PHYS 574: Problem Set

Due: 4:30 pm, 2 February 2012

Note: some problems may be easier if a computer or graphics package is used.

1. (a) Using classical arguments, show that a particle collision with the nucleus of an atom is vary unlikely when compared to collisions with the atomic electrons.

   (b) Semi-classically, the energy transfered to a bound particle with a small velocity can be written as

$$\Delta E = \frac{2z_1^2 z_2^2 e^4}{b^2 v_1^2 m} \,, \tag{1}$$

   where b is the impact parameter of the interaction; $z_1 e$ and v_1 are the charge and speed, respectively, of the incident particle; and $z_2 e$ and m are the charge and mass of the bound particle, respectively. Show that the atomic electrons are responsible for most of the energy loss.

2. Show that the maximum energy transfered to an electron of mass m_e at rest by a much heavier particle of mass M and kinetic energy E in elastic scattering is $4Em_e/M$, and is about 1/500 for protons.

3. Write the Bethe-Bloch equation in terms of dv/dx.

4. Calculate the percentage error which arises from neglecting the relativistic corrections for protons with kinetic energy 20 MeV in an aluminium absorber.

5. Derive a low-energy approximation for $T_{\rm max}$ and state the condition for validity. For a pion in copper, what error does this approximation introduce into dE/dx at 100 GeV.

6. In the minimum ionisation region where $\beta\gamma \approx 3 - 4$, show that the minimum value of $-dE/dx$ can be calculated from the Bethe-Bloch formula and for a particle with unit charge is given approximately by

$$\frac{1}{\rho}\left(-\frac{dE}{dx}\right)_{\rm min} \approx 3.5\frac{Z}{A}\ \text{MeV g}^{-1}\ \text{cm}^2 \,. \tag{2}$$

7. What is the expected mean energy loss of 50 GeV protons in beryllium? How much is this result affected by density effect corrections?

8. The average number $\bar{n}$ of ionising collisions suffered by a fast particle of charge ze in traversing an interval dx (g cm^{-2}) of a medium, and resulting in energy transfer $E' \to E' + dE'$, is

$$\bar{n} = f(E')dE'dx = \frac{2\pi z^2 e^4 N_0 Z}{mv^2 A}\frac{dE'}{(E')^2}\left(1 - \beta^2\frac{E'}{E'_{\rm max}}\right)dx \,, \tag{3}$$

where the symbols are the usual and the maximum transferable energy is $E'_{\rm max} = 2mv^2/(1-\beta^2)$. For individual particles, the distribution in number of collisions n follows the Poisson law, so that $\langle (n-\bar{n})^2 \rangle = \bar{n}$. If we multiply the above equation by $(E')^2$ and integrate, we obtain the mean squared deviation in energy loss, $\varepsilon^2 = \langle (\Delta E - \overline{\Delta E})^2 \rangle$, about the mean $\overline{\Delta E}$. Show that

$$\varepsilon^2 = 0.6 \frac{Z}{A} (mc^2)^2 \gamma \left(1 - \frac{\beta^2}{2}\right) \Delta x \, . \tag{4}$$

9. Calculate the fractional rms deviation $\varepsilon/\overline{\Delta E}$ in energy loss for protons of kinetic energy 500 MeV traversing (a) 0.1, (b) 1.0, and (c) 10 g cm^{-2} of plastic scintillator ($Z/A = 0.5$). Take dE/dx as 3 MeV g^{-1} cm^2.

10. For highly relativistic velocities, the energy loss of electrons is

$$-\frac{dE}{dx} = K \left(\frac{Z}{A}\right) \left(\frac{z}{\beta}\right)^2 \left[\ln \frac{2mc^2}{I} - \frac{3}{2} \ln(1-\beta^2)^{1/2} - \frac{1}{2} \ln 8 + \frac{1}{16}\right] \tag{5}$$

whereas for protons it is

$$-\frac{dE}{dx} = K \left(\frac{Z}{A}\right) \left(\frac{z}{\beta}\right)^2 \left[\ln \frac{2mc^2}{I} + 2 \ln \frac{1}{(1-\beta^2)^{1/2}} - 1\right] . \tag{6}$$

At equal values of β, how much do the two expressions differ and over what energy range?

11. Show that alpha particles and protons of the same initial speed have approximately the same range in any stopping material. Why is this not accurately true? Which particle should have a slightly longer range and why?

12. Show that a deuteron of energy E has twice the range of a proton of energy $E/2$.

13. A gamma ray is Compton-scattered backward ($\theta = 180$ deg). Calculate the energy of the scattered quantum for a primary quantum having $\hbar\omega = 0.01$, 0.1, 1.0, 10, 100, 1000 MeV.

14. Derive

$$N \approx 2^t \tag{7}$$

and

$$E(t) \approx \frac{E_0}{2^t} \tag{8}$$

using the simple Heitler model, but starting with an electron rather than a photon.

15. Describe one method in detail for designing a compensating calorimeter.

16. A 50 GeV electron traverses a stack of iron.

    (a) Using the Heitler shower model, estimate the number of positrons produced after 15 cm.

    (b) What is the average positron energy?

    (c) At what depth in the stack will the number of particles in the shower reach a maximum?

    (d) What is the maximum number of particles present in the shower?

17. (a) Estimate the total depth of a practical 15 GeV electromagnetic calorimeter if we allow an additional $2t_{\mathrm{max}}$ beyond the shower maximum to minimise the probability of escaping particles.

    (b) How does this compare with the total depth of a 15 GeV hadron calorimeter?

18. (a) Show that the minimum opening angle of the two photons in π^0 decay is $2m_\pi/E_\pi$, where E_π is the π^0 energy.

    (b) Assume that the two-photon showers can be resolved if they are separated by two Moliere radii. Estimate the maximum π^0 energy that can be resolved using a practical lead-scintillator calorimeter 2 m from the interaction point.