PHYS 530: Problem Set 10

Due: 4:30 pm, 16 April 2013

If the answer is shown, all the marks will be given for the derivation not for writing down the answer. In your solutions, you may need to make some assumptions. Make sure that you formulate all of them clearly.

1. [10] Solve problem 9.1 in Pathria.

   Use the Hubble expansion relation

   $$v = \frac{da}{dt} = Ha = \sqrt{\frac{8\pi G u}{3c^2}}\,a\,, \tag{1}$$

   the temperature scaling relation $T(t)a(t) =$ constant, and the energy density relation before the electron-positron annihilation

   $$u_{\text{total}}(T) = \left(2 + \frac{21}{4} + \frac{7}{2}\right)\frac{u_\gamma(T)}{2} = \frac{43}{8}u_\gamma(T) \tag{2}$$

   to show that the temperature as a function of time during the first second of the universe was

   $$T(t) \approx 10^{10}\ \text{K}\sqrt{\frac{0.992\ \text{s}}{t}}\,. \tag{3}$$

2. [5] Solve problem 9.8 in Pathria.

   Gold-on-gold nuclear collisions at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory create a quark-gluon plasma with an energy density of about 4 GeV/fm^3. Treat nuclear matter as composed of a noninteracting relativistic gas of quarks and gluons. Include the low-mass up and down quarks and their antiparticles (all spin-1/2), and spin-1 massless gluons. Like photons, the gluons are bosons, have two spin states each and are their own antiparticle. There are eight varieties of gluons that change the three colour states of the quarks. Only the strongly interacting particles need to be considered due to the tiny size of the plasmas. What is the temperature of the quark-gluon plasma?

3. [10] Solve problem 9.9 in Pathria.

   Calculate the energy density versus temperature very early in the universe when the temperatures were above $kT = 300$ MeV. At those temperatures, quarks and gluons were released from individual nuclei. Treat the quark-gluon plasma as a noninteracting relativistic gas. At those temperatures, the species that are in equilibrium with one another are: photons, the three neutrino species, electrons and positrons, muons and antimuons, up and down quarks and their antiparticles (all spin-1/2), and spin-1 massless gluons. Like photons, the gluons are bosons, have two spin states each and are their own antiparticle. There are eight varieties of gluons that change the three colour states of the quarks. The strange, charm, top, and bottom quarks and tau leptons are heavier than 300 MeV, so they do not contribure substantially at this temperature. Using your result and

$$v = \frac{da}{dt} = Ha = \sqrt{\frac{8\pi G u}{3c^2}}a \tag{4}$$

to determine the temperature evolution as a function of the age of the universe during this era and its age when $kT \approx 300$ MeV.