PHYS 200: Final Review

April 2005

1. A plane flies at a speed of 2000 km/hr, for how long must it fly before its clock loses 1 s because of time dilation?

2. A stick of proper length L_P makes an angle θ with the x-axis in frame S. Show that the angle θ' made with the x'-axis in frame S', which is moving along the $+x$-axis with speed v, is given by $\tan\theta' = \gamma \tan\theta$ and that the length of the stick in S' is

$$L' = L_P \left(\frac{1}{\gamma^2} \cos^2\theta + \sin^2\theta \right)^{1/2}.$$

3. Show that if a particle moves at an angle θ with the x-axis with speed u in frame S, it moves at an angle θ' with the x'-axis in S' given by

$$\tan\theta' = \frac{\sin\theta}{\gamma(\cos\theta - v/u)},$$

   where the frame S' is moving with speed v relative to S.

4. When at rest, the Σ^- particle has a lifetime of 0.15 ns before it decays into a neutron and a pion. One particular Σ^- particle is observed to travel 3.0 cm in the lab before decaying. What was its speed?

5. A cosmic-ray proton entering the atmosphere from space has a kinetic energy of 2.0×10^{20} eV.
   (a) What is the kinetic energy in joules?
   (b) If all of the kinetic energy of the proton could be harnessed to lift an object of mass 1.0 kg near Earth's surface, how far could the object be lifted?
   (c) What is the speed of the proton?

6. A nuclear power plant produces an average of 1.0×10^3 MW of power during a year of operation. Find the corresponding change in mass of reactor fuel, assuming all the energy released by the fuel can be converted directly to electrical energy. (In a practical reactor, only a relatively small fraction of the energy can be converted to electricity.)

7. A lump of putty with a mass of 0.240 kg and a speed of 0.980c collides head-on and sticks to an identical lump of putty moving with the same speed. After the collision the system is at rest. What is the mass of the system after the collision?

8. Using $p = mv/\sqrt{1 - v^2/c^2}$ and $E = mc^2/\sqrt{1 - v^2/c^2}$, prove that the total energy of an object is related to its relativistic momentum p according to $E^2 = (pc)^2 + (mc^2)^2$.

9. Two atomic particles approach each other in a head-on collision. Each particle has a mass of 2.16×10^{-25} kg. The speed of each particle is 2.10×10^8 m/s when measured by an observer standing in the laboratory.
   (a) What is the speed of one particle as seen by the other particle?

(b) Determine the relativistic momentum of one particle, as would be observed by the other.

10. An unstable particle is at rest and suddenly breaks up into two fragments. No external forces act on the particle or its fragments. One of the fragments has a velocity of $+0.800c$ and a mass of 1.67×10^{-27} kg, while the other has a mass of 5.01×10^{-27} kg. What is the velocity of the more massive fragment?

11. A pi meson of rest mass m_π decays at rest into muon (rest mass m_μ) and a neutrino of zero rest mass. Show that the kinetic energy of the muon is $KE_\mu = (m_\pi - m_\mu)^2 c^2 / 2m_\pi$.

12. Use the binomial expansion $(1+x)^n = 1 + nx + n(n-1)x^2/2 + \cdots \approx 1 + nx$ and $E^2 = (pc)^2 + (mc^2)^2$ to show that when $pc \ll mc^2$, the total energy is given approximately by

$$E \approx mc^2 + \frac{p^2}{2m}.$$

13. For a particle moving with velocity $\vec{u}$ in frame S, show that its momentum and energy in frame S', which is moving along the x-axis with speed v, are related to its momentum and energy in S by the transformation equations.

$$p'_x = \gamma\left(p_x - \frac{vE}{c^2}\right), \quad p'_y = p_y, \quad p'_z = p_z,$$

$$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \frac{vp_x}{c^2}\right).$$

14. In a simple thought experiment, Einstein showed that there is mass associated with electromagnetic radiation. Consider a box of length L and mass M resting on a frictionless surface. At the left wall of the box is a light source that emits radiation of energy E, which is absorbed at the right wall of the box. According to classical electromagnetic theory, this radiation carries momentum of magnitude $p = E/c$.

    (a) Find the recoil velocity of the box such that momentum is conserved when the light is emitted. (Since p is small and M is large, you may use classical mechanics.)

    (b) When the light is absorbed at the right wall of the box, the box stops, so the total momentum remains zero. If we neglect the very small velocity of the box, the time it takes for the radiation to travel across the box is $\Delta t = L/c$. Find the distance moved by the box in this time.

    (c) Show that if the center of mass of the system is to remain at the same place, the radiation must carry mass $m = E/c^2$.

15. A horizontal turntable rotates with angular speed ω. There is a clock at the center of the turntable and one at the distance r from the center. In an inertial reference frame, the clock at distance r is moving with speed $u = r\omega$.

(a) Show that from time dilation in special relativity, time intervals Δt_0 for the clock at rest and Δt_r for the moving clock are related by

$$\frac{\Delta t_r - \Delta t_0}{\Delta t_0} \approx -\frac{r^2\omega^2}{2c^2} \quad \text{if} \quad r\omega \ll c.$$

(b) In a reference frame rotating with the table, both clocks are at rest. Show that the clock at distance r experiences a pseudoforce $F_r = mr\omega^2$ in this accelerated frame and that this is equivalent to a difference in gravitational potential between r and the origin of $\phi_r - \phi_0 = \frac{1}{2}r^2\omega^2$. Use this potential difference in

$$\frac{\Delta t_2 - \Delta t_1}{\Delta t} = \frac{1}{c^2}(\phi_2 - \phi_1)$$

to show that in this frame the difference in time intervals is the same as in the inertial frame.

16. Use conservation of angular momentum to estimate the angular velocity of a neutron star which has collapsed to a diameter of 10 km, from a star whose radius was equal to that of our Sun (7×10^8 m), of mass 1.5 times that of the Sun, and which rotated (like our Sun) about once a month.

17. A certain pulsar, believed to be a neutron star of mass 1.5 times that of the Sun, with diameter 10 km, is observed to have a rotational speed of 1.0 rev/s. If it loses rotational KE at the rate of 1 part in 10^9 per day, which is all transformed into radiation, what is the power output of the star?