---

**Note that this is a long assignment. Please start it early!!**

The objectives for this assignment are:

1. to develop your analysis skills for continuous time dynamic systems,
2. to develop your skills in converting state-space models to transfer function models,
3. to investigate the effects of sampling time on the quality of the resulting discrete-time model,
4. to continue building your MATLAB / Simulink skills for process simulation.

You may work together in groups to complete the assignment, but you must hand in your own assignment solution. **If you work with a group, please identify the people that you worked with on your solution**. Computer printout may be included with your solution as an appendix, but please do not provide these as your entire solution report.

---

You have been asked by the ACME Fine Chemicals Company to investigate a model that they have for one of their reactors. The reactor is a stirred tank in which a Belousov-Zhabotinsky type reaction takes place. (This is an interesting class of reactions. Have a look at *en.wikipedia.org\wiki\BelousovZhabotinsky_reaction*. There are also some nice YouTube videos available). The continuous-time, state-space model is:

$$\begin{aligned}\frac{dx_1}{dt} &= u_1 - x_1 - \frac{4x_1x_2}{1+x_1^2}\\ \frac{dx_2}{dt} &= u_2 x_1\left(1 - \frac{x_2}{1+x_1^2}\right)\\ \mathbf{y} &= \mathbf{x}\end{aligned}$$

The normal operating conditions are $\mathbf{u} = [10\ 4]^{\mathrm{T}}$ and $\mathbf{y} = [2\ 5]^{\mathrm{T}}$. The *time* unit in this model is minutes. In simulating this reactor, you will have to be careful in selecting your simulation parameters. I would suggest that you reduce the *relative error tolerance* to well below its default value, but not so far as to make the solving computationally expensive. I used a tolerance of $1e-06$.

Please do / answer each of the following, showing all of your work:

1. Begin by simulating the reactor during start-up (*i.e.*, $\mathbf{y}(0) = \mathbf{0}$) for:
   a) The normal value of the input variables (*i.e.*, $\mathbf{u} = [10\ 4]^{\mathrm{T}}$). Plot the the phase portrait (*i.e.*, $x_1$ *vs.* $x_2$) for this operation.
   b) A common, but abnormal value of the input variables (*i.e.*, $\mathbf{u} = [10\ 2]^{\mathrm{T}}$). Plot the the phase portrait (i.e., $x_1$ *vs.* $x_2$) for this operation.
   c) Comment on the main differences in the two modes of operation.
2. Linearize the state-space model around each of the following operating points:
   a) The normal operating mode(*i.e.*, $\mathbf{u} = [10\ 4]^{\mathrm{T}}$ and $\mathbf{y} = [2\ 5]^{\mathrm{T}}$).
   b) The abnormal operating mode (*i.e.*, $\mathbf{u} = [10\ 2]^{\mathrm{T}}$ and $\mathbf{y} = [2\ 5]^{\mathrm{T}}$).
   c) Compare accuracy of each model to the original state-space model. I suggest that you plot the responses of the original model and the linearization on the same plot for each operating mode. One way to do this is to use the *hold* command in MATLAB. (There are other ways). Comment on the quality of the linearization for each operating mode.
3. Convert your linear state-space models into transfer function form and compute the poles of the transfer functions? Determine whether they are the same as the eigenvalues of the state matrix for the corresponding state-space model.
4. Comment on the stability of the reactor for each of the operating modes. Explain.
5. For the normal operating mode (*i.e.*, $\mathbf{u} = [10\ 4]^{\mathrm{T}}$ and $\mathbf{y} = [2\ 5]^{\mathrm{T}}$), convert the continuous-time, state-space model into a linear, discrete-time, state-space model for each of the following three sampling times:
   a) $T_s = 0.5$ minutes,
   b) $T_s = 1.0$ minutes,
   c) $T_s = 2.0$ minutes.
6. Simulate the reactor start-up using each of the discrete-time models. I suggest that you plot each of the simulations on the same plot. You will have to be careful of the time index, as they depend on sampling time. Remember that you are using deviation variables.
7. Convert your discrete-time state-space models into transfer function form and compute the poles of the transfer functions. Determine whether they are the same as the eigenvalues of the state matrix for the corresponding discrete-time, state-space model.
8. Comment on the stability of your reactor model for each sampling rate. Explain.
9. Which sampling rate is the most appropriate. Explain.