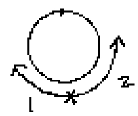
PROBLEM SET 6

CHAPTER 8 : 12, 26, 28, 32, 44

CHAPTER 8 #12 [4 marks]



$\omega_1 = 1.7 \times 10^{-3}$ rad/s
$\omega_2 = 3.4 \times 10^{-3}$ rad/s
$t = ?$

Since they have a constant angular speed, their angular displacements are given by, $\theta = \omega t$

$\theta_1 = \omega_1 t \quad , \quad \theta_2 = \omega_2 t$

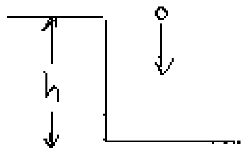
At the time t when they meet, their displacements must add up to 2π

$\therefore \; 2\pi = \theta_1 + \theta_2 = \omega_1 t + \omega_2 t = (\omega_1 + \omega_2) t$

$$t = \frac{2\pi}{\omega_1 + \omega_2} = \frac{2\pi}{(1.7 + 3.4) \times 10^{-3}}$$

$$\boxed{t = 1.2 \times 10^{3}\, s}$$

CHAPTER 8 #26 [4 marks]



$h = 8.3\,m$
$\langle\omega\rangle = 1.6$ rev/s
$\theta = ?$

We first determine the time it takes to fall through hight h

$$y = y_0 + (v_0)_y t + \frac{1}{2}at^2$$
$$0 = h + 0 - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

For an average angular speed, $\langle\omega\rangle$

$$\theta = \langle\omega\rangle t = \langle\omega\rangle\sqrt{\frac{2h}{g}}$$

$$= 1.6\sqrt{\frac{2\times 8.3}{9.8}}\ \text{rev}$$

$$\boxed{\theta = 2.1\ \text{rev}}$$

CHAPTER 8 #28 [4 marks]

$r = 2.2\times10^{20}$ m
$\omega = 1.2\times10^{-15}$ rad/s

(A) $v_T = r\omega = (2.2\times10^{20})(1.2\times10^{-15})$

$$\boxed{v_T = 2.6\times10^{5}\ \text{m/s}}$$

(B) For one revolution $\theta = 2\pi$

For a constant angular speed, $\theta = \omega t$

$$\therefore\ t = \frac{\theta}{\omega} = \frac{2\pi}{\omega} = \frac{2\pi}{1.2\times10^{-15}} = 5.2\times10^{15}\ \text{s}$$

In years, $t = \dfrac{2\pi}{1.2\times10^{-15}}\ \dfrac{1}{60\times60\times24\times365.3}$

$$\boxed{t = 1.7\times10^{8}\ \text{yr}}$$

CHAPTER 8 #32 [8 marks]

$R_E = 6.38 \times 10^6\ m$
$T = 23.9\ h = 23.9 \times 60 \times 60 = 8.60 \times 10^4\ s$

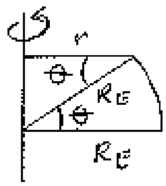
(A) $v_T = ?$

For a constant speed of rotation, $\theta = \omega t$
For one revolution, $t = T$ and $\theta = 2\pi$

$\therefore \omega = \dfrac{2\pi}{T}$

$$v_T = \omega r = \left(\frac{2\pi}{T}\right) R_E = \frac{2\pi \times 6.38 \times 10^6}{23.9 \times 60 \times 60} = \underline{\underline{466\ m/s}}$$

(B) $v_T' = \frac{1}{3} v_T$
$\theta = ?$



for person not living on the equator, $r \neq R_E$

$\dfrac{r}{R_E} = \cos\theta$

also $v_T' = \omega r$
same as part (A) (rigid body)

$$\omega r = v_T' = \frac{v_T}{3} = \frac{\omega R_E}{3}$$

$$\therefore r = \frac{R_E}{3} \Rightarrow \frac{r}{R_E} = \frac{1}{3}$$

$$\therefore \theta = \cos^{-1}\left[\frac{r}{R_E}\right] = \cos^{-1}\left(\frac{1}{3}\right) = \underline{\underline{70.5^\circ}}$$

CHAPTER 8 # 44 [4 marks]

$\omega_0 = 0$
$\alpha = 22.0 \text{ rad/s}^2$

What is t when $a_c = a_T$

$a_T = r\alpha$, $a_c = r\omega^2$

$\therefore\ r\alpha = r\omega^2 \Rightarrow \omega = \sqrt{\alpha}$

But $\omega = \omega_0 + \alpha t$, where $\omega_0 = 0$

$\therefore\ t = \frac{\omega}{\alpha} = \frac{\sqrt{\alpha}}{\alpha} = \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{22.0}}$

$$\boxed{t = 0.213\text{ s}}$$