(So-Called) "Paradoxes of Material Implication"

Recall the truth table for the material conditional:

φ_1	φ_2	$(\varphi_1 \supset \varphi_2)$
T	T	T
T	F	F
F	T	T
F	F	T

There are a number of places where the material conditional ($\supset$) does not comport well with our intuitions about the truth/falsity of conditional sentences [i.e., sentences with (stylistic variants of) "if–then…"]. These tend to be called the Paradoxes of the Material Conditional.

$(p \supset q)$ has the same truth table as $(\neg p \vee q)$

Since everyone(??) agrees with

$$\neg p \models (\neg p \vee q)$$
$$q \models (\neg p \vee q)$$

It follows that you should believe

$$\neg p \models (p \supset q)$$
$$q \models (p \supset q)$$

Or in English, you should believe that these are valid arguments:

Since Bernie Linsky is not Department Chair, if he *is* Department Chair then Paris is not in France.

Paris is in France, so if Paris has seceded from France then Paris is in France.

If Edmonton is south of Calgary, then Venus orbits the sun.

If Edmonton is north of Calgary, then Lady Gaga sings "Born that Way"

If Frank Sinatra sings "Born that Way", then Alpha Centari is inside the orbit of Jupiter

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Indicative, Subjunctive, and Counterfactual Conditionals:

(indicative) If Oswald shot Kennedy, then he was guilty of murder
(counterfactual) If Oswald didn't shoot Kennedy, then someone else did
(subjunctive) If Oswald hadn't shot Kennedy, then someone else would have

Priest rejects the idea that there are different *if–then*s here. He says the difference between these counterfactuals and subjunctives is due to the tense/mood/aspect of the verb phrases *within* the 'if' and 'then' clauses.

Some more counterexamples (from Priest)

The following are valid arguments, formally…check them out with tableaux. But then consider the English versions. Priest also says that Grice-like conversational rules can't help here.

(1) $(A \wedge B) \supset C \vDash (A \supset C) \vee (B \supset C)$
If you close switch x and switch y the light will go on. Hence either: if you close switch x the light will go on, or if you close switch y the light will go on.

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(2) $(A \supset B) \wedge (C \supset D) \models (A \supset D) \vee (C \supset B)$
If John is in Paris he is in France, and if John is in London he is in England. Hence, either: if John is in Paris he is in England, or if John is in England, he is in France.

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(3) $\neg(A \supset B) \vDash A$
It is not the case that if there is a good god the prayers of evil people will be answered. So there is a good god.

Arguments for treating *if–then* as $\supset$

(1) There are 16 possible binary truth functions. Write them all out. We all acknowledge that at the very least, if A is true but B is false, then $A \supset B$ must be false. And we all want to say that Modus Ponens (MP, $\supset$-E) is a valid rule–so, if $A \supset B$ and A are true, then B has to be true. These two considerations rule out a number of the possible truth functions. Let's inspect the rest.

Another argument

(2) (a) Show that: if *if* A *then* B is true, then $A \supset B$ is true: Suppose that *if* A *then* B is true. Also, we know that either $\neg A$ is true or that A is true. In the first case, then $\neg A \vee B$ is true. In the second case, then B is true by MP, and hence $\neg A \vee B$ is true. In either case $\neg A \vee B$ is true–that is, $A \supset B$ is true.

Another argument

(2) (a) Show that: if *if A then B* is true, then $A \supset B$ is true: Suppose that *if A then B* is true. Also, we know that either $\neg A$ is true or that A is true. In the first case, then $\neg A \vee B$ is true. In the second case, then B is true by MP, and hence $\neg A \vee B$ is true. In either case $\neg A \vee B$ is true–that is, $A \supset B$ is true.

(b) Show that: if $A \supset B$ is true then *if A then B* is true. This argument appeals to the claim

> *(*) 'If A then B' is true if there is some true statement, C, such that from C and A together, we can deduce B.*

Now, suppose that $A \supset B$ is true, that is, suppose that $\neg A \vee B$ is true. From this and A we can get B from $\vee$-E. But it then follows from (*) that 'If A then B' is true.