CHE 572 Assn #2 1/

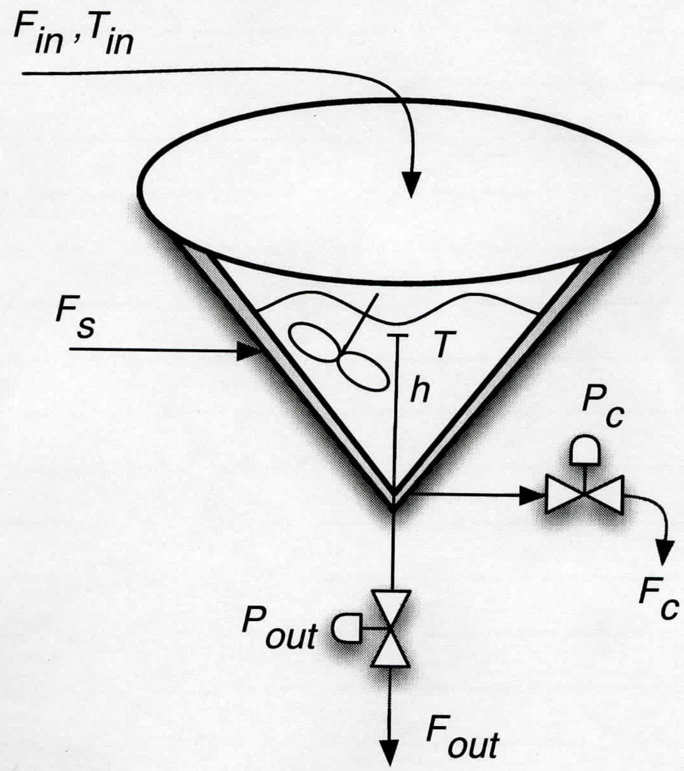

Figure 1: ACME jacketed surge tank

Input variables – things that can change and will affect process operation

Output variables – things that change as a result of process operation.

State variables – those quantities that reflect the "state" of the process.

#1)

	Input		Output			State
	manip.	dist.	meas	unmeas	contr.	
F_c				✓		
F_{in}		✓				
F_{out}				✓		
F_s				✓		
h				✓		
P_c	✓					
P_{out}	✓					
T				✓		
T_{in}		✓				
$\dot{H}_{in}$		✓				
$\dot{H}_{OUT}$				✓		
H						✓
$\dot{H}_s$		✓				
$\dot{H}_c$				✓		
$\dot{m}_{in}$		✓				
$\dot{m}_{out}$				✓		
m						✓

Since I did not indicate any sensors on the diagram, I marked all output variables as unmeasured.

#2/ process model:

- only ~~a~~ mass and energy balances will be required.

a) mass balance (interior of tank)

accumulation = in - out.

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{d\rho V}{dt} = \rho F_{in} - \rho F_{out}.$$

$$\cancel{\rho}\frac{dV}{dt} = \cancel{\rho} F_{in} - \cancel{\rho}\, k P_{out}\sqrt{h} \qquad (1)$$

- we need to express $\underline{V}$ in terms of h.

$$V \equiv \int_0^h \text{area}\, dh = \int_0^h \pi r^2\, dh$$

↳ radius of cone depends on h
↳ this is a linear relationship
↳ $h=0$, $r=0$ and $h=2m$, $r=1m$.

$$\therefore\ r = \frac{1}{2}h$$

$$\therefore\ V = \pi \int_0^h \left(\frac{1}{2}h^2\right) dh = \frac{\pi}{4}\int_0^h h^2\, dh$$

$$\therefore \quad V = \frac{\pi}{12} h^3 + \cancel{C}^{\,0}$$

but $V(0) = 0 \quad \therefore \quad C = 0$

substitute into 1

$$\frac{\pi}{12} \frac{dh^3}{dt} = F_{in} - k P_{out} \sqrt{h}$$

Simplifying.

$$\frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt} = F_{in} - k P_{out} \sqrt{h}$$

$$\frac{dh}{dt} = \frac{4}{\pi h^2} \left[F_{in} - k P_{out} \sqrt{h} \right] \quad (2).$$

b) energy balance (tank interior, not jacket)

↳ changes in kinetic and potential energy can be neglected.

$\cancel{\frac{dH}{dt}}$ accumulation = in − out

$$\frac{dH}{dt} = \underbrace{\dot{H}_{in} - \dot{H}_{out}}_{\text{tank}} + \underbrace{\dot{H}_s - \dot{H}_c}_{\text{jacket}}$$

$$\frac{d\rho V c_p (T - T^0)}{dt} = \rho F_{in} c_p (T_{in} - T^0) -$$

$$\rho F_{out} c_p (T_{out} - T^0) +$$

$$\rho_s F_s \tilde{H}_s - \rho_c F_c \tilde{H}_c$$

where T^o is an arbitrary reference temperature.

Note that I have chosen to represent the energy moving from the heating jacket to the tank as the enthalpy change of the steam/water in jacket.

You could try to use heat transfer relationships. To do this you would require heat transfer coefficients, heat transfer area ....

Some things to understand:

- in most plants, steam is supplied by a common header that is maintained at a constant pressure by the boiler system.

- as condensate is removed from the jacket, steam enters to ensure that pressure is constant in the jacket and is the same as the common steam header

- given the temperature difference between the steam and the warm chocolate, steam will immediately condense on the tank wall and condensate will flow down the tank wall, where it will leave via the condensate line.

- the steam jacket will ~~al~~ always be full.

$\therefore$ we are going to assume that there is no accumulation of mass in the jacket. (i.e., the steam jacket will operate at steady-state).

if you don't assume this, then you would have to model the change of height of condensate in the jacket. Do you think this would make a big difference to your model?

- some convenient assumptions
  - ↳ no heat loss to the environment
  - ↳ no subcooling of the condensate.
    - ↳ why is this reasonable?

given this discussion:

$$\rho_s F_s \tilde{H}_s - \rho_c F_c \tilde{H}_c \approx \rho_c F_c \Delta H_{vap}$$

substituting gives:

$$\rho C_p \frac{dVT}{dt} = \rho C_p F_{in} T_{in} - \rho C_p F_{out} T_{\cancel{out}} + \rho_c F_c \Delta H_{vap}$$

↳ why did I drop the reference temperature.

$$\frac{dVT}{dt} = F_{in} T_{in} - F_{out} T_{\cancel{out}} + \frac{\rho_c \Delta H_{vap}}{\rho C_p} F_c$$

$$V \frac{dT}{dt} + T \frac{dV}{dt} = F_{in} T_{in} - F_{out} T_{\cancel{out}} + \frac{\rho_c \Delta H_{vap}}{\rho C_p} F_c$$

$$\frac{\pi}{12} h^3 \frac{dT}{dt} + T(F_{in} - \cancel{F_{out}}) = F_{in} T_{in} - \cancel{F_{out} T} + \frac{\rho_c \Delta H_{vap}}{\rho C_p} F_c$$

$$\frac{\pi}{12} h^3 \frac{dT}{dt} = F_{in} (T_{in} - T) + \frac{\rho_c \Delta H_{vap}}{\rho C_p} F_c.$$

$$\frac{dT}{dt} = \frac{12}{\pi h^3} \left[F_{in} (T_{in} - T) + \frac{\rho_c \Delta H_{vap}}{\rho C_p} F_c \right].$$

3

$F_c = k_c P_c$

$\therefore$ the nonlinear state space model is:

$$\frac{dh}{dt} = \frac{4}{\pi h^2}\left[F_{in} - k P_{out}\sqrt{h}\right]$$

$$\frac{dT}{dt} = \frac{12}{\pi h^3}\left[F_{in}(T_{in}-T) + \frac{h_c \rho_c \Delta H_{vap}}{\rho C_p} P_c\right]$$

#2

Assumptions:

1) ρ, C_p are constant.
2) chocolate is well-mixed
3) no heat losses
4) kinetic and potential energy can be neglected, no work done on the system.
5) no subcooling of condensate.
6) jacket is operated at steady-state.

Note that I have not included any ~~i~~ output equations and that my state equations now contain only input and output variables.

#3) linearize the model.

state eq^{ns}

$$\begin{bmatrix} \frac{dh}{dt} \\ \frac{dT}{dt} \end{bmatrix} = \begin{bmatrix} \frac{4}{\pi h^2}\left[F_{in} - k P_{out}\sqrt{h}\right] \\ \frac{12}{\pi h^3}\left[F_{in}(T_{in} - T) + \frac{k_c \rho_c \Delta H_{vap}}{\rho C_p} P_c\right] \end{bmatrix}$$

output eq^{ns}

$$\begin{bmatrix} h \\ T \\ F_{OUT} \\ F_c \end{bmatrix} = \begin{bmatrix} h \\ T \\ k P_{OUT}\sqrt{h} \\ k_c P_c \end{bmatrix}$$

- I chose the output variable arbitrarily based on what variables I wanted to investigate the transient response for.

Let:

$$\underline{x} = \begin{bmatrix} h \\ T \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} F_{in} \\ T_{in} \\ P_{out} \\ P_c \end{bmatrix}, \quad \underline{y} = \begin{bmatrix} h \\ T \\ F_{out} \\ F_c \end{bmatrix}.$$

then the nonlinear model is:

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} \frac{4}{\pi x_1^2}\left[u_1 - k u_3 \sqrt{x_1}\right] \\ \frac{12}{\pi x_1^3}\left[u_1(u_2 - x_2) + \frac{h_{fc} \Delta H_{vap}}{\rho c_p} u_4\right] \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} x_1 \\ x_2 \\ k u_3 \sqrt{x_1} \\ k_c u_4 \end{bmatrix}$$

$$\nabla_{\underline{x}} \underline{f} = \begin{bmatrix} -\frac{4}{\pi x_1^2}\left[\underbrace{\frac{2(u_1 - k u_3\sqrt{x_1})}{x_1}}_{\to 0} + \frac{\frac{1}{2} k u_3}{\sqrt{x_1}}\right] & 0 \\ -\frac{36}{\pi x_1^4}\underbrace{\left[u_1(u_2 - x_2) + \frac{h_{fc}\Delta H_{vap}}{\rho c_p} u_4\right]}_{\to 0 \text{ at steady-state}} & \frac{-12 u_1}{\pi x_1^3} \end{bmatrix}$$

$$\nabla_u \underline{f} = \begin{bmatrix} \frac{4}{\pi x_1^2} & 0 & \frac{k 4 \sqrt{x_1}}{\pi x_1^2} & 0 \\ \frac{12}{\pi x_1^3}(u_2 - x_2) & \frac{12 u_1}{\pi x_1^3} & 0 & \frac{12 h_{fc} \Delta H_{vap}}{\pi \rho c_p x_1^3} \end{bmatrix}$$

$$\underline{\nabla_x g} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{-k u_3}{2\sqrt{x_1}} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{\nabla_u g} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k\sqrt{x_1} & 0 \\ 0 & 0 & 0 & k_c \end{bmatrix}$$

using the steady-state data.

$$k = 0.3 \frac{m^3}{min\cdot\sqrt{m}} = 18 \frac{m^3}{hr\sqrt{m}}$$

$$F_6 = \frac{68.75 \text{ kg/hr}}{930 \text{ kg/m}^3} = 7.39 \times 10^{-2} \text{ m}^3/\text{hr}$$

$$k_c = \frac{7.39 \times 10^{-2}}{0.5} = 0.148 \frac{m^3}{hr}$$

$$T = \frac{68.75 \cdot 2160}{1.5 \times 0.15 \times 60 \cdot 1100} + 30 = 40^\circ C$$

$$F_{OUT} = 0.150 \frac{m^3}{min} = 9.0 \text{ m}^3/\text{hr}.$$

$$\frac{h_c \rho_c \Delta H_{vap}}{\rho C_p} = \frac{0.148 \cdot 930 \cdot 2160}{1100 \cdot 1.5} = 180\ ^\circ c$$

Substituting :

$$\underline{\underline{A}} = \begin{bmatrix} -18/\pi & 0 \\ 0 & \frac{-108}{\pi} \end{bmatrix}$$

$$\underline{\underline{B}} = \begin{bmatrix} \frac{4}{\pi} & 0 & \frac{72}{\pi} & 0 \\ -\frac{120}{\pi} & \frac{108}{\pi} & 0 & \frac{2160}{\pi} \end{bmatrix}$$

$$\underline{\underline{C}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{9}{2} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{\underline{D}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 0.148 \end{bmatrix}$$

and the linearized model is

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$$

$$\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$$

$\Rightarrow$ remember that these new $\underline{x}, \underline{u}, \underline{y}$ variables are in deviation form about the steady state.

#3

#4) Degrees of Freedom.

Let's use the linearized model.

\# of variables = 10

\# of time derivatives = 2

\# of equations = 6

\# of initial conditions = 2 (h, T only).

\# DOF = 10 + 2 − 6 − 2 = 4

∴ you must specify all four inputs to get a unique solution.

#4

In order to simulate the nonlinear system, the dee block of Simulink should be used. In Figure 1 the Simulink model and settings of DEE block have been shown. Note that we assume that the system is initially at steady state. Therefore the values of input variables should also be set as the steady state values except for F_{in} which will change from 9 to 9.9 for positive change and to 8.1 for negative change.

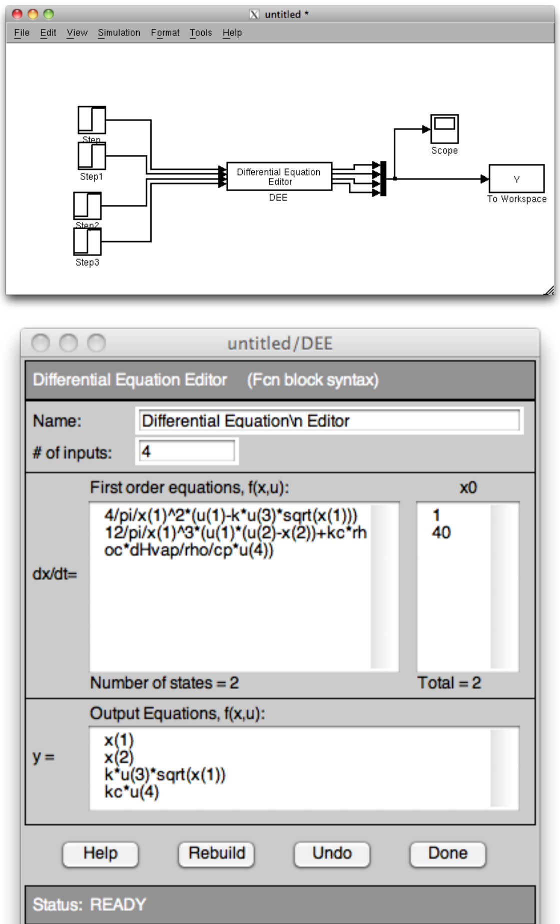

FIGURE 1. Simulink model of nonlinear system

In order to model the linearized system, one can use the DEE block, but there is block called "state space" which can be found under "continuous" in the library browser. The Simulink model and settings of state space block are shown in Figures 2-4.

Note that if you choose to use DEE block to simulate the linear system, the initial values and steady state values of input variables should be set as zero (as the linearized equations are in terms of deviation variables).

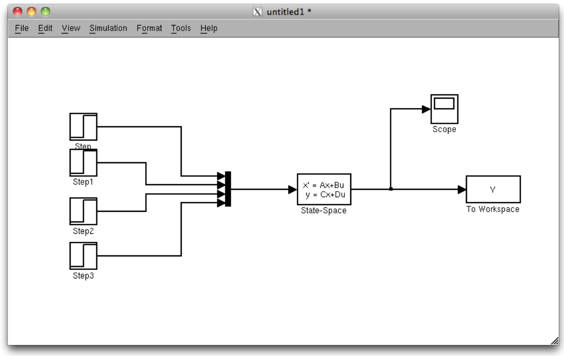

FIGURE 2. Simulink model of linearized system

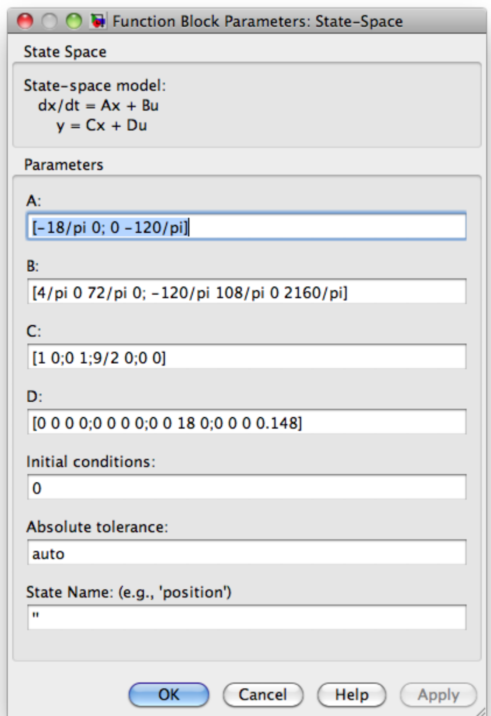

FIGURE 3. State space block settings

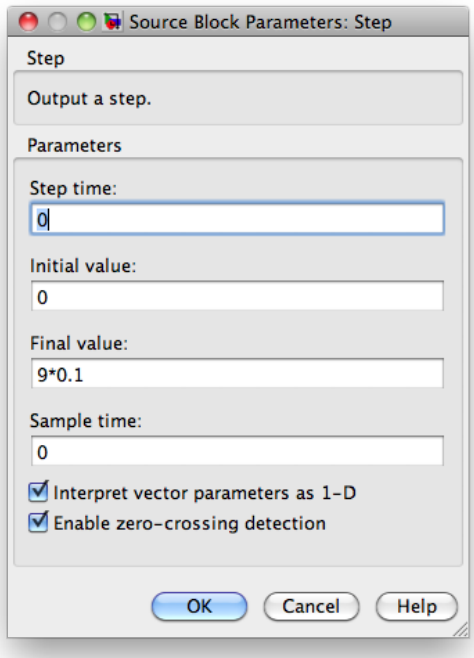

FIGURE 4. Step block settings for linearized system

Figures 5-8 illustrate the simulation results for parts a-d.

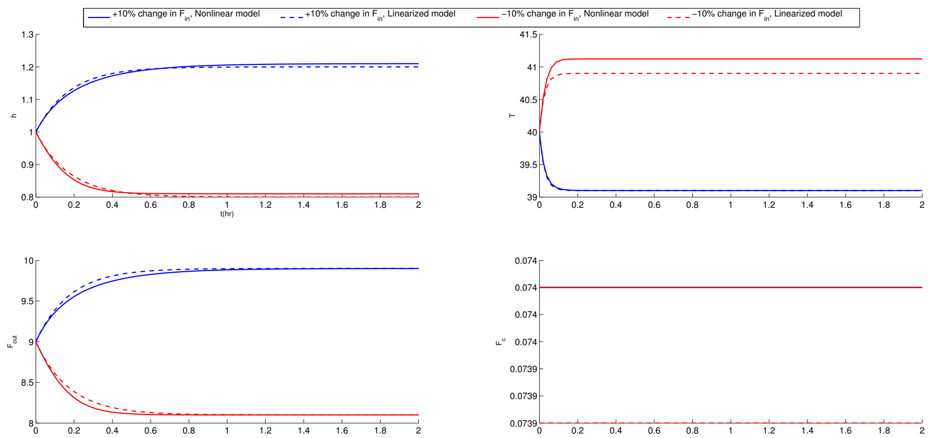

FIGURE 5. $\pm 10\%$ change in F_{in}

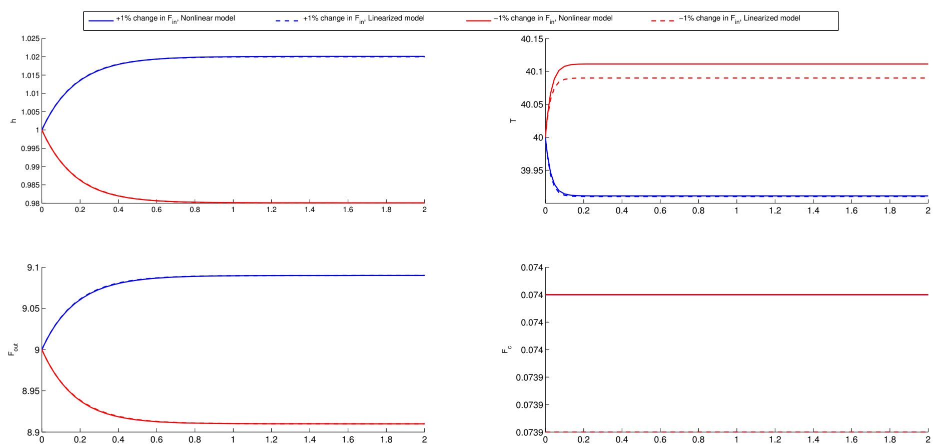

FIGURE 6. $\pm 1\%$ change in F_{in}

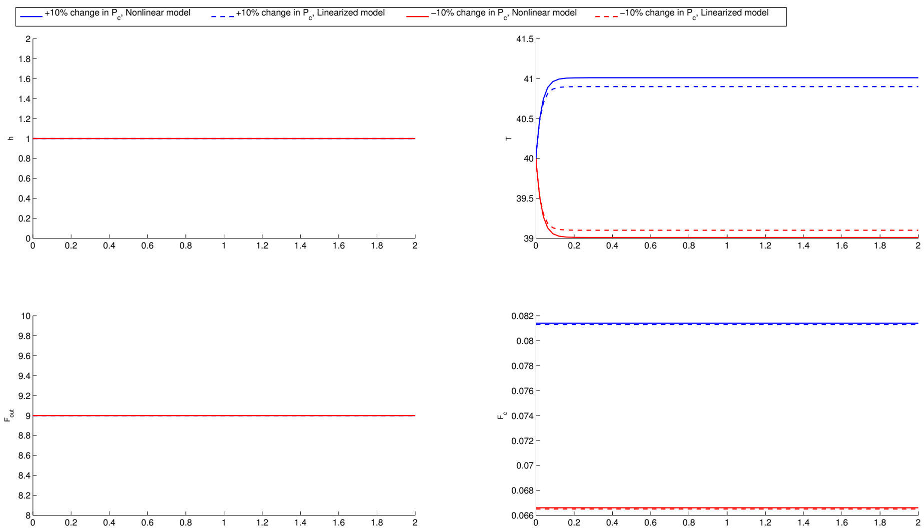

FIGURE 7. $\pm 10\%$ change in P_c

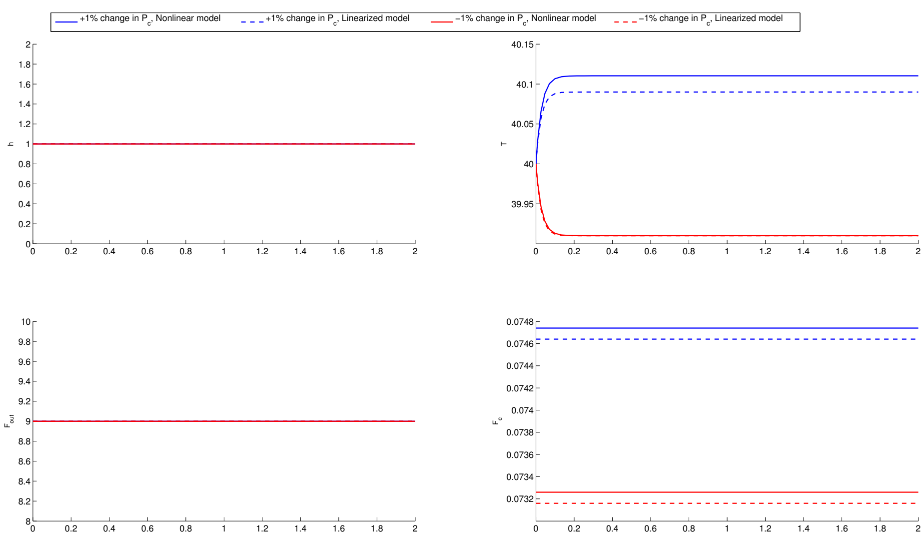

FIGURE 8. $\pm 1\%$ change in P_c

If you have a close look at the figures, the linearized model does surprisingly well for both the small and larger step changes to the inputs. (This is not always the case). As expected, the deviation between the linear and nonlinear models changes based on the step size and direction. You should expect that the linear model will differ more for larger steps. Also, you can see that the difference between the models is exhibited in both steady-state gains and in the transient responses.