PHYS 530: Problem Set 2

Due: 4:30 pm, 31 January 2013

If the answer is shown, all the marks will be given for the derivation not for writing down the answer.

1. [5] Justify
$$\Omega(1, V, \varepsilon) \approx V f(\varepsilon)\,; \tag{1}$$
that is, prove that for a single point-particle of given energy ε contained in a box, the number of accessible states is approximately proportional to the box volume.

2. [7] Prove that the mixing entropy for two identical gases initially at the same temperature is either zero (when the two containers have the same initial density) or positive (when the initial densities are different). In other words, prove that the mixing entropy of identical gases at the same temperature cannot be negative.

3. [5] Solve problem 1.3 in Pathria's book.
Two systems A and B, of identical composition, are brought together and allowed to exchange both energy and particles, keeping volumes V_A and V_B constant. Show that the minium value of the quantity dE_A/dN_A is given by
$$\frac{\mu_A T_B - \mu_B T_A}{T_B - T_A}\,, \tag{2}$$
where the μ's and the T's are respective chemical potentials and temperatures.

4. [8] Solve problem 1.4 in Pathria's book.
In a classical gas of hard spheres (of diameter D) the spatial distribution of the particles is no longer uncorrelated. Roughly speaking, the presence of n particles in the system leaves only a volume $(V - nv_0)$ available for the $(n+1)$th particle; clearly v_0 would be proportional to D^3. Assuming that $Nv_0 \ll V$, determine the dependence of $\Omega(N, V, E)$ on V (compare to $\Omega(N, E, V) \propto V^N$) and show that, as a result of this, V in the ideal-gas law $PV = NkT = nRT$ gets replaced by $(V - b)$, where b is four times the actual volume occupied by the particles.

5. [5] Solve problem 1.6 in Pathria's book.
A cylindrical vessel 1 m long and 0.1 m in diameter is filled with a monatomic gas at $P = 1$ atm and $T = 300$ K. The gas is heated by an electric discharge, along the axis of the vessel, which releases an energy of 10^4 J. What will the temperature of the gas be immediately after the discharge?

6. [22] Solve problem 1.7 in Pathria's book.
Study the statistical mechanics of an extreme relativistic gas characterised by the single-particle energy states
$$\varepsilon(n_x, n_y, n_z) = \frac{hc}{2L}\left(n_x^2 + n_y^2 + n_z^2\right)^{1/2}; \quad n_x, n_y, n_z = 1, 2, 3, \dots\,, \tag{3}$$

instead of $\varepsilon(n_x, n_y, n_z) = h^2/(8mL^2)(n_x^2 + n_y^2 + n_z^2)$, along the lines followed in class. Show that the ratio C_P/C_V in this case is 4/3, instead of 5/3.

7. [8] Solve problem 1.8 in Pathria's book.
Consider a system of quasiparticles whose energy eigenvalues are given by

$$\varepsilon(n) = nh\nu; \quad n = 0, 1, 2, \dots . \tag{4}$$

Obtain an asymptotic expression for the number Ω of this system for a given number N of the quasiparticles and a given total energy E. Determine the temperature T of the system as function of E/N and $h\nu$, and examine the situation for which $E/(Nh\nu) \gg 1$.

8. [10] This problem is inspired by problem 1.11 of Pathria's book. Consider four moles of nitrogen in volume V_1 and one mole of oxygen in volume V_2. Both gases are at $P =$ 1 atm and $T = 300$ K. The gases are initially in separate containers but these containers are connected with a small tube, which allows the two gases to mix uniformly in both containers. Assuming that the two gases are ideal gases (noninteracting particles with no internal degrees of freedom),

   (a) Calculate the change in entropy due to mixing.
   (b) Calculate the heat absorbed or released by the system.
   (c) Calculate the change in total energy of the system.
   (d) Do you expect the heat calculated from the change in entropy from part (a) could be used to drive a heat engine and perform work? Explain your answer in some detail.