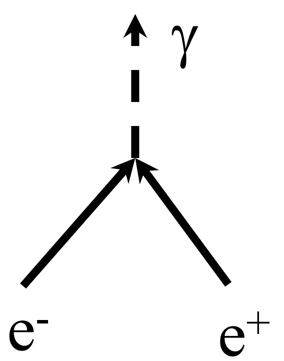
Matter and Antimatter Annihilation

- When particles of matter and antimatter meet, they annihilate one another.
- The result is that the particles cease to exist.
- This satisfies charge conservation, since the net charge of the system is zero before and after the annihilation.
- For energy conservation, the mass of the two particles is converted into two gamma rays.
- Thus, in matter-antimatter annihilations the particles vanish into a burst of radiation.
- This is why there is no natural abundance of anitmatter in our region of the universe.

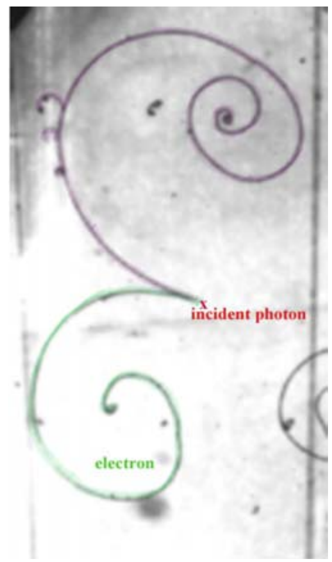
Pair Annihilation and Production

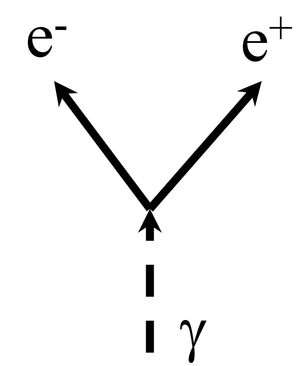
e^- negatively charged electron (mater)
e^+ positively charge positron (antimatter)

electromagnetic
radiation



pair annihilation





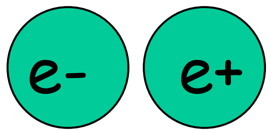
electromagnetic
radiation

pair production

Proton Antiproton Accelerator

Electron-Positron Annihilation

before



A positron is the anti-particle of the electron, same mass but opposite charge.

Total momentum 0, total energy $2mc^2$.

after

γ ← - - - - - - - → γ two photons

E_1, p_1 E_2, p_2

conservation of momentum $\vec{p}_1 + \vec{p}_2 = 0$

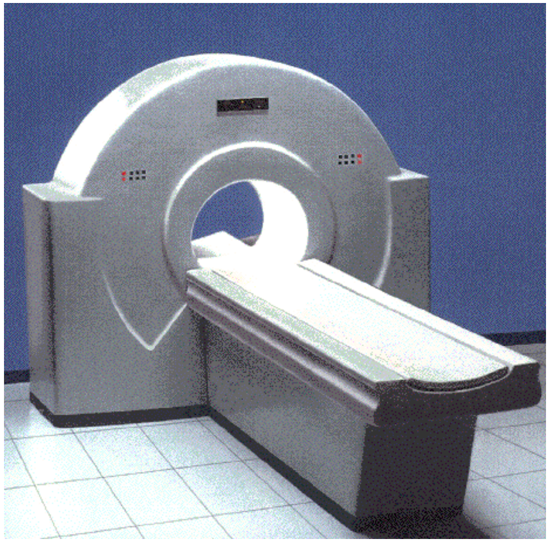
conservation of energy $E_1 + E_2 = 2mc^2$

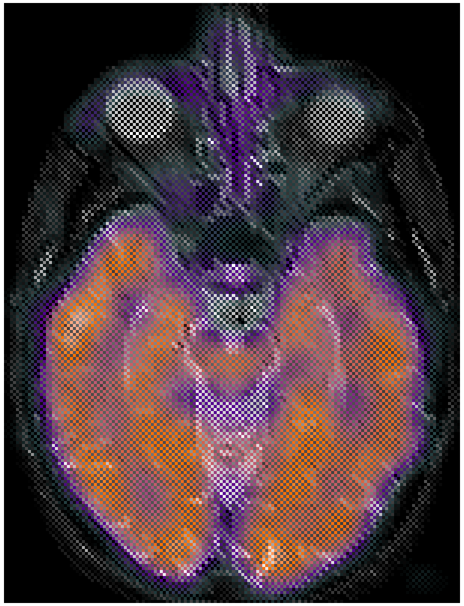
$$E_1 = E_2 = mc^2 = 0.551 \text{ MeV}$$

All the rest energy is 100% converted into electromagnetic energy.

Positron Emission Tomograph (PET)

- Application of electron-positron annihilation.
- Patient is injected with solution containing a positron-emitting radioactive element.
- A suitably chosen solution is attracted to an area of concern, which then emit positrons.
- The positrons quickly annihilate with nearby electrons, and the outcoming photons can be monitored by a ring of detectors.
- This way it is possible to make an accurate map of the areas of interest in the patient.





Module 18

Relativistic Optical Effects

The Drag Effect

- To what extent will a flowing liquid “drag” light along with it?
- We know that flowing air totally drags sound along it.
- Expectation before relativity:
  - Since light is a disturbance of the ether and not of a liquid, there could be no drag effect for light.
- But experiment indicates that there is a drag effect:
  - A liquid seemed to force the ether along with it, but only partially.

Experimental Result of Drag Effect

It was found from experiment that

$$u = u' + kv, \quad k = 1 - \frac{1}{n^2}$$

u speed of light relative to stationary observer,
u' speed of light relative to liquid,
v speed of liquid,
k drag coefficient,
n refractive index of liquid.

recall $u' = \frac{c}{n}$

Relativistic Explanation of the Drag Effect

In relativity the previous experimental result is just the velocity addition formula

$$u = \frac{u' + v}{1 + u'v/c^2}$$

recall $u' = \dfrac{c}{n}$

$$u = \frac{(c/n) + v}{1 + v/cn} = \frac{c}{n}\left(1 + \frac{vn}{c}\right)\left(1 + \frac{v}{cn}\right)^{-1} = \frac{c}{n}\left(1 + \frac{vn}{c}\right)\left(1 - \frac{v}{cn} + \cdots\right)$$

for $v << c$, to first order in v/c

$$u \approx \frac{c}{n}\left(1 + \frac{vn}{c} - \frac{v}{cn}\right) = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)v = u' + kv$$

Classical Doppler Effect

- Characteristic of all wave motion.
- The change in frequency depends on whether it is the source or the receiver that is moving.
  - Such a distinction is possible because there is a medium (the air) relative to which the motion takes place.
  - Such a distinction can not be made for light in a vacuum.
- The frequency, and thus colour, of light will change.

Classical Doppler Effect Formula

- Change in frequency when source and listener/observer are in relative motion.

f_0 frequency emitted by source
f frequency observed
v speed of source or observer

classical moving source

$$\frac{f}{f_0} = \frac{1}{\left(1 \mp \frac{v}{c}\right)}$$

- toward, + away

classical moving observer

$$\frac{f}{f_0} = \left(1 \pm \frac{v}{c}\right)$$

+ toward, - away

Problem with Classical Doppler Effect

classical
moving observer

$$\frac{f}{f_0} = 1 \pm \frac{v}{c}$$

classical
moving source

$$\frac{f}{f_0} = \frac{1}{\left(1 \mp \frac{v}{c}\right)} = \left(1 \mp \frac{v}{c}\right)^{-1} = 1 \pm \frac{v}{c} \mp \left(\frac{v}{c}\right)^2 \pm \cdots \approx 1 \pm \frac{v}{c}$$

The classical result distinguishes between moving source and moving observer to second order in v/c.

This is contrary to the first postulate of special relativity.

Relativistic Doppler Effect

Relativity has an added correction due to time dilation.

relativistic
$$\frac{f}{f_0} = \frac{1}{\gamma\left(1 \mp \frac{v}{c}\right)} = \sqrt{\frac{1 \pm \frac{v}{c}}{1 \mp \frac{v}{c}}}$$
upper sign: approaching
lower sign: receding

$$\frac{f}{f_0} = \left(1 \pm \frac{v}{c}\right)^{1/2}\left(1 \mp \frac{v}{c}\right)^{-1/2} = \left(1 \pm \frac{1}{2}\frac{v}{c} \pm \cdots\right)\left(1 \pm \frac{1}{2}\frac{v}{c} \mp \cdots\right) \approx 1 \pm \frac{v}{c}$$

Same as classical results.

Generalized Doppler Effect

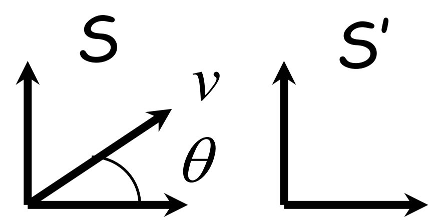
If the relative motion is perpendicular to the line joining the source and observer, there is no classical Doppler effect.

Time dilation still occurs, and thus there is a transverse relativistic Doppler effect.

$$\frac{f}{f_0} = \frac{1}{\gamma}$$

Thus, in general

$$\frac{f}{f_0} = \frac{1}{\gamma\left(1 \mp \frac{v}{c}\cos\theta\right)}$$



where θ is angle between the line joining the source and the observer, and the direction of the motion.

Twin Paradox Revisited

- Two 11-year old twins.
- One twin travels at a speed of $(4/5)c$ over a proper distance of 20.0 ly and returns.
- They compare ages.
  - The astronaut is $11 + 30 = 41$ years old.
  - The stay-at-earth twin is $11 + 50 = 61$ years old.
- Every year on their birthdays, each twin sends each other a greeting in the form of a light pulse. The frequency is, $f = 1$ b/yr.

Birthday Greeting Received by Astronaut

On the outbound trip

$$f_r = f_s\sqrt{\frac{1-\beta}{1+\beta}} = \left(1\frac{\text{b}}{\text{yr}}\right)\sqrt{\frac{1-(4/5)}{1+(4/5)}} = \frac{1}{3}\,\text{b/yr}$$

(15 yr)(1/3 b/yr) = 5 b.

On the inbound trip

$$f_r = f_s\sqrt{\frac{1+\beta}{1-\beta}} = \left(1\frac{\text{b}}{\text{yr}}\right)\sqrt{\frac{1+(4/5)}{1-(4/5)}} = 3\,\text{b/yr}$$

(15 yr)(3 b/yr) = 45 b

Astronaut receives a total 5 + 45 = 50 birthday greetings and expects stay-at-earth twine to be 50 + 11 = 61 year old.

Birthday Greetings Received by Stay-at-earth Twin

On the outbound trip

$$f_r = \frac{1}{3}\,\text{b/yr} \quad \text{but} \quad t = \frac{d}{v} = \frac{20.0c \cdot \text{yr}}{(4/5)c} = 25.0\ \text{yr}$$

When the astronaut turns around the stay-at-earth twin does not immediately start receiving the greetings at a higher frequency.

There are a large number of greetings still in flight on their way back to earth.

Not until the last of the lower frequency greetings has reached earth does the stay-at-earth twin start receiving the higher-frequency flashes.

It takes the last low-frequency flash 20 years to get back to earth.

(25 + 20 yr)(1/3 b/yr) = 15 b.

Birthday Greetings Received by Stay-at-earth Twin

On the inbound trip $f_r = 3\ \text{b/yr}$

But only 50 – 45 = 5 years left.

$(5\ \text{yr})(3\ \text{b/yr}) = 15\ \text{b}$

Stay-at-earth twin receives a total 15 + 15 = 30 birthday greetings and expects astronaut to be 30 + 11 = 41 year old.

Frequencies of Star Light

- A star emits and absorbs light at certain frequencies that are characteristic of the elements in the star.
- Thus, by analyzing the spectrum of light from a star, one can identify which elements it contains.
- By seeing if the characteristic frequencies are shifted up or down (as compared to those from a source at rest in the observatory) one can tell if the star is moving toward or away from us.

Redshift

- Hubble found that the light from distant galaxies is shifted down in frequency, or redshifted (since red is at the low-frequency end of the visible spectrum), indicating that most galaxies are moving away from us.
- Hubble also found that the speeds of recession of galaxies are roughly proportional to their distance from us.
- This implies that the universe is expanding uniformly .
- It also provides a convenient way to find the distance of many galaxies, since measurement of a Doppler shift is usually much easier than the direct measurement of distance.

Redshift: $z = \Delta f / f$

Example: Redshift

The longest wavelength of light emitted by hydrogen in the Balmer series has a wavelength of $\lambda_0 = 656$ nm.

In light from a distant galaxy, this wavelength is measured to be $\lambda = 1458$ nm.

Find the speed at which the distant galaxy is receding from Earth.

$$\frac{f}{f_0} = \sqrt{\frac{1-\beta}{1+\beta}} \quad \Rightarrow \quad \frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\beta = \frac{\left(\frac{\lambda}{\lambda_0}\right)^2 - 1}{\left(\frac{\lambda}{\lambda_0}\right)^2 + 1} = 0.663$$

The galaxy is thus receding at a speed of $0.663c$.

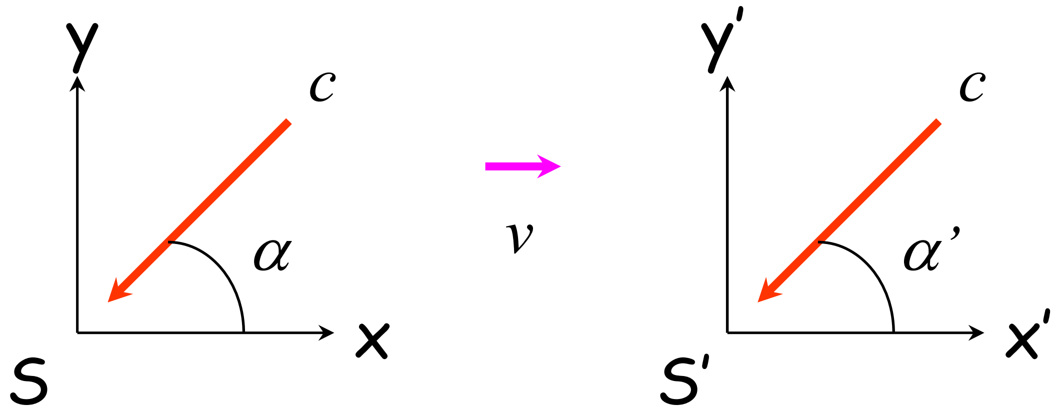
The shift towards longer wavelengths of light from distant galaxies that are receding from us is called the redshift.

Aberration

- If I drive into the rain, it seems to come at me obliquely.
- For similar reasons, if two observers measure the angle which an incoming ray of light makes with their relative line of motion, their measurements will generally not agree.
- This is aberration (well known before relativity).

Aberration Coordinate System

Consider an incoming light signal whose negative direction makes angles α and α' with the x-axes of the usual two frames S and S′, respectively.



$$u_x = -c\cos\alpha \qquad u'_x = -c\cos\alpha'$$

$$u_y = -c\sin\alpha \qquad u'_y = -c\sin\alpha'$$

Aberration Velocity Transformations

$$u'_x = \frac{u_x - v}{1 - vu_x / c^2}$$

velocity transformation formula along x-axis

$$u'_y = \frac{-u_y}{\gamma[1 - vu_x / c^2]}$$

velocity transformation formula along y-axis

$$\cos\alpha' = \frac{\cos\alpha + v/c}{1 + (v/c)\cos\alpha}$$

velocity transformation formula along x-axis

$$\sin\alpha' = \frac{\sin\alpha}{\gamma[1 + (v/c)\cos\alpha]}$$

velocity transformation formula along y-axis

Aberration Formula

Using the trigonometric identity $\tan\frac{\alpha'}{2} = \frac{\sin\alpha'}{1+\cos\alpha'}$

$$\tan\frac{\alpha'}{2} = \sqrt{\frac{1-\beta}{1+\beta}}\tan\frac{\alpha}{2}$$

Visual Appearance of Objects

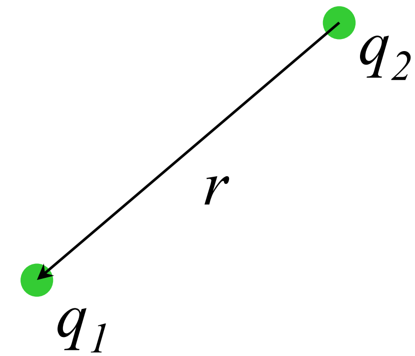
- Aberration implies, that as the earth travels along its orbit, the apparent directions of the fixed stars trace out small ellipses in the course of each year.
- Aberration also causes certain distortions in the visual appearance of extended objects, as the observer moves past the conical pattern of rays converging from objects to the observer's eye, rays from its different points are unequally aberrated.
- Alternatively, from the viewpoint of the observer's rest frame, the light from different parts of the moving object takes different times to reach the eye, and thus it was emitted at different past times: the more distant points of the object consequently appear displaced relative to the nearer points in the direction opposite to the motion.

Module 19

Electromagnetism

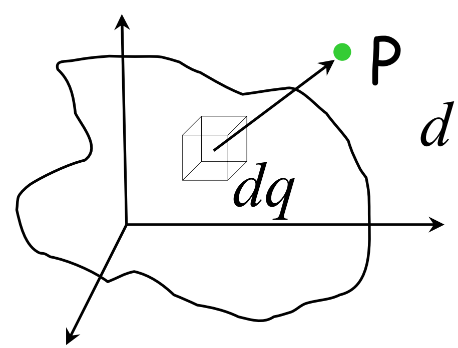
Static Electric Charges

Coulomb's Law $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$



Define electric field $\vec{E} = \frac{\vec{F}}{q_0}$

Distribution of charge

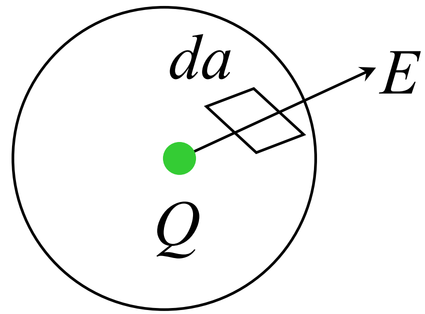


$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_{i0}^2}\hat{r}_{i0} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq}{r^2}\hat{r}$$

Gauss's Law

Gauss's Law $\int_S \vec{E} \cdot \hat{n} da = \frac{Q}{\varepsilon_0}$



Charge density $Q = \int_V \rho dV$

Divergence theorem,
for arbitrary vector A $\int_S \vec{A} \cdot \hat{n} da = \int_V \vec{\nabla} \cdot \vec{A} dV$

In one dimension $\vec{\nabla} = \frac{\partial}{\partial r} \hat{r}$

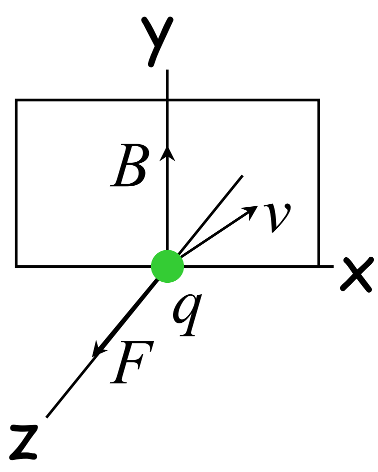
$$\int_S \vec{E} \cdot \hat{n} da = \int_V \vec{\nabla} \cdot \vec{E} dV = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_V \rho dV \quad \Rightarrow \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Magnetic Field

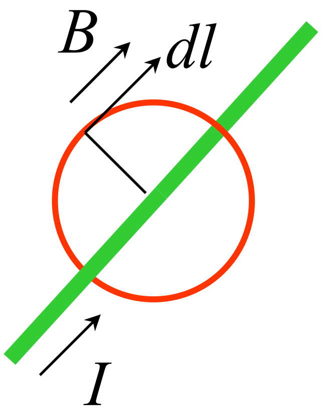
A moving charge produces a magnetic field and in turn exerts a force on other moving charges.

Magnetic field of moving charge $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{q\vec{v} \times \hat{r}}{r^2}$

Magnetic force on a moving charge $\vec{F} = q\vec{v} \times \vec{B}$



Ampere's Law



$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

Current density $I = \int_S \vec{J} \cdot \hat{n}\, da$

Stoke's theorem,
for arbitrary vector A

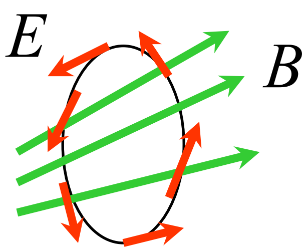
$$\oint_C \vec{A} \cdot d\vec{\ell} = \int_S \left(\vec{\nabla} \times \vec{A}\right) \cdot \hat{n}\, da$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \int_S \left(\vec{\nabla} \times \vec{B}\right) \cdot \hat{n}\, da = \mu_0 I = \mu_0 \int_S \vec{J} \cdot \hat{n}\, da \quad \Rightarrow \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Faraday's Law

Changing magnetic field induces an emf in a coil.

Electromotive force $\text{Emf} = \oint_C \vec{E} \cdot d\vec{\ell}$



Faraday's law $\text{Emf} = \oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$

Magnetic flux $\phi_m = \int_S \vec{B} \cdot \hat{n} da$

$$\oint_C \vec{E} \cdot d\vec{\ell} = \int_S \left(\vec{\nabla} \times \vec{E}\right) \cdot \hat{n} da = -\frac{d\phi_m}{dt} = -\frac{d}{dt}\int_S \vec{B} \cdot \hat{n} da = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} da$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Relativity and Electromagnetism

- There is no nonrelativistic electromagnetism.
- Maxwell's theory of electricity and magnetism has always been relativistic.
- The concepts of electric field (E) and magnetic field (B) are not absolute but are only relative.
- A test charge that is moving with respect to one frame, and therefore subject to a magnetic force, is stationary with respect to some other frame and therefore experiences no magnetic force.
- One observer may say there is a magnetic field, another may not.
- They will also disagree on the value of the electric field.
- A pure E field or a pure B field in one frame has both E and B components in another frame.
- Two observers agree in their measures of force per unit length.
  - The part of the force that is independent of the velocity of a test charge will be due to E.
  - The part of the force dependent on the velocity of the test charge will be due to B.

Invariance of Electric Charge

- We my venture to guess that an electric charge Q coulombs that is stationary with respect reference frame S would appear to carry either $Q\gamma$ or Q/γ coulombs for an observer in frame S′.
- This is wrong, electric charge is invariant, and a charged body carries the same electrical charge for all observers.

Electric and Magnetic Forces

Electric force on test charge q due to
a collection of electric charges at rest.

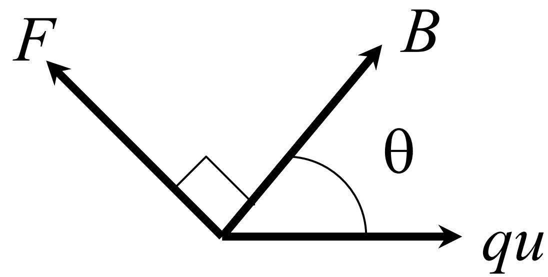
$$\vec{F} = q\vec{E}$$

Collection of moving charge produce magnetic field B.

Magnetic force on test charge q moving
with velocity u due fixed field B.

$$\vec{F} = q\vec{u} \times \vec{B}$$

$$F = quB\sin\theta$$



E and B are defined as fixed relative to the observer.

Electromagnetic Force

Lorentz force $\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$

Consider the case of a uniform magnetic field only $\vec{F} = q(\vec{u} \times \vec{B})$

F is perpendicular to u. u is constant but changes direction. The body moved in a circle perpendicular to B.

$$\frac{d\vec{p}}{dt} = q\vec{u} \times \vec{B}$$

$$\gamma m \frac{d\vec{u}}{dt} = q\vec{u} \times \vec{B}$$

$$\gamma m a = quB$$

$$\gamma m \frac{u^2}{R} = quB$$

$$R = \frac{p}{qB}$$

Same as classical result, where p is now the relativistic momentum.

Convenient way to measure momentum of a particle of known charge q.

Example: Momentum Measurement

A proton of unknown momentum p is sent through a uniform magnetic field $B = 1.0$ tesla (T), perpendicular to p, and is found to move in a circle of radius $R = 1.4$ m.

What are the proton's momentum in MeV/c and energy in MeV?

The proton's charge is $q = +e = 1.6 \text{ x } 10^{-19}$ coulomb (C).

$$p = qBR = (1.6 \times 10^{-19}\,\text{C}) \times (1.0\text{T}) \times (1.4\text{m})$$
$$= 2.24 \times 10^{-19}\,\text{kg} \cdot \text{m/s}$$

$$1\,\text{MeV}/c = 5.34 \times 10^{-22}\,\text{kg} \cdot \text{m/s}$$

$$p = (2.24 \times 10^{-19}\,\text{kg} \cdot \text{m/s}) \times \frac{1\,\text{MeV}/c}{5.34 \times 10^{-22}\,\text{kg} \cdot \text{m/s}} = 420\,\text{MeV}/c$$

$$\text{Since } m = 938\,\text{MeV}/c^2 \quad \Rightarrow \quad E = \sqrt{(pc)^2 + (mc^2)^2} = 1030\,\text{MeV}$$

Measurement of the *e/m* Ratio

$$\frac{e}{m} = \frac{\gamma v}{RB} = \frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \frac{1}{RB}$$

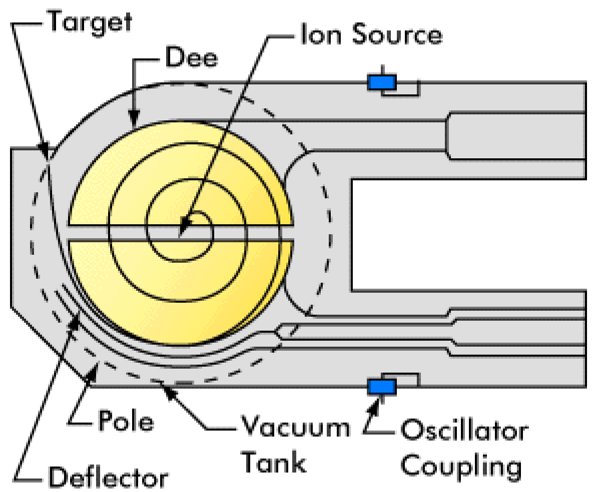
Knowing v, R, B we can measure e/m.

Particle Accelerators

$$v\gamma = \frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{eRB}{m}$$

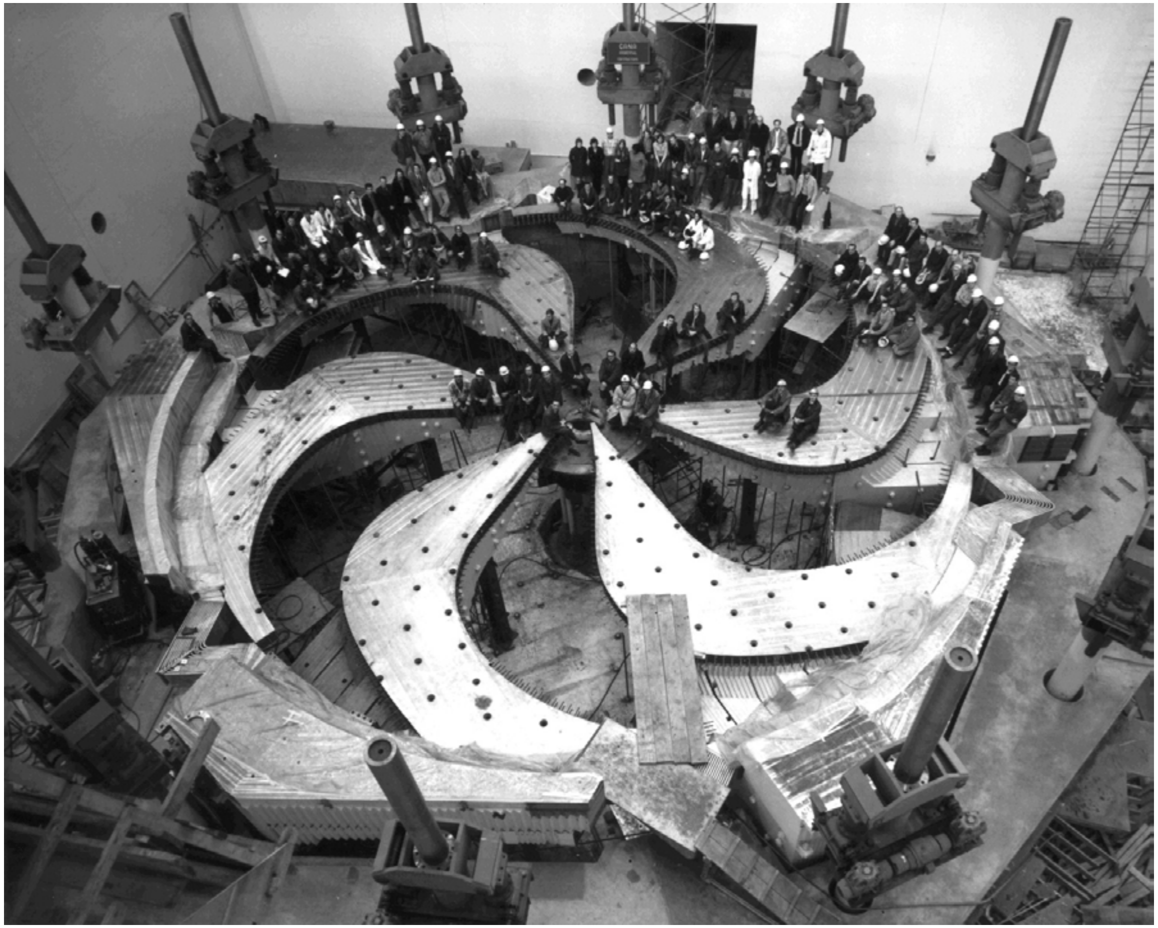
Cyclotron

- The cyclotron was the first particle accelerator (1929).
- A cyclotron consists of two large dipole magnets designed to produce a semi-circular region of uniform magnetic field, pointing uniformly downward.
- The two D's were placed back-to-back with their straight sides parallel but slightly separated.
- An oscillating voltage was applied to produce an electric field across this gap.
- Particles injected into the magnetic field region of a D trace out a semicircular path until they reach the gap.
- The electric field in the gap then accelerates the particles as they pass across it.
- The particles now have higher energy so they follow a semi-circular path in the next D with larger radius and so reach the gap again.
- The electric field frequency must be just right so that the direction of the field has reversed by their time of arrival at the gap.
- The field in the gap accelerators them and they enter the first D again.
- Thus the particles gain energy as they spiral around.



- When particles in a cyclotron gain energy they become more massive.
- This tends to slow them down and throws the acceleration pulses at the gaps between the D's out of phase.

TRIUMF Cyclotron in Vancouver



Built 1972. Currently largest cyclotron in the world

Synchrotron

- A synchrotron is a circular accelerator which has an electromagnetic resonant cavity to accelerate the particles.
- Particles pass through each cavity many times as they circulate around the ring, each time receiving a small acceleration, or increase in energy.
- When either the energy or the field strength changes so does the radius of the path of the particles.
- Thus, as the particles increase in energy the strength of the magnetic field that is used to steer them must be changes with each turn to keep the particles moving in the same ring.
- The change in magnetic field must be carefully synchronized to the change in energy or the beam will be lost.

Particle Accelerators

1931 accelerated protons
to 80 keV
13 cm diameter

2007 accelerate protons
to 14 TeV (14 x 10^9 keV)
27 km circumference

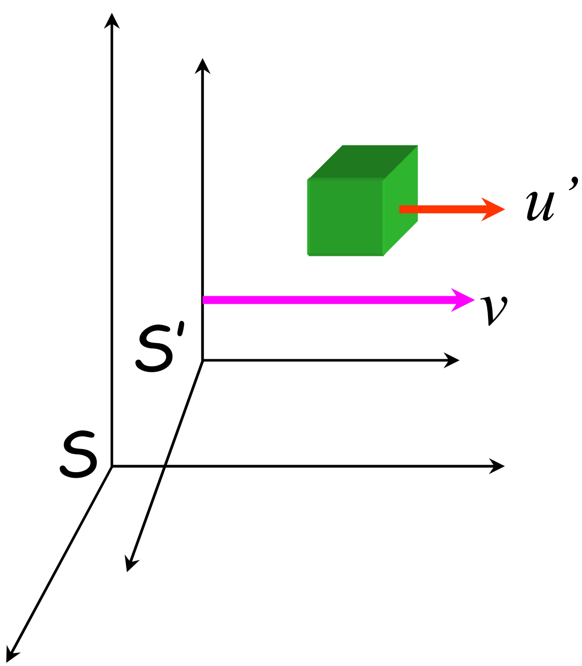
An Element of Volume

- An element of volume at rest has a volume dV for observer S.
- What is the volume for S′?
- Assume the element of volume is a cube of volume l_0^3 in S.
- Element of volume effected by Lorentz contraction only in direction of relative motion, say v along x-axis.

$$dV' = l_0^3\sqrt{1-(v/c)^2} = \frac{l_0^3}{\gamma} = \frac{dV}{\gamma}$$

A Moving Element of Volume

- We shall deal with elements of volume having a velocity u with respect to frame S and u' with respect tor frame S', along x-axis.



$$dV = l_0^3\left[1-(u/c)^2\right]^{1/2}$$

$$dV' = l_0^3\left[1-(u'/c)^2\right]^{1/2}$$

$$u' = \frac{u-v}{1-uv/c^2}$$

$$\boxed{dV' = \frac{dV}{\gamma\left(1-uv/c^2\right)}}$$

Electric Charge Density and Current

- Consider an element of volume that contains dn electric charges of Q coulombs each and that has a velocity u with respect to reference frame S.
- The total charge Qdn in the element of volume is the same for both observers:
  - Individual charges Q are the same for both observers, since Q is an invariant.
  - dn must also be invariant because it is simply a number of objects that both observers can count.

Charge Density

The charge density is $\rho = \frac{Qdn}{dV}$

dV is the element of volume as seen by S

$$dV' = \frac{dV}{\gamma\left(1 - uv/c^2\right)}$$

$$\boxed{\rho' = \rho\gamma\left(1 - uv/c^2\right)}$$

Current Density

- The quantity of ρv is the product of electric charge density, and of the velocity of these charges in the direction parallel to the x-axis.
- The product is the electric charge flowing per second and per square meter at the point considered, in the direction of the x-axis, or the electric current density j in ampers/meter^2.

$$j' = \rho' u' = \rho\gamma\left(1 - uv/c^2\right)\frac{u-v}{1-uv/c^2}$$
$$= \gamma\left(j - v\rho\right)$$

Four-Current Density

Lorentz Transformation of Current

$$\rho' = \gamma\left(\rho - v j_x / c^2\right)$$

$$j_x' = \gamma(j_x - v\rho)$$

$$j_y' = j_y$$

$$j_z' = j_z$$

Current Four-Vector

$$J = (c\rho, \vec{j})$$

Invariant Quantity

$$(J')^2 = (c\rho')^2 - (\vec{j'})^2 = J^2 = (c\rho)^2 - \vec{j}^{\,2}$$

Maxwell's Equations

$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ Gauss's Law

$\vec{\nabla} \cdot \vec{B} = 0$ No monopoles

$\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$ Ampere's Law

$\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$ Faraday's Law

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

$$J^\mu = (c\rho, \vec{j})$$

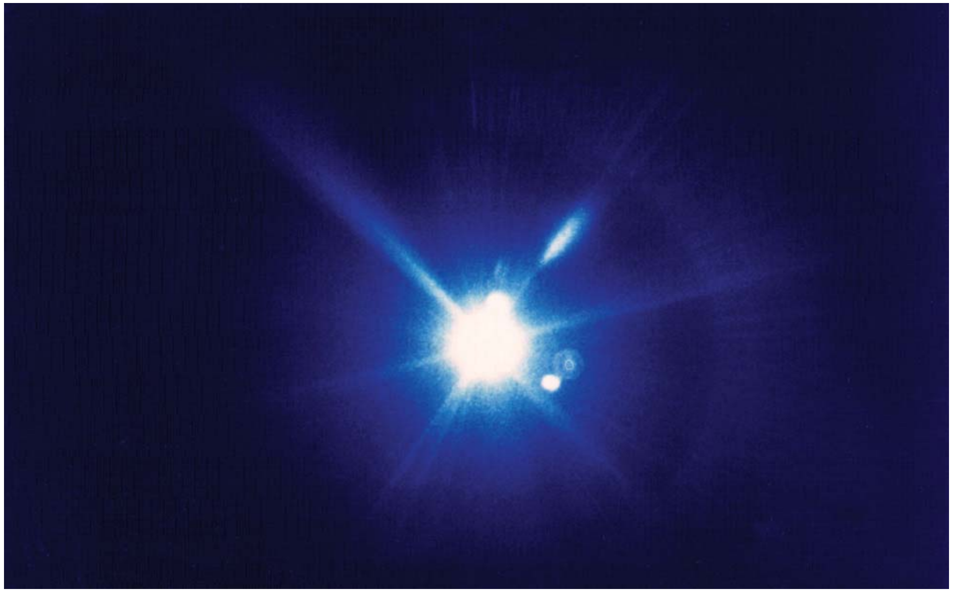
Maxwell field tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

$\partial_\nu F^{\mu\nu} = \frac{4\pi}{c} J^\mu$ Maxwell's Equations

Synchrotron Radiation

- Charged particles moving in circular paths at speeds near the speed of light emit radiation that is largely compressed into brief pulses in the direction of motion of the charged particle.
- In a high energy electron or positron storage ring these photons are emitted with energies ranging from infra-red to energetic (short wavelength) x-rays.



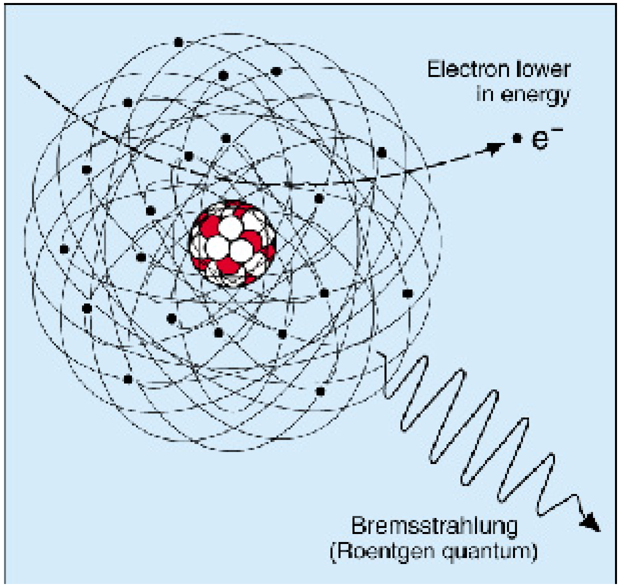
The energy loss for a given electron energy is proportional to γ^3.

Canadian Light Source

The National Synchrotron Facility Saskatoon Saskatchewan

Bremsstrahlung

- Suppose there are charged particles in a piece of matter and a very fast electron comes by.
- Because of the electric field around the atomic nucleus the electron is pulled, accelerated, so that the curve of its motion has a slight kink or bend in it.
- When very energetic electrons move through matter they spit radiation in a forward direction.



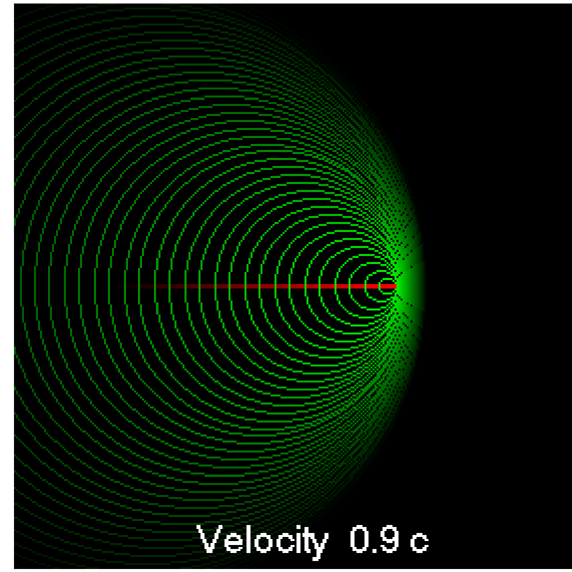
this is call bremsstrahlung

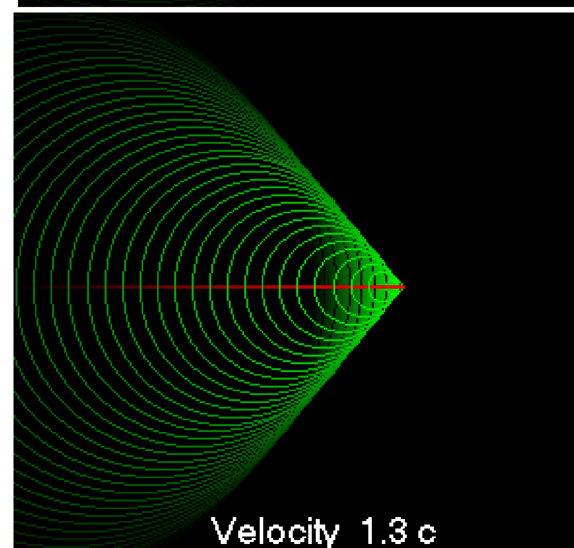
Sonic Boom

- A sonic boom is a shock wave which propagates from an aircraft or other object which is going faster than sound through the air (or other medium).

Sonic Boom for Light

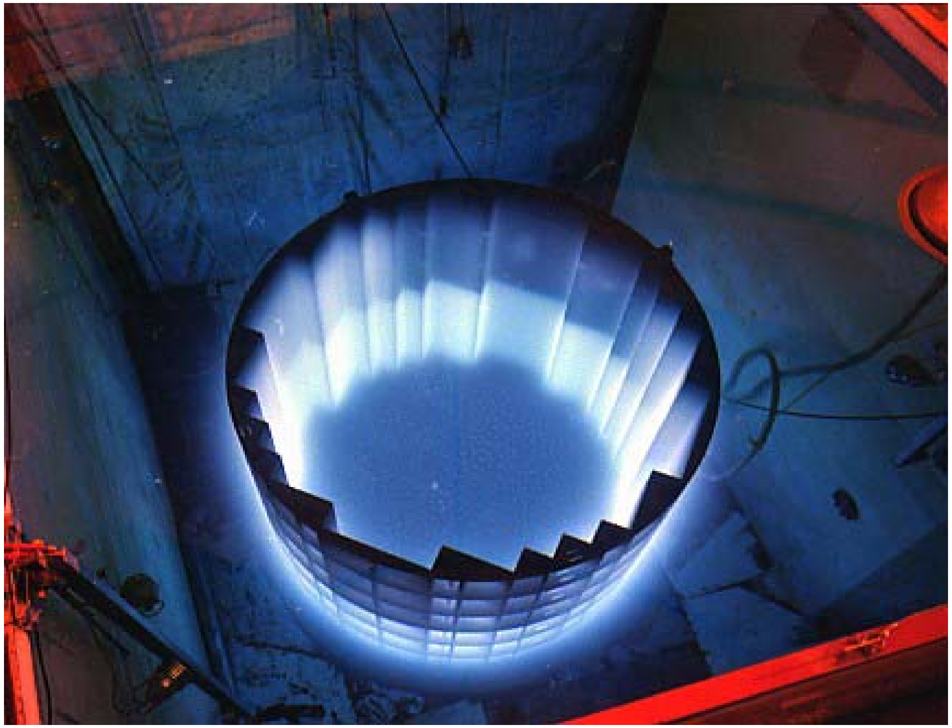
- There is no light equivalent of the sonic boom in a vacuum.
- In a medium such as water, the speed of light is considerably less than the speed of light in vacuum.
- So it is possible for a particle to travel though water or other media at a speed faster than the speed of light in that media.





Cherenkov Radiation

- When a charged particle does so, a faint radiation is produced from the medium.
- The light propagates away in a cone forward of the region.
- The blue glow in the water surrounding nuclear reactors is Cherenkov radiation.
- The water is there to stop neutrons but neutrons are uncharged and do not directly cause the radiation.
- It actually comes form the beta particle (fast electrons) which are emitted by fission produces.



Module 20

General Theory of Relativity

The General Theory of Relativity

- In special relativity we saw that the laws of physics must be the same in all inertial reference frames.
- What is so special about inertial reference systems?
- The inertial reference frames are, in a sense, playing the same role as Newton's absolute space.
- Shouldn't the laws of physics be the same in all coordinate systems, whether inertial or non-inertial?
- The inertial frame should not be such a privileged frame.
- We must show that even all accelerated motions are relative.
- The general theory of relativity: the laws of physics are the same in all frames of reference.
- Developed by Einstein in 1915.
- Extension of special relativity that includes the effects of gravity.

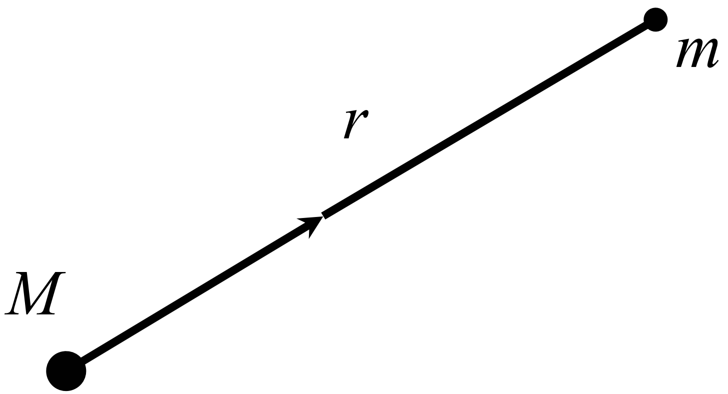
Applicability of General Relativity

- For small masses the differences between Einstein's and Newton's theories are negligible.
- Even at solar masses the differences are very small.
- GR only of practical importance for very large dense masses or situations requiring high precision.
- Important for cosmology, the study of the structure and evolution of the whole universe.

Newtonian Theory of Gravitation

Newton's inverse square law

$$|\vec{F}| = G\frac{mM}{r^2}$$



$$G = 6.670\times10^{-8}\,\mathrm{cm}^3\mathrm{gm}^{-1}\mathrm{s}^{-2}$$

Gravitational Potential

- Only spherically symmetric objects can be idealized as point masses.
- We wish to derive an expression for the gravitational force exerted on a test particle by an extended non-spherical mass distribution.
- Regard matter distribution as creating a gravitational field, which exerts a force on any test particle in its domain.
- Law of superposition: the force exerted on a test particle by a collection of point objects is the sum of the force exerted by the objects separately.

Gravitational Potential

The gravitational force is the gradient of a scalar field.

$$\vec{F} = -m\vec{\nabla}\Phi$$

potential function of the gravitational field

All physics of the situation can be deduced from the potential.

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

gradient operator in Cartesian coordinates

For a spherically symmetric mass M, the exterior gravitational potential is

$$\Phi = -G\frac{M}{r}$$

Example

- If one takes into account the rotation of the sun, there is a flattening at the poles, and one does not have exact spherical symmetry.
- One says the sun has a quadruple moment.
- In this case, a more accurate expression for the gravitational potential is

$$\Phi = -G\left(\frac{M}{r} + \frac{Q}{2r^3}(1 - 3\cos^2\theta)\right)$$

Equation of Motion

- For a test particle of mass m moving in a gravitational field

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}$$

$$\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}\Phi$$

Newtonian's equation of motion for a test particle moving in a gravitational field with potential Φ

The motion of a test particle in a given gravitational field is independent of the mass and composition of the particle.

How does a test particle move in a given gravitational field?

Gravitational Field Equations

- Partial differential equations that the gravitational potential satisfy.

$$\nabla^2\Phi = 0$$ outside matter

$$\nabla^2\Phi = 4\pi G\rho$$ inside matter

ρ is the matter density

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ Laplacian operator in Cartesian coordinates

What is the nature of the gravitational field created by a given matter distribution?

Inertial and Gravitational Mass

- Initial mass: occurs as the ratio between force and acceleration in Newton's second law and thus measures a particle's resistance to acceleration.
- Gravitational mass: gravitational analog of the electric charge.
- For all particles in nature the inertial and gravitational masses are in the same proportion, and are made equal by a suitable choice of units.
- This proportionality is sometimes called the "weak" equivalence principle.

Weak Equivalence Principle

- All particles experience the same acceleration in a given gravitational field.
  - The field times the passive mass give the force.
  - The force divided by the inertial mass give the acceleration.
- Hence the path followed by a particle in space and time is entirely independent of the kind of particle chosen (Galileo's principle).
- This principle is entirely unexplained in Newton's theory. Particles could have different types of masses.

Inertial Forces

Consider two classical frames, an inertial frame S and a noninertial frame S′, accelerating relative to S with acceleration A.

In the inertial frame S, the equation of motion for a mass m is Newton's second law.

In the noninertial frame S′

$$\vec{a}' = \vec{a} - \vec{A} \text{ where } \frac{d\vec{v}}{dt} = \vec{A} \text{ is the acceleration of the frame S'.}$$

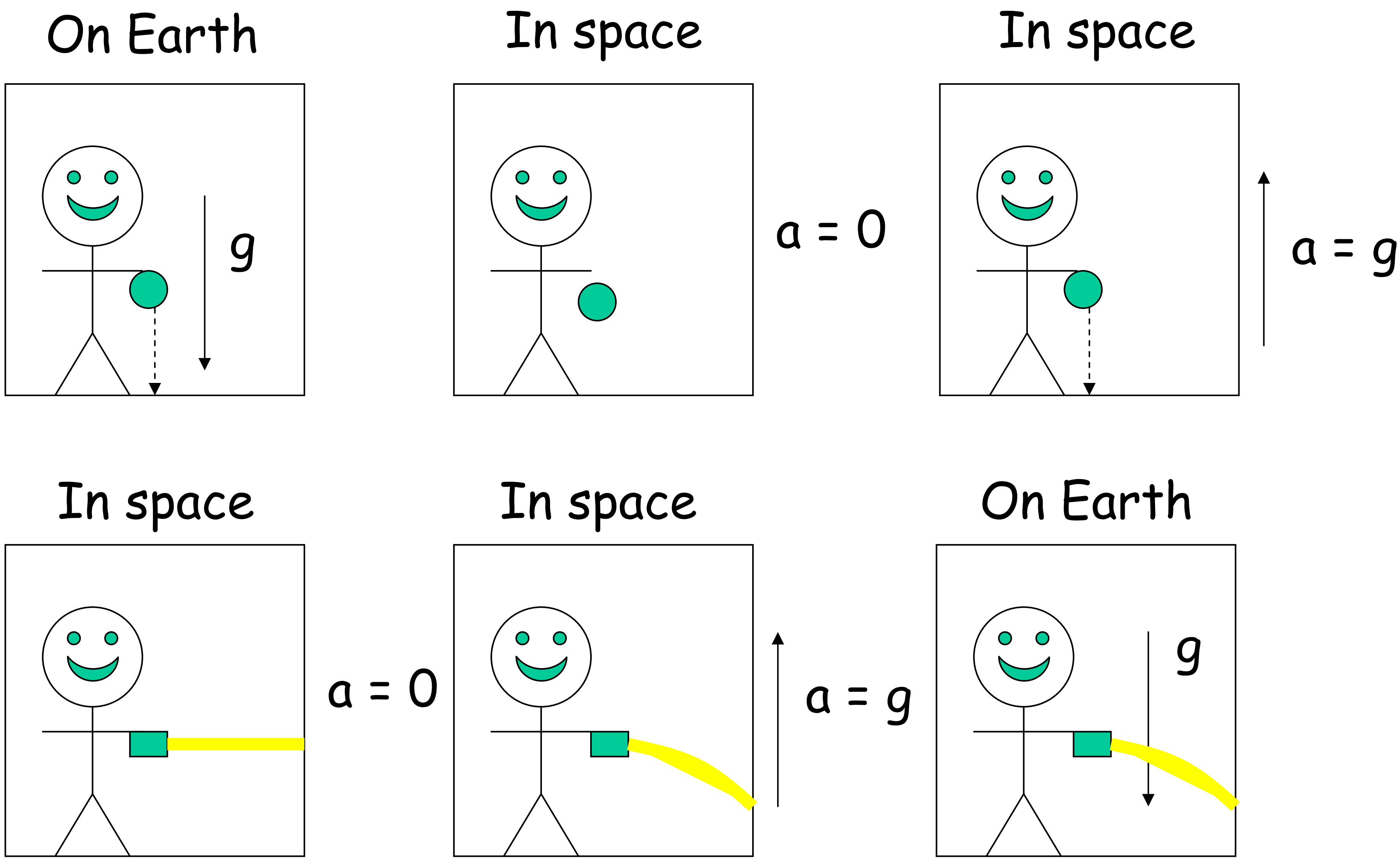
Multiplying by m and putting $m\vec{a} = \sum \vec{F}$, we have $m\vec{a}' = \sum \vec{F} - m\vec{A}$.

There is an additional extra force term in frame S′.

Inertial Forces

- Inertial forces experienced in noninertial frames are familiar in several everyday situations:
  - Aircraft acceleration during take off.
  - Standing in a bus.
  - Car going around a sharp curve (centrifugal force).
- For an observer in an accelerating frame this force is real.

Accelerating Elevators



Equivalence Principle

The inerital force $\vec{F}_{in} = -m\vec{A}$ is proportional to the mass m of the object. The gravitational force $\vec{F}_{gr} = m\vec{g}$ is also proportional to the mass.

Considering the elevator experiment it is not possible to distinguish accelerations from gravity.

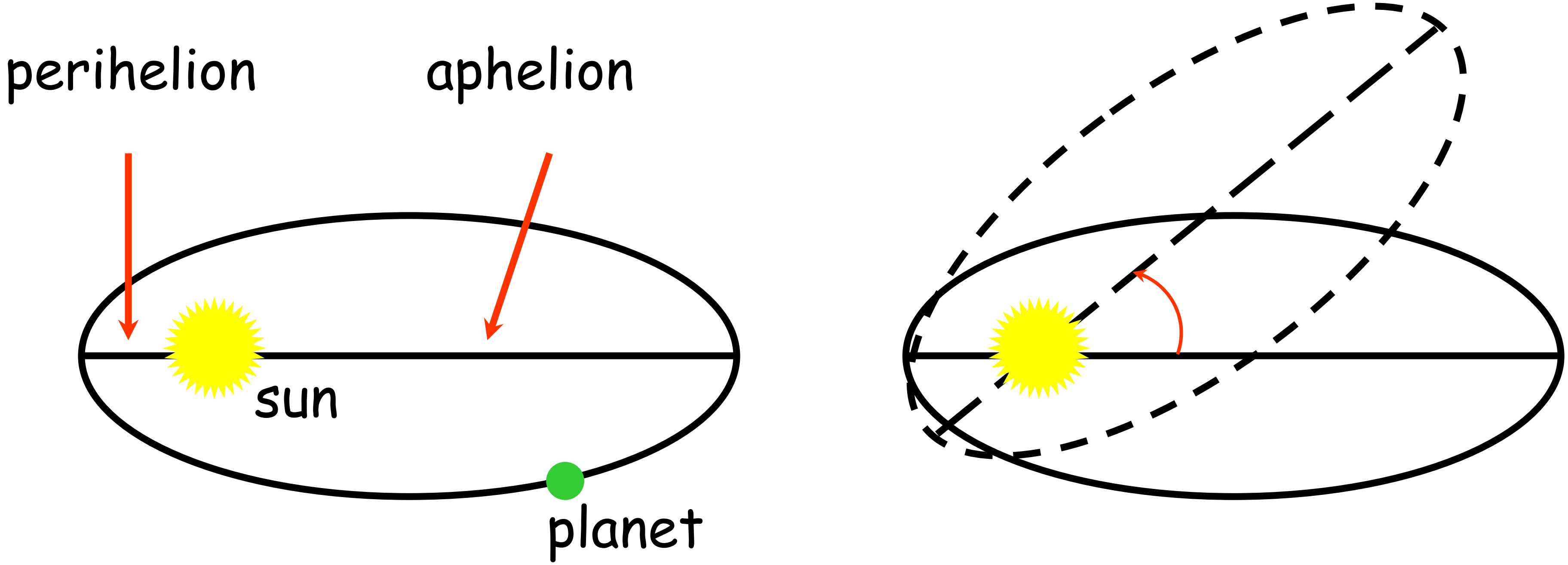
No experiment, mechanical or otherwise, can distinguish between a uniform gravitational field (g) and the equivalent uniform acceleration ($A = -g$).

The general theory of relativity is built on this postulate in much the same way that special relativity is built on its two postulates.

Classical Equivalence

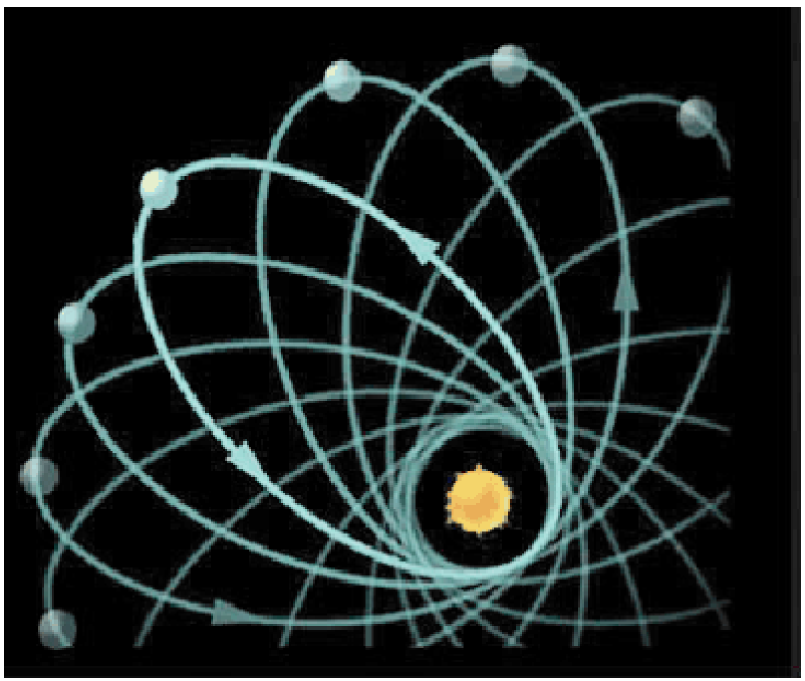
- The effects of gravity can be either created or eliminated by the proper choice of coordinate systems.
- A homogeneous gravitational field is completely equivalent to a uniformly accelerating reference frame.
- This is a way to make accelerations relative.
- This principle arises in Newtonian mechanics because of the apparent identity of gravitational mass and inertial mass.
- The principle of equivalence applies to all physics and not just to mechanics.

Advance of the Perihelion



The Precession of Mercury's Orbit

- First test of general relativity.

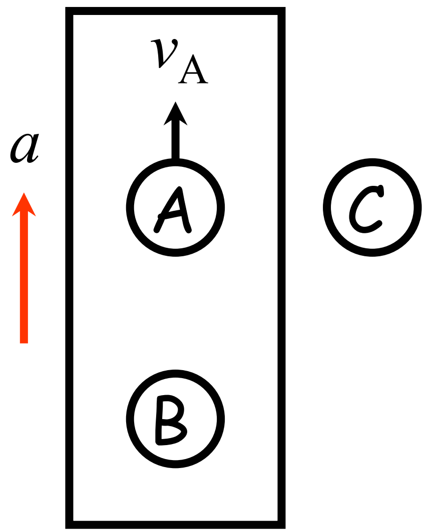


- The planet Mercury does not move in the perfect, fixed ellipse predicted by Newtonian theory; instead, the orbit precesses in such a way that its axis rotates slowly . The perihelion precesses by about 0.01^{o} per century.

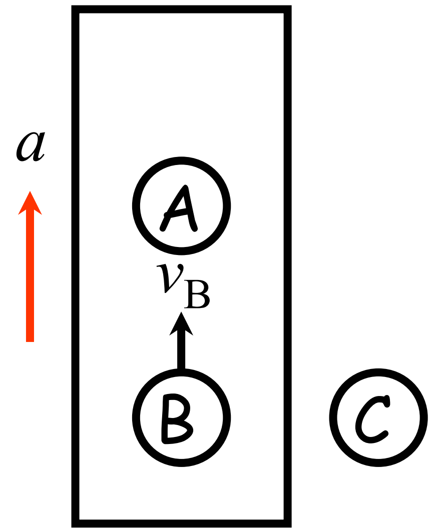
Gravitational Redshift

- Clocks run slower in regions of strong gravitational field.
- The frequency of vibration of an atom near the Sun will be lower than that of the same atom on Earth.
- This shifting towards a lower frequency and therefore a longer wavelength is called the gravitational redshift.
- Light is redshifted as it travels in a strong gravitational field.

Gravitational Redshift



$$\Delta t_C = \frac{\Delta t_A}{\sqrt{1 - v_A^2 / c^2}}$$



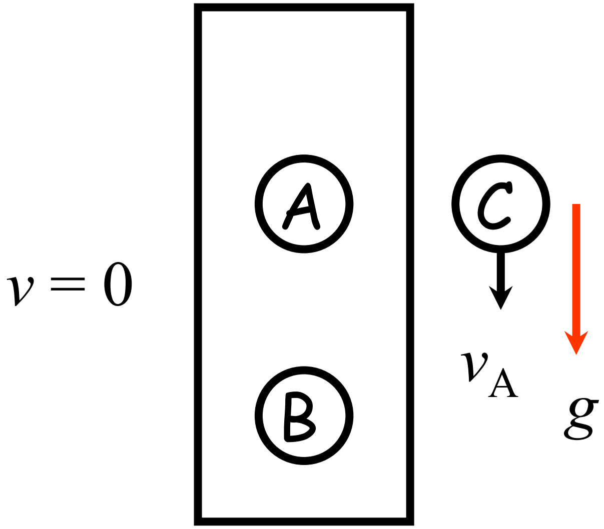
$$\Delta t_C = \frac{\Delta t_B}{\sqrt{1 - v_B^2 / c^2}}$$

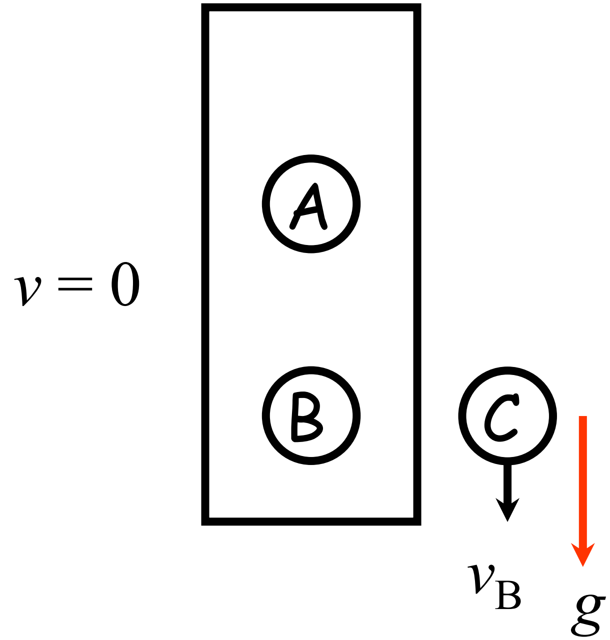
$$\frac{\Delta t_A}{\Delta t_B} = \left(1 - v_A^2 / c^2\right)^{1/2} \left(1 - v_B^2 / c^2\right)^{-1/2}$$

$$= \left(1 - \frac{v_A^2}{2c^2}\right)\left(1 + \frac{v_B^2}{2c^2}\right)$$

$$\approx 1 + \left(\frac{v_B^2}{2} - \frac{v_A^2}{2}\right)\frac{1}{c^2}$$

Using the Principle of Equivalence





$$\frac{1}{2}mv^2 + PE = E = \text{constant}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

$$\frac{v_B^2}{2} - \frac{v_A^2}{2} = gh_A - gh_B = gh_{AB} = gh$$

$$\frac{\Delta t_A}{\Delta t_B} = 1 + \frac{gh}{c^2}$$

Gravitational and Field-Free Space

$\Delta t_B = \Delta t_g$ elapsed time in strong gravitational field
$\Delta t_A = \Delta t_f$ elapsed time in gravitational field - free space

$$\Delta t_f = \Delta t_g \left(1 + \frac{gh}{c^2} \right)$$

Since $(1 + gh/c^2) > 0$, the elapsed time on the clock in the gravitational field-free space Δt_f is greater than the elapsed time on a clock in a gravitational field Δt_g. Hence, a clock in a gravitational field runs slower than a clock in a field-free space.

Effect of Gravity on Excited Atoms

The speed of light for a spectra line $c = \lambda f = \dfrac{\lambda}{T}$

$$T_f = T_g\left(1 + \frac{gh}{c^2}\right) \qquad \lambda_f = \lambda_g\left(1 + \frac{gh}{c^2}\right) \qquad \lambda_f > \lambda_g$$

Wavelength observed in gravity-free space is greater than wavelength emitted from the atom in the gravitational field.

Visible portion of electromagnetic spectrum runs from violet light at low wavelengths to red light at high wavelengths.

Wavelength of the spectral line increases toward the red end of the spectrum, and the entire process of slowing down of clocks in a gravitational field is referred to as the gravitational red shift.

Example: Gravitational Frequency Shift

Find the change in frequency per unit frequency for a γ-ray traveling from the basement, where there is a large gravitational field to the roof of a building, which is $22.5\ \mathrm{m}$ higher, where the gravitational field is weaker.

A similar analysis in terms of frequency can obtain

$$f_g = f_f\left(1 + \frac{gh}{c^2}\right)$$

$$\frac{\Delta f}{f_g} = \frac{gh}{c^2}$$

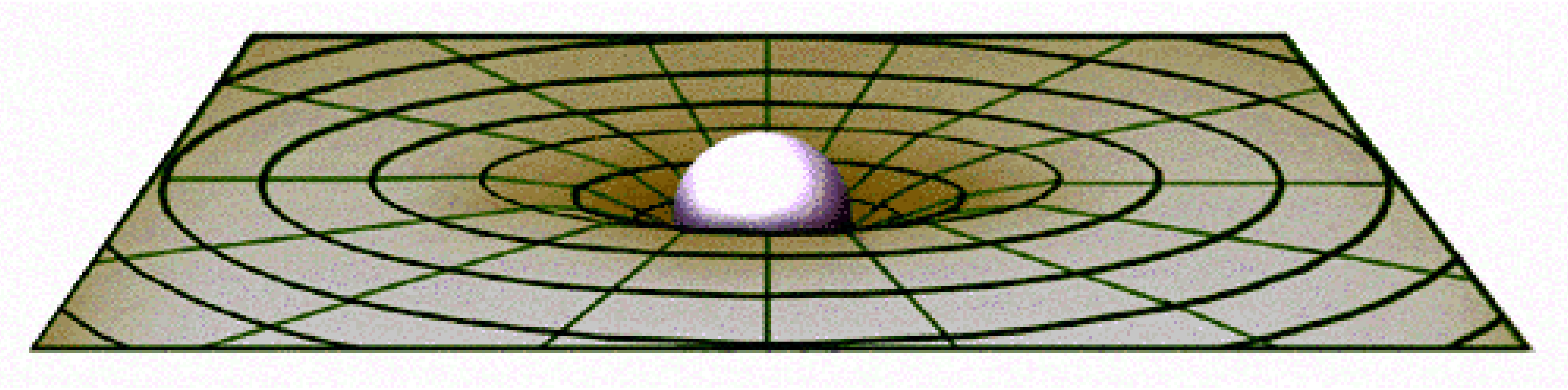
$$= \left(9.80\frac{\mathrm{m}}{\mathrm{s}^2}\right)\left[\frac{22.5\ \mathrm{m}}{\left(3\times 10^8\ \mathrm{m/s}\right)^2}\right]$$

$$= 2.50\times 10^{-15}$$

Geometry of Space

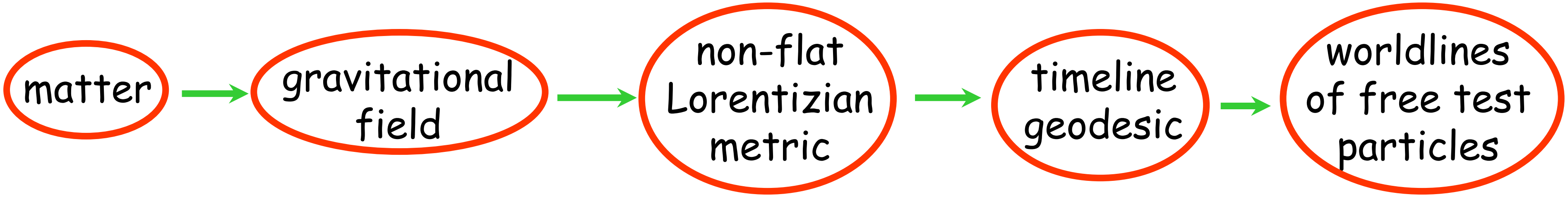
- The effects of gravity are incorporated into the geometry of space.
- The mass that is responsible for causing the gravitational field, must warp spacetime to make the world lines of spacetime curved.
  - Matter warps spacetime
  - Spacetime tells matter how to move.
- Instead of saying that the sun exerts forces on the planets causing them to follow their curved orbits, we say that the gravitational field of the sun causes a curvature of space, and it is this curvature that is responsible for the curved orbits of the planets.
- In the presence of gravitating bodies the Lorentzian metric on spacetime is non-flat.

Curvature of Spacetime



Geodesic Hypothesis

- The worldline of a free particle is a timelike geodesic of the non-flat Lorentzian metric of spacetime.
- The worldline of a light signal (photon) is a null geodesic of the non-flat Lorentzian metric of spacetime.



Null Geodesic

- A light beam must travel by the shortest, most direct, path between two points.
- If it didn't, some other object could travel between the two points in a shorter time and thus have a greater speed than the speed of light.
- If a light beam can follow a curved path, then this curved path must be the shortest distance between the two points, which suggests that space itself is curved and that it is the gravitational field that causes the curvature.

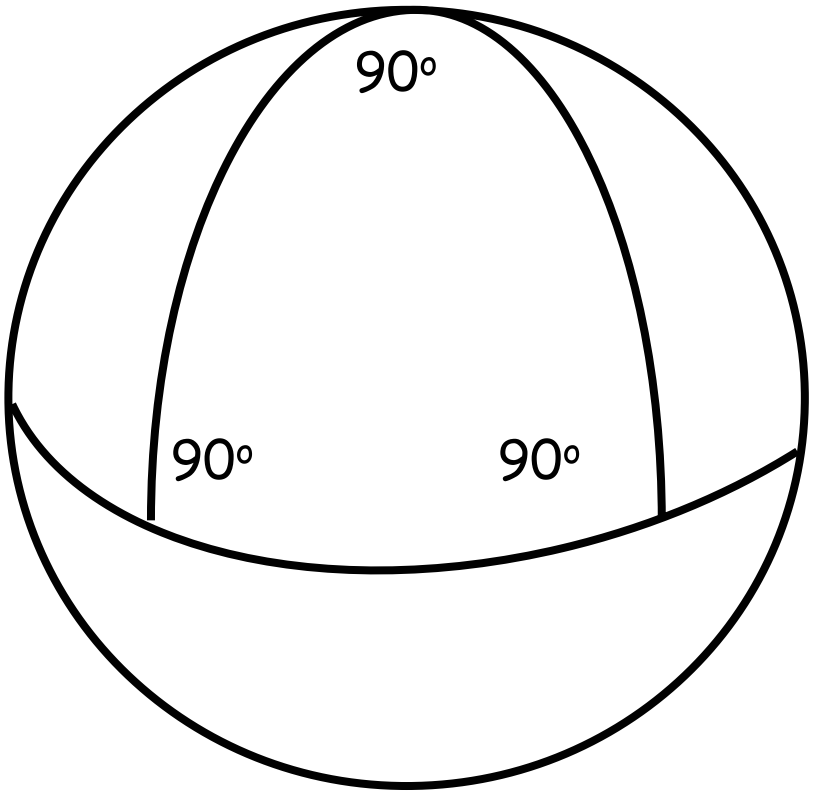
What is Meant by Curved Space?

- Our normal way of viewing the world is via Euclidean plane geometry.
- Example, the sum of the angles of any triangle is 180^o.
- It is hard enough to imagine 3-dimensional curved space, much less curved 4-dimensional spacetime.
- Let us explain the idea of curved space by using 2-dimensional surfaces.

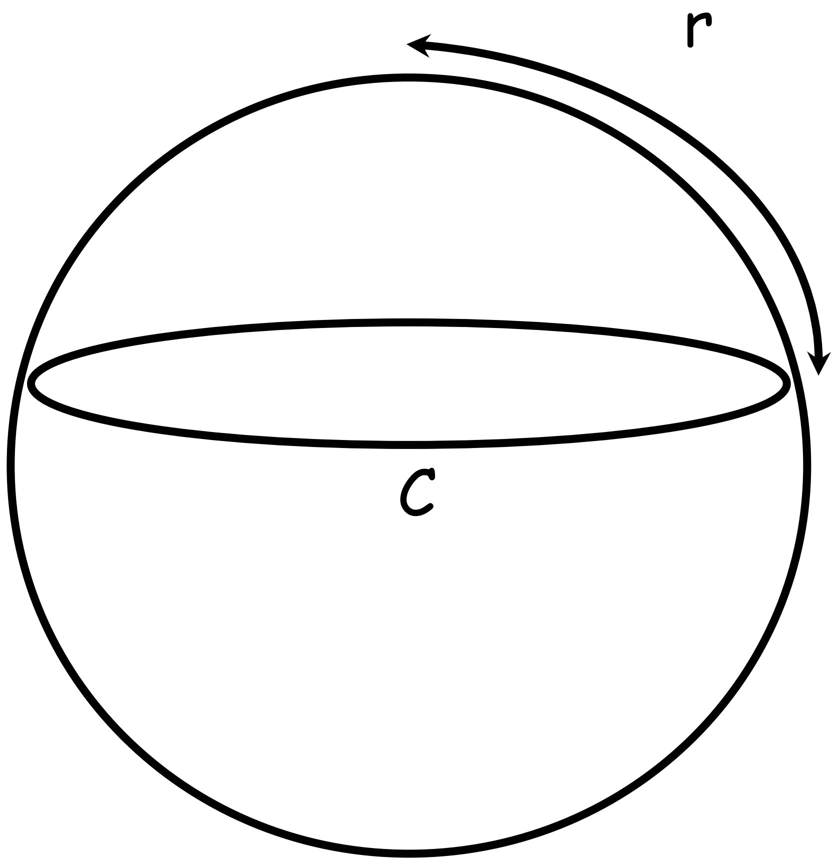
The 2-Dimensional Surface of a Sphere

- How would hypothetical 2-dimensional creatures determine whether their 2-dimensional space were flat (a plane) or curved?
- One way is to measure the sum of the angles of a triangle.
- To construct a triangle on a curved surface, say a sphere, we must use the equivalent of a straight line: that is, the shortest distance between two points, which is called a geodesic.
- On a sphere, a geodesic is an arc of a great circle (an arc contained in a plane passing through the center of the sphere) such as the equator and the longitude lines.
- For a sphere the sum of the angles of a triangle is greater than 180^o, clearly non-Euclidean space.

2-Dimensional Curved Surface



The sum of the angles will be 270o.



Measure radius and circumference of circle

Proportionality Constant of 2π

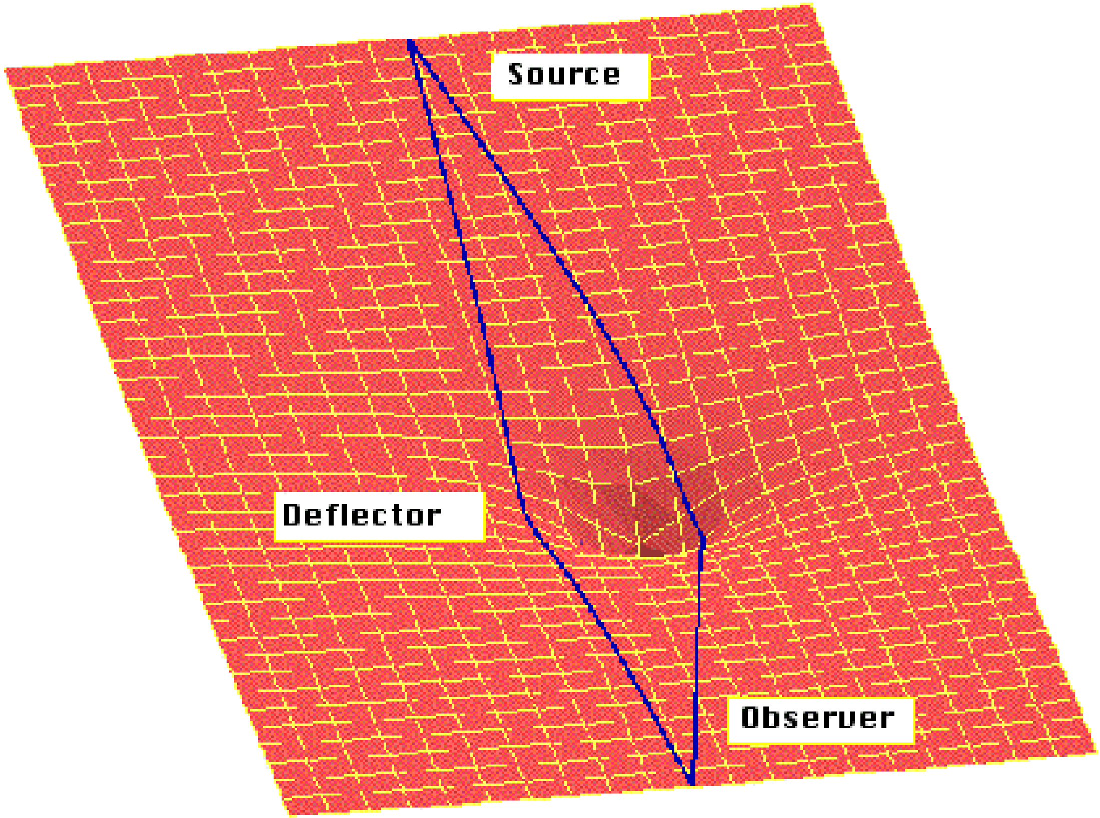
- Another way to test the curvature of space is to measure the radius r and circumference C of a large circle.
- On a plane surface, $C = 2\pi r$.
- But on a 2-dimensional spherical surface, C is less than $2\pi r$.
  - The proportionality constant between C and r is less than 2π.
  - Such a surface is said to have positive curvature.
- On a *saddle-like* surface the circumference of a circle is greater than $2\pi r$, and the sum of the angles of a triangle is less than 180^o.
  - Such a surface is said to have a negative curvature.

Curvature of Our Universe

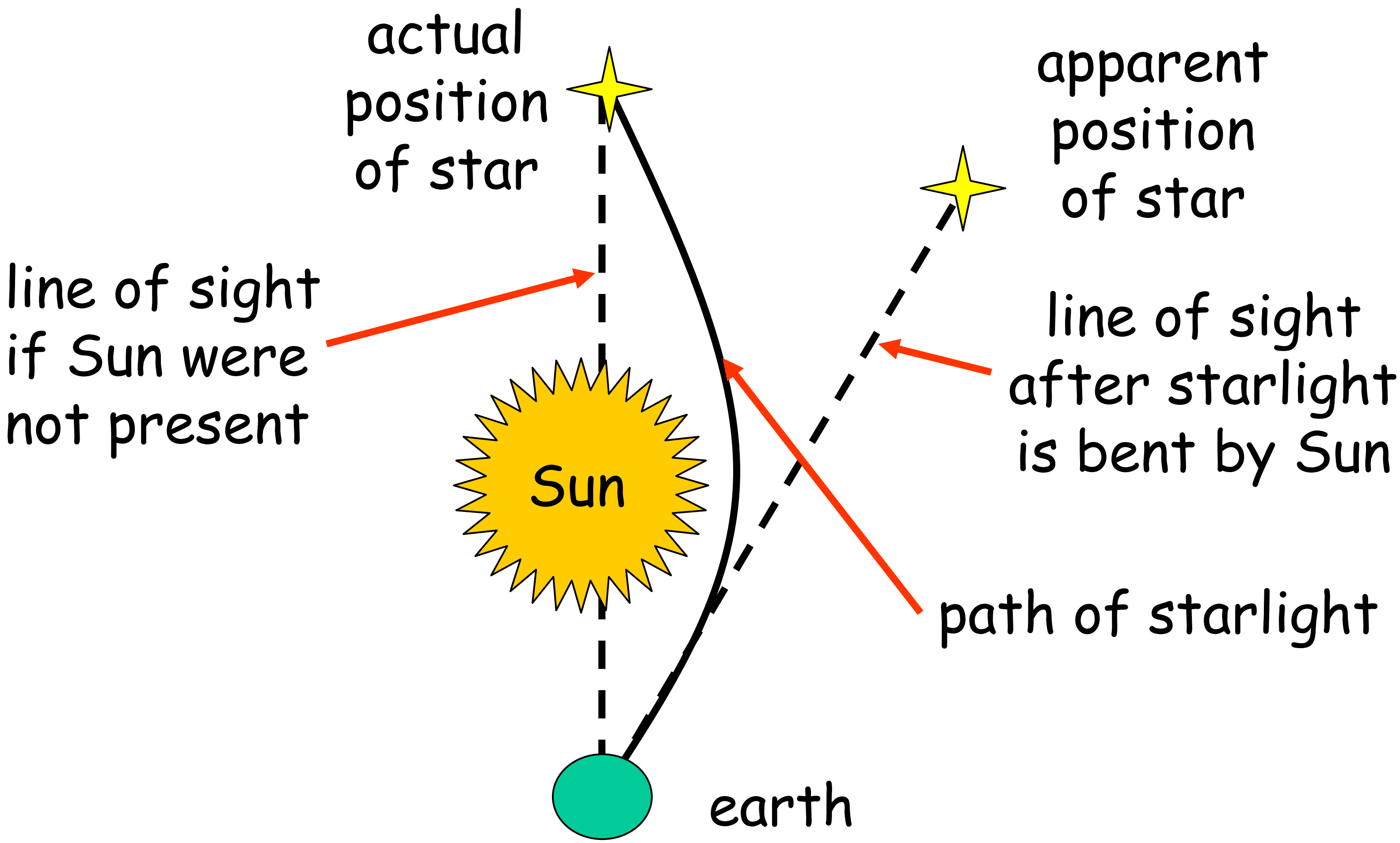
- On a large scale (not just near a large mass), what is the overall curvature of the universe?
- The answer is still unknown.
- If the universe has positive curvature, the universe is finite, or closed.
  - Space would fold back and close on itself.
  - There is no boundary or edge in such a universe.
  - If a particle were to move in a straight line in a particular direction, it would eventually return to the starting point.
  - The space of the universe is all that there is, it is futile to ask what is beyond such a closed universe.
- If the curvature of space is zero or negative, the universe would be open.
  - It would just go on and on and never fold back on itself.
  - An open universe would be infinite.
- If the universe is open or closed depends, in part, on how much total mass there is in the universe.

Deflection of Light

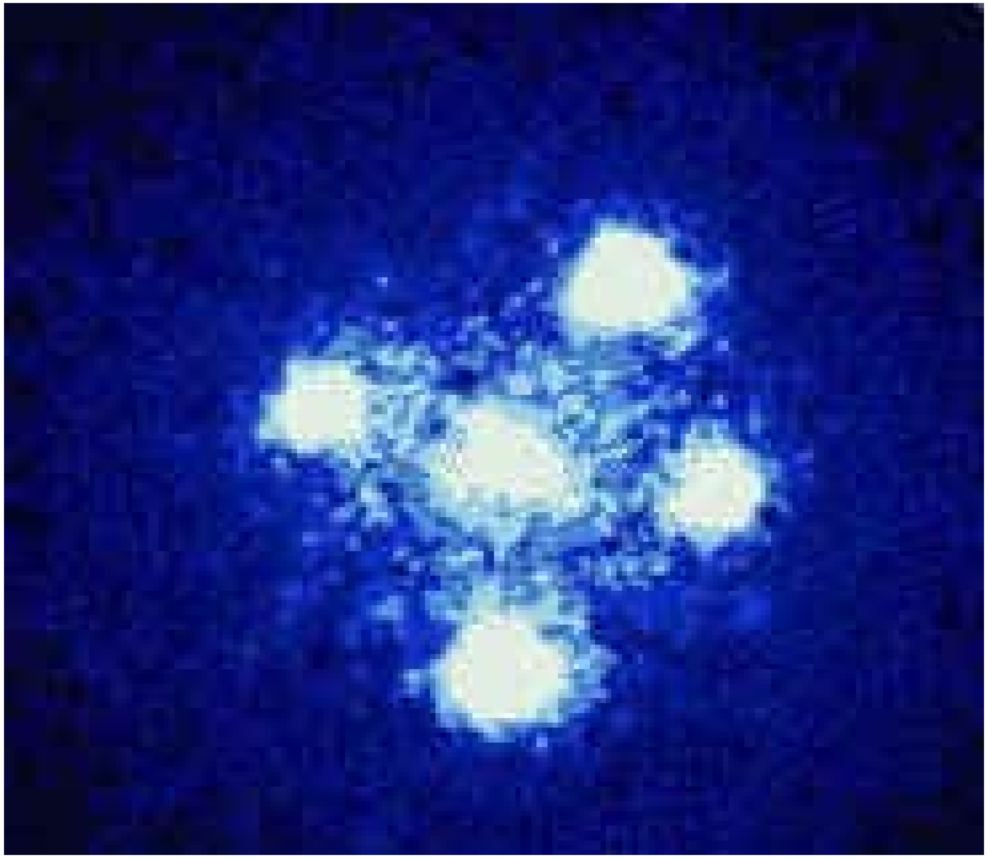
- The equivalence principle tells us that light traveling across the gravitational field of a massive body must be deflected downwards toward the body.



Sun Acts Like a Lens



Gravitational Lensing

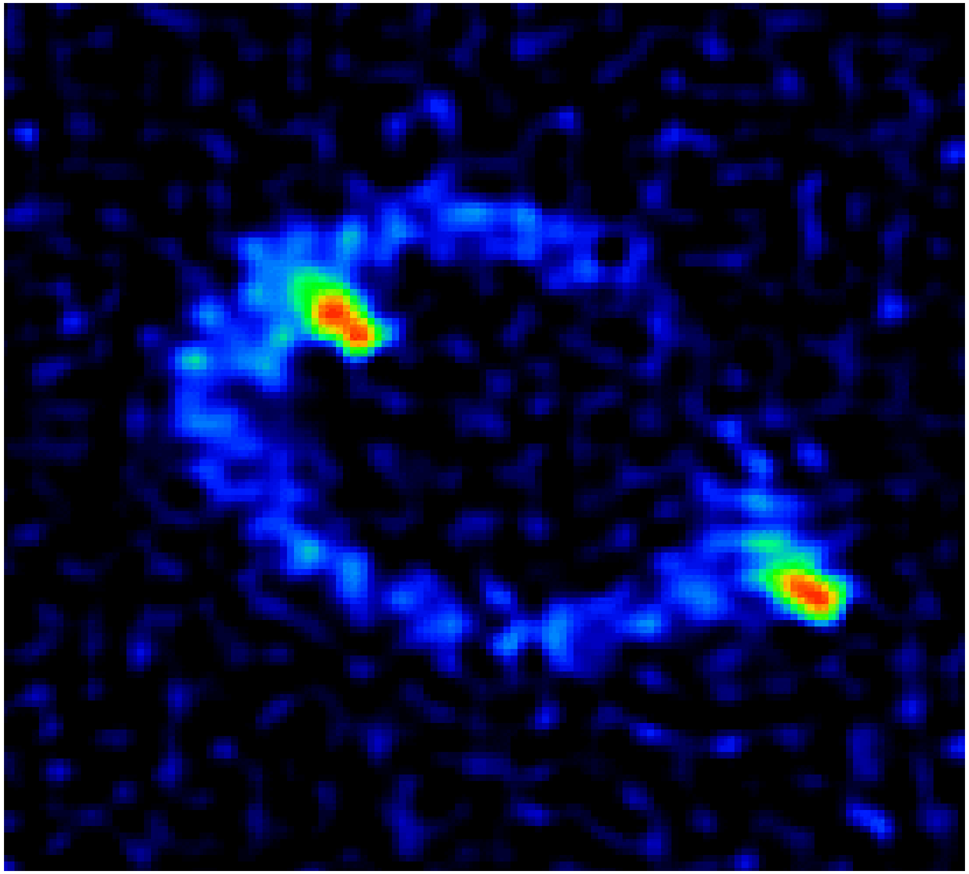


Einstein cross

Lensing galaxy at center is 400 millions light years away.

Quasar whose multiple images surround it is 20 times further away

Gravitational Lensing



Einstein ring (MG1131+0456)

Black Holes

- A very heavy and dense star, whose gravitational field is so strong that no light – or anything else – can escape from its interior.
- A black hole is formed when a star of several solar masses exhausts its supply of nuclear fuel.
- Without the fuel to maintain the pressure needed to support it, the star begins to collapse and continues to do so until its radius approaches a value called the Schwarzschild radius.
- Near the Schwarzshchild radius the rate of collapse slows down (as observed from far away), the redshift of light from the star grows indefinitely, and the light from inside can no longer escape at all.

Black Holes

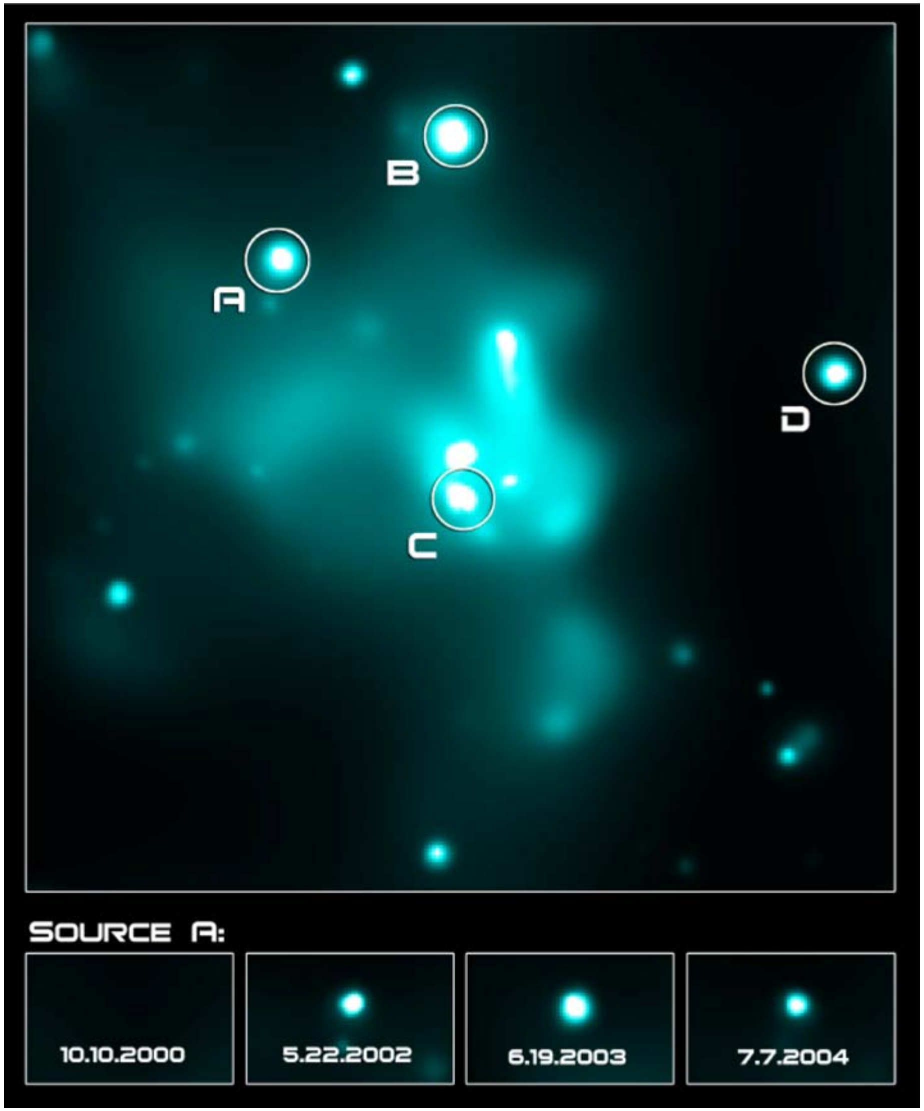
- Since light from inside a black hole cannot escape, direct evidence for black holes is hard to come by.
- If an ordinary star were caught in orbit around a black hole, matter would be torn from the star by the black hole.
- Before it disappears inside the black hole, this falling matter should reach such high energies that its collisions should produce x-rays.
- It is thought that extremely massive black holes may be responsible for intense radiation from the superenergetic and distant objects called quasars.

Black Holes

- The effect of a black hole on objects outside the critical radius is the same as that of any other mass.
- Its gravitational attraction is so great that once inside a critical radius, nothing can escape.
- Nothing that happens inside it can be communicated to the outside.
- For an object with a mass equal to that of our Sun to be a black hole, its radius would have to be about 3 km.

Black Hole Detection

- Since no radiation is emitted from a black hole and its radius is expected to be small, the detection of a black hole is not easy.
- The best chance of detection would occur if a black hole were a companion to a normal star in a binary star system.



Schwarzschild Radius

Escape speed from body of mass M and radius R

$$v = \sqrt{\frac{2GM}{R}}$$

For light not to escape $v = \sqrt{\frac{2GM}{R}} = c$

Radius of black hole $\boxed{R = \frac{2GM}{c^2}}$

Also called the event horizon of the black hole.

Example: Radius of Black Hole

To what radius must the Earth be condensed to for it to be a black hole?

$$R = \frac{2GM_E}{c^2} = \frac{2\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{\left(3.00\times10^{8}\,\mathrm{m/s}\right)^2} = 8.86\,\mathrm{mm}$$

The size of a walnut!

Schwarzschild Metric

Invariant interval is the metric of flat spacetime, that is, one in which there is not mass to warp space.

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

Invariant interval of spacetime curved by the presence of a point mass.

$$(\Delta s)^2 = \frac{(\Delta r)^2}{1 - 2GM/rc^2} - \left(1 - \frac{2GM}{rc^2}\right)(c\Delta t)^2$$

For light world line $\Delta s = 0$

Schwarzschild Solution

$$\frac{(\Delta r)^2}{1-2GM/rc^2} = \left(1-\frac{2GM}{rc^2}\right)(c\Delta t)^2$$

$$\frac{\Delta r}{\Delta t} = \left(1-\frac{2GM}{rc^2}\right)c$$

$$= \left(1-\frac{R_S}{r}\right)c$$

If $r = R_S$, the velocity of light is zero, and no light is able to leave the gravitating body.

$$r = 1000R_S \Rightarrow \frac{\Delta r}{\Delta t} = 0.999c$$

$$r = R_S \Rightarrow \frac{\Delta r}{\Delta t} = 0$$

$$r = \frac{R_S}{10} \Rightarrow \frac{\Delta r}{\Delta t} = -9c$$

Creation of a Black Holes

- A star is essentially a gigantic nuclear reactor converting hydrogen to helium in a process called nuclear fusion.
- Think of the star as millions of hydrogen bombs going off at the same time, thereby producing enormous quantities of energy and enormous forces outward from the star.
- For a star there is an equilibrium between the gravitational forces inward and the forces outward caused by the exploding gas.
- Eventually when all the nuclear fuel is used, there is no longer an equilibrium condition.

Creation of a Black Holes

- The gravitational force causes the star to become very compact.
- If the star is large enough, it is compressed below its Schwarzschild radius and a black hole is formed.
- It must be about 25 times the mass of the sun.
- When a star condenses to a black hole it does not stop at the event horizon but continues to reduce in size until it becomes a singularity, a point mass.
    - That is, the entire mass of the star has condensed to the size of a point.

Consequences of a Black Hole

- If a person were to fall into the black hole he would eventually be crushed due to the enormous gravitational forces.
- Time would slow down for him as he approached the event horizon.
- At the event horizon, time would stand still for him.

Rotating Black Holes

- A more general solution of a black hole should allow for rotation: Kerr black hole.
- A rotating black hole (accelerating black hole) drags spacetime around with it, forming a second event horizon, leaving a space between the first event horizon and the second event horizon.
- It has been speculated that it may be possible to enter the first event horizon, but not the second, and exit somewhere else in either another universe or in this universe in another place and/or time.

White Holes

- It has been speculated that there might also exist white holes in space.
- Mass that is drawn into a black hole would be spewed out of a white hole.
- Some physicists have speculated that a black hole in one universe is a white hole in another universe.

Evaporation of Black Holes

- Black holes evaporate away.
- The matter within them, through the slow quantum-mechanical process of tunneling would “leak out”.
- This process is so slow that it would take on the order of 10^{100} years to evaporate a black hole away.

Gravitational Waves

- Unlike the Newtonian theory, general relativity predicts that accelerating masses should radiate gravitational waves, just as accelerating electric charges radiate electromagnetic waves (like the radio waves from oscillating charges in a radio antenna).
- Even the most violent cosmic events produce very feeble gravitational waves.
- No reproducible, direct observation of gravitational waves to date.
- Strong indirect evidence, in that certain rapidly rotating star systems have been observed to be losing energy at precisely the rate that should result from their gravitational radiation.

LIGO

- Laser Interferometer Gravity-wave Observatory (LIGO).
- Michelson interferometer with arms 4 km long.
- Any passing gravitational wave should change the length of one of the arms enough to cause an observable shift in the interference pattern.
- For a strong gravity wave, expect change in length by less than the diameter of an atomic nucleus.
- One in Washington State and one in Louisiana.
- This is a possible new telescope for studying the universe using gravity waves.

LIGO

4-Notation Again

$$t = x_0, x = x^1, y = x^2, z = x^3$$

$$x^2 = \begin{pmatrix} x_0 & x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$(\Delta s)^2 = \eta_{\mu\nu} \Delta x^\mu \Delta x^\nu$$

sum over repeated indices implied

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Lorentz metric

Lorentz Metric and Geodesics

$\eta_{\mu\nu}$ is the Lorentz metric tensor, which has constant components. We say that the Lorentz metric is flat.

Geodesics of this Lorentz metric are straight lines

The worldline of an inertial (i.e. free) material particle is a timelike *geodesic*.

The worldline of a photon (light signal) is a null *geodesic*.

A flat metric tensor is said to be a flat space.

General relativity theory is based on a non-flat Lorentzian metric on 4-D spacetime.

Module 21

Elementary Particles

History

- In 1932 the electron, proton, neutron were thought to be nature's three elementary particles.
- Basic building block from which all matter is constructed.
- Now we know that several hundred additional particles exist.
- Most new particles are unstable and decay with times between 10^{-6}s and 10^{-23}s.

Neutrinos

- A neutrino accompanies the beta-deay of a radioactive nucleus.
- The neutrino has no charge.
- A very small mass.
- Travels at speeds close to the speed of light.
- Created in abundance in nuclear reactors and particle accelerators.
- Are thought to be plentiful in the universe.

Positron

- Same mass but opposite charge of electron.
- Example of antiparticle.

Muons and Pions

- Muon (μ), like an electron but 200 times more mass.
- Particle is μ^-, antiparticle is μ^+.
- Muons interact via the weak nuclear force.
- Muons are unstable.
- Pions (π) come in 3 types
- Pions interact via the strong nuclear force.
- Pions are unstable.

$$\mu^- \rightarrow e^- + \nu_\mu + \overline{\nu}_e$$

$$\mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e$$

$$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^0 \rightarrow \gamma + \gamma$$

Classification of Forces

- Four fundamental forces.
- Unified force
  - Gravitational force (10^{-43}s)
  - Grand Unified Theory force
    - Strong nuclear force (10^{-35}s)
    - Electroweak force (10^{-10}s)
      - Weak nuclear force
      - Electromagnetic force

Four Fundamental Forces of Nature

Force	Relative Strength	Range
strong	1	~1 fm
electromagnetic	10^{-2}	infinite ($1/r^2$)
weak	10^{-6}	~10^{-3} fm
gravitational	10^{-43}	infinite ($1/r^2$)

Classification of Particles

- 3 families: photon, leptons, hadrons.
- Photon: photon only
  - Interact via electromagnetic force only.
- Leptons: electron, muon, tau, 3 neutrions.
  - Interact via weak nuclear force.
  - Also gravitational (if massive) and electromagnetic (if charged).
- Hadrons:
  - Interact via strong and weak nuclear force.
  - Also gravitational (if massive) and electromagnetic (if charged).
  - Mesons: pions, kaons, others
  - Baryons: proton, neutron, lambda, sigma, others.

Leptons

- Interact weakly but not strongly
- Six known leptons, all fundamental
- 6 anti leptons
- E and e nutrino stable

Hadrons

- Experience both weak and strong force
- Hundreds of known hadrons
- Only proton stable
- Composed of 2 or 3 quarks
- 2 quarks are meson
- 3 quarks are baryons

Quarks

- Since there are so many particle perhaps they are not fundamental.
- Postulate that hadrons are made of quarks.
- Quarks are now fundamental, along with photon and leptons.
- Free quarks have never been found (quark confinment).
- Fractional charged
- 6 quarks and 6 anti-quarks

Standard Model

- Interactions can occur via one of 4 forces:
  - Gravitational force
  - Strong nuclear force
  - Weak nuclear force
  - Electromagnetic force
- Building blocks of matter:
  - Molecules made of atoms.
  - atoms made of orbital electrons around nucleus.
  - Nucleus made of neutrons and protons.
  - Neutrons and protons made of quarks.

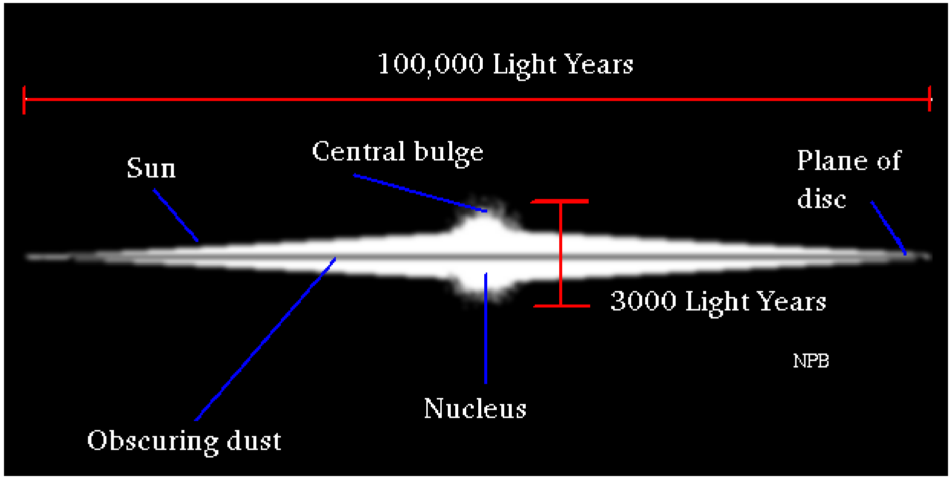
Module 22

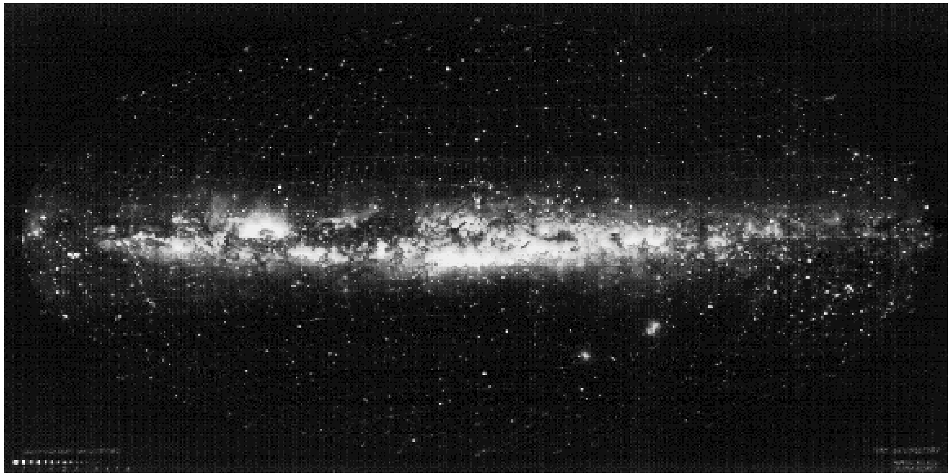
Astrophysics

Stars and Our Galaxy

- The distances involved are so great that we specify them in terms of the time it takes light to travel the given distance, example:
  - 1 light-second = $(3\times10^8$ m/s$)(1$ s$) = 3\times10^8 = 3\times10^5$ km
  - 1 light-year = $(3\times10^8$ m/s$)(3\times10^7$ s/yr$) \approx 10^{13}$ km
- Examples
  - Sun is 8.3 light-minutes away.
  - Nearest star is 4.3 light-years away (10,000 times further than the furthest planet (Pluto).
- The milky way is our Galaxy:
  - diameter 100,000 light-years.
  - thickness 2,000 light-years.
  - about 10^{11} stars.
  - Sun 28,000 ly from center (little more than halfway from center).

The Milky Way Galaxy

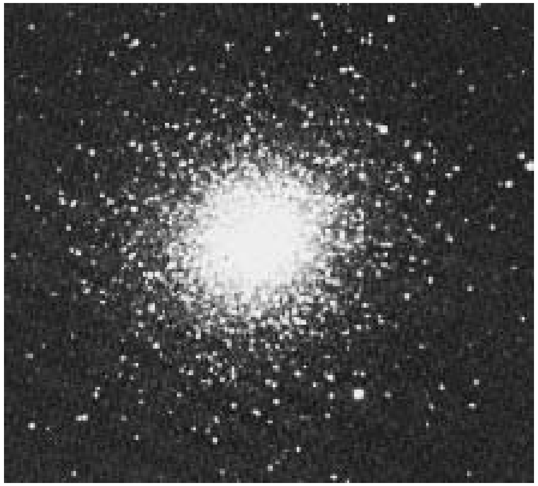




Galaxies and Other Objects

- Star clusters, groups of stars that are so numerous they appear to be a cloud.
- Nebula, glowing clouds of gas or dust.
- Extragalactic, outside our galaxy
- Galaxies, example Andromeda is over 2 million light-years (20 times greater than the diameter of or galaxy), now know of 10^{11} galaxies.
- Galaxies tend to group into galaxy clusters
- Clusters are organized into even larger aggregates, called superclusters

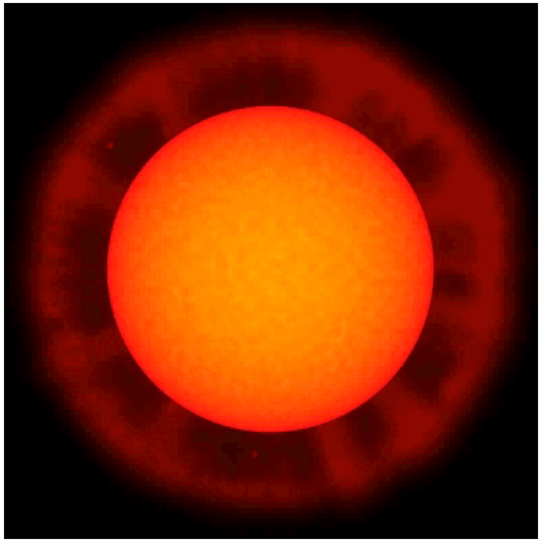
Star Clusters and Superclusters of Galaxies



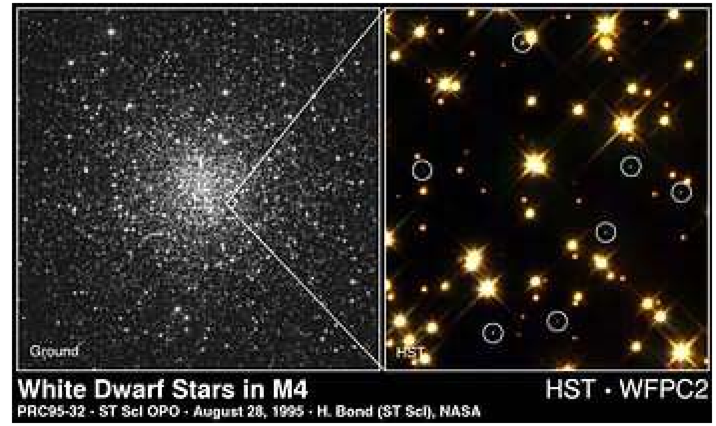
Other Interesting Objects

- Red giants, white dwarfs, neutron stars, black holes, novae and supernovae (exploding stars).
- Quasars (quasi-stellar radio sources).
- Active galaxies.
- Background radiation.

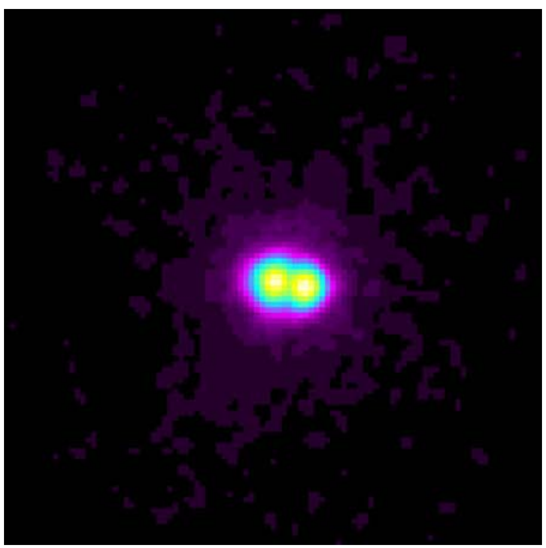
Death of Stars



red giant



white dwarfs

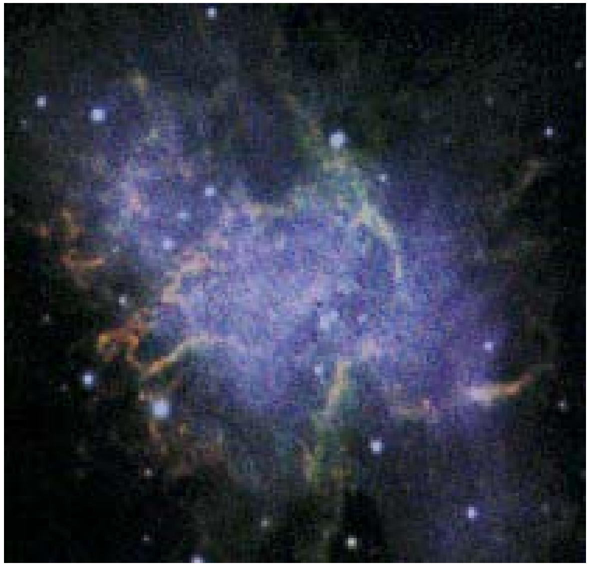


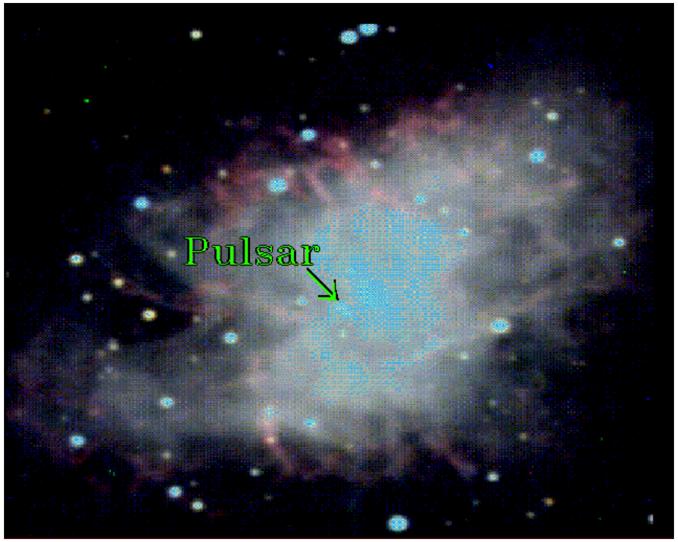
neutron
start

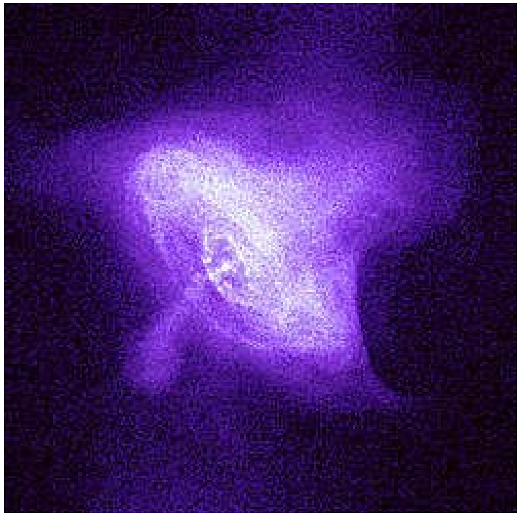
M15: two x-ray binary system

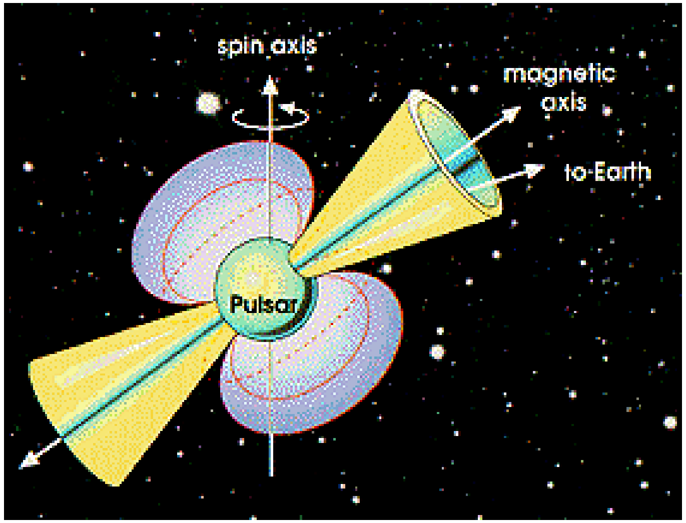
Super Nova 1987A

The Crab Pulsar



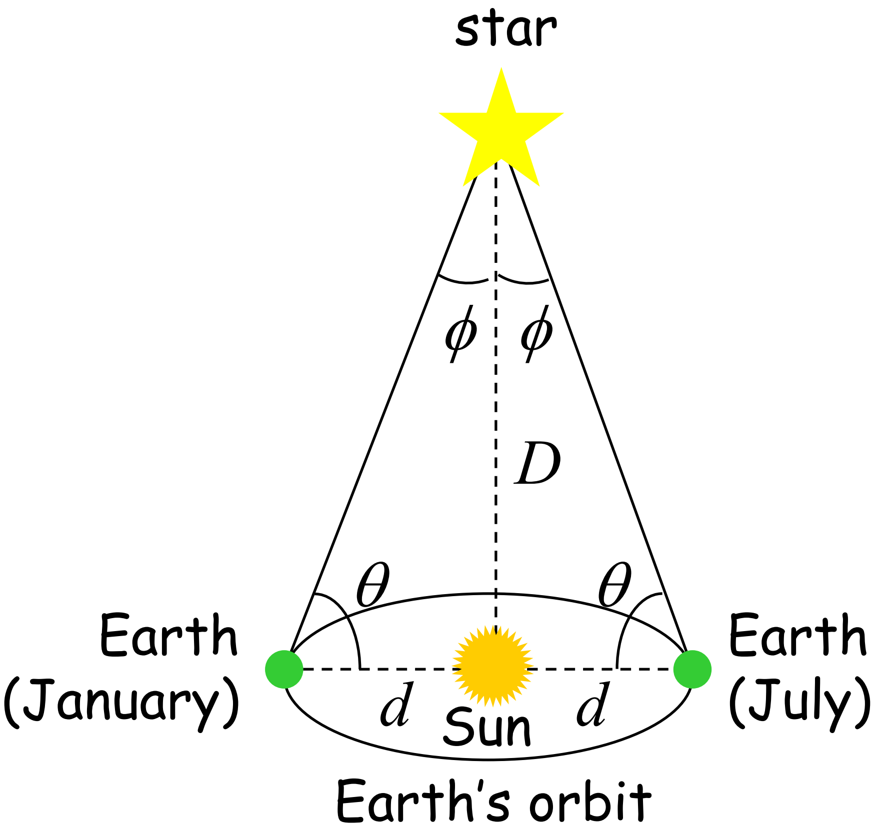






Distance Measurement by Parallax

- How do we measure the vast distances?
- Parallax: the apparent motion of a star against the background of more distant stars, due to the Earth's motion about the Sun.
- The sighting angle of a star relative to the plane of the Earth's orbit (θ) is measured at different times of the year.
- Since we know d from Earth to Sun, we can reconstruct the right-angle triangles and determine the distance D to the star.



Example: Star Distance

Estimate the distance D to a star if the angle θ is 89.99994°.

The angle ϕ is 0.00006° or 1.0×10^{-6} rad.

Since ϕ is very small $\tan\phi \approx \phi$

$$D = \frac{d}{\tan\phi} = \frac{d}{\phi} = \frac{1.5\times10^{8}\,\text{km}}{1.0\times10^{-6}\,\text{rad}} = 1.5\times10^{14}\,\text{km}$$

or about 15 ly.

Distances in Angles and Parces

- Distances to stars are often specified in terms of parallax angle given in seconds of arc.
  - 1 minute of arc is 1/60 of a degree
  - 1 second (") is 1/60 of a minute (') of arc.
- parsec (pc) **par**allax angle in **sec**onds of arc
  - is $1/\phi$ and ϕ is given in seconds.
- Example: $\phi = (6\times10^{-5})^{o}\times3600 = 0.22''$ of arc. The star is at a distance of $1/0.22'' = 4.5$ pc.
- 1 pc = 3.26 ly

Other Distance Measurement Methods

- Beyond 100 ly (30 parsecs) parallax angles are too small to measure.
- Can compare apparent brightness of galaxies and use the inverse square law (intensity drops off as a square of the distance).
  - Not precise since we can not expect all galaxies to have the same intrinsic brightness
  - Assume brightest star in any galaxy the same and can use brightest star as *apparent* brightness.
- Estimate distance via the red-shift in the spectra of elements.
- As we look farther and farther away, the measurement techniques are less and less reliable.

Brightness of Stars

- The large range of brightness of stars is due both to the differences in the amount of light stars emit and to their distances from us.
- Absolute luminosity L total power radiated in watts.
- Apparent brightness l power crossing unit area perpendicular to the path of the light at the Earth.
- Since energy is conserved, and ignoring absorption in space, the total emitted power L at a distance d from the star will be spread over a sphere of surface area $4\pi d^2$, where d is the distance from the star to the Earth.

$$l = \frac{L}{4\pi d^2}$$

Example: Star Brightness

Suppose a particular star has absolute luminosity equal to that of the Sun but is 10 pc away from Earth.

By what factor will it appear dimmer than the Sun?

The luminosity L is the same for both stars, so the apparent brightness depends only on their relative distances.

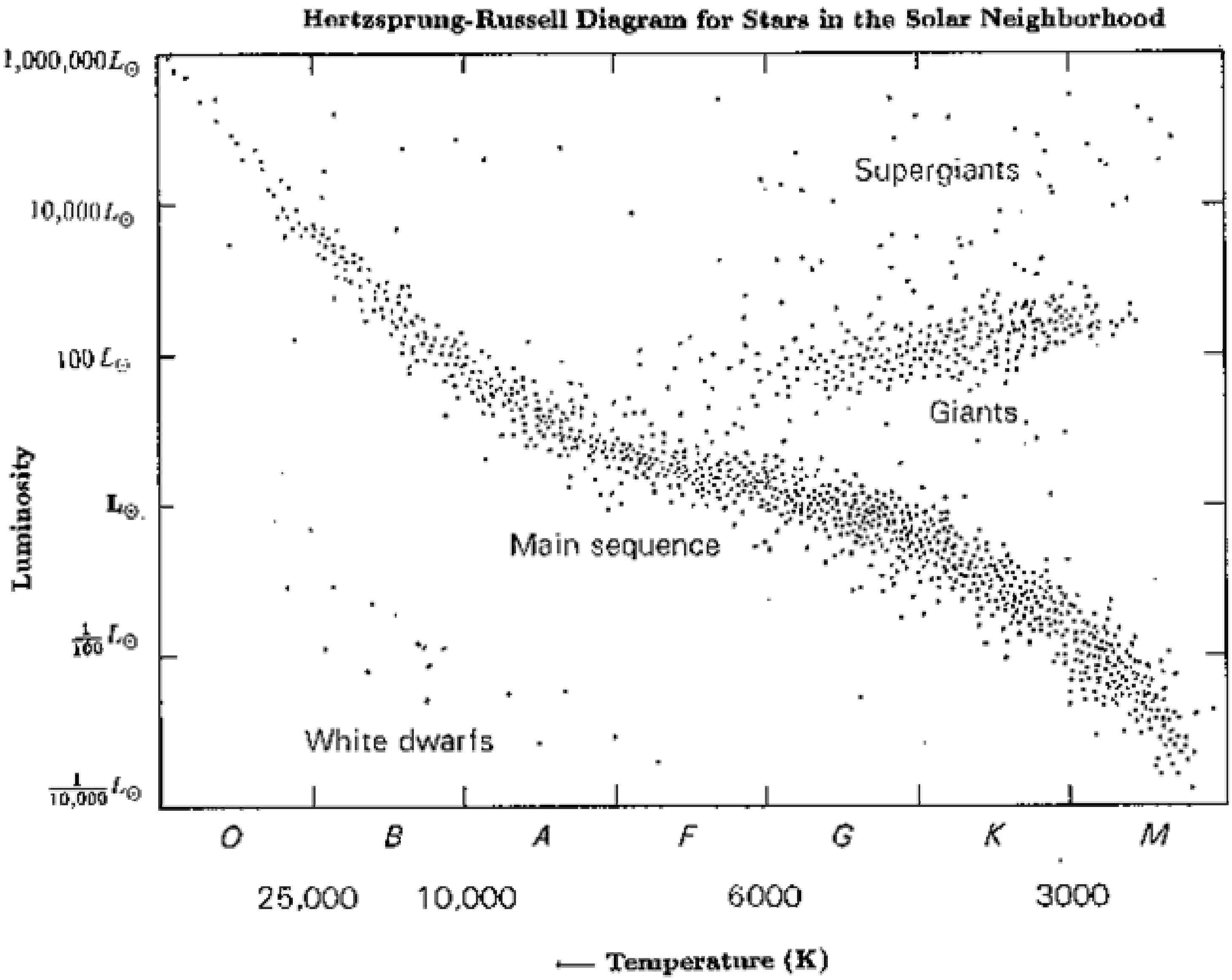
Let d_1 and d_2 be the distances from Earth to the star and the Sun, respectively.

$$\frac{l_1}{l_2} = \frac{d_2^2}{d_1^2} = \frac{\left(1.5\times 10^8\,\text{km}\right)^2}{(10\,\text{pc})^2(3.26\,\text{ly/pc})^2(10^{13}\,\text{km/ly})^2} \approx 10^{-13}$$

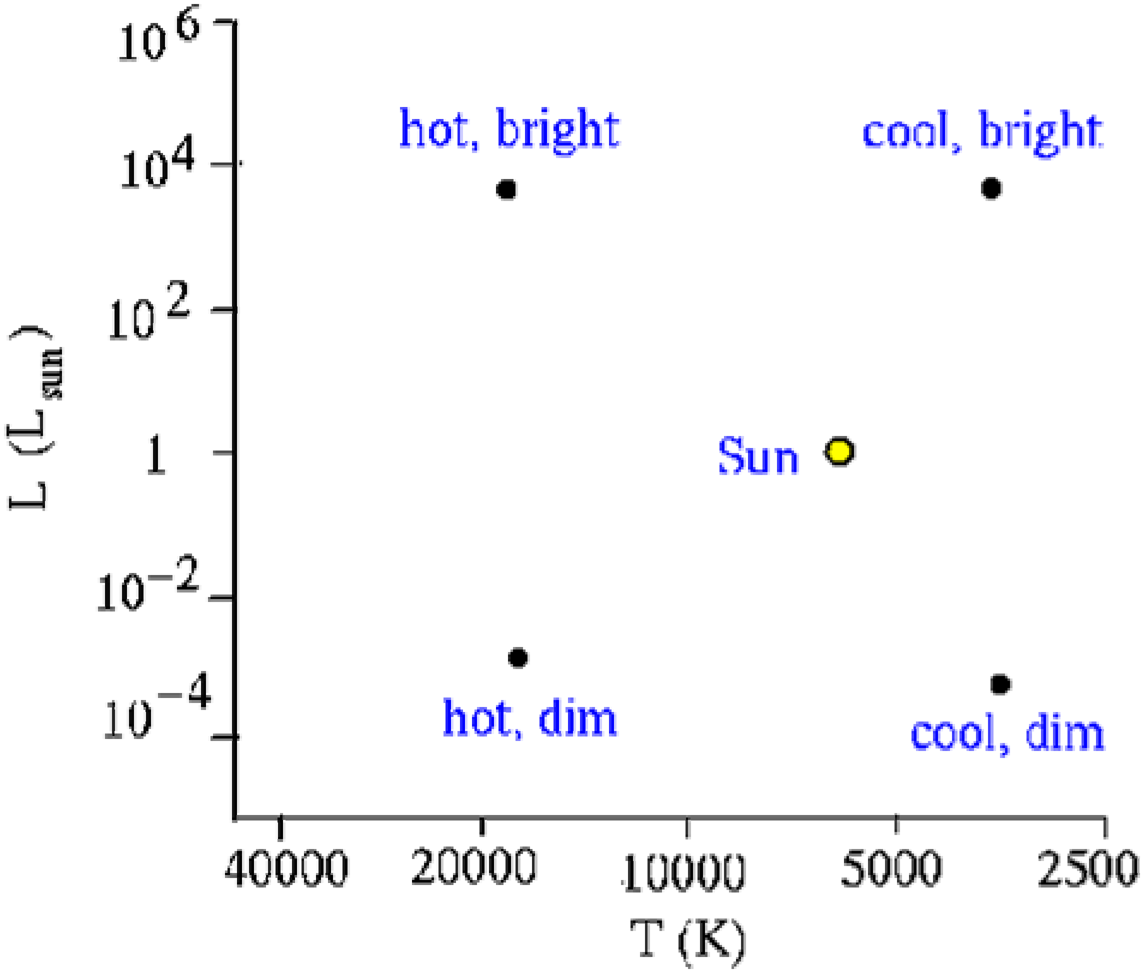
Luminosity Increases with Star Mass

- For most stars the absolute luminosity depends on the mass.
- The more massive the star, the greater its luminosity.
- Another important parameter of a star is its surface temperature.
  - The temperature can be determined from the spectrum of electromagnetic frequencies it emits, just as a blackbody.
  - The spectrum of hotter and hotter bodies shifts from predominately lower frequencies (red) to higher frequencies (blue).
  - Surface temperatures of stars typically range from 3,500 K (reddish) to 50,000 K (bluish).
- For most stars, the colour is related to the absolute luminosity and therefore to the mass.

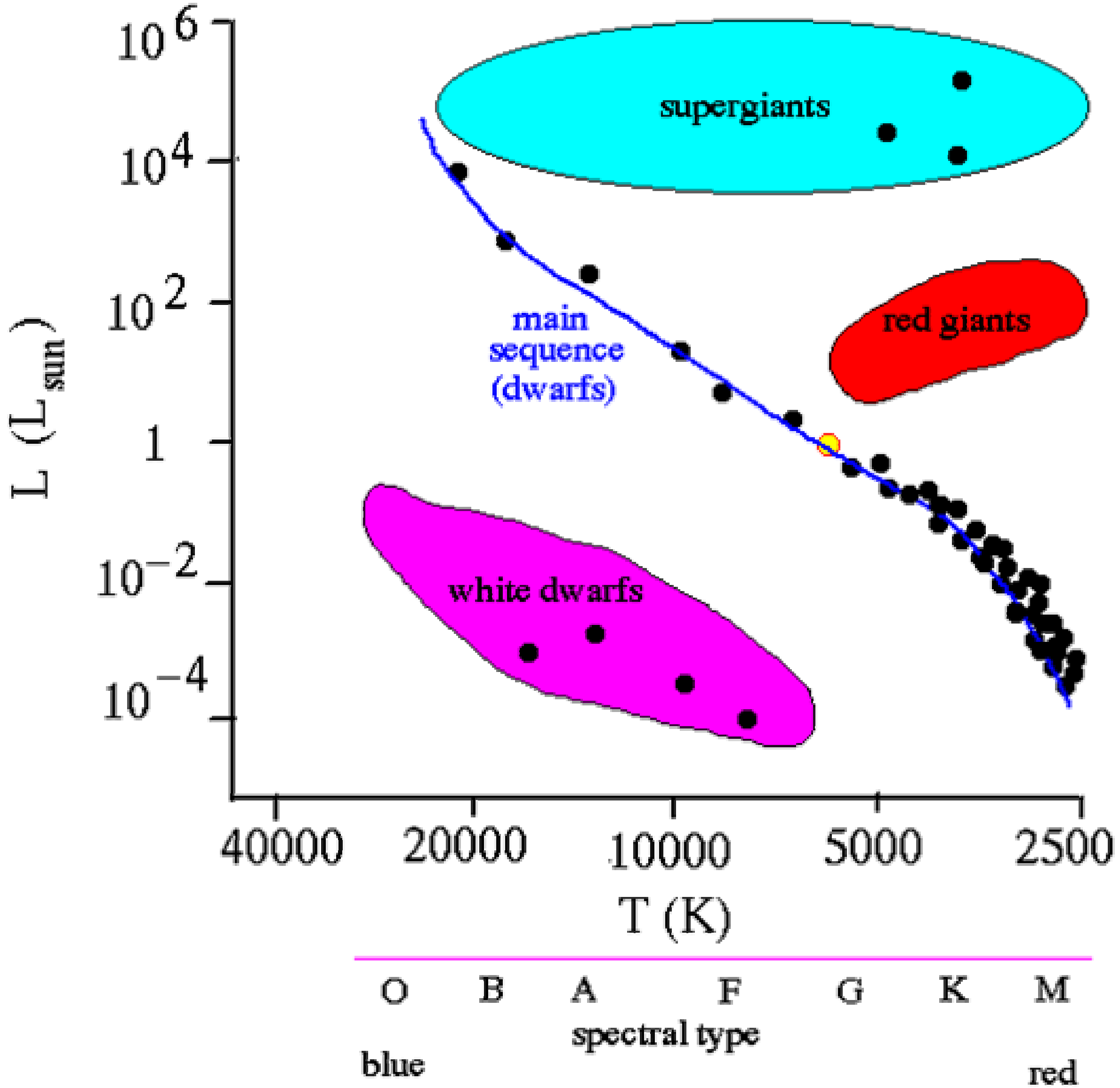
Hertzsprung-Russell (H-R) Diagram



H-R Diagram Regions



Red Giants and White Dwarfs



Stellar Evolution

- Each different type of star represents a different age in the life cycle of a star.
- To reach the main sequence requires perhaps 30 million years.
- It will remain on the main sequence for about 10 billion years (10^{10} yr).
- From the main sequence a star moves to the red giant stage.
- The star goes though a stage of nucleosynthesis until nickel is reached.
- What happens next depends on the mass of the star.

Birth of a Star

- Gaseous clouds (mostly hydrogen) contract due to the pull of gravity.
- As the particles of such a protostar accelerate inward, their kinetic energy increases.
- When the kinetic energy is sufficiently high, the Coulomb repulsion that keeps the hydrogen nuclei apart can be overcome and nuclear fusion can take place.
- The tremendous release of energy in these fusion reactions produces a pressure sufficient to halt the gravitational contraction, and our protostar, now really a young star, stabilizes on the main sequnece.
- Exactly where the star falls along the main sequence depends on its mass.

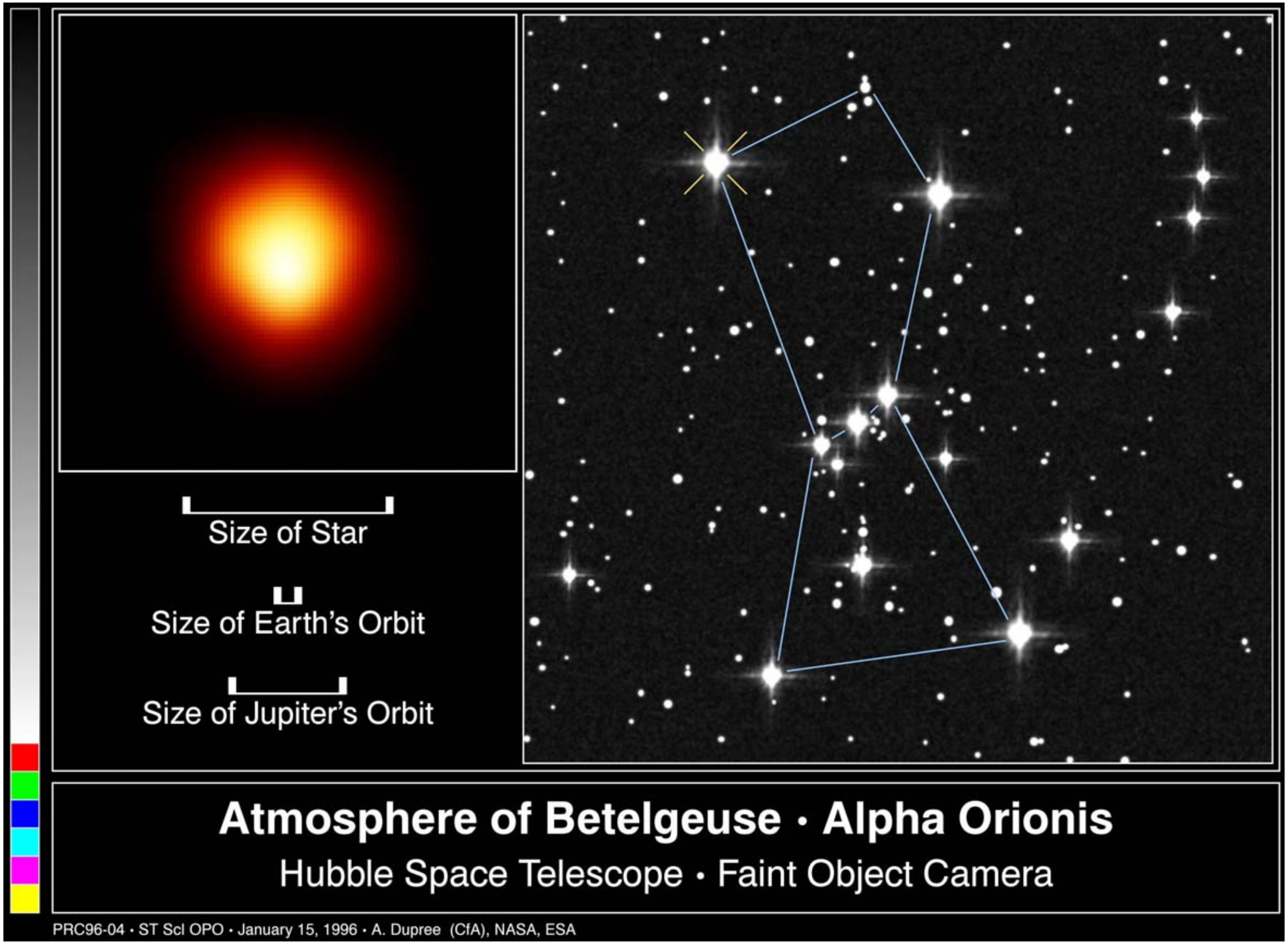
On the Main Sequence

- As hydrogen burns helium is formed at the central core.
- When much of the hydrogen within the core has been consumed, the production of energy decreases and is no longer sufficient to prevent the huge gravitational forces from once again causing the core to contract and heat up.
- The outer envelope expands and cools.
- The surface temperature has reduced, producing a spectrum of light that peaks at longer wavelength (reder).
- The star has left the main sequence and entered the red giant stage.

Red Giants

- Red giants are form from main sequence stars.
- When our Sun leaves the main sequence, it is expected to grow in size (as a red giant) until it occupies all the volume out to approximately the present orbit of Earth.
- Eventually the helium in the core undergoes fusion.
- The star moves to the “horizontal branch”.
- Further reactions are possible creating elements of higher and higher Z through nucleosynthesis.
- Nucleosynthesis ends at nickel.
- What happens next depends on the mass of the star.

Red Supergiant Betelgeuse



White Dwarf.

- If a star has a residual mass less than about 1.4 solar masses, no further fusion energy can be obtained and the star collapses under the action of gravity.
- As the star shrinks and cools it becomes a white dwarf.
- A white dwarf with the mass of the Sun would have the size of the earth.
- A white dwarf continues to lose internal energy until its light goes out.
  - It has become a black dwarf, a dark cold chunk of ash.

Neutron Stars

- Residual mass greater than 1.4 solar masses follow a quite different scenario.
- The star contracts under gravity and heats up even further.
- As the core contracts the mass becomes essentially an enormous nucleus made up almost exclusively of neutrons.
- The star forms a neutron star.
- The density of a neutron star is 10^{14} times that of normal solids and liquids on Earth.
- A neutron star with mass 1.5 Suns would have a diameter of 10 km.
- If the mass of a neutron star is less than about two or three solar masses, its subsequent evolutions is thought to be similar to that of a white dwarf.
- If the mass is greater, a black hole could result.

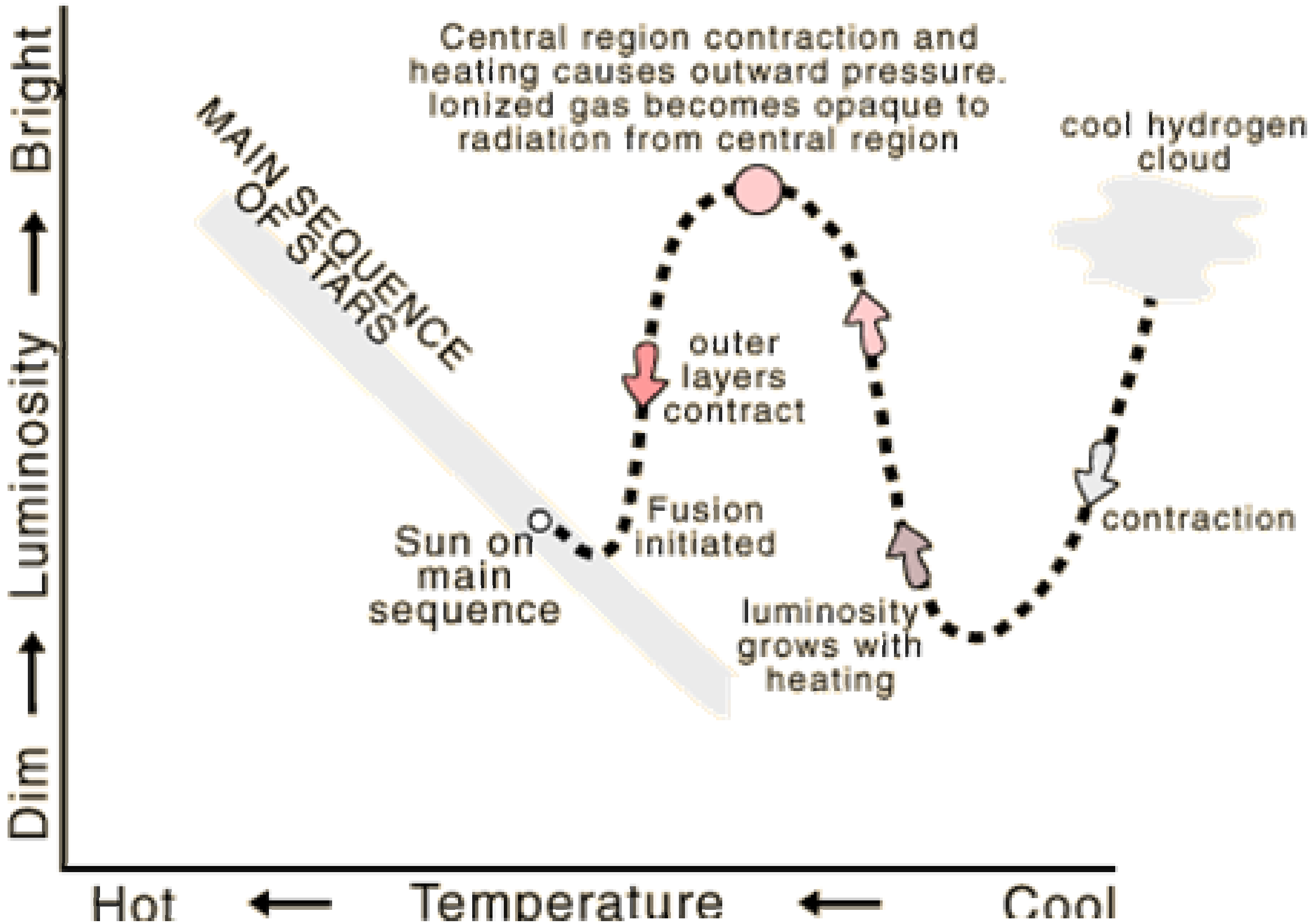
Supernova

- The final core collapse to a neutron star may be accompanied by a catastrophic explosion whose tremendous energy could blow away the entire outer envelope of the star, spreading its contents into interstellar space.
- Such explosions are believed to produce one type of observed supernova.
- Such supernova are thought to be the origin of heavy elements in our solar system, the stuff of which we and our environment are made.
- In a supernova explosion, a star's brightness is observed to suddenly increase billions of times in a period of just a few days and then fade away over the next few months.
- The last supernova visible to the naked eye was in 1987, SN1987a.
  - First since the invention of the telescope.
  - The last was in 1604 (Crab Nebula).

Pulsar

- The Crab Nebula is in the midst of a pulsar.
- Pulsars are astronomical objects that emit sharp pulses of radiation at regular intervals, on the order of seconds.
- They are believed to be neutron stars which increase greatly in rotational speed as their moment of inertia decreases during their contraction.
- The intense magnetic field of such a rapidly rotating star could trap and accelerate charged particles, which then give off radiation in a beam that rotates with the star.

Birth of Our Sun



Module 23

Cosmology

The Study of Cosmology

- General relativity serves as the foundation for modern cosmology, which is the study of the universe as a whole.
- Cosmology builds a framework to understand the observed universe, its origin, and its future.
- Questions posed:
  - Has the universe always existed, or did it have a beginning in time?
  - What about the size of the universe? Is it infinite in size? Or is it finite in size?

Cosmology

- The object is to find a model of the universe as a whole which is a solution to Einstein's equation.
- Before the advent of general relativity, cosmology was in a rather primitive state in spite of the capability of Newtonian mechanics to give a reasonable description.
- The obstacle was to understand why the universe would not simply collapse into the middle of space as a result of the mutual gravitational attractions of the stars.
- Everyone was assuming the universe was in a state of static equilibrium.
- Newton has supposed the universe must be infinite and that the symmetry of the infinite system must mean that there could be no preferred directions and hence no resultant gravitational field at any point.

The Basic Facts

- The first concern of cosmologists must be with the spatial distribution of stars and galaxies.
- Stars:
  - Thermonuclear reactors going through reasonably well understood life cycles.
  - About 10^{11} stars are contained in a typical galaxy.
  - Stars within a galaxy are very sparsely distributed, separated by the order of 10 light years.
- Galaxies:
  - Galaxy has radius of 3×10^4 light years.
  - 3×10^6 light years from its nearest neighbour.

The Basic Facts

- Galaxy clusters:
  - Single galaxies are the exception.
  - The clusters may form superclusters.
- Isotropy:
  - Galaxies appear to be distributed isotropically around us.
  - They all recede from us equally in all directions.
  - No direction in the sky seems preferred over any other.
- Homogeneity:
  - The distant regions of the universe are similar to our own.
  - We see distant regions as they were billions of years ago.
  - Homogeneity in our immediate vicinity prevail among regions of about 10^8 light years in diameter.

The Basic Facts

- The Universe expands:
  - Hubble's constant gives the expansion rate 50 (km/s)/Mpc.
  - 1% every 2×10^8 yr.
  - 1 parsec (pc) = 3.26 light years.
  - Hubble's constant may vary in time.
  - Could even change from positive to negative (contraction).
- Density:
  - Average density 2-6 x 10^{-31} gm/cm^3.
  - This is a lower bound, may be undetected intergalactic matter.
  - Critical datum:
    - Universe positively or negatively curved.
    - Infinitely expanding or doomed to re-collapse.

The Basic Facts

- Age of Universe (if it has an age): 6-20x10^9 years.

Pre-Relativistic Cosmology

- Why does the universe not simply collapse into the middle of space as a result of the mutual gravitational attraction of the stars?
- It was assumed the universe was in a state of static equilibrium.
- Newton supposed the universe must be infinite and that the symmetry of the infinite system must mean that there could be no preferred direction and hence no resultant gravitational field at any point.
- With Hubble's observations that the universe is not static but expanding, the problem immediately goes away because one no longer has to find a mechanism to resist the attractive forces of gravity.

Cosmological Principle

- Modern cosmology is based on the cosmological principle which derives from astronomical observations.
- The universe is homogeneous and isotropic on a scale greater than 10^8 light years.
- On a smaller scale, the universe is certainly not homogeneous but exhibits clustering of stars into galaxies and galaxies into clusters of galaxies.
- The cosmological principle is the assumption of large-scale homogeneity and isotropy.
- There is nothing special about the Earth on a cosmological scale; our large-scale observations are no different from those that might be made elsewhere in the universe.

Difficulties with the Cosmological Principle

- The universe is isotropic (looks the same in all directions).
- The universe is homogeneous (would look the same if we were located elsewhere).
- The expansion of the universe is consistent with the cosmological principle.
- The uniformity of the cosmic microwave background radiation supports this.
- But matter (i.e. galaxies), even on the largest scale, seems not to be homogeneous but tends to clump.
- There is now some doubt about its validity.
- One possible resolution might be that over 90 percent of the universe may be non-luminous dark mater, which might be uniformly distributed.

Hubble's Law

Light from a distant galaxy is red-shifted according to

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c} = \frac{HL}{c}$$

L is the distance of the galaxy.

Thus, from Hubble's law, one infers that the universe is expanding and one has a measure of its expansion rate.

Hubble's law does not work well for nearby galaxies – in fact some are actually moving toward us; but this is believed to merely represent random motions of the galaxies.

For more distant galaxies, the velocity of recession is much greater than that of random motion, and so it dominates.

Hubble's Constant

- The value of Hubble's constant is not known very precisely.
- $H \approx 50$ km/s/Mpc (15 km/s per million light-years)
- Range of 40 km/s/Mpc to 100 km/s/Mpc

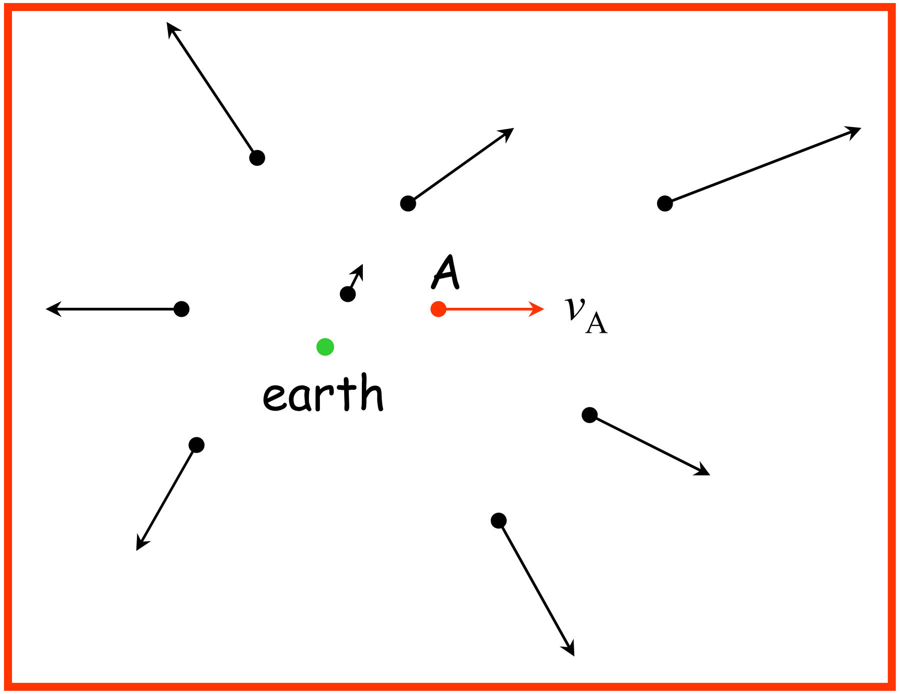
The Expanding Universe

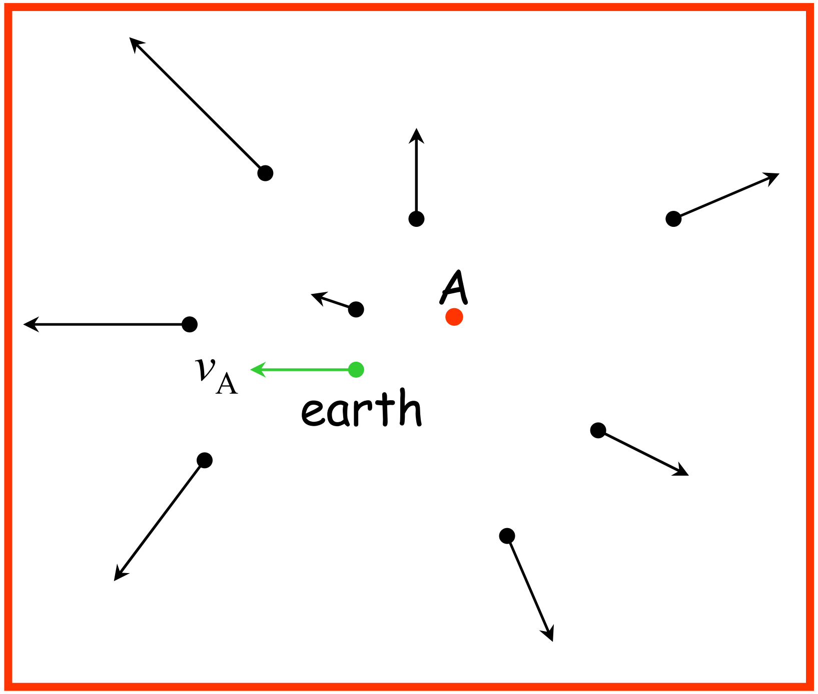
- The evolution of stars suggests that the universe as a whole evolves.
- Hubble suggests an expanding universe.
- It is as if there had been a great explosion at some distant time in the past.
- And that at first sight we seem to be in the middle of it all. But we are not, necessarily.
- The expansion appears the same from any other point in the universe.
- All galaxies are racing away from each other at an average rate of 50 km/s per megaparsec of distance between them.

Milne Model

- Unrealistic since it assumes spacetime is flat, thus ignoring gravity.
- Consider a creation event O in flat spacetime at which a whole system of particles are thrown out isotropically in all directions and with a velocity distribution such that every particle thinks that it is at the centre of the universe.
    - Each particle may consider itself at rest and the velocity distribution that it sees all the other particles moving radially away from it isotropically in all directions.

Expansion Looks the Same at Any Point





Age of the Universe

- The expansion of the universe suggest that the galaxies must have been closer together in the past than they are now.
- This is the basis of the *Big Bang theory* of the origin of the universe.
- The time required for the galaxies to arrive at their present separations would be

$$t_0 = \frac{d}{v} = \frac{1}{H} = \frac{(10^6\,\text{ly})(10^{13}\,\text{km/ly})}{(15\,\text{km/s})(3\times10^7\,\text{s/yr})} \approx 20\times10^9\,\text{yr}$$

20 billion years.

A better estimate is 10 to 18 billion years.

Steady-State Model

- Assumes the universe is infinitely old and on the average looks the same now as it always has.
- No large scale changes have taken place in the universe as a whole.
- To maintain this view in the face of the recession of galaxies away from each other, mass-energy conservation must be violated.
- Matter must be assumed to be created continuously, keeping the density of the universe constant.
- The required rate of mass creation is one nucleon per cubic meter every 10^9 years.
- The discovery of the microwave background radiation favours the Big Bang model.

Big Bang

- If the universe is finite, the explosion would have taken place in a tiny volume approaching a point.
  - This point of extremely dense matter is not to be thought of as a concentrated mass in the midst of a much larger space around it.
  - The initial dense point was the universe – the entire universe. There wouldn't have been anything else.
- If the universe is infinite, then the explosion would have occurred at all points in the universe since an infinite universe, even if smaller at an earlier time, would still have been infinite.
- In either case, we say the universe was once smaller than it is now – the separation of galaxies was less.
- It is thus the size of the universe itself that increase since the Big Bang.

Evidence for the Big Bang

- The age of he universe as calculated from the Hubble expansion, from stellar evolution, and from radioactivity, all point to a consistent time of the origin of the universe.
- The discovery of the cosmic microwave background radiation (CMB).

Cosmic Microwave Background

- In the early stages of the universe, matter and electromagnetic radiation would be in thermal equilibrium and would both cool together as the universe expanded.
- At < 4000 K, when free electrons become permanently attached to nuclei to form atoms, the strong coupling of matter and radiation, via the excitation and de-excitation of atomic electrons, vanishes and the matter and radiation subsequently evolve and cool independently.
- As the universe expands the 4000 K black body radiation would become cosmologically redshifted until at the present size of our universe it would correspond to the 3 K black-body radiation observed.
- Furthermore, on the basis of the cosmological principle, it should be isotropically distributed.

CMB

- The intensity of the radiation does not vary in time; nor does it depend on direction; it comes from all directions in the universe with equal intensity.
- It is concluded that this radiation comes from beyond our Galaxy, from the universe as a whole.
- There needs to be some small inhomogeneities in the CMB that would have provided "seeds" around which galaxy formation could have started.
- Such small inhomogeneities, on the order of parts per million, were finally detected in 1992.
- The intensity of the radiation corresponds to blackbody radiation of 2.7 K.

Importance of CMB

- At the Big Bang the temperature must have been extremely high, so high that there could not have been any atoms in the very early stages of the universe.
- The universe must have consisted solely of radiation (photons) and elementary particles.
- The universe would have been opaque – the photons in a sense trapped since as soon as they were emitted they would have been scattered or absorbed, primarily by electrons.
- The microwave background radiation is evidence that matter and radiation were once in equilibrium at a very high temperature.

The Universe Expands

- As the universe expanded, the energy would have spread out over an increasingly larger volume and the temperature would have dropped.
- Only when the temperature had reached about 3000 K some 300,000 years later, could nuclei and electrons have stayed together as atoms.
- With the disappearance of free electrons, as they combined with nuclei to form atoms, the radiation would have been freed – decoupled from matter – spread throughout the universe.
- As the universe expanded, so too the wavelengths of the radiation expanded, redshifted to longer wavelengths that correspond to lower temperature until they would have reached the 2.7 K background radiation we observe today.

The Universe Today

- The total energy associated with radiation is small compared to the energy, and mass associated with matter.
- Radiation is believed to make up less than 1/1000 of the energy of the universe.
- Today the universe is *matter-dominated*.
- The CMB radiation suggest that the early universe was *radiation-dominated*.
- But the radiation-dominated era lasted less than 1/10000 of the history of the universe (thus far).

Standard Cosmological Model

- It is agreed that the evolution of the universe must have been determined in the first few moments of the Big Bang.
- Before 10^{-43}s after the Big Bang:
  - To make predictions earlier than this would require a theory of the quantum effects on gravity.
  - Prior to this period the four forces of nature were unified – there was only one force.
  - 10^{32}K, particles move every which way with average kinetic energy of 10^{19}eV.

Standard Cosmological Model

- At 10^{-43}s phase transition
  - Gravitational forces, in effect, condensed out as a separate force.
  - The symmetry of the four forces was broken, but the strong, weak, and electromagnetic forces were still unified.
- After 10^{-43}s the grand unified (GUT) era
  - There was no distinction between quarks and leptons; baryon and lepton numbers were not conserved.
  - Very shortly afterward, as the universe expanded considerably and the temperature had dropped to about 10^{27}K, there was another phase transition during which the strong force condensed out.
- At 10^{-35}s
  - Now the universe is filled with a soup of leptons and quarks.
  - The quarks are initially free (something we have not seen in our present universe), but soon they began to condense into more normal particles: nucleons and the other hadrons and their antiparticles.

Standard Cosmological Model

- After 10^{-35}s the hadron era
  - We can think of this soup as a grand mixture of particles and antiparticles as well as photons – all roughly in equal numbers – colliding with one another frequently and exchanging energy.
  - By this time the universe was only 10^{-6}s old, it had cooled to about 10^{13}K, corresponding to an average kinetic energy of 1 GeV, and the vast majority of hadrons disappeared.
  - The process of nucleon creation could not continue but the process of annihilation could continue.

Standard Cosmological Model

- 10^{-35}s
  - A slight excess of quarks over antiquarks was formed.
  - This would result in a slight excess of nucleons over antinucleons.
  - These leftover nucleons is what we are made of today.
  - The excess of nucleons over antinucleons was about one part in 10^9.
  - Protons, neutrons and all other heavier particles were tremendously reduced in number by about 10^{-6}s.
  - The lightest hadrons, pions, disappeared as the nucleons had, they were the last hadrons to go around 10^{-4}s
  - Lighter particles, including electrons, positrons, neutrinos, photons – in roughly equal numbers – dominated the universe during the lepton era.

Standard Cosmological Model

- 10^{-6}s
  - The electrons and positrons annihilate away until a slight excess of electrons is left.
- 10s
  - The universe enters the radiation era.
  - The major constituents are photons and neutrinos.
  - The universe becomes radiation dominated.
  - After 2-3 minutes nuclear fusion began to occur.
  - Deuterium, helium and very tiny amounts of lithium were probably made.
  - The universe cooled too quickly and larger nuclei were not made.
  - After less than $\frac{1}{4}$ hr nucleosynthesis stopped, not to start again for millions of years (in stars).
  - After the first hour, the universe matter consisted mainly of bare nuclei of hydrogen, helium, and electrons. But radiation continued to dominate.

Standard Cosmological Model

- 300,000 years
  - The universe had expanded to 1/1000 of its present size and the temperature had cooled to 3000 K.
  - Electrons could orbit the bare nuclei and remain there, thus forming atoms.
  - The total energy of radiation becomes equal to the total matter energy.
  - As the universe continued to expand, the radiation cooled further to 2.7 K today.
  - The mass of material particles did not decrease and the universe became matter dominated as it remains today.
  - Shortly after the birth of atoms, stars and galaxies formed – probably by self-gravitation around mass concentrations (inhomogeneities).
  - This transpired about a million years after the Big Bang.
  - The universe continued to evolve until today, 10 to 18 million years later.

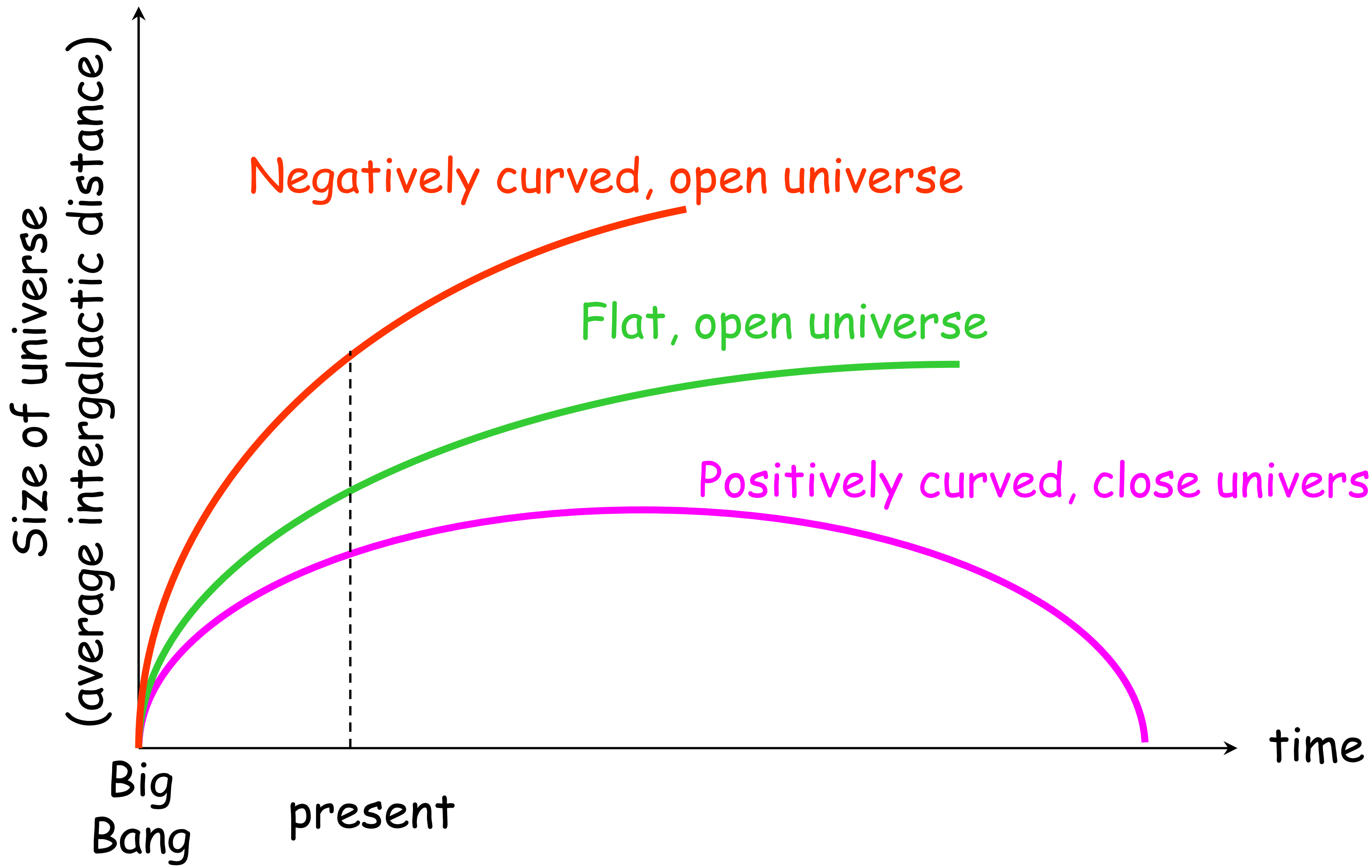
Inflation

- At the earliest stages of cosmological evolution, around 10^{-35}s after the Big Bang, the universe underwent a very rapid exponential expansion associated with the phase transition (symmetry breaking) that separated the strong force from the electroweak force.
- After the brief inflationary period, the scenario settles back to the standard expansion already discussed.
- Inflation provides natural explanations for why the universe is as close to being as flat as it seems to be, and why the cosmic microwave background radiation is so uniform.

Future of the Universe

- Will the universe continue to expand forever?
- We do not know the answer.
- If the curvature of the universe is negative, the expansion of the universe will never stop, although it should decrease due to the gravitational attractions of its parts.
  - Such a universe would be open and infinite.
- If the universe is flat (no curvature), it will still be open and infinite but its expansion will slowly approach a zero rate
- If the universe has positive curvature, it will be closed and finite.
  - The effect of gravity in this case would be strong enough so that the expansion would eventually stop and the universe would then begin to contract.
  - All matter eventually would collapse back onto itself in a big crunch.

Future Possibilities for the Universe



Critical Density of the Universe

- We could determine the future of the universe by knowing the average mass density in the universe.
- If the average mass density is above a critical value known as the critical density, $\rho_c = 10^{-26}$ kg/m^3 (a few nucleons/m^2), then gravity will prevent expansion from continuing forever, and will eventually pull the universe back into a big crunch.

Density of the Universe

- If $\rho > \rho_c$ there would be sufficient mass that gravity would give spacetime a positive curvature
- If $\rho = \rho_c$ the universe will be flat and open.
- If $\rho < \rho_c$ the universe will have a negative curvature and be open, expanding forever.

Measuring the Critical Density

- Estimates of the amount of visible matter in the universe put the actual density at between one and two orders of magnitude less than the critical density.
- This suggests an open universe.
- There is evidence for a significant amount of non-luminous matter in the universe.
- This is often referred to as missing matter or dark matter.
- There may be enough dark matter to bring the density to almost the critical value.

Nature of Dark Matter

- If there is non-luminous matter in the universe, what might it be?
- Could be previously unknown weakly interacting particles (WIMPS).
- Primordial black holes made in the early stages of the universe.
- Massive compact halo objects (MACHOS)
  - Chunks of matter in the form of large planets or stars that are too small to sustain fusion (brown dwarfs).
  - Evidence for MACHOS was found in 1993.

Possibility of Neutrino Mass

- Another possibility for the missing mass is neutrinos.
- Neutrinos may have nonzero rest mass.
- Since there are so many neutrinos, even a small mass could help bring the actual density of the universe up to the critical density.
- The masses of the neutrinos are still not known.

Deceleration Parameter

- Measure of the rate at which the expansion of the universe is slowing down.
- Currently this parameter can not be measured accurately enough.

Future of an Open Universe

- Galaxies would have much of their matter knocked away and scattered throughout the universe by collisions with other stars.
- The remaining matter would eventually condense into massive “galactic black holes”.
- Clusters of these would coalesce into extremely massive “super black holes”.
- The black holes themselves would “evaporate”.
- The universe would then be a thin gas of electrons, positrons, neutrinos, and photons.

Future of a Closed Universe

- If the universe is closed, it might turn around and contract even before all stars have burnt out.
- As the universe contracts the background radiation would increase in energy and temperature.
- The universe might retrace its steps, if it weren't for black holes.
- As the density increases, and the universe rushes toward its inevitable end in the "Big Crunch", black holes might gobble up more and more matter until the entire universe coalesced into a single supermassive black hole – which would then be the universe.

After the Big Crunch

- If the universe is closed, what happens after the big crunch?
- A "bounce" is possible.
- The dense fiery nucleus of the big crunch might explode again, resulting once more in an expanding universe.
- Thus the universe might be cyclic.
- Such a cyclic or pulsating universe proposes a possible answer to one of our favourite unanswerable questions: what happened before the Big Bang?
- In this model there was simply a previous cycle.
- But when did it all begin?

Anthropic Principle

- Calculations on the formation and evolution of the universe have been performed that deliberately varied the values – just slightly – of certain fundamental physical constants.
- The result is a universe in which life as we know it could not exist.
- For example, if the fundamental unit of charge e were different by even a small fraction, of a percent, long-lived main sequence stars like our sun could not exist.
- Such results have given rise to the so-called Anthropic principle.
- If the universe were even a little different than it is, we couldn't be here.
- It's as if the universe were exquisitely tuned, almost as if to accommodate us.

Bibliography

- Physics, Cutnell & Johnson
- Relativity Visualized, Lewis Carroll Epstein, Insight Press.
- Relativity Simply Explained, Martin Gardner, Dover.
- Space and Time in Special Relativity, David Mermin, Waveland.
- Relativity and Common Sense, Hermann Bondi, Dover.
- The Principle of Relativity, Einstein and others, Dover.
- Fundamentals of Modern Physics, Eisberg, Wiley.
- Foundations of Modern Physics, Tipler, Worth.
- The Feynman, Lectures on Physics, Feynman, Leighton, Sands, Addison Wesley.