PHYS 530: Problem Set 8

Due: 4:30 pm, 4 April 2013

If the answer is shown, all the marks will be given for the derivation not for writing down the answer. In your solutions, you may need to make some assumptions. Make sure that you formulate all of them clearly.

1. [10] Solve problem 7.5 in Pathria.

   (a) Show that the isothermal compressibility κ_T and the adibatic compressibility κ_S of an ideal Bose gas are given by

   $$\kappa_T = \frac{1}{nkT}\frac{g_{1/2}(z)}{g_{3/2}(z)} \quad \text{and} \quad \kappa_S = \frac{3}{5nkT}\frac{g_{3/2}(z)}{g_{5/2}(z)}, \tag{1}$$

   where $n(= N/V)$ is the particle density in the gas. Note that, as $z \to 0$, κ_T and κ_S approach their respective classical values, namely $1/P$ and $1/\gamma P$. How do they behave as $z \to 1$?

   (b) Making use of the thermodynamic relations

   $$C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_P = TV\kappa_T\left(\frac{\partial P}{\partial T}\right)_V^2 \tag{2}$$

   and

   $$\frac{C_P}{C_V} = \frac{\kappa_T}{\kappa_S}, \tag{3}$$

   derive

   $$\gamma \equiv \frac{C_P}{C_V} = 1 + \frac{4}{9}\frac{C_V}{Nk}\frac{g_{1/2}(z)}{g_{3/2}(z)} \tag{4}$$

   and

   $$\gamma = \frac{5}{3}\frac{g_{5/2}(z)g_{1/2}(z)}{[g_{3/2}(z)]^2}. \tag{5}$$

2. [6] Solve problem 7.23 in Pathria.

   The sun may be regarded as a black body at a temperature of 5800 K. Its diameter is about 1.4×10^9 m while its distance from the earth is about 1.5×10^{11} m.

   (a) Calculate the total radiant intensity (in W/m^2) of sunlight at the surface of the earth.

   (b) What pressure would it exert on a perfectly absorbing surface placed normal to the rays of the sun?

   (c) If a flat surface on a satellite, which faces the sun, were an ideal absorber and emitter, what equailibrium temperature would it ultimately attain?

3. [12] Consider an isolated system in a volume $V_0 = 10^{10}$ m^3, composed of ^{4}He (Bose-Einstein particles with spin zero), in thermodynamic equilibrium with background black body radiation at temperature $T_0 = 1500$ K. Assuming that the pressure associated with the helium gas is equal to that of the black body radiation, calculate the following:

   (a) The density n_0 of the helium gas; that is, the number of helium atoms per unit volume.

   (b) The total energy in the system.

   (c) The entropy of the helium gas, the entropy of the black body radiation, as well as the total entropy of the system.

   The volume of the system is now slowly reduced by a factor two. The change in the volume is sufficiently slow for the entropy of the gas and that of the black body radiation, taken separately, to remain constant. Let us assume, however, that the compression is sufficiently fast that the ^{4}He and photon gases do not equilibrate; that is, each species (^{4}He and photons) evolve as if the other one did not exist. Calculate the resulting

   (d) Temperature of each component (^{4}He and photons).

   (e) Total work done on the system in the compression process.

   Finally, calculate the final temperature of the system and the total entropy long after the compression, after the two species have had time to come to equilibrium with one another.

4. [7] The following problem explores energy balance between a star and a small planet, or satellite, gravitating on a circular orbit. Let us assume the following:

   The star has a radius R_0 and a surface temperature T_0.

   The star radiates as a black body.

   The small planet has a radius r and an albedo α.

   The distance between the centre of the planet and that of the star is d.

   The planet has some internal mechanism of temperature regulation, so that the surface temperature on the planet is uniform.

   The planet radiates as a black body.

   (a) Derive an expression for the steady state temperature at the surface of the planet, as a function of the other parameters of the problem.

   (b) Make use of the fact that the surface temperature of the sun is $T_0 = 5800$ K, that its radius is $R_0 = 695,000$ km, that the albedo of the earth to solar radiation is approximately 31%, and that the earth orbits at a distance of approximately $d = 150$ millions of km from the centre of the sun, calculate the corresponding surface temperature of the earth.

(c) The average temperature at the surface of the earth is approximately 14.5°C. Explain qualitatively why this average temperature differs from the estimate that you made in part (b). Comment on the validity of each assumption made in deriving the expression in part (a). In each case, indicate how the assumption should be modified in order to be more realistic and, qualitatively, explain how this modification (in the assumption) would help reconcile the estimated average temperature to the observed value.