"Formal philosophy" is a way of doing philosophy. Like most philosophical schools, its members refuse to call their method a school, and prefer to point to the differences among themselves. Nonetheless, we can point to certain tendencies in this cult: Frege is regarded as a patron saint (although some would want to trace their history to Leibniz, or the mediaeval grammarians, or even to Aristotle); Russell was (at times) a prophet; Tarski, Carnap, Church and Quine are latter-day saints; Davidson, Hintikka, Kaplan, Kripke, Lewis, Moravcsik, Scott and van Fraassen (to name a few) are present-day practitioners. Their methodology can be seen as two-fold: to apply results of logic in the solution of philosophical problems, and to extend the apparatus of logic and metamathematics so that it can comprehend under its purview more philosophical matters. Richard Montague was a member of this school; indeed, he may some day come to be regarded as one of its latter-day saints.

"This posthumous volume contains all of Richard Montague's papers that were felt to have application to philosophical and linguistic problems," to quote Thomason's introduction. It also contains a very long introduction (69 pp., longer than any of the Montague papers) by the editor explaining "to readers who are acquainted with the rudiments of set theory, and whose knowledge of symbolic logic includes at least the first-order predicate calculus and its semantic interpretation" what Montague's conception of a philosophy of language was. It also has an almost complete bibliography of Montague's work (for the qualification, see below "errata"). The papers reprinted in this volume are (1) "Logical Necessity, Physical Necessity, Ethics, and Quantifiers" (2) "'That'" (with Donald Kalish) (3) "Pragmatics" (4) "Pragmatics and Intensional Logic" (5) "On the Nature of Certain Philosophical Entities" (6) "English as a

Papers 2, 9, 10, 11 apply formal techniques to a variety of philosophical problems. Papers 1, 3, 4 develop a new group of logical systems: those called pragmatics and those called intensional logic. These systems are capable of formalizing modal statements (including physical, ethical and logical modalities), belief and knowledge statements, and indexical expressions (tenses, deixis, etc.). Paper 5 deals with the ontological status, in this language, of events, pains, sense data, tasks, obligations, etc. Papers 6, 7, 8 are applications of the semantico-mathematical apparatus developed in 1, 3, 4, 5 to the analysis of a natural language, English.

Paper (2), which dates from 1959, is an attempt to treat the problems which arise from substitution into “non-referential” contexts, in particular contexts generated by ‘that’. I think this is the least satisfactory of the papers in the volume, not fitting well with the general position taken in other papers (despite Montague’s claim to the contrary in (1) p. 81). The authors object to the two popular treatments of non-referential contexts: (a) assign unusual denotations (e.g., senses or concepts) to occurrences of terms in such contexts, (b) construe such contexts as “improper” (they are no more independent terms in such contexts than ‘x’ is in ‘xylophone’). The authors find (a) difficult because it is excessively complex and because there is no adequate account of intensions known. They find (b) implausible because it is sometimes necessary to have occurrences of variables in the scope of ‘that’ to express ordinary states of affairs and also to state important logical results, as ‘If \( \phi \) is any theorem of arithmetic, then it is provable that \( \phi \) is a theorem’.

The solution advocated by the authors is to treat ‘that’ as expressing a relation between a person and a certain sentence. The novelty of this approach which sets it apart from others, e.g., Davidson’s, is that ‘that’ is regarded as ambiguous. The ‘that’ can be made unambiguous, that is, can be made to tell us which of the names are to be used in this sentence, by attaching a naming function to it. Taking as a simple case only the natural numbers, we might consider two naming functions, \( N^a \) and \( N' \). The former is a function which assigns to the k-th positive integer the k-th numeral; the latter is identical to \( N^a \).
except that \( N' (9) = \text{'the number of planets'} \). Thus an English sentence like

(1) It is provable in arithmetic that 9 is not prime is ambiguous. Using \( N^0 \) as the naming function we get

(2) It is provable in arithmetic that—\( N^0 \) 9 is not prime

which amounts to

(2') The result of replacing ‘a’ by \( N^0 \) (9) in ‘a is not prime’ is provable in arithmetic.

Using \( N^1 \) as the naming function we get

(3) It is provable in arithmetic that—\( N^1 \) 9 is not prime

which amounts to

(3') The result of replacing ‘a’ by \( N^1 \) (9) in ‘a is not prime’ is provable in arithmetic

or equivalently

(3'') ‘The number of planets is not prime’ is provable in arithmetic.

The truth value of (1) would depend on the way the ambiguity is resolved.

For reasons I shall indicate shortly, this proposal is inadequate; but in discussing the naming function, an important observation is made, viz., that certain naming functions are more “natural” than others. Indeed, the function \( N^\theta \) might even be viewed as giving the standard name of the natural numbers. Within a formalized language there are at least three ways of giving the standard name of an expression. One is to enclose it in quotation marks; a second is to form the Gödel-number of it and identify the standard name with the numeral associated with that number according to \( N^\theta \); a third way could be to form the structural-descriptive name of the expression.\(^1\) The notion of a standard name is important for the discussion in papers (9) and (10); it has also played a role in discussions of essentialism and epistemology (vide Hintikka, Kripke, Plantinga, etc.).

Three difficulties attend this solution. The first is that it at least needs to be generalized to allow one and the same occurrence of ‘that’ in ordinary discourse to have simultaneously two distinct naming functions associated with it in a single analysis. It is obvious that in (4) the two occurrences of ‘the

\(^1\)Another method is taken in “intensional logic.” Vide p. 135.
length of his yacht' are understood as naming different numerals (or lengths) at least in the sense in which it is true.

(4) Tom thought that the length of his yacht was longer than it was.

Secondly, there's the well-known objection that treating (say) belief contexts as relations between people and sentences leads to unwanted falsity when the believer doesn't know the language or cases in which the beliefs concern objects for which the believer has no name (cf. Paper (4) fn. 11). The only comment made in (2) is that the notion of synonymy is of no help with this. Thirdly, and most ironically, is that the analysis proffered in (2) runs afoul of the results of papers (9) and (10). It seems that the analysis of 'that' as expressing a relation between (say) a speaker and the standard name of a sentence (for 'says that') or between a believer and the standard name of a sentence (for 'believes that'), etc., will, just as between a knower and the standard name of what is known, result in an inconsistent theory if only certain very simple other conditions are fulfilled (as they will be in some of these special cases, anyway). This inconsistency can be verified by the reader after considering papers (9) and (10).

Together, (9) and (10) form an excellent example of the application of formal methods to a traditional philosophical puzzle (of epistemology) and show how these methods can lead to new and important results both in logic and philosophy. The puzzle is that of the "unexpected hangman", where a prisoner is condemned to die at noon on one of a specified number of days but on the condition that he shall not know beforehand which one. The prisoner reasons that it cannot be the last day (since he then would know beforehand, on the evening of the second-to-last day), and then successively eliminates the second-to-last, third-to-last, etc., and concludes that he will not be hanged. He is rudely surprized, however, when in the middle of the week the hangman unexpectedly arrives. Is this a genuine paradox? Quine (1953) dismissed it as being merely a case of fallacious reasoning on the part of the prisoner (as have other writers); and in its usual formulation it is (as Kaplan and Montague clearly show). But the real interest in the puzzle is to see if there is a formulation which is genuinely paradoxical and find out if that formulation has any effects on theories of knowledge. (In this they are guided by Tarski's formulation of the liar paradox which has effects on theories of truth).

The authors give a simple formulation of the judge's decree,
issued on Sunday, involving only the days Monday, Tuesday, and Wednesday for the hanging. Even so, there are a considerable number of assumptions needed to bring out the paradox; but it is a paradox, both the hangman and the prisoner are correct in inferring respectively that the prisoner will (will not) be hanged. And each of the assumptions is plausible; “indeed, before discovering the present paradox, we should certainly have demanded of an adequate formalization of epistemology that it render the conjunction of [the eight assumptions], if not necessary, at least not impossible.” But the paradox shows that these assumptions are incompatible with elementary syntax.

Simpler formulations of the paradox are also possible: e.g., only one possible date of execution. In this case the judge’s decree is taken as follows. Unless K knows on Sunday that the present decree is false, the following condition will be fulfilled: K will be hanged on Monday noon, but on Sunday he will not know on the basis of this decree that he will be hanged on Monday noon. And more simply still, the number of possible dates of execution can be reduced to zero, bringing out more clearly the self-reference which was implicit in the original paradox. Here the decree is: K knows on Sunday that the present decree is false.

Using the device of “standard names” again (in any of the formulations of the formulations of this notion mentioned above), the standard name of E is designated E. If S2 is derivable from S1 in elementary syntax, there is a formula of elementary syntax which expresses this fact (this is from the work of Gödel). Let I be the relation of derivability so expressed; then the assertion that S1 ⊨ S2 is expressible as I (S1, S2). Let K (x) be the statement that K knows on Sunday that x. Then the decree (D) in the zero-day case can be formulated as

(1)  D ≡ K(¬D)

and the paradox rests on these three assumptions

(a)  K(¬D) ⊨ ¬D
(b)  K(¬D) ⊨ ¬D
(c)  [I(K(¬D), ¬D) & K(¬D)] ⊨ K(¬D)

Except for (a), these assumptions are perhaps not universally valid remarks about the concept of knowledge. For example,
(b) might require a philosopher: one who knows that everything he knows is the case. But surely some instances of them must be allowed. And furthermore, notice that any predicate 'K' which satisfies anything like (a) — (c) is susceptible to this paradox.

One way around this Knower paradox would be to make 'knows' a predicate of the metalanguage. This would make such sentences as 'Socrates knows there are things which Socrates doesn't know' be meaningless. Or we could invoke a hierarchy of metalanguages. This makes 'knows' be ambiguous, depending on the number of embedded 'knows'. The least satisfactory solution (according to the authors) would be to reject some part of elementary syntax—such as self-reference.

Notice that 'K,' in the above formulation is not a sentential operator; it is a predicate which is true of objects (namely sentences). It is that feature which allows Montague in (10) to criticize attempts by Carnap and Quine to analyze "necessity" as a predicate true of names of sentences rather than as a sentential operator. In this paper Montague uses Gödel numbering as his device for forming standard names, and considers any one-place formula-maker 'N' (e.g., 'K knows that p' or 'p is necessary') instead of the 'K,' of (9). The following theorems about any interpretation of the predicate 'N' (but especially 'necessity') are of interest.

*Theorem 1.* Given any theory containing elementary syntax, if for all sentences \( \phi, \psi \)

(i) \( \vdash N[\phi] \supset \phi \)

(ii) \( \vdash N[N[\phi] \supset \phi] \)

(iii) \( \vdash N[\phi \supset \psi] \supset [N[\phi] \supset N[\psi]] \)

(iv) \( \vdash N[\phi], \text{if } \phi \text{ is a logical axiom} \)

then the theory is inconsistent.

*Theorem 3.* Given any theory containing elementary syntax, if

(i) \( \vdash N[\phi] \supset \phi \)

(v) \( \vdash N[\phi], \text{ whenever } \vdash \phi \)

then the theory is inconsistent.

What conclusions should be drawn from this? Observe that (i) — (v) are provable in all the well-known systems of first order modal logic where 'N' is taken as a sentential operator.
Conditions (i) — (iv) are provable even in the weakest S-system, $S_1$. So if necessity is to be treated syntactically—as a predicate of sentencesala Carnap and Quine, all of modal logic, even the weak $S$, will have to be scrapped. This also holds for various other ‘that’ contexts which one might try to analyze as predicates of names of sentences, e.g., ‘it is provable that’, ‘Kepler is aware that’ (which presupposes the truth of the complement), etc. Thus the proposal of paper (2) will, in some cases at least, lead to inconsistency.

The remainder of (10) shows (a) that embedding such theories into a stronger theory will not alter this picture and (b) that various extensions of elementary syntax are not finitely axiomatizable, if consistent.

The paper (11) is a refreshing change from the usual vague remarks prevalent in the philosophy of science, especially when those writers discuss the concept of determinism. This paper ought to be required reading for all graduate students who intend to do work in the topics of scientific theories or determinism (that way we might stem the flood of imprecise articles to which we have been subjected over the course of the last 30 years).

The present article begins with a discussion of the theory of theories. A (scientific) theory is a couple $<L, V>$ where $L$ is a language and $V$ are the axioms of the theory. A model for $L$ will be a triple $<D, <E^1, \ldots, E^k, \ldots>, <D^1, \ldots, D^n, \ldots>>$ where $D$ is the (non-empty) domain, each $E^i$ is the 'meaning' assigned to the "abstract constants" (= theoretical constructs, including the sets of real and natural numbers), each $D^i$ is the 'meaning' assigned to the "elementary constants" (= observational terms, state-variables) of the language. The history of a theory is identified with the sequence $<D^1, \ldots, D^n, \ldots>$ of a standard model. A history, then, provides interpretations for the elementary or observational constants. A formula $\phi$ of $L$ is realized by a history $S$ if there is a standard model $M$ of which $S$ is a history and $\phi$ is true in $M$. We are next given a complete and elegant formulation of classical particle mechanics (PM) and Newtonian celestial mechanics (CM), including all the mathematical background necessary. The elementary constants of PM are $P$ (the set of particles), $m^i$ (the mass of), $s^i$ (the position of), $v^i$ (the velocity of), and $f$

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2This should not be taken to imply that Montague believed there to be any hard and fast distinction drawn between observational and theoretical terms. He explicitly rejects this, p. 306.
(the force acting on . . . by . . .). The abstract constants are \( R \) (the set of reals), \( N \) (the set of naturals), \(+\), \(-\), and various mathematical constants. If we add \( g' \) (gravitational force on) to the elementary constants and make \( f' \) an abstract constant, we get the language of CM. The entire discussion of PM and CM is carried on at the level of one dimension, but as Montague indicates (pp. 318-19) it would be trivial to extend it to \( n \) dimensions. A novelty (I believe) of the formulation of the axioms for PM is that the Second Law of Motion (about the net force acting on a particle) is stated without recourse to the variable binding operator \( \Sigma \). The First Law of Motion is redundant in PM; if we replace the Third Law of Motion by the Law of Gravitation, we arrive at CM (we also of course need axioms insuring that the gravitational constant operating on \( x \) is independent of time and \( x \)).

We are next introduced to various definitions of 'determinism'. If \( S \) is a history of a theory, the state of \( S \) at \( t \), \( st_\alpha (t) \), is defined as \( \langle D^1 (t), \ldots, D^n (t) \rangle \). A theory is futuristically deterministic iff a given state uniquely determines all later states; it is historically deterministic iff a given state uniquely determines all earlier states; it is (simply) deterministic iff it is both futuristically and historically deterministic. Besides categorical determinism, we might consider conditional determinism: if we are given a state \( S \) and the complete history of certain state variables, the complete history of the remaining state variables is determined. In such a case it is deterministic in those state variables. Various other definitions of deterministic theories and histories are canvassed, and the notion of a history is narrowed now to apply to theories in which one of the observation terms is a set—as in the case of PM and CM, the set of particles \( P \). A history is then of the form \( \langle C, D^1, \ldots, D^n \rangle \) where \( C \) is the class of objects whose history is provided by \( D^1, \ldots, D^n \). It is natural to define determinism in such cases as: whenever we have two histories \( \langle C, D^1, \ldots, D^n \rangle \) and \( \langle C^*, D^*1, \ldots, D^*n \rangle \) which realize the theory, together with a 1–1 function between elements of \( C \) and \( C^* \) such that at a given instant each element of \( C \) has the same state (according to \( S \)) as the corresponding element of \( C^* \) (according to \( S^* \)), then also at any other instant each element of \( C \) will have the same state as the corresponding element of \( C^* \). Montague proves that this seemingly more stringent conception of determinism is equivalent to the earlier one (under the assumption that the set \( C \) is not the set of real numbers).
It is sometimes claimed that PM is deterministic. As Montague notes, this is trivially false since we can imagine distinct histories of PM which coincide at an instant of time with respect to position, velocity, mass, etc., of particles. He provides an exact proof that PM is deterministic in the force-function (that is, once the force-function is given a complete history). Since one way of specifying the force-function is by the Law of Gravitation, one would expect that CM is deterministic. Whether it is or not is a non-trivial problem which is shown to be equivalent (but otherwise left open) to a problem in the theory of differential equations.

Some writers have held determinism to be associated with a periodic history, or periodicity. Montague proves that there are deterministic theories which are not periodic and periodic histories which are not deterministic. A history is sometimes said to be deterministic if none of the functions which it comprises exhibits a random sequence of values. Following Church (1940), Montague defines the notion of a random sequence of values, and conjectures that there are deterministic theories which exhibit random histories.

The final section of this paper concerns the notions of 'explicit definability' and 'provable determinism' (which are shown to be equivalent). This is a difficult but rewarding section, especially as it shows how to construct solutions for special cases of the n-body problem for k-dimensions. As mentioned above, CM can be extended to hold in an arbitrary number k of dimensions. Let these theories be called CM_{k}. Furthermore we can add to CM an axiom which has the effect of asserting that the number of particles is n. Call this theory CM_{k,n}. By Montague's theorem 16, the n-body problem for k dimensions is soluble iff the theory CM_{k,n} is provably deterministic. Furthermore, in the cases where CM_{k,n} has been effectively shown to be provably deterministic, Montague's methods will lead to an actual solution of the problem.

The remaining papers are glimpses at the fifteen years of development of Montague's ideas starting with treatments of modality (in (1)), the development of intensional logic (3) – (4), the treatment of the ontological status of certain entities in this language (5), and its application to English (6) – (8). A book was being prepared by Montague, to be called The Analysis of Languages and which which was to make the set theoretic foundations of intensional logic more precise and presumably also to extend the range of English constructions.
which were under consideration. The papers (6) – (8) therefore have to be read as parts of this work which was left unfinished by his premature death. The introduction by Thomason provides a convenient exposition of Montague's general conception of formal languages, including English, and explains various points not made explicit in Montague's published work. While the introduction is quite worthwhile, it perhaps starts at too advanced a level for people with only a minimal knowledge of logic. (That is, I think Thomason's comment that it presupposes only the rudiments of set theory and semantics of first-order languages is somewhat misleading—one needs a reasonably good grasp of such areas). However, it is a valuable introduction and will help anyone who has the perseverance to carefully study it. It is unfortunate that there is no comparable introduction for linguists in this book, but perhaps some of the work of Partee's will remedy this (see bibliography of works on Montague's theory of language).

In (1), written in 1955, we are introduced to an analysis of modal and deontic logic in terms of something essentially like possible worlds. This is surely one of the earliest in the modern revival of modality, along with Kanger (1957 a, b, c), Hintikka (1961), and Kripke (1959); it is also related to the earlier work of Carnap (1946). The main difference between all of these others and that of Kripke (1959) is what the "accessibility relation" holds between. In Kripke, it is "possible worlds" (= "points of reference"); in the others it is a relation between models.

Consider the following schemata:

(1a) If $\phi$ is tautologous, then $\phi$ holds

(1b) If $\phi$ is tautologous, then $N\phi$ holds

(2a) $N(\phi \supset \psi) \supset (N\phi \supset N\psi)$ holds

(2b) $N(N(\phi \supset \psi) \supset (N\phi \supset N\psi))$ holds

(3a) $N\phi \equiv NN\phi$ holds

(3b) $N(N\phi \equiv NN\phi)$ holds

(4a) $\neg N\phi \equiv N\neg N\phi$ holds

(4b) $N(\neg N\phi \equiv N\neg N\phi)$ holds

Now whether we interpret 'N' as 'for all $x$', 'it is logically necessary that', 'it is physically necessary that', or 'it is
obligatory that', we find that (1a) – (4b) hold. Montague exploits this analogy, along with Tarski's definition of satisfaction (and truth) for quantifiers, to give a Tarski style semantic definition of truth for modal and deontic logic. The "accessibility relations" between models are defined for the above four interpretations (in order) as follows (where a model is a triple \(<D, R, f>\) and D is a non-empty set (the domain), R is an assignment of appropriate elements of D to the constants of the language, and f is a function which assigns to each variable an element of D):

(Q) \(<D, R, f> A_q <D', R', f'> iff D = D', R = R', f (a) = f' (a) for all variables a other than 'x'\)

(L) \(<D, R, f> A_l <D', R', f'> iff D = D' and f = f'\)

(P) \(<D, R, f> A_p <D', R', f'> iff D = D', f = f', and <D', R', f'> satisfies all members of K\)

(O) \(<D, R, f> A_o <D', R', f'> iff D = D', f = f', and <D', R', f'> \in I\)

Here, the accessibility relation \(A_q\) is the familiar Tarski relation of finding an assignment which is exactly like another except for its assignment to the variable 'x'. It is clear how to define 'satisfaction' from this. \(A_l\) reflects the intuition that it is logically necessary that \(\phi\) holds under every assignment of extensions to the descriptive constants (i.e., for every R). To interpret \(A_p\) we find a set K of "physical laws", given in advance, and without any occurrences of 'it is physically necessary that' in them. Then the relation \(A_p\) holds between models just like \(A_l\) did, except that the second model has to satisfy all physical laws also. Finally, for \(A_o\) we set up a class of "ideal models" I wherein all constants have the extensions they ought have; two models are in the \(A_o\) relation iff D = D', f = f', and the second model is ideal. It is obvious that we can define 'satisfaction' and hence 'truth' for any of these interpretations of 'N', given the appropriate accessibility relation. And equally obviously we can do the same for a language with many different such modal operators in them (e.g., quantified modal deontic logic).

\[ N \phi \rightarrow \phi \]

is absent from the general characterization. But it clearly holds in the quantifier and logical necessity cases, as can be seen from the definitions of \(A_q\) and \(A_l\) (they are reflexive in their field). It does not hold for physical necessity or obligation.
It turns out that logical necessity under Montague's interpretation is a kind of universal quantification over all descriptive constants contained in the sentence following. This cannot be expressed in a first-order language, because it would require quantifiers over predicates. However, its force can be seen from the following example. The inference from \( (x) \exists y L(x = y) \) to \( (\exists y) L(c = y) \) is invalid, Montague claims, since \( L \) (in formulae with constants) is to be interpreted as a quantifier over the constants. In the premise here, this quantification is vacuous since there are no constants, but the conclusion would become \( (\exists y) (c) (c = y) \). And the fallacy in the original inference (the apparent application of UI) is seen to involve nothing more mysterious than a violation of the normal restrictions of "clashing quantifiers". This, Montague claims, is also the fault in the example \( L \) (the morning star = the evening star) therefore \( (\exists x) L \) (the morning star = x). So quantified modal logic is not incoherent; the problem is in the statement of Leibniz's Law.

(A) \( (x) (y) (x = y \supset (\phi x \equiv \phi y)) \)
is satisfied by every model, but

(B) \( c = d \supset (\phi c \equiv \phi d) \)
does not hold in general (when \('c' and 'd' are individual constants).

In modal logic the key idea is that the truth-value of formulae containing modal operators is to be evaluated at different possible worlds. Intuitively speaking, the extension of a sentence at a possible world is a truth-value; the intension of a sentence is a function on possible worlds to truth-values (i.e., the characteristic function of the set of worlds at which the extension = T). Another way of putting this is that the intension of a sentence is a specification of exactly under what circumstances the sentence will be true; and that has often been associated with the meaning of the sentence. Pragmatics is a logic which generalizes these first-order intuitions to the ultimate degree. Instead of merely considering the one-place modal operators, we are given a method for evaluating n-place sentence operators; and instead of the ordinary possible worlds, we are given the more general notion of indices (or points of reference). For certain applications of the resulting generalized intensional language, the indices will be identical with the ordinary notion of possible world; for other applications they
will be moments of time (tense logic), "ideals" (deontic logic), contain information about the speaker, place, etc., of utterance (logic of pronouns, indexical adverbs, deixis, dates, etc.), fields of probability (inductive logic), and various combinations (which yield rather natural readings for "the future subjunctive conditional" and the phrase 'it was predictable at some time in the past that it would now be the case that φ'). While Montague did not consider the possibility, it seems that "relevance logics" would also be subsumed as a special case.

An interpretation (model structure) for a pragmatic language is <I, U, F> where I is the set of all indices, U is the set of all possible objects, and F is a function which maps elements of the language into appropriate entities at each index. We are also given a special predicate 'E' which tells us (at each index) what elements of U exist at that index. Such treatments of non-existing (but possible) objects are likely to put a lot of philosophers off: it treats non-existents exactly like existents except that the predicate 'E' doesn't apply truly (at a given index). Not only does it make 'exists' a predicate (which some people dislike, but which I shall not defend here), but also seems to raise such difficult questions as (a) what are the properties of (say) mythical characters not explicitly mentioned in the myths? (b) (In the field of tense logic) at a given time Smith may be dead, so what does it amount to in saying Jones now remembers him? Remembers him alive? Crying? Smiling? (c) What about Russell's difficulties with Meinongian theories (which bear a strong resemblance to the present proposal)?

I think we can ignore the Meinongian difficulties; they arose mostly in connection with definite descriptions and Montague's theory does not treat definite descriptions as names in the way required for the Russellian objections (and anyway it is not obvious that Meinong's theory is really vulnerable even to these—see Parsons (1974)). As for difficulty (a), each index will indeed contain all information about, say, Zeus, and every predicate or its negation will be applicable to him (similarly with n-place relations). So far there is no incoherence; the difficulty comes in when we try to specify which is the actual world—we are unable to because the totality of our knowledge is compatible with many different indices, ones where Zeus is taking adult education courses at Harvard and ones where he is not. But this indeterminacy can cause us no difficulty and is surely not philosophically objectionable. Similar remarks apply to difficulty (b). Montague's arguments in favor of the
all-inclusive domain can be found on p. 100 (repeated on p. 126), p. 103 (repeated on p. 126), p. 124 (repeated on p. 153). The strongest argument, however, is that the treatment works for describing the semantics of a large class of intensional contexts while no other approach has come close.

In particular applications of pragmatics, we may not be interested in all possible models, but only in a certain subclass; we similarly might not be interested in all but only a restricted set of the indices. For example, let 'y' be an individual constant naming yesterday and 'P' be a predicate constant for 'is past'. When the sentence 'Py' is uttered, we use the time of utterance i both to fix the referent of 'y' and to evaluate whether this referent is an element of the extension of 'P' at that index. Clearly this formula is semantically invalid: there are indices where 'y' denotes sets of moments which are not previous to i. Nonetheless the formula is pragmatically valid: it cannot be uttered at such an index. (That is to say, we shall not allow such indices to play a part in determining the validity in certain circumstances. We shall restrict our attention only to those "normal" indices where 'y' denotes a set of moments previous to i.) A similar restriction for pronouns might be to demand that we consider only indices where the pronoun-symbol denotes something that exists at the index of utterance. If this (rather reasonable) restriction were done, then (but only then) sentences like 'I exist' would be valid. So we see how pragmatic validity differs from semantic validity and gives sense to both Descartes' claim "'I am, I exist' is necessarily true every time I utter it" and to his opponents' claim that it is not semantically valid.

In (4) a kind of second-order extension of pragmatics is given and called intensional logic. In this language we can, for example, quantify over predicates (viewed as denoting functions from sets of indices into appropriate entities: e.g., 1-place predicates are functions into sets (of things that have that property), 2-place predicates are functions into sets of ordered pairs (of things that bear that relation to one another), 0-place predicates (= propositional constants) denote functions into the set of truth values, etc.) The resulting language includes the calculus of lambda-conversion but has more expressive power since any intensional entity can be named even the intension of variables and individual constants. (The sign \( ^\wedge \) concatenated before any expression \( \phi \) gives the intension of \( \phi \). The sign \( ^\wedge \) over a variable is set abstraction on that
variable with respect to the condition following. And \( ^{\wedge} \) written over a variable \( x \) is an abbreviation for \( ^{\wedge}x \). I hope such informal explanations are helpful in following Montague's demanding notation.) In this language, Frege's functionality principle applies in the following manner: the intension of a complex expression is purely a function of the intensions of its components. The extensions of expressions (e.g., the truth value of a sentence) sometimes depend on the intensions of the components, and so the language does not embody the functionality principle for extensions. Nonetheless, we never have to consider intensions of intensions in addition to ordinary intensions and ordinary extensions.

Montague regards intensional logic as "the first fully adequate treatment of belief contexts and the like" (p. 116). Many philosophers, including Quine (1960) and elsewhere, have pointed out difficulties involved in quantifying into "belief contexts" when these are construed as expressing relations between individuals and propositions. Attempts to meet Quine's objections—e.g., Kaplan (1964, 1969) have involved difficulties in expressing the iteration of belief. Montague gives the following example involving both difficulties to show how these problems are treated in intensional logic: 'There is an object of which Jones believes that Robinson believes it to be perfectly spherical'.

\[(\exists x)(\text{Ex} \cdot \beta [j,^\wedge (v,^\wedge Sx)])\]

Where 'E' is our special predicate of existence, 'j' and 'r' are individual constants naming Jones and Robinson, ' β ' is the believes relation which holds between an individual and a proposition (which is indicated by the \( ^{\wedge} \)), and 'S' is a predicate constant whose extension is the class of perfectly spherical objects.\(^3\)

Some philosophers are likely to have doubts as to the validity of this analysis, however. Leibniz's Law, remember, holds in the form

\[(x) \ (y) \ (x = y \supset ( \Phi x \supset \Phi y))\]

That is, if 'c' and 'd' name the same entity in all possible worlds then Leibniz's Law is applicable no matter what the context. 'Tully' and 'Cicero', it might be argued (ala Kripke 1973) are such names and so any beliefs one has invoking Tully are

\(^3\)This translation would be handled differently in paper (8). In that paper one would get this translation only after applying certain "meaning postulates."
therefore (under the present analysis) also beliefs about Cicero. And similarly for logically equivalent statements—one believes all propositions equivalent to ones he believes. Montague's argument in favor of this (p. 139) might not sound very convincing even to philosophers who hold the Kripke position.

Paper (5) is a discussion "of such dubious epistemological, metaphysical, and ethical entities as pains, tasks, events, and obligations." Montague thinks that intensional logic is "an exact and convenient language in which to speak of them, and [by which to] analyze the pertinent notion of logical consequence." (p. 149).

The language of intensional logic "fully meets the objections of Quine and others, possesses a simple structure as well as a close conformity to ordinary language, and concerning the adequacy of which . . . no serious doubts can be entertained" (pp. 155-6). In this language we have the now-familiar explanation of what predicates are: they are functions from all possible worlds to individuals—namely the set of individuals which in that world have the property in question. For example, the property of being red is the function which assigns to each possible world the set of possible individuals which in that world are red. More generally, an n-place predicate of individuals is a function on possible worlds which gives sets of n-place sequences of possible individuals which exemplify that relation. So, for example, the 3-place relation ". . . gives . . . to . . ." is a function which in each possible world has as its value the set of ordered triples <a, b, c> such that a gives b to c in that world.

A sentence, e.g., 'the sun rose at eight', is considered to express that a certain (generic) event (the rising of the sun) occurred at a particular moment of time. The event is taken to be the property expressed by the formula 'the sun rose at t', that is, the property of being a moment of time at which the sun rises. Events in general are a certain class of properties of moments of time. A task R is taken to be the intension of the relation R born by x to t just in case x does R at t. Experiences (including pains and sense data—the latter being discussed quite extensively) are also held to be the intension of a certain class of relations between people and times. The fact that one's presystematic intuitions about logical consequence can be easily mirrored under this analysis surely counts strongly in its favor.

Papers (6) – (8) form an important contribution to linguistic
theory. In fact, it is my opinion that, for the near future, these three papers (and especially (8)) will be Montague's most widely read and appreciated works. One has merely to look at the recent upsurge of articles and dissertations in linguistics to appreciate this fact. However, this makes a detailed discussion of Montague here always behind the times, as refinements and extensions of his treatment are always going on. I therefore shall try to give only the flavor of his work in this field, and leave it to the reader to consult the works listed below in the bibliography.

This manifesto appears at the beginning of (7), a similar one is at the beginning of (6):

This is clearly a radical departure from both the "ordinary language" and the "positivist" traditions. It is also against what has been the overwhelming beliefs in linguistic theory (see Katz and Fodor 1963), although Montague sometimes seems to think otherwise (see the beginning of (7)).

The basic aim of semantics, says Montague, is to characterize such notions as truth under an interpretation, entailment, etc. That of syntax is to characterize various syntactic categories. It is possible, therefore, that there be many equally good, descriptively adequate syntactic theories, only some of which will provide a suitable basis for semantics. Montague "fail[s] to see any great interest in syntax except as a preliminary to semantics." (p. 223 fn)

For the "ordinary" sentences of English—those treated impressionistically by elementary first-order logic—we know what the appropriate semantics should be. It is exactly what the usual model-theoretic account gives to their translations into first order logic. The problem with simply adopting this is that elementary logic gives no algorithm for translating these "ordinary" sentences into first order logic; and even if one were to somehow codify the informal hints given in elementary logic texts, there is no guarantee that this will mesh well with extensions to more complex areas of English. Furthermore, the phenomenon of ambiguity is not easily accommodated by such an approach; and finally, there is a very large number of
sentence-types which cannot be handled in any first-order language.

Montague's approach (in (8)) is to (a) disambiguate each sentence under consideration, (b) give an algorithm for translating each disambiguated reading of a sentence into a sentence of intensional logic, and (c) use the model theory for intensional logic to give the semantics for the disambiguated sentences of English. A few words on each of these steps are in order. With respect to (a), the approach in (8) is to consider a "fragment" of English. We are given a set of "basic terms" each of which is associated with a syntactic category. We are also given syntactic rules by means of which we can construct non-basic phrases of the various syntactic categories. This is done by constructing "analysis trees", starting at the "bottom" (the leaves) and continuing "up" the tree in accordance with the syntactic rules (which combine branches) until the "top" (the root) is reached. Each node will be some English phrase, together with an indication of the syntactic category to which it belongs and the rule used in obtaining that node. The "top" will be the English sentence under consideration; the sentence together with the tree is a disambiguated sentence. A sentence with more than one non-equivalent analysis tree is ambiguous.

With respect to (b), we are given a number of translation rules. These rules are sensitive to the type of syntactic combination employed at each node of the analysis tree. For every such syntactic rule, there is a translation rule. One "works up" the tree to finally get a translation of the sentence at the top. Hence every disambiguated sentence has a translation into intensional logic. These translations may now be "reduced" to an equivalent, simpler form by means of theorems of intensional logic. With respect to (c), Montague has a number of "meaning postulates" which restrict the range of admissible models. This is necessary because of the rather baroque translation scheme used; the scheme makes, for example, proper names of English be translated into terms which denote an individual concept (= function on possible worlds to individuals) rather than an individual. In some contexts (e.g., belief contexts) this seems desirable; in others we want also to know that we are talking about the individual itself. A meaning postulate might, for example, say: in such-and-such

\*Verb Phrase, Common Noun Phrase, Adverb Phrase, etc., all of which are defined in terms of two elementary categories—Entity Phrases (e) and Declarative Sentence (t), for neither of which does Montague find any basic phrases in English.
contexts, a sentence which says that an individual concept possesses a certain property may be replaced by a sentence which says that a certain individual is a member of a certain set. This procedure of Montague's makes it look very much like he believes English to be entirely intensional, and only certain contexts generating extensionality. Most philosophers of logic hold the reverse position.

The test of correctness for the present fragment are these. First, this approach agrees in truth conditions with first order logic on all sentences that they both treat. Second, inferences intuitively judged to be valid (invalid) are validated (invalidated) by this method. Third, intuitions about the truth conditions of complex sentences containing, say, 'worships', 'believes', 'seeks', 'tries', etc., are appropriately mirrored. And fourth, intuitions about syntactic ambiguity are captured by the syntactic analyses.

On the other hand, while the types of contexts handled by (8) include the trickiest philosophers are aware of, the range of sentences actually produced by the syntactic component is quite small, and to some extent quite stilted English. However, linguists have picked up on Montague's approach and are devoting considerable energy to extending the range of sentences treated (see bibliography). It is to be hoped that Montague's insights here can be fruitfully combined with those of transformational grammarians to produce a reasonably comprehensive account of a natural language.

There are three (or four?) omissions from the "Bibliography of Works by Richard Montague" at the end of Formal Philosophy.

3. The Proper Treatment of Mass Terms in English—A Response to Julius Moravcsik. Ditto version of #2 above with various changes.

\(^5\)E.g., 'a horse such that it speaks is a entity such that Jones finds it', 'every man is a entity such that it loves a woman such that she loves it.'
There are also various misprints in the book. Since Montague's notation is already difficult to follow, and since I think this book to be very important, I think it worthwhile to include a rather complete list of them. Many of these errors were compiled by members of the "Montague Grammar" course at the 1974 Summer Linguistics Institute, taught by Barbara Hall Partee. An underline on the line number means to count up from the bottom.

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\[ 256 \quad 2 \quad \Lambda u \phi \quad \Lambda u \phi \]
\[ 257 \quad 2 \quad D_{\alpha, \lambda, A, I, I} \quad D_{\alpha, \lambda, A, I, I} \]
\[ 260 \quad 1 \quad \phi \square a \quad \phi_B \]
\[ 260 \quad 4 \quad [\Lambda u \phi ] \quad [\Lambda u \phi ] \]
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in T1(e) and T2 make all P's italic except for P (cf.260:5)

in T1(e) \^ \quad \^ \quad \^ \quad 262\quad 11\quad [\phi \Lambda \psi] \quad [\phi \Lambda \psi^*] \]
\[ 262 \quad 12 \quad [\phi \vee \psi] \quad [\phi \vee \psi^*] \]
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\[ 262 \quad 11 \quad \hat{\gamma} \quad \hat{\gamma} \]

263 \quad 2 \quad P \quad P \text{ (make italic)} \]
\[ 265 \quad 11 \quad \text{TV} \quad \text{IV} \]

266-7 \quad \text{make P's in translations italic} \]
\[ 289 \quad 10 \quad \alpha (\Delta_{nr}(\phi)) \quad \alpha (\Delta_{nr}(\phi)) \]
\[ 292 \quad 2 \quad Q(\alpha) \quad Q(\alpha) \]
\[ 305 \quad 8 \quad \text{four} \quad \text{five} \]
\[ 308 \quad 15 \quad \text{D}_m \quad \text{D}_m \]
\[ 308 \quad \text{fn} \quad \text{by} \]
\[ 317 \quad 13 \quad \Sigma_{m=0}^{\alpha-1} \quad \Sigma_{m=0}^{\alpha-1} \]
\[ 317 \quad 19 \quad (s)_{\alpha-1} \quad (s)_{\alpha-1} \]
\[ 317 \quad 5 \quad (f) \quad (f) \]
\[ 325 \quad 6 \quad \text{points,} \quad \text{points out,} \]
\[ 337 \quad 17 \quad \text{Theorem 3.} \quad \text{Theorem 5.} \]
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