PHYS 530: Problem Set 6

Due: 4:30 pm, 12 March 2013

If the answer is shown, all the marks will be given for the derivation not for writing down the answer. In your solutions, you may need to make some assumptions. Make sure that you formulate all of them clearly.

1. [7] Prove that the trace of an operator is independent of the representation, provided that the set of basis functions used in that representation is orthonormal. That is, prove that

$$\mathrm{Tr}(A) = \sum_{k=1}^{N} \langle \phi_k | A | \phi_k \rangle \tag{1}$$

   is independent of the set of basis functions $\{\phi_k\}$, as long as these basis functions satisfy $\langle \phi_k | \phi_l \rangle = \delta_{kl}$.

2. [5] Suppose that you were presented with an actual realisation of a statistical ensemble. That is, you entered a very large warehouse with, say, one billion similar systems nicely placed on shelves. How would you physically determine whether the ensemble (the large warehouse of systems) is in a pure state or not?

3. [10]

   (a) Prove the Bohr van Leuven theorem, that is, prove that classical systems of charged particles cannot exhibit diamagnetism (see problem 3.43 in Pathria).

   (b) Read the short article entitled "Is diamagnetism possible classically?" in Europhysics, Vol. 40, no. 4 2009 (go to www.europhysicsnews.org). Summarise your understanding of this article in no more than half a page.

4. [10] Use the density matrix formulation to derive an expression for the expectation value of each component (x, y, z) of the electron magnetic moment when the magnetic field is given by

$$\vec{B} = B_0 \left(\hat{x} \cos\theta + \hat{y} \sin\theta \right). \tag{2}$$