

Searching for a Lower Dimensional Spacetime Structure with the ATLAS Detector

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Abstract

Dimensional reduction (DR) is a phenomenological feature in which the effective number of spatial dimensions decreases at high energies. If the transition scale from a $(3 + 1)$ - to a $(2 + 1)$ -spacetime is set at the low TeV scale, $\Lambda_3 \sim \text{TeV}$, then the effects of this spatial transition may be observable in the event topologies of high-energy proton-proton collisions. We develop a simplified DR Monte Carlo event generator simulating tree-level QCD and $2 \rightarrow 2, 3, 4, 5$ massless parton phase space. We show that the event shape observables aplanarity, A , and biplanarity, B , effectively distinguish $(2 + 1)$ DR events from $(3 + 1)$ backgrounds, as $(2 + 1)$ events are fully confined to $A = 0$ and $B = 1$ respectively, with planarity as a function of event invariant mass serving as a metric to differentiate DR scales. Evaluating this framework under ATLAS Run 2 conditions (centre-of-mass energy of 13 TeV and an integrated luminosity of 140 fb^{-1}), we find that jet-level smearing causes up to 70% of events classified as planar at parton level to migrate outside the planar aplanarity and biplanarity regions. While parton-level DR perfectly differentiates planar topologies with sharp transitions in planarity at the DR scale, jet reconstruction smooths these features consequentially restricting experimental sensitivity to $\Lambda_3 \in [2, 7] \text{ TeV}$.

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Statement of Originality

Starter scripts for the dimensional reduction Monte Carlo model and ATLAS software were provided by Dr. Douglas Gingrich. The work presented in this thesis is original and is the author's own, except where denoted by citations.

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List of Abbreviations

ALICE	A Large Ion Collider Experiment
ATLAS	A Toroidal LHC Apparatus
BSM	Beyond the Standard Model
BEH	Brout-Englert-Higgs
CoM	Centre-of-mass
DR	Dimensional Reduction
DRTOM	Dimensional Reduction Toy Model event generator
GR	General Relativity
ID	Inner Detector
IP	Interaction point
LAr	Liquid Argon
LHC	Large Hadron Collider
LHCb	Large Hadron Collider beauty
LHE	Les Houches Event
MC	Monte Carlo
ME	Matrix element
NN	Neural network
PDF	Parton Distribution Function
QCD	Quantum Chromodynamics
QED	Quantum Electrodynamics
QFT	Quantum Field Theory
ReLU	Rectified linear unit
SM	Standard Model of particle physics
TDAQ	Trigger and Data Acquisition

Chapter 1

Introduction

The nature of space and time form a foundational component in the development of physics. General relativity (GR) describes a classical theory of gravity. The Standard Model of particle physics (SM) offers a quantum description of the microscopic world. Both, despite the difference in the scale they operate at, utilize the notion of a four-dimensional spacetime. These theories, however, face significant limitations as there is currently no accepted unified theory of quantum gravity. General relativity does not fit comfortably within the framework of quantum fields, and, conversely, the SM fails to account for gravity and suffers from unresolved issues such as the fine-tuning problem. The incompatibility between these two well-tested yet fundamentally distinct theories motivates the search for a unifying framework in a theory beyond the SM (BSM).

Dimensional reduction (DR) is a phenomenological feature across many high-energy physics theories that offers a path towards a unification scheme by proposing a new description of spacetime [1]. It suggests that at high energy scales (short length scales), the number of effective spatial dimensions decrease. Spacetime is thus a fundamental $(1+1)$ - or $(2+1)$ -structure that evolves into a $(3+1)$ -spacetime at sufficiently low energy scales. This proposition is compelling as it offers a natural resolution to both the hierarchy problem and the challenges of quantum gravity while retaining a $(3+1)$ description of the Universe [2, 3, 4]. If such a transition from $(3+1)$ to $(2+1)$ exists, the effects could be observed in high-energy particle collisions. Assuming that the characteristic energy scale between three- and two-spatial dimensions is on the order of $\sim \text{TeV}$, the Large Hadron Collider (LHC), with a centre-of-mass (CoM) collision energy at 13.0 TeV in Run

2, becomes an ideal environment for experimental investigation of DR [5]. In particular, the topology of particle collisions in the A Toroidal LHC Apparatus (ATLAS) detector can offer insight into the hypothesis; collision events should exhibit either three- or two-dimensional characteristics depending on whether a dimensional transition actually occurs.

We build and utilize a simplified Monte Carlo (MC) simulation model of dimensional reduction (DRToM), along with standardized SM ATLAS event generators, to investigate the effects of dimensional reduction in ATLAS-like detectors. Chapter 2 provides an overview of spacetime in the SM and GR, and a conceptual discussion of dimensional reduction. Chapter 3 explores proton–proton collisions, with a focus on tree-level quantum chromodynamic (QCD) events (as this is the basis for DRToM), and solidifies our description of DR. Chapter 4 describes the ATLAS detector, covering relevant hardware and software components. Chapter 5 outlines the computational operation scheme of DRToM. Chapter 6 then shows the kinematic outputs of DRToM and introduces topological event shape variables as a tool to distinguish between spherical and planar events. In Chapter 7 we extend our analysis techniques to study the effects of dimensional reduction (after QCD showering, hadronization, and heavy quark decay) on jets. Finally, Chapter 8 summarizes the results and conclusions of the analysis.

Chapter 2

Theoretical Foundation

2.1 The Standard Model of Particle Physics

Of the four fundamental forces, three are described by particle interactions: the electromagnetic force through quantum electrodynamics (QED), the weak nuclear force through the electroweak theory, and the strong nuclear force through quantum chromodynamics. These are called quantum field theories (QFTs).

A force in a QFT is an interaction mediated by interaction particles (called gauge bosons) and dictated by an elementary quantum number of the matter particles (fermions) that must be conserved across the interaction. In QED, the force carrier is the photon, γ , and the quantum number is the electric charge. For the weak nuclear force, the force carriers are the W and Z bosons and the quantum number is the weak hypercharge. In QCD the force carriers are gluons, g , and the quantum number is colour charge. There are three colours (red, green, blue) and three anticolours (antired, antigreen, antiblue)—if all three colours are combined, the state is colourless. A quark carries a single colour, an antiquark a single anticolour, and gluons carry both a colour and anticolour that cannot be the same type (e.g. no red-antired pairings). In QCD, colour confinement prevents particles that carry colour charge from existing as isolated particles. All particles described by the SM can be seen in Fig. 2.1.

These three QFTs are neatly contained in the SM Lagrangian [7, 8, 9]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (i\bar{\psi}\not{D}\psi + \text{h.c.}) + (\psi y_{ij}\psi_j\phi + \text{h.c.}) + |D_\mu\phi|^2 - V(\phi). \quad (2.1)$$

Standard Model of Elementary Particles

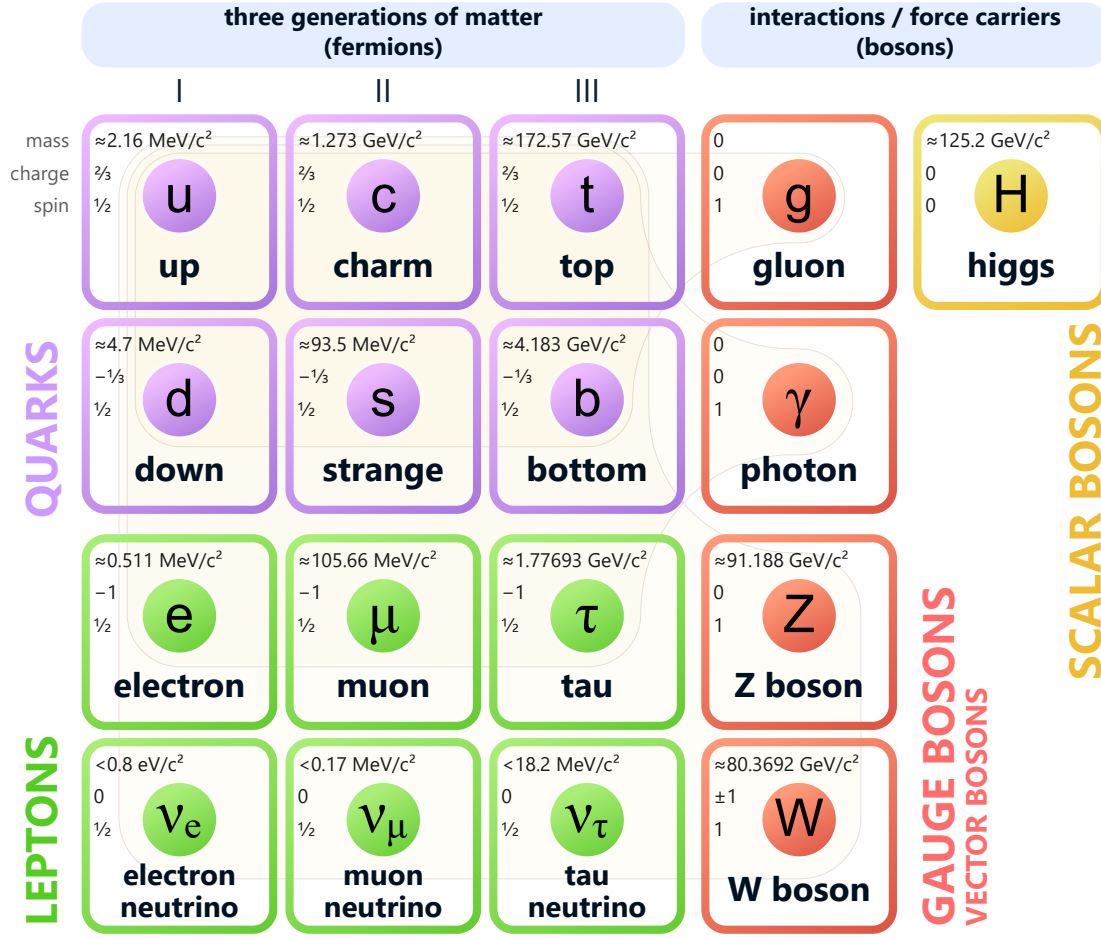


Figure 2.1: The particles that make up the Standard Model. Each fermion (except neutrinos) has a corresponding antifermion with the same mass but opposite charge [6].

The first term, $F_{\mu\nu}F^{\mu\nu}$, is the scalar product of the field strength tensor, $F_{\mu\nu}$, which itself encodes the behaviour of all interaction particles, including self interactions, except the Higgs boson.

The second term, $i\bar{\psi}\not{D}\psi$, describes the kinetic behaviour of the matter fields and their interactions with the gauge bosons. Here, ψ and $\bar{\psi}$ represent the (anti)quark and (anti)lepton fields, while $\not{D} = \gamma^\mu D_\mu$ denotes the covariant derivative acting on fermion fields. Since fermions carry weak isospin, weak hypercharge, and (for quarks) colour charge, this derivative introduces their interactions with the electroweak gauge bosons and gluons. The notation h.c. is shorthand for hermitian conjugate and is used to ensure that the Lagrangian remains real-valued. Although the fermion kinetic term is already

self-adjoint, the hermitian conjugate is sometimes included for consistency of notation.

The third term, $\bar{\psi}_i y_{ij} \psi_j \phi + \text{h.c.}$, describes the Yukawa interactions between the matter fields and the Brout-Englert-Higgs (BEH) field, ϕ . The entries of the Yukawa matrix, y_{ij} , determine the strength of the coupling between the fermion fields and the BEH field. Following spontaneous symmetry breaking, the BEH field acquires a non-zero vacuum expectation value, and this term generates mass terms for the fermions. The h.c. is necessary here as it is required to describe how antimatter particles interact with the BEH field.

The fourth term, $|D_\mu \phi|^2$, is the kinetic term of the BEH field. Here D_μ is the covariant derivative acting on the BEH field rather than the fermion fields. Because the BEH field transforms as a weak isospin doublet and carries hypercharge but no colour charge, this derivative contains only the electroweak gauge fields. After spontaneous symmetry breaking, this term generates mass terms for the $W^\pm$ and Z^0 bosons. The photon remains massless because the electromagnetic symmetry remains unbroken, while the gluons remain massless because the BEH field does not carry colour charge and therefore does not couple to the gluon fields. The final term, $V(\phi)$, is the potential of the BEH field. Its form causes spontaneous electroweak symmetry breaking and results in the BEH field acquiring a non-zero vacuum expectation value. This vacuum expectation value allows fermion masses to arise through the Yukawa interaction term and the $W^\pm$ and Z^0 boson masses to arise through the BEH kinetic term. The potential also determines the mass of the physical Higgs boson and describes its self-interactions.

How interaction particles couple to matter particles, and how they interact with each other, are encoded in Eq. 2.1 but are visualized through Feynman diagrams and rules. Examples of such diagrams and the fundamental interactions they represent are shown in Fig. 2.2, Fig. 2.3, Fig. 2.4, and Fig. 2.5.

From the perspective of the SM, spacetime is a flat manifold on which interactions occur. What we consider particle forces—those mediated by interaction carriers—obey relativistic principles by existing within spacetime. They are inherently quantum in nature and are described by probabilistic fields.

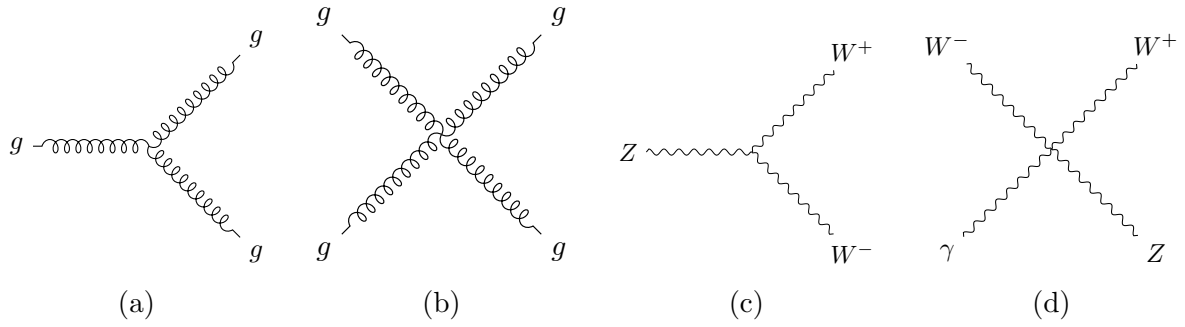


Figure 2.2: Some interaction terms from the field strength tensor $F_{\mu\nu}$. The left two diagrams (a and b) are QCD processes with gluon self interactions, and the right two diagrams (c and d) are weak process involving the W and Z bosons and photons.

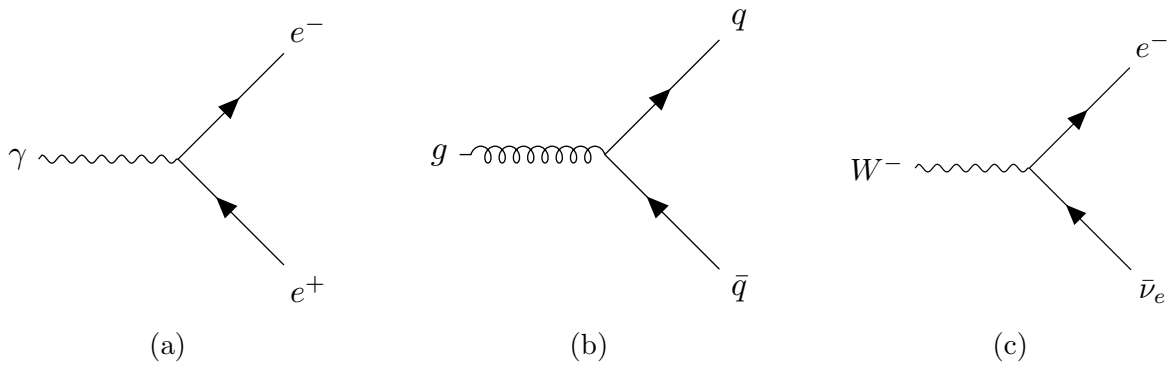


Figure 2.3: Basic Feynman interactions for matter and force carriers. (a) Depicts the electron (in general, the lepton) annihilation and creation scheme, (b) the quark annihilation and creation, and (c) possible neutrino weak interactions.

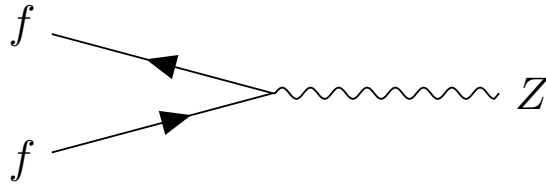


Figure 2.4: Z boson as a mediator for matter particles. It couples to all fermions, f .

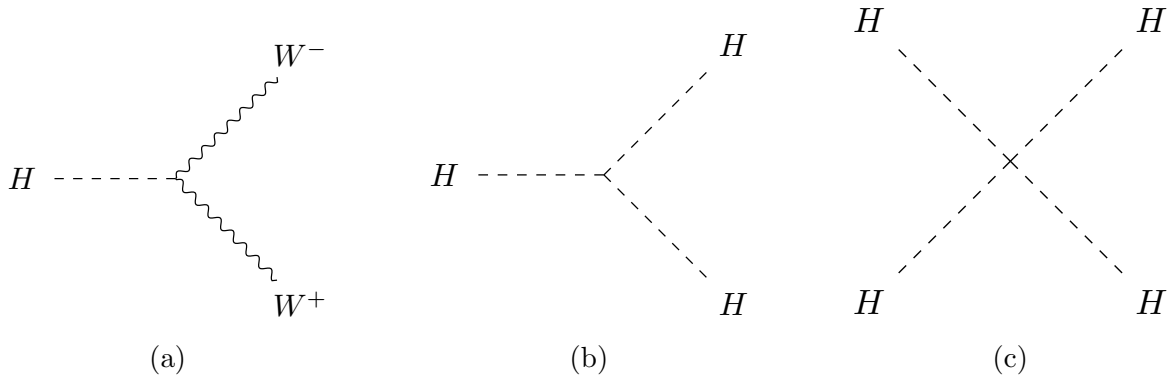


Figure 2.5: Possible Higgs boson couplings. The left (a) is Higgs coupling to W bosons, whereas the right two (b and c) show Higgs coupling to itself.

2.2 General Relativity

The final fundamental force, gravity, stands distinct from the particle interactions of the SM. Consider the Lagrangian

$$\mathcal{L} = \left[\frac{1}{2\kappa}(R - 2\Lambda) + \mathcal{L}_M \right] \sqrt{-g}, \quad (2.2)$$

where $\kappa = 8\pi G/c^4$ is a constant, R is the Ricci scalar which is the contraction of the Ricci tensor $R_{\mu\nu}$, Λ is the cosmological constant, $g = \det(g_{\mu\nu})$ where $g_{\mu\nu}$ is the metric tensor, and $\mathcal{L}_M$ is the Lagrangian of the matter fields. Varying the action of Eq. 2.2 with respect to the inverse metric results in the Einstein field equations [10, 11]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}. \quad (2.3)$$

The left-hand side, commonly denoted as $G_{\mu\nu}$, is the Einstein tensor which encodes the geometric structure of spacetime. The right-hand side is the stress-energy tensor,

$$T_{\mu\nu} = -2 \frac{\delta \mathcal{L}_M}{\delta g^{\mu\nu}} + g_{\mu\nu} \mathcal{L}_M, \quad (2.4)$$

which represents the distribution of energy and momentum.

Equation (2.3) tells us that spacetime is a geometric entity that bends in response to matter. Alternatively, it shows that matter and curvature—i.e. the classical gravitational field—are fundamentally the same, rendering spacetime itself a dynamic quantity. Thus, gravity as a force is the manifestation of spacetime itself.

2.3 Unification

We see that there are differing notions as to the role of forces and spacetime in our fundamental theories. Unification schemes are grouped into two categories: covariant and canonical [12, 13]. Covariant approaches regard the curved spacetime of GR as an effective modification of a flat background geometry by the graviton (the proposed force carrier of gravity) field. In this view, particle physics and QFT form the natural

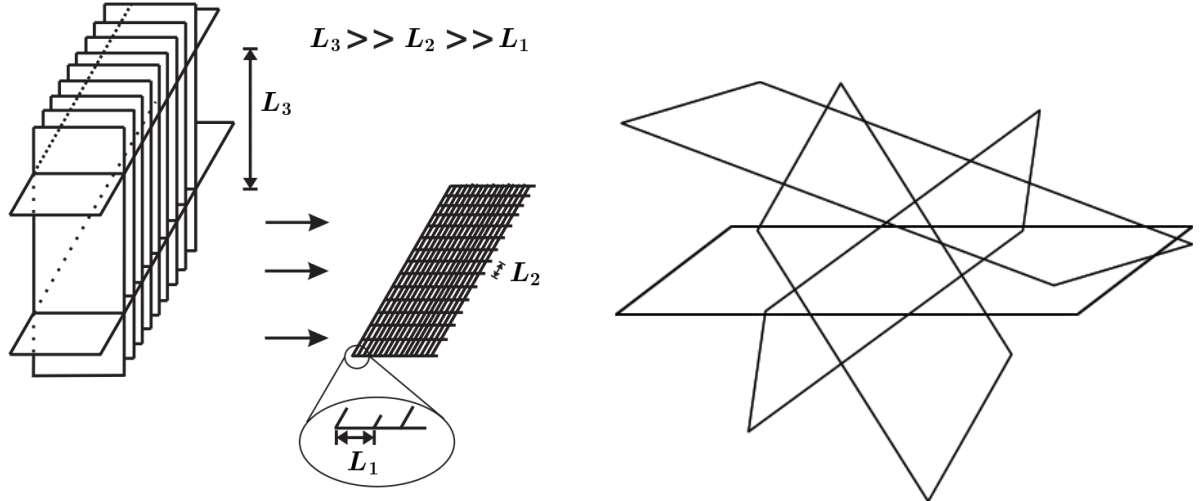
framework for developing a quantum theory of gravity (e.g. string theory). In contrast, canonical approaches treat the entire spacetime metric as a field and aim to quantize it directly without decomposing it into a flat background plus perturbations. In other words, canonical approaches attempt to construct a quantum theory in which the phase space of GR is replaced by an appropriate Hilbert space (e.g. loop quantum gravity).

2.4 Dimensional Reduction

Dimensional reduction is a phenomenon where the number of dimensions required to describe a theory reduces with an increase in the energy scale. This effect has been seen to happen in high temperature string theory, short distance Wheeler-DeWitt equation, loop quantum gravity, Horava-Lifshitz gravity, and causal dynamical triangulations. Although it is clear that the phenomena is present, it is not clear as to what the underlying dynamics are or if the dimensions are physical or mathematical [1].

Moving forward, for a definite and physical notion of dimensional reduction, we require that spacetime be distinguished into $(n + 1)$ -slices, where $n \in [1, 3]$. The dimensionality is ordered based on the energy scales Λ_n , where $\Lambda_1 > \Lambda_2 > \Lambda_3$, with corresponding length scales, L_n , such that $L_1 < L_2 < L_3$ [5]. Figure 2.6a proposes a possible construction of a universe that is ordered as a rigid lattice. Another possibility is a layered foliation of lower dimensional sheets as depicted in Fig. 2.6b. We note that this hierarchy could, in principle, continue indefinitely with the possibility of $n \geq 4$ (given that Λ_3 may not be the fundamental energy scale of the universe), however, we do not require this to be the case for our discussion.

As an instructive example, physics observed at energies $\Lambda < \Lambda_3$ —equivalently, at length scales $L > L_3 \approx \Lambda_3^{-1}$ —appears as if in three spatial dimensions. The term appears does not imply that three-dimensionality is illusory, but rather that, if the fundamental spacetime is $(1+1)$ or $(2+1)$, the observations of $(3+1)$ -dimensionality reflect an emergent structure. Now, in any valid physical process, if the energy scale rises to $\Lambda_3 < \Lambda < \Lambda_2$ (or the corresponding length scale decreases to $L_2 < L < L_3$), spacetime transitions from



(a) Layered structure of spacetime with each dimension corresponding to a different energy/length scale [5].

(b) Foliation of two-dimensional sheets that would give rise to a three-dimensional continuum at large length scales [3].

Figure 2.6: Illustrations of dimensional reduction.

(3 + 1)- to (2 + 1)-dimensions. In this scheme, Λ_1 has no explicit transition meaning.

We note that the energy scales, Λ_n , represent the effective energies at which spacetime transitions out of $(n + 1)$ on average. These scales depend on the specifics of the dimensional structure, particularly how each domain is connected within the larger higher-dimensional framework. For example, the spacetime hierarchy may originate from a (non-specified) tension of the $(1 + 1)$ -dimensional spacetime. A larger tension would result in a longer length scale over which the line curves or intersects itself (as a higher tension resists bending, requiring a greater distance for the structure to wrap) to produce the effective two-dimensional spacetime. The transition from two- to three-dimensional space would then depend on the effective tension of the two-dimensional sheet. There is no strict prescription for how the foliation into higher dimensions must occur, and for the purposes of this investigation, we leave such structural details unspecified.

Dimensional reduction, as we have described it, offers an alternative method to the standard unification schemes in Sec. 2.3. This proposition of a progressively evolving dimensional spacetime intends to represent a global dynamical structure on which mathematical frameworks are formulated in specific regions [2, 5]. While its parametrized evolution can vary, the essential result is the distinct dynamics that emerge within de-

fined Λ_n spacetimes. Thus, the physics in $(3+1)$ —i.e. the physics of GR and the SM—are proper theories for a $(3+1)$ -spacetime, but they fail at unification since the universe is not truly $(3+1)$ at high-energies. The goal of this model is not to alter the mathematical framework of modern physics, but to redefine the background (that being spacetime) it lives on in hopes of offering a new approach to unresolved problems [2, 3, 4, 5].

Chapter 3

Proton-Proton Collisions

3.1 Tree Level Kinematics

Our discussion starts with two incoming particles in the CoM frame, A and B , colliding such that there are two outgoing particles, C and D . This is a $2 \rightarrow 2$ collision process, as seen in Fig. 3.1, with each particle having a three-momentum $\vec{P}_N$ for $N = A, B, C, D$. The interaction can either be elastic or inelastic. In an elastic collision, A and B collide and scatter such that C and D remain the same particles. In an inelastic collision, the particles are broken apart by the interaction and C and D may represent any new constituent particle (or they can be broken up into many outgoing particles, which is not represented in the figure below). Collisions are further classified as soft or hard depending on the momentum transfer. A soft collision refers to small momentum exchange while a hard collision refers to a large momentum exchange. This classification applies to both elastic and inelastic processes.

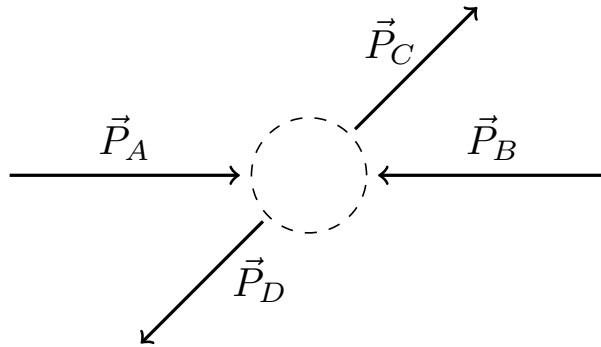


Figure 3.1: A general $2 \rightarrow 2$ ($AB \rightarrow CD$) collision in the CoM frame.

When two protons collide, we must take into account that protons are not elementary particles but are composed of constituent quarks bound together by gluons—collectively referred to as partons. The valence quarks in a proton are two up quarks and one down quark. In addition to the valence quarks, protons contain a sea of virtual quark-antiquark pairs generated by both the gluons that bind the valence quarks and from vacuum fluctuations. As a result, proton-proton collisions are actually parton-parton collisions with interactions between valence quarks, sea quarks, and gluons from each proton. At sufficiently high collision energies, the proton structure is broken apart and we deal with inelastic collisions. At the partonic level, however, high-momentum transfers are most relevant and elastic collisions are possible.

Let us consider the collision of two protons, A and B , each with four-momentum $P_N^\mu = (E_N, \vec{P}_N)$. For this thesis, a capital P will refer to the whole particle (where the constituent particles come from, here a proton); a subscript will denote the particle, in this case $N = A, B$; a superscript denotes the four-vector component, $\mu = 0, 1, 2, 3$; and a vector denotes the three-momentum, $\vec{P} = (P^1, P^2, P^3)$. Moving forward, we work in natural units, $c = \hbar = G = 1$, and adopt the mostly negative Minkowski metric convention.

Each parton in a proton carries a fraction of the total proton energy, denoted by x . We therefore consider parton a from proton A interacting with parton b from proton B . The four-momentum for constituents of the whole particle will be denoted with a lowercase p , thus the partons have four-momenta given by p_n^μ for $n = a, b$ and $\mu = 0, 1, 2, 3$. A superscript will only be used if a specific component of the four-momentum is required, otherwise the four-momentum will be left as P or p . The momentum fractions of each parton are defined as

$$x_a = \frac{E_a}{E_A}, \quad x_b = \frac{E_b}{E_B}. \quad (3.1)$$

For massless particles, where $E = |\vec{p}|$, then

$$x_a = \frac{|\vec{p}_a|}{|\vec{P}_A|}, \quad x_b = \frac{|\vec{p}_b|}{|\vec{P}_B|}. \quad (3.2)$$

After the collision, two outgoing partons (c and d) are produced with a scattering angle

θ in the CoM frame as seen in Fig. 3.2. We note that the dashed circle is used as an arbitrary collision representation; two partons go in, a collision of some-sort happens (the specifics of which we do not care about), then two partons emerge.

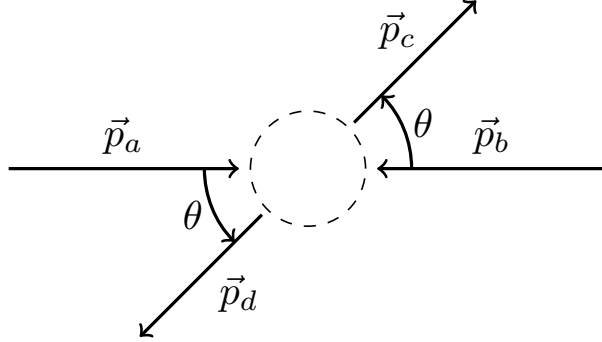


Figure 3.2: A $2 \rightarrow 2$ ($ab \rightarrow cd$) parton-parton collision in the CoM frame with scattering angle θ .

To describe the kinematics of the collision, we introduce the Mandelstam variables:

$$\hat{s} = (p_a + p_b)^2, \quad \hat{t} = (p_a - p_c)^2, \quad \hat{u} = (p_a - p_d)^2. \quad (3.3)$$

The hats indicate that the variables refer to parton-level interactions. The definitions of these variables are directly related to the Feynman diagrams process: $\hat{s}$ for the s -channel, $\hat{t}$ for the t -channel, and $\hat{u}$ for the u -channel. Moreover, in the CoM frame, $\hat{s}$ is defined as the total energy squared (since $\vec{p}_a = -\vec{p}_b$). By definition, these satisfy the relation

$$\hat{s} + \hat{t} + \hat{u} = m_a^2 + m_b^2 + m_c^2 + m_d^2. \quad (3.4)$$

At the LHC, each colliding proton carries half of the CoM energy and its momentum is aligned with the beam axis, which we define as the z -axis. At a CoM energy of 13 TeV, each proton has an energy of exactly 6.5 TeV. Given the proton's rest mass of 0.938 GeV, we work in the ultra-relativistic limit where the mass of the proton is negligible and the energy comes solely from its momentum. In this limit, the Mandelstam relation reduces to $\hat{s} + \hat{t} + \hat{u} = 0$. We note, however, that this is only the case for high-momentum fraction (e.g. high x_a, x_b) collisions. At low momentum fraction collisions, we cannot ignore the top quark mass. In general, top quarks (with a mass of 173 GeV) cannot be neglected in general TeV-scale processes as they can still exist in the proton's virtual sea of particles.

Consider a general particle collision in Cartesian coordinates. The transverse momentum of an outgoing particle is given by

$$p_T = \sqrt{(p^x)^2 + (p^y)^2}, \quad (3.5)$$

which is invariant under Lorentz boosts along the beam axis. Recall that we are opting to show coordinate positions as superscripts and set the beam axis to the z -axis. Then, it follows

$$p^x = p_T \cos \phi, \quad p^y = p_T \sin \phi, \quad (3.6)$$

where ϕ is the azimuthal angle measured in the xy -plane and covers a 2π range. The rapidity is defined as

$$y = \frac{1}{2} \ln \frac{E + p^z}{E - p^z}. \quad (3.7)$$

For a massless particle, this can be related to the polar angle θ between the particle's three-momentum and the beam axis,

$$y \approx \frac{1}{2} \ln \frac{1 + \cos \theta}{1 - \cos \theta} = -\ln \tan \frac{\theta}{2}. \quad (3.8)$$

The pseudorapidity is then defined as

$$\eta = -\ln \tan \frac{\theta}{2}, \quad (3.9)$$

so that $y \rightarrow \eta$ in the massless limit [14, 15]. Particles close to the beam in the positive direction have large rapidity (as $\theta \rightarrow 0(\pi)$ then $\eta \rightarrow \infty(-\infty)$), while particles with momentum purely transverse to the beam ($p^z = 0$ or $\theta = \pi/2$) have zero rapidity. Due to conservation of momentum, in the CoM frame the outgoing partons are back-to-back with polar angle θ and $\pi - \theta$. This means that $y_c = -y_d$, so their magnitudes are equivalent $|y_c| = |y_d| = y$; that is, they have equal transverse momentum magnitudes and opposite transverse momentum vectors. Under a boost along the beam axis, the rapidity of each particle shifts by a constant, Y ,

$$y_c = y + Y, \quad y_d = -y + Y, \quad (3.10)$$

making rapidity differences boost-invariant observables. This is important because col-

lisions at the LHC are in the lab frame, whereas the kinematics of the theory and our generator are in the CoM frame then boosted to the lab. Rearranging Eq. 3.10 gives

$$y = \frac{1}{2}(y_c - y_d), \quad Y = \frac{1}{2}(y_c + y_d). \quad (3.11)$$

With the rapidity variable, we can define the energy and z -momenta component as

$$E = m_T \cosh y, \quad p^z = m_T \sinh y, \quad (3.12)$$

where $m_T = \sqrt{p_T^2 + m^2}$ is the effective mass/energy of a particle when ignoring its motion along the beam axis (and m is the mass of the particle).

A general form of p_T can be found in terms of rapidity and outgoing masses. Let the outgoing particles have four-momentum $p_c = (E_c, \vec{p}_c)$ and $p_d = (E_d, \vec{p}_d)$. Because $\hat{s}$ is the total partonic CoM energy squared, we define the invariant mass of the event as

$$\hat{s} = M^2. \quad (3.13)$$

From this, we get

$$\begin{aligned} M^2 &= (p_c + p_d)^2 \\ &= p_c^2 + p_d^2 + 2p_c \cdot p_d \\ &= m_c^2 + m_d^2 + 2(p_T^2 + m_{T,c}m_{T,d} \cosh(2y)). \end{aligned} \quad (3.14)$$

Rearranging gives

$$p_T^2 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}, \quad (3.15)$$

where

$$A = \cosh^2(2y) - 1, \quad B = (m_c^2 + m_d^2) \cosh^2(2y) + 2a, \quad C = (m_c m_d)^2 - a^2, \quad (3.16)$$

and

$$a = \frac{M^2 - m_c^2 - m_d^2}{2}. \quad (3.17)$$

We note that for the cases of $m_c = m_d = 0$ and $m_c = m_d = m$ it is easier to start from

Eq. 3.14 and simply substitute to find

$$p_T = \frac{M}{2 \cosh y}, \quad \text{for } m_c = m_d = 0, \quad (3.18)$$

and

$$p_T = \frac{1}{2 \cosh y} \sqrt{M^2 - 4m^2 \cosh^2 y}, \quad \text{for } m_c = m_d = m. \quad (3.19)$$

If we define s as the proton CoM energy squared, $s = (P_A + P_B)^2$, and introduce

$$\tau = \frac{M^2}{s}, \quad (3.20)$$

then the parton momentum fractions are [16]

$$x_a = \sqrt{\tau} e^Y, \quad x_b = \sqrt{\tau} e^{-Y}, \quad (3.21)$$

such that the Mandelstam variables become

$$\hat{s} = M^2, \quad \hat{t} = -\frac{M^2}{2} \frac{e^{-y}}{\cosh y}, \quad \hat{u} = -\frac{M^2}{2} \frac{e^y}{\cosh y}. \quad (3.22)$$

3.2 Parton Distribution Functions

As we are dealing with parton-parton collisions, at high momentum transfer we turn to a discussion on parton distribution functions (PDFs). PDFs describe how the energy of a hadron is shared among its constituent partons. They represent the probability density $x f_i(x, Q)$ to find a constituent parton i carrying an energy fraction x at a given energy scale Q [17]. Note that we will drop the prefactor x for this chapter.

The forms of PDFs are extracted from experimental data (one function per flavour of parton for a total of thirteen). A common approach begins by parametrizing the x -dependence of the parton distributions at a relatively low value of Q . These inputs are then evolved to higher energy scales using the DGLAP equations [17, 18]. The QCD factorization theorem (which expresses the hadronic cross section as a convolution of the parton distribution functions and perturbatively calculated partonic cross sections) is then used to relate the evolved distributions to observable cross sections, which are compared to experimental measurements. By fitting these predictions to data, the free

parameters describing the initial PDF parameterization parameters are determined, ultimately yielding a grid of parton distributions. Different analysis processes (e.g. Drell–Yan dilepton production, jet production, and electroweak boson production) are used to extract PDFs as each process probes different aspects of the proton’s structure [19, 20]. The results are then combined into global fits that provide a more complete and accurate determination of the proton’s structure.

For this analysis we use the CTEQ PDF set, as ATLAS uses the CTEQ6L1 PDF set for gravity related physics searches [21]. Depicted in Fig. 3.3, this PDF is smooth up to $x \approx 0.97$ after which discontinuities occur for gluons. We do not explicitly regulate or remove this discontinuity. This is further discussed in Chapter 6. We note that t and $\bar{t}$ are not represented in the set and hence we do not consider them for our analysis.

3.3 QCD Dynamics

Regardless of the end state, a collision process starts from two incoming particles. Because we are considering a simplified model for our generator, we only utilize tree level (leading order) processes to determine collision dynamics and ignore higher order loops. The colour and spin averaged tree-level QCD processes and their corresponding spin- and colour-averaged squared matrix elements (MEs), $|\mathcal{M}|^2$, are given in Table 3.1. To obtain the full spin- and colour-averaged squared MEs, two additional considerations are required. First, if the squared ME is not symmetric in $\hat{t}$ and $\hat{u}$, then we must add a symmetric term. Second, if the process has the same outgoing particles (e.g. two gluons or two identical (anti)quarks), then we must divide by two for indistinguishability.

The likelihood of one of the twelve tree-level QCD processes taking place is determined by the cross section, σ . This is the effective area of a collision and, for classical particles, is of the order of magnitude of the size of particles involved (measured in barns). The differential cross section is defined as the scattering rate per unit solid angle, Ω . In QFT it can be expressed in terms of the squared ME and the CoM energy squared. For massless

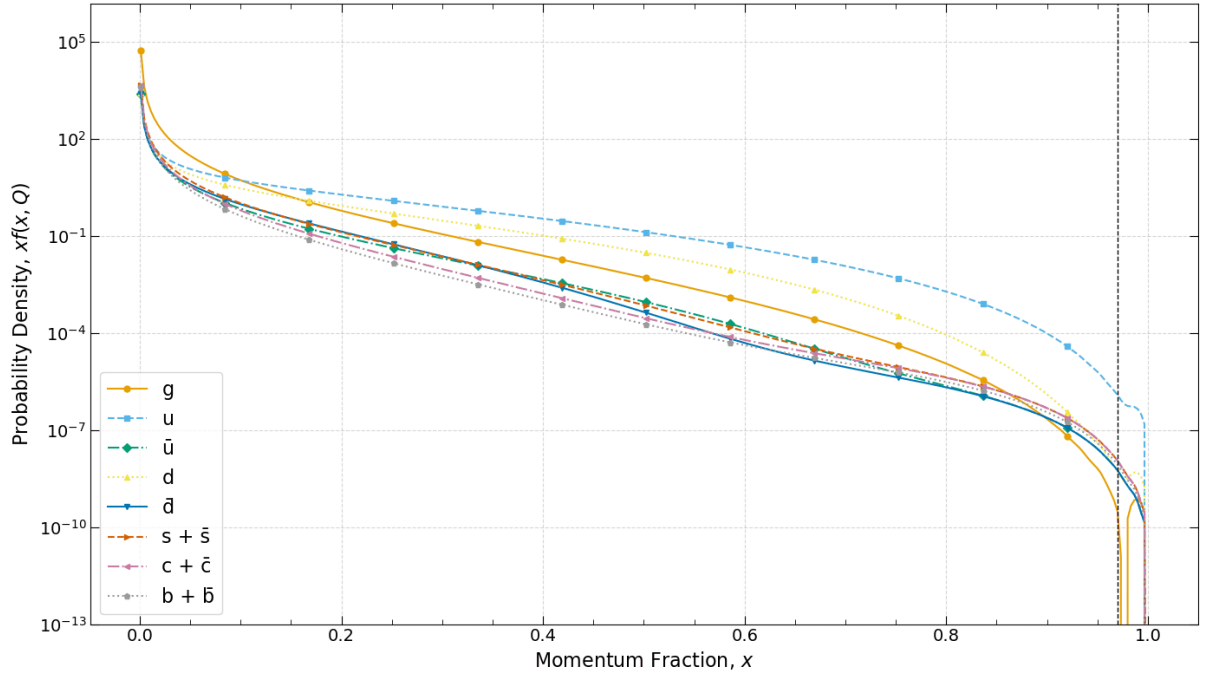
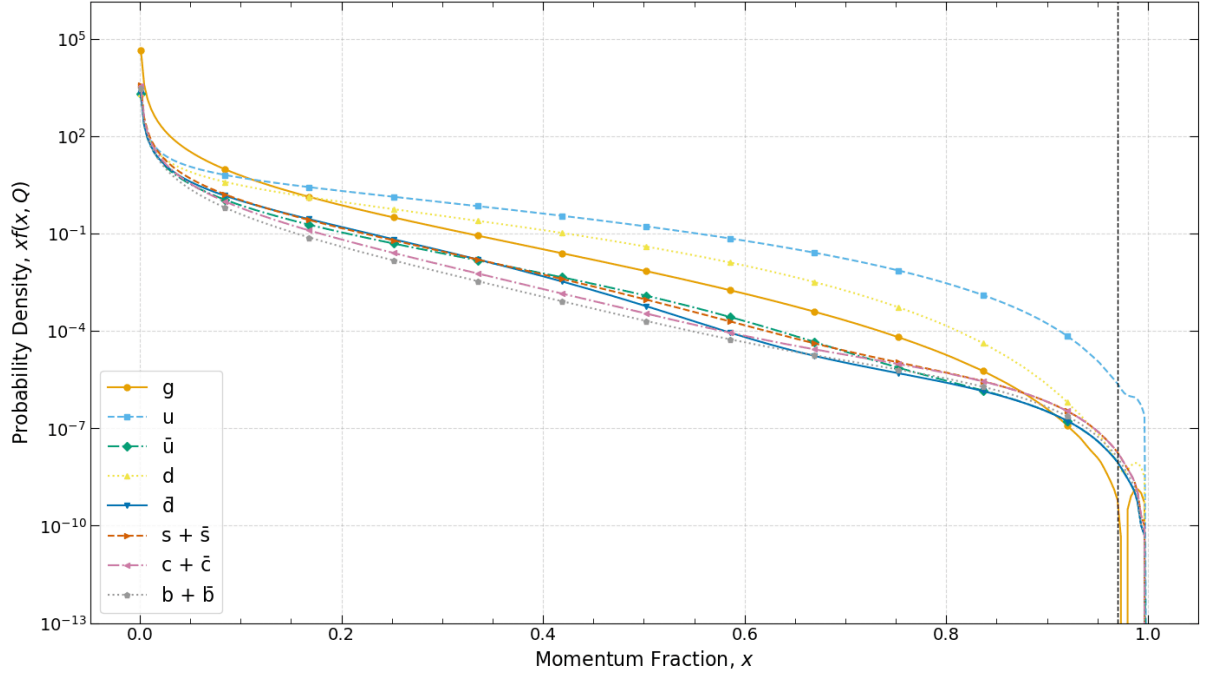


Figure 3.3: Comparison of CTEQ6L1 PDF at $Q = 1$ TeV (top) and $Q = 10$ TeV (bottom). The vertical black line represents the $x \approx 0.97$ gluon discontinuity. This is not explicitly accounted for in our generator/analysis and discussed further in Chapter 6.

Table 3.1: Tree-level QCD processes with their corresponding squared matrix elements [22, 23]. A prime indicates a separate quark from the un-primed quark, and $\mathbf{g}$ in the second column is the QCD running coupling constant.

$ab \rightarrow cd$ Process	Squared Matrix Element $\frac{ \mathcal{M} ^2}{\mathbf{g}}$
$gg \rightarrow gg$	$\frac{9}{2} \left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} \right)$
$gg \rightarrow q\bar{q}$	$\frac{1}{6} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{3}{8} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$q\bar{q} \rightarrow gg$	$\frac{32}{27} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$gq \rightarrow gq$	$-\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} + \frac{\hat{u}^2 + \hat{s}^2}{\hat{t}^2}$
$g\bar{q} \rightarrow g\bar{q}$	$-\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} + \frac{\hat{u}^2 + \hat{s}^2}{\hat{t}^2}$
$qq' \rightarrow qq'$	$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q\bar{q}' \rightarrow q\bar{q}'$	$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$\bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'$	$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q\bar{q} \rightarrow q'\bar{q}'$	$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$qq \rightarrow qq$	$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2} \right) - \frac{8}{27} \frac{\hat{s}^2}{\hat{u}\hat{t}}$
$\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}$	$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2} \right) - \frac{8}{27} \frac{\hat{s}^2}{\hat{u}\hat{t}}$
$q\bar{q} \rightarrow q\bar{q}$	$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right) - \frac{8}{27} \frac{\hat{u}^2}{\hat{s}\hat{t}}$

$2 \rightarrow 2$ parton scattering,

$$\frac{d\hat{\sigma}(\hat{s})}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2\hat{s}}. \quad (3.23)$$

We must take into account that collisions between protons are really collisions between partons. As we have seen with PDFs, we cannot assume equal likelihood for parton interactions as not every parton is equally abundant in a proton. Collision dynamics therefore have a dependency on the likelihood of the constituents colliding. This is achieved by applying the QCD factorization theorem where we convolve the partonic cross section with the PDFs of the colliding partons and sum over all possible parton flavours,

$$\sigma(s) = \sum_{a,b} \int \int dx_a dx_b f_1(x_a, M) f_2(x_b, M) \hat{\sigma}_{ab}(\hat{s}), \quad (3.24)$$

where $a, b = u, \bar{u}, d, \bar{d}, \dots$ [24]. Here, $\hat{\sigma}_{ab}(\hat{s})$ is the partonic cross section for each $ab \rightarrow cd$ process obtained by integrating the differential cross section Eq. 3.23 over the relevant phase space, and $f_{1,2}(x_{a,b}, M)$ are the PDFs of parton 1, 2 of flavour a, b . We set $Q = M$ since the PDF should be evaluated at the same scale as the hard scattering. For us, M is the natural physical scale of the process. If the colliding partons are of different flavour, then another term with flipped x_a and x_b must be included since they are equally likely to come from either beam.

We investigate the differential cross section with respect to M , given by [25, 26]

$$\frac{d\sigma(s)}{dM} = M\tau \sum_{a,b} \int \int dY dy f_1(x_a, M) f_2(x_b, M) \frac{|\mathcal{M}(ab \rightarrow cd)|^2}{16\pi^2\hat{s} \cosh^2 y}. \quad (3.25)$$

Although the hadronic cross section is Lorentz invariant, experimental measurements impose kinematic requirements defined in the lab frame, in particular on the outgoing rapidities. Setting an upper bound on the absolute value of the outgoing rapidities in the lab frame, $|y_c|, |y_d| < y_{\max}$, with the requirement that $x_a, x_b < 1$, we get an upper bound on Y as [26]

$$Y_{\max} = \min \left\{ \ln \left(\frac{1}{\sqrt{\tau}} \right), y_{\max} \right\}. \quad (3.26)$$

The differential cross section then becomes [23, 27, 26]

$$\frac{d\sigma(s)}{dM} = M\tau \sum_{a,b} \left(\int_{-Y_{\max}}^{Y_{\max}} dY f_1(x_a, M) f_2(x_b, M) \right) \int_{-(y_{\max}-|Y|)}^{y_{\max}-|Y|} dy \frac{|\mathcal{M}(ab \rightarrow cd)|^2}{16\pi \hat{s} \cosh^2 y}. \quad (3.27)$$

3.4 Dimensional Reduction in Proton Collisions

As we are observing and analyzing high-energy proton-proton collisions, we focus on the lower-dimensional crossover from a $(3+1)$ - to $(2+1)$ -spacetime dictated by Λ_3 . We assume:

- (i) Isotropic reduction, such that there is no preferred direction of spacetime when the reduction occurs,
- (ii) The transition from $(3+1)$ to $(2+1)$ always happens when $\sqrt{\hat{s}}$ exceeds Λ_3 ,
- (iii) The proton-proton QCD integrated cross section does not change above Λ_3 .

These assumptions offer us a simplified approach to the theory, but can be easily built upon (e.g. a skewed spacetime with a preferred spatial direction, probabilistic reduction, and an altering of cross section for scattering processes to some new value as $\sqrt{\hat{s}}$ becomes comparable with Λ_3). All together, then, we assume that any $2 \rightarrow n$, $n \in [2, 3, \dots]$, scattering process that surpasses Λ_3 will have its final state completely confined to a plane in the CoM frame [2]. If that is the case, the event is called planar. Otherwise, events where the momenta cannot be defined on a single plane are called spherical.

Although we make the claim for all $2 \rightarrow n$, $n \in [2, 3, \dots]$, scattering processes, our analysis focuses on four-parton final states for the following reasons. Two-parton final states, while common, are insufficient as two non-collinear vectors always define a plane. Moreover, in a collision the two outgoing momenta will always be collinear (back-to-back) in the CoM frame (which is not a unique plane). For three outgoing partons, the reasoning isn't as trivial but follows from a simple argument. Consider the parton with the largest momentum. With no pre-established coordinate system, in the CoM frame we are free to say that it's momentum components lie in an $(x, y, 0)$ plane, let's say $(x_a, y_a, 0)$. Then,

the other two parton momenta take on a general form of (x_b, y_b, z_b) and (x_c, y_c, z_c) . Now, conservation of momentum requires that each component sums to zero in the CoM frame. As such, the z condition simply becomes $z_2 = -z_3$. This extra z -dimension can be seen as an arbitrary rotation of the second and third parton into a new (arbitrary) dimension since the most general solution is $z_2 = z_3 = 0$.

Because two and three parton final states are planar, they alone are insufficient in investigating dimensional reduction. We thus must consider multi-parton final states, states with greater than three outgoing partons, as they can be naturally spherical and planar. The $2 \rightarrow 4$ process is therefore the baseline we use to probe dimensional reduction from a $(3+1)$ - to $(2+1)$ -spacetime where $2 \rightarrow 2$ and $2 \rightarrow 3$ act as background to $2 \rightarrow 4$ planar events.

3.5 Showering and Hadronization

To recap, when two protons collide at high energies, the hard scattering process occurs between their constituent partons. At these energy scales, the strong coupling is relatively small (the strong force becomes weaker) owing to asymptotic freedom. The interaction can therefore be described using perturbative QCD, with a tree-level calculation providing the lowest order approximation. Following the hard scattering, the outgoing partons may radiate gluons in a manner analogous to the radiation of photons by electrically charged particles. These emissions are predominantly collinear with the direction of the parent parton. Since gluons themselves carry colour charge, they may also radiate additional gluons or split into quark-antiquark pairs. Successive emissions and splittings give rise to a branching process known as parton showering. As the shower evolves, the characteristic energy scale decreases and the strong coupling increases. Eventually perturbative QCD is no longer applicable and colour confinement becomes important. The coloured partons must therefore combine into colour-neutral hadrons through a process known as hadronization. The resulting particles include hadrons such as pions, kaons, and D and B mesons. The combined effects of parton showering and hadronization produce a

collimated cluster of hadrons. Consequently, rather than observing an isolated parton in a detector, one observes a collimated spray of particles known as a jet.

Parton showering and subsequent hadronization occurs very rapidly and over a short distance so there is no way to know what partons are involved in the process. Additionally, heavy flavour decays (e.g. $b \rightarrow c \rightarrow s \rightarrow u$) throughout the process give rise to leptons in the final state. The only information we can gather are the final state hadrons (and leptons, neutrinos, and photons) which act as the source of information to what interaction occurred. Experimental evidence suggests that the process of hadronization does not significantly impact the momenta of partons which means a highly energetic quark will result in a similarly energetic jet of hadrons. Another way to say this: the jet-like structure of a hadronic event can be attributed to the properties of the radiated gluons from the coloured particles [28].

Chapter 4

The ATLAS Detector at the LHC

4.1 The Large Hadron Collider

The LHC, as depicted in Fig. 4.1, is the world’s largest particle accelerator and collider. It has a circumference of 26.7 km and four large detectors where the two beams cross to allow for collisions [29]. The ATLAS and the Compact Muon Solenoid (CMS) detectors are general-purpose detectors used for precision SM tests, Higgs precision measurements, and searches for BSM physics [30, 31]. The A Large Ion Collider Experiment (ALICE) is dedicated to heavy-ion experiments designed for studying quark-gluon plasmas; and LHC beauty (LHCb) explores matter-antimatter asymmetry by focusing on bottom-quark physics and CP violation [32, 33].

Throughout its lifetime, the LHC has undergone three major operational runs. Run 1 (2009–2013) achieved a CoM collision energy of 8 TeV and a total integrated luminosity of 29.2 fb^{-1} at the ATLAS and CMS experiments [30, 31]. Run 2 (2015–2018) operated at a CoM energy of 13 TeV, achieving a total integrated luminosity of 159.8 fb^{-1} at ATLAS and CMS [35, 36]. Run 3, which began in 2022 and concluded in June 2026, achieved a CoM energy of 13.6 TeV [29, 37]. The LHC is now entering its third Long Shutdown (which happens after every Run) to undergo upgrades and become the High Luminosity LHC [38, 39].

The CERN accelerator complex *Complexe des accélérateurs du CERN*

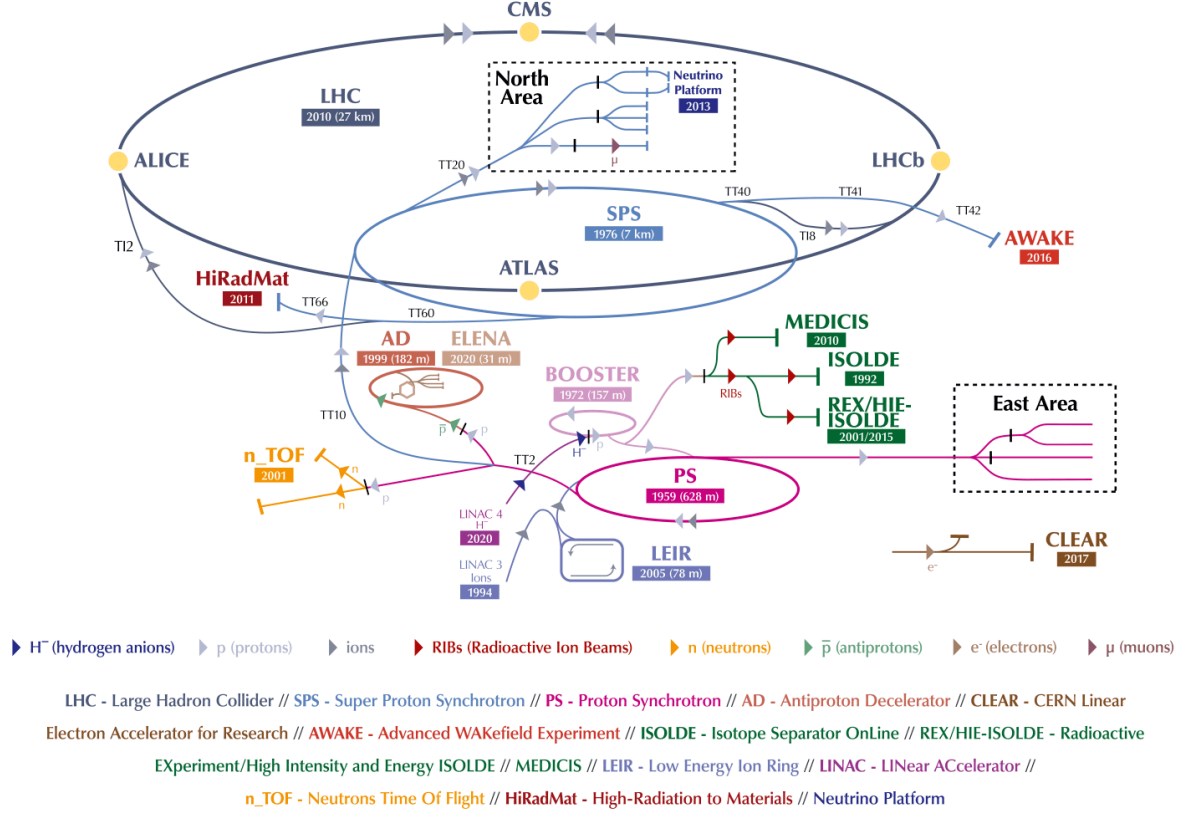


Figure 4.1: The configuration of the LHC. It begins with protons accelerated in the Booster, followed by further acceleration in the PS and the SPS. The protons are then split into two beams travelling in opposite directions within the LHC's circular tunnel. Along the LHC's circumference are the locations of the four main experiments [34].

4.1.1 Circular Accelerator Physics

Accelerators are composed of three main components. The first is the beam which transports a current of charged bunches (a grouping of particles) through a vacuum. The second component is the accelerating structure which is responsible for accelerating the particles. Finally, there are the magnets, which guide the beams along well-defined paths while compressing them into small transverse areas.

In colliders, the proton CoM invariant mass is given by

$$\sqrt{s} = \sqrt{4E_a E_b} = 2E_a, \quad (4.1)$$

where E_a and E_b are the energies of the two beams, and we use that the LHC always has $E_a = E_b$ [40, 41]. At the parton level, only a fraction of the proton momentum participates in the hard scattering, so the relevant energy scale is

$$\hat{s} = x_a x_b s. \quad (4.2)$$

The luminosity is determined by the properties of the beams and characterizes their ability to produce particle collisions. With two particle bunches that collide with frequency f , the luminosity is given by

$$\mathcal{L} = f n \frac{N^2}{4\pi\sigma^x\sigma^y}, \quad (4.3)$$

where n is the number of bunches in the beam, N is the number of particles per bunch in each beam, and $\sigma^{x,y}$ are the transverse sizes of the beam in the x and y directions (assuming identical transverse profiles for both beams). At the LHC, $f = 11$ kHz, $n = 2808$, $N = 10^{11}$, and $\sigma^{x,y} = 16 \mu\text{m}$ [40, 42]. Equation 4.3 defines the instantaneous luminosity, which describes the collision-producing capability of the beams at a given moment in time. Accelerators aim to maximize luminosity to increase the likelihood of observing interesting physical events.

Integrating the instantaneous luminosity over a period of time gives the integrated luminosity,

$$L = \int \mathcal{L}(t) dt. \quad (4.4)$$

When this integration is performed over the live time during which the detector and trigger systems were active, it gives the recorded integrated luminosity for the data-taking period. The integrated luminosity is used to estimate the total number of events produced for a particular process,

$$N_{\text{event}} = \sigma \int \mathcal{L}(t) dt = \sigma L \approx \sigma \mathcal{L} \Delta t. \quad (4.5)$$

Here, σ is the cross section for the physical process being considered and the final approximation assumes that the instantaneous luminosity remains approximately constant over the time interval Δt [40, 41].

4.2 The ATLAS Detector

The ATLAS detector is the largest detector at the LHC. It measures 46 m in length, 25 m in height, 25 m in width, and weighs approximately 7,000 tonnes. ATLAS covers nearly the entire solid angle around the collision/interaction point (IP). It consists of several layers: an inner tracking detector, surrounded by a thin superconducting solenoid, followed by electromagnetic and hadronic calorimeters, and finally a muon spectrometer. The general configuration of ATLAS is depicted in Fig. 4.2. Interesting events—those deemed rare and relevant for study—are selected by the Trigger and Data Acquisition (TDAQ) system.

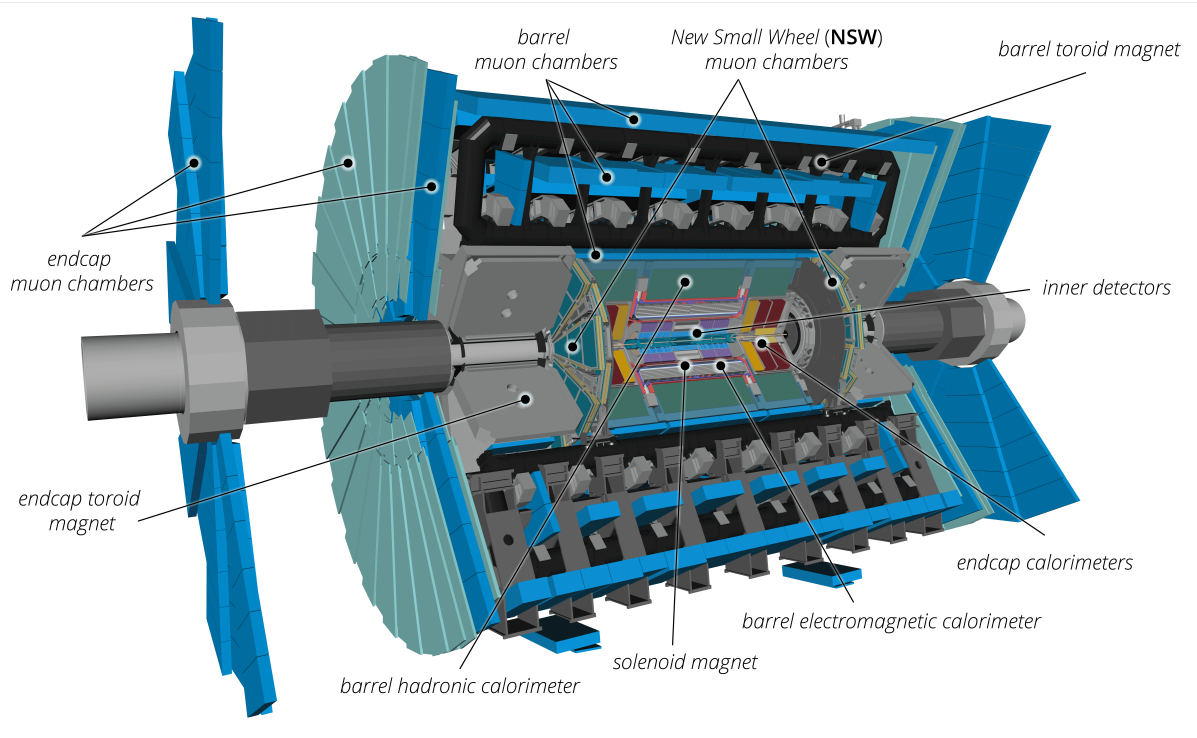


Figure 4.2: Configuration of the ATLAS detector for Run 3 with the major components of larger sub-systems indicated [29].

4.2.1 Inner Detector

The Inner Detector (ID) system, depicted in Fig. 4.3, is a tracking detector designed to provide precise tracking of charged particles. It is crucial for identifying the vertices of

particle interactions including the primary vertex, where particles from the beams collide, and secondary vertices, which indicate particle decays. The ID measures 6.2 m in length, has a diameter of 2.1 m and operates within a 2 T axial magnetic field generated by the central solenoid. This magnetic field bends the paths of charged particles enabling the measurement of their momenta. The ID covers the pseudorapidity range $|\eta| < 2.5$ and consists of three main subsystems: (1) The high-granularity Pixel detector, located closest to the IP, provides up to four precise measurements per track. This includes the Insertable B-Layer, a cylindrical layer added before Run 2 to enhance vertexing performance. (2) The semiconductor tracker, which surrounds the Pixel detector, consists of silicon micro-strip detectors, typically providing eight measurements per track and ensure high-resolution tracking in the central region. (3) The transition radiation tracker, positioned as the outermost component of the ID, uses layers of thin, gas-filled cylindrical detectors containing a central readout wire to extend track reconstruction up to $|\eta| = 2.0$. It also aids in electron identification by detecting transition radiation through hits with higher energy deposits. Together, these three systems ensure precise vertex reconstruction, momentum measurement, and particle identification [29, 30, 40].

4.2.2 Calorimeters

ATLAS calorimeters are sampling calorimeters, consisting of layers of absorbing high-density material (that stop incoming particles) interwoven with layers of active media that measure the particle energies. The absorbers are materials such as lead, copper, or steel, while the active media include liquid argon (LAr) and plastic scintillators that produce electrical or optical signals when particles pass through them. These calorimeters are designed to measure the energy of both charged and neutral particles. Their goal is to stop incoming particles within the detector material, converting the particle's energy into measurable signals through ionization or scintillation in the active layers. As particles lose energy in the detector material they produce cascades of secondary particles, known as particle showers. These showers are categorized as either electromagnetic or hadronic depending on whether they are initiated by electromagnetic interactions (e.g. electrons

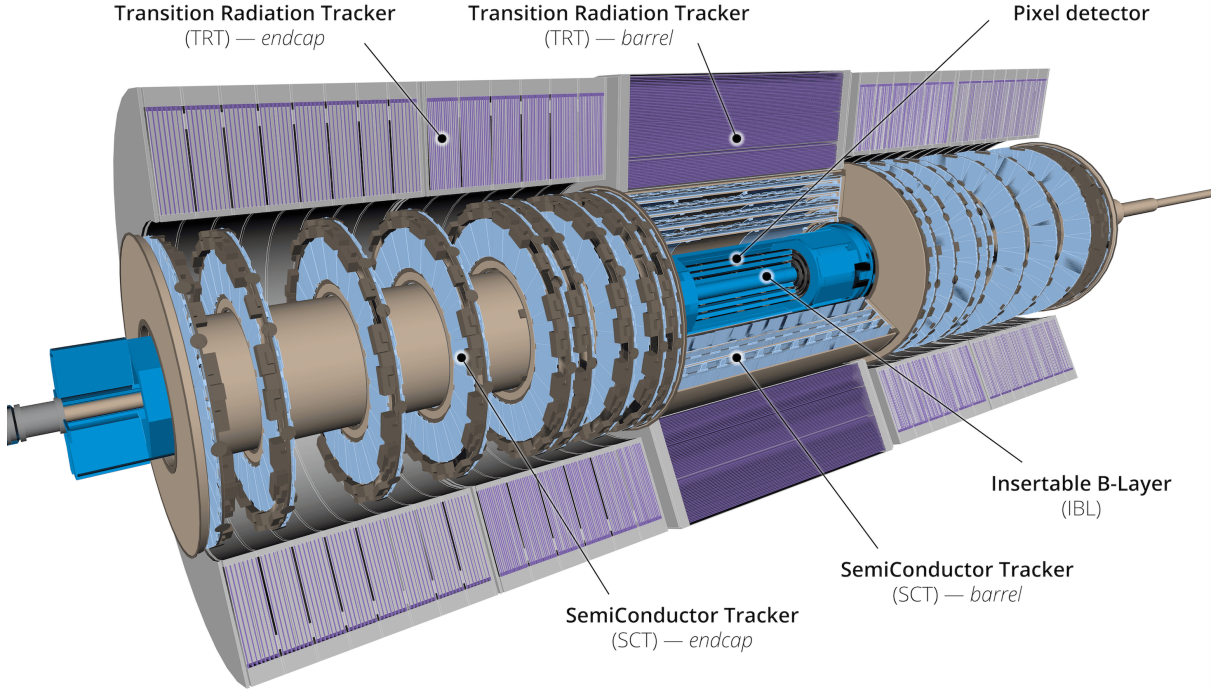


Figure 4.3: The configuration of the ATLAS ID for Run 3. The ID provides full coverage in ϕ and operates within $|\eta| < 2.5$. The segmenting of the different trackers along the barrel and both endcaps ensures comprehensive tracking performance [29].

or photons) or by hadronic interactions of strongly interacting particles [29, 40].

The calorimeter system, shown in Fig. 4.4, covers the pseudorapidity range $|\eta| < 4.9$, providing both electromagnetic and hadronic energy measurements. Within the region $|\eta| < 3.2$, electromagnetic calorimetry is provided by high-granularity lead/LAr calorimeters, which include a barrel section ($|\eta| < 1.475$) and endcap sections ($1.375 < |\eta| < 3.2$). To correct for energy loss in material upstream of the calorimeters, an additional thin LAr slice is used in the region $|\eta| < 1.8$ [29, 30].

Hadron calorimetry is provided by the steel/scintillator-tile calorimeter, segmented into three barrel structures within $|\eta| < 1.7$, and by two copper/LAr hadron endcap calorimeters. Additionally, the forward calorimeter modules are optimized for both electromagnetic and hadronic energy measurements, covering the regions $3.1 < |\eta| < 3.9$. These forward calorimeter modules consist of copper/LAr for electromagnetic measurements and tungsten/LAr for hadronic energy measurements.

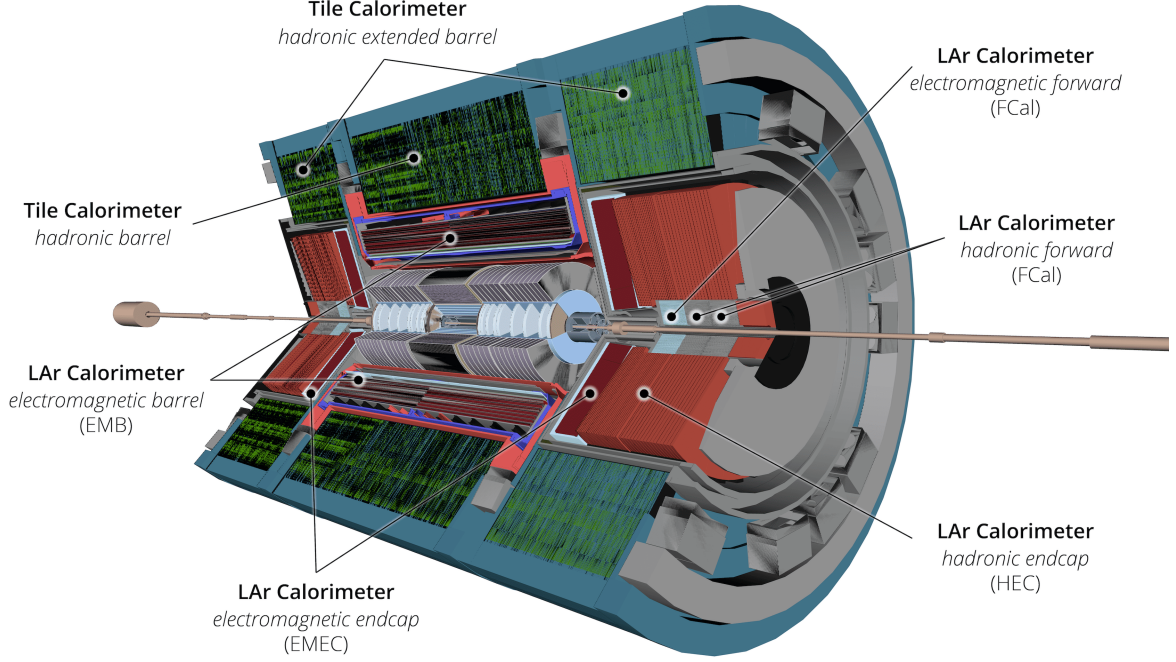


Figure 4.4: The configuration of the ATLAS calorimeter system for Run 3. It is designed to measure the energies and positions of charged and neutral particles through interleaved absorber and active layers. It measures out to $|\eta| < 4.9$ [29].

4.2.3 Muon Spectrometer

The muon spectrometer, shown in Fig. 4.5, is located as the outermost layer around the calorimeters. It is designed to detect charged particles that exit the barrel and endcap calorimeters and measure their momentum within the pseudorapidity range $|\eta| < 2.7$. It comprises separate trigger and high-precision tracking chambers that measure the deflection of muons in a magnetic field generated by superconducting air-core toroidal magnets. The field integral of the toroid ranges between 2.0 and 6.0 T·m across the detector [29, 30].

The muon spectrometer utilizes large superconducting magnets to deflect muon tracks and high-precision tracking chambers to record measurements. The large barrel toroid operates over the range $|\eta| < 1.4$, while muon tracks in the region $1.6 < |\eta| < 2.7$ are bent by two smaller endcap magnets. In the region $1.4 < |\eta| < 1.6$, bending is provided by a combination of both barrel and endcap magnets. Precision measurements of track coordinates are provided by monitored drift tubes across $|\eta| < 2.7$, while cathode strip

chambers are used in the region $1 < |\eta| < 2.7$. Muon track identification is accomplished using resistive plate chambers in the barrel and thin gap chambers in the endcap regions.

Three stations of precision chambers, each consisting of monitored drift tubes, cover the region $|\eta| < 2.7$. In the innermost station of the endcaps, where $|\eta| > 1.3$, the new small wheel detectors replace those used in Runs 1 and 2. The muon trigger system covers $|\eta| < 2.4$, with resistive plate chambers in the barrel ($|\eta| < 1.0$), thin gap chambers in the endcap regions outside the toroids, and the new small wheel detectors situated between the cryostats of the endcap calorimeters and the endcap toroids.

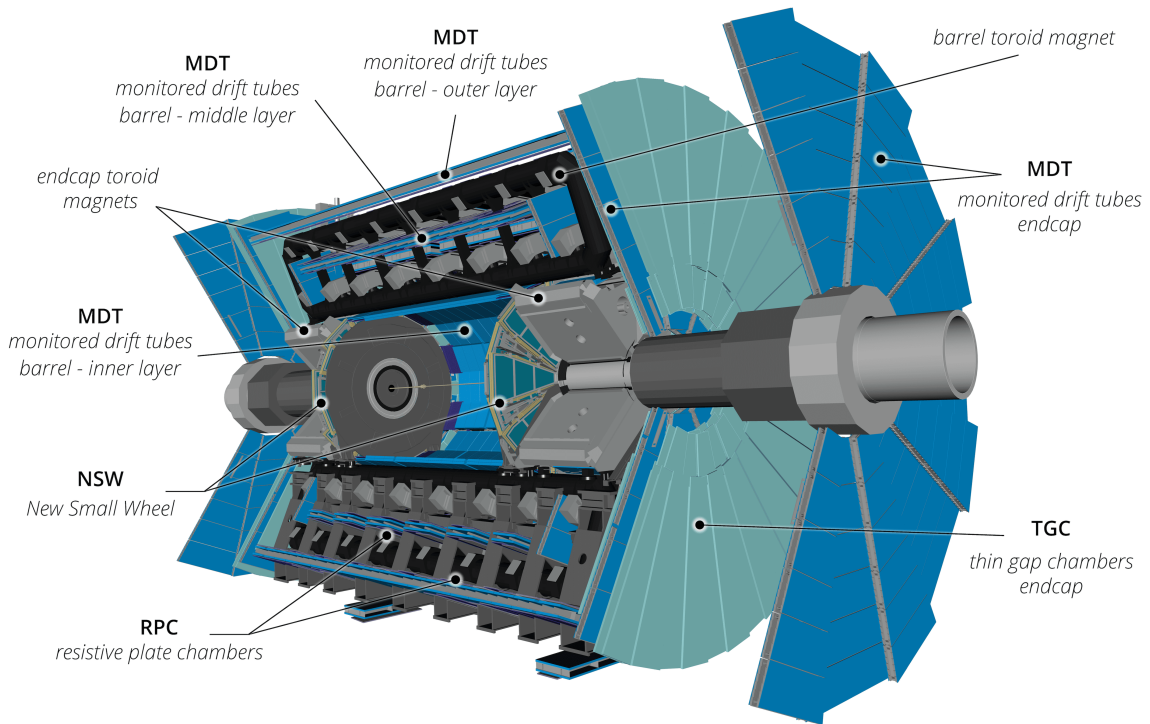


Figure 4.5: The configuration of the ATLAS muon spectrometer for Run 3. It is designed to detect charged particles exiting the barrel and endcap calorimeters, measuring their momentum in the range $|\eta| < 2.7$ [29].

4.2.4 Forward Detectors

Four smaller detector systems cover the ATLAS forward region. The Luminosity Cherenkov Integrating Detector is located ± 17 m from the IP and detects inelastic proton-proton scattering. It is the main online and offline luminosity monitor for ATLAS. After that, the Zero Degree Calorimeters is located at ± 140 m from the IP. It consists of layers of alternating quartz rods and tungsten plates which measure neutral particles at $|\eta| \geq 8.2$

and plays a key role in determining the centrality of heavy-ion collisions. The ATLAS Forward Proton detector located at ± 210 m from the IP. It is designed to study soft, hard, and central (exclusive) diffractive events at low luminosities using a silicon-based tracker for momentum measurements. The furthest out is the Absolute Luminosity for ATLAS Roman Pot detector located ± 240 m from the IP. It consists of scintillating fibre trackers housed inside Roman pots and is used in dedicated low luminosity running of the LHC and can provide a measurement of the total cross section for proton-proton interactions [29].

4.2.5 Trigger and Data Acquisition System

With multiple proton-proton collisions occurring rapidly in each bunch crossing, and with particle tracks being associated with vertices created from these collisions, the TDAQ system plays a crucial role in reducing the data recorded by the detector. It is responsible for selecting events with distinguishing characteristics (e.g. energetic leptons, photons, hadronic jets, and missing energy) that are of interest for physics analysis, filtering out unnecessary data for efficient offline analysis. The trigger system works by applying a two-level event selection process: the Level-1 Trigger and the High-Level Trigger [29, 30].

The Level-1 trigger, a custom built hardware system, uses reduced-granularity information from the calorimeters and muon spectrometer to search for signatures of interesting events. It has a maximum acceptance rate of 1×10^5 events per second. For each bunch crossing, the trigger decision must be produced within a strict latency of $2.54 \mu\text{s}$. The Level-1 trigger identifies regions-of-interest in the detector and passes this information to the second stage of the trigger process. The High-Level Trigger, a software-based system implemented on commercial computers, refines the selections made by the Level-1 trigger using the regions-of-interest data, and applies further kinematic selections to reduce the data size. It aims to reproduce the offline selection process as closely as possible. After further processing, the event filter categorizes events according to ATLAS-defined data streams, such as electrons, photons, or jets.

Chapter 5

Dimensional Reduction Event Generator

Using the kinematics of proton-proton collisions outlined in Chapter 3, we build a Monte Carlo dimensional reduction toy model (DRToM) event generator to investigate the effects of a $(3 + 1)$ - to $(2 + 1)$ -spacetime reduction. DRToM is capable of generating tree-level $2 \rightarrow 2$ QCD interactions, and $2 \rightarrow n$, $n \in [2, 5]$, massless parton phase space kinematics.

5.1 Numerical Integration and Event Generation

The core event generation pipeline of DRToM remains the same regardless of the chosen physical framework (QCD or phase space). We start by considering the numerical evaluation of the hadronic differential cross section of Eq. 3.27. For a fixed invariant mass M , we write the differential cross section for partonic channel k as

$$\frac{d\sigma_k}{dM}(M) = \int_{\Omega(M)} F_k(M, Y, y) dY dy, \quad (5.1)$$

where $\Omega(M)$ denotes the kinematically allowed region in the (Y, y) plane. The integrand is

$$F_k(M, Y, y) = x_a f_{1,k}(x_a, M) x_b f_{2,k}(x_b, M) \frac{|\mathcal{M}_k|^2 M}{16\pi \hat{s}^2 \cosh^2 y}. \quad (5.2)$$

Recall that x_a and x_b are the incoming parton momentum fractions, $f_{1,k}$ and $f_{2,k}$ are the corresponding PDFs for each beam, and $|\mathcal{M}_k|^2$ is the squared ME for channel k . When the incoming partons are different, the beam-interchanged PDF contribution is also included. The k channels include the processes listed in Table 3.1 and their allowed incoming-parton combinations.

5.1.1 Grid Point Integration

The integral in Eq. 5.1 is evaluated using a deterministic midpoint Riemann sum. The allowed Y -interval is divided into N_Y intervals of width

$$\Delta Y = \frac{Y_{\max} - Y_{\min}}{N_Y}. \quad (5.3)$$

The midpoint of interval i is therefore

$$Y_i = Y_{\min} + \left(i - \frac{1}{2}\right) \Delta Y, \quad i = 1, \dots, N_Y. \quad (5.4)$$

Because the allowed range of y may depend on Y , the y -step must be evaluated separately for each Y_i

$$\Delta y_i = \frac{y_{\max}(Y_i) - y_{\min}(Y_i)}{N_y}. \quad (5.5)$$

The corresponding midpoint values are

$$y_{ij} = y_{\min}(Y_i) + \left(j - \frac{1}{2}\right) \Delta y_i, \quad j = 1, \dots, N_y. \quad (5.6)$$

The channel differential cross section is then approximated by

$$\frac{d\sigma_k}{dM}(M) \approx \sum_{i=1}^{N_Y} \sum_{j=1}^{N_y} F_k(M, Y_i, y_{ij}) \Delta Y \Delta y_i. \quad (5.7)$$

The dependence of Δy_i on Y_i is important because the allowed integration region is not generally rectangular.

5.1.2 Total Differential Cross Section

At each value of M , the total differential cross section is obtained by summing the contributions from all $k = 1, \dots, K$ active partonic channels

$$\frac{d\sigma_{\text{Tot}}}{dM}(M) = \sum_{k=1}^K \frac{d\sigma_k}{dM}(M). \quad (5.8)$$

This remains differential in M . The cross section over the generated mass range is

$$\sigma_{\text{Tot}} = \int_{M_{\min}}^{M_{\max}} \frac{d\sigma_{\text{Tot}}}{dM}(M) dM. \quad (5.9)$$

5.1.3 Monte Carlo Integration

The generator samples N_{Events} independent mass values uniformly over the selected range

$$M_n \sim \mathcal{U}(M_{\min}, M_{\max}), \quad n = 1, \dots, N_{\text{Events}}. \quad (5.10)$$

Consequently, Eq. 5.9 is estimated using one-dimensional MC integration

$$\sigma_{\text{Tot}} = \frac{M_{\max} - M_{\min}}{N_{\text{Events}}} \sum_{n=1}^{N_{\text{Events}}} \frac{d\sigma_{\text{Tot}}}{dM}(M_n), \quad (5.11)$$

where we must take the average value across all events in the range, and the prefactor is a MC integration factor. The cross section is initially expressed in GeV^{-2} , so the result is converted to picobarns according to

$$\hat{\sigma}_{\text{Tot}}[\text{pb}] = \hat{\sigma}_{\text{Tot}}[\text{GeV}^{-2}] (0.389379 \times 10^9) \text{ pb}. \quad (5.12)$$

5.1.4 Partonic Channel Selection

Once M_n has been generated and the contribution from each channel has been evaluated, inverse cumulative distribution sampling is used to select a partonic channel. Specifically, channel k is selected with probability

$$P(k \mid M_n) = \frac{d\sigma_k}{dM}(M_n) \bigg/ \sum_{\ell=1}^K \frac{d\sigma_{\ell}}{dM}(M_n), \quad (5.13)$$

where $k \mid M_n$ means: k given M_n . This selection ensures that channels with larger differential cross sections at the generated mass are selected more frequently.

To implement Eq. 5.13, the cumulative channel contributions are defined as

$$S_k(M_n) = \sum_{\ell=1}^k \frac{d\sigma_{\ell}}{dM}(M_n), \quad S_{\text{Tot}}(M_n) = S_K(M_n). \quad (5.14)$$

A random number

$$r_1 \sim \mathcal{U}(0, S_{\text{Tot}}(M_n)) \quad (5.15)$$

is generated, and the selected channel is the unique channel k satisfying

$$S_{k-1}(M_n) < r_1 \leq S_k(M_n), \quad S_0 = 0. \quad (5.16)$$

5.1.5 Generation of Phase Space Variables

After selecting channel k , the phase space variables (Y, y) are generated according to the channel integrand $F_k(M_n, Y, y)$ using acceptance–rejection sampling. First, Y is drawn uniformly from its allowed range

$$Y \sim \mathcal{U}(-Y_{\max}(M_n), Y_{\max}(M_n)). \quad (5.17)$$

For a given value of Y , using Eq. 3.10 with the constraint $-y_{\max} \leq y_{a,b} < y_{\max}$, the allowed range of y is

$$-y_{\lim}(Y) \leq y \leq y_{\lim}(Y), \quad y_{\lim}(Y) = y_{\max} - |Y|. \quad (5.18)$$

The variable y is then drawn conditionally according to

$$y \mid Y \sim \mathcal{U}(-y_{\lim}(Y), y_{\lim}(Y)), \quad (5.19)$$

This procedure ensures that every proposed point lies within the kinematically allowed region $\Omega(M_n)$.

Because the width of the interval from which y is sampled depends on Y , the proposal density is not uniform over the two-dimensional (Y, y) region. Up to an overall constant, the quantity used in the acceptance–rejection test is therefore

$$G_k(M_n, Y, y) = y_{\lim}(Y) F_k(M_n, Y, y). \quad (5.20)$$

The factor $y_{\lim}(Y)$ corrects for the Y -dependent proposal density, ensuring that the accepted points are distributed proportionally to $F_k(M_n, Y, y)$.

For each mass and selected channel, an envelope is estimated by evaluating G_k on a finite midpoint grid over the allowed phase space region. If $\max((G_{k,\text{scan}}(M_n)))$ is the largest value found by this scan, the envelope is defined as

$$H_k(M_n) = c_{\text{safety}} \max(G_{k,\text{scan}}(M_n)), \quad (5.21)$$

where c_{safety} is a safety factor included to account for the possibility that the finite scan

does not sample the exact maximum. It was chosen arbitrarily to be 1.5.

To avoid bias, for every proposed phase space point, a random height is drawn

$$r_2 \sim \mathcal{U}(0, H_k(M_n)). \quad (5.22)$$

The point is accepted when

$$r_2 < G_k(M_n, Y, y). \quad (5.23)$$

If this condition is not satisfied, a new pair (Y, y) is proposed. Since a finite scan cannot rigorously guarantee that the envelope bounds the corrected integrand everywhere, each proposed point is also checked explicitly:

$$G_k(M_n, Y, y) \leq H_k(M_n). \quad (5.24)$$

Once a phase space point (Y, y) is accepted, the invariant mass M and the system rapidity Y are used to calculate the incoming parton momentum fractions, x_a and x_b . The rapidities of the two outgoing partons are determined from the accepted values of Y and y . These quantities are subsequently used to construct the incoming and outgoing parton four-momenta for QCD.

5.1.6 Monte Carlo Integration Uncertainty

Since the invariant masses M_n are sampled randomly, the estimated integrated cross section has a statistical MC uncertainty [43, 44]. We define

$$f_n = \frac{d\sigma_{\text{Tot}}}{dM}(M_n), \quad \bar{f} = \frac{1}{N_{\text{Events}}} \sum_{n=1}^{N_{\text{Events}}} f_n, \quad (5.25)$$

such that the cross section estimator is

$$\hat{\sigma}_{\text{Tot}} = (M_{\text{max}} - M_{\text{min}}) \bar{f}. \quad (5.26)$$

Its estimated standard error is

$$\delta \hat{\sigma}_{\text{Tot}} = \frac{M_{\text{max}} - M_{\text{min}}}{\sqrt{N_{\text{Events}}}} \sqrt{\frac{1}{N_{\text{Events}} - 1} \sum_{n=1}^{N_{\text{Events}}} (f_n - \bar{f})^2}. \quad (5.27)$$

This uncertainty estimates the statistical precision of the MC integration over M . It does not account for systematic effects such as PDF uncertainties, scale variations, finite midpoint-grid resolution, or an invalid acceptance-rejection envelope.

5.1.7 Final States

If considering a $2 \rightarrow 2$ process, one must specify whether the QCD or phase space variant is used. If considering a $2 \rightarrow 3$, $2 \rightarrow 4$, or $2 \rightarrow 5$ process, the generator chooses the final-state partons based on the selected $2 \rightarrow 2$ QCD cross section. For example, if the generator selects a $gg \rightarrow gg$ collision, this determines the allowed possible outputs. Say we are considering a $2 \rightarrow 4$ process. Looking at Table 5.1, which lists each partonic collision input and DRToM's possible $2 \rightarrow 4$ final states, we see there are four choices:

$$gg \rightarrow gggg, \quad gg \rightarrow ggg\bar{q}, \quad gg \rightarrow q\bar{q}q\bar{q}, \quad gg \rightarrow q\bar{q}q'\bar{q}'. \quad (5.28)$$

Here, a prime denotes a quark flavour different from the unprimed flavour. At this point, the specific $2 \rightarrow 4$ final state is selected randomly from the available mappings (the higher-multiplicity final states are not selected using separate $2 \rightarrow 4$ QCD). The $2 \rightarrow 2$ collisions are those listed in Table 3.1, while the final states of DRToM for $2 \rightarrow 3$ and $2 \rightarrow 5$ tables are given in Appendix A.

With a chosen collision process, the next step is to generate the event kinematics and colour flow. The momentum configuration is first generated in the partonic CoM frame. The incoming lab frame momenta are then constructed using the selected momentum fractions x_a and x_b , and the complete event is Lorentz boosted into the lab frame. The colour flow is selected from the available colour-flow configurations generated using MadGraph [45]. These are further discussed in Sec. 5.2, and Sec. 5.3, respectively.

Table 5.1: Allowed $2 \rightarrow 4$ partonic end state processes of DRToM.

Input ($2 \rightarrow 2$)	Possible output ($2 \rightarrow 4$)
$gg \rightarrow gg$	$gg \rightarrow gggg, \quad gg \rightarrow ggg\bar{q}, \quad gg \rightarrow q\bar{q}q\bar{q}, \quad gg \rightarrow q\bar{q}q'\bar{q}'$
$gg \rightarrow q\bar{q}$	$gg \rightarrow ggg\bar{q}, \quad gg \rightarrow q\bar{q}q\bar{q}, \quad gg \rightarrow q\bar{q}q'\bar{q}'$
$gq \rightarrow gq$	$gq \rightarrow gqgg, \quad gq \rightarrow gqq\bar{q}, \quad gq \rightarrow gqq'\bar{q}'$
$g\bar{q} \rightarrow g\bar{q}$	$g\bar{q} \rightarrow g\bar{q}gg, \quad g\bar{q} \rightarrow g\bar{q}q\bar{q}, \quad g\bar{q} \rightarrow g\bar{q}q'\bar{q}'$
$qq \rightarrow qq$	$qq \rightarrow qqgg, \quad qq \rightarrow qq\bar{q}\bar{q}, \quad qq \rightarrow qqq'\bar{q}'$
$qq' \rightarrow qq'$	$qq' \rightarrow qq'gg, \quad qq' \rightarrow qq'q\bar{q}, \quad qq' \rightarrow qq'q'\bar{q}', \quad qq' \rightarrow qq'q''\bar{q}''$
$\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}$	$\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}gg, \quad \bar{q}\bar{q} \rightarrow \bar{q}\bar{q}q\bar{q}, \quad \bar{q}\bar{q} \rightarrow \bar{q}\bar{q}q'\bar{q}'$
$\bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'$	$\bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'gg, \quad \bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'q\bar{q}, \quad \bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'q'\bar{q}', \quad \bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'q''\bar{q}''$
$q\bar{q}' \rightarrow q\bar{q}'$	$q\bar{q}' \rightarrow q\bar{q}'gg, \quad q\bar{q}' \rightarrow q\bar{q}'q\bar{q}, \quad q\bar{q}' \rightarrow q\bar{q}'q'\bar{q}', \quad q\bar{q}' \rightarrow q\bar{q}'q''\bar{q}''$
$q\bar{q} \rightarrow gg$	$q\bar{q} \rightarrow gggg, \quad q\bar{q} \rightarrow ggq\bar{q}, \quad q\bar{q} \rightarrow ggq'\bar{q}', \quad q\bar{q} \rightarrow q\bar{q}q\bar{q}, \quad q\bar{q} \rightarrow q\bar{q}q'\bar{q}',$ $q\bar{q} \rightarrow q'\bar{q}'q'\bar{q}', \quad q\bar{q} \rightarrow q'\bar{q}'q''\bar{q}''$
$q\bar{q} \rightarrow q\bar{q}$	$q\bar{q} \rightarrow q\bar{q}gg, \quad q\bar{q} \rightarrow q\bar{q}q\bar{q}, \quad q\bar{q} \rightarrow q\bar{q}q'\bar{q}'$
$q\bar{q} \rightarrow q'\bar{q}'$	$q\bar{q} \rightarrow q'\bar{q}'gg, \quad q\bar{q} \rightarrow q\bar{q}q'\bar{q}', \quad q\bar{q} \rightarrow q'\bar{q}'q'\bar{q}', \quad q\bar{q} \rightarrow q'\bar{q}'q''\bar{q}''$

5.2 Massless Partonic Phase Space

In our discussion in Chapter 3 we focused on the kinematics of $2 \rightarrow 2$ QCD and mentioned why we are interested in $2 \rightarrow 4$ partonic processes. Given that the proton-proton collision energies we consider are much higher than the mass of the interaction proton (≈ 6.5 TeV vs. ≈ 0.938 GeV, respectively), and since we are not considering the top quark, we let the partons be massless. Then, for a simplified model, we work in massless phase space which allows us to investigate the properties of $2 \rightarrow n$ spherical and planar partonic processes, where $n \in [2, 3, \dots]$.

5.2.1 Outgoing Four Parton Topology Requirements

Consider a (non-specified and fictitious) mother particle at rest in the CoM frame with energy equal to its rest mass, $E = M$, such that it has a four-momentum $P = [M, 0, 0, 0]$. Now, let the mother particle decay into four massless particles, $p_{\mathfrak{d}}, \mathfrak{d} = a, b, c, d$, which we call the daughter particles and denote by $\mathfrak{d}$.

Because each daughter particle is massless, for each $\mathfrak{d}$ we require

$$m_{\mathfrak{d}}^2 = p_{\mathfrak{d}} \cdot p_{\mathfrak{d}} = 0. \quad (5.29)$$

The relativistic energy relation $E_{\mathfrak{d}}^2 = m_{\mathfrak{d}}^2 + |\vec{p}_{\mathfrak{d}}|^2$ simplifies to $E_{\mathfrak{d}} = |\vec{p}_{\mathfrak{d}}|$ which tells us that the energy of each daughter particle must equal the magnitude of its spatial momentum. The four-momentum for an arbitrary massless particle can thus be given by

$$p = (p^0, p^1, p^2, p^3) = (E, Ev^x, Ev^y, Ev^z), \quad (5.30)$$

where $(v^x)^2 + (v^y)^2 + (v^z)^2 = 1$ since a massless particle must travel at the speed of light.

Using Eq. (5.30), the relativistic dot product is

$$p \cdot p = E^2 - \vec{p} \cdot \vec{p} = E^2 (1 - (v^x)^2 - (v^y)^2 - (v^z)^2) = 0, \quad (5.31)$$

as required.

We now impose the physical constraints required to construct allowed phase space

configurations. First, four-momentum must be conserved: $P^\mu = \sum_{\mathfrak{d}=a}^d p_{\mathfrak{d}}^\mu$. In the CoM frame, the total four-momentum is $P^\mu = [M, 0, 0, 0]$, such that $M = \sum_{\mathfrak{d}=a}^d E_{\mathfrak{d}}$, and $\sum_{\mathfrak{d}=a}^d \vec{p}_{\mathfrak{d}} = 0$. Second, each daughter particle is required to be massless, $p_{\mathfrak{d}}^2 = 0$. Four-momentum conservation and the massless condition hold in both the CoM and lab frames. However, in the lab frame, the total spatial momentum does not need to vanish and the total energy need not equal M , where M remains the invariant mass of the system.

5.2.2 Planar and Spherical Topologies

Basic planar configurations, as shown in Fig. 5.1, are straightforward to construct using the constraints outlined in Sec. 5.2.1. Spherical events can also be constructed using the above constraints, however the complexity significantly increases given the addition of a third spatial dimension. Instead of configuring possible planar and spherical topologies by hand, we follow a general phase space construction in the CoM frame [46].

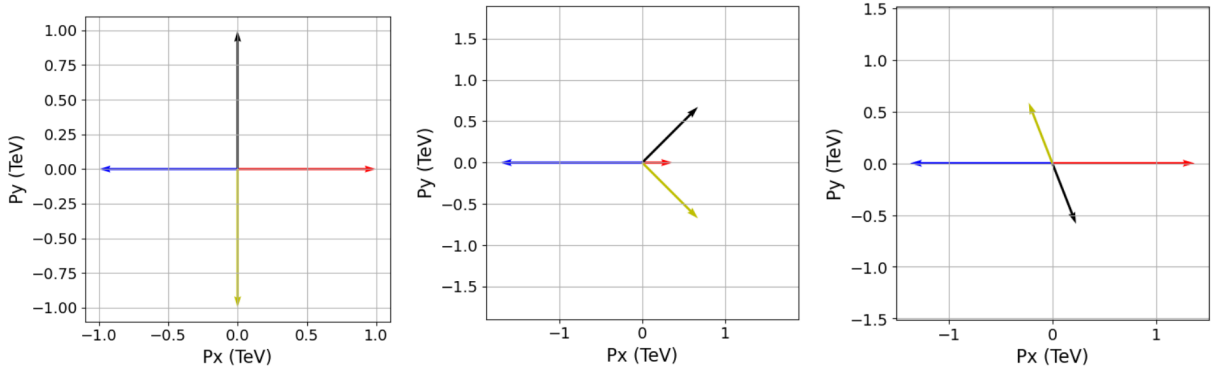


Figure 5.1: Three possible planar configurations in the CoM frame. Each vector corresponds to one daughter particle $\mathfrak{d} = a, b, c, d$; the colours hold no meaning. The total energy of each orientation is 4 TeV. These orientations can be rotated into 3D space and will still retain their planar configuration.

Consider a system of four massless particles with four-momentum $q_{\mathfrak{d}}^\mu$ that are not constrained by momentum conservation but have finite momenta. We define the overall momentum of the set of $q_{\mathfrak{d}}$'s by Q^μ , ($Q^\mu = \sum_{\mathfrak{d}=a}^d q_{\mathfrak{d}}^\mu$). To start, we generate four independent massless particles with four-momentum $q_{\mathfrak{d}}$ that have an isotropic angular distribution and energies $q_{\mathfrak{d}}^0$ according to the density distribution $q_{\mathfrak{d}}^0 e^{-q_{\mathfrak{d}}^0} dq_{\mathfrak{d}}^0$. The spatial

components are

$$q_{\mathbf{d}}^1 = q_{\mathbf{d}}^0 \sqrt{1 - c_{\mathbf{d}}^2} \cos \phi_{\mathbf{d}}, \quad (5.32a)$$

$$q_{\mathbf{d}}^2 = q_{\mathbf{d}}^0 \sqrt{1 - c_{\mathbf{d}}^2} \sin \phi_{\mathbf{d}}, \quad (5.32b)$$

$$q_{\mathbf{d}}^3 = q_{\mathbf{d}}^0 c_{\mathbf{d}}, \quad (5.32c)$$

where, letting $\mathcal{U}_i$ be a random number uniformly distributed in $[0,1)$ for $i = 1, 2, 3, 4$,

$$c = 2\mathcal{U}_1 - 1, \quad (5.33a)$$

$$\phi = 2\pi\mathcal{U}_2, \quad (5.33b)$$

$$q^0 = -\ln(\mathcal{U}_3\mathcal{U}_4). \quad (5.33c)$$

Next, with the arbitrary, yet confined, four-momenta of each particle, we must relate these four-vectors to physical vectors $p_{\mathbf{d}}^\mu$ with a corresponding momentum set P^μ . To do this, we perform a conformal transformation from Q^μ into P^μ that satisfies energy and momentum conservation. The conformal mapping is a Lorentz boost with a scaling transformation given by

$$p_{\mathbf{d}}^0 = x \left(\gamma q_{\mathbf{d}}^0 + \vec{b} \cdot \vec{q}_{\mathbf{d}} \right), \quad (5.34a)$$

$$\vec{p}_{\mathbf{d}} = x \left(\vec{q}_{\mathbf{d}} + \vec{b} q_{\mathbf{d}}^0 + a (\vec{b} \cdot \vec{q}_{\mathbf{d}}) \vec{b} \right). \quad (5.34b)$$

We define the scaling factor mass (which can be thought of as the mass of the unconserved momentum system) as $\tilde{M} = \sqrt{Q^2}$ such that

$$\vec{b} = -\frac{\vec{Q}}{\tilde{M}}, \quad (5.35a)$$

$$x = \frac{w}{\tilde{M}} \quad (5.35b)$$

$$\gamma = \frac{Q_0}{\tilde{M}} = \sqrt{1 + \vec{b}^2} \quad (5.35c)$$

$$a = \frac{1}{1 + \gamma}. \quad (5.35d)$$

In this scheme, w is the CoM energy (the energy of the mother particle in its rest frame, $P = [w, 0, 0, 0]$).

With this method, we can effectively generate arbitrary spherical events for n -massless

particles, $n \in [2, 3, \dots]$, although we only generate $n \in [2, 5]$. Figure 5.2 shows three random four-particle orientations that were generated using this method. After generation, the event can be randomly rotated but will still retain its spherical status.

This algorithm also provides a way to generate events in a random plane. If we choose $c_d = 0$, then $q_d^3 = 0$ and the generation of Q^μ (and therefore boosting to P^μ) properly preserves energy and momentum for massless particles in the $p^1 p^2$ plane. These can then be randomly rotated so that they do not occupy the same plane but are still planar events.

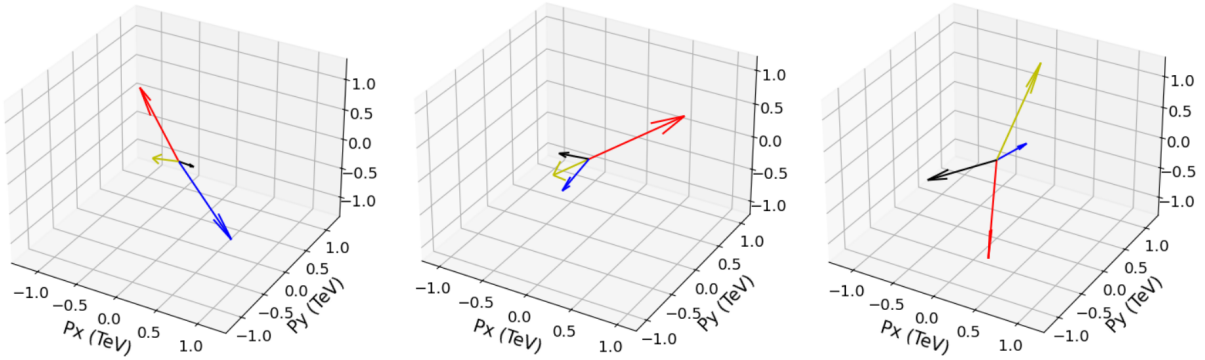


Figure 5.2: Three random CoM spherical orientations of four massless particles generated according to Sec. 5.2.2. The angle of depiction may make it seem like the total momentum does not sum to zero, this is simply a limit of the graphic and not the algorithm as it was validate with simple summing. Each vector corresponds to one daughter particle $d = a, b, c, d$; the colours hold no meaning. The total energy of each orientation is 4 TeV.

5.2.3 Lorentz Boost

The massless phase space events are generated in the CoM frame and must be boosted to the lab frame for comparison with collision data. The incoming parton four-momenta in the CoM frame are

$$p_a = \frac{1}{2}(M, 0, 0, M), \quad p_b = \frac{1}{2}(M, 0, 0, -M), \quad (5.36)$$

where $M = \sqrt{\hat{s}} = \sqrt{x_a x_b s}$. The total incoming four-momentum in the CoM frame is therefore $P_{\text{CoM}} = p_a + p_b = (M, 0, 0, 0)$. In the lab frame, the incoming parton four-

momenta are

$$p_a = \frac{1}{2} (x_a \sqrt{s}, 0, 0, x_a \sqrt{s}), \quad p_b = \frac{1}{2} (x_b \sqrt{s}, 0, 0, -x_b \sqrt{s}). \quad (5.37)$$

Consequently, the total incoming four-momentum in the lab frame is

$$P_{\text{lab}} = p_a + p_b = \frac{\sqrt{s}}{2} (x_a + x_b, 0, 0, x_a - x_b). \quad (5.38)$$

For an arbitrary boost velocity $\vec{\beta}$, the energy and three-momentum transformations are given by [47]

$$E' = \gamma (E + \vec{p} \cdot \vec{\beta}), \quad (5.39)$$

$$\vec{p}' = \vec{p} + (\gamma - 1) (\vec{p} \cdot \hat{\beta}) \hat{\beta} + \gamma E \vec{\beta}, \quad (5.40)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}. \quad (5.41)$$

Here, the primed quantities denote the four-momentum components after the boost.

Since the incoming partons have no net transverse momentum, the partonic CoM frame is boosted relative to the lab frame only along the beam axis. The boost vector is therefore

$$\vec{\beta} = (0, 0, b), \quad (5.42)$$

where

$$b = \frac{P_{\text{lab}}^z}{E_{\text{lab}}} = \frac{x_a - x_b}{x_a + x_b}, \quad -1 < b < 1. \quad (5.43)$$

Thus, the boost is along the positive z -direction when $x_a > x_b$ and along the negative z -direction when $x_b > x_a$. If $x_a = x_b$, then $b = 0$ and the partonic CoM frame coincides with the lab frame. Although the total energy and longitudinal momentum change under the boost, By definition, the invariant mass is preserved,

$$P_{\text{lab}}^2 = E_{\text{lab}}^2 - (P_{\text{lab}}^z)^2 = x_a x_b s = M^2. \quad (5.44)$$

Lorentz invariance also ensures that each massless daughter particle remains massless,

$$(p)_{\text{lab}}^2 = (p)_{\text{CoM}}^2 = 0.$$

5.3 Colour Flow

Possible colour flows for select $2 \rightarrow 4$ processes are shown in Fig. 5.3. Gluons are displayed with no kinematic properties and their colour properties are the most immediate lines above and below the coils. The arrows on the coloured lines do not signify a direction in space, they only represent colour flow. On the other hand, the kinematic properties of quarks and their colour are both indicated in a single line; the arrows signal an incoming/outgoing quark/antiquark. If we look at the top right quark of Fig. 5.3b the arrow is outgoing hence it is a quark with a red colour charge, whereas the one below is an anti-quark with a anti-red colour charge.

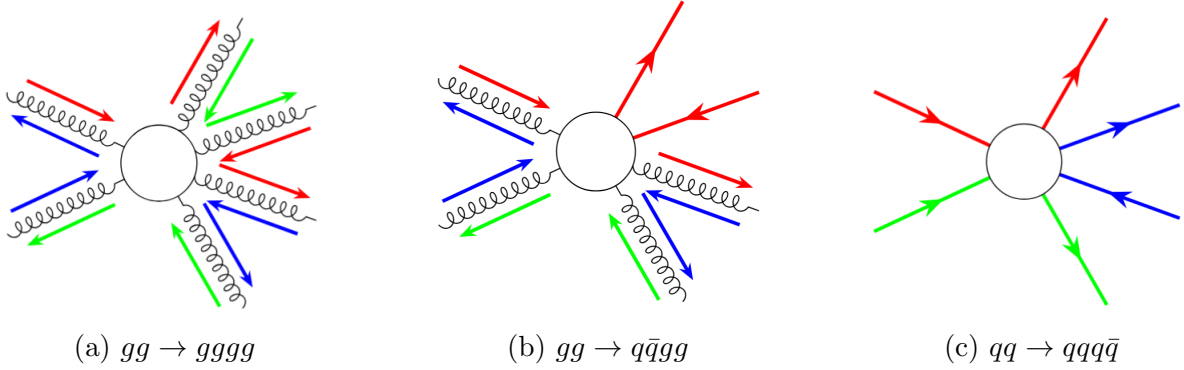


Figure 5.3: Possible colour flow orientations for three subprocesses of $2 \rightarrow 4$. Time is read left to right so the two incoming particles are on the left of the circle. The choice of colours in these figures is only one possible representation. If we consider (c), the red/green incoming quarks were a choice and therefore must have a outgoing red/green quark. The blue quarks could be chosen to be any colour.

The possible $2 \rightarrow 2, 3, 4, 5$ colour flows were obtained by generating 10,000 events for each process using the MadGraph event generator and recording both the individual flows and their frequencies [45]. DRToM then uses these files to determine the colour assignment for a given process based on the relative occurrence of each flow in the MadGraph samples. For example, for $gg \rightarrow gggg$, 10,000 events were generated in MadGraph, the resulting colour flows were counted, and the unique flows were stored in a dedicated file. DRToM accesses this file to perform a weighted random choice of the colour flow when $gg \rightarrow gggg$ is selected. This was done for all partonic outputs of DRToM (from $2 \rightarrow 2$ up to $2 \rightarrow 5$).

5.4 Summary

The computation of DRToM consists of several coupled numerical steps: (1) uniform sampling of the invariant mass M ; (2) midpoint quadrature over the phase space variables (Y, y) for each partonic channel; (3) inverse cumulative distribution sampling to select a channel according to its integrated cross section; (4) acceptance–rejection sampling to generate (Y, y) according to the selected channel’s differential cross section integrand; and (5) Monte Carlo estimation of the cross section over the sampled invariant mass range.

We note that for a phase space generation, the user can manually select spherical or planar momenta configurations. When the DR probability option is enabled, the generator can instead use a Bernoulli trial to select probabilistically between the spherical and planar phase space routines. This is not used in our analysis as we are considering a step-like behaviour for DR.

With a specific process chosen, the final-state particle IDs and colour flow are selected, and the momentum configuration is generated in the partonic CoM frame. The event is then boosted into the lab frame using the selected values of x_a and x_b . Once an event has been fully processed, including its differential cross section (with uncertainties), collision process, colour flow, and momentum four-vectors, the final event configuration is written in the standard Les Houches Event (LHE) file format [\[48\]](#).

Chapter 6

Parton Level Event Simulation

6.1 Kinematic Validation

With an algorithm in place for $2 \rightarrow 2$ QCD, and for $2 \rightarrow 2$ up to $2 \rightarrow 5$ massless partonic phase space of spherical and planar events, we now investigate the outputs of DRToM. We start by considering the QCD cross sections as a function of event invariant mass, then we investigate the kinematics of the processes. The kinematic variables we consider are the event total energy E , the event total momentum magnitude $|\vec{p}|$, the transverse momentum p_T , the individual momentum components p_x, p_y, p_z ; as well as the angle distributions of the azimuthal angle ϕ . We do not apply any weighting to the kinematics (we do not simulate data). For illustrative purposes we will only show two cases: $2 \rightarrow 2$ QCD for $gg \rightarrow gg$ (based on the kinematics in Chapter 3, i.e. not uniform phase space), and $2 \rightarrow 4$ massless phase space with all tree-level processes from Table 3.1 active (based on Sec 5.2.2) for spherical and planar events in the CoM frame. Additional plots (in the lab frame and additional angles) for $2 \rightarrow 4$ are shown in Appendix D.

We use the CTEQ6L1 PDF set, which are evaluated at a factorization scale of $Q = M$ [49]. We impose a maximum absolute final-state rapidity requirement of $|y_{1,2}| < 4$ in the lab frame to restrict the forward region and regulate the collinear divergences of the squared QCD MEs. This corresponds to the region $|Y| + |y| < 4$, and, since we are working in the massless limit, it is really a requirement of $|\eta| < 4$. The choice of PDF set, factorization scale, and rapidity requirement can all be changed by the user.

The event generation follows a stratified sampling scheme: events were generated over

0.1 TeV intervals between the mass range of 2-11 TeV with a CoM energy of 13.0 TeV for a total of 9.9×10^6 events. A summary of the MC generation scheme for the plots in this section is given in Appendix C.

6.1.1 QCD Cross Sections

We show the generated cross section as a function of event invariant mass. The invariant mass of an event is calculated as

$$M = \sqrt{E_{\text{Tot}}^2 - ((p_{\text{Tot}}^x)^2 + (p_{\text{Tot}}^y)^2 + (p_{\text{Tot}}^z)^2)}. \quad (6.1)$$

Figure 6.1 shows the cross sections as a function of event invariant mass for all QCD processes in Table 3.1. Multiplying this plot by luminosity gives the expected number of events of these types produced by that amount of luminosity. We see that each process appropriately spans the entire generated invariant mass range and there are no discontinuities or abrupt spikes present.

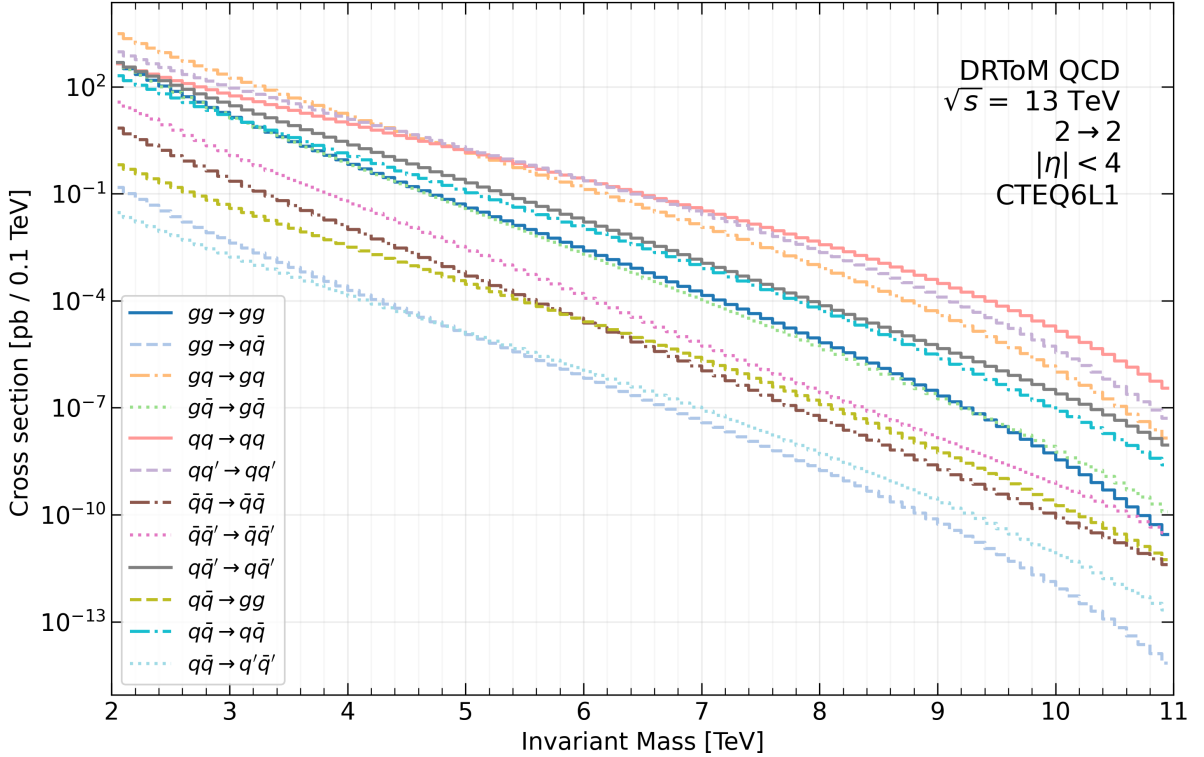


Figure 6.1: Generated cross sections for all QCD processes with the CTEQ6L1 PDF set.

6.1.2 Gluons to Gluons QCD

We consider $gg \rightarrow gg$ in the CoM frame as a means to investigate the QCD kinematics of DRToM. In Sec. 3.2, we saw that the chosen PDF contains a discontinuity in the gluon distribution at approximately $x \sim 0.97$. DRToM does not explicitly regulate or remove this discontinuity. Configurations in this region are naturally suppressed because values of x close to unity occupy a restricted region of the available phase space and the gluon PDF decreases rapidly at high x . Adding the discontinuity, relatively few generated events contain an incoming gluon with a momentum fraction in the affected region and no events will be generated for x -values that go to zero.

This behaviour is visible in Fig. 6.2, where the number of events in the $x \sim 0.97$ region, the $0.96 < x_{a,b} < 0.98$ bins, decreases for both incoming partons. The small number of events in this region limits its effect on the overall event sample. The kinematic distributions discussed below and the calculated cross sections do not exhibit any clear unphysical features associated with the discontinuity. Within the statistical precision and kinematic range considered in this work, the effect of the discontinuity appears to be negligible.

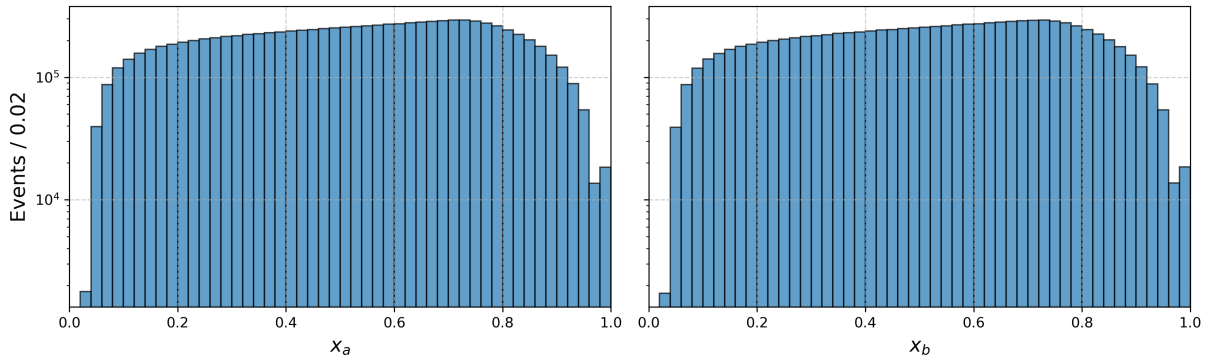


Figure 6.2: Momentum fraction distribution for two colliding gluons. We use the CTEQ6L1 PDF set, setting the energy scale Q to the invariant mass of the event.

Figure 6.3 shows the total event energy, the magnitude of event momentum, and the event transverse momentum; and the p_x , p_y , and p_z distributions. The plots show the fraction of events (each plot is normalized to the total number of events used) for the leading and subleading parton. Normally, leading and subleading are distinguished

between the highest and lowest p_T parton. Since the kinematics of the two outgoing gluons are identical, we choose the leading to be the first outgoing occurrence in the event record and subleading to be the second. The two outgoing partons in $2 \rightarrow 2$ are back-to-back, hence the overlapping distributions across all plots. We see that the energy and momenta-magnitude plots are identical (even though they are not on an overlapping plot) which is expected as we are dealing with massless particles. Moreover, their distributions are flat across the energy range since we generated the same amount of events across each 0.1 TeV slice and are not weighting the events. Momentum conservation for each collisions was checked (for p_T, p_x, p_y , and p_z) and found to sum to zero in each case.

The azimuthal angle, ϕ , of each parton is given by

$$\phi = \arctan\left(\frac{p^y}{p^x}\right), \quad -\pi \leq \phi \leq \pi. \quad (6.2)$$

We consider the difference in ϕ between the two outgoing gluons,

$$\Delta\phi = \phi_{\text{leading}} - \phi_{\text{subleading}}, \quad -\pi \leq \Delta\phi \leq \pi. \quad (6.3)$$

This is an ideal variable to plot because (as mentioned above) we know that the two particles in the CoM frame (and, consequently, the lab frame given the boost along the beam axis) must lie back-to-back. Hence, the angular difference between the two must (only) peak a π apart. This is exactly what we see in the fractional distribution of Fig. 6.4 which is a good indication that the azimuthal kinematics of the generator are appropriately configured (up to the chosen bin width).

The last plot we discuss is a comparison of DRToM with a validated MC generator, STRINGS [44]. STRINGS is a MC event generator for the production and decay of first and second string resonances through $2 \rightarrow 2$ partonic and 2-parton $\rightarrow \gamma +$ parton scatterings in proton-proton collisions. The generator can also be used to produce QCD diparton processes. For our comparison we consider the polar scattering angle θ ,

$$\theta = \arccos\left(\frac{p^z}{\sqrt{(p^x)^2 + (p^y)^2 + (p^z)^2}}\right), \quad (6.4)$$

which represents the particle's angle between its momentum vector and the beam axis.

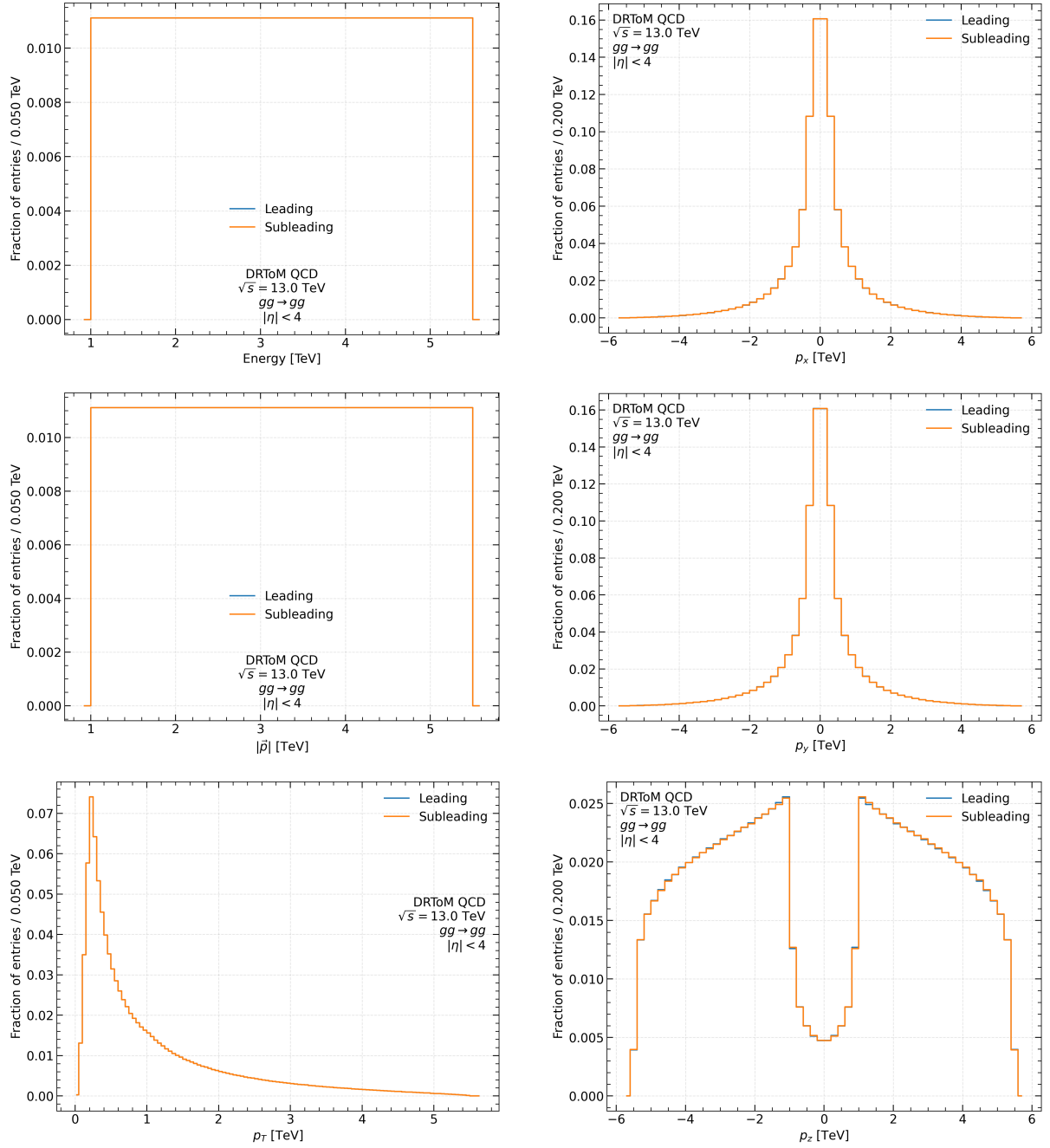


Figure 6.3: E , $|\vec{p}|$, p_T , p_x , p_y , and p_z distributions for QCD $gg \rightarrow gg$ events in the CoM frame. The left column shows, from top to bottom, the E , $|\vec{p}|$, and p_T distributions, while the right column shows the corresponding p_x , p_y , and p_z distributions. Since the partons have the same p_T , leading indicates the first occurrence in the event record and subleading the second. All lines use the same thickness and are directly overlapped on one another. The distributions are not weighted (they do not simulate data).

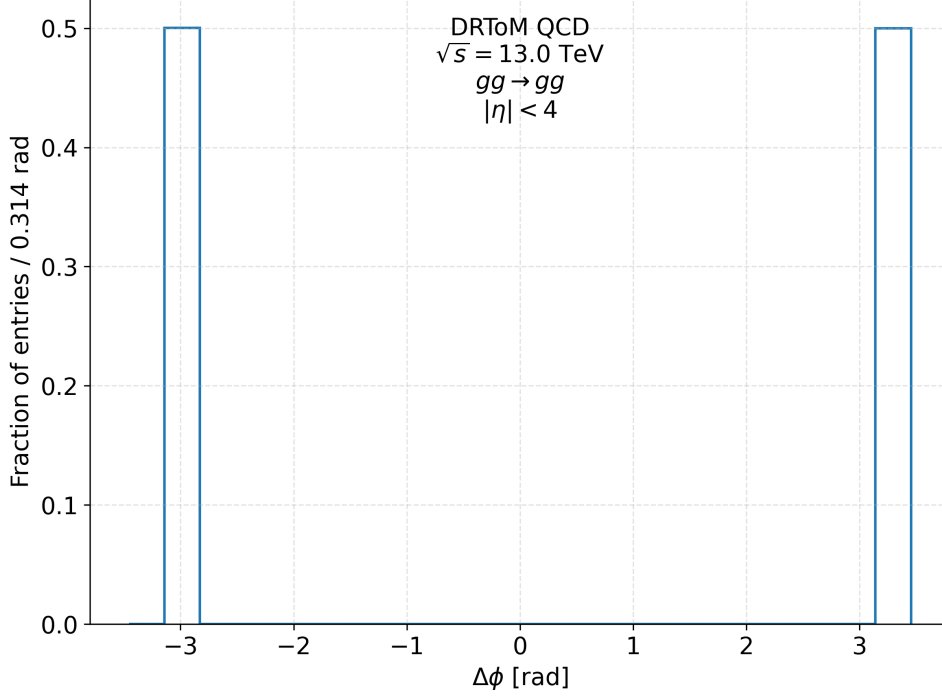


Figure 6.4: $\Delta\phi$ for the leading and subleading partons of QCD $gg \rightarrow gg$ events in the CoM frame. Note that the binning asymmetry is an artifact of automatic axis padding. Because the plotted range slightly exceeds the physical $[-\pi, \pi]$ limits, the fixed-width bins do not perfectly align with the expected geometric symmetry.

Figure 6.5 overlays $\cos\theta$ of $gg \rightarrow gg$ QCD collisions generated from STRINGS (orange) vs. those from DRToM (blue) in the lab frame (not normalized). We note that this plot was made using 100,000 events between 2.0 TeV and 2.1 TeV and was done for all possible diparton processes that STRINGS offers. We see that, up to statistical fluctuations, the generators agree which acts as another layer of validation for DRToM.

6.1.3 Four Outgoing Massless Partonic Phase Space

In order to investigate $2 \rightarrow 4$ processes, the generator starts from the kinematics of tree-level QCD. Then, once a tree-level collision is chosen, a random $2 \rightarrow 4$ process is selected and the phase space kinematics are generated. As such, the cross section vs. invariant mass distribution (and hence the number of predicated events after multiplying by luminosity) of a $2 \rightarrow 4$ process will be equivalent to that of the QCD $2 \rightarrow 2$ process (since the cross section is determined from the QCD tree-level state and not the final state $2 \rightarrow 3, 4, 5$ state). For instance, if looking solely at $gg \rightarrow gggg$, the only QCD

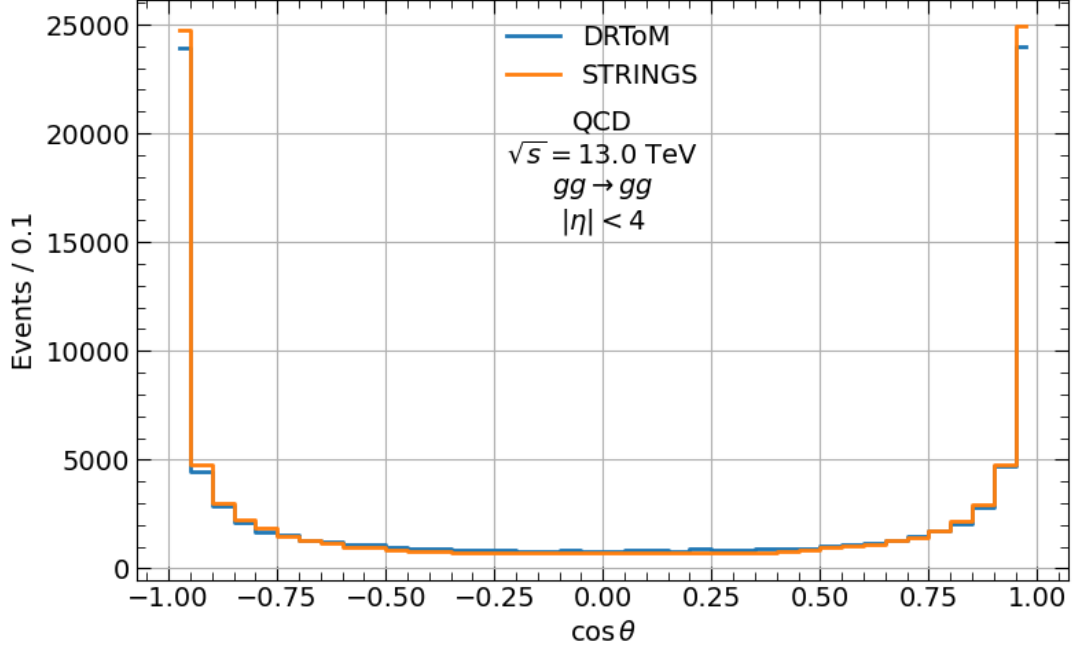


Figure 6.5: Comparison of $\cos \theta$ for QCD $gg \rightarrow gg$ of DRTOM and STRINGS in the lab frame.

process that builds this final phase space state is $gg \rightarrow gg$ (as seen in Table 5.1) so the cross section vs. event invariant mass for $gg \rightarrow gggg$ is equivalent to Fig. 6.1 for $gg \rightarrow gg$.

For all figures and generation after this point, we are letting all processes from Table 5.1 be active so that there is no one-to-one $2 \rightarrow 2$ cross section relation. Figure 6.6 shows the cross section vs. invariant mass plot for $2 \rightarrow 4$ massless phase space spherical events in the CoM frame following the generation of Appendix C. We see the distribution covers the whole range and that it follows the same shape as the tree-level processes.

We again consider the fractional distributions of the total event energy, the event momentum magnitude, the transverse momentum, and p_x, p_y, p_z in the CoM for spherical and planar events as shown in Fig. 6.7 and Fig. 6.8, respectively. We consider the distributions for the leading, subleading, tertiary, and fourth parton (ordered by p_T where the leading parton has the highest p_T). Compared to the $2 \rightarrow 2$ case, the leading two partons no longer have to be back-to-back and do not have identical angular distributions. This is exemplified in Fig. 6.9 where we see that the leading and subleading partons no longer need to be 2π apart as the third and fourth particles offer means to make up the required energy and momentum conservation for the configuration. Momentum conservation for

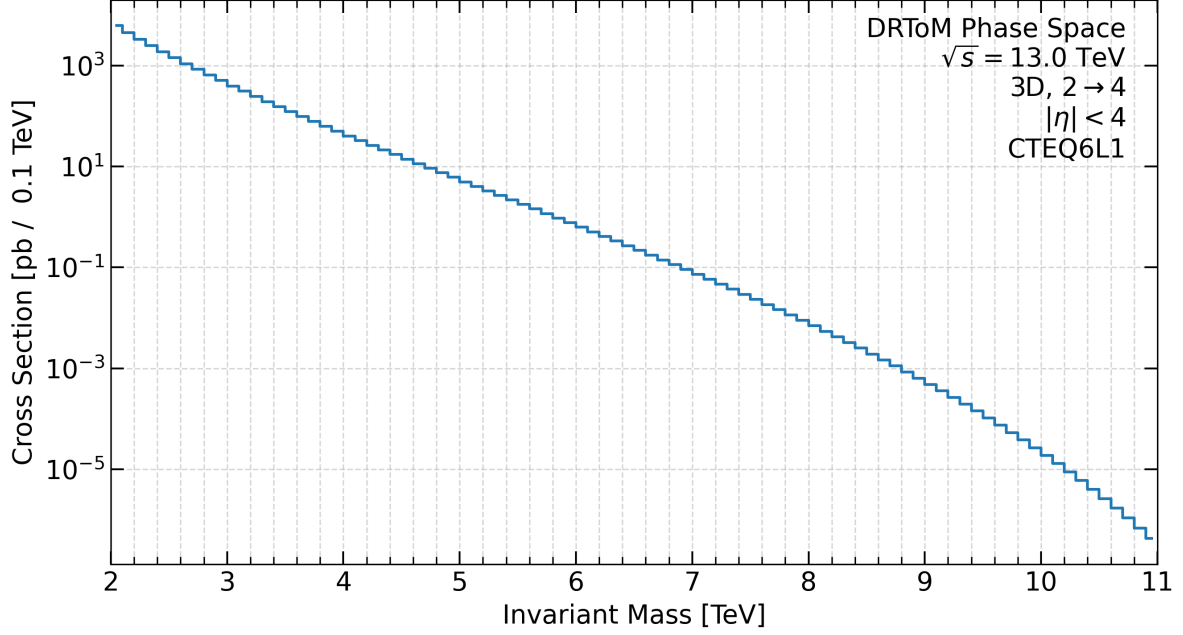


Figure 6.6: Cross section of $2 \rightarrow 4$ DRTom generation for spherical events in the CoM frame with all QCD processes active. We use the CTEQ6L1 PDF set, setting the energy scale Q to the invariant mass of the event.

each collisions was checked (for p_T, p_x, p_y , and p_z) and found to sum to zero in each case.

Comparing between the spherical and planar event samples, we find that the single-particle kinematic distributions (E , $|\vec{p}|$, p_T , p_x , p_y , and p_z) are identical within statistical fluctuations. This is expected because the parton momenta are generated according to the same underlying phase space distribution in both samples. Consequently, these observables alone do not provide a clear distinction between the two event topologies.

The $\Delta\phi$ distributions also have the same general shape. Both exhibit peaks near $\Delta\phi = \pm\pi$, corresponding to the leading and subleading partons being approximately back-to-back in the transverse plane, followed by a symmetric decrease toward $\Delta\phi = 0$. However, the prominence of the peaks differs between the two samples. For spherical events, the peak bins contain approximately 23% of the events, while the adjacent bins contain approximately 15%, corresponding to a decrease of about 8%. For planar events, the corresponding values are approximately 30% and 12%, giving a decrease of about 18% (a larger peak and steeper drop off). The sharper peaks in the planar events indicate that the leading and subleading partons are more strongly concentrated around a back-to-back azimuthal configuration. This is consistent with the restricted geometry of planar events.

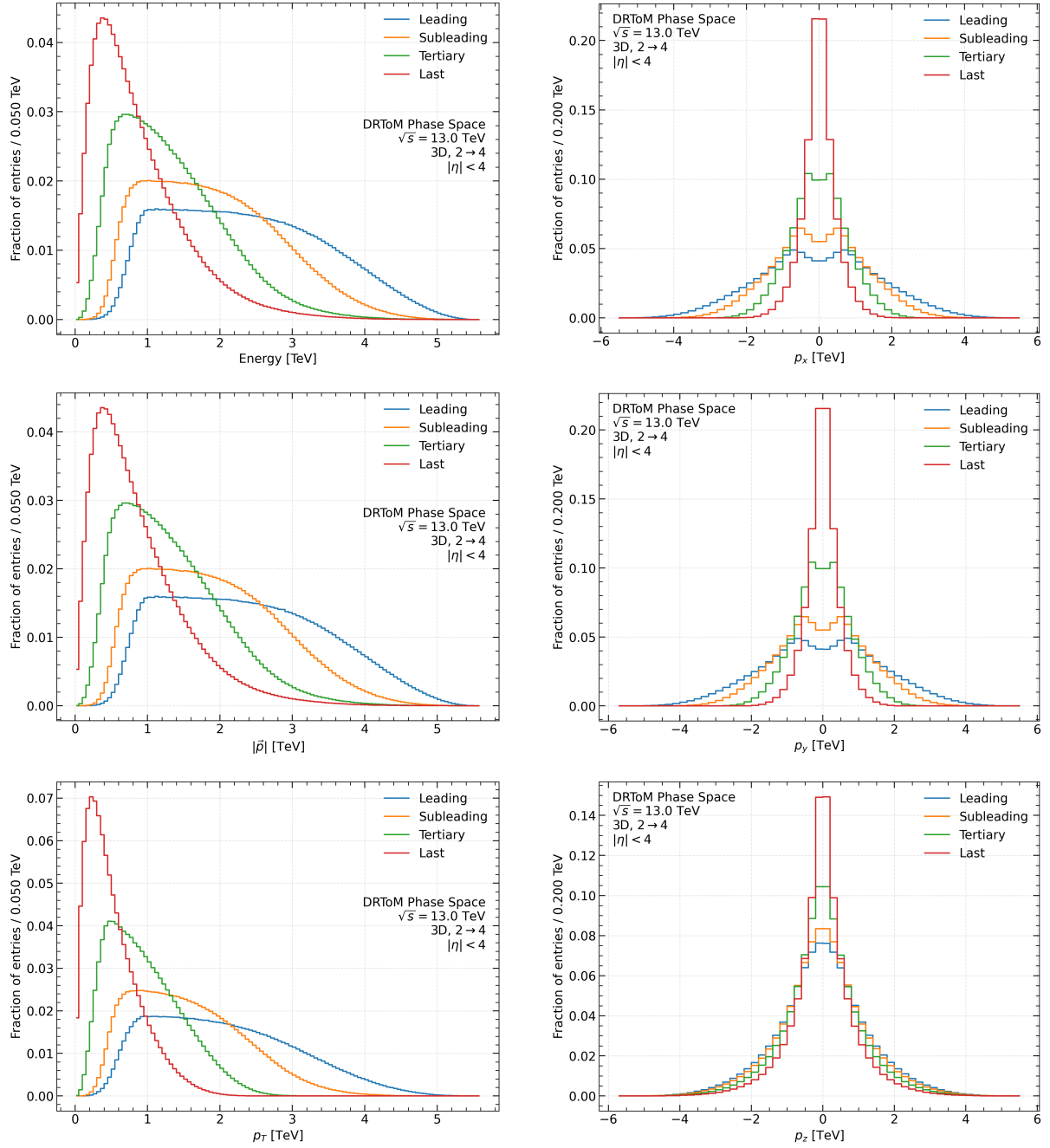


Figure 6.7: E , $|\vec{p}|$, p_T , p_x , p_y , and p_z distributions for $2 \rightarrow 4$ massless partonic spherical events in the CoM frame. The left column shows, from top to bottom, the E , $|\vec{p}|$, and p_T distributions, while the right column shows the corresponding p_x , p_y , and p_z distributions. The partons are ordered by p_T where leading indicates the parton with the highest p_T , subleading the second highest p_T , tertiary the third highest p_T , and last the final parton. The events are not weighted (they do not simulate data).

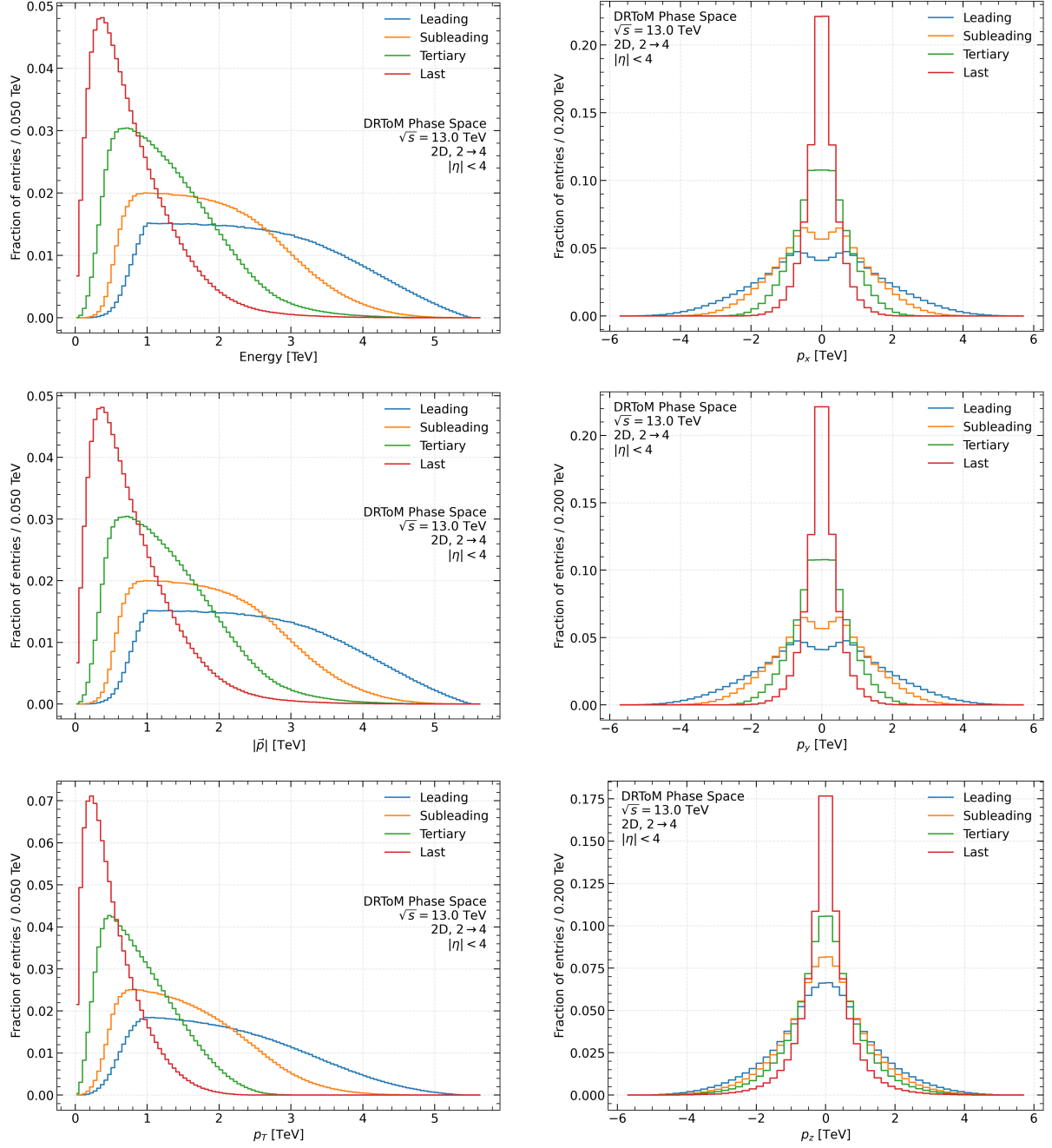
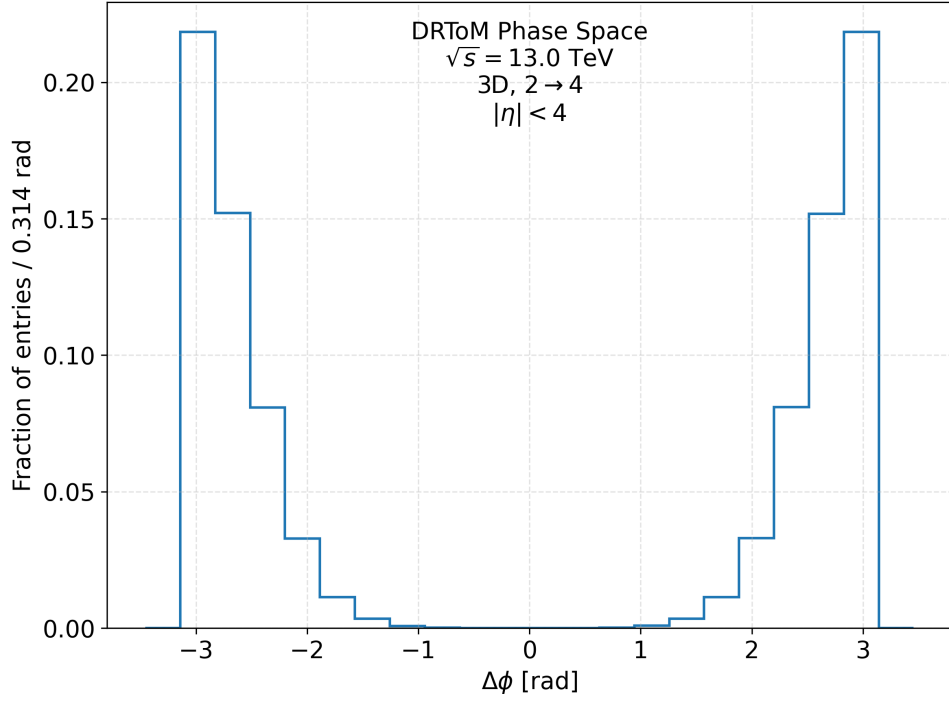
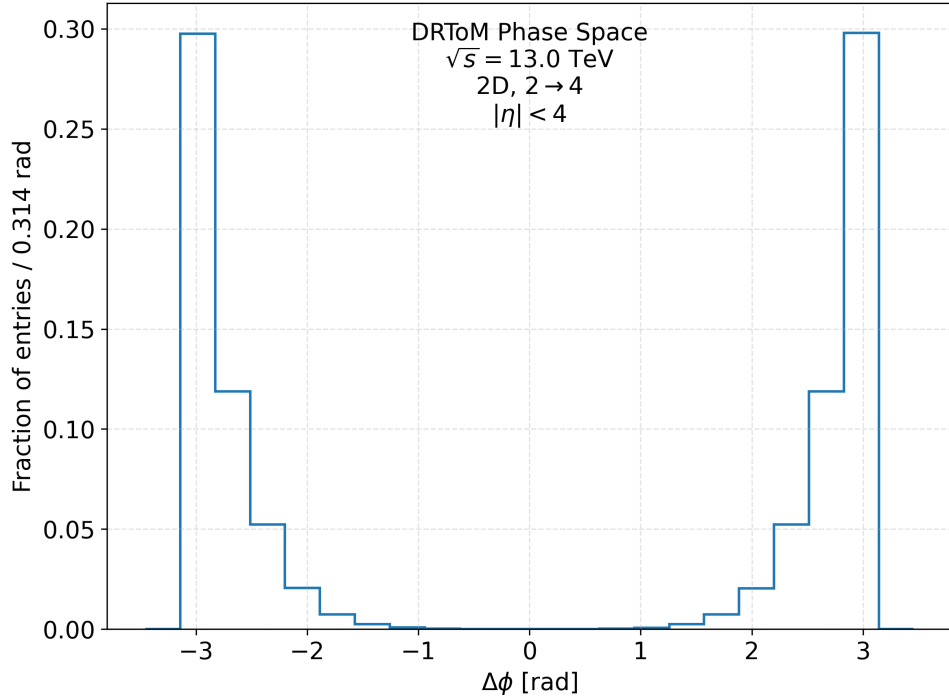


Figure 6.8: E , $|\vec{p}|$, p_T , p_x , p_y , and p_z distributions for $2 \rightarrow 4$ massless partonic planar events in the CoM frame. The left column shows, from top to bottom, the E , $|\vec{p}|$, and p_T distributions, while the right column shows the corresponding p_x , p_y , and p_z distributions. The partons are ordered by p_T where leading indicates the parton with the highest p_T , subleading the second highest p_T , tertiary the third highest p_T , and last the final parton. The events are not weighted (they do not simulate data).



(a) Spherical events.



(b) Planar events.

Figure 6.9: $\Delta\phi$ between the leading and subleading partons for massless phase space $2 \rightarrow 4$ events in the CoM frame. The spherical event distribution is shown on top, while the planar event distribution is shown on the bottom.

6.2 Topological Event Shape Variables

With a method to generate $2 \rightarrow 4$ massless phase space events, we now turn to classification schemes for event topologies in space. These variables are what allow us to quantify topological differences between spherical and planar events.

6.2.1 Momentum Tensor

Consider a collision with i outgoing particles. We define the momentum tensor as [50]

$$M^{xyz} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{1}{|\vec{p}_i|} \begin{pmatrix} (p_i^x)^2 & p_i^y p_i^x & p_i^z p_i^x \\ p_i^y p_i^x & (p_i^y)^2 & p_i^z p_i^y \\ p_i^z p_i^x & p_i^z p_i^y & (p_i^z)^2 \end{pmatrix}. \quad (6.5)$$

The tensor is normalized with ordered eigenvalues

$$e_1 + e_2 + e_3 = 1, \quad \text{and} \quad 0 \leq e_1 \leq e_2 \leq e_3. \quad (6.6)$$

These eigenvalues are used to quantify the topology of the event through event shape variables. The first variable we discuss is sphericity, S , defined as

$$S = \frac{3}{2}(e_1 + e_2). \quad (6.7)$$

Sphericity is a measure of how isotropic (sphere-like) or collimated (jet-like) the particle distribution is within an event, helping to characterize the event's overall shape. Sphericity ranges from $0 \leq S \leq 1$ where higher values indicate a more uniform particle distribution across all directions, and lower values suggest a collimated distribution where particles align along specific axes.

The second event shape variable we discuss is aplanarity, A , defined as

$$A = \frac{3}{2}e_1, \quad (6.8)$$

which is normalized such that $0 \leq A \leq 0.5$. Aplanarity quantifies how far the particle momentum distribution deviates from being planar. If $A = 0$, then the momentum of the particle is confined to a plane, whereas $A = 0.5$ indicates a fully three-dimensional,

isotropic distribution. This makes aplanarity a top candidate to investigate DR since $A = 0$ concretely defines planar events, and $A \neq 0$ has some spherical component. The distribution, then, for events that are fully planar (those that have undergone dimensional reduction) would all be in a single bin (the first bin) in the CoM frame, whereas fully spherical events (a case where there is no dimensional reduction) would have a full distribution over the range. This is exactly what we see in Fig. 6.10. For every aplanarity plot we use 50 bins across it's range such that it has a bin width of 0.01.

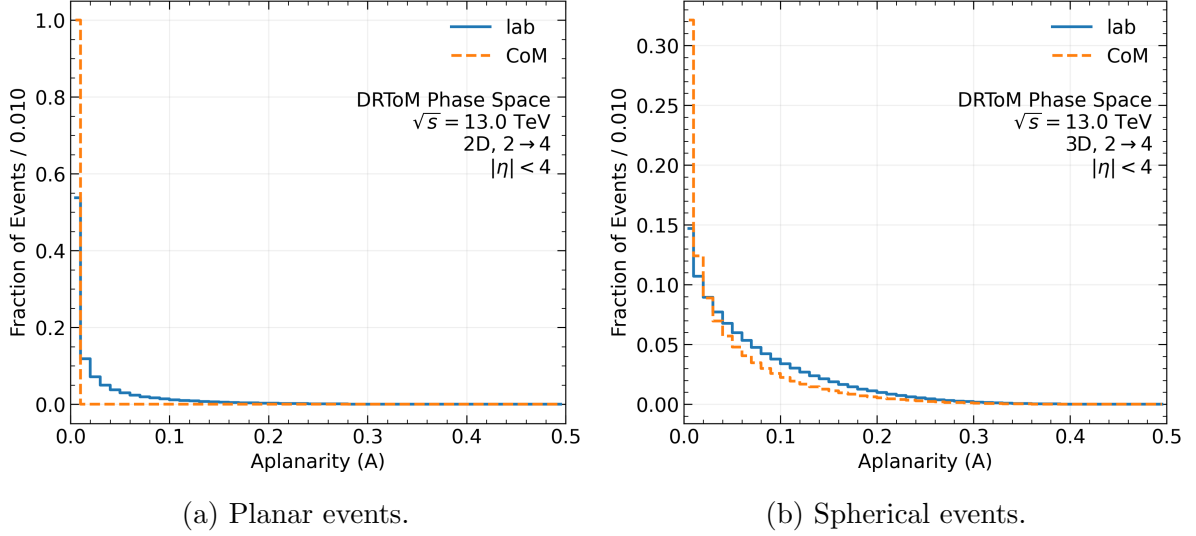


Figure 6.10: Aplanarity distributions for $2 \rightarrow 4$ massless phase space in the lab and CoM frame. The spherical plot (right) does not have the same y -axis as the planar plot (left).

While sphericity captures overall isotropy and aplanarity identifies out-of-plane activity, we introduce a complementary variable, Y , which quantifies the energy asymmetry within the planar distribution of the event,

$$Y = \frac{\sqrt{3}}{2}(e_2 - e_1). \quad (6.9)$$

This quantity measures the uneven distribution of momentum in the plane perpendicular to e_3 . A large Y ($e_2 \gg e_1$) indicates a flat, pancake-like distribution, while a small Y ($e_2 \approx e_1$) suggests a more uniform, isotropic distribution.

Because we often measure in the lab frame, transverse objects are usually considered.

Taking the transverse components of Eq. 6.5 gives the transverse momentum tensor

$$M^{xy} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{1}{|\vec{p}_i|} \begin{pmatrix} (p_i^x)^2 & p_i^y p_i^x \\ p_i^y p_i^x & (p_i^y)^2 \end{pmatrix}. \quad (6.10)$$

It is equivalently normalized with eigenvalues

$$u_1 + u_2 = 1, \quad \text{and} \quad u_1 < u_2. \quad (6.11)$$

The transverse sphericity event shape variable is defined as

$$S_T = \frac{2u_1}{u_1 + u_2}, \quad (6.12)$$

and is bounded between 0 and 1 with small (large) values indicating more collimated (isotropic) events in the transverse plane.

A summary of the most used event shape variables derived from the momentum tensor are given in Table 6.1. An example of calculating the momentum tensor for a basic planar event with some variables is worked through in Appendix E. Distributions of each variable for spherical and planar events in the CoM and lab frames are shown in Appendix F.

Table 6.1: Event shape observables derived from the eigenvalues of the momentum tensor.

Event shape variable	Definition	Range
Sphericity, S	$\frac{3}{2} (e_1 + e_2)$	[0,1]
Transverse Sphericity, S_T	$\frac{2u_1}{u_1+u_2}$	[0,1]
Aplanarity, A	$\frac{3}{2} e_1$	[0, 0.5]
Y-parameter	$\frac{\sqrt{3}}{2} (e_2 - e_1)$	[0,1]
C-parameter	$3(e_1 e_2 + e_1 e_3 + e_2 e_3)$	[0,1]
D-parameter	$27(e_1 e_2 e_3)$	[0,1]

6.2.2 Thrust

The thrust variables provide insight into the spatial orientation of particles (i.e. are they close together or not). We note that thrust is intended for jets, but for now we will define them at parton level. For each event, we can define the thrust axis $\hat{n}_T$ as the direction

with respect to which the projection of the parton momenta is maximized. Given a set of three-momentum, $\{\vec{p}_i\}$, where i runs over all partons in the event, the thrust is calculated as [50, 51]

$$T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}_T|}{\sum_i |\vec{p}_i|}. \quad (6.13)$$

If momentum conservation gives $\sum_i p_i = 0$ (regardless of frame) then the thrust is maximized at $T = 1$ (all momenta are parallel or anti-parallel), and is minimized at $T = 1/2$ (when the momenta is distributed isotopically). We can also build the transverse thrust, $T_\perp$, and it's minor component, T_m , as

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_T|}{\sum_i |\vec{p}_{T,i}|}, \quad T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_T|}{\sum_i |\vec{p}_{T,i}|}. \quad (6.14)$$

The minor component is bound by $0 \leq T_m < 2/\pi$ with large values indicating a large energy flow outside the plane spanned by the thrust and beam axis. We define the transverse thrust deficit as

$$\tau_\perp = 1 - T_\perp, \quad 0 \leq \tau_\perp < 1 - 2/\pi. \quad (6.15)$$

The lower values of $\tau_\perp$ indicate a more collimated-like configuration whereas higher values indicate a larger energy flow orthogonal to the thrust axis. A summary of the thrust variables is given in Table 6.2. Distributions of the thrust variables for spherical and planar events in the CoM and lab frames are shown in Appendix F.

Table 6.2: Event shape thrust variables.

Event shape variable	Definition	Range
Thrust, T	$\max_{\hat{n}} \frac{\sum_i \vec{p}_i \cdot \hat{n}_T }{\sum_i \vec{p}_i }$	$[1/2, 1)$
Transverse thrust, $T_\perp$	$\frac{\sum_i \vec{p}_{T,i} \cdot \hat{n}_T }{\sum_i \vec{p}_{T,i} }$	$(2/\pi - 1, 1]$
Thrust minor, T_m	$\frac{\sum_i \vec{p}_{T,i} \times \hat{n}_T }{\sum_i \vec{p}_{T,i} }$	$[0, 2/\pi)$
Transverse thrust deficit, $\tau_\perp$	$1 - T_\perp$	$[0, 1 - 2/\pi)$

6.2.3 Biplanarity

As we are interested in the nature of events with four outgoing particles, we define an event shape variable that is specifically designed for this case. Consider four outgoing particles with their-momenta $(\vec{p}_a, \vec{p}_b, \vec{p}_c, \vec{p}_d)$ ordered by largest to smallest p_T in the lab frame (i.e. $\vec{p}_a$ has the largest p_T). Figure 6.11 shows a general orientation for these three-momenta.

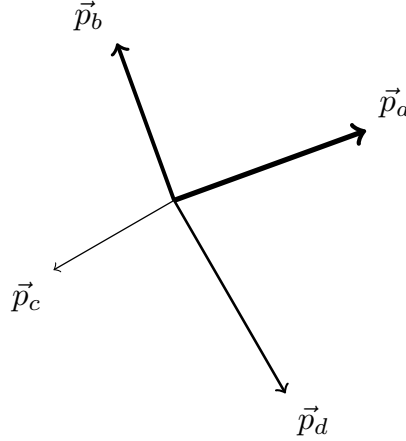


Figure 6.11: General orientation of four three-momenta in space. The thickness of the lines hold no meaning and are there to aid in a 3D visualization.

Let $\vec{n}_1$ be the cross product of the pair of momenta with the largest angular separation, and $\vec{n}_2$ to be the cross product of the remaining pair. With these two planes, a visual depiction shown in Fig. 6.12, we can ask: are they coplanar? Assuming neither cross product vanishes, mathematically this can be written as

$$B = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|} = |\cos \theta_{12}|, \quad 0 \leq B \leq 1. \quad (6.16)$$

This definition of biplanarity was chosen based on a study discussed in Appendix G.

From Eq. 6.16, B measures the angle between the two event-plane normals. If the four three-momenta are all embedded in the same plane, the normals are parallel and the two planes coincide and $B = 1$. Otherwise, the normals define distinct planes indicating that the three-momenta cannot be confined to a single plane. Because $B = 1$ indicates a planar event, and $0 \leq B < 1$ indicates a spherical event, biplanarity acts as a distinct marker for the underlying topology of an event with four outgoing particles.

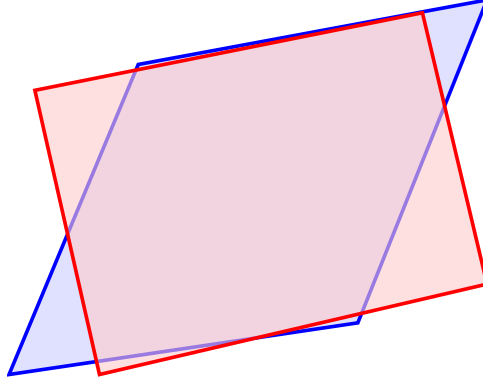


Figure 6.12: Visualization of overlapping planes. The colours hold no meaning.

Figure 6.13 shows the biplanarity distributions of $2 \rightarrow 4$ massless partonic phase space events following the generation of Appendix C. For every biplanarity plot we use 50 bins across its range such that it has a bin width of 0.02. Starting with the planar events in Fig. 6.13a, we see that in the CoM frame the events are all in the final bin which is what we expect. In the lab frame, however, the distribution becomes smeared where the final bin is still the most populated, but no longer contains all the events as before (a bin percent of about 40%). If we now consider the spherical events, shown in Fig. 6.13b, we see the distributions cover the full range in both the CoM and lab frames where the $B = 1$ bin has the highest fraction of values (at 13% and 5%, respectively).

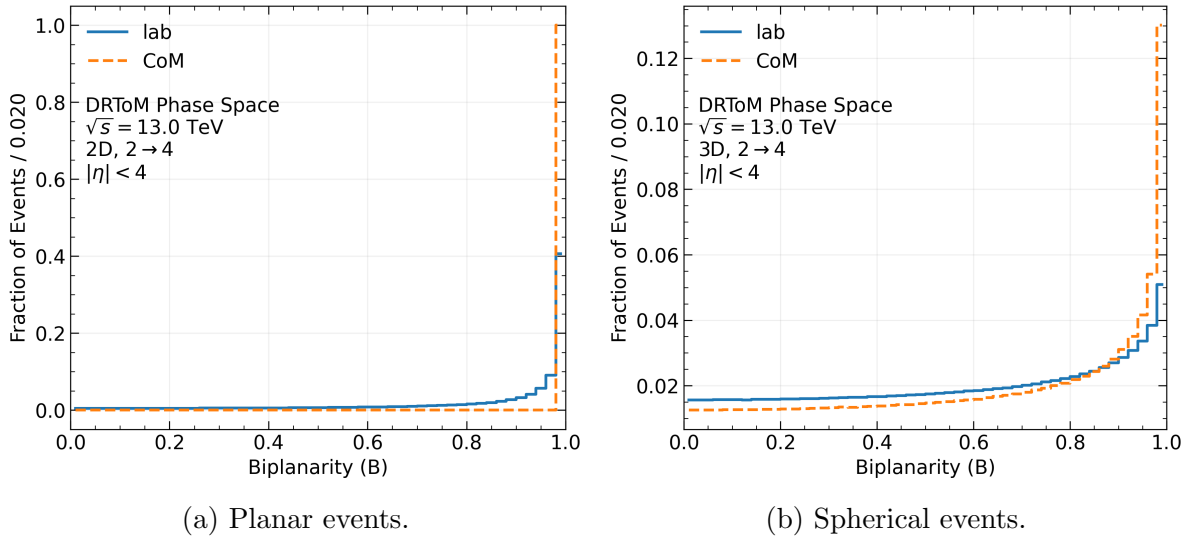


Figure 6.13: Biplanarity distributions for $2 \rightarrow 4$ massless partonic phase space events in the lab and CoM frame. The spherical plot (right) does not have the same y -axis as the planar plot (left).

6.3 Chosen Event Shape Variables for Analysis

Given our discussion on topological event shape variables, we only further analyze aplanarity and biplanarity. Due to their mathematical structure, they have distinct markers for planar events ($A = 0$ and $B = 1$). As such, they offer a baseline to further study the topological effects of DR instead of using every defined event shape variable. The correlation between A and B is discussed below.

For both, we note a limitation in our binning scheme. We see that the spherical event distributions peak in their respective planar bins. This peak does not imply that the majority of events are planar (as for both the bin percent is less than 50 in the CoM frame), it is just an artifact of binning since there is only a single bin to delineate planar from spherical events. This brings us to the main limitation. We arbitrarily chose a total of 50 bins across the distribution (such that the bin widths are 0.01 for A and 0.02 for B) as the distributions were well behaved. Given this choice, events that are not exactly $A = 0$ or $B = 1$ are registered as planar. For example, choosing $0.98 < B \leq 1$ to represent planar events means that events with $B \in (0.98, 0.999\dots]$ are registered as planar even though they are spherical. For us, what is important is not necessarily the bins themselves, but it is the difference in the plots from the CoM to the lab frame. Moreover, this limitation is only relevant at parton level where the kinematics are exact ($A = 0$ and $B = 1$ signal planar events). Any finite resolution in the events would deviate the values of the variables from a planar configuration despite the events being inherently planar. Thus, we choose our bins such that they are not too finely grained or too broad.

6.3.1 Correlation Between Aplanarity and Biplanarity

We investigate the relationship between A and B using spherical $2 \rightarrow 4$ massless partonic events generated in the CoM frame according to Appendix C. Figure 6.14 shows the two-dimensional histogram of biplanarity as a function of aplanarity. The event density is concentrated at small A and large B , demonstrating a clear inverse association between the two observables; i.e. they are correlated, with small A correlating to large B . This is

consistent with our definitions of the variables: low A indicates a more planar event which should (and does) translate to a large B (as a planar event should be marked planar in both cases). As A increases, the distribution shifts toward smaller values of B , indicating that events farther from planarity according to A also tend to have lower biplanarity. The sparsely populated bins at larger A should be interpreted cautiously due to their limited event counts. Although A and B are correlated, we retain both observables in the following analysis to compare how their different definitions characterize planarity.

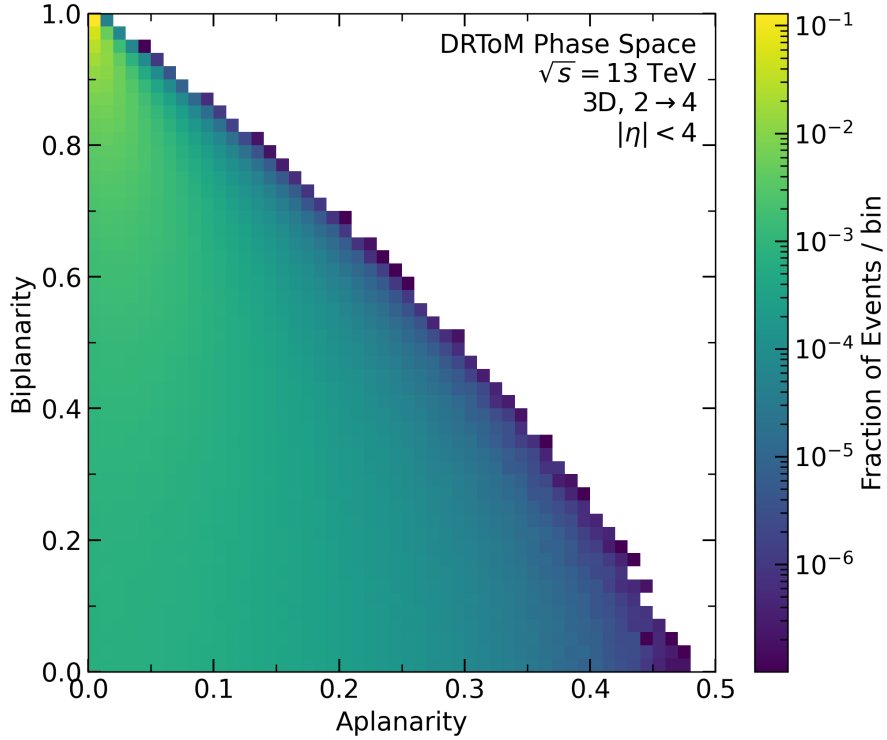


Figure 6.14: Correlation of aplanarity and biplanarity on a 2D histogram.

6.4 Planarity as a Function of Event Invariant Mass

With the choice of aplanarity and biplanarity as the variables of interest, we now ask: how do the number of planar events (that of the $0 \leq A < 0.01$ bin and $0.98 < B \leq 1$ bin) change as a function of the event invariant mass? The event invariant mass acts as the energy scale for each event allowing us to relate the topology of the event directly to the energy. By setting a Λ_3 scale, we know that events with invariant mass less than Λ_3

will behave as spherical whereas above Λ_3 they must be planar.

As we increase from the mass lower bound of 2 TeV up to 11 TeV, we calculate the fraction of events in the planar bin over the total count in all bins. The results for the CoM and lab frame are shown in Fig. 6.15. The fully spherical event generation is the case when $\Lambda_3 = 11$ TeV (as we choose an upper bound at 11 TeV). This is the SM representation. The fully planar generation is when $\Lambda_3 = 2$ TeV. We show the case for nine different Λ_3 values, where in reality (in data) only one Λ_3 could be present.

Let us consider the case where $\Lambda_3 = 10$ TeV. In the CoM frame, from 2-10 TeV invariant mass the fraction of planar events matches the SM. This is to be expected since below an invariant mass of 10 TeV the events have not undergone any change in dimensionality and we are using the same phase space configuration in all generations. At an invariant mass of about 10 TeV, however, the fraction rises to 1. Thus, after the $\Lambda_3 = 10$ TeV scale is passed, all events are planar, as expected. In the lab frame, the same rise occurs at $\Lambda_3 = 10$ TeV, but the planar fraction does not rise to 1. In comparing A and B , we note that the biplanarity distribution is broader (the fraction starts at 0.1 compared to 0.3) and is hence more sensitive to the topological effects of DR.

From these figures we see that in the CoM frame the number of planar events for the SM (a fully spherical generation, when $\Lambda_3 = 11$ TeV) stays approximately constant across the mass range. That is, as we generate more events at higher energies, there is no preference for an increase or decrease in the fraction of planar events. Introducing dimensional reduction, $\Lambda_3 \in [2, 11)$ TeV, we see that (for each case) as the event invariant mass increases, the number of events in the planar bins sharply rise until all the events become planar at the Λ_3 scale, exhibiting a step-like behaviour. This is to be expected because all events after Λ_3 must be planar, and hence must lie in the planar bin. Then, if we look at the full planar case ($\Lambda_3 = 2$ TeV) we see that the fraction starts exactly at 1. We do see this same step-like trend in the lab frame, but the rise is not as dominant (it does not go up to 1) and the distributions themselves are no longer flat. Although there are statistical fluctuations in our generation, we do not place any uncertainty into the bins as we are considering this as an idealistic representation.

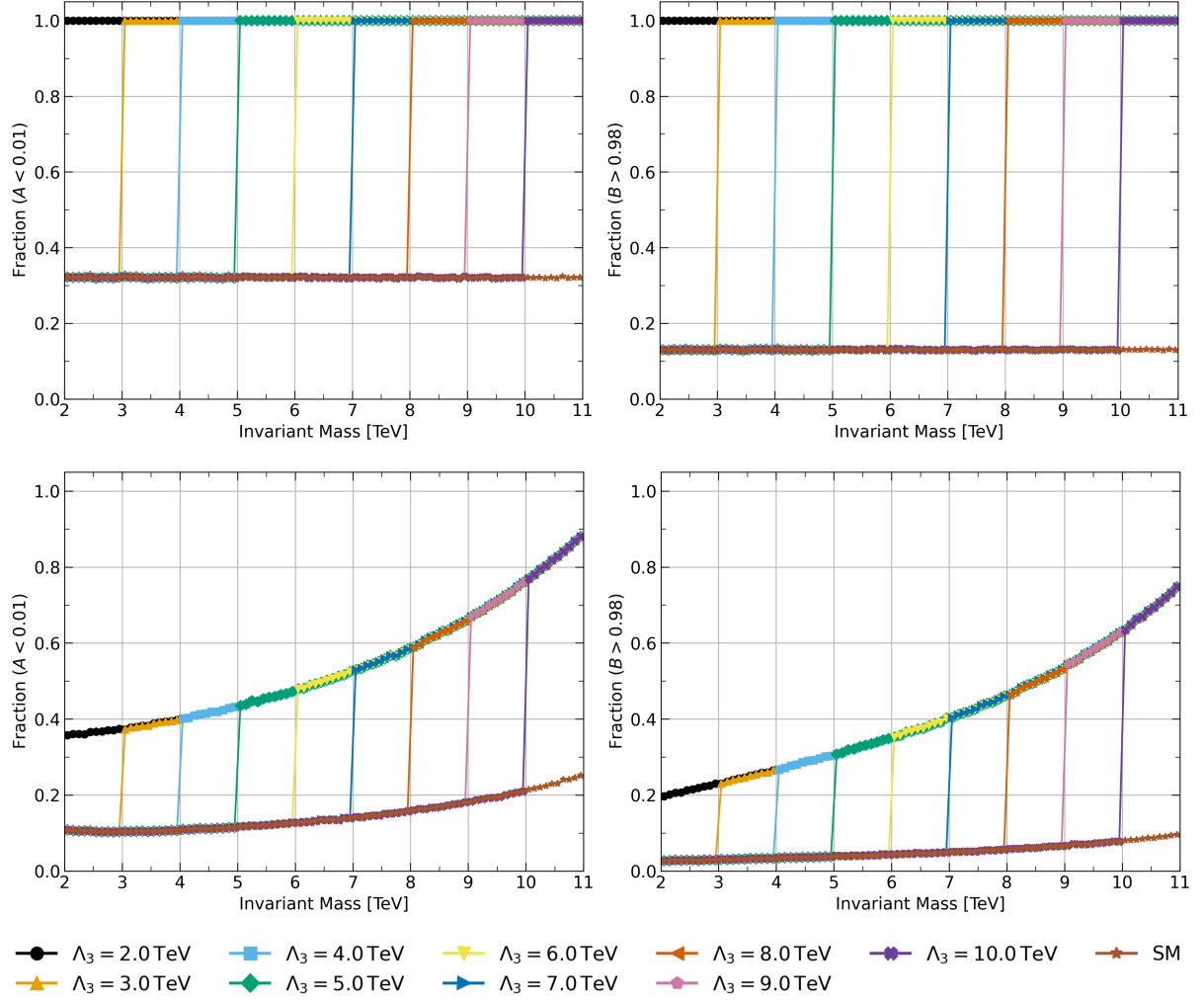


Figure 6.15: Aplanarity (left) and biplanarity (right) planar bins as a function of invariant mass in the CoM frame (top) and lab frame (bottom) for various Λ_3 scales. We consider $2 \rightarrow 4$ massless partonic events following the partonic generation parameters of Appendix C. The Standard Model (SM) is $\Lambda_3 = 11$ TeV by construction. There are no error bars. We plot the fractions every 0.1 TeV.

Regardless of the frame, aplanarity and biplanarity exhibit the same tendency, again proving their reliability as dual variables of interest. We consider biplanarity in the CoM frame, due to its more sensitive nature of DR, as the discriminating variable. What's important here is not necessarily the specific behaviour of the dimensionally reduced events, but that there is a clear difference in the behaviour between fully spherical events (the SM) and those that have undergone dimensional reduction. While the fraction of planar events stay constant throughout the SM, the fraction of planar events in dimensional reduction do not. Because of this, planarity vs. event invariant mass will be another variable relationship used to investigate DR moving forward.

Chapter 7

Particle Level Event Simulation

Up to this point we have investigated the observable effects of DR (in kinematic plots and event shape variables) at the parton level using idealized fully spherical and planar $2 \rightarrow 4$ massless phase space events. From our discussion on jets in Chapter 3 we know that proton-proton collisions produce gluons and quarks that must obey confinement. Thus, after a collision they undergo showering and hadronization to form collimated groupings of particles called jets. Experimentally, then, quarks and gluons manifest as these collimated groupings rather than individual particles. This means that our topological event shape variables from the Chapter 6 are not applied to individual partons in an event but to jets after an appropriate jet finding algorithm (which may have more or less jets than partons). For example, if we consider biplanarity, we would no longer be using four outgoing partons, but instead the four leading outgoing jets which each have their own four-momentum and invariant mass. Our notion of dimensional reduction effecting the dimensionality of an event does not change, we simply need to shift our analysis technique from partons to jets.

7.1 Jet Final States

We outline how we propagate from partonic states into jet-like states for analysis. The ATLAS software framework, Athena, provides the infrastructure for processing simulated events and is accessed through transformation scripts [52]. A Data Set Identifier is assigned to each mass slice and then the LHE files—formatted according to the AT-

LAS evgen standard—are processed through the `Gen_tf` transformation. This step interfaces with Pythia 8, a high-energy Standard Model event generator, to perform parton showering (adds initial- and final-state QCD radiation), hadronization (converts coloured partons into colourless hadrons), and hadron decays (transform unstable hadrons into final-state particles, including leptons and photon) [53]. The resulting fully simulated particle-level events are stored in EVNT files, which contain the complete generator record, including all intermediate particles and their detailed history. The MC generation statistics used in this chapter is given in Appendix C.

As the EVNT files are large and contain more information than required for analysis, they are further processed within Athena using the `Derivation_tf` transformation to produce DAOD-TRUTH3 datasets. This derivation step applies a predefined set of algorithms to retain only relevant truth-level information such as final-state particles, and event weights, while discarding unnecessary details from the full event record. The resulting datasets also include truth jets reconstructed with the FastJet package [54]. No detector simulation is performed at this stage; rather, the event data is restructured and reduced to a more compact, analysis-friendly format. The resulting outputs are ROOT files used in subsequent analysis.

7.1.1 Jet Finding Algorithm

A jet finding algorithm is utilized to group and define what a jet is for the purposes of reconstructing the properties of the original quarks and gluons. It runs on particle four-vectors and not on detector objects. The algorithm starts by introducing a distance d_{ij} between objects i and j (e.g. particles and/or pseudojets), as well as a distance d_{iB} between entity i and the beam, B . The inclusive clustering proceeds by identifying the smaller of the distances (of either d_{ij} or d_{iB}) and giving them a label. If the smaller of the distances is d_{ij} , then i and j are recombined and put back into the entity list, whereas if d_{iB} is the smallest, then i is removed from the list of entities and called a jet. The distances are then recalculated and the procedure repeats until there are no entities left

in the list. The distance measures are given by,

$$d_{ij} = \min(p_{T,i}^{2k}, p_{T,j}^{2k}) \frac{\Delta_{ij}^2}{R^2}, \quad (7.1)$$

$$d_{iB} = p_{T,i}^{2k}, \quad (7.2)$$

where $\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$ with $p_{T,i}$, y_i , and ϕ_i as the transverse momentum, rapidity, and azimuth of particle i , respectively. Additionally, R is a radius parameter (chosen to be 0.4), and k governs the relative power of the energy versus geometrical (Δ_{ij}) scales. The anti- k_t algorithm is used by ATLAS where $k = -1$ [55].

7.2 Jet Migration

As we move to jet level analysis, we are no longer dealing with simple $2 \rightarrow n$, $n \in [2, 5]$, output partonic processes. Instead, our outputs give us a range of jets along with leptons, photons, and missing energy in neutrinos. For example, Fig. 7.1 shows the normalized jet multiplicity distribution after a spherical $2 \rightarrow 4$ massless partonic generation following Appendix C was run through hadronization. At this stage there are additional requirements placed on the jets. The first jet must satisfy $p_T > 480$ GeV, and every other jet in the event must satisfy $p_T > 20$ GeV¹. The way we convey the individual jet p_T requirements in text is [480, 20, 20, 20] (or $p_T > [480, 20, 20, 20]$ GeV, as seen on plots), where the first number indicates the leading jet requirement, the second number for the second jet, and so on (in units of GeV). We also apply a pseudorapidity requirement of $|\eta| < 2.5$ on all jets to match the inner detector range. With these requirements, we see that the distribution ranges from 2 to 19 jets, with the most probable value at 5.

The number of reconstructed jets does not necessarily correspond one-to-one with the four outgoing partons produced in the hard-scattering process. Events containing more than four jets may arise from additional QCD radiation during the parton shower, while events containing fewer than four jets can result from jets falling below the transverse-momentum threshold, lying outside the detector acceptance, or being merged by the

¹The leading p_T requirement of 480 GeV was suggested by Zhijie Li and Jerry Wu.

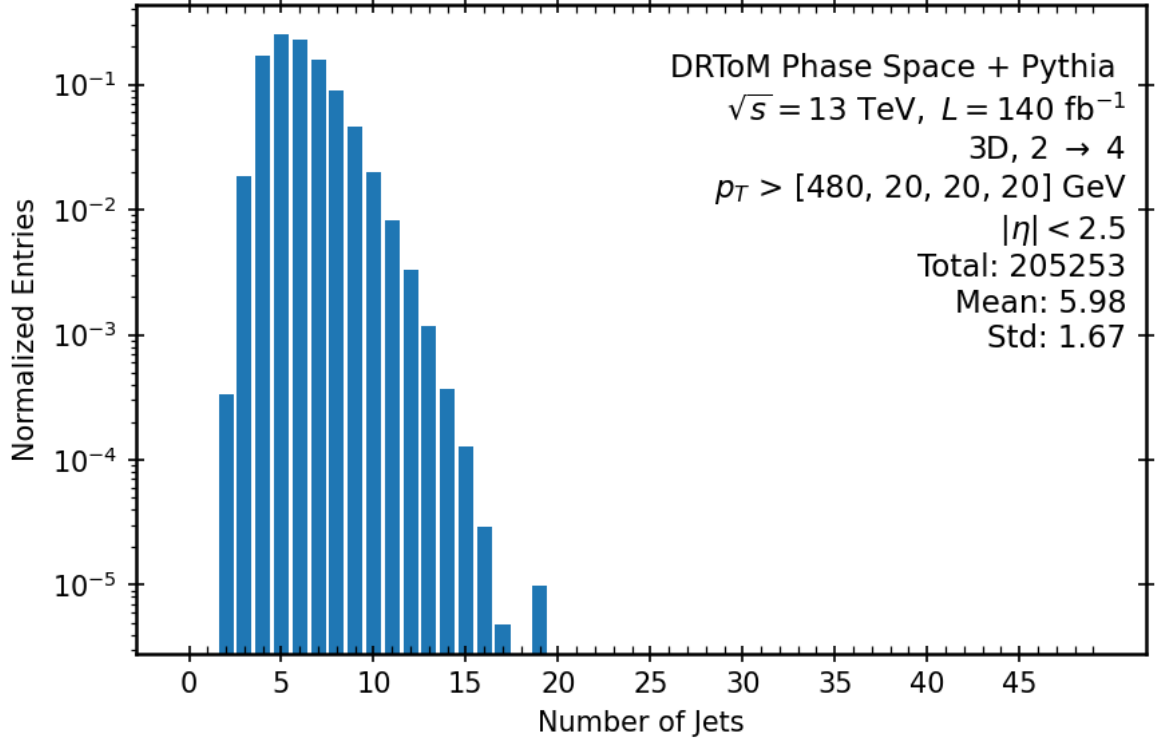


Figure 7.1: Jet distribution of a $2 \rightarrow 4$ spherical partonic event after hadronization. The plot is normalized to 1. The most probable value is 5.

jet-clustering algorithm. This range/deviation in jet multiplicity from four partons is problematic as we want to ensure that the outputs we are using in analysis are direct products from hard scattering. That is, if we are investigating a $2 \rightarrow n$ -partonic process, then we know the n outgoing quarks or gluons must form n direct jets that we want to use. Any other jet is a byproduct that can skew our variables, and as such we do not want to incorporate these jets in the analysis (or, ideally, we could correct for them or have an adjusted jet-algorithm). This is important for $2 \rightarrow 4$ -partonic events because both $2 \rightarrow 2$ and $2 \rightarrow 3$ partonic events can give 4 jets that are not real $2 \rightarrow 4$ partonic outputs (i.e. they are background, which we want to eliminate).

In order to inquire if $2 \rightarrow n$ -partonic processes stay as n -jet events, we perform a jet migration analysis. As DRTOM deals with $2 \rightarrow 2$ up to $2 \rightarrow 5$ partonic massless phase space events, we must consider requirements on the first five jets of an event. We only generate spherical events and order the jets from largest to smallest p_T before applying any requirement. Given our generation requirements from Appendix C, there is a parton

level event invariant mass requirement of 2-11 TeV. Because we are looking at events with up to 5 jets, we choose to apply the fourth jet p_T requirement to the fifth jet (and every other jet in the event as well). Since 480 GeV is the requirement on the leading jet, the highest combination we consider is a p_T requirement of [480, 480, 480, 480] (where, again, the final 480 GeV is applied to the fifth and all other jets). The lowest combination we consider is a p_T requirement of [480, 20, 20, 20], where 20 GeV is required to exclude pileup from multiple interactions which we do not include in the simulation.

To study jet migration, we use migration matrices. In Fig. 7.2 we show the case of $p_T > [480, 80, 80, 80]$. The columns show the simulated parton event information, they are the n in our $2 \rightarrow n$, $n \in [2, 5]$, generation. We will call this the input. The rows correspond to how many jets, m , are in the event after hadronization followed by the anti- k_t algorithm on the parton four-vectors. We will call this the output. There are two values in each box. The left indicates the percentage after normalizing per row, and the right is the percentage after normalizing per column. As such, the left number gives the likelihood that an m -jet output corresponds to an n -parton input, and the right is the percent value that an n -parton input gives an m -jet output. We want to minimize the 4-jet outputs that are tied to 2- and 3-parton inputs while maximizing the 4-jet outputs that corresponds to 4-parton inputs. Thus the colour scale of the heat map is tied to the left value, $P(n\text{-input} | m\text{-output})$, where we fix jet multiplicity, m , and see what input parton multiplicity, n , it corresponds to (the left value). We note that this migration matrix is based on all event masses and is likely to vary with event invariant mass.

We show the case of $p_T > [480, 80, 80, 80]$ as it is the first set of requirements with a $P(4 | 4)$ percent above 50 and we can see that the largest values are concentrated along the diagonal migration matrix elements (except for $P(5 | 5)$). Figure 7.3 shows the normalized jet distributions (the right value in the correlation matrix) of each parton input multiplicity. The mean jet multiplicity is 2.4 for $2 \rightarrow 2$, 3.8 for $2 \rightarrow 3$, 4.6 for $2 \rightarrow 4$, and 5.8 for $2 \rightarrow 5$ which means our 4-jet events are more likely to correlate to events with 4-parton events. We note that despite the mean values centring on their parton outputs, the distributions still range from 2 up to 12 jets (process dependent).

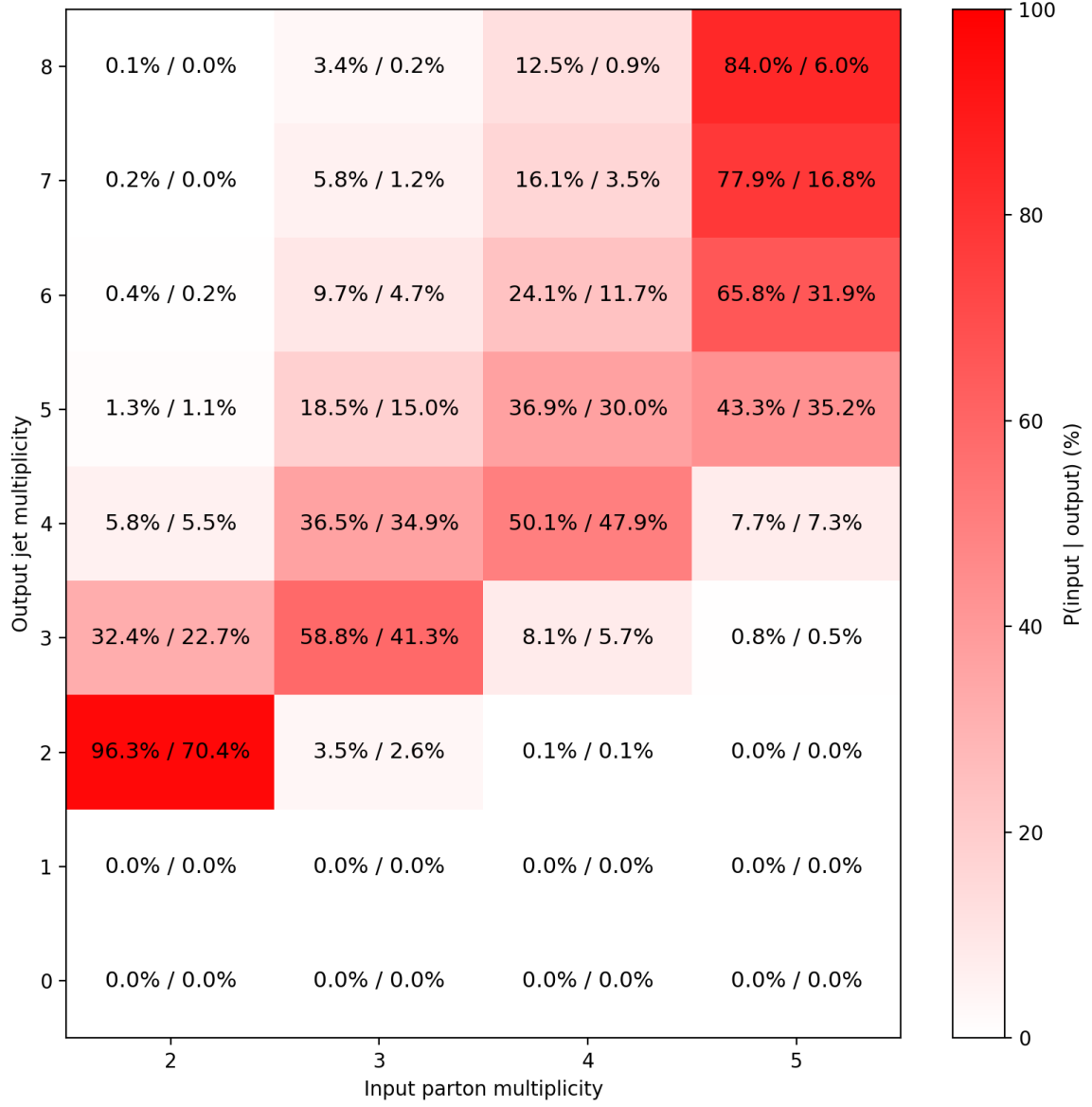


Figure 7.2: Jet migration matrix with $p_T > [480, 80, 80, 80]$.

Also relevant to the choice of requirements is the most probable value (i.e. the bin with the most events). In each case we see that the most probable value directly aligns with the parton process: n for $2 \rightarrow n$, where $n \in [2, 5]$.

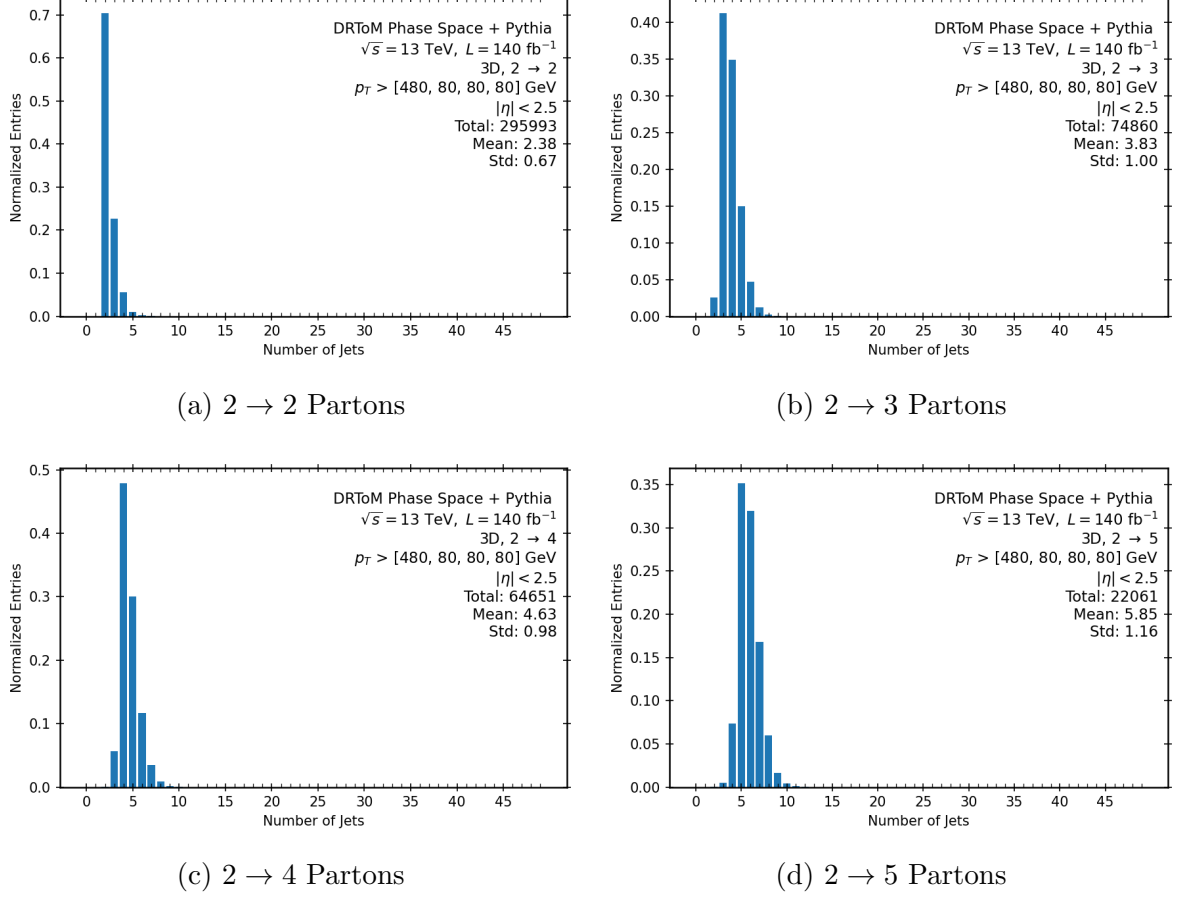


Figure 7.3: Jet multiplicity distributions for $p_T > [480, 80, 80, 80]$.

7.2.1 Jet Structure in Data

We want to estimate how many 4-jet outputs come from $2 \rightarrow 2, 3, 4, 5$ parton inputs in data. We utilize the cross section of $2 \rightarrow 2, 3, 4, 5$ processes from MadGraph, σ_{MG} ². We take the row values of $P(n | 4)$ of Fig. 7.2 and multiply it with σ_{MG} to get a weighted representation of how 4-jets are constructed from data. The results are in Table 7.1.

What we see is that the number of expected events for 2- and 3-parton collisions is about an order of magnitude larger than for 4-parton collisions. For these specific p_T

²The corresponding MadGraph cross section samples were calculated by Jerry Wu. They were generated using 90,000 events (which are not sliced by scalar sums of p_T) for each case. They should be considered preliminary values.

requirements, this therefore predicts that 4-jet events in data will be mostly be made of 2- and 3-parton events with 4-jet events from 4-parton events the least abundant (which is not ideal).

Table 7.1: Jet migration structure of 4-jet events from data.

	n=2	n=3	n=4	n=5
$P(n 4)$	0.058	0.365	0.501	0.433
$\sigma_{\text{MG}} \text{ (fb)}$	9.43×10^{10}	5.50×10^9	1.01×10^9	N/A
$P(n 4) \sigma_{\text{MG}} \text{ (fb)}$	5.47×10^9	2.01×10^9	0.51×10^9	N/A

7.2.2 Jet Momenta Requirements

We find that as we increase the subleading (all jets except the first) jet p_T requirement (from 20 GeV to higher values), the diagonal migration matrix values become more probable. What we are interested in is the requirement that reduces the 2-,3- partonic input to 4-jet output contamination while retaining the most 4-parton to 4-jet output. However, the cost of increasing the p_T requirements comes at reducing the event statistics available for further analysis. Table 7.2 shows the percent values of 2-parton to 4-jet contamination, $P(2 | 4)$, 3-parton to 4-jet contamination, $P(3 | 4)$, 4-parton to 4-jet events, and the resulting total number of (unweighted, as this represents the MC statistics we have) $2 \rightarrow 4$ spherical events (that span the generation mass range) after applying various chosen requirements, $N_{2 \rightarrow 4}$.

Table 7.2: Event statistics based on the jet p_T requirements.

Jet p_T Requirements (GeV)	$P(2 4) \text{ (%)}$	$P(3 4) \text{ (%)}$	$P(4 4) \text{ (%)}$	$N_{2 \rightarrow 4}$
[480, 20, 20, 20]	29.9	39.9	27.4	205,253
[480, 80, 80, 80]	5.9	36.5	50.1	64,651
[480, 140, 140, 140]	2.4	29.8	57.3	47,871
[480, 200, 200, 200]	1.3	25.0	61.2	39,919

The ideal requirement would result in no contamination from 2-parton and 3-parton

collisions, and a $P(4 | 4)$ of 100% with no decrease in MC statistics. The leading requirement of $p_T > 480$ GeV already reduces our event pool by roughly 98% (as we started with 9×10^6 $N_{2 \rightarrow 4}$ events). As we increase the requirements on the subleading jets, the diagonal migration matrix elements becomes more inline with our ideal case, but the number of events is reduced by an order of magnitude (from $[480, 20, 20, 20]$ to $[480, 80, 80, 80]$), and steadily drops after that (by definition, it cannot rise). For our case, although a higher p_T requirement better approximates 4-parton events, we will use the requirement of $p_T > [480, 20, 20, 20]$ to ensure we have enough statistics to make meaningful predictions. We note that the leading p_T requirement does not need to be set at 480 GeV. We could test cases with values larger than this, however we choose to keep this requirement as increasing it will reduce the number of events available even further.

This jet migration study, and hence the p_T requirements, were obtained by considering massless $2 \rightarrow 4$ partonic spherical events generated between 2-11 TeV. As a next step, we could treat these p_T requirements as a function of the invariant mass scale. Since the invariant mass correlates to the overall topology of an event (that of spherical or planar) within a generation range, we could set exact p_T configuration requirements on the orientations of jets to more efficiently isolate proper 4-parton events. Then, the overall energy of an event would come with it's own set of p_T requirements based on the allowed topologies at that energy scale instead of a universal requirement. This would allow for a more precise investigation as to the behaviour of jets for each invariant mass scale.

7.3 Event Invariant Mass Requirement

Although the jet p_T requirements can reduce the overall jet distributions (centring them on their parton equivalents), we saw in Fig. 7.3 that there is still a distribution of jets ranging from 2 to 12 depending on the process. Moreover, since we are using the p_T requirements of $[480, 20, 20, 20]$, we know from Fig. 7.1 that the $2 \rightarrow 4$ jet distribution is from 2 to 19 with a mean of about 6. Because of this, we apply a mass requirement.

We could consider the total invariant mass of the event. This takes all objects (all the

jets, leptons, and photons) and calculates the total invariant mass of the system. Instead, we utilize the leading 4-jet invariant mass, M_{4j} , where leading indicates p_T ordered jets. This is chosen for two reasons. The first, for our study we only want to investigate jets and thus want to ignore the mass of any other outgoing particle types in the event. The second, we are requiring that our signal events have at least 4 jets in them as, following our discussion in Sec 3.4, this is the baseline number of jets needed to distinguish between planar and spherical topologies. Specifically, biplanarity only uses the leading 4-jets and therefore the topology of a collision is solely stored (for our purposes) in the first four jets. This choice also eliminates events with less than 4 jets which arise from poor reconstruction and as such are events we do not want to consider.

7.3.1 Weighting and Scale Factor

Because each QCD process has a different cross section, the inclusive cross section within a given invariant mass interval is obtained by summing the contributions from all processes. Events in each mass interval are then assigned the appropriate cross section and luminosity weight factor, allowing the simulated distributions to represent the expected event yields in data, such as the ATLAS dataset, rather than the number of generated MC events. After jet reconstruction there is a mass migration, in which the invariant mass of the interval may fall outside its original generation range. The weight factor must therefore be applied to the event in its original mass configuration, and not its new mass.

The weight factor is dependent on three elements. The first is the associated cross section of the generation slice, σ_{Slice} , the second is the integrated luminosity recorded by ATLAS for a standard trigger, L (in our case 140 fb^{-1}), and the final is the total number of events for that slice, N_{Slice} . All together, the weight factors are given by:

$$w_{\text{Slice}} = \frac{\sigma_{\text{Slice}} L}{N_{\text{Slice}}}. \quad (7.3)$$

This gives a slice weight for events at a particular luminosity. If we remove L , then we get the slice weight of the production cross section.

If we know how many events are in real ATLAS data for a specific set of jet requirements, N_{Data} , then we can normalize our weighted MC events to this data using the identical jet requirements. Letting N_w represents the total generator events after hadronization and weighting by w , the scaling factor, S_F , is simply

$$N_{\text{Data}} = S_F \cdot N_w. \quad (7.4)$$

An ideal scaling factor would be ~ 1 as this would mean our generator produces the same number of events as seen in data. It does not mean, however, that the distribution is identical to that of data. If the scale factor is not 1, then we are rescaling the area under our generated curves while retaining their original shapes.

From data, there are $N_{\text{Data}} \sim 2.8 \times 10^7$ 4-jet events ³. This value was obtained by using Run 2 data with a p_T requirement of 480 GeV on the leading jet, 200 GeV on the subleading jet, and 70 GeV on the third and fourth jet, as well as a leading jet pseudorapidity requirement of $|\eta| < 3$, and a scalar sum of the 4-jets $p_T > 1$ TeV. Applying the Run 2 jet requirements to our weighted events, we get a total of 5.6×10^8 events. This gives a scale factor of 0.05, which is far from the ideal value of 1.

After scaling our MC generation first by w then S_F to match the number of events in data (over the entire mass range, not per slice), in order to get proper data-like histograms in the low count regions (the tails) we must round the fractional counts as counts in data must be integers. In total we can consider plots without any weights (raw event counts), the weighted plots (w applied), and the data-normalized plots (w , S_F , and rounding applied). Appendix H has biplanarity and leading 4-jet invariant mass distributions for all three variants of plots. It also outlines the statistical uncertainty estimate used in this chapter.

The leading jet $p_T > 480$ GeV requirement must be kept in our analysis as it gives a 100% trigger efficiency for single-jet triggers in Run 2 data ⁴. We could increase the leading p_T requirement, but following our discussion in Sec 7.2 we will keep it at 480 GeV.

³This was given by Zhijie Li and should be considered as an approximation.

⁴Suggested by Zhijie Li and Jerry Wu. The choice of the [200, 70, 70] GeV requirements for the subleading jets were used to reduce pileup effects in data.

All other requirements (the p_T requirements on other jets, the η requirements on the jets, and the scalar sum of p_T for the four leading jets) are free variables. We will apply an $|\eta| < 2.5$ requirement on all jets such that we only retain jets that stay inside the inner detector range, and we will remove the p_T scalar sum requirement since we are only generating events with $M_{4j} \geq 2$ TeV.

7.3.2 Leading 4-Jet Invariant Mass Distribution

For our jet migration analysis we considered the effects of $2 \rightarrow 2, 3, 4, 5$ parton events. Moving forward, our analysis will be conducted solely on $2 \rightarrow 4$ parton events following the generation of Appendix C. We do not consider 4-jet background from $2 \rightarrow 2, 3$ parton processes.

After applying our kinematic requirements ($p_T > [480, 20, 20, 20]$, $|\eta| < 2.5$), weighing the events by w , applying the scale factor S_F , and rounding fractional events, we get the distribution of M_{4j} shown in Fig. 7.4 for $2 \rightarrow 4$ spherical (top) and planar (bottom) events. In both cases the distribution ranges from 1 TeV to about 10 TeV. We note that the $M_{4j} < 2$ TeV events are the results of poorly reconstructed jets and will not be used. At high invariant masses, the event yield decreases rapidly; for example, only approximately 20 events remain near 8 TeV. This is insufficient to populate the regions of the A and B distribution used (which have a total of 50 bins). In particular, approximately 10 events are required in the narrow planar region for statistical significance. Thus we require at least a total yield of approximately 100 events. The event yield falls below this threshold at $\sim M_{4j} = 7$ TeV. We therefore restrict the analysis to $M_{4j} \in [2, 7]$ TeV.

7.4 Planarity Features

We consider the effects of using jets on aplanarity and biplanarity, and the planar bins of A and B as a function of leading 4-jet event invariant mass for both fully spherical and planar jet events.

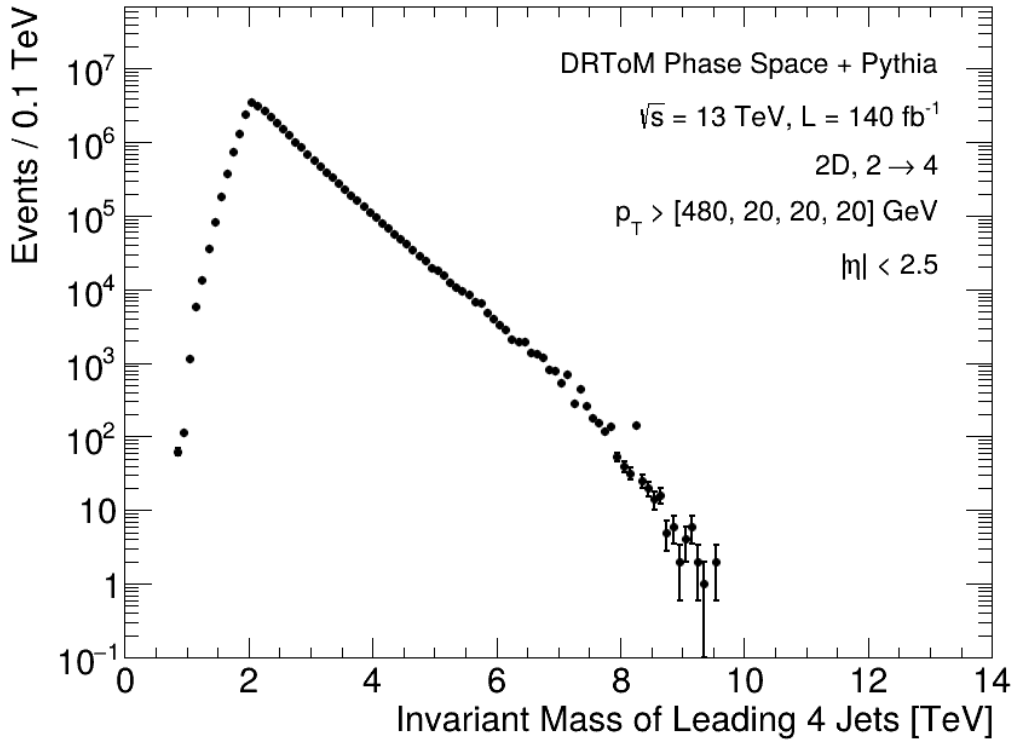
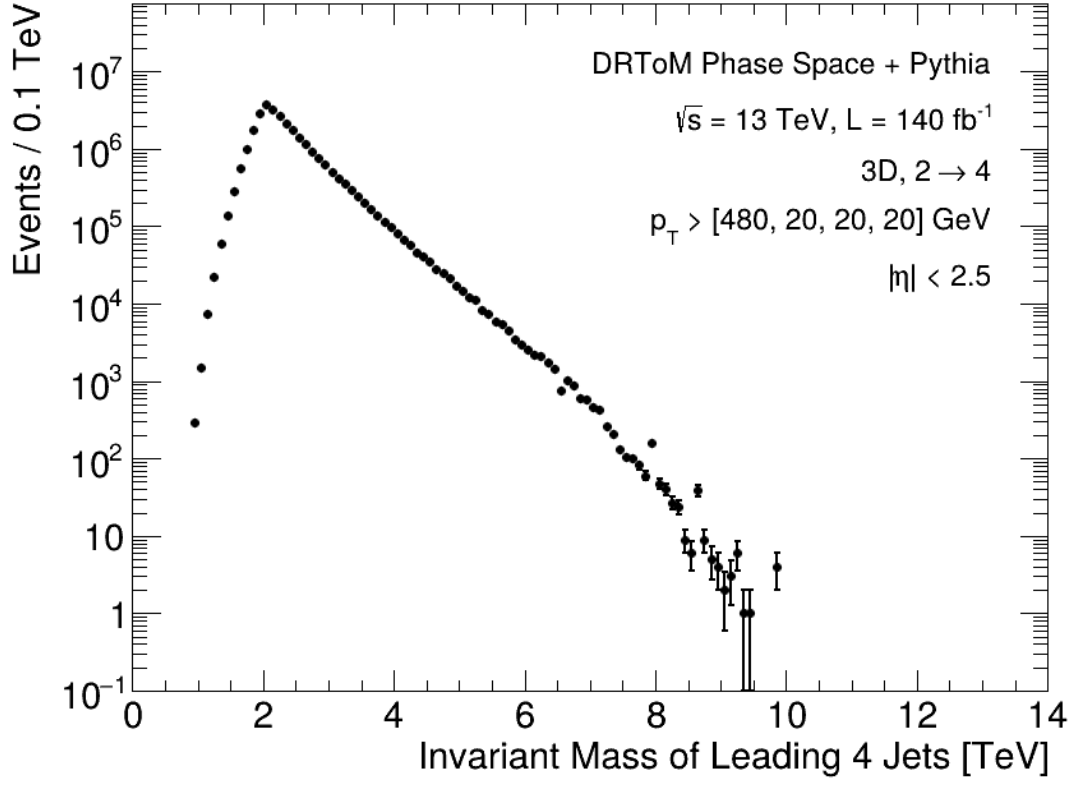


Figure 7.4: Leading 4-jet invariant mass distributions normalized to data for fully spherical (top) and fully planar (bottom) events.

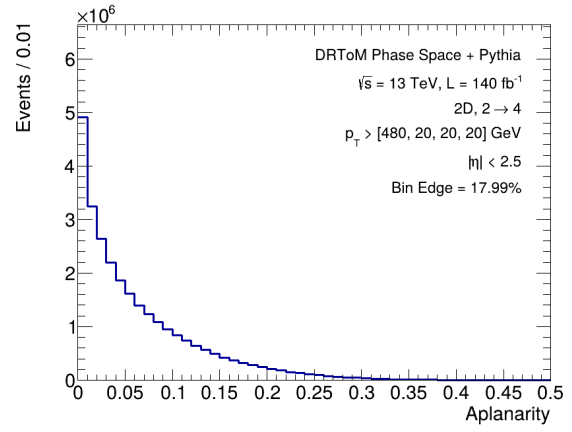
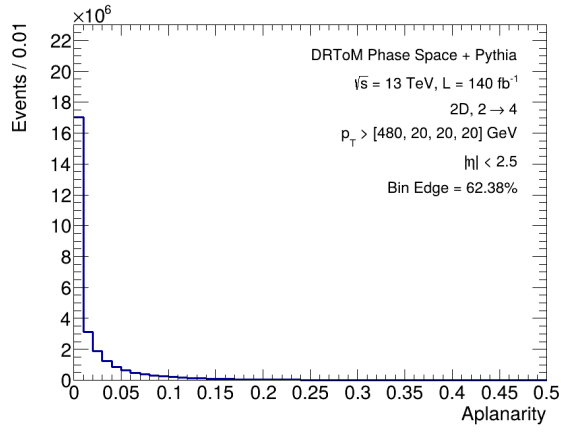
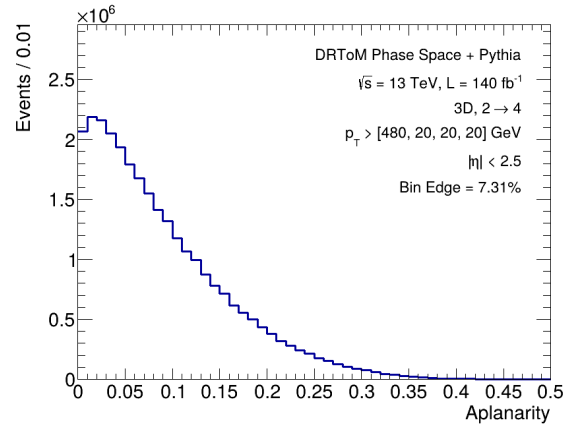
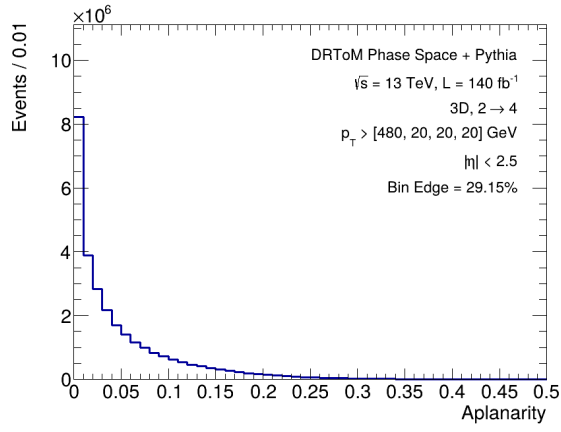
7.4.1 Aplanarity and Biplanarity Distributions

Starting with the spherical event generation, we see that the aplanarity and biplanarity (top rows of Fig. 7.5 and Fig. 7.6, respectively) distributions are similar in shape/behaviour to those of parton level in both frames (there is a distribution over the full domain with a peak in the planar bin). This is because at parton level most events in the generation were fully spherical, and as such after hadronization those events remain spherical. What we do see is that the planar bins (the first bin for A and last bin for B) are reduced/smeared out in comparison to before. For aplanarity, at parton level the spherical events peaked at $\sim 33\%$ in the planar bin for the CoM frame and 15% in the lab frame (see Fig. 6.10). Here they are at 30% and 7% , respectively. The spherical events of biplanarity peaked at about 13% in the CoM frame and 5% in the lab frame (see Fig. 6.13), whereas here they are at about 6% and 2% , respectively.

Looking at the planar event generation, we notice a drastic difference in the aplanarity and biplanarity (bottom rows of Fig. 7.5 and Fig. 7.6, respectively) CoM frame distributions. Where at parton level all events would be in the planar bin, that is no longer the case with jets; the smearing effect is exemplified as about 30% of planar events are now registered as spherical for aplanarity and 70% for biplanarity. The exact predictive nature of the CoM frame has naturally weakened as the fully planar events at parton level are no longer reconstructed as planar at jet level. Before this effect was only seen at parton level when boosting from the CoM frame to the lab frame. In this regard, we also see that the aplanarity and biplanarity distributions in the lab frame with jets are again more smeared. For aplanarity, at parton level in the lab frame the planar events peaked at about 55% . For biplanarity, the peak was at about 40% . At jet-level, these values dropped by about 40% for aplanarity and 35% for biplanarity.

We note an additional feature in A and B that we did not see at parton level. For spherical events in the lab frame (the top right plot in their respective figures), the distributions have a dip in their respective planar bin instead of a peak. These are distinct lab frame features that could be further utilized to differentiate between spherical and

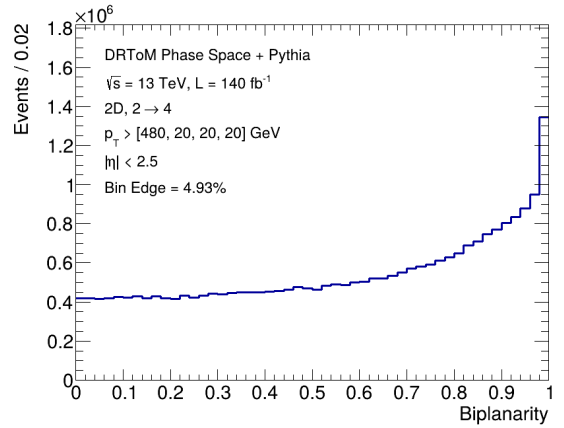
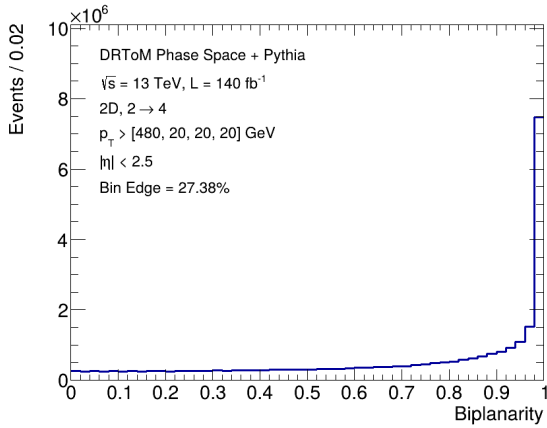
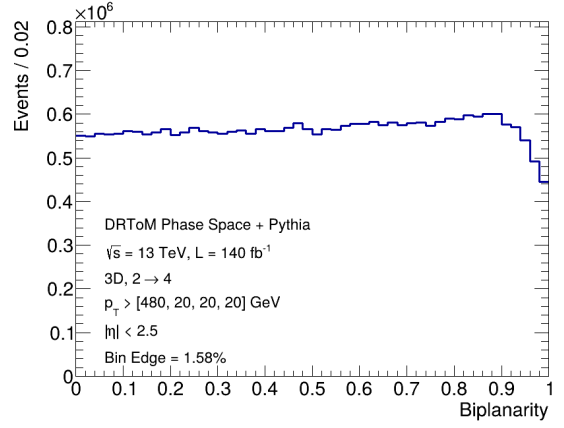
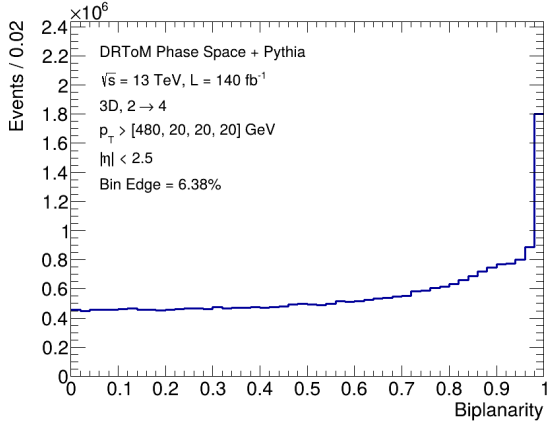
planar events. Moreover, if we take the planar distribution to be signal and the spherical distribution to be background, we can investigate the signal-to-background ratio of the planar bin. Starting with aplanarity, $0 \leq A < 0.01$, the CoM frame has about 17×10^6 planar events for the signal distribution, and about 8×10^6 planar events in the background distribution. This gives a signal-to-background ratio of $17/8 \sim 2$. In the lab frame, the values are about 5×10^6 and 2×10^6 , respectively, for a signal-to-background ratio of about 2.5. This tells us that the lab frame has a better delineation between signal and background for aplanarity. For biplanarity, in the CoM frame the signal planar bin has about 7.5×10^6 events in the $0.98 < B \leq 1$ bin, and a background value of 1.8×10^6 , for a signal-to-background value of 4. In the lab frame, the values are 1.35×10^6 and 0.45×10^6 , respectively, for a signal-to-background value of 3. Then, if considering the biplanarity variable, we see that the CoM frame is the better choice for distinguishing between signal and background planar events.



(a) CoM Frame

(b) Lab Frame

Figure 7.5: Aplanarity distributions of spherical (top row) and planar (bottom row) $2 \rightarrow 4$ jet events. The left column plots are in the CoM frame, and the right column plots are in the lab frame.



(a) CoM Frame

(b) Lab Frame

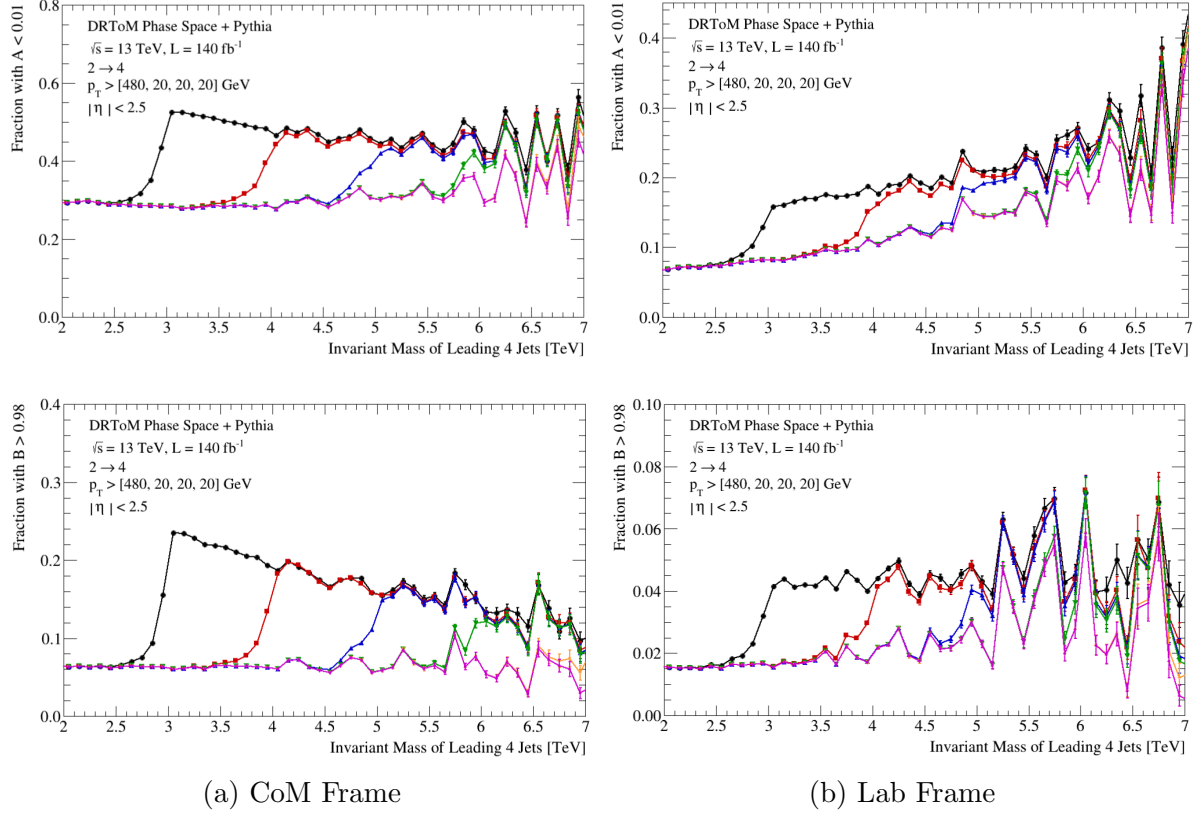
Figure 7.6: Biplanarity distributions of spherical (top row) and planar (bottom row) $2 \rightarrow 4$ jet events. The left column plots are in the CoM frame, and the right column plots are in the lab frame.

7.4.2 Planarity as a Function of 4-Jet Invariant Mass

As we did in Sec. 6.4, we now investigate how the planar bins of aplanarity ($0 \leq A < 0.01$) and biplanarity ($0.98 < B \leq 1.0$) behave as a function of the leading 4-jet invariant mass. This is shown in Fig. 7.7.

Given our discussion in Sec. 7.3.2, we only consider $M_{4j} \in [2, 7]$ TeV. We also only choose the integer cases of $\Lambda_3 \in [3, 7]$ TeV to investigate the transition from spherical to planar, as well as the SM generation. In comparing to the planarity vs. event invariant mass plot at parton level, in the CoM frame (left column), low Λ_3 values do have the characteristic step-like behaviour to suggest an increase in planar events. However, the increase in fractional planar events does not go up to 1. For aplanarity, the $\Lambda_3 = 3$ TeV distribution starts at about 0.3 and goes to 0.5 with the 0.5 peak happening at $\Lambda_3 = 3$ TeV. It then stays at 0.5 out until 5.5 TeV in which the low event counts produce fluctuations. For the SM, the distribution starts at about 0.3 and stays at this fraction up until about 5.5 TeV after which it starts to fluctuate. For biplanarity we see the same trend. The $\Lambda_3 = 3$ TeV distribution starts at about 0.06 and increases to a peak fraction of 0.24 at $\Lambda_3 = 3$ TeV. However, instead of staying constant, the fraction has a steady decrease throughout the rest of the range with fluctuations happening past 5.5 TeV. In the CoM, then, both aplanarity and biplanarity exhibit the same step-like behaviour and fractional increase (they both have an increase of about 0.2). Biplanarity has the advantage that it behaves better in the high mass region and should therefore be considered the discriminating variable at jet-level.

For the lab frame, for both aplanarity and biplanarity, we see that there is a visible notion of step-like distributions. After about 5 TeV, though, the fluctuations become large and the curves become difficult to differentiate. For this reason, the low Λ_3 values, those of $\Lambda_3 \approx 3, 4$ TeV, can be aided by the lab frame distributions, but the lab frame should not be used as the main frame of analysis.



● $\Lambda_3 = 3.0$ TeV
 ■ $\Lambda_3 = 4.0$ TeV
 ▲ $\Lambda_3 = 5.0$ TeV
 ▼ $\Lambda_3 = 6.0$ TeV
 ★ $\Lambda_3 = 7.0$ TeV
 ◆ SM

Figure 7.7: Planarity as a function of the leading 4-jet invariant mass. The top plots are aplanarity ($0 \leq A < 0.01$), and the bottom plots biplanarity ($0.98 < B \leq 1$). The left column plots are in the CoM frame and the right column plots in the lab frame. The statistical uncertainty used for this is outlined in Appendix H. By construction, $\Lambda_3 = 11$ TeV is the Standard Model (SM).

7.5 Jet Level Analysis Results

At jet level, the predictive power of DR is diminished relative to the idealized parton-level case. The jet migration study demonstrated that transverse momentum requirements must be imposed on the jets to improve the agreement between the partonic and jet multiplicities, although an exact correspondence could not be achieved. Increasing the p_T thresholds improved this agreement but substantially reduced the number of events. Consequently, relatively low thresholds were required to retain sufficient statistics, meaning that many of the selected events did not represent ideal 4-parton to 4-jet hard-scattering configurations. This limitation carried through to the aplanarity and biplanarity distributions. For fully planar CoM events, the planar bins were fully populated at parton level. At jet level, however, the populations of these characteristic planar bins decreased by approximately 30% for A and 70% for B . This migration reflects the smearing of the idealized partonic geometry during jet reconstruction.

The limited events due to the p_T requirements also restricted the leading 4-jet invariant mass analysis to $\Lambda_3 \in [2, 7]$ TeV despite the generated mass range extending from 2 to 11 TeV and the highest data event at about 10.5 TeV. In the CoM frame, biplanarity provided the stronger discrimination between planar and spherical events as a function of the leading 4-jet invariant mass. The planar samples exhibited pronounced enhancements across the investigated mass range. Although these enhancements did not reach 1, they remained clearly distinguishable from the SM. Similar behaviour was observed for aplanarity, although the separation was less pronounced. In the lab frame, noticeable enhancements were primarily observed at the lower DR scales, $\Lambda_3 = 3$ and 4 TeV, beyond which the discriminating power was substantially reduced.

Overall, aplanarity and biplanarity are less effective indicators of event geometry at jet level than at parton levels. Despite this, there are clear differences between the SM and DR samples. Most importantly, the dependence of these observables on the leading 4-jet invariant mass retains sufficient sensitivity to distinguish between different DR scales.

Chapter 8

Summary

The Standard Model of particle physics describes three of the four fundamental forces as quantum field theories that exist on a flat spacetime. General relativity describes gravity, the fourth fundamental force, as the curvature of spacetime. These two theories, which offer the most comprehensive description of physics, do not have an accepted unified description. Dimensional reduction is one such solution to this problem. It suggests that GR and the SM are proper theories in a $(3+1)$ -spacetime, however they cannot be unified as $(3+1)$ theories as the fundamental geometry of the Universe is lower dimensional. At high energies, spacetime should undergo a transition from $(3+1)$ to $(2+1)$ (and possibility from $(2+1)$ to $(1+1)$) dictated by energy scales Λ . If the energy scale for a transition from a $(3+1)$ - to $(2+1)$ -spacetime is on the order of a TeV, $\Lambda_3 \sim \text{TeV}$, this effect could be observable at the LHC.

Utilizing tree-level QCD, we developed a simplified Monte Carlo simulation event generator (DRToM) for massless $2 \rightarrow n$ partonic processes, $n \in [2, 5]$, that produces both spherical and planar phase space configurations. Considering parton-parton collisions in the energy range of 2–11 TeV at a proton-proton CoM collision energy of 13 TeV, we first investigated the effects of parton-level dimensional reduction. We found that the event shape variables aplanarity and biplanarity offered the most discrimination between spherical and planar events, where, in the CoM frame, planar events would be fully confined to a single bin ($0 \leq A < 0.01$ and $0.98 < B \leq 1$). We then considered the fraction of planar events as a function of event invariant mass for varying Λ_3 scales. In the CoM frame we found that fully spherical events stay at a constant fraction of about

0.3 and 0.15 (for A and B , respectively) across the mass scale, whereas generations that have undergone DR (irregardless of the choice of Λ_3) exhibited an enhancement from the spherical distribution up to all events being planar at the respective Λ_3 value.

With the idealized parton level kinematics investigated, we extended the study to truth-level jets. Applying the Run 2 data requirements of $p_T > [480, 200, 70, 70]$, a leading jet requirement of $|\eta| < 3$, a scalar sum of $p_T > 1$ TeV, and an integrated luminosity of 140 fb^{-1} , we found that the scale factor for DRToM is 0.05. Switching to the requirement of $p_T > [480, 20, 20, 20]$ and $|\eta| < 2.5$ for all jets after an independent jet migration analysis, we used data-normalized distributions of aplanarity and biplanarity and found that the sensitivity to DR effects significantly degraded. There was a smearing effect in which planar events at parton level were no longer planar at jet level. Specifically, we saw the CoM planar bin reduce by 30% for aplanarity, and 65% for biplanarity. This effect carried to the planar vs. leading 4-jet invariant mass plots in which the distinctive step function behaviour was present, but it did not rise up to 1. Due to limited events, we could only reliably evaluate the effects of DR out to about 7 TeV, but, despite this, we clearly saw a difference between the SM and DR in the CoM frame for both A and B .

While dimensional reduction produces clear signatures at parton level, these effects are significantly diluted when considering jets, posing a major challenge for experimental observation. To improve this analysis there are several things that can be done. As a preliminary step, we can increase the number of generated events in the high mass region so that we are not limited by statistics. As is, we are using requirements with a $P(4 | 4)$ value of 27% and only about 200,000 events. These events are then smeared resulting in even less planar statistics. If we generated MC events that initially pass a leading p_T requirement of 480 GeV, this could extend our statistics and offer a more accurate depiction of DR effects. A part of this issue also lies in the global scale factor. A value of 0.05 suggests that there is a possible misconfiguration of DRToM (in the QCD cross sections themselves) and should be investigated further. Another reason for this poor scale factor could be due to our comparison methods. Right now we are comparing the cross sections of tree-level QCD from DRToM with $2 \rightarrow 4$ QCD events from data. For

a proper comparison, it may be better to use DRToM cross sections against $2 \rightarrow 2$ data events. Regardless, a scale factor closer to 1 would naturally improve the resolution of the high mass region.

Beyond statistics, additional requirements to p_T , η , and mass, such as the angles between jets and the energy of the jets, could be used to further refine the selection of events. In tandem, we could improve the jet finding algorithm and broaden the usage of particle outputs from the events (i.e. instead of using just jets, consider the photons, leptons, and neutrinos of the event as well). Furthermore, studies on the sensitivity of the planar variables (A and B) and M_{4j} should be conducted to determine where the low sensitivity of DR is coming from. We note that the use of M_{4j} should also be refined. Instead of using any event with 4-jets, we should only consider 4-jet events where the 4-jet invariant mass is similar to the total invariant mass of the event such that the energy scale of the collision aligns more closely with notion of Λ . To improve event shape topology classification at jet level, a first step would be to perform more systematic studies on other event shape observables as they may exhibit a more stable transition from parton- to jet-level events. Once the $2 \rightarrow 4$ analysis is exhausted, as a next step one could incorporate $2 \rightarrow 2$, $2 \rightarrow 3$, and $2 \rightarrow 5$ events to run background studies and see how the event shape plots change. The final step would be to quantify the limits of DR.

Bibliography

- [1] S Carlip. “Dimension and dimensional reduction in quantum gravity”. In: *Class. Quantum Gravity* 34.19 (Sept. 2017), p. 193001. ISSN: 1361-6382. DOI: [10.1088/1361-6382/aa8535](https://doi.org/10.1088/1361-6382/aa8535). URL: <http://dx.doi.org/10.1088/1361-6382/aa8535>.
- [2] Luis A. Anchordoqui et al. “Searching for the layered structure of space at the LHC”. In: *Phys. Rev. D* 83.11 (June 2011). ISSN: 1550-2368. DOI: [10.1103/PhysRevD.83.114046](https://doi.org/10.1103/PhysRevD.83.114046). URL: <http://dx.doi.org/10.1103/PhysRevD.83.114046>.
- [3] Jonas Mureika and Dejan Stojkovic. “Detecting Vanishing Dimensions via Primordial Gravitational Wave Astronomy”. In: *Phys. Rev. Lett* 106.10 (Mar. 2011). ISSN: 1079-7114. DOI: [10.1103/PhysRevLett.106.101101](https://doi.org/10.1103/PhysRevLett.106.101101). URL: <http://dx.doi.org/10.1103/PhysRevLett.106.101101>.
- [4] Dejan Stojkovic. “Vanishing dimensions: theory and phenomenology”. In: *Rom. J. Phys.* 57 (2012). Ed. by Luis Alvarez-Gaume et al., pp. 992–1001. arXiv: [1304.6444](https://arxiv.org/abs/1304.6444) [hep-th].
- [5] Luis Anchordoqui et al. “Vanishing Dimensions and Planar Events at the LHC”. In: *Mod. Phys. Lett. A* 27.04 (Feb. 2012), p. 1250021. ISSN: 1793-6632. DOI: [10.1142/S0217732312500216](https://doi.org/10.1142/S0217732312500216). URL: <http://dx.doi.org/10.1142/S0217732312500216>.
- [6] A. B. Arbuzov. “Quantum Field Theory and the Electroweak Standard Model”. In: (2017). Ed. by Martijn Mulders and Giulia Zanderighi, pp. 1–34. DOI: [10.23730/CYRSP-2017-004.1](https://doi.org/10.23730/CYRSP-2017-004.1). arXiv: [1801.05670](https://arxiv.org/abs/1801.05670) [hep-ph].
- [7] David J Griffiths. *Introduction to elementary particles*. Physics textbook. New York, NY: Wiley, 2008. URL: <https://cds.cern.ch/record/111880>.
- [8] Julia Woithe, Gerfried J Wiener, and Frederik F Van der Veken. “Let’s have a coffee with the Standard Model of particle physics!” In: *Phys. Educ.* 52.3 (Mar. 2017), p. 034001. DOI: [10.1088/1361-6552/aa5b25](https://doi.org/10.1088/1361-6552/aa5b25). URL: <https://dx.doi.org/10.1088/1361-6552/aa5b25>.
- [9] R. L. Workman et al. “Review of Particle Physics”. In: *Prog. Theor. Exp. Phys.* 2022 (2022), p. 083C01. DOI: [10.1093/ptep/ptac097](https://doi.org/10.1093/ptep/ptac097).
- [10] Sean M. Carroll. *Spacetime and Geometry: An Introduction to General Relativity*. Cornwall, United Kingdom: Cambridge University Press, 2019.
- [11] Hans C. Ohanian. *Gravitation and Spacetime*. New York City, New York: Norton, 1976.
- [12] Carlo Rovelli. *Notes for a brief history of quantum gravity*. Presented at the 9th Marcel Grossmann Meeting in Roma, July 2000. 2001. arXiv: [gr-qc/0006061](https://arxiv.org/abs/gr-qc/0006061) [gr-qc].
- [13] Carlo Rovelli. *Strings, loops and others: a critical survey of the present approaches to quantum gravity*. Plenary lecture on quantum gravity at the GR15 conference, Pune, India. 1998. arXiv: [gr-qc/9803024](https://arxiv.org/abs/gr-qc/9803024) [gr-qc].

- [14] A. Banfi. *Hadronic jets: An introduction*. San Rafael, CA, USA: Morgan & Claypool Publishers and Institute of Physics, 2016. DOI: [10.1088/978-1-6817-4073-7](https://doi.org/10.1088/978-1-6817-4073-7).
- [15] C.Y. Wong. *Introduction to High-energy Heavy-ion Collisions*. Introduction to High-energy Heavy-ion Collisions. World Scientific, 1994. ISBN: 9789810202637. URL: <https://books.google.ca/books?id=Fnxvrdj2NOQC>.
- [16] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to quantum field theory*. Reading, USA: Addison-Wesley, 1995. ISBN: 978-0-201-50397-5. DOI: [10.1201/9780429503559](https://doi.org/10.1201/9780429503559).
- [17] John Campbell, Joey Huston, and Frank Krauss. *The Black Book of Quantum Chromodynamics : a Primer for the LHC Era*. Oxford University Press, 2018. ISBN: 978-0-19-965274-7. DOI: [10.1093/oso/9780199652747.001.0001](https://doi.org/10.1093/oso/9780199652747.001.0001).
- [18] Alan D. Martin. *Proton structure, Partons, QCD, DGLAP and beyond*. 2008. arXiv: [0802.0161 \[hep-ph\]](https://arxiv.org/abs/0802.0161). URL: <https://arxiv.org/abs/0802.0161>.
- [19] Ringaile Placakyte. *Parton Distribution Functions*. 2011. arXiv: [1111.5452 \[hep-ph\]](https://arxiv.org/abs/1111.5452). URL: <https://arxiv.org/abs/1111.5452>.
- [20] Davison E. Soper. “Parton distribution functions”. In: *Nucl. Phys. B Proc. Suppl.* 53.1–3 (Feb. 1997), pp. 69–80. ISSN: 0920-5632. DOI: [10.1016/S0920-5632\(96\)00600-7](https://doi.org/10.1016/S0920-5632(96)00600-7). URL: [http://dx.doi.org/10.1016/S0920-5632\(96\)00600-7](http://dx.doi.org/10.1016/S0920-5632(96)00600-7).
- [21] Pavel M. Nadolsky et al. “Implications of CTEQ global analysis for collider observables”. In: *Phys. Rev. D* 78 (1 July 2008), p. 013004. DOI: [10.1103/PhysRevD.78.013004](https://doi.org/10.1103/PhysRevD.78.013004). URL: <https://link.aps.org/doi/10.1103/PhysRevD.78.013004>.
- [22] B. L. Combridge, J. Kripfganz, and J. Ranft. “Hadron Production at Large Transverse Momentum and QCD”. In: *Phys. Lett. B* 70 (1977), p. 234. DOI: [10.1016/0370-2693\(77\)90528-7](https://doi.org/10.1016/0370-2693(77)90528-7).
- [23] E. Eichten et al. “Supercollider physics”. In: *Rev. Mod. Phys.* 56 (4 Oct. 1984), pp. 579–707. DOI: [10.1103/RevModPhys.56.579](https://doi.org/10.1103/RevModPhys.56.579). URL: <https://link.aps.org/doi/10.1103/RevModPhys.56.579>.
- [24] Stefan Höche. *Introduction to parton-shower event generators*. 2015. arXiv: [1411.4085 \[hep-ph\]](https://arxiv.org/abs/1411.4085). URL: <https://arxiv.org/abs/1411.4085>.
- [25] Manami Hashi and Noriaki Kitazawa. “Signatures of low-scale string models at the LHC”. In: *J. High Energy Phys* 2012.2 (Feb. 2012). ISSN: 1029-8479. DOI: [10.1007/JHEP02\(2012\)050](https://doi.org/10.1007/JHEP02(2012)050). URL: [http://dx.doi.org/10.1007/JHEP02\(2012\)050](http://dx.doi.org/10.1007/JHEP02(2012)050).
- [26] E. Eichten et al. “Supercollider physics”. In: *Rev. Mod. Phys.* 56 (4 Oct. 1984), pp. 579–707. DOI: [10.1103/RevModPhys.56.579](https://doi.org/10.1103/RevModPhys.56.579). URL: <https://link.aps.org/doi/10.1103/RevModPhys.56.579>.
- [27] Luis A. Anchordoqui et al. “String resonances at hadron colliders”. In: *Phys. Rev. D* 90 (6 Sept. 2014), p. 066013. DOI: [10.1103/PhysRevD.90.066013](https://doi.org/10.1103/PhysRevD.90.066013). URL: <https://link.aps.org/doi/10.1103/PhysRevD.90.066013>.
- [28] Andrea Banfi. *Hadronic Jets (Second Edition)*. 2053-2563. IOP Publishing, 2022. ISBN: 978-0-7503-4737-2. DOI: [10.1088/978-0-7503-4737-2](https://doi.org/10.1088/978-0-7503-4737-2). URL: <https://doi.org/10.1088/978-0-7503-4737-2>.

- [29] Aad Aabot et al. “The ATLAS experiment at the CERN Large Hadron Collider: a description of the detector configuration for Run 3”. In: *J. Instrum.* 19.05 (May 2024), P05063. ISSN: 1748-0221. DOI: [10.1088/1748-0221/19/05/p05063](https://doi.org/10.1088/1748-0221/19/05/p05063). URL: <http://dx.doi.org/10.1088/1748-0221/19/05/P05063>.
- [30] Aad Aabot et al. “The ATLAS Experiment at the CERN Large Hadron Collider”. In: *J. Instrum.* 3.08 (Aug. 2008), S08003. DOI: [10.1088/1748-0221/3/08/S08003](https://doi.org/10.1088/1748-0221/3/08/S08003). URL: <https://dx.doi.org/10.1088/1748-0221/3/08/S08003>.
- [31] S. Chatrchyan et al. “The CMS Experiment at the CERN LHC”. In: *J. Instrum.* 3 (2008), S08004. DOI: [10.1088/1748-0221/3/08/S08004](https://doi.org/10.1088/1748-0221/3/08/S08004).
- [32] K. Aamodt et al. “The ALICE experiment at the CERN LHC”. In: *J. Instrum.* 3 (2008), S08002. DOI: [10.1088/1748-0221/3/08/S08002](https://doi.org/10.1088/1748-0221/3/08/S08002).
- [33] A. Augusto Alves Jr. et al. “The LHCb Detector at the LHC”. In: *J. Instrum.* 3 (2008), S08005. DOI: [10.1088/1748-0221/3/08/S08005](https://doi.org/10.1088/1748-0221/3/08/S08005).
- [34] Ewa Lopienska. *The CERN accelerator complex, layout in 2022. Complexe des accélérateurs du CERN en janvier 2022*. Ed. by N/A. General Photo. 2022. URL: <https://cds.cern.ch/record/2800984>.
- [35] The ATLAS collaboration. “Operation of the ATLAS trigger system in Run 2”. In: *J. Instrum.* 15.10 (Oct. 2020), P10004–P10004. ISSN: 1748-0221. DOI: [10.1088/1748-0221/15/10/p10004](https://doi.org/10.1088/1748-0221/15/10/p10004). URL: <http://dx.doi.org/10.1088/1748-0221/15/10/P10004>.
- [36] Eija Tuominen and {CMS Collaboration}. “Performance of the CMS Level-1 trigger in proton-proton collisions at $\sqrt{s} = 13$ TeV”. English. In: *J. Instrum.* 15.10 (Oct. 2020). ISSN: 1748-0221. DOI: [10.1088/1748-0221/15/10/P10017](https://doi.org/10.1088/1748-0221/15/10/P10017).
- [37] A. Hayrapetyan et al. “Development of the CMS detector for the CERN LHC Run 3”. In: *J. Instrum.* 19.05 (May 2024), P05064. ISSN: 1748-0221. DOI: [10.1088/1748-0221/19/05/p05064](https://doi.org/10.1088/1748-0221/19/05/p05064). URL: <http://dx.doi.org/10.1088/1748-0221/19/05/P05064>.
- [38] Oliver Brüning and Markus Zerlauth. *LHC operation and the High-Luminosity LHC upgrade project*. 2025. arXiv: [2505.03535](https://arxiv.org/abs/2505.03535) [physics.acc-ph]. URL: <https://arxiv.org/abs/2505.03535>.
- [39] ATLAS and CMS Collaborations. *Highlights of the HL-LHC physics projections by ATLAS and CMS*. 2025. arXiv: [2504.00672](https://arxiv.org/abs/2504.00672) [hep-ex]. URL: <https://arxiv.org/abs/2504.00672>.
- [40] Heather M. Gray. *TASI 2022 lectures on LHC experiments*. Ed. by Heather M. Gray. 2023. arXiv: [2307.01894](https://arxiv.org/abs/2307.01894) [hep-ex]. URL: <https://arxiv.org/abs/2307.01894>.
- [41] R. L. Workman et al. “Review of Particle Physics”. In: *Prog. Theor. Exp. Phys.* 2022 (2022), p. 083C01. DOI: [10.1093/ptep/ptac097](https://doi.org/10.1093/ptep/ptac097).
- [42] B.J. Holzer. “Introduction to Particle Accelerators and their Limitations”. en. In: *CERN Yellow Reports* (2016), Vol 1 (2016): Proceedings of the 2014 CAS–CERN Accelerator School: Plasma Wake Acceleration. DOI: [10.5170/CERN-2016-001.29](https://doi.org/10.5170/CERN-2016-001.29). URL: <https://e-publishing.cern.ch/index.php/CYR/article/view/212>.

- [43] Dirk P. Kroese, Thomas Taimre, and Zdravko I. Botev. *Handbook of Monte Carlo Methods*. Wiley Series in Probability and Statistics. John Wiley & Sons, 2011. ISBN: 9780470177938.
- [44] Pourya Vakilipourtakalou and Douglas M. Gingrich. *Monte Carlo event generator for the production and decay of string resonances in proton-proton collisions*. 2018. arXiv: [1811.07458 \[hep-ph\]](https://arxiv.org/abs/1811.07458). URL: <https://arxiv.org/abs/1811.07458>.
- [45] Johan Alwall et al. “MadGraph 5: going beyond”. In: *J. High Energy Phys* 2011.6 (June 2011). ISSN: 1029-8479. DOI: [10.1007/jhep06\(2011\)128](https://doi.org/10.1007/jhep06(2011)128). URL: [http://dx.doi.org/10.1007/JHEP06\(2011\)128](http://dx.doi.org/10.1007/JHEP06(2011)128).
- [46] R Kleiss, W.J Stirling, and S.D Ellis. “A new Monte Carlo treatment of multi-particle phase space at high energies”. In: *Comput. Phys. Commun.* 40.2 (1986), pp. 359–373. ISSN: 0010-4655.
- [47] Douglas M. Gingrich. *Computational Methods for Physics in Python*. N/A, 2024.
- [48] J. Alwall et al. “A standard format for Les Houches Event Files”. In: *Comput. Phys. Commun.* 176.4 (Feb. 2007), pp. 300–304. ISSN: 0010-4655. DOI: [10.1016/j.cpc.2006.11.010](https://doi.org/10.1016/j.cpc.2006.11.010). URL: <http://dx.doi.org/10.1016/j.cpc.2006.11.010>.
- [49] Andy Buckley et al. “LHAPDF6: parton density access in the LHC precision era”. In: *Eur. Phys. J. C* 75.3 (Mar. 2015). ISSN: 1434-6052. DOI: [10.1140/epjc/s10052-015-3318-8](https://doi.org/10.1140/epjc/s10052-015-3318-8). URL: <http://dx.doi.org/10.1140/epjc/s10052-015-3318-8>.
- [50] G. Aad et al. “Measurement of hadronic event shapes in high-pT multijet final states at $\sqrt{s} = 13$ TeV with the ATLAS detector”. In: *J. High Energy Phys* 2021.1 (2021). ISSN: 1029-8479. DOI: [10.1007/jhep01\(2021\)188](https://doi.org/10.1007/jhep01(2021)188). URL: [http://dx.doi.org/10.1007/JHEP01\(2021\)188](http://dx.doi.org/10.1007/JHEP01(2021)188).
- [51] Matteo Cacciari. *A thrust to trust minimum thrust*. 2025. arXiv: [2512.04244 \[hep-ph\]](https://arxiv.org/abs/2512.04244). URL: <https://arxiv.org/abs/2512.04244>.
- [52] ATLAS Collaboration. *Athena*. Version 21.0.127. May 2021. DOI: [10.5281/zenodo.4772550](https://doi.org/10.5281/zenodo.4772550). URL: <https://doi.org/10.5281/zenodo.4772550>.
- [53] Christian Bierlich et al. *A comprehensive guide to the physics and usage of PYTHIA 8.3*. 2022. arXiv: [2203.11601 \[hep-ph\]](https://arxiv.org/abs/2203.11601). URL: <https://arxiv.org/abs/2203.11601>.
- [54] Matteo Cacciari, Gavin P. Salam, and Gregory Soyez. “FastJet user manual: (for version 3.0.2)”. In: *The European Physical Journal C* 72.3 (Mar. 2012). ISSN: 1434-6052. DOI: [10.1140/epjc/s10052-012-1896-2](https://doi.org/10.1140/epjc/s10052-012-1896-2). URL: <http://dx.doi.org/10.1140/epjc/s10052-012-1896-2>.
- [55] Matteo Cacciari, Gavin P. Salam, and Gregory Soyez. “The anti-kt jet clustering algorithm”. In: *J. High Energy Phys.* 2008.04 (Apr. 2008), p. 063. DOI: [10.1088/1126-6708/2008/04/063](https://doi.org/10.1088/1126-6708/2008/04/063). URL: <https://doi.org/10.1088/1126-6708/2008/04/063>.
- [56] Diego Casadei. *How to measure efficiency*. 2022. arXiv: [0908.0130 \[physics.data-an\]](https://arxiv.org/abs/0908.0130). URL: <https://arxiv.org/abs/0908.0130>.
- [57] T. Ullrich and Z. Xu. *Treatment of Errors in Efficiency Calculations*. 2012. arXiv: [physics/0701199 \[physics.data-an\]](https://arxiv.org/abs/physics/0701199). URL: <https://arxiv.org/abs/physics/0701199>.

- [58] Vinod Nair and Geoffrey E. Hinton. “Rectified linear units improve restricted boltzmann machines”. In: *Proceedings of the 27th International Conference on International Conference on Machine Learning*. ICML’10. Haifa, Israel: Omnipress, 2010, pp. 807–814. ISBN: 9781605589077.
- [59] Abien Fred Agarap. *Deep Learning using Rectified Linear Units (ReLU)*. 2026. arXiv: [1803.08375](https://arxiv.org/abs/1803.08375) [cs.NE]. URL: <https://arxiv.org/abs/1803.08375>.
- [60] Jason Ansel et al. “PyTorch 2: Faster Machine Learning Through Dynamic Python Bytecode Transformation and Graph Compilation”. In: *Proceedings of the 29th ACM International Conference on Architectural Support for Programming Languages and Operating Systems, Volume 2*. ASPLOS ’24. La Jolla, CA, USA: Association for Computing Machinery, 2024, pp. 929–947. ISBN: 9798400703850. DOI: [10.1145/3620665.3640366](https://doi.org/10.1145/3620665.3640366). URL: <https://doi.org/10.1145/3620665.3640366>.
- [61] Diederik P. Kingma and Jimmy Ba. *Adam: A Method for Stochastic Optimization*. 2017. arXiv: [1412.6980](https://arxiv.org/abs/1412.6980) [cs.LG]. URL: <https://arxiv.org/abs/1412.6980>.

Appendix A

DRTToM Collision Tables

In this appendix we show the additional interactions possible by DRTToM. Table A.1 shows the possible $2 \rightarrow 3$ processes and Table A.2 shows the possible $2 \rightarrow 5$ processes given each QCD tree-level process. A g indicates a gluon, a q a quark, a $\bar{q}$ an antiquark; a prime denotes a quark that is of a different flavour from q , and a double prime indicates a quark that is of a different flavour than q and q' . The tree-level process are in Table 3.1 in Chapter 3, and the $2 \rightarrow 4$ processes are in Table 5.1 in Chapter 5.

Table A.1: DRToM allowed $2 \rightarrow 3$ partonic processes.

Input ($2 \rightarrow 2$)	Possible output ($2 \rightarrow 3$)
$gg \rightarrow gg$	$gg \rightarrow ggg, \quad gg \rightarrow q\bar{q}g$
$gg \rightarrow q\bar{q}$	$gg \rightarrow q\bar{q}g$
$gq \rightarrow gq$	$gq \rightarrow gqg, \quad gq \rightarrow q\bar{q}q, \quad gq \rightarrow q'\bar{q}'q$
$g\bar{q} \rightarrow g\bar{q}$	$g\bar{q} \rightarrow g\bar{q}g, \quad g\bar{q} \rightarrow q\bar{q}\bar{q}, \quad g\bar{q} \rightarrow q'\bar{q}'\bar{q}$
$qq \rightarrow qq$	$qq \rightarrow qqg$
$qq' \rightarrow qq'$	$qq' \rightarrow qq'g$
$\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}$	$\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}g$
$\bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'$	$\bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'g$
$q\bar{q}' \rightarrow q\bar{q}'$	$q\bar{q}' \rightarrow q\bar{q}'g$
$q\bar{q} \rightarrow gg$	$q\bar{q} \rightarrow ggg, \quad q\bar{q} \rightarrow q\bar{q}g, \quad q\bar{q} \rightarrow q'\bar{q}'g$
$q\bar{q} \rightarrow q\bar{q}$	$q\bar{q} \rightarrow q\bar{q}g$
$q\bar{q} \rightarrow q'\bar{q}'$	$q\bar{q} \rightarrow q'\bar{q}'g$

Table A.2: DRToM allowed $2 \rightarrow 5$ partonic processes.

Input ($2 \rightarrow 2$)	Possible output ($2 \rightarrow 5$)
$gg \rightarrow gg$	$gg \rightarrow gggq\bar{q}, \quad gg \rightarrow q\bar{q}q\bar{q}g, \quad gg \rightarrow q\bar{q}q'\bar{q}'g$
$gg \rightarrow q\bar{q}$	$gg \rightarrow gggq\bar{q}, \quad gg \rightarrow q\bar{q}q\bar{q}g, \quad gg \rightarrow q\bar{q}q'\bar{q}'g$
$gq \rightarrow gq$	$gq \rightarrow gqggg, \quad gq \rightarrow gq q\bar{q}g, \quad gq \rightarrow gq q'\bar{q}'g, \quad gq \rightarrow q\bar{q}q\bar{q}q,$ $gq \rightarrow q\bar{q} q'\bar{q}'q, \quad gq \rightarrow q'\bar{q}' q'\bar{q}'q, \quad gq \rightarrow q'\bar{q}' q''\bar{q}''q$
$g\bar{q} \rightarrow g\bar{q}$	$g\bar{q} \rightarrow g\bar{q}ggg, \quad g\bar{q} \rightarrow g\bar{q} q\bar{q}g, \quad g\bar{q} \rightarrow g\bar{q} q'\bar{q}'g, \quad g\bar{q} \rightarrow q\bar{q}q\bar{q}\bar{q},$ $g\bar{q} \rightarrow q\bar{q} q'\bar{q}'\bar{q}, \quad g\bar{q} \rightarrow q'\bar{q}' q'\bar{q}'\bar{q}, \quad g\bar{q} \rightarrow q'\bar{q}' q''\bar{q}''\bar{q}$
$qq \rightarrow qq$	$qq \rightarrow qqggg, \quad qq \rightarrow qq q\bar{q}g, \quad qq \rightarrow qq q'\bar{q}'g$
$qq' \rightarrow qq'$	$qq' \rightarrow qq'ggg, \quad qq' \rightarrow qq' q\bar{q}g, \quad qq' \rightarrow qq' q'\bar{q}'g, \quad qq' \rightarrow qq' q''\bar{q}''g$
$\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}$	$\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}ggg, \quad \bar{q}\bar{q} \rightarrow \bar{q}\bar{q} q\bar{q}g, \quad \bar{q}\bar{q} \rightarrow \bar{q}\bar{q} q'\bar{q}'g$
$\bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'$	$\bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}'ggg, \quad \bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}' q\bar{q}g, \quad \bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}' q'\bar{q}'g, \quad \bar{q}\bar{q}' \rightarrow \bar{q}\bar{q}' q''\bar{q}''g$
$q\bar{q}' \rightarrow q\bar{q}'$	$q\bar{q}' \rightarrow q\bar{q}'ggg, \quad q\bar{q}' \rightarrow q\bar{q}' q\bar{q}g, \quad q\bar{q}' \rightarrow q\bar{q}' q'\bar{q}'g, \quad q\bar{q}' \rightarrow q\bar{q}' q''\bar{q}''g$
$q\bar{q} \rightarrow gg$	$q\bar{q} \rightarrow ggggg, \quad q\bar{q} \rightarrow gg q\bar{q}g, \quad q\bar{q} \rightarrow gg q'\bar{q}'g, \quad q\bar{q} \rightarrow q\bar{q}q\bar{q}g,$ $q\bar{q} \rightarrow q\bar{q} q'\bar{q}'g, \quad q\bar{q} \rightarrow q'\bar{q}' q'\bar{q}'g, \quad q\bar{q} \rightarrow q'\bar{q}' q''\bar{q}''g$
$q\bar{q} \rightarrow q\bar{q}$	$q\bar{q} \rightarrow q\bar{q}ggg, \quad q\bar{q} \rightarrow q\bar{q} q\bar{q}g, \quad q\bar{q} \rightarrow q\bar{q} q'\bar{q}'g$
$q\bar{q} \rightarrow q'\bar{q}'$	$q\bar{q} \rightarrow q'\bar{q}'ggg, \quad q\bar{q} \rightarrow q\bar{q} q'\bar{q}'g, \quad q\bar{q} \rightarrow q'\bar{q}' q'\bar{q}'g, \quad q\bar{q} \rightarrow q'\bar{q}' q''\bar{q}''g$

Appendix B

Colour Flow Diagrams

As seen in Chapter 3, Chapter 5, and Appendix A, there are many collision processes we consider from $2 \rightarrow 2$ up to $2 \rightarrow 5$. We show one possible end product colour orientation for each process allowed by DRToM. Figure B.1 shows colour flow diagrams for $2 \rightarrow 2$. Figure B.2, Fig B.3, and Fig. B.4 show allowed colour flows for $2 \rightarrow 3, 4, 5$, respectively. In the figure captions we note the specific end state we are displaying as well as the tree-level process that it originated from. A g indicates a gluon and is represented by a coiled line. They are displayed with no kinematic properties and their colour properties are the most immediate lines above and below the coils (i.e. the arrows on the coloured lines do not signify a direction in space, they are only representing colour orientation). A q indicates a quark, a $\bar{q}$ an antiquark; a prime denotes a quark that is of a different flavour from q , and a double prime indicates a quark that is of a different flavour than q and q' . All quarks are represented by arrows. The kinematic properties of quarks and their colour are both indicated in a single line; the arrows signal an incoming/outgoing quark/antiquark. In all plots time moves from left to right and the centre circle is used to represent the fact that we do not care about the specific collision dynamics, only the incoming and outgoing partons.

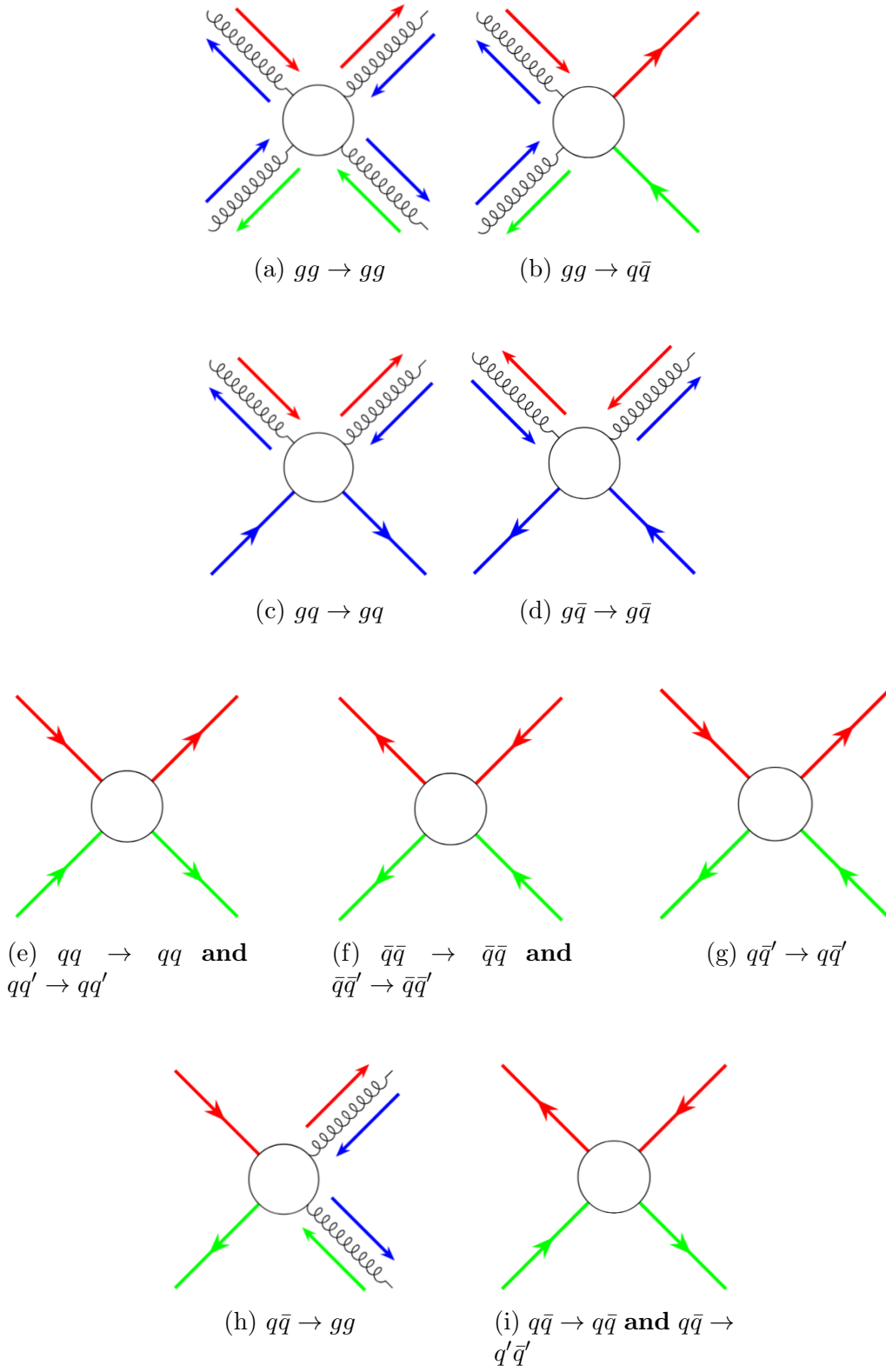


Figure B.1: Possible colour flow orientations for each $2 \rightarrow 2$ QCD process.

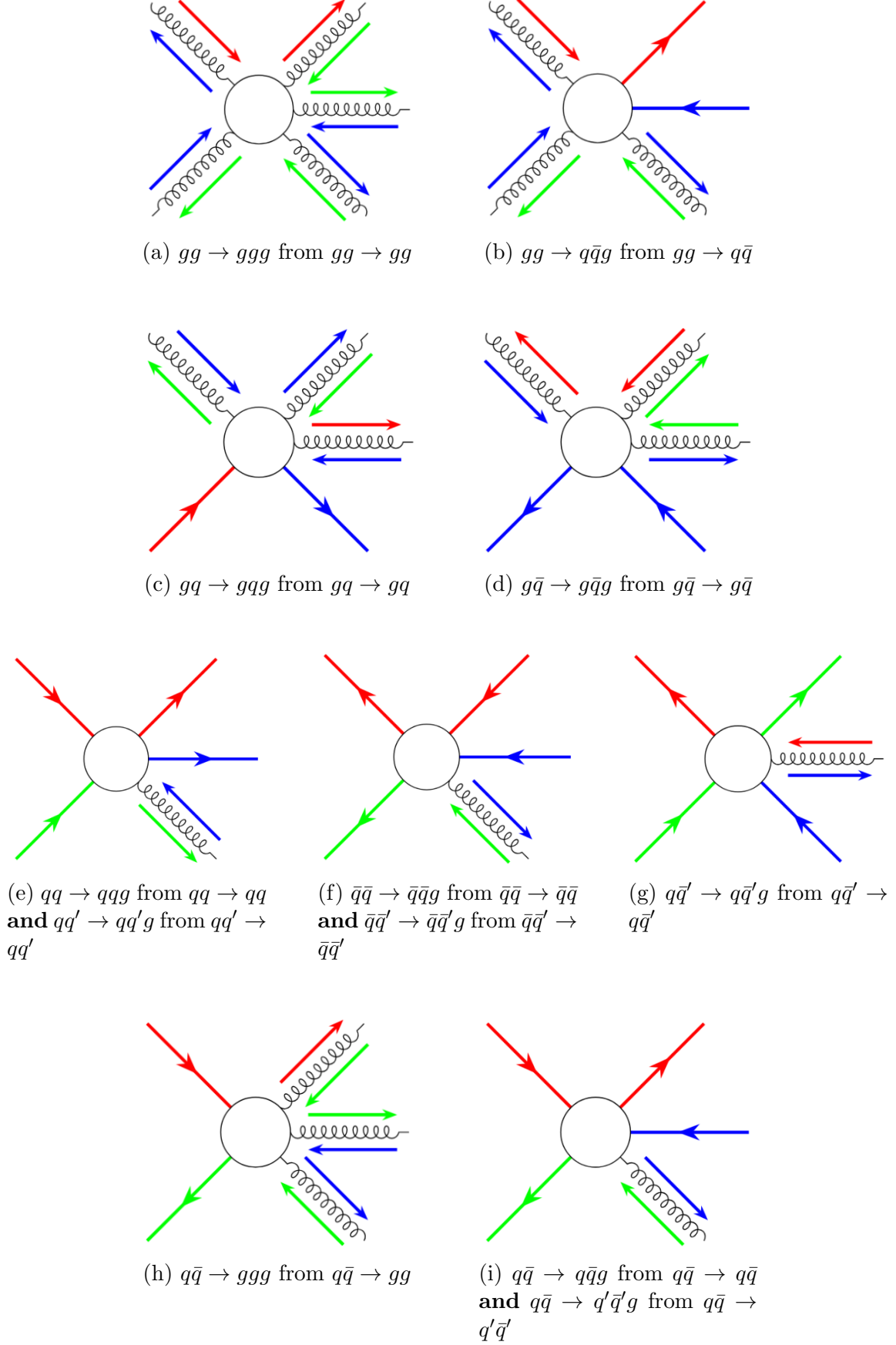


Figure B.2: Possible colour flow orientations for each $2 \rightarrow 3$ process. In the captions of each we note the specific end state and which QCD process it came from.

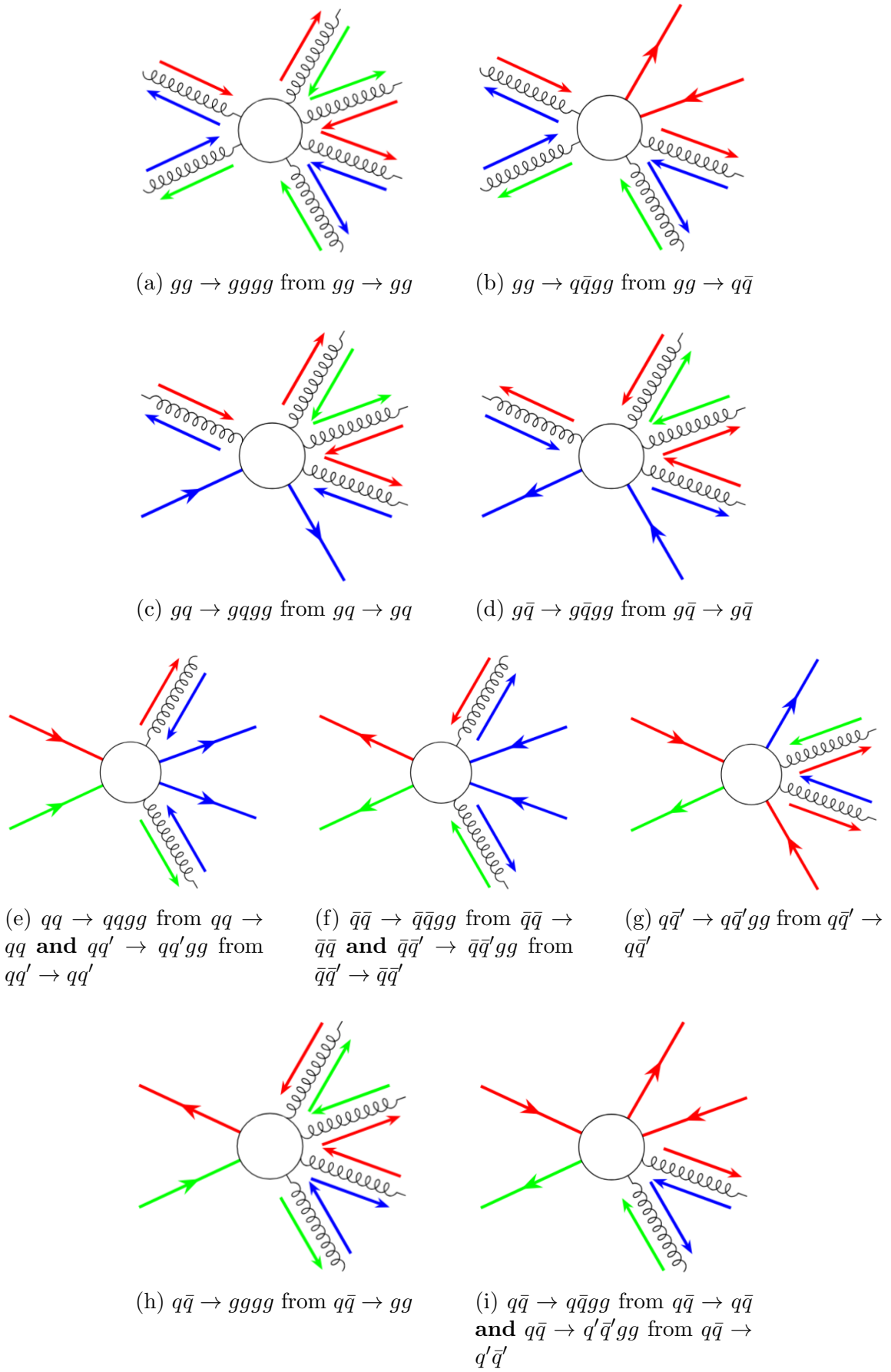


Figure B.3: Possible colour flow orientations for each $2 \rightarrow 4$ process. In the captions of each we note the specific end state and which QCD process it came from.

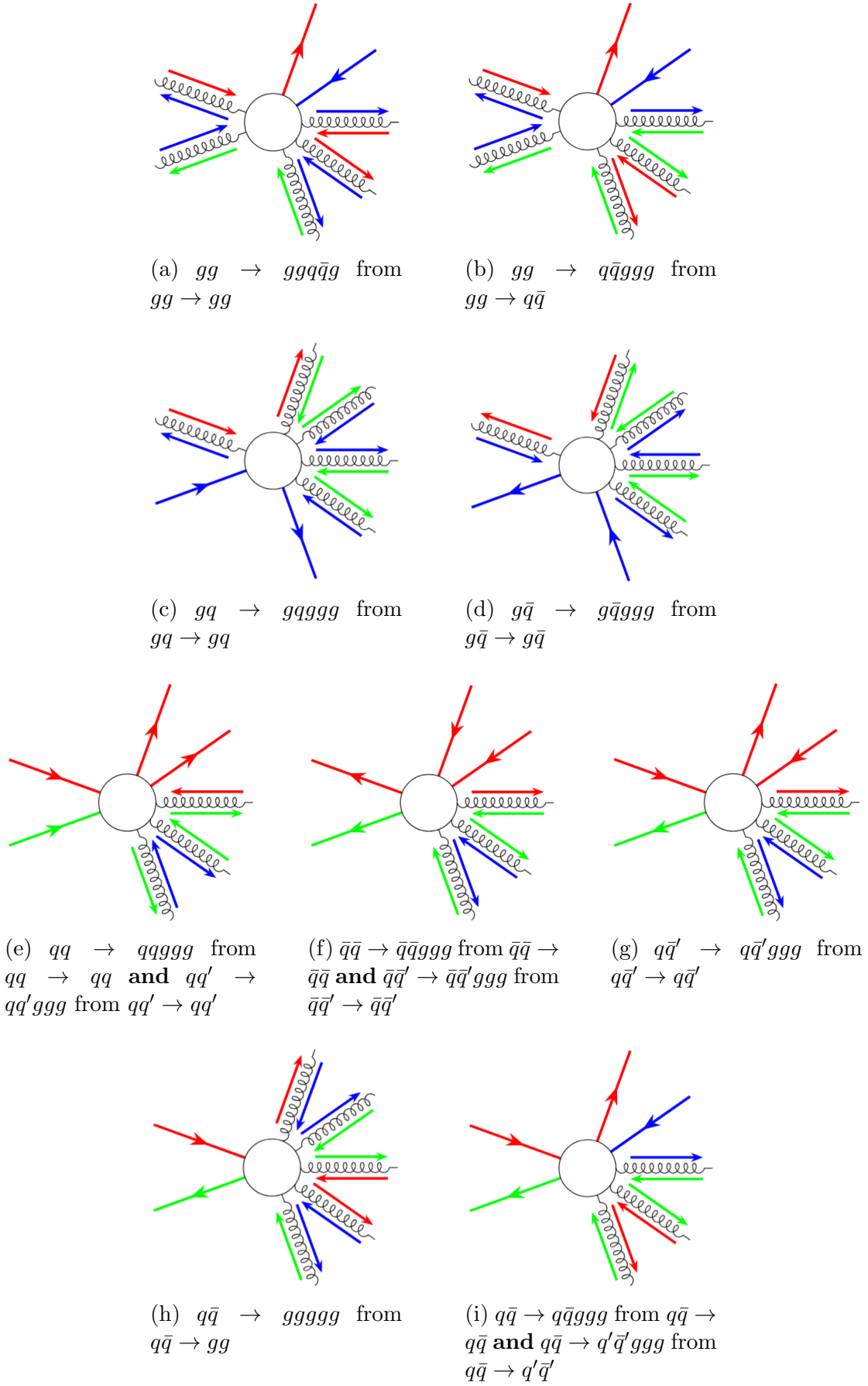


Figure B.4: Possible colour flow orientations for each $2 \rightarrow 5$ process. In the captions of each we note the specific end state and which QCD process it came from.

Appendix C

Event Generation Table

The statistics used for the parton level DRToM generation (used in Chapter 6, Appendix D, and Appendix F), regardless of dimensionality or frame choice, are given in Table C.1. The first row is the proton CoM collision energy at 13 TeV, and the second row is the lab frame pseudorapidity requirement applied to each parton used to avoid QCD divergences ($|\eta| < 4$ was an arbitrary choice that proved to be consistent in avoiding QCD divergences). The ‘Lowest/Highest CoM Mass’ rows represent the mass range of the generation, for us this is 2-11 TeV. We chose 11 TeV as the upper bound because the highest 4-jet invariant mass from data was found to be around 10.5 TeV (and rounded up). The lower bound choice of 2 TeV was based on the peak bin count from data ¹. ‘Mass Slice Range’ indicates how we are splitting the mass range for stratified sampling. We choose slices of 0.1 TeV which gives us a total of 90 slices across the mass range. ‘Parton Events per Slice’ tells us how many events (at the parton level from DRToM) are generated per mass slice. There is no limit to how many events can go into an LHE file, but for computational purposes we chose 22,000. For each mass slice we generate multiple LHE files that each have Parton Events per Slice events. Five files were chosen here so that each mass slice has over 100,000 events. This is what ‘Files per Slice’ represents. Lastly, ‘Total Parton Events’ is the total number of parton events in the generation.

This stratified sampling scheme was necessary because the generator preferentially samples processes with larger cross sections. If events were generated over the full

¹These data ranges were suggested by Dr. Douglas Gingrich. They provide a safe range to use 100% trigger efficient events.

2–11 TeV range in a single sample, the steeply falling QCD cross section would cause the events to be concentrated near the lower mass bound, leaving the high-mass region sparsely populated. Dividing the full range into narrower mass slices and generating an equal number of events in each slice therefore ensures adequate MC statistics across the entire range.

Moving to jet level analysis in Chapter 7 (and for plots in Appendix G and Appendix H), we use the same parton generation of Table C.1, but we consider additional requirements which are given in Table C.2. First, in our software ² we require a 10% event buffer which is why ‘Jet Events per Slice’ is at 20,000. Then, the ‘Total Jet Events’ is how many jet-level events are generated (and used in the jet analysis before any jet requirements are applied). The final four rows consist of the integrated luminosity used for weighting (based on Run 2 of ATLAS), and our analysis requirements.

Table C.1: Parton event generation and requirements.

$\sqrt{s}$	13.0 TeV
$ \eta $ for all Partons	< 4
Lowest CoM Mass	2 TeV
Highest CoM Mass	11.0 TeV
Mass Slice Range	0.1 TeV
Parton Events per Slice	22, 000
Files per Slice	5
Total Parton Events	9,900,000

Table C.2: Jet event generation and requirements.

Jet Events per Slice	20,000
Total Jet Events	9,000,000
L	140 fb^{-1}
p_T	$> [480, 20, 20, 20] \text{ GeV}$
$ \eta $ for all Jets	< 2.5

²To get to evgen we used AthGeneration version 23.6.50, and to get to TRUTH level we used Athena version 25.0.28.

Appendix D

DRTToM Kinematic Plots

We show additional kinematic plots for $2 \rightarrow 4$ partonic massless phase space. We consider both spherical and planar events in the the CoM and lab frame. When considering momenta plots, we only show spherical event generation since the momenta kinematics were found to be identical in Chapter 6. The generation parameters are those outlined in Appendix C for the partonic generation. The overlay plots take each parton distribution and plots their fraction separately. The differences figures (those of $\Delta\eta$, $\Delta\phi$, and $\Delta\theta$) show the difference between the leading and subleading parton values.

The partons are ordered by p_T where leading indicates the parton with the highest p_T , subleading the second highest p_T , tertiary the third highest p_T , and last the final parton. The events are not weighted (they do not simulate data), and every distribution is normalized to the number of generated events. Momentum conservation was checked independently and is not shown here.

D.1 CoM Frame

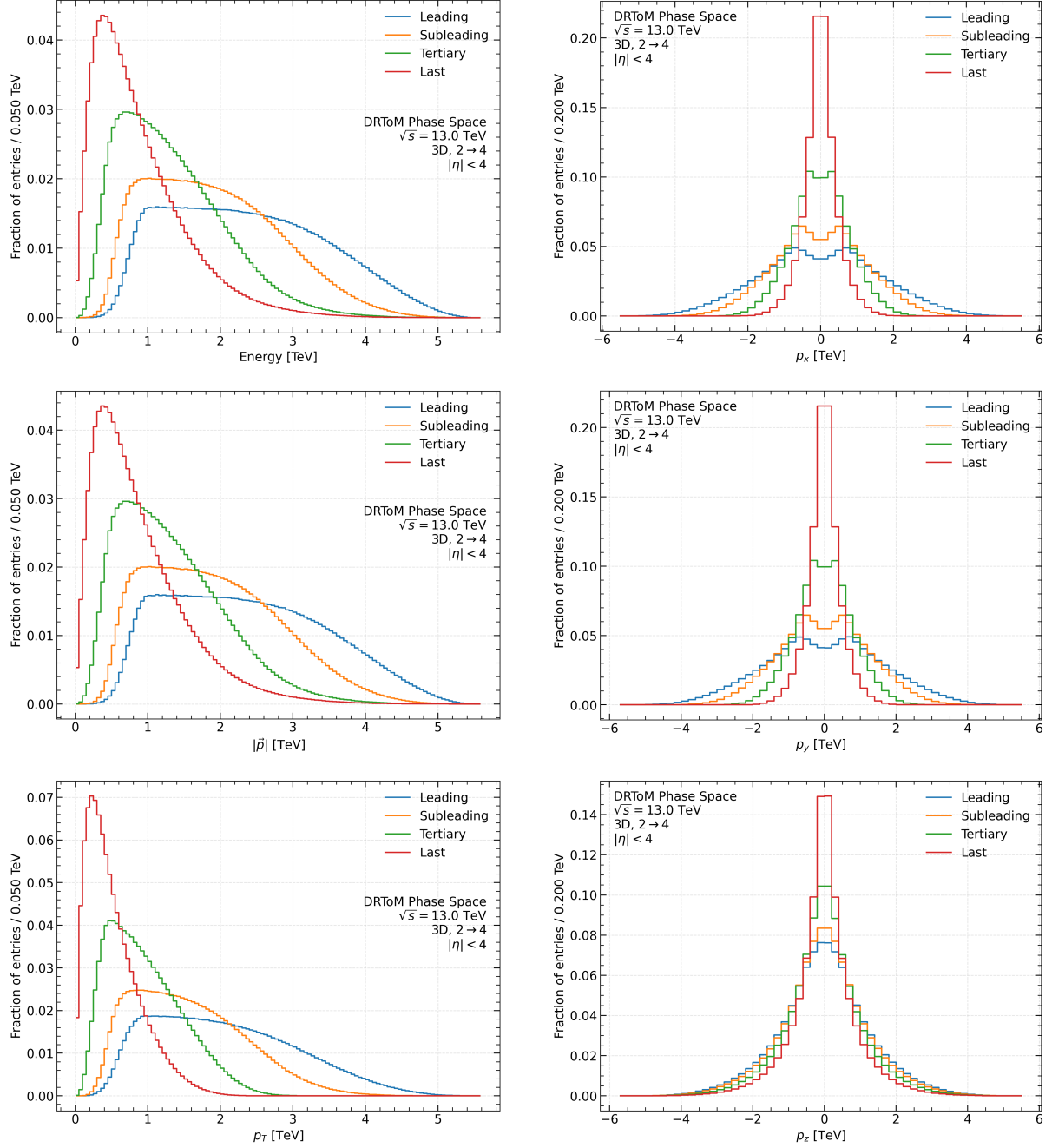
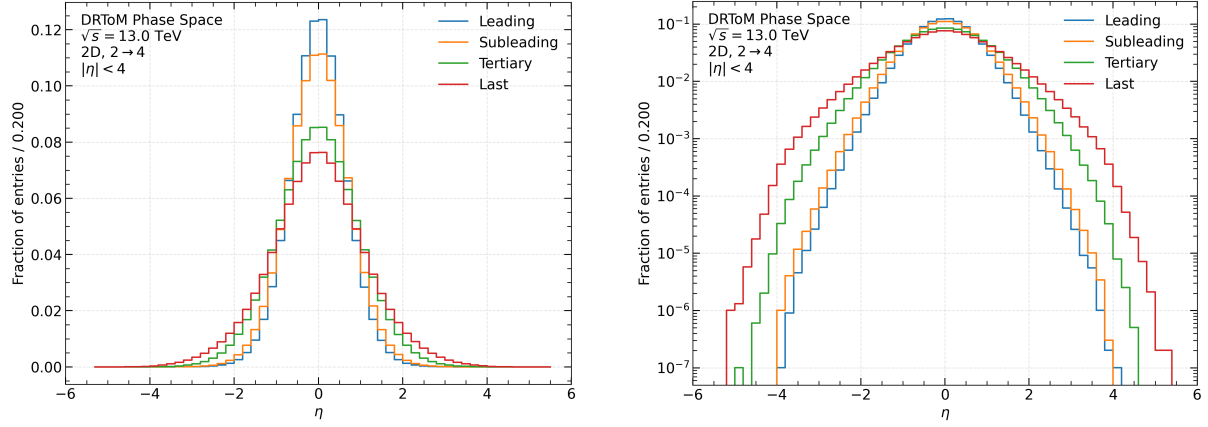
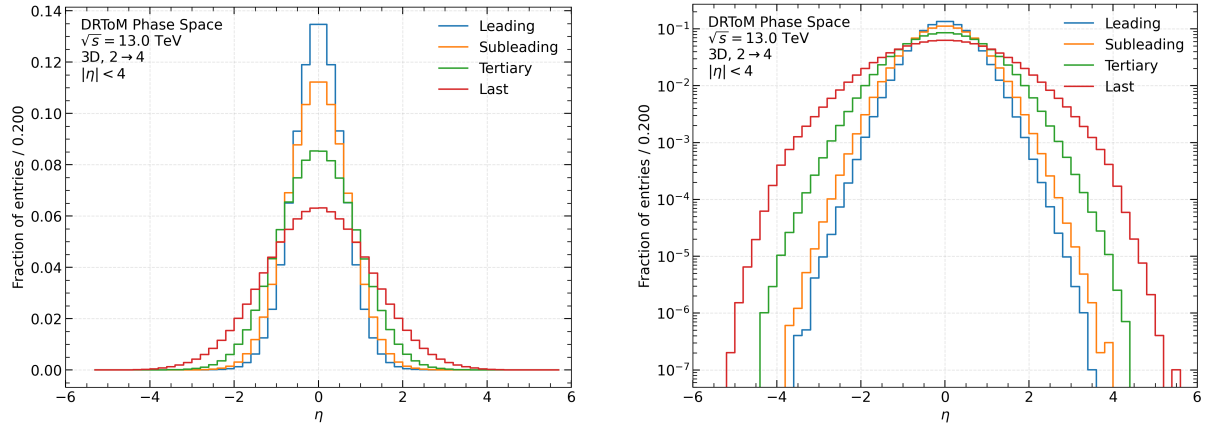


Figure D.1: E , $|\vec{p}|$, p_T , p_x , p_y , and p_z distributions for 2 $\rightarrow$ 4 massless partonic spherical events in the CoM frame. The left column shows, from top to bottom, the E , $|\vec{p}|$, and p_T distributions, while the right column shows the corresponding p_x , p_y , and p_z distributions.

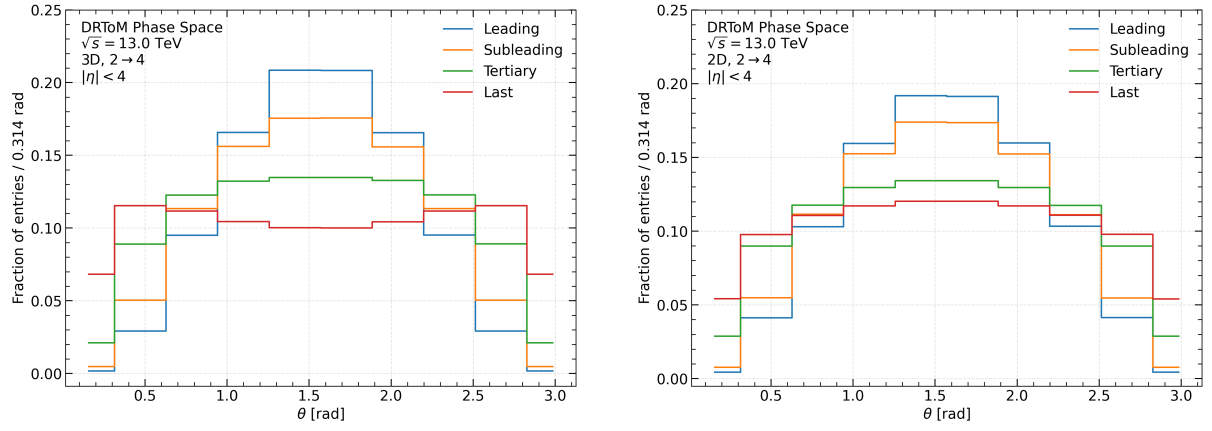


(a) Planar events.

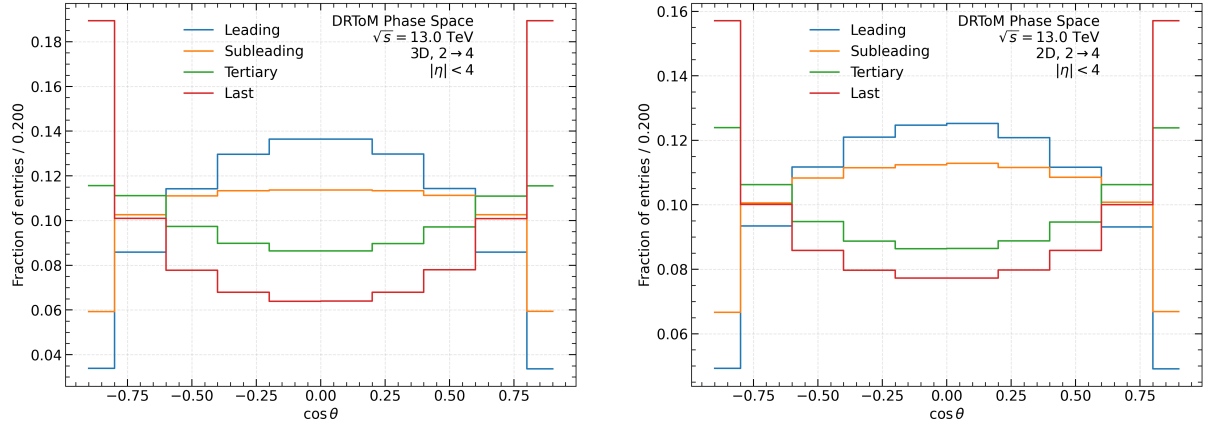


(b) Spherical events.

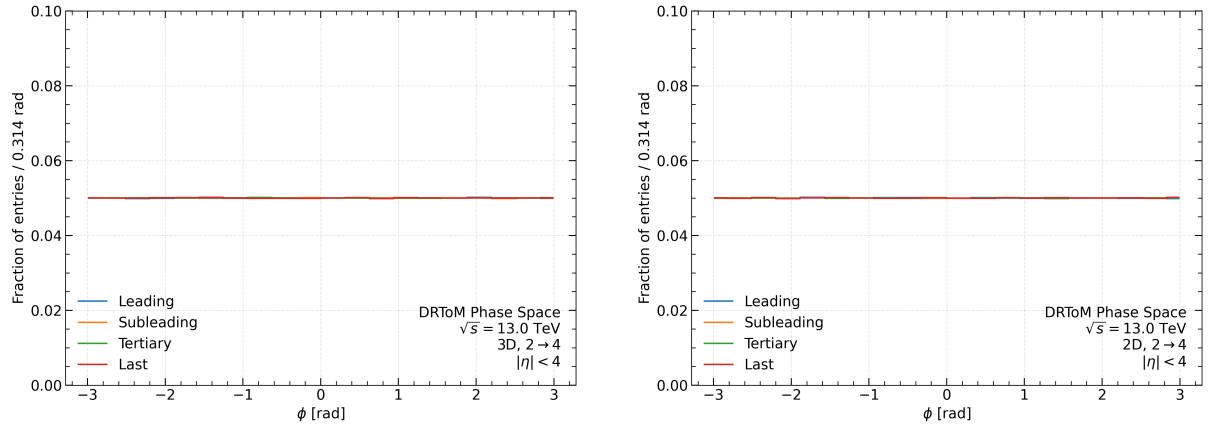
Figure D.2: CoM frame overlays of the η distributions. The top row shows planar events, while the bottom row shows spherical events. The plots on the left use a linear scale, while those on the right use a logarithmic scale. The distributions are symmetric about $\eta = 0$. Although a bound of $|\eta| < 4$ was imposed, boosting causes some events to lie outside this range.



(a) θ distributions for spherical events (left) and planar events (right).

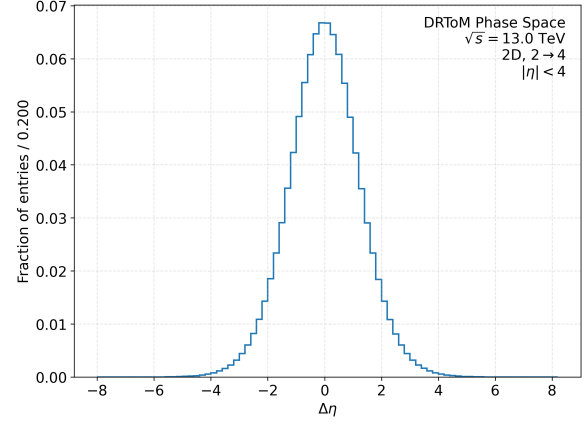
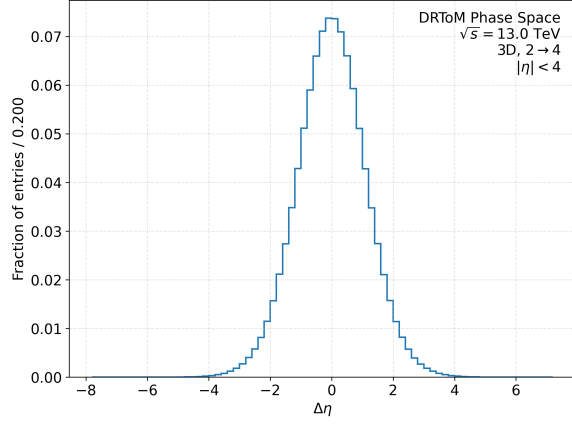


(b) $\cos \theta$ distributions for spherical events (left) and planar events (right).

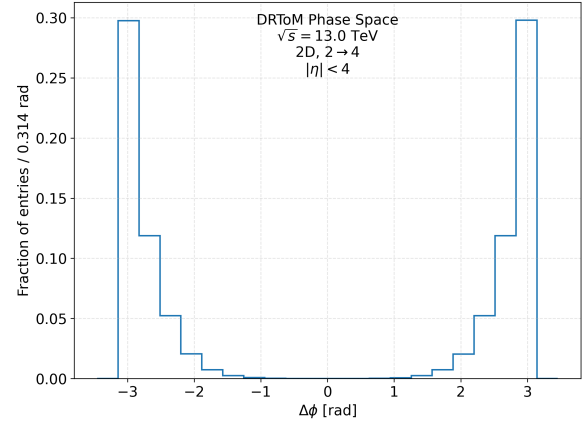
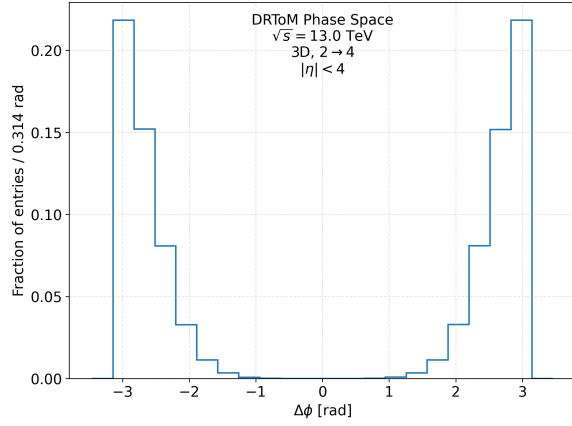


(c) ϕ distributions for spherical events (left) and planar events (right).

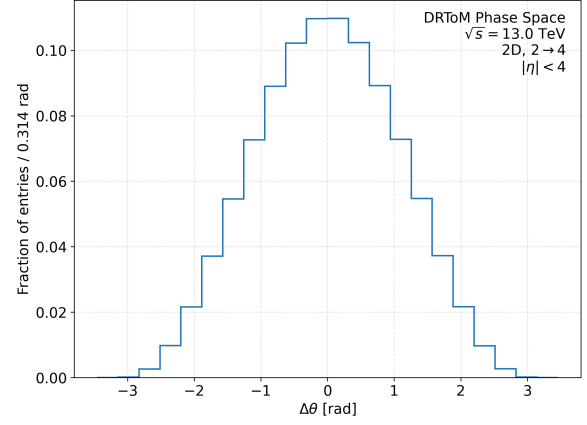
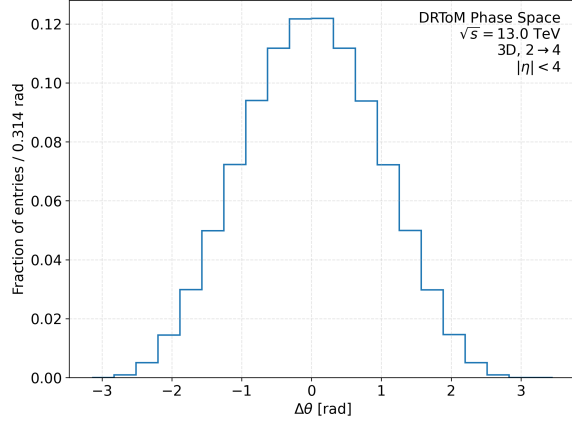
Figure D.3: CoM frame overlay of the θ (top), $\cos \theta$ (middle), and ϕ (bottom) distributions. The left plots are for spherical events and the right plots for planar events.



(a) $\Delta\eta$ distributions for spherical events (left) and planar events (right).



(b) $\Delta\phi$ distributions for spherical events (left) and planar events (right).



(c) $\Delta\theta$ distributions for spherical events (left) and planar events (right).

Figure D.4: Differences in η , ϕ , and θ in the CoM frame. The plots on the left are for spherical events and the plots on the right are for planar events.

D.2 Lab Frame

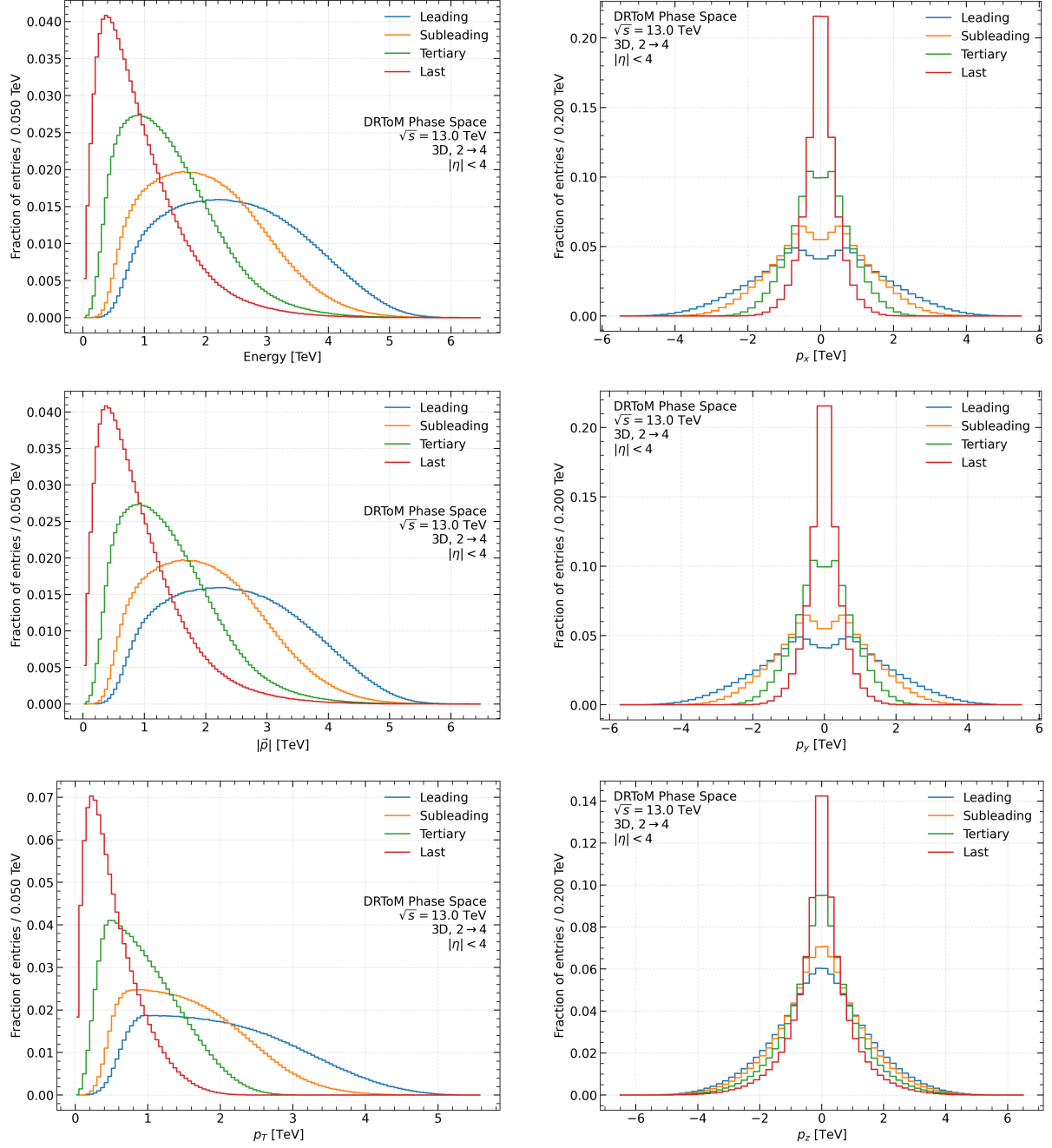
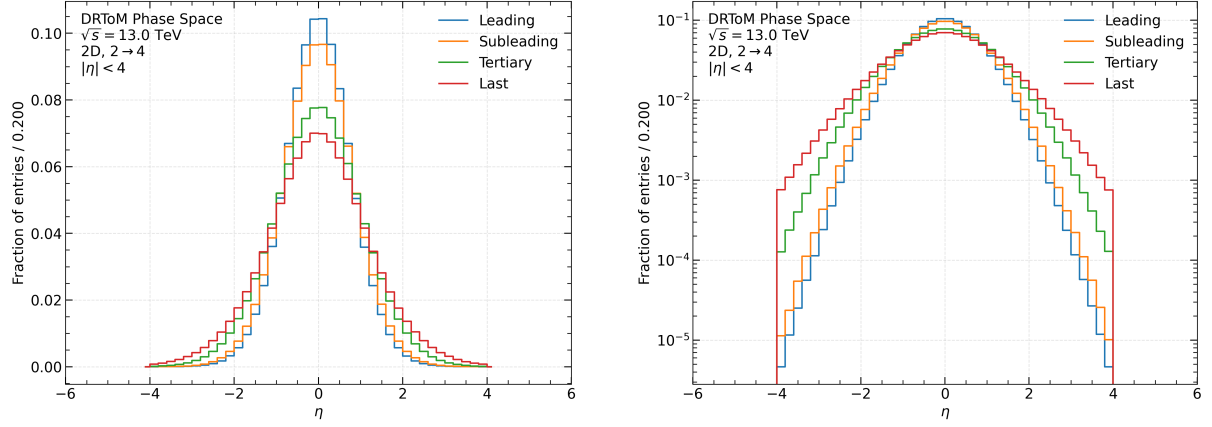
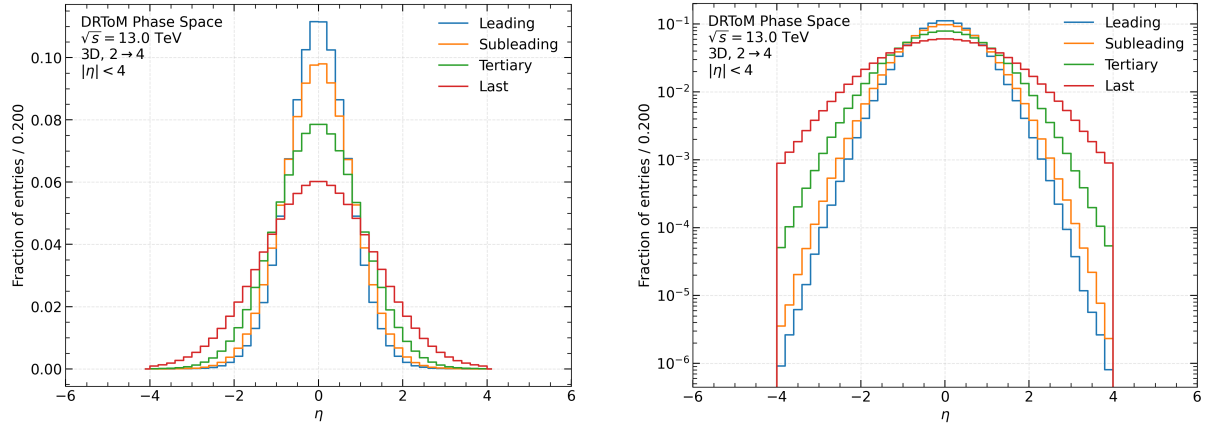


Figure D.5: E , $|\vec{p}|$, p_T , p_x , p_y , and p_z distributions for $2 \rightarrow 4$ massless partonic spherical events in the lab frame. The left column shows, from top to bottom, the E , $|\vec{p}|$, and p_T distributions, while the right column shows the corresponding p_x , p_y , and p_z distributions.

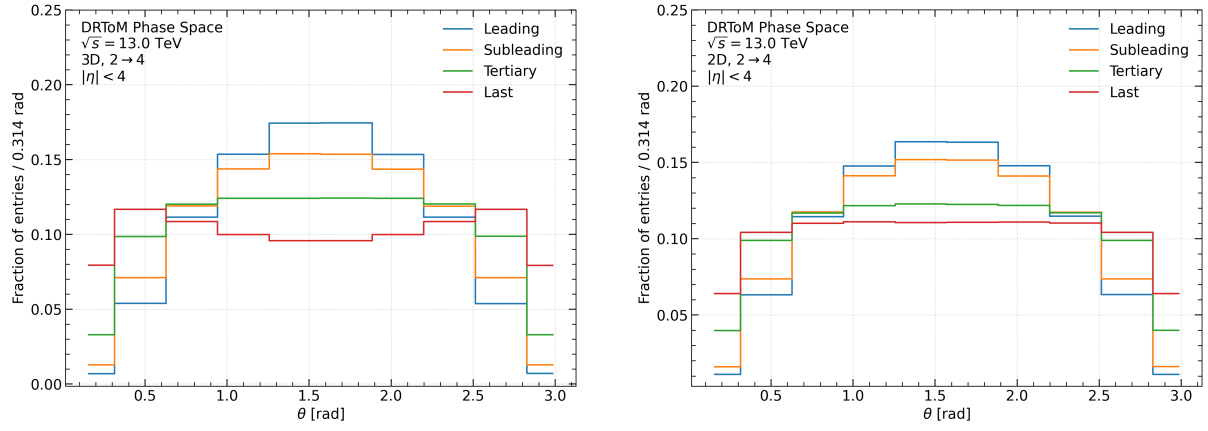


(a) Planar events.

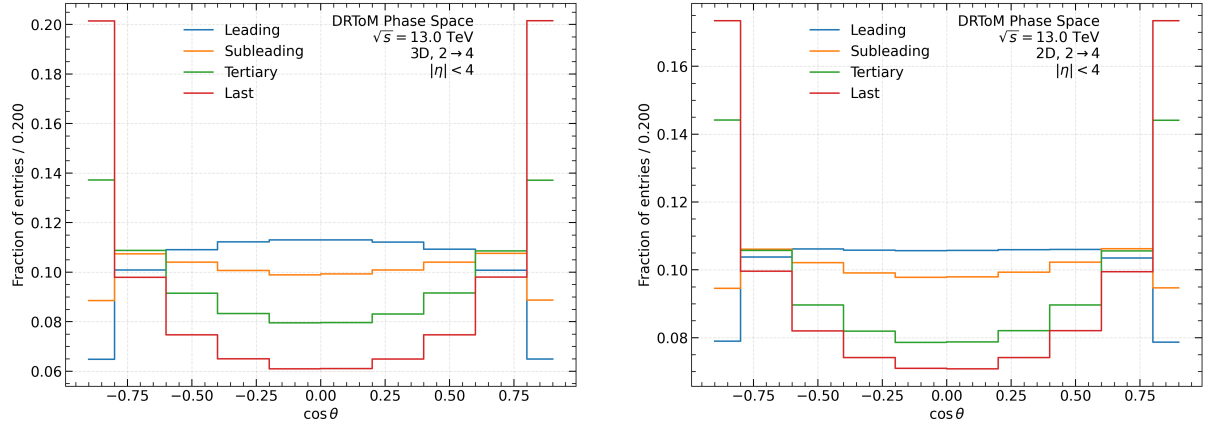


(b) Spherical events.

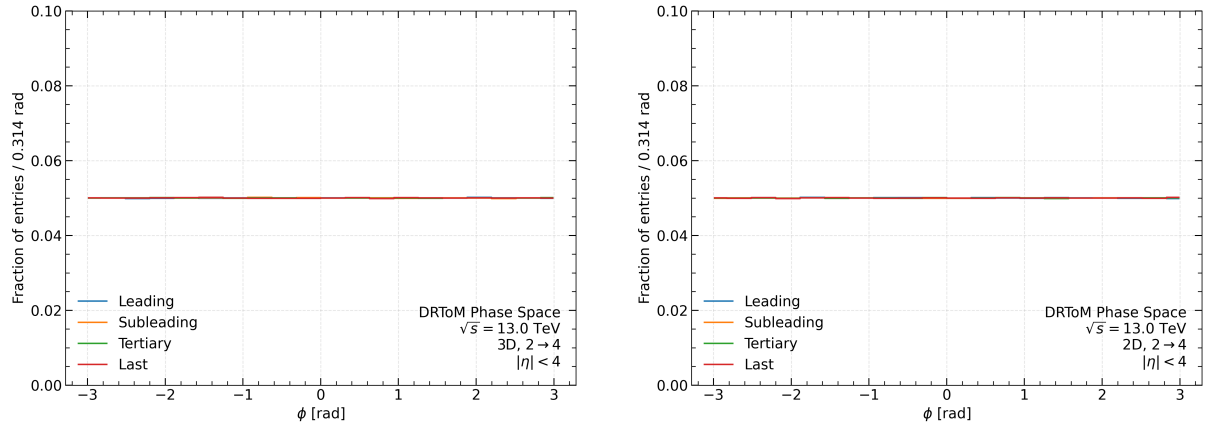
Figure D.6: Lab frame overlays of the η distributions. The top row shows planar events, while the bottom row shows spherical events. The plots on the left use a linear scale, while those on the right use a logarithmic scale. The distributions are symmetric about $\eta = 0$ and we see the $|\eta| < 4$ requirement as there are no events outside of that range.



(a) θ distributions for spherical events (left) and planar events (right).

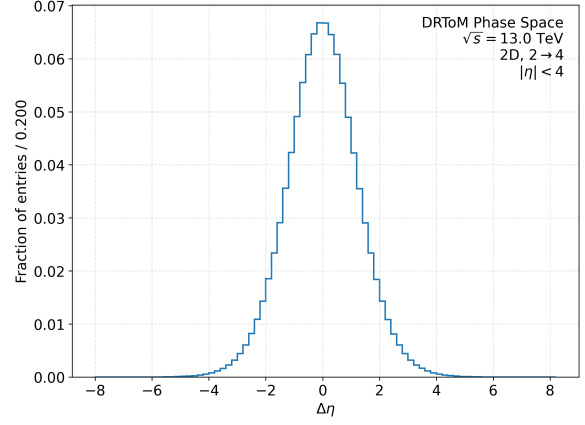
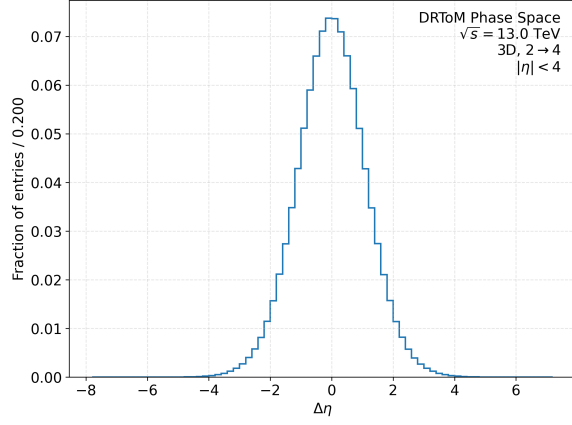


(b) $\cos \theta$ distributions for spherical events (left) and planar events (right).

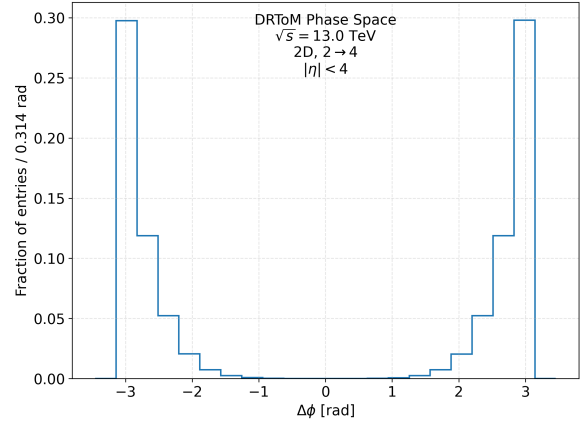
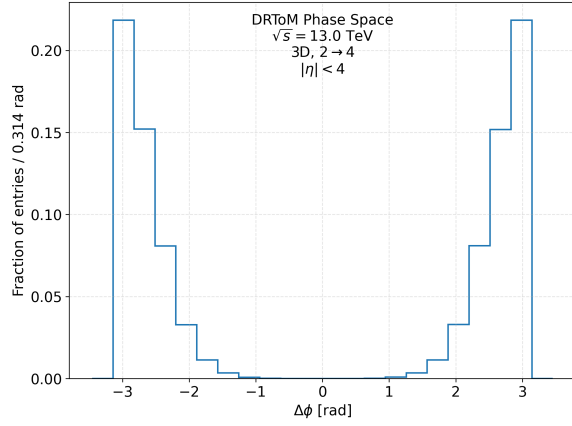


(c) ϕ distributions for spherical events (left) and planar events (right).

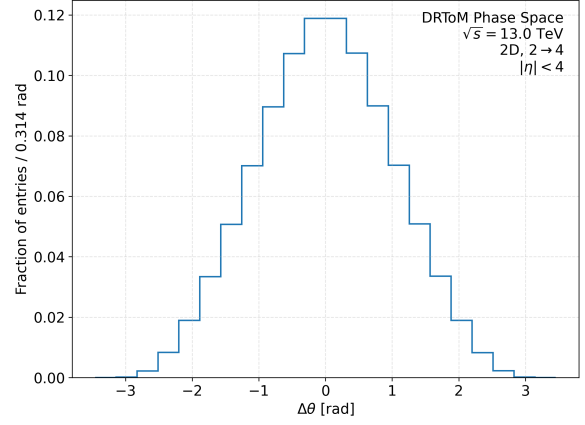
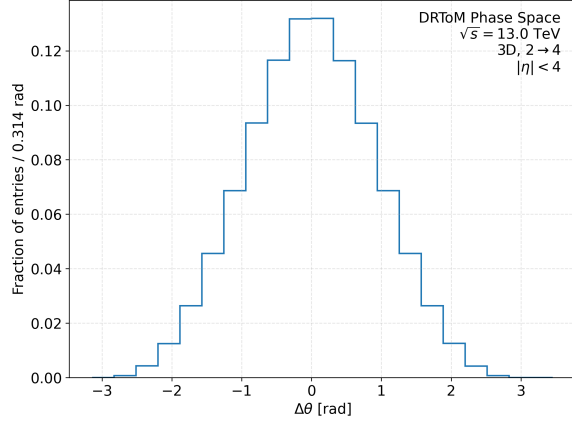
Figure D.7: Lab frame overlay of the θ (top), $\cos \theta$ (middle), and ϕ (bottom) distributions. The left plots are for spherical events and the right plots for planar events.



(a) $\Delta\eta$ distributions for spherical events (left) and planar events (right).



(b) $\Delta\phi$ distributions for spherical events (left) and planar events (right).



(c) $\Delta\theta$ distributions for spherical events (left) and planar events (right).

Figure D.8: Differences in η , ϕ , and θ in the lab frame. The plots on the left are for spherical events and the plots on the right are for planar events.

Appendix E

Momentum Tensor Example

Consider a mother particle with four-momentum $P = [4, 0, 0, 0]$ that decays into a planar event with daughter particles $\mathfrak{d} = a, b, c, d$,

$$p_a = [1, 1, 0, 0], \quad p_b = [1, 0, 1, 0], \quad p_c = [1, -1, 0, 0], \quad p_d = [1, 0, -1, 0]. \quad (\text{E.1})$$

For the purposes of the main text, energy units can be considered in TeV; however, this detail is not necessary for this illustration. To calculate the normalized momentum tensor and the event shape variables, we only need the spatial components of the event,

$$\vec{p}_a = [1, 0, 0], \quad \vec{p}_b = [0, 1, 0], \quad \vec{p}_c = [-1, 0, 0], \quad \vec{p}_d = [0, -1, 0], \quad (\text{E.2})$$

The magnitude of each spatial momenta is 1, so

$$\sum_{\mathfrak{d}=a}^d |\vec{p}_{\mathfrak{d}}| = |\vec{p}_a| + |\vec{p}_b| + |\vec{p}_c| + |\vec{p}_d| = 1 + 1 + 1 + 1 = 4, \quad (\text{E.3})$$

and

$$\sum_{\mathfrak{d}} \frac{1}{|\vec{p}_{\mathfrak{d}}|} \begin{pmatrix} (p_{\mathfrak{d}}^x)^2 & p_{\mathfrak{d}}^y p_{\mathfrak{d}}^x & p_{\mathfrak{d}}^z p_{\mathfrak{d}}^x \\ p_{\mathfrak{d}}^y p_{\mathfrak{d}}^x & (p_{\mathfrak{d}}^y)^2 & p_{\mathfrak{d}}^z p_{\mathfrak{d}}^y \\ p_{\mathfrak{d}}^z p_{\mathfrak{d}}^x & p_{\mathfrak{d}}^z p_{\mathfrak{d}}^y & (p_{\mathfrak{d}}^z)^2 \end{pmatrix} = \sum_i \begin{pmatrix} p_{x,i}^2 & p_{y,i} p_{x,i} & p_{z,i} p_{x,i} \\ p_{y,i} p_{x,i} & p_{y,i}^2 & p_{z,i} p_{y,i} \\ p_{z,i} p_{x,i} & p_{z,i} p_{y,i} & p_{z,i}^2 \end{pmatrix} \quad (\text{E.4})$$

$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (\text{E.5})$$

The momentum tensor is thus

$$M^{xyz} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\text{E.6})$$

with ordered eigenvalues that satisfy normalization

$$e_1 = 0, \quad e_2 = e_3 = \frac{1}{2}, \quad \text{with} \quad e_1 + e_2 + e_3 = 1. \quad (\text{E.7})$$

Sphericity and aplanarity are calculated as

$$S = \frac{3}{2}(e_1 + e_2) = \frac{3}{4}, \quad \text{and} \quad A = \frac{3}{2}e_1 = 0, \quad (\text{E.8})$$

respectively, noting that $A = 0$ as expected. Because this event orientation is inherently a plane (regardless of rotation) it will always have the same eigenvalues and hence the same S/A values. This is true for any orientation and event shape variable made from M^{xyz} .

Considering the same momenta in Eq. E.2, we can also calculate the biplanarity using Eq. 6.16. In this case, the angle between each is the same in the CoM and lab frame, so we simply do our cross products with the ordered momentum,

$$\vec{n}_1 = \vec{p}_a \times \vec{p}_b = [0, 0, 1], \quad \text{and} \quad \vec{n}_2 = \vec{p}_c \times \vec{p}_d = [0, 0, -1]. \quad (\text{E.9})$$

With this, the angle between these two planes is zero and $B = 1$, as expected. If we instead chose $\vec{n}_1 = \vec{p}_a \times \vec{p}_c$, then we would get $\vec{n}_1 = [0, 0, 0]$ which would give $B = 0$. What this tells us is that if the cross products are non-zero, then any pairing of momenta will work. However, if a cross product is zero, then the vectors are collinear. The momenta is still in a plane, but it is not represented by $B = 1$. We do not take this specialized case into account in our analysis. A natural next step would be to account for it by adding a condition that $B = 0$ events are really $B = 1$ events so that we could incorporate them into our methods with no additional changes to the analysis.

Appendix F

Event Shape Distributions

We show the event shape variable distributions (in order) for the variables from Table 6.1, Table 6.2, and biplanarity, for $2 \rightarrow 4$ massless partonic phase space events. The DRTom generation follows Appendix C. We show distributions for both fully spherical and fully planar events in the lab and CoM frames. As we are interested in the overall shapes, each plot is normalized by the total number of events in the generation. Note: when comparing between side-by-side figures, the y -axis ranges may not be the same.

Figure F.3 and Fig. F.10 show the aplanarity and biplanarity distributions. For the fully spherical generation in the CoM frame, their shapes mirror each other with the most notable feature being a peak in the planar bin (the $A=0$ and $B=1$ bin). Moving to the lab frame for spherical events, we see that the aplanarity and biplanarity distributions are not as enhanced in their planar bins. Looking at the fully planar CoM frame distributions, it is clear why aplanarity and biplanarity are the variables of choice for this study: their distributions are completely distinct when all events are planar vs. spherical. When all the events are planar, then, in the CoM frame, all events fall into the planar bin. Finally, boosting to the lab frame for fully planar events, we see that the aplanarity and biplanarity distributions lose their one-bin state, but their peaks are noticeably in the first and final bin, respectively.

The remaining figures show the sphericity, transverse sphericity, Y , C , D , $T_\perp$, T_m , and τ distributions. What's interesting here is that the D variable takes on a similar shape to aplanarity with the same peak and drop-off. If both A and D encode the same information (and behave the same way) then we are free to simply choose aplanarity

to investigate. We do see differences in the distributions of Y , C , the sphericities and thrusts in regards to frame and planar type, but their distinguishing differences are far less obvious than aplanarity and biplanarity. As such, we do not consider them in this analysis.

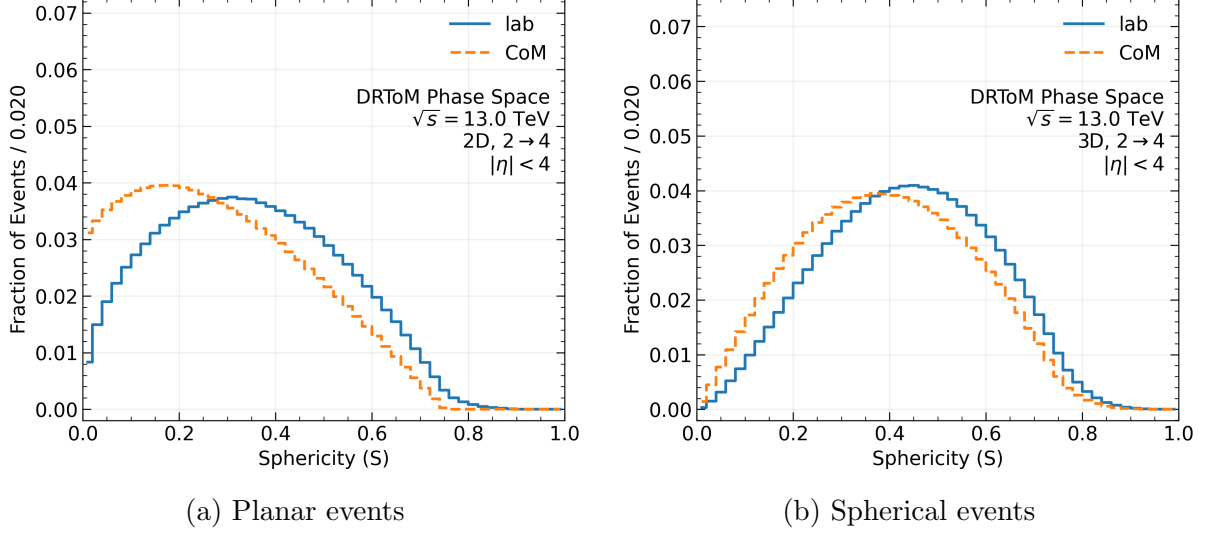


Figure F.1: Sphericity distributions.

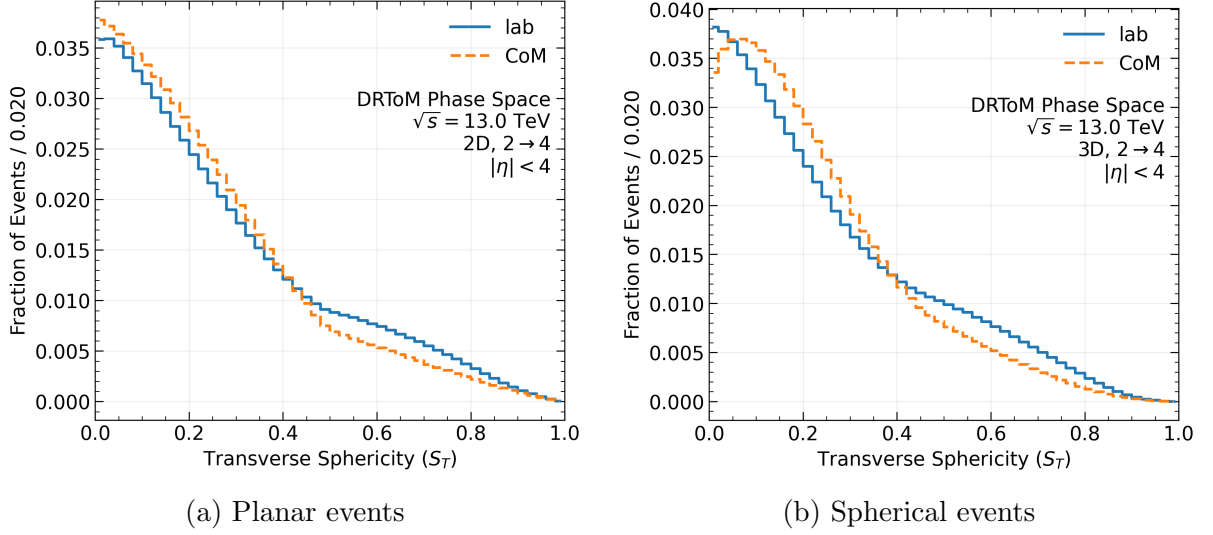
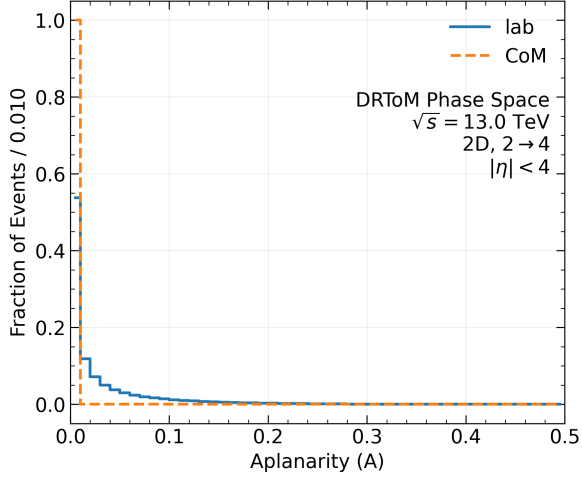
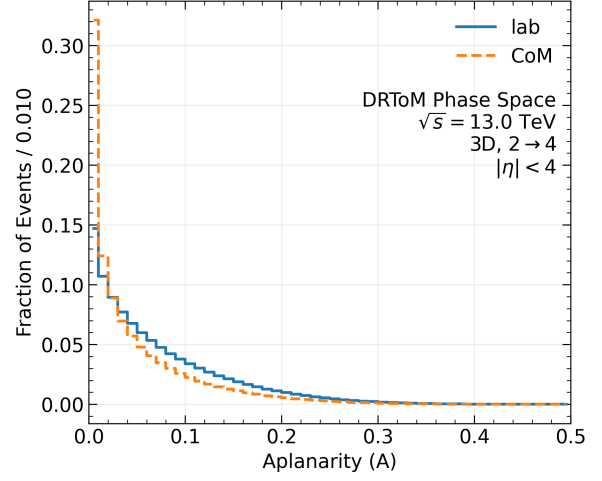


Figure F.2: Transverse sphericity distributions.

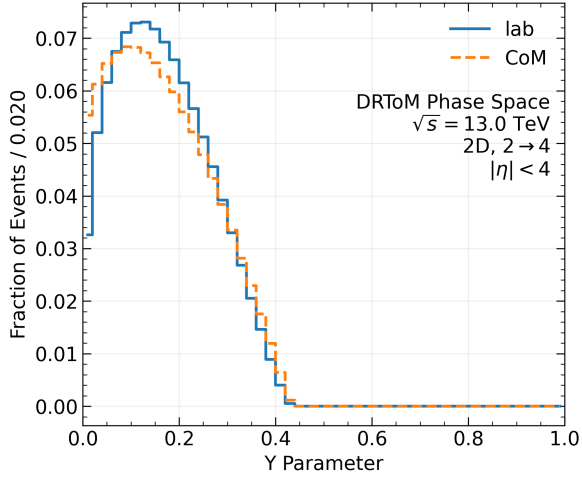


(a) Planar events

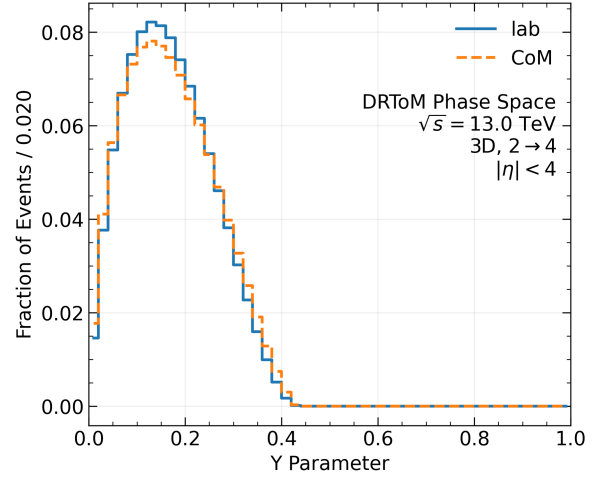


(b) Spherical events

Figure F.3: Aplanarity distributions.

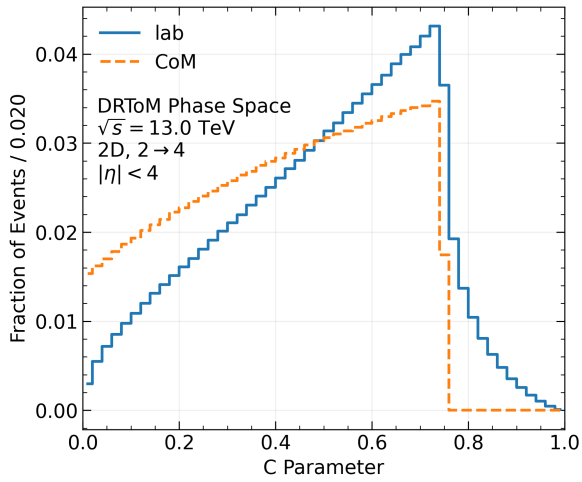


(a) Planar events

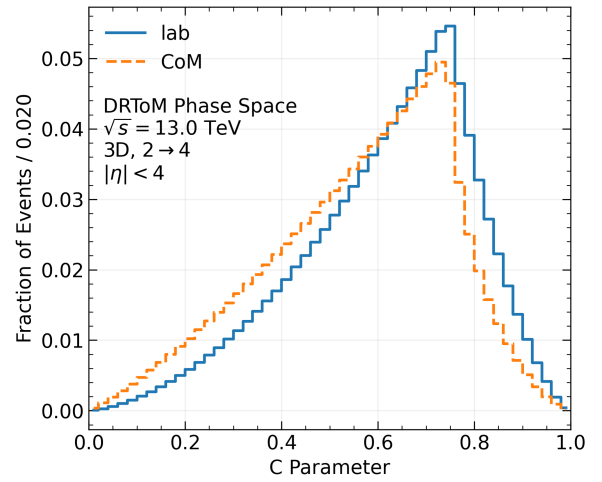


(b) Spherical events

Figure F.4: Y distributions.

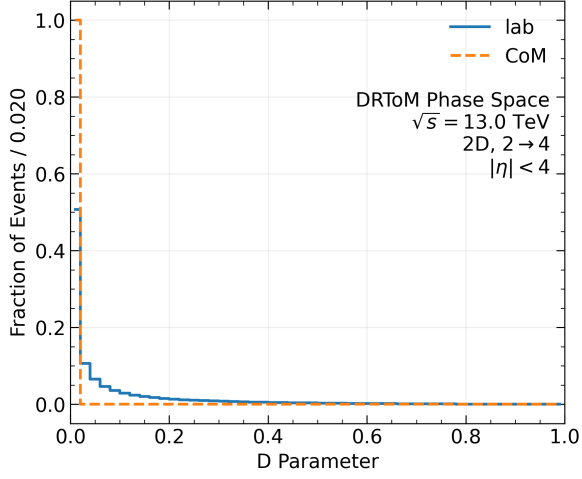


(a) Planar events

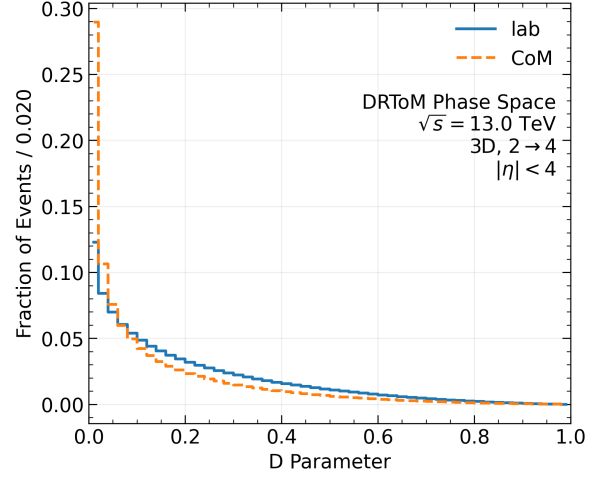


(b) Spherical events

Figure F.5: C distributions.

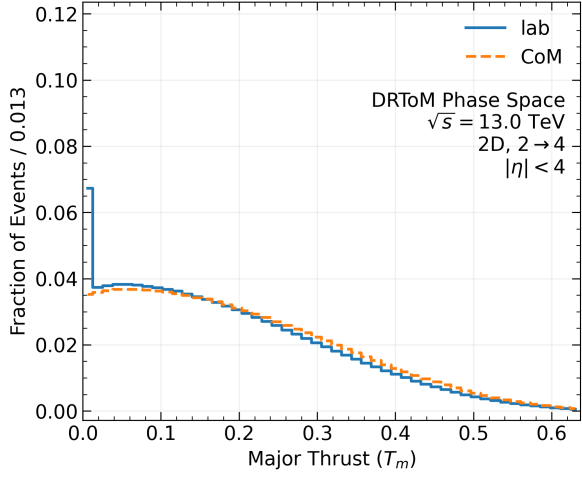


(a) Planar events

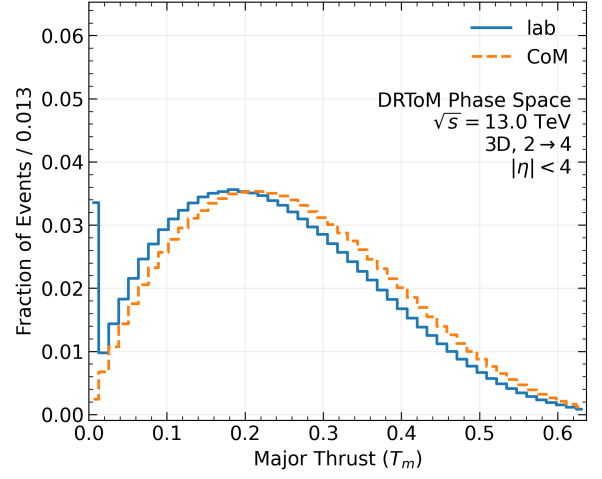


(b) Spherical events

Figure F.6: D distributions.

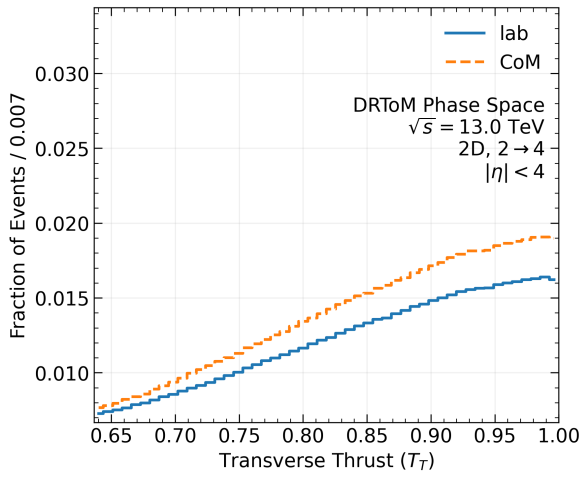


(a) Planar events

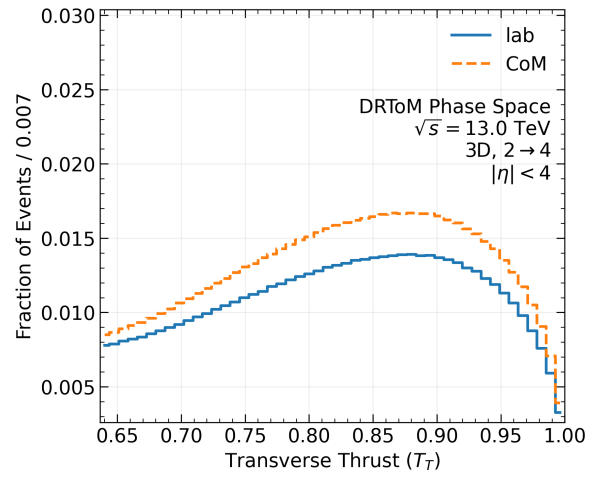


(b) Spherical events

Figure F.7: T_m distributions.

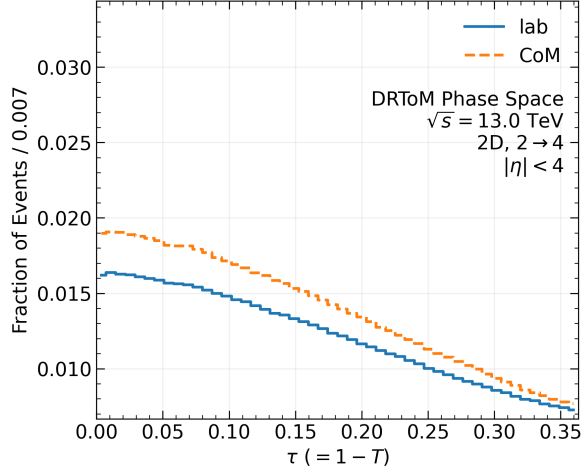


(a) Planar events

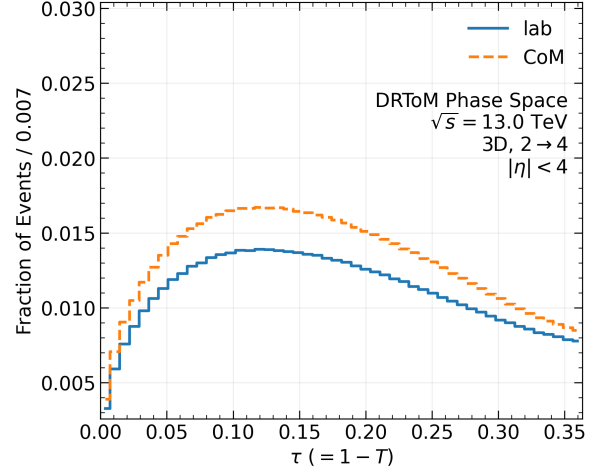


(b) Spherical events

Figure F.8: T_T distributions.

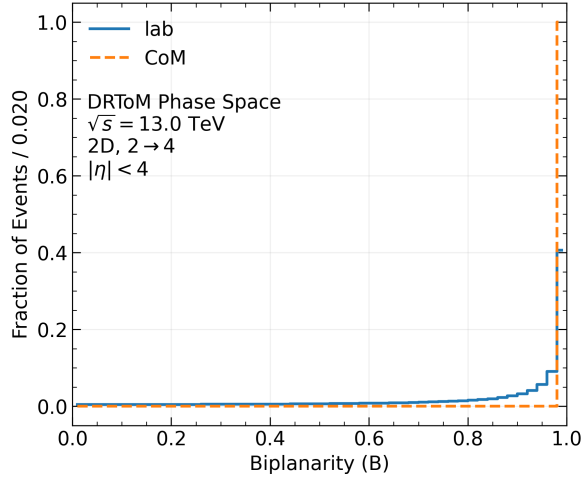


(a) Planar events

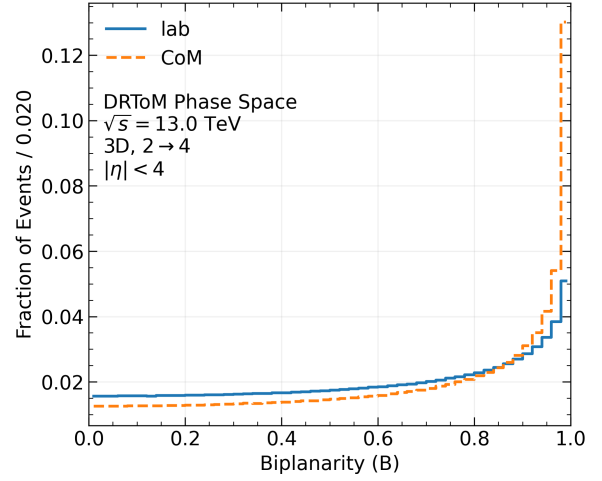


(b) Spherical events

Figure F.9: τ distributions.



(a) Planar events



(b) Spherical events

Figure F.10: Biplanarity distributions.

Appendix G

Biplanarity Study

Biplanarity is defined as

$$B = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|} = |\cos \theta_{12}|, \quad 0 \leq B \leq 1. \quad (\text{G.1})$$

where $\vec{n}_i$, $i = 1, 2$, are vectors (or planes) defined by the cross product of momentum vectors, e.g. $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$, and $\vec{n}_2 = \vec{p}_c \times \vec{p}_d$ where a, b, c, d are distinct vectors. What we want to do is find which orientation of these momentum vectors results in a minimal spherical event final bin, while maximizing the planar final bin (i.e. maximizing the signal while minimizing background).

Given our definitions of $\vec{n}_1$ and $\vec{n}_2$, we consider three cases of momentum vector ordering:

1. Order momentum vectors by p_T ,
2. Order momentum vectors by $|\vec{p}|$,
3. Order momentum vectors by their relative angle to each other.

The first two cases are simple as we define $\vec{p}_a$ as the vector with the largest $p_T/|\vec{p}|$, and $\vec{p}_d$ as the vector with the smallest $p_T/|\vec{p}|$. Then, we can consider three different combinations (ignoring sign) of cross products:

1. $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$ and $\vec{n}_2 = \vec{p}_c \times \vec{p}_d$,
2. $\vec{n}_1 = \vec{p}_a \times \vec{p}_c$ and $\vec{n}_2 = \vec{p}_b \times \vec{p}_d$,

3. $\vec{n}_1 = \vec{p}_a \times \vec{p}_d$ and $\vec{n}_2 = \vec{p}_b \times \vec{p}_c$.

The third ordering case, ordering by relative angular separation, is performed as follows. First, the jets are ordered by transverse momentum p_T . The angular separation between two jets is then defined using

$$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}, \quad (\text{G.2})$$

where η is the pseudorapidity and ϕ is the azimuthal angle. Using the four leading jets, the three unique pairings into two dijet systems are constructed:

$$(\vec{p}_a\vec{p}_b)(\vec{p}_c\vec{p}_d), \quad (\vec{p}_a\vec{p}_c)(\vec{p}_b\vec{p}_d), \quad (\vec{p}_a\vec{p}_d)(\vec{p}_b\vec{p}_c). \quad (\text{G.3})$$

For each pairing configuration, the angular separations of the two jet pairs are calculated. For example, for the pairing $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$ and $\vec{n}_2 = \vec{p}_c \times \vec{p}_d$, the quantities ΔR_{ab} and ΔR_{cd} are computed, and a total separation score is defined as

$$S = \Delta R_{ab} + \Delta R_{cd}. \quad (\text{G.4})$$

The three possible pairing configurations are then ordered according to this score from largest to smallest. For each ordered pairing configuration, the planes are defined as

$$\vec{n}_1 = \vec{p}_i \times \vec{p}_j, \quad \vec{n}_2 = \vec{p}_k \times \vec{p}_l, \quad (\text{G.5})$$

where (ij) and (kl) correspond to the two jet pairs in that configuration. Thus:

1. the first configuration corresponds to the pairing with the largest total angular separation score,
2. the second configuration corresponds to the intermediate score,
3. the third configuration corresponds to the pairing with the smallest total angular separation score.

This study was done in two parts: at the parton and jet level. The generation parameters for both are given in Appendix C. At jet level, we use the weighting scheme of w outlined in Sec. 7.3.1 (not normalized to data). We show select biplanarity distributions

and summary tables. For all three ordering cases, the subfigure captions show which $\vec{n}_1$ ordering is being considered. We note that for the angle separation ordering, following the p_T and $|\vec{p}|$ ordering we will consider $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$ to be the case with the largest separation score, $\vec{n}_1 = \vec{p}_a \times \vec{p}_c$ to be the intermediate case, and $\vec{n}_1 = \vec{p}_a \times \vec{p}_d$ to be the smallest separation score.

G.1 Parton Level

At the parton level, we only generate $2 \rightarrow 4$ massless spherical and planar events. In the CoM frame, the ordering and cross product scheme does not matter since a planar event will register the same across all options. They therefore all have the same distribution, that of Fig. G.1. Figure G.2 shows the biplanarity distributions for different $\vec{n}_1$ definitions in the lab frame using the p_T ordering scheme for planar events.

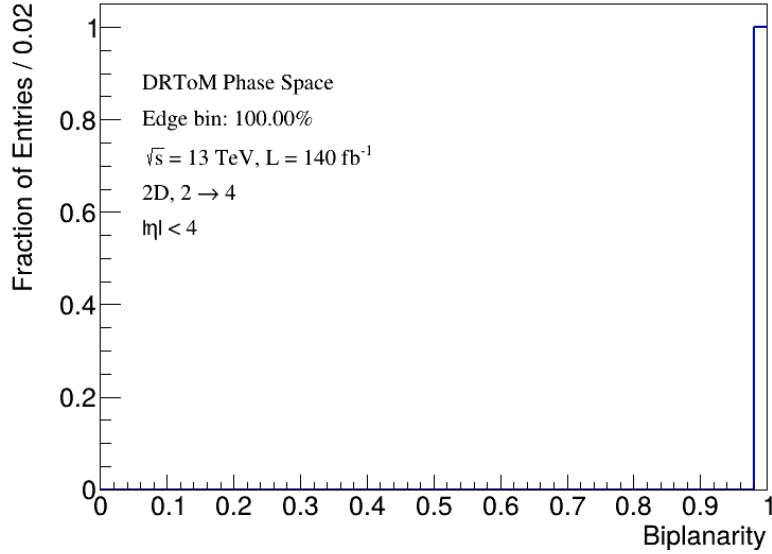
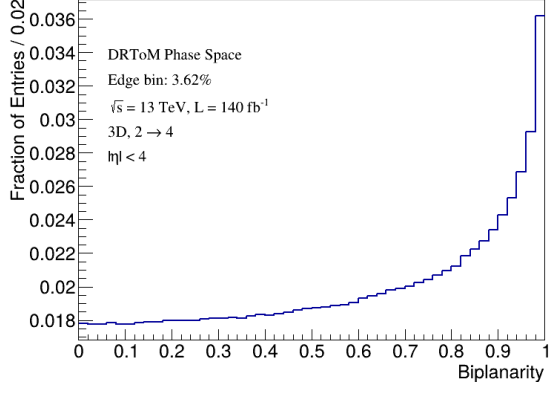
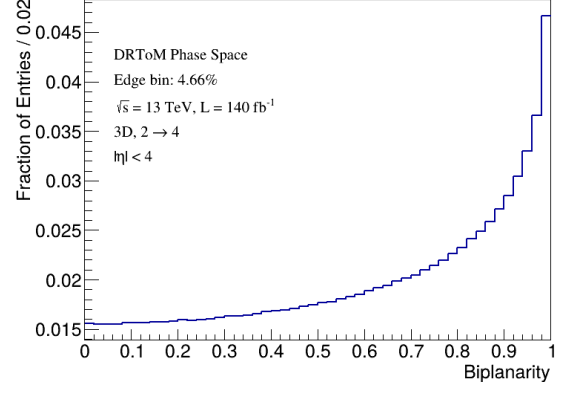


Figure G.1: Biplanarity distribution in the CoM frame for $2 \rightarrow 4$ massless partonic planar events. The distribution is the same regardless of the ordering scheme and $\vec{n}_1$ choice.

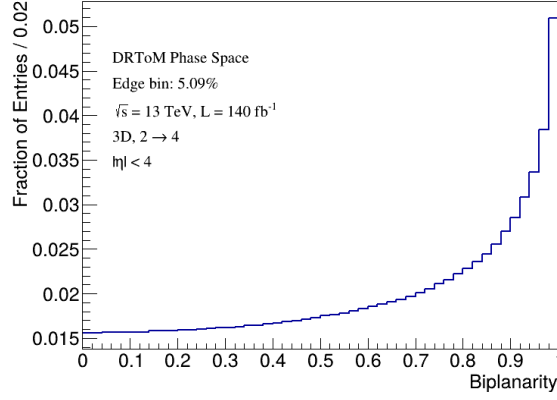
As a summary of all ordering schemes and pairing types at parton level, Table G.1 shows the final bin percentages for the lab frame and CoM frame of planar and spherical events. For the planar events we see that the angle ordering with $\vec{n}_1 = \vec{p}_a \times \vec{p}_d$ ($\vec{n}_1$ as the pair with the smallest angular separation) gives the highest final bin percentage. For



(a) $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$



(b) $\vec{n}_1 = \vec{p}_a \times \vec{p}_c$



(c) $\vec{n}_1 = \vec{p}_a \times \vec{p}_d$

Figure G.2: Biplanarity distributions for different $\vec{n}_1$ definitions in the lab frame using the p_T ordering scheme of $2 \rightarrow 4$ massless partonic spherical events.

spherical events, we see that in the both frames the angle ordering with $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$ ($\vec{n}_1$ as the pair with the largest angular separation) results in the smallest final bin percentage.

G.2 Jet Level

At jet level, we consider four massless parton input cases: $2 \rightarrow 2$ spherical, $2 \rightarrow 3$ spherical, and $2 \rightarrow 4$ spherical and planar. The first two act as a means to investigate the containment from non 4-parton events so we want to use the definition that minimizes the planar bin of these cases (i.e. minimize the spherical/background contribution). The $2 \rightarrow 4$ spherical and planar cases act as the next logical step after the parton level study. In each case use $p_T > [480, 20, 20, 20]$, and all jets have the requirement $|\eta| < 2.5$.

Table G.1: Final bin percentage for $2 \rightarrow 4$ massless partonic spherical and planar events in the CoM and lab frame. There are no p_T requirements on the partons and they were generated between 2 and 11 TeV with $|\eta| < 4$. The optimal values for each case are in bold.

	$\vec{p}_a \times \vec{p}_b$	$\vec{p}_a \times \vec{p}_c$	$\vec{p}_a \times \vec{p}_d$
Planar, Lab			
p_T	24.28	28.85	36.07
$ \vec{p} $	25.40	30.40	33.39
Angle	21.24	28.13	39.83
Spherical, CoM			
p_T	12.32	12.94	13.02
$ \vec{p} $	12.78	12.82	12.68
Angle	8.72	18.69	10.87
Spherical, Lab			
p_T	3.33	4.27	4.55
$ \vec{p} $	3.58	4.19	4.39
Angle	2.97	5.24	3.95

Instead of showing plots, we skip to the results which are shown in Table G.2. The ‘Table’ column shows which process and frame the results are from. The next three columns, Best $p_T/|\vec{p}|$ /Angle Pair, show which cross product of the momentum vectors $\vec{p}_i$ for $i = a, b, c, d$ results in the final bin with the smallest value (for $2 \rightarrow 2, 3, 4$ spherical events), and the final bin with the highest value (for $2 \rightarrow 4$ planar events). In these columns is also the final bin percentage in brackets. For example, if we are looking at $2 \rightarrow 2$ CoM, then the cross product that results in the smallest final bin for all three ordering cases is when $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$. The final column, Best Ordering, shows which of the three ordering options (of p_T , $|\vec{p}|$, and angle) are the best.

It’s clear that the best ordering parameter is angle. If working in the CoM frame, then $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$ (the largest angle separation) minimizes the final bin for spherical events, whereas in the lab frame $\vec{n}_1 = \vec{p}_a \times \vec{p}_d$ (the smallest angle separation) is optimal. If trying to maximize planar events, then it’s best to again order by angle but use $\vec{n}_1 = \vec{p}_a \times \vec{p}_c$ (the intermediate angle separation) in the CoM frame. If in the lab frame then the $|\vec{p}|$ ordering is best with $\vec{n}_1 = \vec{p}_a \times \vec{p}_d$.

Taking a holistic approach to our definition of biplanarity, it is obvious to choose the

angle ordering as this, across all cases, is the best ordering choice. Since we are mainly working in the CoM frame (this is where we are defining our discriminating variables) and the ab pairing minimizes the background across all three spherical cases, we use $\vec{n}_1 = \vec{p}_a \times \vec{p}_b$. If working in the lab frame, one should use the ad pairing, whereas if the goal is to solely maximize the planar bin for $2 \rightarrow 4$ events in the CoM frame, then it is best to use the ac pairing.

Table G.2: Jet-level biplanarity results with $p_T > [480, 20, 20, 20]$ and $|\eta| < 2.5$ for all jets. The optimal values for each row are in bold.

Table	Best p_T Pair	Best $ \vec{p} $ Pair	Best Angle Pair	Best Ordering
$2 \rightarrow 2$				
CoM	ab (14.47)	ab (16.18)	ab (12.72)	Angle
Lab	ab (7.12)	ab (9.43)	ad (5.73)	Angle
$2 \rightarrow 3$				
CoM	ab (12.57)	ab (13.50)	ab (9.84)	Angle
Lab	ab (3.30)	ad (5.03)	ad (3.09)	Angle
$2 \rightarrow 4$ Spherical				
CoM	ab (11.23)	ab (12.06)	ab (6.79)	Angle
Lab	ab (2.17)	ab (3.51)	ad (2.13)	Angle
$2 \rightarrow 4$ Planar				
CoM	ac (47.37)	ac (41.18)	ac (54.58)	Angle
Lab	ad (8.61)	ad (9.79)	ad (8.58)	$ \vec{p} $

Appendix H

Three Types of Jet Level Plots

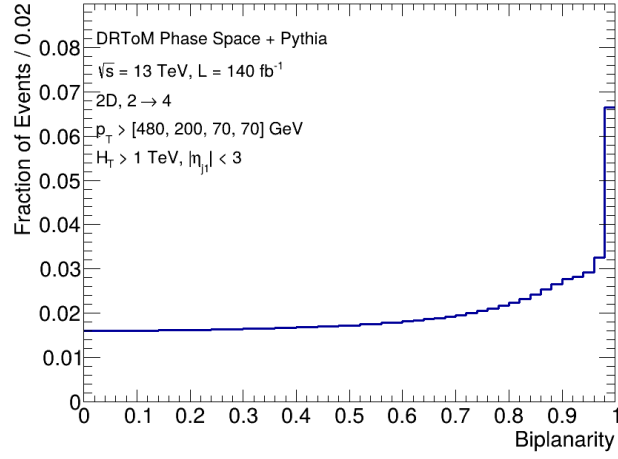
This appendix aims to highlight the differences in plotting schemes from Sec. 7.3.1 (i.e. raw counts, weighted counts, and normalized to data). We only show distributions for spherical $2 \rightarrow 4$ massless partonic phase space events, and we only consider the (event normalized) biplanarity event shape variable in the CoM frame and leading 4-jet invariant mass distributions. The events were generated based on Table C.1 and Table C.2. Across all three plots we use the requirements applied to Run 2 data: a leading jet p_T requirement of $p_T > 480$ GeV and $|\eta| < 3$, the second jet a requirement of $p_T > 200$ GeV, the third and fourth jet a requirement of $p_T > 70$ GeV, and there is p_T scalar sum across all four jets to be greater than 1 TeV, $H_T > 1$ TeV (as shown on the plots).

Figure H.1 shows the biplanarity distributions across each plotting scheme (50 total bins used for each for a bin width of 0.02). We see that all three distributions retain their overall shape irregardless of the weighting scheme applied. What does change is the number of events that the bins correspond to. For the raw case, we see that the $B = 1$ bin has fraction of about 0.066; then for the weighted case and normalized to data case (which is scaled by the scale factor 0.05 and fractional events rounded) it is at 0.062 (which shows us that the planar events are not being eliminated after normalizing to data). One can also note a difference between the raw and the weighted/data normalized distributions. This is most notable just before the $B = 1$ where we see that the bins lose some of the overall smoothness as they have become properly adjusted to the actual number of events (e.g. high mass events may not be as relevant, whereas low mass events get increased). Although the percent is the same, the number of values in each bin is

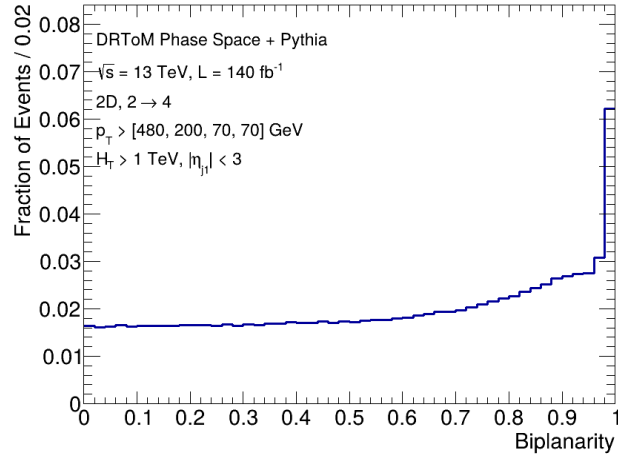
different due to the scaling factor and rounding.

Figure H.2 shows the leading 4-jet invariant mass plots (left column) and the biplanarity vs. leading 4-jet invariant mass (right column) for the different weighting schemes. Starting with the leading 4-jet invariant mass, we see that the raw counts are naturally integer events and that the distribution ranges from about 1 TeV to 12 TeV with a flat event count from 2 to 11 TeV. The reconstructed events, those from 1 TeV to 2 TeV and 11 TeV to 12 TeV are scarce, whereas our chosen generation range consistently has 10^5 events which is what we expect (from Appendix C). Scaling the events by the weight, w , we get our expected invariant mass distribution. We see that after ~ 8 TeV the distribution loses its linear drop-off for a steeper curve. After normalizing to data, the biggest difference is that (as expected) there are no more fractional events. Because of this, and due to our rescaling by the scale factor, the distribution goes out to about 8 TeV with few events going out to about 10 TeV. This is anticipated because we are scaling the events by 0.05 which means we will lose high-mass events.

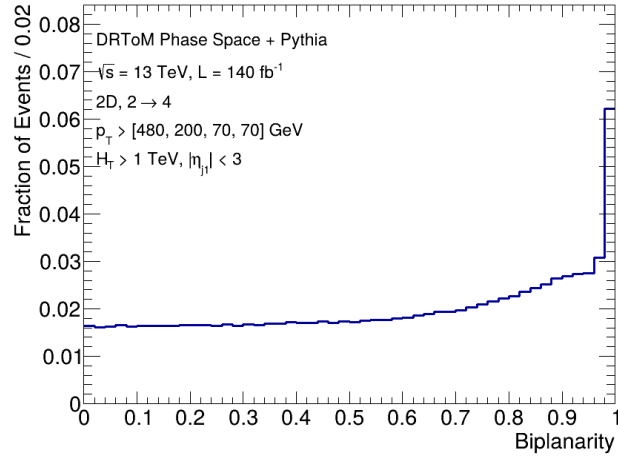
The last piece we consider is the final bin of biplanarity ($0.98 < B \leq 1.0$) as a function of the leading 4-jet invariant mass, shown in the right column of Fig. H.2. Starting with the raw counts, we see that the fraction of planar events stays at a constant ~ 0.08 up until around 11 TeV where the lack of statistics in the higher invariant mass region causes some fluctuations. For the weighted case, the constant ~ 0.08 only goes out to about 6 TeV before the weighted statistics causes more fluctuations and higher uncertainty. Lastly, after normalizing to data, the distribution resembles that of the weighted case out to about 9 TeV, where the errors are not as large due to the definite statistical measurement. However, due to the scaling factor and rounding of fractional events, the distributions ends near 9 TeV where the weighted sampling continued out to 12 TeV.



(a) Raw Counts

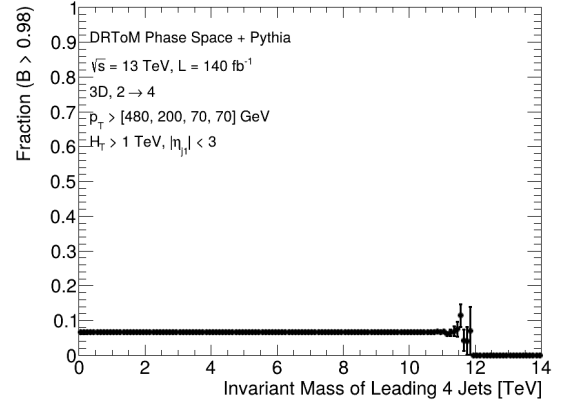
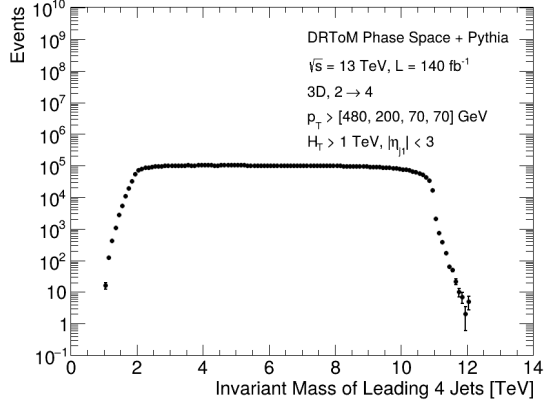


(b) Weighted Counts

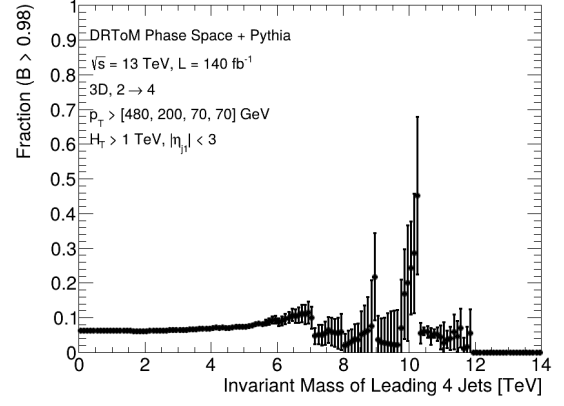
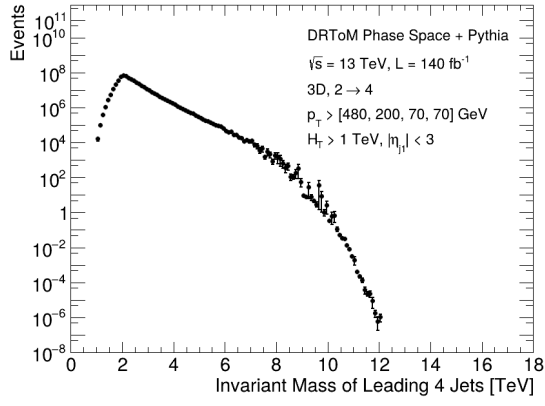


(c) Normalized to Data

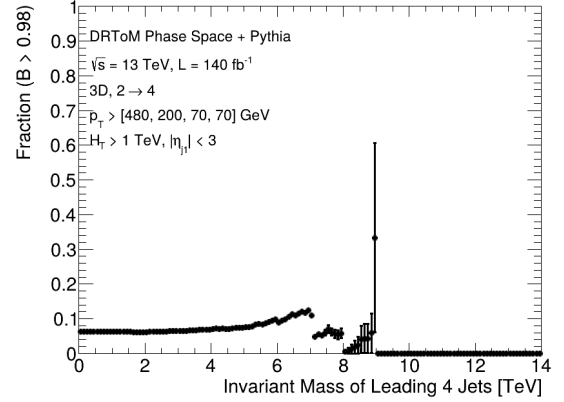
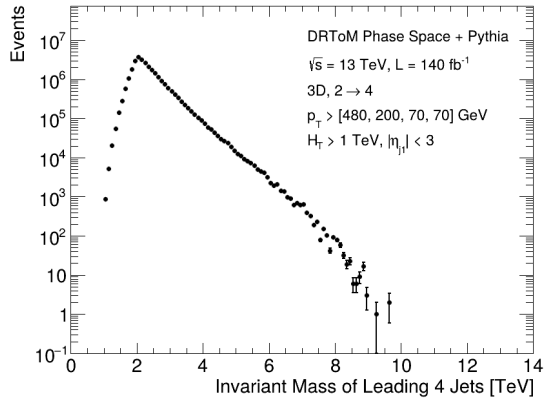
Figure H.1: Biplanarity distributions of $2 \rightarrow 4$ spherical events after hadronization in the CoM frame for different plotting/weighting schemes.



(a) Raw counts.



(b) Weighted by w .



(c) Normalized to data.

Figure H.2: Left column: leading 4-jet invariant mass distributions of $2 \rightarrow 4$ spherical events after hadronization. Right column: biplanarity as a function of total jet invariant mass of $2 \rightarrow 4$ spherical events in the CoM frame after hadronization.

H.1 Statistical Uncertainty

For the leading 4-jet invariant mass distributions, if N is the number of counts in a bin, then the uncertainty is taken to be the standard Poisson counting uncertainty [56],

$$\sigma_N = \sqrt{N}. \quad (\text{H.1})$$

For the fraction plots, the quantity of interest is the fraction of events above a given leading 4-jet invariant mass threshold that also satisfy a specific event shape requirement [57]. For aplanarity, the plotted fraction is

$$f_{A/B}(M_{\text{cut}}) = \frac{N(A_{\text{cut}}/B_{\text{cut}}, M_{4j} \geq M_{\text{cut}})}{N(M_{4j} \geq M_{\text{cut}})}. \quad (\text{H.2})$$

where, if looking at aplanarity, we use A_{cut} as $A < 0.01$, and for biplanarity B_{cut} is $B > 0.98$. In both cases, N is used to represent the number of events that pass the requirement within the brackets, M_{4j} is the leading 4-jet invariant mass, and M_{cut} is the invariant mass threshold the cumulative fraction is being conducted at. More generally, the plotted fraction can be written as

$$f = \frac{S}{T}, \quad (\text{H.3})$$

where S is the selected yield passing the event shape cut and invariant mass threshold, and T is the total yield above the invariant mass threshold. Since the numerator is a subset of the denominator, we treat this as a binomial pass/fail quantity in spite of the denominator being a random variable.

For an unweighted sample with exactly N_{Tot} total events, the standard binomial uncertainty on a measured fraction is

$$\sigma_f = \sqrt{\frac{f(1-f)}{N_{\text{Tot}}}}. \quad (\text{H.4})$$

For a weighted sample, the total yield above the invariant mass threshold is

$$T = \sum_i w_i, \quad (\text{H.5})$$

where the event weights w_i are the weights of slices from Eq. 7.3 for the corresponding parton level mass slice. The MC statistical variance of this yield is estimated as

$$\sigma_T^2 = \sum_i w_i^2. \quad (\text{H.6})$$

An effective number of entries can consequently be defined by matching the relative statistical uncertainty of the weighted sample to that of an unweighted sample

$$\frac{\sigma_T}{T} = \frac{1}{\sqrt{N_{\text{eff}}}}, \quad (\text{H.7})$$

which gives

$$N_{\text{eff}} = \frac{(\sum_i w_i)^2}{\sum_i w_i^2}. \quad (\text{H.8})$$

This quantity represents the number of unweighted events that would give the same relative statistical uncertainty as the weighted sample. In the unweighted limit, where every event has weight $w_i = 1$, we obtain

$$T = N_{\text{Tot}}, \quad \sigma_T^2 = N_{\text{Tot}}, \quad (\text{H.9})$$

and therefore

$$N_{\text{eff}} = \frac{N_{\text{Tot}}^2}{N_{\text{Tot}}} = N_{\text{Tot}}. \quad (\text{H.10})$$

Thus, the weighted expression correctly reduces to the usual unweighted event count.

Using the effective number of entries, the uncertainty on the fraction is taken to be

$$\sigma_f = \sqrt{\frac{f(1-f)}{N_{\text{eff}}}}. \quad (\text{H.11})$$

Then, Eq. H.11 is the same as Eq. H.4 for the unweighted case. For the data-like plots, each bin is scaled and rounded such that N_{eff} is the total rounded data-like counts above the given invariant mass threshold.

We note that this is a simplified approach to the uncertainty statistics. Because jets are undergoing mass migration, each bin of M_{4j} has entries from several $2 \rightarrow 4$ parton bins each with their own (different) weight. A more in depth uncertainty propagation starting from parton level must be conducted and carried through to TRUTH3.

Appendix I

Neural Network

In Chapter 7 we found that the planar features of DR in topological event shape variables diminishes when using jets compared to partons. Because of this, we want another method to investigate and classify the topology of events, mainly in differentiating between planar and spherical events. This problem, in classifying events between two distinct classes, is ideal for neural networks (NN).

Using PyTorch, we consider a fully connected multilayer perceptron with rectified linear unit (ReLU) activations applied to all hidden layers. The ReLU activation function is defined as

$$\text{ReLU}(x) = \max(0, x), \quad (\text{I.1})$$

where x is the input to the activation function. The negative inputs are mapped to zero, while positive inputs are passed through unchanged, introducing nonlinearity into the network (so that the layers don't reduce to a single linear transformation) while remaining computationally efficient [58, 59]. We choose the input parameters of the NN to be a combination of momentum three vectors of the particles as this is all that is needed to classify the topology of an event (as seen in Sec. 6.2). The specific choice will be discussed in more detail below. Because the task is binary classification, where spherical events are labelled as 0 and planar events as 1, the network outputs a single value between 0 and 1 through a sigmoid activation function,

$$\sigma(x_f) = \frac{1}{1 + e^{-x_f}}, \quad (\text{I.2})$$

which can be interpreted as the probability that an event is planar. Here, x_f is the (final)

output of the neural network before the sigmoid activation, and $\sigma(x_f)$ is the corresponding probability. The probability that an event is spherical is therefore $1 - \sigma(x_f)$.

We study two cases. First we train and validate the model at parton-level as an idealized investigation of the problem. From this we get a baseline network structure and then consider the predictive nature of the network at jet-level (a simplified case where the inputs are jet momenta instead of parton momenta, ignoring any additional degrees of freedom from jets). The specific training parameters (and model evaluation tools) for both cases are outlined in Appendix J.

I.1 Hidden Layer Structure From Parton Level

At parton-level we can consider training on $2 \rightarrow n$, for $n \in [2, 5]$, massless partonic phase space, or $2 \rightarrow 2$ QCD. Given our discussion in Sec. 3.4, we want to focus on events with four or more outgoing particles, so we choose to solely investigate $2 \rightarrow 4$ massless partonic phase space events. As a base choice (one that is simple yet contains the full topological information of an event) we choose the input parameters to be the CoM unit-vector three-momentum of each particle, $[(\hat{p}^x, \hat{p}^y, \hat{p}^z)_i], i = a, b, c, d$, resulting in twelve input features per event. The parton generation follows Table C.1 with no p_T requirements applied. However, we do not use all 9×10^6 events. When using a different number of events, it is noted in the text below.

After investigating two and three hidden layer structures (discussed in Appendix I) we use a network that has three hidden layers with 128 nodes each. In this appendix we consider two model metrics: the validation accuracy and the validation loss. The accuracy is defined as the fraction of events that are assigned the correct class label, and therefore measures the overall classification performance of the network. For this model it was found to be 0.964 (if using a total of 1×10^6 spherical and planar events, or 0.917 for 1×10^5 spherical and planar events). Loss quantifies the disagreement between the predicted probabilities of the NN and the true event labels (of planar or spherical). Lower values correspond to predictions that are both more accurate and more confident, while

increasing validation loss (as in: it worsens) despite continued improvement in training loss can indicate that the model is beginning to overfit the training data. We use binary cross-entropy with logits loss which is defined as

$$L = -\frac{1}{N} \sum_i^N (y_i \log(p_i) + (1 - y_i) \log(1 - p_i)), \quad (\text{I.3})$$

where y_i denotes the true class label (0 or 1), $p_i = \sigma(x_i)$ is the predicted probability for the class, and N is the total number of samples in the dataset (per batch) [60]. We use Adam to optimize the network's weights and biases to minimize the loss function [61].

Figure I.1a shows the training and validation accuracy of the model after each epoch, and Fig. I.1b shows the training and validation loss after each epoch. We note that the validation steps were not performed after each epoch, but after a validation interval of ten epochs. So, although the training data has five hundred epochs, the validation only has fifty. We use 1×10^6 spherical and planar events for a total of 2×10^6 events. What we take away from these plots is that the training accuracy and loss improve up to about the one hundredth epoch before plateauing, but the best epoch (vertical dashed line) is found at the upper range. This tells us that with more epochs, the model could continue to improve.

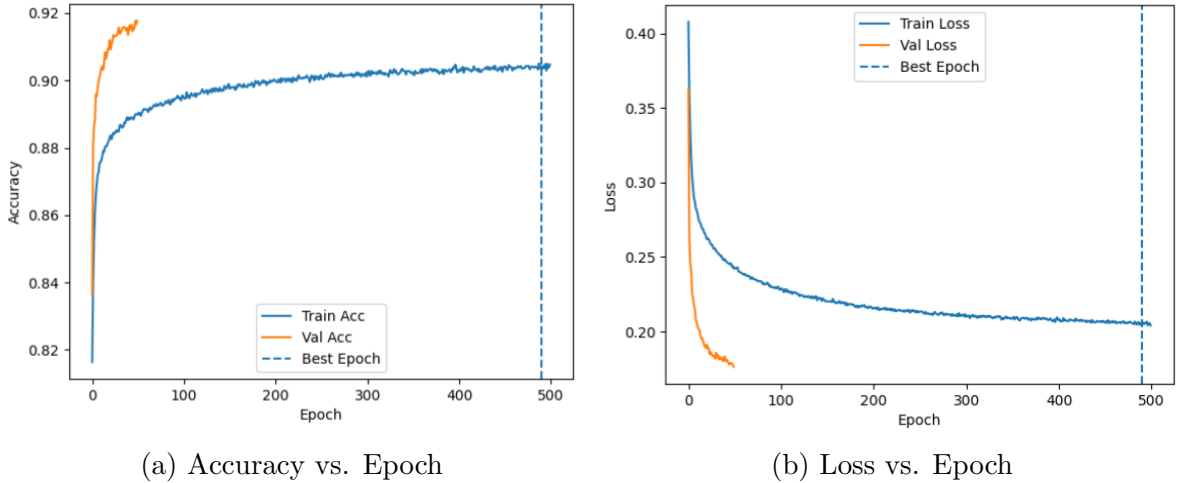


Figure I.1: Parton-level training and validation accuracy and loss. We use 1×10^6 random $2 \rightarrow 4$ massless partonic spherical and planar events with invariant mass between 2 and 11 TeV. There are no p_T requirements but they must satisfy $|\eta| < 4$.

The next plot we consider is the validation probability distribution in Fig. I.2. The

histograms display the unit-normalized predicted probability for each class. The predicted probability means that if the output is above 0.5, then it is predicting the correct class, but the level of confidence is worse the closer it is to the mid point. We see that both classes have distinct peaks, but there is overlap across the entire range (concentrated in the middle region). This tells us the model is overall confident at distinguishing between the classes, where (looking at the log plot) we see that it is more likely for a planar event to be classified as spherical than a spherical as planar (which aligns with our expectations from the main text given the smearing of planar events).

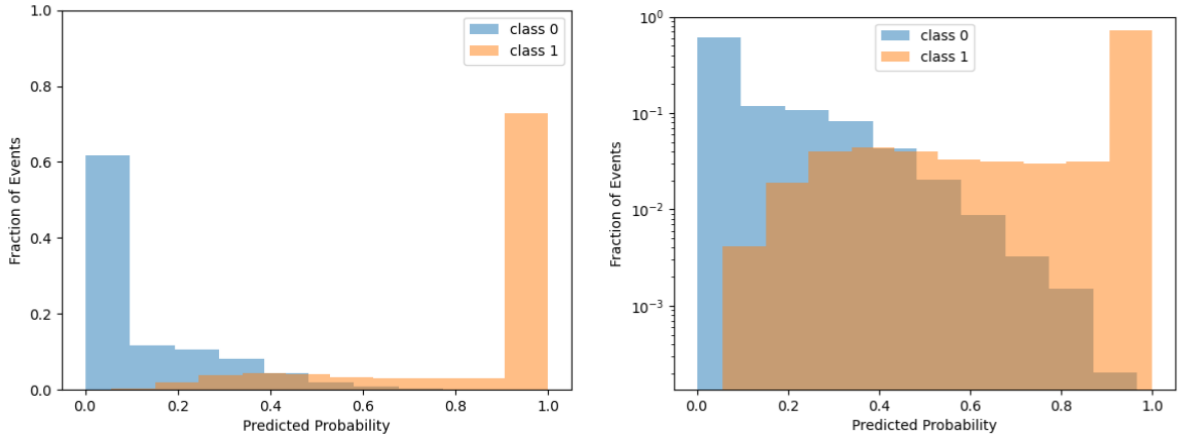


Figure I.2: Parton-level probability class histogram normalized to unity. The left plot is on a linear scale, and the right is on a logarithmic scale. We use 1×10^6 random $2 \rightarrow 4$ massless partonic spherical and planar events with invariant mass between 2 and 11 TeV. There are no p_T requirements but they must satisfy $|\eta| < 4$.

The last figure we consider is the representation of the exact fractions of true and false positive classifications in Fig. I.3. This is called the (row normalized) confusion matrix. The confusion matrix highlights the exact number of false positives (the top right being how many spherical events are labelled as planar events and the bottom right how many planar events as spherical events) and true hits for each class. We see that the spherical class (the top left panel) is accurately chosen 97% of the time, and planar events (the bottom right) 86% of the time. As such, and as we would anticipate, the NN is better at classifying spherical events than planar events.

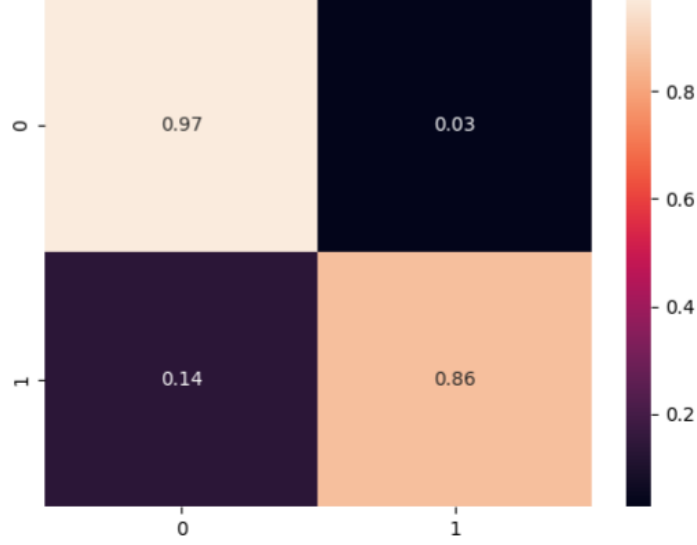


Figure I.3: Parton-level row normalized confusion matrix. We use 1×10^6 random $2 \rightarrow 4$ massless partonic spherical and planar events with invariant mass between 2 and 11 TeV. There are no p_T requirements but they must satisfy $|\eta| < 4$.

I.2 Jet Level Results

With a hidden layer structure from parton-level, we now apply the full NN to jet level inputs. At jet level we know (from Sec. 7.4) there is a smearing effect of planar events that makes them appear spherical. What we want, then, is to see if the NN can properly parse the smearing of $2 \rightarrow 4$ massless phase space events to accurately classify planar events. There is no weighting factor applied to the jets, and we use the same events from parton level (after being hadronized with no additional requirements).

As we move away from the idealized parton input, we first investigate the NN for various momenta input parameters. We consider jets with no p_T requirements, as well as specific p_T requirements (discussed below), where in both cases the jets are ordered from largest to smallest p_T . We test the NN on four different jet momenta orientations all in the CoM frame. The first jet momenta input we consider is the unit-vector three-momentum (as was done at parton-level); the second is the four-momentum; the third is the four-momentum where the three-momenta are the unit vectors and the energy is divided by the total energy of the event (E_{Tot}); and the fourth is the three-momentum in cylindrical coordinates. A summary of the different momenta input is given in Table I.1.

Table I.1: Input parameters at jet-level; i runs over the four leading p_T ordered jets.

Unit three-momentum	$[(\hat{p}^x, \hat{p}^y, \hat{p}^z)_i]$
Four-momentum	$[(p^x, p^y, p^z, E)_i]$
Normalized four-momentum	$[(\hat{p}^x, \hat{p}^y, \hat{p}^z, \frac{E_i}{E_{\text{Tot}}})_i]$
Cylindrical three-momentum	$[(p_T, \eta, \phi)_i]$

The same learning parameters, events, and network evaluation metrics that were used at parton level are used to train and validate the model at jet level (see Appendix I). Starting with the p_T requirement of $[0, 0, 0, 0]$ (following the convention of Chapter 7, where here we are only applying these requirements to the first four jets of an event—the final value is not applied to all subsequent jets after the fourth) in Table I.2, we see that the highest validation accuracy comes from the unit three-momentum and normalized four-momentum inputs, around 65%. The four-momentum and cylindrical three-momentum have a validation accuracy of about 50% which means that the NN is just guessing the output and will not be further considered.

Table I.2: Jet validation accuracy and loss with $p_T > [0, 0, 0, 0]$ for different momenta inputs. The jets satisfy $|\eta| < 4$ and are not weighted.

Input	Accuracy	Loss
Unit three-momentum	0.650	0.633
Four-momentum	0.498	0.693
Normalized four-momentum	0.662	0.623
Cylindrical three-momentum	0.504	0.693

We now apply a p_T requirement of $p_T > [480, 480, 480, 480]$ and retain the $|\eta| < 4$ requirement on all jets to resemble as closely the parton level study. This p_T requirement choice gives a $P(4 | 4)$ value of 67% and contamination from $2 \rightarrow 2, 3$ of 12%. The aim of this is to only feed the network events that we know are built from $2 \rightarrow 4$ massless parton events to further evaluate the model. We use 1×10^6 events for both planar and spherical events. Table I.3 shows the validation and loss results for the unit three-momentum and normalized four momentum inputs given these p_T requirements. We see that the

validation accuracy significantly improve (by roughly ~ 0.14 for both), approaching the parton level values (which were at 0.964 and 0.122, respectively). We note that as an additional step, different hidden layer structures were tested (e.g. four or five hidden layers with different node structures) but the results shown here are the best across all tests. We also considered the case when the loss was updated based on the weight of each event, w_i ,

$$L = -\frac{1}{N} \sum_i^N w_i (y_i \log(p_i) + (1 - y_i) \log(1 - p_i)) \quad (\text{I.4})$$

but found no additional improvement using this method (in fact, it was found to be worse).

Table I.3: Jet validation accuracy and loss with p_T requirements [480, 480, 480, 480] for different momenta inputs. Every jet satisfies $|\eta| < 4$ and are not weighted.

Input	Accuracy	Loss
Unit three-momentum	0.777	0.493
Normalized four-momentum	0.779	0.491

For the sake of completion, and to compliment the parton level results, we again show the four plots of accuracy and loss (unweighted case) after each epoch, the probability histograms, and the confusion matrix, for the unit three-momentum input with a requirement of $p_T > [480, 480, 480, 480]$. Looking at Fig. I.4, we see that the validation accuracy and loss has a steep increase (by about 0.02 better than the training curve in both cases), with slight improvement after which is similar to what we saw at parton-level. We also note that the best model (the best epoch) is located near 400 which is slightly sooner than the parton case. This means that the learning of classes given jet momenta happens sooner than that of partons, but can still be improved upon.

Figure I.5 shows the normalized class histogram distribution. It is clear that the NN is not as confident in separating the two classes (compared to parton-level) as a large number of events are overlapping in the middle section instead of distinct class 0 and 1 spikes at their respective ends. This aligns with our expectation of the smearing of planar events into spherical events, however we are seeing the the network is applying this

same degradation to spherical events (by mistaking spherical events for planar events). Lastly, Fig. I.6 is the row normalized confusion matrix where we again see that the NN is (partially) better at classifying spherical events over planar ones (only by 2% here).

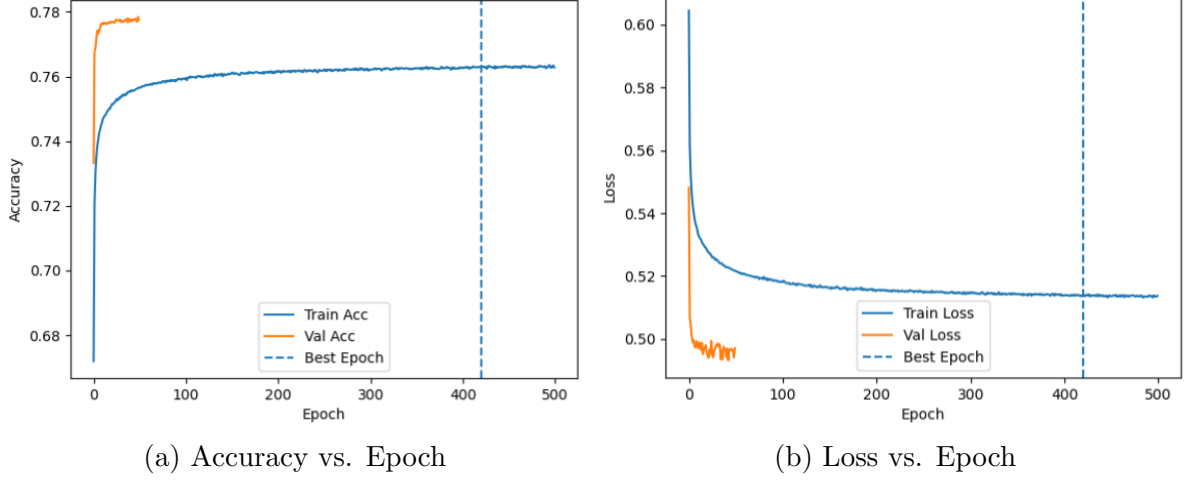


Figure I.4: Jet-level training and validation accuracy and loss (for unweighted jets). We use 1×10^6 random $2 \rightarrow 4$ massless partonic spherical and planar events with invariant mass between 2 and 11 TeV. We use p_T requirements of $[480, 480, 480, 480]$, and the all jets must satisfy $|\eta| < 4$.

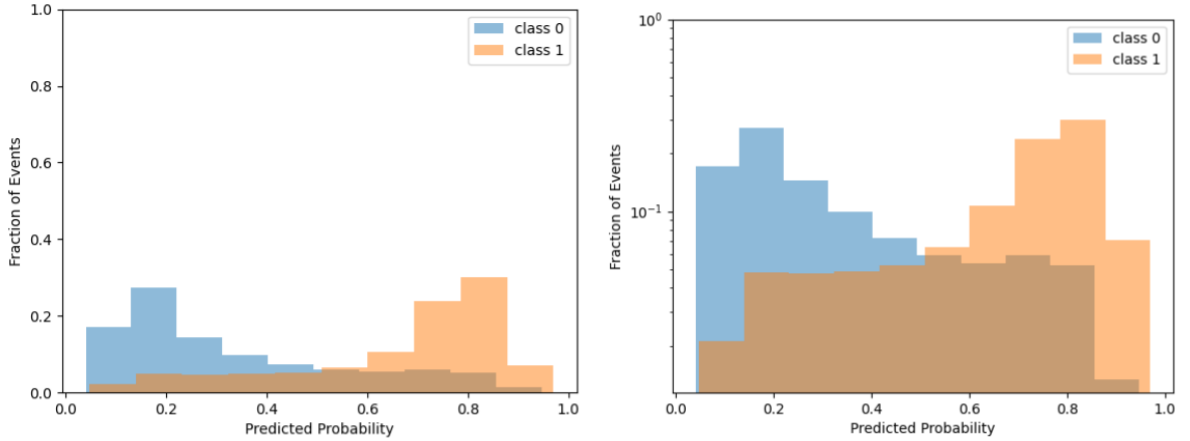


Figure I.5: Normalized jet-level probability class histogram (for unweighted jets). The left plot is on a linear scale, and the right is on a logarithmic scale. We use 1×10^6 random $2 \rightarrow 4$ massless partonic spherical and planar events with invariant mass between 2 and 11 TeV. We use p_T requirements of $[480, 480, 480, 480]$, and all jets must satisfy $|\eta| < 4$.

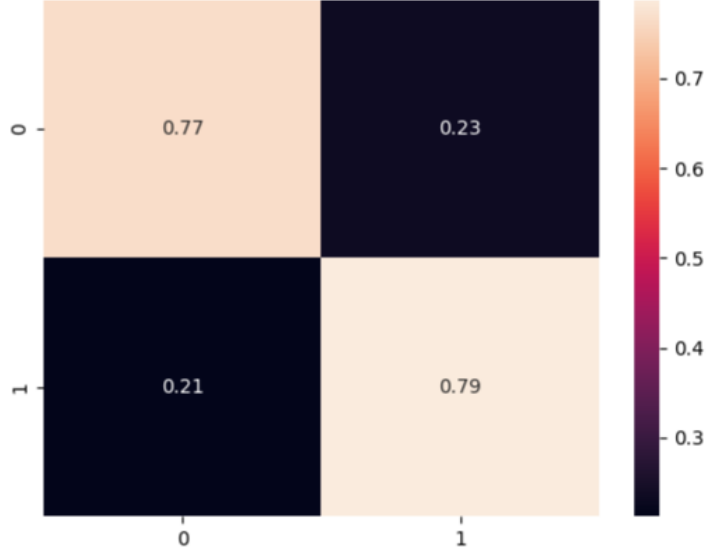


Figure I.6: Jet-level row normalized confusion matrix (for unweighted jets). We use 1×10^6 random $2 \rightarrow 4$ massless partonic spherical and planar events with invariant mass between 2 and 11 TeV. We use p_T requirements of $[480, 480, 480, 480]$, and all jets must satisfy $|\eta| < 4$.

I.3 Next Steps

From our studies, we see that the NN does not distinguish between jet-level planar and spherical events at the same accuracy that it does for partonic events. This is the same trend we saw for the event shape variables. It is clear, then, that after showering and hadronization the momenta of jets for planar events are scattered such that they appear spherical. Regardless of what they started as, if the momenta are not in a plane then the NN (in it's current state and solely using momenta vectors as input) will not be able to learn and thus not be able to identify planar from spherical events at a sufficient level ($> 90\%$). Thus, the downside of using event shape variables as parameters for jet event topologies transfers to this structure of NN.

As a means to improving the structure, one could implement more input parameters along with the current best choices of momenta vectors. Information such as the angles between the jets, the jet mass, and all possible topological event shape variables of the events at parton and jet level, could help aid in the class distinction.

Appendix J

Neural Network Extended

In this Appendix we further outline and discuss the results of the NN at parton and jet level. First we show the results of our hidden layer study (to find the baseline structure for the NN). We investigate two hidden layer and a three hidden layer structure with differing number of neurons per layer. The dataset used (for this) consisted of random 1×10^6 spherical and 1×10^6 planar $2 \rightarrow 4$ massless partonic events with no p_T requirements, but satisfied $|\eta| < 4$, and had an invariant mass between 2 and 11 TeV. The dataset was split into 80% training, and 20% validation sets. We used a learning rate of 0.01, training was performed using mini-batch sizes of 4096, and early stopping based on validation loss was employed to prevent overfitting with the best-performing model over the 500 epochs retained.

The overall performance of the models were evaluated using the training accuracy (Train. Acc.), validation accuracy (Val. Acc.), and the validation loss (Val. Loss) from the best model, as well as the area under the receiver operating characteristic curve (AUC). The AUC quantifies the ability of the classifier to distinguish between the two classes across all possible classification thresholds (where we have been using 0.5). The receiver operating characteristic curve is obtained by plotting the true positive rate against the false positive rate as the decision threshold is varied. The AUC corresponds to the area under this curve, with a value of 0.5 indicating performance equivalent to random guessing and a value of 1.0 corresponding to perfect separation between the classes. Table J.1 and Table J.2 show the values of the two and three hidden layer tests, respectively. We see that the hidden layer structures with the most promising results

(governed by the largest values of Train. Acc., Val. Acc, and AUC) are hidden layers [128, 128, 128] and [256, 256, 256] with validation accuracies of ~ 0.96 . Since the former is less complex (in terms of number of nodes) and gives approximately the same results, we consider this our baseline structure.

Table J.1: Two hidden layers summary chart. We use 1×10^6 random $2 \rightarrow 4$ massless partonic spherical and planar events with invariant mass between 2 and 11 TeV. There are no p_T requirements but they must satisfy $|\eta| < 4$.

Hidden Layers	Train. Acc.	Val. Acc.	Val. Loss	AUC
32, 32	0.796	0.796	0.431	0.865
32, 64	0.838	0.837	0.369	0.898
64, 32	0.828	0.826	0.384	0.890
64, 64	0.866	0.863	0.327	0.918
128, 12	0.833	0.831	0.379	0.893
128, 128	0.884	0.880	0.299	0.927
192, 192	0.889	0.886	0.287	0.933
256, 256	0.860	0.858	0.338	0.913

Table J.2: Three hidden layers summary chart. We use 1×10^6 random $2 \rightarrow 4$ massless partonic spherical and planar events with invariant mass between 2 and 11 TeV. There are no p_T requirements but they must satisfy $|\eta| < 4$.

Hidden Layers	Train. Acc.	Val. Acc.	Val. Loss	AUC
32, 64, 32	0.902	0.900	0.261	0.941
64, 64, 64	0.920	0.918	0.229	0.952
128, 128, 128	0.968	0.964	0.122	0.979
128, 256, 128	0.899	0.898	0.258	0.941
192, 192, 192	0.906	0.902	0.255	0.943
256, 256, 256	0.970	0.967	0.111	0.982

To investigate the network further, we test the model on a reduced dataset of 1×10^5 planar events and 1×10^5 spherical events with a training set of 80% (20% validation set), mini-batch sizes of 512, and a learning rate of 0.001. The results of the best model are shown in Table J.3, noting an AUC value of 0.982. Table J.4 has extensions (including the training accuracy and AUC) of Table I.2 and Table I.3 from Appendix I, respectively.

Table J.3: Results for the parton data generation with 1×10^5 planar and spherical events.

Train. Acc.	Val. Acc.	Val. Loss	AUC
0.935	0.917	0.174	0.982

Table J.4: Jet level training for different p_T requirements and momenta inputs. The jets are not weighted and all jets satisfy $|\eta| < 4$.

Input	Train. Acc.	Val. Acc.	Val. Loss	AUC
$p_T > [0, 0, 0, 0]$				
Unit three-momentum	0.670	0.650	0.633	0.699
Four-momentum	0.501	0.498	0.693	0.500
Normalized four-momentum	0.684	0.662	0.623	0.712
Cylindrical three-momentum	0.499	0.504	0.693	0.500
$p_T > [480, 480, 480, 480]$				
Unit three-momentum	0.779	0.777	0.493	0.844
Normalized four-momentum	0.782	0.779	0.491	0.845

As a final means of discussion, we note that the best epoch for the normalized four-momentum was at 490 (which is better than the normalized three-momentum input). This suggests that with more training, and more events, this structure of NN could continue to improve. The AUC could extend beyond 0.9 and we could begin to see accuracies greater than 90%. A test case of 1000 epochs and 2×10^6 planar and spherical events was run and the AUC was 0.84 suggesting the NN structure itself needs to be updated, or we need to increase the p_T requirements (e.g. to $[500, 500, 500, 500]$).