Instructions:

- The only aids allowed for this exam are: your own **calculator** and a **single** $8\frac{1}{2} \times 11$ **sheet of handwritten notes**.
- Please read the entire exam before beginning. There are 7 pages in this examination.
- Please try to answer all questions as fully as you can.
- The marks allotted for each question are given with the question and there are a total of 100 marks for this exam.

1. (10 points) Given the model:

$$\begin{aligned} \frac{d^2x}{dt} - x &= 3u \\ y &= 3x + u \end{aligned}$$

Please do / answer each of the following:

a) Convert this model into the general continuous-time, LTI (Linear Time Invariant) state space form. *Show your work.*

b) Is the model stable? *Explain.*

2. (15 points) For the general Box-Jenkins model form:

$$y_t = \frac{\omega_s(z^{-1})}{\delta_r(z^{-1})} z^{-b-1}\ u_t + \frac{\theta_q(z^{-1})}{\phi_p(z^{-1})\,(1 - z^{-1})^d}\ \epsilon_t$$

Please do/ answer each of the following:

a) Indicate the stochastic and deterministic portions of the model.

b) Indicate which terms are the autoregressive, moving average and integrating portions of the model.

c) Describe the procedure for developing a process model of this form.

d) Given the plots in Figures 1 through 4, estimate: r, s, b and p, d, q? *Explain why you chose these estimates.*

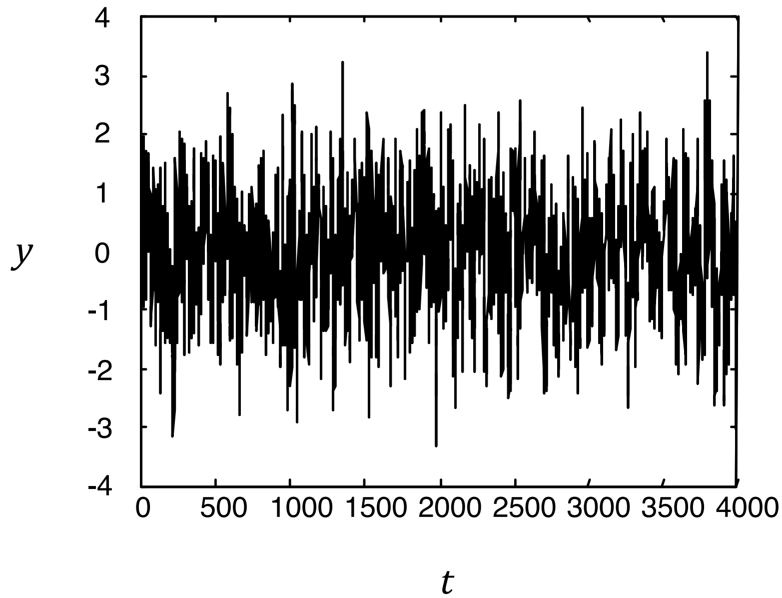

Figure 1: Output data

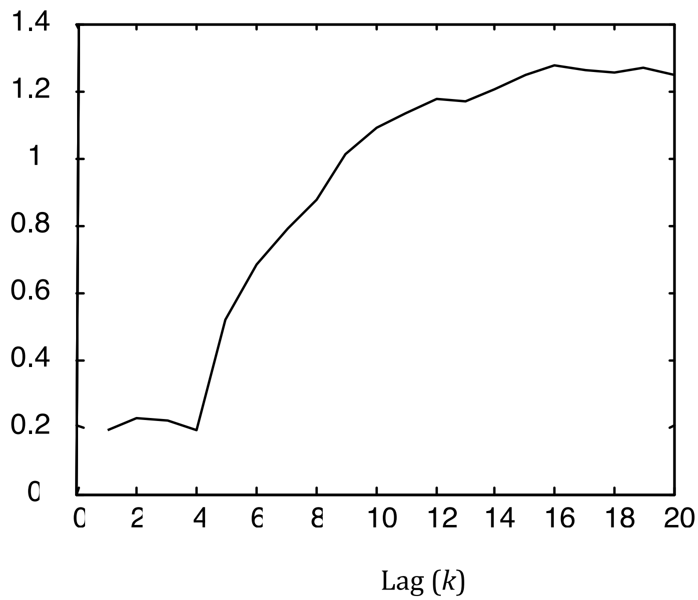

Figure 2: Estimated step response weights

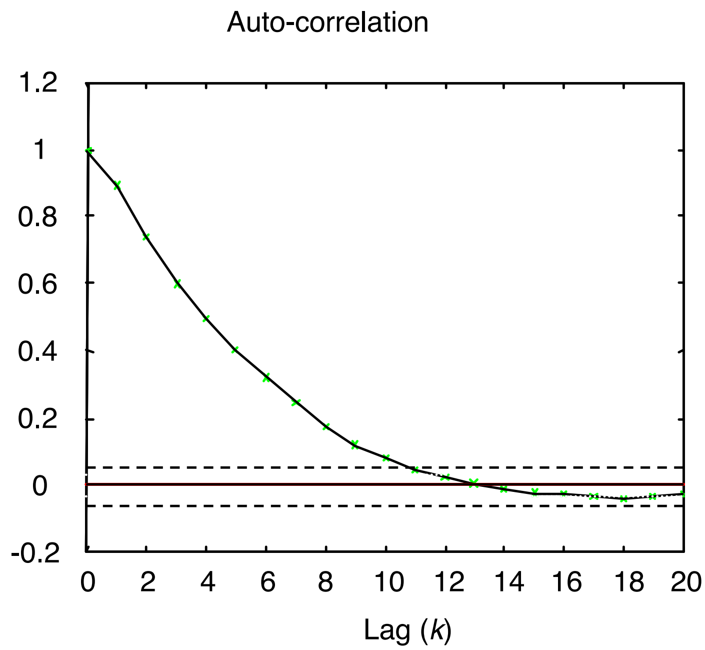

Figure 3: Autocorrelation function

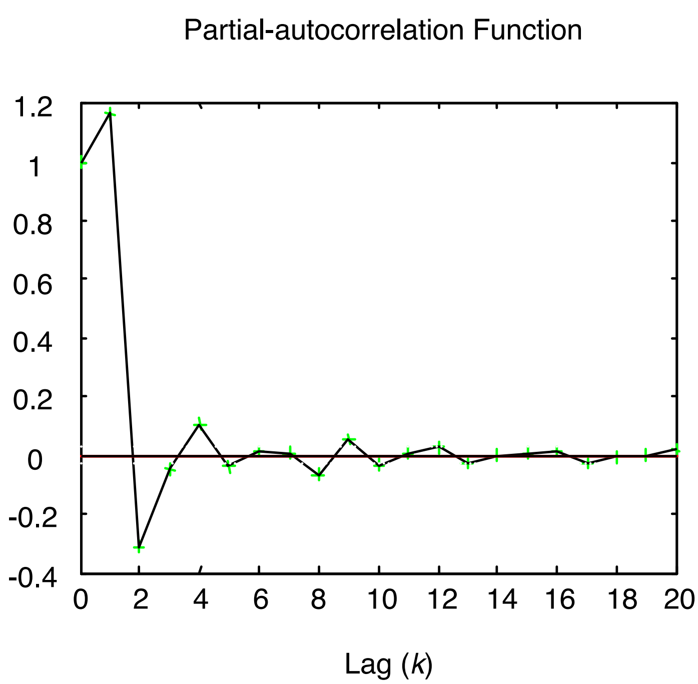

Figure 4: Partial autocorrelation function

3. (10 points) Given the following plot of experimental data:

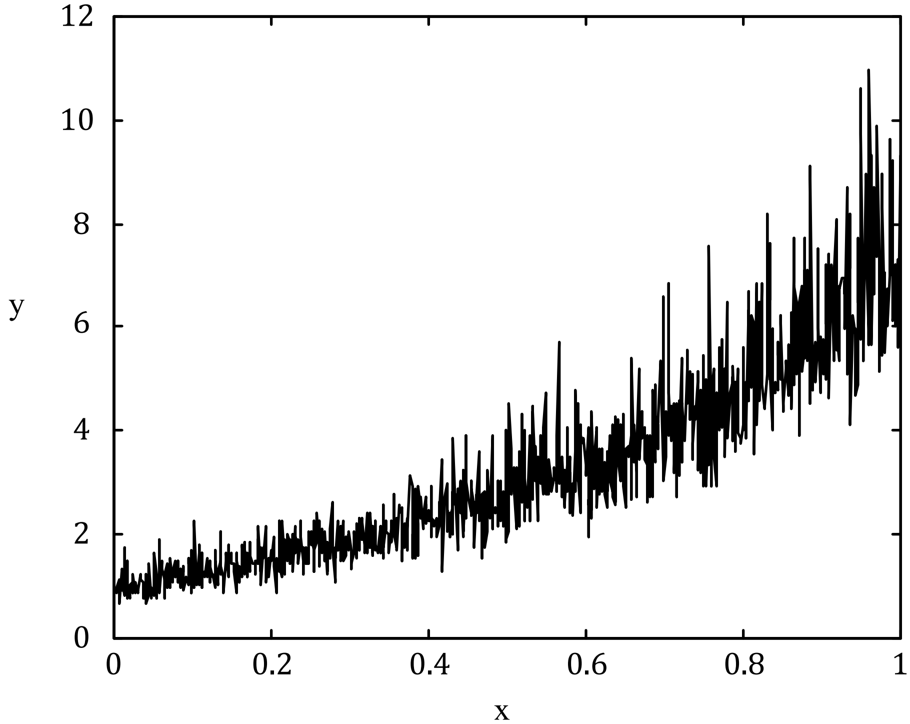

Figure 5: Regression data

You have been asked to estimate, using Least Squares Regression, the parameters for the model:

$$y = mx + b$$

State the underlying assumptions of Least Squares Regression that you will violate in this case and explain why they will be violated.

4. (10 points) For both continuous and discrete transfer function models, and using a sketch of pole and zero locations in the complex plane, describe what dynamic behaviour arises from the range of possible pole and zero locations.

5. (15 points) Given the continuous-time, LTI state space system:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & -1 \\ -1000 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x}$$

Please answer each of the following questions:

a) Choose an appropriate sampling time (T_s) for this system. *Explain your choice.*

b) Convert the given system to a discrete-time, LTI system. *Show your work.*

c) Convert the given system to a discrete-time, transfer function model. *Show your work.*

d) Are there as many poles in your transfer function as you expected? *Please explain what you expected and why, and any discrepancy with your expectation.*

6. (10 points) Given the following step response data:

t	0	1	2	3	4	5	6	7	8	9	10
u	1	3	3	3	3	3	3	3	3	3	3
y	1.29	0.55	1.38	1.90	3.45	3.60	4.95	5.40	4.56	4.09	5.86

and the ARX model form:

$$y_t = \frac{k(1 + \omega_1 z^{-1}) z^{-b-1}}{1 + \delta_1 z^{-1} + \delta_2 z^{-2}}\, u_t + \frac{1}{1 + \delta_1 z^{-1} + \delta_2 z^{-2}}\, \epsilon_t$$

Do each of the following:

a) How much *dead-time* does this step response exhibit?

a) Set up the $\mathbf{y}$ vector and the $\mathbf{X}$ matrix to solve the parameter estimation problem. **Do not do the calculations!**

b) Do you expect to get accurate parameter estimates from the given step test data? *Explain.*

7. (25 points) As many of you may have heard, several floors of our Chemical-Materials Engineering Building have been completely renovated and are now lab floors. During the design phase for this building project, many laboratory safety issues were carefully considered. One of the key concerns was the length of time required for contaminant concentrations in the lab to return to safe levels after a catastrophic gas cylinder leak.

   The design basis for the a small lab module is:

lab width	5.0 m
lab length	8.0 m
lab height	2.5 m

   The ventillation system was designed to provide 6 complete air exchanges per hour in each lab module.

   To ensure that good decisions will be made, you have been asked to develop a model for the transient behaviour of a contaminant gas concentration within a lab module. Please answer each of the following questions, showing all of your work:

   a) What assumptions will you make in developing your model?

   b) Develop a state-space model that describes the behaviour of a contaminant gas within a lab module.

   c) Is your model linear or nonlinear? *Briefly explain.*

   d) Develop a continuous-time transfer function model from your state space model.

   e) The design engineers claim that the concentration of a contaminant gas will return to safe levels after 10 minutes, should a gas bottle vent accidentally. Assume that the gas bottle contains 46.4 l of carbon monoxide (CO) at 2216 $psia$. (14.7 psi $\equiv$ 101.324 kPa $\equiv$ 1.0 Atm). The molecular weight of CO is 28.0 $g/gmol$ and air is approximately 29.0 $g/gmol$. Clinical symptoms for CO poisoning begin to appear at approximately 100 ppmV (parts per million by volume). **Are the design engineers correct**? *Show your work.*

8. (5 points) Please answer both of the following questions about the course.

   a) If you could add one topic to CH E 572 to improve the course, what would it be? *Explain.*

   b) If you could remove one topic from CH E 572 to improve the course, what would it be? *Explain.*

Some useful formulae:

Inverse of a 2 × 2 matrix $\rightarrow$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

matrix exponential $\rightarrow$ $e^{\mathbf{A}t} = \mathbf{V}^{-1}e^{\mathbf{\Lambda}t}\mathbf{V}$, where $\mathbf{V}$ and $\mathbf{\Lambda}$ are the eigenvectors and eigenvalues of $\mathbf{A}$, respectively.

convolution integral $\rightarrow$ $\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}\tau}\mathbf{B}\mathbf{u}(t-\tau)d\tau$.

ideal gas law $\rightarrow$ $PV = nRT$ and $R = 8.3145\frac{J}{mol \cdot K}$.

relationship between the complex variables s and z $\rightarrow$ $z = e^{T_s s}$.