Instructions:

- You may use your own calculator, but no other aids are allowed during this exam.
- Please read the entire exam before beginning. There are 3 pages of questions.
- Please try to answer all questions as fully as you can.
- The marks allotted for each question are given with the question and there are a total of 50 marks for this exam.

---

1. (10 points) Briefly compare and contrast the following terms:

   a) transient versus steady-state behaviour,

   b) lumped parameter versus distributed parameter system,

   c) deterministic versus stochastic system,

   d) linear versus nonlinear model,

   e) state versus output variable.

2. (5 points) Given the linear, time-invariant state-space model:

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}\end{aligned}$$

   Please answer each of the following questions:

   a) Which matrix is the *state matrix*?

   b) Which equation is the *state equation*?

   c) Is the output equation a *differential* or an *algebraic* equation?

   d) Write the expression for the *state transition matrix* Φ?

3. (10 points) Given the state space system:

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -1000 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathbf{x}$$

Please answer each of the following questions:

a) Will the system exhibit linear or nonlinear behaviour?

b) Is this system stable? **Briefly explain**.

c) Are there any special characteristics of this system that you will have to consider when you simulate it? **Briefly explain**.

d) Will the *process gain* for the inputs be positive or negative? **Briefly explain**.

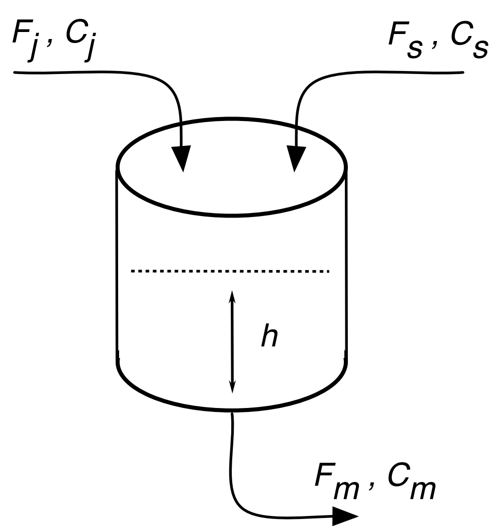

Figure 1: OMH Co. mixing tank

4. (25 points) The Old Mother Hubbard's Jelly Company uses a cylindrical tank to mix fruit juice and concentrated sugar solution, before sending the mixture for packaging. You have been asked to develop a model for control system studies. The tank, as shown in Figure 1, is 2.0 m tall and has a constant cross-sectional area of 1.0 m^2. The fruit juice and the concentrated sugar solution are fed to the tank at volumetric flow-rates of F_j m^3/hr and F_s m^3/hr, respectively. This fruit juice flow-rate depends on an upstream process operation. The sugar solution flow-rate is manipulated to ensure that the minimum sugar content specification for the mixture is respected. The tank is gravity draining and the out-flow can be assumed to follow a *square root law*:

$$F_m = k\sqrt{h}$$

The densities of the various streams are $\rho_s = 1012\ kg/m^3$, $\rho_j = 1015\ kg/m^3$ and $\rho_m = 1013\ kg/m^3$. The value of key process variables at normal steady-state operation for this surge tank is:

variable	steady-state value
F_j	100.0 m^3/hr
F_m	110.0 m^3/hr
h	1.0 m
C_j	1.0 kg/m^3
C_s	10.0 kg/m^3

Please answer each of the following questions, showing all of your work:

a) What assumptions will you make in developing your model?

b) What *balances* will be included in your model?

c) Develop the state-space model that describes the behaviour of the level in the mixer and sugar content of the effluent from the mixer.

d) Linearize the model around the normal steady-state operation.

e) Determine the stability of the model at the normal steady-state operation. **Briefly explain**.