PHYS 590: Problem Set 1

Due: 4:30 pm, 15 January 2009

1. Show that the matrices $L_\mu^{\ \nu}$ corresponding to proper Lorentz transformations form a group.

2. We have seen that the Lorentz boost can be represented in terms of the relativistic scale factor $\gamma(\beta)$ or the rapidity using $\beta = \tanh\theta$. An alternative expression for the Lorentz boost uses the relativistic Doppler factor $\lambda(\beta)$ defined by

$$\lambda(\beta) = \sqrt{\frac{1+\beta}{1-\beta}}.$$

   (a) Derive the relationship between λ and γ.

   (b) Derive an expression for β in terms of λ.

   (c) Derive an expression for λ in terms of the hyperbolic functions of the rapidity parameter, and

   (d) in terms of the exponential as a function of the rapidity parameter.

   (e) Show that the following identities are satisfied:

$$\lambda(\beta)\lambda(-\beta) = 1, \quad \lambda(\beta) + \lambda(-\beta) = 2\gamma(\beta), \quad \lambda(\beta) - \lambda(-\beta) = 2\beta\gamma(\beta).$$

3. Choose some direction (usually the beam direction) for the z-axis; then the energy and momentum of a particle can be written as

$$E = m_T \cosh y, \quad p_x, \quad p_y, \quad p_z = m_T \sinh y\,,$$

   where m_T is the transverse mass

$$m_T^2 = m^2 + p_x^2 + p_y^2\,,$$

   and the rapidity y is defined by

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = \ln\left(\frac{E+p_z}{m_T}\right) = \tanh^{-1}\left(\frac{p_z}{E}\right)\,.$$

   Under a boost in the z-direction to a frame with velocity β, show that $y \to y - \tanh^{-1}\beta$. Hence the shape of the rapidity distribution dN/dy is invariant.

   Show that the invariant cross section may also be rewritten as

$$E\frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi dy p_T dp_T}\,.$$

For $p \gg m$, show that the rapidity may be approximated as

$$y \approx -\ln\tan(\theta/2) \equiv \eta\,,$$

where $\cos\theta = p_z/p$. The pseudorapidity η is approximately equal to the rapidity y for $p \gg m$ and $\theta \gg 1/\gamma$, and in any case can be measured when the mass and momentum of the particles is unknown.

From the definition of pseudorapidity obtain the following identities.

$$\sinh\eta = \cot\theta, \quad \cosh\eta = 1/\sin\theta, \quad \tanh\eta = \cos\theta\,.$$

4. Problem B.1 in Cottingham and Greenwood
5. Problem B.2 in Cottingham and Greenwood