
The objectives for this assignment are:

1. review some material from your math and numerical methods courses,
2. get started on first-principles process modelling,
3. continue to build your MATLAB / Simulink skills.

You may work together in groups to complete the assignment, but you must hand in your own assignment solution. If you work with a group, please identify the people that you worked with on your solution. Computer printout may be included with your solution as an appendix, but please do not provide these as your entire solution report.

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1. (10 points) Determine the second-order Taylor series expansion of each following functions showing all of your work:

   a) The Arrhenius rate expression $r(T) = k\ e^{-\frac{B}{T}}$ about T_0,
   b) The flow relationship $F(h) = k\ \sqrt{h}$ about h_0,
   c) The common function $f(x) = x\ ln(x)$ about x_0.

   You may use a computer to check your work, but please do the derivations by hand.

2. (5 points) Calculate the gradient of the following function showing all of your work:

   $$f(\mathbf{x}) = k\ e^{\frac{-B}{x_1}}\ x_2^2\ x_3$$

   This function is a common reaction rate expression. You may use a computer to check your work, but please do the derivations by hand.

3. (5 points) Calculate the Jacobian of the following set of equations showing all of your work:

   $$\mathbf{f}(\mathbf{x}) = \left[\begin{array}{c} x_1^2 + 2x_1x_2 + x_2^2 + 4x_1 + 4x_2 + 8 \\ x_1x_2 - x_1 - x_2 \end{array} \right] = \mathbf{0}$$

   This set of equations would commonly come from a regression analysis. You may use a computer to check your work, but please do the derivations by hand.

4. (10 points) The primary numerical ordinary differential equation solvers in Matlab/Simulink use the Runge-Kutta technique. Please answer each of the following:

   a) briefly describe how the Runge-Kutta technique works.

   b) briefly explain the main differences between MATLAB's ODE23 and ODE45 routines. Which is more accurate? Why.

   c) how are these techniques modified to handle *stiff* ODEs.

5. (45 points) The ACME Chocolate Company has a conical surge tank in the production line where they make their new line of truffles. As shown in Figure 1, warm, liquid chocolate is fed into the tank near the top of the tank and exits the tank at the bottom, through a valve, where it is used to coat the truffles as they pass by on a conveyor belt.

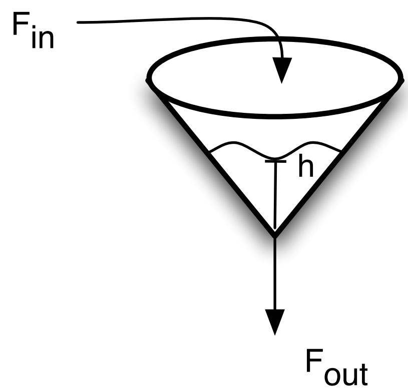

Figure 1: ACME surge tank

The tank is 2 m in height and 2 m in diameter at the tank top. The outlet flow follows a square root relationship, $F_{out} = k\sqrt{h}$. The density of the warm chocolate is $\rho = 1100\ \frac{kg}{m^3}$. At the normal steady-state operation for this surge tank the inlet flow is $F_{in} = 150.0\ \frac{l}{min}$ and the level in the tank is $h = 1.0\ m$.

Please answer each of the following questions, showing all of your work:

   a) classify the process variables,

   b) develop a dynamic model of the process,

   c) determine the constant k for the outlet flow relationship,

   d) plot the transient response of the level of liquid chocolate in the tank to a $\pm 10\%$ change in inlet flow rate F_{in}. Is this transient behaviour linear? Explain.