April 19, 1999

# Helicity Amplitude Calculation of Vector Leptoquark Production and Decay in Electron-Positron Annihilation

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A full calculation of vector leptoquark production and decay in $e^+e^-$ annihilation is presented. Explicit results for the contribution to the production of each initial and final state helicity is give. These expressions can thus be implemented in a Monte Carlo event generator for vector leptoquark production and decay.

## 1 Introduction

We start with an effective Lagrangian describing the interactions of leptoquarks with the known particles. As before [1], we require that the effective Lagrangian satisfy baryon and lepton number conservation, have non-derivative and family-diagonal couplings to lepton-quark pairs, and respect the $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ symmetry of the standard model. With these requirements, the possible representations of the leptoquarks with respect to the gauge groups and the couplings to the gauge bosons are completely determined. Only the leptoquark masses and strengths of the leptoquark-fermion couplings remain as free parameters.

## 2 Feynman Rules

Based on the effective Lagrangian of reference [1], we explicitly state the Feynman rules that will be used in the following calculations. The lepton-quark interactions with vector leptoquarks, $V_\mu$, are given by terms in the effective Lagrangian of the generic form

$$\mathcal{L}_{lqV} \sim g_\tau \overline{q}_{\tau'} \gamma^\mu l_\tau V_\mu, \tag{1}$$

where $\tau = L(R)$ and $\tau' = L(R)$ denote the $\mathrm{SU(2)_L}$ lepton and quark doublets (singlets) respectively. The $g_\tau$'s are arbitrary coupling constants. These terms in the effective Lagrangian give rise to the well known vertex factor

$$-ig_\tau\gamma_\mu. \tag{2}$$

The interactions of vector leptoquarks with the photon and Z-boson are given by terms in the effective Lagrangian of the generic form

$$\mathcal{L}_{VV\gamma(Z)} = -\frac{1}{2}[D_\mu V_\nu - D_\nu V_\mu]^\dagger[D^\mu V^\nu - D^\nu V^\mu], \tag{3}$$

where $D_\mu = \partial_\mu - ieQ^\gamma A_\mu - ieQ^Z Z_\mu$ is the covariant derivative. The charges are given by $Q^\gamma = Q_{em}$ and $Q^Z = (T_3 - Q_{em}\sin^2\theta_W)/(\cos\theta_W \sin\theta_W)$, where $Q_{em}$ and $T_3$ are the electromagnetic charge and third component of isospin of the given vector leptoquark.

The relevant Feynman rule for the vertex involving a pair of vector leptoquarks and a gauge boson is

$$-ieQ^{\gamma(Z)}[(k-k')_\mu g_{\alpha\beta} - k_\alpha g_{\beta\mu} + k'_\beta g_{\alpha\mu}], \tag{4}$$

where $k$ and $k'$ are the four-momenta of the outgoing vector leptoquark and anti-leptoquark respectively. This vertex factor differs from the three-gauge-boson vertex of the standard model. As noted in reference [1], this is necessary as it unlikely that light leptoquarks are gauge bosons.

The contributions from the Higgs-boson exchange to the processes considered here are ignored since they are suppressed by the ratio of the fermion mass to leptoquark mass.

# 3 Kinematics

We now explicitly state the kinematic expressions that will be used in the calculation of the process (figure 1)

$$e^+(p_+,\kappa_+) + e^-(p_-,\kappa_-) \to V(k_+,\lambda_+) + \overline{V}(k_-,\lambda_-). \tag{5}$$

The arguments of this expression indicate the momenta and helicities of the incoming fermions ($\kappa = \pm\frac{1}{2}$) and outgoing vector leptoquarks ($\lambda = -1, 0, +1$).

We assign the initial-state electron and positron to have the four-momentum[1]

$$p^\mu_\pm = \frac{\sqrt{s}}{2}(1; 0, 0, \mp 1) \tag{6}$$

in the centre-of-mass system, where $s = (p_+ + p_-)^2 = (k_+ + k_-)^2$ as usual. The final-state pair of vector leptoquark four-momentum are defined as

$$k^\mu_\pm = \frac{\sqrt{s}}{2}(1; \mp\beta\sin\theta, 0, \mp\beta\cos\theta), \tag{7}$$

where $\beta = \sqrt{1 - 4M^2/s}$ is the velocity of the leptoquarks with mass $M$ and $\theta$ is the scattering angle between the $e^+$ and $V$.

[1] We set $m_l = m_q = 0$ throughout this paper.

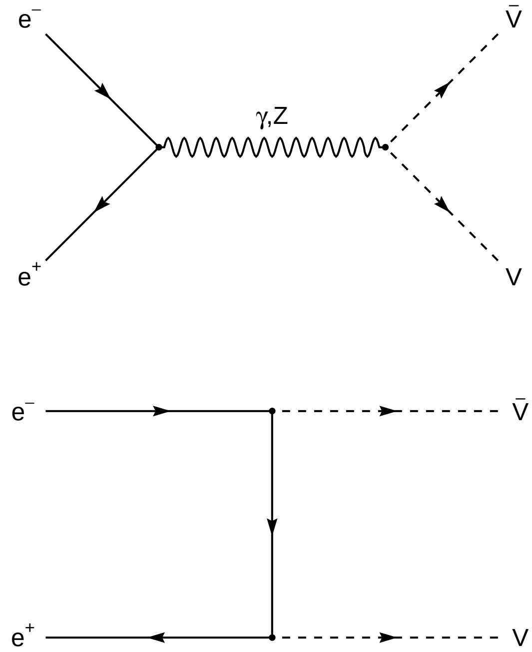


Figure 1: Lowest-order Feynman diagrams for leptoquark pair-production in $e^+e^-$ annihilations.

Because we are neglecting the electron mass, the helicity of the positron is opposite to the helicity of the electron. We may write

$$\kappa_- = -\kappa_+ = \kappa. \tag{8}$$

This reduces the number of helicity-matrix elements from 36 to 18.

The polarization vectors of the leptoquarks are

$$\epsilon^{*\mu}_{\pm}(k_\pm, \lambda = +1) = \frac{1}{\sqrt{2}}(0; \mp\cos\theta, -i, \pm\sin\theta), \tag{9}$$

$$\epsilon^{*\mu}_{\pm}(k_\pm, \lambda = -1) = \frac{1}{\sqrt{2}}(0; \mp\cos\theta, i, \pm\sin\theta), \tag{10}$$

$$\epsilon^{*\mu}_{\pm}(k_\pm, \lambda = 0) = \frac{1}{\sqrt{1-\beta^2}}(\beta; \mp\sin\theta, 0, \mp\cos\theta). \tag{11}$$

Normalization and orthogonality of the polarization vectors[2] is shown in table 1.

The dot products of the particle momentum vectors with the polarization vectors are given in table 2.

[2] We have dropped the $k_\pm$ from the argument of the polarization vectors since it is implicit in the subscript of the polarization vectors.

| $\epsilon_\pm(\lambda)\cdot\epsilon(\lambda')$ | $\epsilon_\pm(+)$ | $\epsilon_\pm(-)$ | $\epsilon_\pm(0)$ | $\epsilon_\mp(+)$ | $\epsilon_\mp(-)$ | $\epsilon_\mp(0)$ |
|---|---|---|---|---|---|---|
| $\epsilon_\pm(+)$ | $-1$ | 0 | 0 | 0 | 1 | 0 |
| $\epsilon_\pm(-)$ | 0 | $-1$ | 0 | 1 | 0 | 0 |
| $\epsilon_\pm(0)$ | 0 | 0 | $-1$ | 0 | 0 | $\frac{1+\beta^2}{1-\beta^2}$ |

Table 1: Normalization and orthogonality of the polarization vectors.

| $k, p\cdot\epsilon_\pm(\lambda)$ | $\epsilon_\pm(+)$ | $\epsilon_\pm(-)$ | $\epsilon_\pm(0)$ |
|---|---|---|---|
| $k_\pm$ | 0 | 0 | 0 |
| $k_\mp$ | 0 | 0 | $\beta\sqrt{\frac{s}{1-\beta^2}}$ |
| $p_\pm$ | $+\frac{1}{2}\sqrt{\frac{s}{2}}\sin\theta$ | $+\frac{1}{2}\sqrt{\frac{s}{2}}\sin\theta$ | $\frac{1}{2}\sqrt{\frac{s}{1-\beta^2}}(\beta-\cos\theta)$ |
| $p_\mp$ | $-\frac{1}{2}\sqrt{\frac{s}{2}}\sin\theta$ | $-\frac{1}{2}\sqrt{\frac{s}{2}}\sin\theta$ | $\frac{1}{2}\sqrt{\frac{s}{1-\beta^2}}(\beta+\cos\theta)$ |

Table 2: Dot products of four-momentum and polarization vectors.

Some useful dot products between the momenta vectors are

$$p_\pm\cdot p_\pm = 0, \tag{12}$$
$$p_\pm\cdot p_\mp = \frac{s}{2}, \tag{13}$$
$$k_\pm\cdot k_\pm = \frac{s}{4}(1-\beta^2), \tag{14}$$
$$k_\pm\cdot k_\mp = \frac{s}{4}(1+\beta^2), \tag{15}$$
$$p_\pm\cdot k_\pm = \frac{s}{4}(1-\beta\cos\theta), \tag{16}$$
$$p_\pm\cdot k_\mp = \frac{s}{4}(1+\beta\cos\theta). \tag{17}$$

# 4 Helicity Amplitudes

We represent the helicity-matrix elements as $\mathcal{M}(\kappa,\lambda_+,\lambda_-,s,t)$. CP invariance implies the relation

$$\mathcal{M}(\kappa,\lambda_+,\lambda_-,s,t) = \mathcal{M}(\kappa,-\lambda_-,-\lambda_+,s,t), \tag{18}$$

which holds at the tree level for vector leptoquark production. Consequently one effectively has only 12 independent helicity-matrix elements instead of 18.

The general helicity amplitude can be decomposed into a series of basic matrix elements, $\mathcal{M}_i^\kappa(\lambda_+,\lambda_-)$, multiplied by invariant functions, $\mathcal{F}_i^\kappa(s,t)$:

$$\mathcal{M}(\kappa,\lambda_+,\lambda_-,s,t) = \sum_i \mathcal{M}_i^\kappa(\lambda_+,\lambda_-)\mathcal{F}_i^\kappa(s,t). \tag{19}$$

The basic matrix elements are purely kinematical objects and contain the complete dependence on the leptoquark polarization. The invariant functions contain the dynamical information (i.e. the model dependence) but are independent of the leptoquark polarization.

We now calculate the helicity-matrix elements for the s-channel and t-channel separately.

## 4.1 Production in the s-channel

The matrix element for s-channel production is

$$\begin{aligned} \mathcal{M}_S(\kappa, \lambda_+, \lambda_-, s, t) &= i\frac{e^2}{s}\mathcal{K}_\kappa(s)\overline{v}(p_+)\gamma_\mu u(p_-)[(\epsilon_+^* \cdot \epsilon_-^*)(k_+ - k_-)^\mu \\ &\quad -(\epsilon_-^* \cdot k_+)\epsilon_+^{*\mu} + (\epsilon_+^* \cdot k_-)\epsilon_-^{*\mu}], \end{aligned} \tag{20}$$

where

$$\mathcal{K}_\kappa(s) = Q_\kappa^\gamma(e) Q^\gamma(V) + Q_\kappa^Z(e)\frac{s}{s - M_Z^2 + iM_Z\, ,_Z} Q^Z(V), \tag{21}$$

$,_Z$ is the total width of the Z-boson and

$$Q_{L,R}^\gamma(e) = -1, \tag{22}$$

$$Q_L^Z(e) = \frac{-1/2 + \sin^2\theta_W}{\cos\theta_W \sin\theta_W}, \qquad Q_R^Z(e) = \tan\theta_W, \tag{23}$$

$$Q^\gamma(V) = Q_{em}, \tag{24}$$

$$Q^Z(V) = \frac{T_3 - Q_{em}\sin^2\theta_W}{\cos\theta_W \sin\theta_W}. \tag{25}$$

We may identify

$$\mathcal{F}_S^\kappa(s,t) = i\frac{e^2}{s}\mathcal{K}_\kappa(s) \tag{26}$$

and

$$\mathcal{M}_S^\kappa(\lambda_+, \lambda_-) = \overline{v}(p_+)\gamma_\mu u(p_-)[(\epsilon_+^* \cdot \epsilon_-^*)(k_+ - k_-)^\mu - (\epsilon_-^* \cdot k_+)\epsilon_+^{*\mu} + (\epsilon_+^* \cdot k_-)\epsilon_-^{*\mu}]. \tag{27}$$

Calculation gives [2]

$$\mathcal{M}_S^\kappa(\pm, \mp) = 0, \tag{28}$$

$$\mathcal{M}_S^\kappa(\pm, \pm) = s\beta\sin\theta, \tag{29}$$

$$\mathcal{M}_S^\kappa(\pm, 0) = \mathcal{M}_S^\kappa(0, \mp) = -\frac{s\beta}{\sqrt{2}\sqrt{1-\beta^2}}(\cos\theta \mp 2\kappa), \tag{30}$$

$$\mathcal{M}_S^\kappa(0,0) = -s\beta\sin\theta. \tag{31}$$

### 4.2 Production in the t-channel

We define the momentum transfer

$$q = k_- - p_- = p_+ - k_+ \tag{32}$$

so that the matrix element for t-channel production can be written as

$$\mathcal{M}_T(\kappa, \lambda_+, \lambda_-, s, t) = 4i\frac{g_\kappa^2}{st}\overline{v}(p_+)\ \not\epsilon_+^* \not q\ \not\epsilon_-^* u(p_-), \tag{33}$$

where we have made use of $q^2 = -st/4$ and $t = 1 + \beta^2 - 2\beta\cos\theta$.
We may identify

$$\mathcal{F}_T^\kappa(s,t) = 4i\frac{g_\kappa^2}{st} \tag{34}$$

and

$$\mathcal{M}_T^\kappa(\lambda_+, \lambda_-) = \overline{v}(p_+)\ \not\epsilon_+^* \not q\ \not\epsilon_-^* u(p_-). \tag{35}$$

Calculation gives [2]

$$\mathcal{M}_T^\kappa(\pm,\mp) = -\frac{s}{2}\sin\theta(\cos\theta \mp 2\kappa), \tag{36}$$

$$\mathcal{M}_T^\kappa(\pm,\pm) = -\frac{s}{2}\sin\theta(\cos\theta - \beta), \tag{37}$$

$$\mathcal{M}_T^\kappa(\pm,0) = \mathcal{M}_T^\kappa(0,\mp) = -\frac{\sqrt{2}}{4}\frac{s}{\sqrt{1-\beta^2}}(\cos\theta \mp 2\kappa)[2\beta - 2\cos\theta \mp 2\kappa(1-\beta^2)], \tag{38}$$

$$\mathcal{M}_T^\kappa(0,0) = -\frac{s}{2(1-\beta^2)}\sin\theta(3\beta - \beta^3 - 2\cos\theta). \tag{39}$$

# 5 Production Cross Section

From the helicity amplitudes, the differential cross section for explicit vector leptoquark polarization and various degrees of initial-state polarization can be constructed. For definite degrees of polarization, $P^\pm$, of the leptons[3] and definite degrees of polarizaton $(\lambda_+, \lambda_-)$ of the vector leptoquarks, we write

$$\begin{aligned}\frac{d\sigma}{d\cos\theta}(\lambda_+,\lambda_-) &= \frac{3\beta}{32\pi s}\sum_\kappa \frac{1}{4}(1-2\kappa P^+)(1+2\kappa P^-)|\mathcal{M}(\kappa,\lambda_+,\lambda_-,s,t)|^2 \\ &= \frac{3\pi\alpha^2}{8m^2}\beta\frac{1-\beta^2}{4s^2}\sum_\kappa(1-2\kappa P^+)(1+2\kappa P^-)\left[|\mathcal{K}_\kappa(s)|^2\mathcal{M}_S^\kappa(\lambda_+,\lambda_-)^2\right.\end{aligned} \tag{40}$$

[3] $P^- = \pm 1$ corresponds to purely right- and left-handed electrons, respectively.

$$
\begin{aligned}
&+ \quad 2\left(\frac{g_\kappa}{e}\right)^2 \left(\frac{4}{t}\right) \mathcal{R}e[\mathcal{K}_\kappa(s)] \mathcal{M}^\kappa_S(\lambda_+,\lambda_-)\mathcal{M}^\kappa_T(\lambda_+,\lambda_-) \qquad (41)\\
&+ \quad \left(\frac{g_\kappa}{e}\right)^4 \left(\frac{4}{t}\right)^2 \mathcal{M}^\kappa_T(\lambda_+,\lambda_-)^2 \Bigg].
\end{aligned}
$$

The products of the basic matrix elements are as follows. For s-channel production:

$$
\begin{aligned}
\mathcal{M}^\kappa_S(\pm,\mp)^2 &= 0, && (42)\\
\mathcal{M}^\kappa_S(\pm,\pm)^2 &= s^2\beta^2\sin^2\theta, && (43)\\
\mathcal{M}^\kappa_S(\pm,0)^2 = \mathcal{M}^\kappa_S(0,\mp)^2 &= \frac{s^2\beta^2}{2(1-\beta^2)}(\cos^2\theta \mp 4\kappa\cos\theta + 1), && (44)\\
\mathcal{M}^\kappa_S(0,0)^2 &= s^2\beta^2\sin^2\theta. && (45)
\end{aligned}
$$

For t-channel production:

$$
\begin{aligned}
\mathcal{M}^\kappa_T(\pm,\mp)^2 &= \frac{s^2}{4}\sin^2\theta(\cos^2\theta \mp 4\kappa\cos\theta + 1), && (46)\\
\mathcal{M}^\kappa_T(\pm,\pm)^2 &= \frac{s^2}{4}\sin^2\theta(\cos^2\theta - 2\beta\cos\theta + \beta^2), && (47)\\
\mathcal{M}^\kappa_T(\pm,0)^2 = \mathcal{M}^\kappa_T(0,\mp)^2 &= \frac{1}{8}\frac{s^2}{1-\beta^2}(\cos\theta \mp 2\kappa)^2\left[2\beta - 2\cos\theta \mp 2\kappa(1-\beta^2)\right]^2, && (48)\\
\mathcal{M}^\kappa_T(0,0)^2 &= \frac{s^2}{4(1-\beta^2)^2}\sin^2\theta(3\beta - \beta^3 - 2\cos\theta)^2. && (49)
\end{aligned}
$$

The interference contributions are

$$
\begin{aligned}
\mathcal{M}^\kappa_S(\pm,\mp)\mathcal{M}^\kappa_T(\pm,\mp) &= 0, && (50)\\
\mathcal{M}^\kappa_S(\pm,\pm)\mathcal{M}^\kappa_T(\pm,\pm) &= \frac{s^2}{4}\sin^2\theta\left[t - (1-\beta^2)\right], && (51)\\
\mathcal{M}^\kappa_S(\pm,0)\mathcal{M}^\kappa_T(\pm,0) &= \mathcal{M}^\kappa_S(0,\mp)\mathcal{M}^\kappa_T(0,\mp) = \frac{s^2\beta}{4(1-\beta^2)}\Big[4\beta \mp 4\kappa(1-\beta^2) \pm 8\kappa \\
&+ \cos\theta[-4 \mp 8\beta\kappa + 2(1-\beta^2)] && (52)\\
&+ \sin^2\theta[-2\beta + 2\cos\theta \pm 2\kappa(1-\beta^2) \mp 8\kappa]\Big],\\
\mathcal{M}^\kappa_S(0,0)\mathcal{M}^\kappa_T(0,0) &= \frac{s^2}{2(1-\beta^2)}\sin^2\theta\left[t - (1-\beta^2)^2\right]. && (53)
\end{aligned}
$$

# 6 Cross Section for Unpolarized Leptons

The differential cross section for unpolarized electrons and positrons, but for explicit vector leptoquark polarizations can be written as

$$\begin{aligned}
\frac{d\sigma}{d\cos\theta}(\lambda_+,\lambda_-) &= \frac{3\beta}{32\pi s}\sum_\kappa \frac{1}{4}|\mathcal{M}(\kappa,\lambda_+,\lambda_-,s,t)|^2 && (54)\\
&= \frac{3\pi\alpha^2}{8m^2}\beta\frac{1-\beta^2}{4s^2}\sum_\kappa \Big[|\mathcal{K}_\kappa(s)|^2\mathcal{M}_S^\kappa(\lambda_+,\lambda_-)^2\\
&+ 2\left(\frac{g_\kappa}{e}\right)^2\left(\frac{4}{t}\right)\mathcal{R}e[\mathcal{K}_\kappa(s)]\mathcal{M}_S^\kappa(\lambda_+,\lambda_-)\mathcal{M}_T^\kappa(\lambda_+,\lambda_-) && (55)\\
&+ \left(\frac{g_\kappa}{e}\right)^4\left(\frac{4}{t}\right)^2\mathcal{M}_T^\kappa(\lambda_+,\lambda_-)^2\Big].
\end{aligned}$$

The sum of the products of the basic matrix elements are as follows. For s-channel production:

$$\begin{aligned}
\sum_\kappa |\mathcal{K}_\kappa(s)|^2\mathcal{M}_S^\kappa(\pm,\mp)^2 &= 0, && (56)\\
\sum_\kappa |\mathcal{K}_\kappa(s)|^2\mathcal{M}_S^\kappa(\pm,\pm)^2 &= s^2\beta^2\sin^2\theta\sum_\kappa|\mathcal{K}_\kappa(s)|^2, && (57)\\
\sum_\kappa |\mathcal{K}_\kappa(s)|^2\mathcal{M}_S^\kappa(\pm,0)^2 &= \sum_\kappa |\mathcal{K}_\kappa(s)|^2\mathcal{M}_S^\kappa(0,\mp)^2 = \frac{s^2\beta^2}{2(1-\beta^2)}\Big[\\
& (1+\cos^2\theta)\sum_\kappa|\mathcal{K}_\kappa(s)|^2 \mp 4\cos\theta\sum_\kappa \kappa|\mathcal{K}_\kappa(s)|^2\Big], && (58)\\
\sum_\kappa |\mathcal{K}_\kappa(s)|^2\mathcal{M}_S^\kappa(0,0)^2 &= s^2\beta^2\sin^2\theta\sum_\kappa|\mathcal{K}_\kappa(s)|^2. && (59)
\end{aligned}$$

For t-channel production:

$$\begin{aligned}
\sum_\kappa\left(\frac{g_\kappa}{e}\right)^4\mathcal{M}_T^\kappa(\pm,\mp)^2 &= \frac{s^2}{4}\sin^2\theta\left[(1+\cos^2\theta)\sum_\kappa\left(\frac{g_\kappa}{e}\right)^4 \mp 4\cos\theta\sum_\kappa\kappa\left(\frac{g_\kappa}{e}\right)^4\right], && (60)\\
\sum_\kappa\left(\frac{g_\kappa}{e}\right)^4\mathcal{M}_T^\kappa(\pm,\pm)^2 &= \frac{s^2}{4}\sin^2\theta(\cos^2\theta+t-1)\sum_\kappa\left(\frac{g_\kappa}{e}\right)^4, && (61)\\
\sum_\kappa\left(\frac{g_\kappa}{e}\right)^4\mathcal{M}_T^\kappa(\pm,0)^2 &= \sum_\kappa\left(\frac{g_\kappa}{e}\right)^4\mathcal{M}_T^\kappa(0,\mp)^2 = \frac{s^2}{8(1-\beta^2)}\Big[\big[(1+\cos^2\theta)[(2\beta-2\cos\theta)^2\\
&+ (1-\beta^2)^2] + 4\cos\theta(1-\beta^2)(2\beta-2\cos\theta)\big]\sum_\kappa\left(\frac{g_\kappa}{e}\right)^4\\
&\mp 4\cos\theta[4\beta^2-4\cos^2\theta+(1-\beta^2)^2-8\beta\cos\theta]\sum_\kappa\kappa\left(\frac{g_\kappa}{e}\right)^4 && (62)\\
&\mp 8(1+\cos^2\theta)(1-\beta^2)(\beta-\cos\theta)\sum_\kappa\kappa\left(\frac{g_\kappa}{e}\right)^4\Big],\\
\sum_\kappa\left(\frac{g_\kappa}{e}\right)^4\mathcal{M}_T^\kappa(0,0)^2 &= \frac{s^2}{4(1-\beta^2)^2}\sin^2\theta(3\beta-\beta^3-2\cos\theta)^2\sum_\kappa\left(\frac{g_\kappa}{e}\right)^4. && (63)
\end{aligned}$$

The interference contributions are

$$\sum_\kappa \left(\frac{g_\kappa}{e}\right)^2 \mathcal{R}e[\mathcal{K}_\kappa(s)]\mathcal{M}_S^\kappa(\pm,\mp)\mathcal{M}_T^\kappa(\pm,\mp) = 0, \tag{64}$$

$$\sum_\kappa \left(\frac{g_\kappa}{e}\right)^2 \mathcal{R}e[\mathcal{K}_\kappa(s)]\mathcal{M}_S^\kappa(\pm,\pm)\mathcal{M}_T^\kappa(\pm,\pm) = \frac{s^2}{4}\sin^2\theta\left[t-(1-\beta^2)\right]\sum_\kappa \left(\frac{g_\kappa}{e}\right)^2, \tag{65}$$

$$\begin{aligned}
\sum_\kappa \left(\frac{g_\kappa}{e}\right)^2 \mathcal{R}e[\mathcal{K}_\kappa(s)]\mathcal{M}_S^\kappa(\pm,0)\mathcal{M}_T^\kappa(\pm,0) &= \sum_\kappa \left(\frac{g_\kappa}{e}\right)^2 \mathcal{R}e[\mathcal{K}_\kappa(s)]\mathcal{M}_S^\kappa(0,\mp)\mathcal{M}_T^\kappa(0,\mp) \\
&= \frac{s^2\beta}{4(1-\beta^2)}\Big[\Big[4\beta+\cos\theta[-4+2(1-\beta^2)] \\
&+ \sin^2\theta[-2\beta+2\cos\theta]\Big]\sum_\kappa \left(\frac{g_\kappa}{e}\right)^2 \\
&\pm \Big[8-4(1-\beta^2)-8\beta\cos\theta \\
&- \sin^2\theta[8-2(1-\beta^2)]\Big]\sum_\kappa \kappa\left(\frac{g_\kappa}{e}\right)^2\Big],
\end{aligned} \tag{66}$$

$$\sum_\kappa \left(\frac{g_\kappa}{e}\right)^2 \mathcal{R}e[\mathcal{K}_\kappa(s)]\mathcal{M}_S^\kappa(0,0)\mathcal{M}_T^\kappa(0,0) = \frac{s^2\sin^2\theta}{2(1-\beta^2)}\left[t-(1-\beta^2)^2\right]\sum_\kappa \left(\frac{g_\kappa}{e}\right)^2. \tag{67}$$

# 7 Cross Section for Unpolarized Leptoquarks

For vector leptoquark pair production in $e^+e^-$ annihilation with unpolarized electrons and positrons, we may write the differential cross section as

$$\frac{d\sigma}{d\cos\theta} = \frac{3\beta}{32\pi s}\sum_{\kappa,\lambda_+,\lambda_+}\frac{1}{4}|\mathcal{M}(\kappa,\lambda_+,\lambda_-,s,t)|^2 \tag{68}$$

$$\begin{aligned}
&= \frac{3\pi\alpha^2}{8m^2}\beta\frac{1-\beta^2}{4s^2}\sum_{\kappa,\lambda_+,\lambda_-}\Big[|\mathcal{K}_\kappa(s)|^2\mathcal{M}_S^\kappa(\lambda_+,\lambda_-)^2 \\
&+ 2\left(\frac{g_\kappa}{e}\right)^2\left(\frac{4}{t}\right)\mathcal{R}e[\mathcal{K}_\kappa(s)]\mathcal{M}_S^\kappa(\lambda_+,\lambda_-)\mathcal{M}_T^\kappa(\lambda_+,\lambda_-) \\
&+ \left(\frac{g_\kappa}{e}\right)^4\left(\frac{4}{t}\right)^2\mathcal{M}_T^\kappa(\lambda_+,\lambda_-)^2\Big].
\end{aligned} \tag{69}$$

Summing over all possible helicity states gives

$$\sum_{\kappa,\lambda_+,\lambda_-}\mathcal{M}_S^\kappa(\lambda_+,\lambda_-)^2 = \frac{4s^2\beta^2}{1-\beta^2}\left[1+\frac{1}{4}(1-3\beta^2)\sin^2\theta\right], \tag{70}$$

$$\begin{aligned}
\sum_{\kappa,\lambda_+,\lambda_-}\mathcal{M}_S^\kappa(\lambda_+,\lambda_-)\mathcal{M}_T^\kappa(\lambda_+,\lambda_-) &= \frac{s^2t}{2(1-\beta^2)}\left[2\left[1-\frac{1-\beta^2}{t}\right](1-\beta^2)\right. \\
&+ \left. 4\beta^2-\beta^2\left[1-2\frac{1-\beta^2}{t}\right]\sin^2\theta\right],
\end{aligned} \tag{71}$$

$$\sum_{\kappa,\lambda_+,\lambda_-} \mathcal{M}_T^\kappa(\lambda_+,\lambda_-)^2 = \frac{s^2t^2}{4(1-\beta^2)}\Big[4 + \frac{\beta^2}{4}\left[(1-\beta^2)\left(\frac{4}{t}\right)^2 + \frac{4}{1-\beta^2}\right]\sin^2\theta\Big]. \tag{72}$$

These final results are identical to those in reference [1].

# 8 Vector Leptoquark Decay

We now consider the decay of a vector leptoquark with four-momentum $k^\mu$ and helicity $\lambda$:

$$V(k,\lambda) \to l(l) + \overline{q}(q), \tag{73}$$

where the arguments $l$ and $q$ indicate the four-momentum of the lepton and quark respectively.

The four-momentum of the particles are

$$k^\mu = M(1;0,0,0), \tag{74}$$

$$l^\mu = \frac{M}{2}(1,+\beta_l\sin\theta,0,+\beta_l\cos\theta), \tag{75}$$

$$q^\mu = \frac{M}{2}(1,-\beta_q\sin\theta,0,-\beta_q\cos\theta), \tag{76}$$

where $\beta^2_{l(q)} = 1 - 4m^2_{l(q)}/M^2$.

The helicities of the leptoquark in its rest frame are

$$\epsilon^\mu(\lambda=0) = (0;0,0,1), \tag{77}$$

$$\epsilon^\mu(\lambda=\pm1) = \frac{1}{\sqrt{2}}(0;-1,\mp i,0). \tag{78}$$

We will make use of the following dot products:

$$l\cdot q = \frac{M^2}{2}\left[1-\frac{m_l^2+m_q^2}{M^2}\right], \tag{79}$$

$$\epsilon(0)\cdot\epsilon^*(0) = \epsilon(\pm)\cdot\epsilon^*(\pm) = -1 \tag{80}$$

$$\epsilon(0)\cdot l = \epsilon^*(0)\cdot l = -\beta_l\cos\theta, \tag{81}$$

$$\epsilon(0)\cdot q = \epsilon^*(0)\cdot q = +\beta_l\cos\theta, \tag{82}$$

$$\epsilon(\pm)\cdot l = \epsilon^*(\pm)\cdot l = +\beta_l\sin\theta, \tag{83}$$

$$\epsilon(\pm)\cdot q = \epsilon^*(\pm)\cdot q = -\beta_l\sin\theta. \tag{84}$$

For definiteness we will consider a leptoquark that couples to a left handed lepton with coupling $g_L$. The matrix element is

$$\mathcal{M}(\lambda) = -\frac{i}{2} g_L \overline{u}(l)\gamma_\mu(1-\gamma_5)v(q)\epsilon^\mu(k,\lambda). \tag{85}$$

Squaring the matrix element and summing over final fermion spin states gives

$$\begin{aligned} |\mathcal{M}(\lambda)|^2 &= \frac{g_L^2}{4}\mathrm{tr}[\not\epsilon(1-\gamma_5)(\not p - m_q)(1+\gamma_5)\ \not\epsilon^*(\not l + m_l)] && (86)\\ &= 2g_L^2[(\epsilon\cdot q)(\epsilon^*\cdot l) + (\epsilon\cdot l)(\epsilon^*\cdot q) - (\epsilon\cdot\epsilon^*)(l\cdot q)]. && (87) \end{aligned}$$

For the different polarizations

$$\begin{aligned} |\mathcal{M}(0)|^2 &= g_L^2 M^2\left[1 - \left(\frac{m_l^2+m_q^2}{M^2}\right) - \beta_l\beta_q\cos^2\theta\right], && (88)\\ |\mathcal{M}(\pm)|^2 &= g_L^2 M^2\left[1 - \left(\frac{m_l^2+m_q^2}{M^2}\right) - \frac{1}{2}\beta_l\beta_q\sin^2\theta\right]. && (89) \end{aligned}$$

The differential decay widths are

$$\frac{d,\ (\lambda)}{d\Omega} = \frac{|\mathcal{M}(\lambda)|^2}{64\pi^2 M}\left[1-\left(\frac{m_l+m_q}{M}\right)^2\right]^{1/2}\left[1-\left(\frac{m_l-m_q}{M}\right)^2\right]^{1/2}. \tag{90}$$

In the limit of negligible lepton and quark masses

$$\begin{aligned} \frac{d,\ (0)}{d(\cos\theta)} &= \frac{g_L^2}{32\pi}M(1-\cos^2\theta), && (91)\\ \frac{d,\ (\pm)}{d(\cos\theta)} &= \frac{g_L^2}{32\pi}M\frac{1+\cos^2\theta}{2}. && (92) \end{aligned}$$

Integrating any of the partial widths give

$$,\ = \frac{g_L^2}{24\pi}M. \tag{93}$$

# 9 Results

All amplitudes vanish in the forward and the backward directions except those for one longitudinal and one transverse polarized leptoquark. This is a direct consequence of angular momentum conservation. The s-channel amplitudes vanish at threshold[4] and the t-channel graph dominates in the threshold region.

Asymptotically, for $s$ larger than the particle masses, the pure s-channel contribution and the interference of the s-and t-channel terms approach a finite value, whereas the pure t-channel contribution grows proportional to $s$. This behaviour is reminiscent of the t-channel contribution to $e^+e^- \to W^+W^-$. However, as is well-known, in the latter case the

[4] This holds for arbitrary CP-conserving s-channel contributions in the limit of vanishing electron mass [2].

potentially dangerous term is canceled by corresponding contributions from the s-channel graphics. This is due to the special structure of the three-gauge-boson couplings which differ from the coupling used here. Yet, in the spirit of an effective theory, the behaviour of the cross section is perfectly acceptable as long as the energy is kept sufficiently low. At high energies it is assumed that the effective description provided by the Lagrangian is superseded by a more fundamental theory in which the increase of the cross section with $s$ is eventually dampened and unitarity is preserved.

Numerical predictions are presented and discussed for two interesting CMS energies: $\sqrt{s} = 2m_W$ and 1 TeV.

# References


[1] J. Blümlein & R. Rückl, Phys. Lett. **B304** (1993) 337-346.

[2] W. Beenakker & A. Denner, Int. J. Mod. Phys. **A9** (1994) 4837-4919.