
Note that this is a long assignment. Please start it early.

The objectives for this assignment are:

1. to develop your analysis skills for continuous time dynamic systems,
2. to develop your skills in converting state-space models to transfer function models,
3. to investigate the effects of sampling time on the quality of the resulting discrete-time model,
4. to continue building your MATLAB / Simulink skills for process simulation.

You may work together in groups to complete the assignment, but you must hand in your own assignment solution. If you work with a group, please identify the people that you worked with on your solution. Computer printout may be included with your solution as an appendix, but please do not provide these as your entire solution report.

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1. You have been contracted by the ACME Chocolate Company to assess the quality of the model that they have for a semi-batch bioreactor that is used to destroy contaminants in their waste water. The company hopes to use this model for process control. The state space model is:

$$\begin{aligned}\frac{dx_1}{dt} &= 10 - x_1 - \frac{4x_1x_2}{1+x_1^2}\\ \frac{dx_2}{dt} &= ux_1\left(1 - \frac{x_2}{1+x_1^2}\right)\\ \mathbf{y} &= \mathbf{x}\end{aligned}$$

   The initial conditions for the reactor, at the start of semi-batch operations, are $\mathbf{x} = [0\ 2]^{\mathrm{T}}$. The normal operating input value for the reactor, during operations, is $u = 3.5$ and the process has an equilibrium point at $\mathbf{x} = [2\ 5]^{\mathrm{T}}$. In this assignment you will consider the behaviour of the process under three different operations:

   i) during normal operation where $u = 3.5$,
   ii) after a step change to the input $\Delta u = 0.5$ at time $t = 0$,
   iii) after a step change to the input $\Delta u = -0.5$ at time $t = 0$.

   Please do / answer each of the following, showing all of your work:

   a) Confirm that $\mathbf{x} = [2\ 5]^{\mathrm{T}}$ is an equilibrium point for the process model.

b) Simulate the process behaviour under the three given operations. You will have to ensure that your simulation is sufficiently long for the system to settle to its final steady state. You will also have to carefully tune your simulation with respect to solver type, accuracy, etc. to ensure your results are reliable.

c) For each of the operations, plot the time trace of each state variable (i.e., x_i *vs.* t).

d) For each of the operations, plot the phase portrait (i.e., x_1 *vs.* x_2).

e) Linearize the model around the equilibrium point $\mathbf{x} = [2\ 5]^{\mathrm{T}}$. Comment on whether you believe it is appropriate to do this linearization? **Briefly explain**.

f) Convert your state space model to continuous transfer function form.

g) Determine the poles of your transfer function model.

h) Is the transfer function model stable? **Briefly explain**.

i) Compare the time traces and phase portraits of the transfer function model to those for the nonlinear process model.

j) Based on your work, comment on the quality of the ACME's bioreactor and whether you believe it will be useful for control purposes.

2. Based on your work on the previous bioreactor model, the ACME Chocolate Company has asked you to evaluate a second bioreactor model. The alternative model is:

$$\frac{dx_1}{dt} = x_1 - 2x_2$$
$$\frac{dx_2}{dt} = 2x_1 - x_2 + u$$
$$y_1 = x_1 + x_2$$
$$y_2 = x_1 - x_2$$

The process is initially at steady state (*i.e.*, $\mathbf{x}(0) = [0\ 0]^{\mathrm{T}}$). In this question, you will investigate the effect of sampling time on discrete representations of this continuous model. The three sample times you will use are:

i) $T_s = \frac{2\pi}{\sqrt{3}}$ seconds,

ii) $T_s = \frac{\pi}{\sqrt{3}}$ seconds,

iii) $T_s = \frac{\pi}{5\sqrt{3}}$ seconds,

Do / answer each of the following:

a) Determine whether the continuous time model is stable. **Briefly explain**.

b) Based on the eigenvalues of the state matrix, what type of behaviour do you expect from the given continuous-time model to step and impulse inputs.

c) Plot the time trace of each output variable (i.e., y_i *vs.* t) and the phase portrait for the output variables (i.e., y_1 *vs.* y_2) in the continuous model for a unit step input.

d) Develop the continuous transfer function matrix for the process model. Show your work.

e) Develop a discrete-time state space model for each of the given sampling times.

f) Plot the eigenvalues of the discrete model as a function of sampling time. Comment on the behaviour you expect from the discrete model corresponding to each of the given sampling times.

g) Are the discrete models stable? **Briefly explain**.

h) Develop discrete-time transfer function models based on each sampling time. Show your work.

i) For each of the discrete transfer function models, plot the response to a unit step input.

j) Compare the responses of the given continuous model and each of the discrete transfer function models to the unit step input. Comment on the effect of sampling rate on the accuracy of the discrete models.