PHYS 530: Problem Set 5

Due: 4:30 pm, 5 March 2013

If the answer is shown, all the marks will be given for the derivation not for writing down the answer. In your solutions, you may need to make some assumptions. Make sure that you formulate all of them clearly.

1. [10] Solve problem 4.1 in Pathria's book.
   Show that the entropy of a system in the grand canonical ensemble can be written as

$$S = -k \sum_{r,s} P_{r,s} \ln P_{r,s} \,, \tag{1}$$

   where $P_{r,s}$ is given by

$$P_{r,s} = \frac{\exp(-\alpha N_r - \beta E_s)}{\sum_{r,s} \exp(-\alpha N_r - \beta E_s)} \,. \tag{2}$$

2. [20]

   (a) Use the grand canonical formalism to derive an approximate expression for the Saha equation for a neutral hydrogen plasma

$$\frac{n_i}{n_n} = \frac{e^{-\phi/(kT)}}{n_i \lambda^3}, \tag{3}$$

   where n_i and n_n are respectively the density of ionised and neutral hydrogen, ϕ is the ionisation potential of hydrogen (13.6 eV), and λ is the electron thermal wavelength. In this problem, for simplicity, assume that the neutral hydrogen atom can only exist in its ground state. That is, neglect all excited states in the construction of the partition functions.

   (b) Discuss what difficulty would arise if you tried to include all possible excited states of the neutral hydrogen atom in the calculation of the partition function. Recall that the degeneracy of energy level n in a hydrogen atom is $g_n = 2n^2$.

   (c) How would you resolve this difficulty in practice? Justify your proposed solution.

   (d) Using the Saha equation, calculate the density at which 99% of hydrogen would be ionised at temperature $T = 0.1$ eV.

   (e) In many laboratory and space plasmas, the ionisation level is different from what is predicted with the Saha equation. Briefly explain why that is possible.

   Hint: Assume that each proton is similar to an adsorption site to which an electron can stick (to form a neutral atom) or not (to remain ionised) and use the similarity with problem 4.10 in Pathria's book.