PHYS 485: Problem Set 5

Due: 4:30 pm, 9 March 2010

If the answer is shown, all the marks will be given for the derivation not for writing down the answer.

1. [3] Griffiths Problem 7.3.
2. [2] Griffiths Problem 7.5.
3. [4] Griffiths Problem 7.20.
4. [5] Griffiths Problem 7.27.
5. [2] Prove the following without using an explicit representation for the γ matrices.
   (a) $Tr[\not{a}\not{b}] = 4a \cdot b$
   (b) $Tr[\gamma^5] = 0$
6. [10] If we go to a system moving with speed βc in the x-direction, a Dirac spinor transforms according to

$$\psi \to \psi' = S\psi\,,$$

where S is given by

$$S = a_+ + a_-\gamma^0\gamma^1\,,$$

with $a_\pm = \pm\sqrt{\frac{1}{2}(\gamma \pm 1)}$ and $\gamma = (1-\beta^2)^{-1/2}$, as usual.

   (a) Calculate $S^\dagger$.
   (b) Show that the inverse of S is $S^{-1} = a_+ - a_-\gamma^0\gamma^1$.
   (c) Calculate $S^\dagger S$ in terms of γ and β.
   (d) Show that $\gamma^0 S^\dagger \gamma^0 = S^{-1}$.
   (e) Show that $S^{-1}\gamma^\mu S = \gamma^\mu$ for $\mu = 2, 3$ and $S^{-1}\gamma^\mu S = \gamma^\mu S^\dagger S$ for $\mu = 0, 1$.
   (f) Show that $\bar{\psi}\gamma^\mu\psi$ is a four-vector by confirming that its components transform as a vector according to a special Lorentz transformation.
   (g) Check that it transforms as a (polar) vector under parity (that is, the time component is invariant, whereas the spatial components change sign).