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Green Patents in an Oligopolistic Market with Green Consumers

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Abstract

We analyze the impact of patent policies and emission taxes on green innovation. We allow for strategic interactions of firms in a duopolistic market in the presence of green-conscious consumers. We identify a paradoxical effect of increasing emission taxes beyond a certain threshold which results in an increase in emissions. Decreasing patenting costs mitigates this paradox, while the impact of tightening patentability requirements is more complex. Moreover, we show that the greater the proportion of green-conscious consumers, the less likely firms are to license a green patent, which results in higher emissions levels. With green consumers, the lowest emissions occur for an intermediate range of taxes for which licensing does occur. Finally, we find that while tax increases lead to a switch from overinvestment to underinvestment in the absence of green-conscious consumers, they have the reverse effect in their presence.

Keywords: Patent, Green Innovation, Pollution

JEL Classification = O34, L13, Q50.

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1 Introduction

While governments and private sectors across the world have been investing in developing and implementing clean technologies over the last two decades,[1] the impending urgency of further speeding up the innovation process is being keenly felt by policy makers and environmentally concerned citizens as we struggle to transition to a world with net-zero emissions of greenhouse gases.[2] Indeed, while the number of green patents issued in China has increased particularly rapidly,[3] that in most OECD countries has plateaued in recent years (Acemoglu et al., 2023). This has generated much debate regarding which are the most effective policy tools to foster green innovation.

Two market failures can explain underinvestment in environmental innovation. The first externality, knowledge externality, which is common to all innovations due to the public good nature of knowledge, results from imperfect appropriability of knowledge. Innovators have too little incentive to invest in R&D because they cannot appropriate all the benefit from their innovation. This externality in knowledge production can be addressed with patents.[4] Second, there is also an environmental externality that is specific to green innovation. Since environmental benefits are not appropriately valued by markets, firms and consumers have no incentive to reduce emission without policy intervention. The relevant literature, therefore, examines the impact of combining environmental regulations with specific policies addressing the relevant knowledge market failure (e.g., Carraro and Siniscalco, 1994; Carraro and Soubeyran, 1996; Katsoulacos and Xepapadeas, 1996; Popp, 2006, 2019; Fischer and Newell, 2008; Gerlagh et

[1] According to Bloomberg, global clean energy spending has surged by 17% to a record $1.8 trillion in 2023. These include investments to install renewable energy, buy electric vehicles, build hydrogen production systems and deploy other technologies (see: https://www.bloomberg.com/news/articles/2024-01-30/china-leads-global-clean-energy-spending-which-record-1-8-trillion-in-2023). International organizations, such as the United Nations (UN) and G8, have also pro-actively encouraged countries to fund the development of clean technologies.

[2] See, e.g., Barrett (2009) and Galiana and Green (2009).

[3] China has emerged as the world leader with $890bn investment in clean-energy sectors in 2023 (see: https://www.carbonbrief.org/analysis-clean-energy-was-top-driver-of-chinas-economic-growth-in-2023/).

[4] A patent confers its owner a temporary right to exclude others from exploiting the innovation. In exchange for the exclusionary right, the patent holder must disclose his innovation. For surveys of the patent literature, see Langinier and Moschini (2002), Rockett (2010), and Eckert and Langinier (2014).

al., 2009; Acemoglu et al., 2012; Hepburn et al., 2018; Lehmann and Söderholm, 2018; Gugler, Szücs and Wiedenhofer, 2024).

Most of these papers focus on R&D subsidies as the policy tool that corrects for knowledge market failure while implicitly assuming the existence of intellectual property rights. Indeed, green patent count is used, for the most part, as an instrument for measuring green innovation by studies that evaluate other policy tools such as emission taxes and R&D subsidies. We contribute to this literature by shifting the focus on the role of patent policy tools rather than R&D subsidies. More specifically, we analyze the impact of changing patentability requirements and patenting costs and how these tools work in conjunction with an emission tax to impact firms' patenting, licensing and investment decisions, and the ensuing impacts on emission and welfare levels. As such we contribute to the ongoing debate about the effectiveness of the patent system in promoting green innovation. On the one hand, international organizations advocate royalty-free compulsory licensing of green technologies, excluding green technologies from patenting and even revoking existing patent rights on them (UNFCCC, 2009).[5] On the other hand, many countries actively facilitate patenting of green innovation by lowering the cost of obtaining patents in an attempt to increase private sector investment.[6]

While undertaking our analysis, we allow for strategic behaviour of firms within an oligopolistic setting and for heterogeneity across consumers regarding the degree to which they care about the environment. It is important to take into account environmentally friendly consumers given the increasing environmental consciousness of citizens globally, which is, for instance, reflected in widely used eco-labeling schemes internationally.[7] Most of the contributions that study optimal environmental policies in the presence of environmentally friendly consumers do not address

[5]Such provisions are also incorporated in the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) (Derclaye, 2008; Rimmer, 2011).

[6]In the US, in 2023, the USPTO created different programs to accelerate the examination of renewable technologies patent applications such as Patents for Humanity program for green innovation and the Climate change mitigation pilot program (https://www.uspto.gov/about-us/news-updates/uspto-announces-new-patents-humanity-green-energy-category).

[7]For example, in countries like Sweden about 50% of the market share for certain products consists of the environmentally friendly variant. Green marketing is also frequently used to influence consumer behavior in transportation and electricity markets (Kraftborsen, 2001).

green innovation (Arora and Gangopadhyay, 1995; Cremer and Thisse, 1999; Moraga-Gonzalez and Padron-Fumero, 2002; Bansal and Gangopadhyay, 2003; Lombardini-Riipinen, 2005; Bansal, 2008; ANA, P1). An exception is Langinier and Ray Chaudhuri (2020), and Gil-Moltó and Varvarigos (2013) who examine the adoption of clean technologies in the presence of green consumers.

Moreover, recent evidence shows that environmental willingness to pay varies significantly across regions, and firms' innovation strategies are dependent on the different degrees of environmental willingness to pay they face from their consumer bases (Aghion et al., 2023). Typically, the presence of green-conscious consumers is thought to induce firms to invest more in green innovation since firms realize that such consumers are willing to pay a premium for products produced using cleaner technologies. By endogenizing not only investment but also, crucially, the patenting and licensing decisions, we show that the relationship between the proportion of green-conscious consumers and innovation is, in fact, more nuanced within oligopolistic settings.

Our framework analyzes a product's market, the production of which causes pollution. Implementing a cleaner technology is assumed to reduce the emission per unit of output ratio (similar to, for example, Benchekroun and RayChaudhuri, 2014, 2015). We further assume that the product is vertically differentiated in terms of its emission-output ratio, with green-conscious consumers preferring products with lower emission-output ratios (similar to Bansal and Gangopadhyay, 2003; Ibañez and Grolleau, 2008). We incorporate heterogeneity across consumers in terms of their degree of environmental friendliness. Our model is applicable to a wide variety of products, the production processes of which are becoming cleaner. Consider, for example, a variety of manufactured products such as toys, furniture and packaging, the "greenness" of which may depend on the proportion of recycled inputs used in the production process.[8] Another example is energy production, the "greenness" of which may depend on the proportion of renewable energy sources used in the production process.

To model the patenting cost, we introduce a lump sum cost associated with obtaining and keeping a patent in force and examine the impact of lowering this cost.[9] In practice, policy

[8] Over the period 2005-2016, the volume of recycling in Europe increased by 34%.

[9] Several components constitute the overall cost of obtaining and keeping patents in force. First, the monetary

makers use different means to reduce patenting costs. One such method is fast-tracking or expediting the review process of green patent applications.[10] Evidence shows that fast-tracking programs have accelerated the diffusion of knowledge in green technologies in the short run and reduced the time from application to grant by up to 75% (Dechezleprêtre, 2013).

To model patentability requirements, we consider a minimum investment threshold level that must be satisfied to obtain a patent, in line with Crampes and Langinier (2009). We then vary this investment threshold to examine whether a stricter patentability requirement fosters more green innovation. This reflects that to be patentable, an innovation must be novel (not already in the public domain), non-obvious (to a person with ordinary skills in a particular field), and useful (to have at least one application). The relevant requirements vary across jurisdictions and are currently stricter in the EU than in the U.S. (Eckert and Langinier, 2014).[11]

We consider a Bertrand duopoly framework which has three stages. In the first stage, each firm decides its level of investment in R&D. In the second stage, if a firm has discovered an innovation, it decides whether to obtain a patent. If both firms invest, discover an innovation

fees associated with the application process range, on average, between $5000-$15000. Second, the average waiting time for patents to be granted is about three years, implying that the opportunity cost in terms of lost profits during the waiting period can be high (Eckert and Langinier, 2014). Third, the potential litigation costs of enforcing a patent may be large enough to deter small firms from obtaining patents in several industries. According to the American Intellectual Property Law Association, the cost of an average patent lawsuit, where $1 million to $25 million is at risk, is $1.6 million through the end of discovery and $2.8 million through final disposition. Fourth, renewal costs are monetary fees payable regularly during the patent's lifetime. In the U.S., a patent must be renewed three times (at age 3, 7 and 14 years), whereas in Europe and Canada, the renewal fee is charged annually for 20 years.

[10] Fast-tracking has been implemented by several countries, including Australia, Brazil, Canada, China, UK, U.S., Japan and Korea (see https://www.ic.gc.ca/eic/site/cipointernet-internetopic.nsf/eng/wr02462.html). More recently, the U.S. created different programs as explained in Footnote 6.

[11] Some studies have illustrated that strong Intellectual Property Rights may not necessarily enhance innovation (Green and Scotchmer, 1995; Gallini, 2002; Bessen and Maskin, 2009). Even though in a static world (single innovation), patents of appropriate scope can encourage innovations (Klemperer, 1990; Gilbert and Shapiro, 1990), this is no longer the case in the case of cumulative innovations. In this latter case, strict patentability requirements may even discourage follow-on innovations (Scotchmer, 1991; Bessen and Maskin, 2009). We abstract away from these issues and present an alternative mechanism through which stronger patentability requirements affect innovation.

and apply for a patent, only one gets the patent with a probability of 1/2. The patent holder then decides whether to sell a license to the other firm to use the green innovation. The firm that does not obtain the patent may decide to purchase the license and produce the cleaner product, produce the dirty product, or not produce anything. In the third stage, the firms are engaged in price competition in the product market.

Within this setting, we have three main sets of findings, as follows. First, we identify a paradoxical effect of increasing emission taxes beyond a certain threshold which results in an increase in emissions, contrary to the Porter Hypothesis. This is because we introduce a fixed patenting cost to capture the various costs associated with the patenting process. As the emission tax increases, the profits generated in the product market become insufficient to outweigh this fixed patenting cost, thereby making the investment in green innovation unprofitable. Thus, decreasing patenting costs mitigates this paradox by increasing the tax threshold beyond which the paradox occurs. The impact of tightening patentability requirements is more complex. If the patentability requirement is initially lax, making it stricter mitigates this paradox, whereas if it is already very strict, making it even stricter exacerbates it.[12]

Second, we find that the key role played by the green-conscious consumers is to introduce heterogeneity across consumers within our setting, which in turn makes it profitable to engage in product differentiation. This yields an equilibrium without licensing for low tax rates (in contrast to the case without green consumers where licensing always occurs), while at sufficiently high tax rates, we retrieve the same paradoxical result as in the absence of green-conscious consumers. Thus, with green consumers, the lowest emissions occur for an intermediate range of taxes for which licensing occurs and yet the tax is not so high as to make investment unprofitable.

Third, we find that while tax increases lead to a switch from overinvestment to underinvestment in the absence of green-conscious consumers, they have the reverse effect in their presence.

[12] This is in sharp contrast to Langinier and RayChaudhuri (2020) who find that the lowest level of emissions, in the presence of green consumers, is reached with very high emission tax rates combined with high patenting costs. The difference arises since Langinier and RayChaudhuri (2020) consider a monopoly framework and abstract away from licensing, while we consider a Bertrand duopoly which enables us to endogenize the licensing decision. Gerlagh et al., (2014) and ANA (P2) examine patent policies for green technologies in which they focus on the lifetime of patents issued to green innovations.

Since with green-conscious consumers, for low tax rates, there is no licensing, at least one of the firms chooses to produce the dirty good leading to underinvestment for low tax rates. While licensing reduces pollution, the product price rises due to the royalty fees charged by the patent holder, thereby reducing consumer surplus. For sufficiently high tax rates, the latter effect outweighs the reduction in pollution damage such that we have overinvestment relative to the socially desirable level.

The paper is organized as follows. In Section 2, we present our model. In Section 3, we present the benchmark case without green-conscious consumers. In Section 4, we derive the equilibrium and policy implications for the case with green-conscious consumers. In Section 5, we analyze the total welfare and determine the conditions under which firms over or underinvest. Section 6 presents our concluding remarks.

2 Model Setting

We consider a three-stage model in which two firms, firms 1 and 2, sell a final good to consumers, the production of which is polluting and has a marginal cost c. For each firm i, $i = 1, 2$, the emission of the pollutant generated per unit of production is given by

$$\gamma_i \equiv \frac{e_i}{q_i}, \tag{1}$$

where e_i denotes emission and q_i denotes output. Firm i can invest I_i to reduce its emission-output ratio, γ_i. Each firm i has a probability $\rho_i(I_i, I_j)$ to discover an innovation that reduces the emission-output ratio, where $\partial\rho_i(I_i, I_j)/\partial I_i \geq 0$ (the higher firm i's investment, the higher its probability of discovering an innovation), $\partial\rho_i(I_i, I_j)/\partial I_j \leq 0$ (the higher firm j's investment, the lower its probability of making the discovery), and $\partial^2\rho_i(I_i, I_j)/\partial I_i \partial I_j \leq 0$. For simplicity, we assume that I_i is a discrete variable that can take two values: 0 and $I_P > 0$. We assume that[13]

$$\rho_i(I_i, I_j) = \begin{cases} 1 & \text{if } I_i = I_P > I_j = 0 \\ \frac{1}{2} & \text{if } I_i = I_j = I_P \\ 0 & \text{if } I_i = 0 \leq I_j \end{cases}$$

The emission-output ratio is given by

$$\gamma_i = \begin{cases} \gamma_H & \text{if } I_i = 0 \\ \gamma_P & \text{if } I_i = I_P \end{cases} \tag{2}$$

where γ_H represents the emission-output ratio associated with the dirty product (i.e., the product produced using the technology with the higher emission-output ratio, γ_H), γ_P represents the emission-output ratio of the cleaner product (i.e., the product produced using the technology with the lower emission-output ratio, γ_P), with $\gamma_H \equiv \gamma_P + \Delta$, where $\Delta > 0$. We note that I_P is a function of Δ with $I_P'(\Delta) > 0$, i.e., a higher level of investment is required to obtain a larger decrease in the emission-output ratio. We also assume that $I_P''(\Delta) > 0$. Henceforth, for notational convenience, we omit the argument of the function I_P unless it plays a role in our analysis.

2.1 The Demand Side

The demand side consists of a continuum of N consumers. Each of them buys either 0 or 1 unit of the good. There exists a fraction λ of 'green-conscious' consumers, whose utility is increasing in the greenness of the product, that is, decreasing in γ_i, and a fraction $(1 - \lambda)$ of 'non-green-conscious' consumers, whose utility is independent of the greenness of the product.

Let G denote the degree of environmental friendliness of a consumer, with G being uniformly distributed over the interval $[\underline{G}, \overline{G}]$ with $\underline{G} > 0$. We assume that consumers can observe how green a product is. This is equivalent to assuming that consumers can observe γ_i.[14] We normalize N such that $N = 1$, and we assume that $\overline{G} - \underline{G} = 1$. Let $P(e)$ denote the pollution damage to each consumer, which is a function of total emissions, $e = e_1 + e_2$. Following Bansal and

[13] We make the simplifying assumption that an innovation will be discovered with probability 1. Alternatively, if we assumed that an innovation would be discovered with an exogenously given probability of less than 1, this would make the notation and analysis more cumbersome without generating any new insights.

[14] This is a relevant scenario to consider in the presence of effective eco-labeling programs.

Gangopadhyay (2003), Ibañez and Grolleau (2008) and Langinier and RayChaudhuri (2020), we assume that the pollution level generated by total production is exogenous to each consumer, regardless of their consumption level. Let p_1 and p_2 denote the prices set by firm 1 and firm 2, respectively.

A green-conscious consumer has the following utility function

$$U_G = \begin{cases} v - \gamma_i G - p_i - P(e) & \text{from buying the product from firm } i \\ -P(e) & \text{from not buying} \end{cases} \tag{3}$$

for $i = 1, 2$. The term $-\gamma_i G$ in (3) reflects that the greener the product from firm i, that is, the lower is γ_i, the better off the green-conscious consumer. Also, v represents the gross utility of consuming one unit of the good. A green-conscious consumer does not buy the product if $v - \gamma_i G - p_i < 0$. Note that if $\gamma_1 = \gamma_2 = \gamma$, a green-conscious consumer has a utility $U_G = v - \gamma G - \min\{p_1, p_2\} - P(e)$ from buying the product.

A non-green-conscious consumer has the following utility function

$$U_{NG} = \begin{cases} v - \min\{p_1, p_2\} - P(e) & \text{from buying the product} \\ -P(e) & \text{from not buying} \end{cases} \tag{4}$$

A non-green-conscious consumer buys the product only if $v \geq \min\{p_1, p_2\}$. Henceforth, we assume that $P(e) = e$.

If firm i sells the greener product, such that $\gamma_i = \gamma_P < \gamma_j = \gamma_H$, a green-conscious consumer G buys the greener product from firm i rather than the dirty product from firm j as long as $v - \gamma_P G - p_i - P(e) > v - \gamma_H G - p_j - P(e)$, that is, as long as $G \in (\widetilde{G}_1, \overline{G}]$, with

$$\widetilde{G}_1 \equiv \frac{p_i - p_j}{\Delta}.$$

Thus, there exists a green-conscious consumer, $\widetilde{G}_1$, who is indifferent between buying the greener product and the dirty product. A green-conscious consumer buys the green product from firm i rather than not buying anything as long as $v - \gamma_P G - p_i - P(e) > -P(e)$, that is, as long as $G < \widetilde{G}_2$, with

$$\widetilde{G}_2 \equiv \frac{v - p_i}{\gamma_P}.$$

Thus, there exists a green-conscious consumer, $\widetilde{G}_2$, who is indifferent between buying the greener product and not buying anything. To have $\widetilde{G}_1 < \widetilde{G}_2$, the condition $p_i < (\Delta v + \gamma_H p_j)/\gamma_P$ must be satisfied. As long as $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$, some green-conscious consumers buy the dirty product (a fraction $\widetilde{G}_1 - \underline{G}$ cannot afford the cleaner product, so they buy the dirty product), some buy the greener product (a fraction $\widetilde{G}_2 - \widetilde{G}_1$), and others buy nothing (a fraction $\overline{G} - \widetilde{G}_2$).

However, in equilibrium, the conditions $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$ are not always satisfied.[15] If $\underline{G} < \widetilde{G}_1 < \overline{G} < \widetilde{G}_2$, the market is entirely covered, and both firms share the demand from green-conscious consumers (firm i gets a fraction $\overline{G} - \widetilde{G}_1$ and firm j gets a fraction $\widetilde{G}_1 - \underline{G}$). If $\widetilde{G}_1 < \underline{G} < \widetilde{G}_2 < \overline{G}$, only firm i, producing the cleaner product, gets some demand from green-conscious consumers (a fraction $\widetilde{G}_2 - \underline{G}$), and firm j does not get any demand from green-conscious consumers. If $\widetilde{G}_1 < \underline{G} < \overline{G} < \widetilde{G}_2$, firm i, producing the cleaner product, gets the entire demand from green-conscious consumers (a fraction $\overline{G} - \underline{G}$) while firm j gets no demand from them. If $\widetilde{G}_1 > \overline{G}$, there is no demand from green-conscious consumers for the cleaner good, while if $\widetilde{G}_2 < \underline{G}$, there is no demand at all from green-conscious consumers.

Thus, if $\gamma_i = \gamma_P < \gamma_j = \gamma_H$, such that firm i produces the cleaner product, as long as $\widetilde{G}_1 < \overline{G}$ and $\widetilde{G}_2 > \underline{G}$, we have the following demand functions[16]

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda(\min\{\widetilde{G}_2, \overline{G}\} - \max\{\widetilde{G}_1, \underline{G}\}) & \text{if } p_i > p_j \text{ and } p_i \leq (\Delta v + \gamma_H p_j)/\gamma_P \\ \lambda(\min\{\widetilde{G}_2, \overline{G}\} - \underline{G}) + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda(\min\{\widetilde{G}_2, \overline{G}\} - \underline{G}) + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases} \tag{5}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} \lambda(\max\{\widetilde{G}_1, \underline{G}\} - \underline{G}) + (1-\lambda) & \text{if } p_i > p_j \text{ and } p_i \leq (\Delta v + \gamma_H p_j)/\gamma_P \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases} \tag{6}$$

If $p_i > p_j$, firm j with the lower price attracts some green-conscious consumers and all the non-green-conscious consumers, whereas firm i with the higher price only attracts a fraction of the green-conscious consumers. If $p_i = p_j$, each firm attracts half of the demand from non-green-conscious consumers and firm i captures all the demand from the green-conscious consumers. That is, if $p_i = p_j$ and firm i produces a greener product, green-conscious consumers only buy

[15] Here we assume that $\underline{G}$ and $\overline{G}$ are given, but as indifferent consumers change, it affects the demand functions. Cassagnard and Espinosa (2022) study how changing $\underline{G}$ and $\overline{G}$ affect the demands.

[16] The details of the calculation of the demands are presented in Appendix A1.

from firm i. If $p_i < p_j$, given that firm i's product is cleaner and cheaper, nobody buys from firm j.

If both firms sell the same product such that $\gamma_1 = \gamma_2 = \gamma$, a green-conscious consumer buys the good if $v - \gamma G - \min\{p_1, p_2\} - P(e) > -P(e)$, or $G < \widetilde{G}_3$, where

$$\widetilde{G}_3 \equiv \frac{v - \min\{p_1, p_2\}}{\gamma}. \tag{7}$$

Thus, there exists a green-conscious consumer, $\widetilde{G}_3$, who is indifferent between buying the product and not buying anything. As long as $\underline{G} < \widetilde{G}_3 < \overline{G}$, some, but not all, green-conscious consumers buy the product. If $\underline{G} < \overline{G} < \widetilde{G}_3$, all the green-conscious consumers buy the only good that is offered, while if $\widetilde{G}_3 < \underline{G} < \overline{G}$, none of them buy the good. Thus, as long as $\widetilde{G}_3 > \underline{G}$, we have the following demand function for firm i, for $i, j = 1, 2$ and $i \neq j$

$$D_i(p_i, p_j; \gamma) = \begin{cases} \lambda(\min\{\widetilde{G}_3, \overline{G}\} - \underline{G}) + (1 - \lambda) & \text{if } p_i < p_j \leq v \\ \frac{1}{2}\lambda(\min\{\widetilde{G}_3, \overline{G}\} - \underline{G}) + \frac{1}{2}(1 - \lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i > p_j > v \end{cases} \tag{8}$$

When $\gamma_i = \gamma_j$, if $p_i > p_j$, firm i does not attract any consumers as only prices matter. If $p_i = p_j$, firms split the market equally. If $p_i < p_j$, firm i gets the entire demand.

Furthermore, the following assumption ensures that there is some demand from green-conscious consumers

$$\tau < \overline{\tau} \equiv \frac{v - c}{\gamma_H} - \underline{G}. \tag{9}$$

2.2 Policy Tools

We consider a combination of policy tools. On the R&D side, we model a patent policy for green innovations. On the environmental side, we assume that the firms must pay a tax, τ, per unit of emission. Thus, the tax bill faced by each firm i is given by $\tau\gamma_i D_i(.)$, where, by (1), $\gamma_i D_i(.)$ represents the emissions generated by the firm.

The patent policy is such that firm i must discover a sufficiently novel innovation to be able to obtain a patent. We assume that the novelty of the innovation is increasing in the investment level, where Δ represents the inventive step, that is, the difference between the level of emission-output ratio before and after innovation. To obtain a patent, firm i must reduce γ_i by at least

Δ. A weak patentability requirement corresponds to a small value of Δ, whereas a stronger patentability requirement corresponds to a larger value of Δ. Henceforth, we hold γ_H constant and allow Δ to vary to reflect changes in the patentability requirement. Effectively, a stronger patentability requirement corresponds to a lower value of γ_P.

Initially, both firms produce the dirty product with γ_H. As per (2), a firm must invest I_P to reduce the emission-output ratio from γ_H to $\gamma_P \equiv \gamma_H - \Delta$. If both firms invest the same amount and thereby obtain γ_P, only one obtains the patent with a probability of 1/2.

To obtain a patent, each firm must also incur an exogenously given cost C_P, which is broadly defined to include a monetary application fee payable by the firm to the patent office, the opportunity cost in terms of lost profits incurred while waiting for the patent to be granted, renewal costs which are monetary fees payable regularly during the lifetime,[17] as well as potential litigation costs for enforcing the patent right. There are several ways in which policy makers may reduce C_P, including by implementing a fast-track patent system for green technologies that reduces the patent application processing time for green innovations. We assume that the firm that does not obtain the patent does not incur the cost C_P, regardless of its choice of investment level. This is to reflect that typically patenting costs are higher for the patent holder than for other firms that apply but do not get a patent since some of the elements of C_P, such as renewal costs and potential litigation costs, are only incurred by the firm that obtains the patent. Therefore, for simplicity, we assume that the patenting cost for the firm that does not obtain the patent is 0.[18]

2.3 Timing

The timing of the three-stage game is as follows. In the first stage, each firm i, $i = 1, 2$, decides its investment level in the green technology, $I_i \in \{0, I_P\}$. In the second stage, if an innovation has been discovered by firm i, it decides whether to obtain a patent. Once firm i has obtained a patent, firm i decides whether to license the patent. If a license is offered, firm j decides whether

[17] In the U.S., the average renewal fees are US\$2000, US\$3760 and US\$7700 (USPTO website, 2024).

[18] Adding a positive application fee for the firm that applies for but does not obtain the patent would not generate any new insights.

to accept it or refuse it. In the third stage, firms choose the prices of their products, p_1 and p_2.

We solve for the equilibrium through backward induction. For given investment levels, patenting and licensing decisions, we first determine the firms' pricing strategy. Then, we determine the licensing decisions, the patenting decisions, and the investment levels in equilibrium.

As a benchmark case, we first consider the scenario where there exist only non-green-conscious consumers ($\lambda = 0$) in Section 3. We then enrich our analysis by including green-conscious consumers ($0 < \lambda \leq 1$) in Section 4. We find that the key difference between the two scenarios is that without green-conscious consumers there is always licensing in equilibrium. However, this changes when green-conscious consumers are introduced. This drives our main results, as shown in the following sections.

3 Benchmark Case: Non-Green-Conscious Consumers

3.1 Price Competition, Licensing and Patenting Decisions

In the scenario without green-conscious consumers, we show that licensing always occurs in equilibrium. We find that, when licensing occurs, the patent holder can set the royalty rate such that it obtains monopoly profits, and therefore it has a strong incentive to offer a license. At the same time, the non-patent holder's outside option (i.e., its payoff if it rejects the license) is zero such that the non-patent holder cannot do any better by rejecting the license.

To see this, let us begin with the third stage of the game. The non-green-conscious consumers care only about prices and buy from the firm that offers the lower price. From (5), the demand for firm i with $i \neq j$ for $i, j = 1, 2$ is given by

$$D_i(p_i, p_j) = \begin{cases} 1 & \text{if} \quad p_i < p_j \text{ and } p_i \leq v \\ \frac{1}{2} & \text{if} \quad p_i = p_j \leq v \\ 0 & \text{if} \quad p_i > \min\{p_j, v\} \end{cases} \tag{10}$$

For any combination of investment levels in the first stage, and patenting and licensing decisions in the second stage, we determine the firms' pricing strategies in the third stage. Recall that firms choose from two levels of investment: 0, in which case the dirty good is produced, and I_P, which is the requirement to patent the innovation of size $\Delta = \gamma_H - \gamma_P$. Three possible cases may arise: neither firm invests, one firm invests and the other does not, and both firms invest. If firm i offers a license to firm j and firm j accepts it, both firms produce the cleaner good γ_P. Firm i offers a license (r, F) where r is a per-unit royalty rate, and F is a fixed fee.[19] The findings are summarized in the following Lemma.

Lemma 1 *Firms' investment, patenting and licensing decisions lead to the following cases:*

1. *If $I_1 = I_2 = 0$, Bertrand competition in homogenous (dirty) product ensues with $p^* = p_1^* = p_2^* = c + \tau\gamma_H$ and both firms getting zero payoffs.*

2. *If one or both firms invest and patent holder i does not license, Bertrand competition with heterogeneous marginal costs ensues with $p^* = c + \tau\gamma_H - \varepsilon$. The payoff of the patent holder is $\Delta\tau$, and the non-patent holder, j, does not produce.*

3. *If one or both firms invest and patent holder i does license, we have $r^* = v - c - \tau\gamma_P$, $F^* = 0$, and $p^* = v$. The payoff of the patent holder is $v - c - \tau\gamma_P$, and of the non-patent holder is 0.*

Proof: Let us consider each case in turn.

In the first case, when none of the firms invest ($I_1 = I_2 = 0$), we obtain the classical finding of Bertrand competition; both firms set prices at marginal cost, $p^* = p_1^* = p_2^* = c + \tau\gamma_H$, and, thus, their payoffs are zero. They both produce the dirty good.

In the second case, when firm j decides not to invest and firm i decides to invest I_P, and apply for a patent, firm i must decide whether to license its patent to firm j. If firm i **does not license**, or if firm j refuses the license agreement, firm j produces the dirty good γ_H, and firm i produces the cleaner good γ_P. The payoff of firm i is $D_i(p_{iP}, p_j)(p_{iP} - c - \tau\gamma_P)$, and the payoff of firm j is $D_j(p_{iP}, p_j)(p_j - c - \tau\gamma_H)$ where the demands are defined by (10), p_{iP} is firm i's price (where subscript P stands for patent), and p_j is firm j's price.

Each firm chooses the price that maximizes its payoff. The Nash equilibrium is $p_{iP}^* = c + \tau\gamma_H - \varepsilon$ such that firm j gets a payoff of zero (and does not produce) while firm i obtains[20]

[19] Our findings with a Nash Bargaining solution would be qualitatively similar.

[20] We omit ε in the profit function of firm i.

$\Delta\tau - C_P$, and produces the cleaner product, for $i, j = 1, 2$ and $i \neq j$. In the case where only firm i invests I_P, it patents if $\tau \geq C_P/\Delta$, and, thus, firm i produces a cleaner product while firm j does not produce. However, if $\tau < C_P/\Delta$, firm i does not patent, both firms produce the dirty product and get zero payoffs.

If **licensing occurs**, for any license (r, F) accepted by firm j, firm i obtains the following payoff

$$D_i(p_{iP}, p_j)(p_{iP} - c - \tau\gamma_P) + \underbrace{rD_j(p_{iP}, p_j) + F}_{\text{License revenues}},$$

and firm j obtains the following payoff

$$D_j(p_{iP}, p_j)(p_j - c - \tau\gamma_P) - \underbrace{(rD_j(p_{iP}, p_j) + F)}_{\text{License payments}}.$$

We find that for any license (r, F), there is a unique Nash equilibrium in prices such that $p^* = p^*_{iP} = p^*_j = c + \tau\gamma_P + r$, and firm i obtains the payoff $(r + F)$ while firm j obtains the payoff $-F$ (see Appendix A2 for the proof).

In the second stage of the game (patenting and licensing decision), after deciding to offer a license, firm i will choose (r, F) that maximize its profit $(r + F)$ subject to the constraint that $-F \geq 0$ and $c + \tau\gamma_P + r \leq v$. Thus, the optimal license is $F^* = 0$, and $r^* = v - c - \tau\gamma_P$. If firm i offers a license $(r^* = v - c - \tau\gamma_P, F^* = 0)$, then the equilibrium price is $p^* = v$, so that the payoff of firm i is $v - c - \tau\gamma_P$, while the payoff of firm j is null.

In the third case, if both firms decide to invest I_P, and both firms decide to apply for a patent, one firm gets the patent (and pays the cost C_P) with probability 1/2, and the analysis is similar to the above one. ■

Lemma 1 has implications for the licensing decision of the patent holder, as follows.

Lemma 2 *Without any green-conscious consumers, if one or both firms invest and one of the firms obtains a patent, licensing always occurs in equilibrium.*

Proof: From Lemma 1 it follows that if firm i is the patent holder, it decides to license if $v - c - \tau\gamma_P \geq \Delta\tau$, which is always satisfied by assumption (9).[21] Therefore, in the case where

[21] Furthermore, for firm i to make non-negative profits in equilibrium, the condition $v - c - \tau\gamma_P \geq 0$ must be satisfied, which is always the case by assumption (9).

one firm invests and the other one does not, if the firm that invests patents its innovation, it will always license it to the other firm. Finally, firm i decides to patent as long as $C_P \leq v - c - \tau\gamma_P$. Firm j gets a null payoff whether it accepts the license or not. We assume that if the non-patent holder is indifferent between accepting or rejecting the license, it will accept it. Therefore, licensing occurs in equilibrium. ■

Lemma 2 shows that licensing always occurs in equilibrium in the absence of green consumers. This is because, when licensing occurs, the patent holder sets the royalty rate, r^*, such that it obtains monopoly profits, which is greater than the payoff generated by not licensing, $\Delta\tau$, as shown by Lemma 1. Therefore, the patent holder has a strong incentive to offer a license. At the same time, the non-patent holder's outside option (i.e., the payoff of the non-patent holder if it rejects the license) is zero such that the non-patent holder cannot do any better by rejecting the license, as shown by Lemma 1. Thus, we always have licensing in equilibrium as long as there are no green-conscious consumers. This affects the investment decisions in the first stage.

Before analyzing the investment decisions, it is useful to note that if both firms decide to invest I_P, and both firms decide to apply for a patent, one firm gets the patent (and pays the cost C_P) with probability 1/2. As we have seen above, the patent holder always prefers to license its innovation to the other firm. Thus, the expected payoff of firm i for $i = 1, 2$ is

$$\frac{1}{2}\underbrace{[D_i(p_{iP}, p_j)(p_{iP} - c - \tau\gamma_P) - C_P]}_{(I)} + \frac{1}{2}\underbrace{D_i(p_i, p_{jP})(p_i - c - \tau\gamma_P)}_{(II)},$$

where (I) represents the payoff of firm i if it gets the patent, and (II) represents the payoff if it does not get the patent. In the third stage, the equilibrium price is $p^* = v$. Therefore, if both firms invest, only one obtains a patent and licenses its technology to the other firm. Thus, each firm gets $(v - c - \tau\gamma_P - C_P)/2$, such that the condition to apply for a patent is given by $C_P \leq v - c - \tau\gamma_P$.

3.2 Investment Decisions

We now turn to the firms' investment decisions in the first stage. Since for $C_P > v - c - \tau\gamma_P$, none of the firms apply for a patent, they would earn zero payoffs even if they were to invest. For this reason, firms have no incentive to invest as long as $C_P > v - c - \tau\gamma_P$. To summarize the analysis of the previous subsection, if none of the firms invest, they each obtain a null payoff. If one firm invests I_P and the other does not, the one that invests licenses its innovation and has a payoff of $v - c - \tau\gamma_P - C_P - I_P$, while the other has a null payoff. If both of them invest I_P, each of them obtains $\frac{1}{2}(v - c - \tau\gamma_P - C_P) - I_P$. Let

$$C_1(\tau) \equiv v - c - \tau\gamma_P - I_P, \tag{11}$$

and

$$C_2(\tau) \equiv v - c - \tau\gamma_P - 2I_P, \tag{12}$$

with $C_1(\tau) > C_2(\tau)$ for any τ.

If $C_P \in [0, C_2(\tau)[$, the best response of firm i to any investment I_j by firm j is $BR_i(I_j) = I_P$. If $C_P \in [C_2(\tau), C_1(\tau)[$, the best response of firm i to any investment I_j by firm j is

$$BR_i(I_j) = \begin{cases} I_P & \text{if } I_j = 0 \\ 0 & \text{if } I_j = I_P \end{cases}$$

If $C_P \geq C_1(\tau)$, the best response of firm i to any investment I_j by firm j is $BR_i(I_j) = 0$.

Thus, for $C_P \in [0, C_2(\tau)[$, the unique Nash equilibrium of the first stage is (I_P, I_P), that is, both firms invest. For $C_P \in [C_2(\tau), C_1(\tau)[$, there exist two Nash equilibria $(0, I_P)$ and $(I_P, 0)$ such that one firm does not invest while the other invests I_P. For $C_P \geq C_1(\tau)$, there exists a unique Nash equilibrium $(0, 0)$ in which none of the firms invest.

In Figure 1, we represent the functions $C_1(\tau)$ and $C_2(\tau)$ in a graph where the horizontal axis represents the emission tax τ, and the vertical axis represents the patenting cost C_P. Recall that $\bar{\tau}$ is as defined in assumption (9).

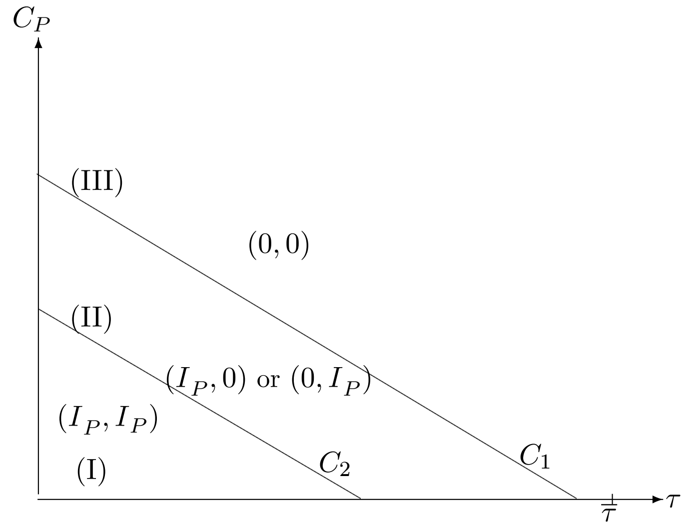

Figure 1: Equilibrium investments with non-green-conscious consumers

For a given patenting cost C_P, we obtain a paradoxical result whereby increasing the emission tax beyond a certain threshold leads to less innovation. Moreover, the higher the patenting cost, the greater the range of taxes for which the paradox occurs. As illustrated by Figure 1, for a given patenting cost C_P, as τ increases, firms invest less. Note that this finding runs counter to the Porter Hypothesis which posits that higher emission taxes induce more innovation. This paradox occurs because we have introduced a fixed cost C_P in our model. As τ increases, it becomes less likely that this fixed cost will be overcome. Consequently, as τ increases, the less likely it becomes for the investment to be profitable.

The Subgame Perfect Nash Equilibrium is presented in the following Proposition.

Proposition 1 *For a given τ, the Subgame Perfect Nash Equilibrium is the following:*

1. *if $C_P \in [0, C_2(\tau)[$, both firms invest $(I_1^*, I_2^*) = (I_P, I_P)$, but only one of them gets a patent and licenses it to the other firm ($r^* = v - c - \tau\gamma_P, F^* = 0$); both firms produce the cleaner product at price $p^* = v$;*

2. *if $C_P \in [C_2(\tau), C_1(\tau)[$, firm i invests while firm j does not $(I_i^*, I_j^*) = (I_P, 0)$ for $i, j = 1, 2$ and $i \neq j$; firm i gets a patent and licenses it to firm j ($r^* = v - c - \tau\gamma_P, F^* = 0$); both firms produce the cleaner product at price $p^* = v$;*

3. if $C_P \geq C_1(\tau)$, none of the firms invest $(I_1^*, I_2^*) = (0, 0)$, and they both produce the dirty product at price $p^* = c + \tau\gamma_H$.

3.3 Equilibrium Emission levels

Before presenting the equilibrium emission levels in the next Proposition, we rewrite (11) and (12) as thresholds on τ such that

$$\tau_1 \equiv \frac{v - c - C_P - I_P}{\gamma_P}, \tag{13}$$

and

$$\tau_2 \equiv \frac{v - c - C_P - 2I_P}{\gamma_P},$$

with $\tau_1 > \tau_2$, where both τ_1 and τ_2 are decreasing with C_P and I_P. For any $\tau > \tau_1$, none of the firms invest, while for $\tau \leq \tau_1$, at least one of the firms invest (both invest if $\tau \leq \tau_2$).

Proposition 2 *In equilibrium, the total emission level is given by*

$$e^* = \begin{cases} \gamma_P & \textit{for } \tau \leq \tau_1 \\ \gamma_H & \textit{for } \tau > \tau_1 \end{cases}$$

Figure 2a illustrates the equilibrium emission level. For $\tau > \tau_1$, neither firm invests, which results in both firms producing the dirty product. Each firm has half the demand, such that the emission generated by each firm is $e_i^* = \gamma_H/2$ for $i = 1, 2$. Thus, the total emission is $e^* = e_1^* + e_2^* = \gamma_H$. For $\tau \in [\tau_2, \tau_1[$, only one firm invests, gets a patent and licenses it to the other firm, such that both firms produce the cleaner technology, and $e_i^* = \gamma_P/2$ for $i = 1, 2$. Thus, the total emission is $e^* = \gamma_P$. For $\tau \leq \tau_1$, both firms invest, and one of them obtains a patent and licenses it to the other firm such that we have $e^* = \gamma_P$.

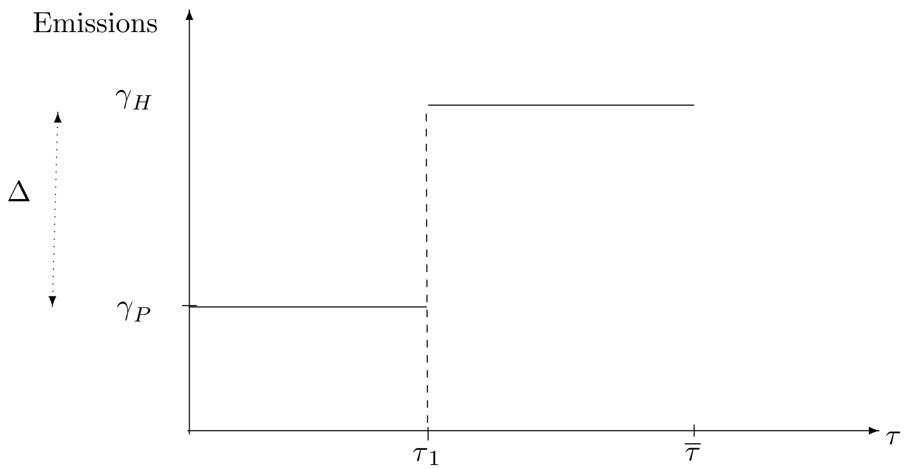

Figure 2a: Equilibrium emission with non-green-conscious consumers

By Proposition 2 and as illustrated by Figure 2a, for a given patenting cost C_P, we obtain a paradoxical result as highlighted in the following Corollary.

Corollary 1 *Increasing the emission tax beyond the threshold* τ_1 *leads to a discrete increase in the emission level.*

We now examine the role of the patent policy instruments in impacting the emission level. In particular, we show how varying C_P and γ_P affects the paradox summarized by Corollary 1.

Reducing patenting costs

As illustrated by Figure 2b, the higher the patenting cost, the greater the range of taxes for which the paradox occurs as τ_1 is a decreasing function of C_P. Thus, reducing the patenting cost, C_P, increases τ_1, such that a lower emission level is realized for a larger range of taxes. In other words, reducing patenting costs mitigates the paradox mentioned in Corollary 1.

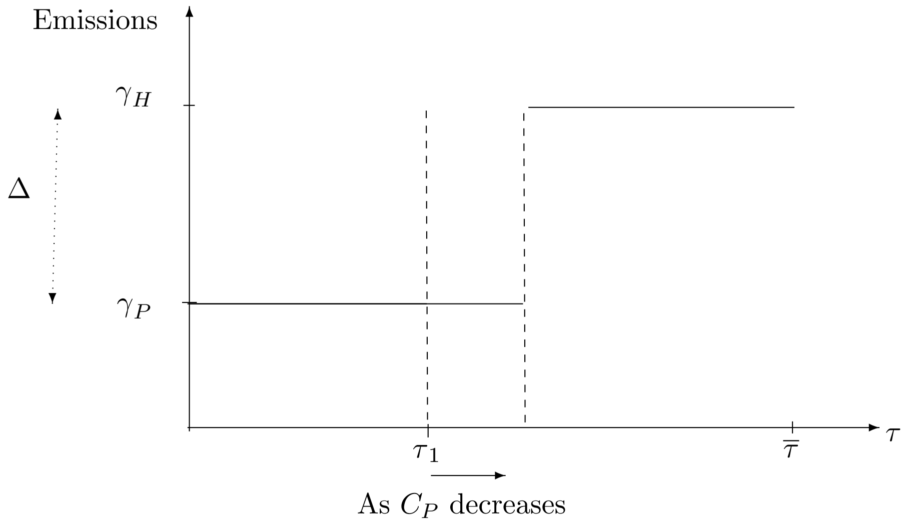

Figure 2b: Equilibrium emission with non-green-conscious consumers as C_P decreases

Making patentability requirement stricter

Unlike reducing the patenting cost, making the patentability requirement more stringent by reducing the level of γ_P has an ambiguous effect on τ_1. As shown in Appendix A2, a decrease in γ_P increases τ_1 if and only if $\tau_1 > -\partial I_P/\partial\gamma_P$. In other words, as long as the elasticity of investment with respect to γ_P is sufficiently small, a decrease in γ_P increases τ_1. In those sectors where a small increase in investment is able to achieve large reductions in the emission-output ratio, making the patentability requirement more stringent mitigates the paradox mentioned in Corollary 1, similar to the impact of a reduction in C_P. Moreover, since γ_P represents the emission level itself, a lower emission level is achieved for $\tau < \tau_1$ by making the patentability requirement more stringent compared to that obtained by reducing the patenting cost C_P, as shown in Figure 2c.

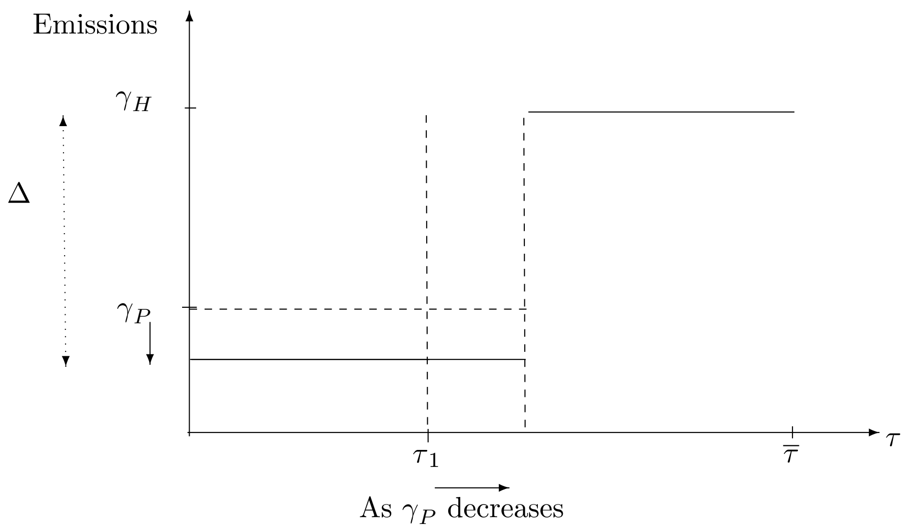

Figure 2c: Equilibrium emission with non-green-conscious consumers as γ_P decreases and $\tau_1 > -\partial I_P/\partial \gamma_P$

However, in sectors where a large increase in investment is required to achieve a small reduction in the emission-output ratio, i.e., $\tau_1 < -\partial I_P/\partial \gamma_P$, making the patentability requirement more stringent exacerbates the paradox mentioned in Corollary 1, by decreasing τ_1. At the same time, a lower emission level is achieved for $\tau < \tau_1$ by making the patentability requirement more stringent compared to that obtained by reducing the patenting cost C_P, as shown in Figure 2d. Thus, in such sectors, the impact of tightening the patentability requirement on the emission level is ambiguous. While at low levels of τ tightening the patentability requirement lowers the emission level, it also hinders the smooth functioning of emission taxes by exacerbating the paradox at higher levels of τ, specifically for $\tau > \tau_1$.

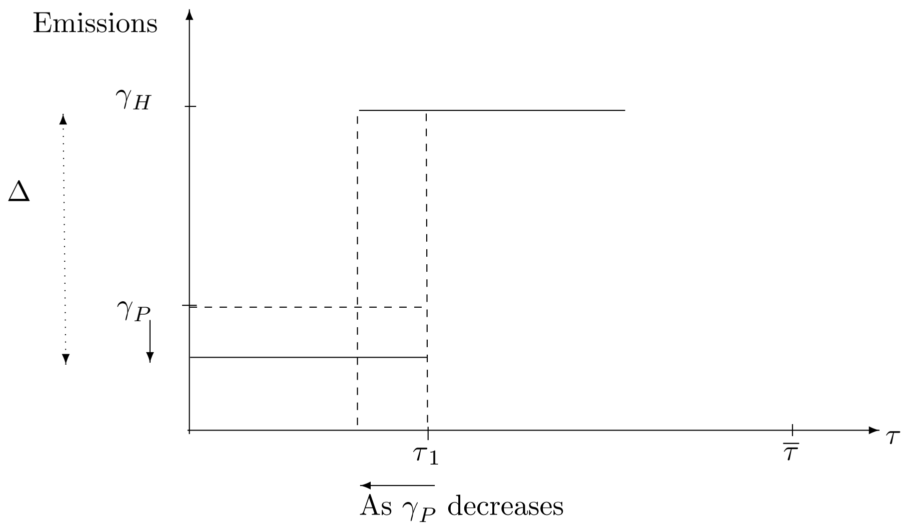

Figure 2d: Equilibrium emission with non-green-conscious consumers as γ_P decreases and $\tau_1 < -\partial I_P/\partial\gamma_P$

We summarize our results in the following Proposition.

Proposition 3 *With only non-green-conscious consumers, i.e.,* $\lambda = 0$*,*

(i) reducing patenting cost C_P *leads to a higher* τ_1*, thereby reducing the emission level from* γ_H *to* γ_P *for a greater range of emission taxes.*

(ii) making the patentability requirement more stringent by reducing the level of γ_P *leads to a higher* τ_1*, thereby reducing the emission level from* γ_H *to* γ_P *for a greater range of emission taxes if* $\tau_1 > -\partial I_P/\partial\gamma_P$*.*

(iii) making the patentability requirement more stringent by reducing the level of γ_P *leads to a lower* τ_1*, thereby increasing the emission level from* γ_P *to* γ_H *for a greater range of emission taxes if* $\tau_1 < -\partial I_P/\partial\gamma_P$*.*

Proposition 3(i) follows from (13) which shows that τ_1 is decreasing in C_P. From (13) we see that while the direct effect of reducing γ_P is to increase τ_1, there also exists an indirect effect in the counter direction since a reduction in γ_P increases the investment level. As long as the the elasticity of investment with respect to γ_P is sufficiently small, a policy maker may use either instrument (reducing C_P or making the patentability requirement more stringent) or some combination of both to mitigate the paradoxical result associated with increasing the emission tax. That is, in industries where it is sufficiently cheap to develop cleaner technologies, both these patent policy instruments work in conjunction with the emission tax to allow the emission tax to function more smoothly. However, in industries where it is sufficiently expensive to develop cleaner technologies, making the patentability requirement more stringent decreases τ_1, thereby leading to a higher emission level for a greater range of emission taxes, as shown by Proposition 3(*iii*).

At the same time, Figures 2b-2d show that at a given value of τ, such as $\tau = \tau_1 + \varepsilon$ with $\varepsilon > 0$, making the patentability requirement more stringent is more effective in terms of reducing emission levels than reducing the patent cost. Moreover, at a given value of $\tau < \tau_1$, only making the patentability requirement more stringent reduces the emission level by reducing γ_P, whereas reducing C_P is ineffective and does not affect the emission level. This is summarized in the following Corollary.

Corollary 2 *With only non-green-conscious consumers, i.e.,* $\lambda = 0$*, for* $\tau_1 > -\partial I_P/\partial \gamma_P$*, at any given level of* τ*, making the patentability requirement more stringent is more effective in terms of reducing emission levels than reducing the patent cost,* C_P*.*

Corollary 2 highlights the finding that in industries where it is sufficiently cheap to develop cleaner technologies, making the patentability requirement more stringent unambiguously leads to a better environmental outcome than reducing patenting costs. At the same time, in industries where it is sufficiently expensive to develop cleaner technologies, it is unclear which policy achieves a lower emission level.

It is useful to compare our findings with Langinier and RayChaudhuri (2020). The main differences between this paper and Langinier and RayChaudhuri (2020) are the following. First, the market structure is different since Langinier and RayChaudhuri (2020) consider an incumbent monopolist who faces potential entry if it does not innovate. Second, they do not allow for the licensing of the green innovation. Third, they model investment as a continuous variable while we model investment as a discrete variable.

In the absence of any green-conscious consumers, Langinier and RayChaudhuri (2020) also obtain the paradoxical result that we have reported in Proposition 2, as well as a similar effect of reducing patenting costs. However, it is important to note that this does not imply that the degree of competition (i.e., duopoly as in this paper versus monopoly as in Langinier and RayChaudhuri, 2020) does not matter in determining policy implications. Indeed, we would not obtain the paradoxical result similar to Langinier and RayChaudhuri (2020) in our duopoly setting if we did not allow for licensing. While Langinier and RayChaudhuri (2020) do not allow for licensing, in our duopolistic setup, licensing effectively enables the innovator to obtain monopoly rents.[22] Without licensing, we would obtain very different results as firms would be engaged in tough price competition. In both papers, as the tax rate increases, the cost to the innovator increases (Effect 1). This is the only effect of tax increase captured by Langinier and RayChaudhuri (2020), which results in the innovation becoming unprofitable beyond a certain tax rate. In this paper, there is a second effect of a tax increase arising due to duopolistic competition. The price of the cleaner product is set at the marginal cost (which includes the marginal tax, $\tau\gamma_H$) of the firm producing the dirty product. This implies that the price of the cleaner product increases in τ (Effect 2) in the absence of licensing, which results in equilibrium investment and emission levels that differ from Proposition 1. However, because of the license, Effect 2 vanishes since prices are set to v and only Effect 1 matters. For this reason, we obtain results similar to Langinier and RayChaudhuri (2020) after allowing for licensing, despite the different market structure.

At the same time, we note that the paradoxical result, whereby increasing emission tax may increase emissions, is not restricted to cases where investment is modeled as a discrete variable. As long as the product price does not increase with emission tax, regardless of whether investment is modeled as a continuous or discrete variable, the paradox may occur if the fixed cost C_P is sufficiently large. This is because, as τ increases, if firms' profits cannot outweigh C_P, investment in the green technology becomes unprofitable.

[22]Recall that with licensing prices are set to v, which corresponds to the monopoly price. Also, the patent holder extracts all the surplus from the licensee, effectively obtaining monopoly profits.

4 Green and Non-Green-Conscious Consumers

In this section, we continue our analysis with the general model introduced in Section 2 with green-conscious consumers, i.e., $0 < \lambda \leq 1$. The key departure from the benchmark case without green-conscious consumers is that, when the latter are included, licensing does not always occur in equilibrium. This is because in the presence of heterogenous consumers, product differentiation becomes a profitable strategy for firms. In the following analysis, we determine the conditions under which licensing does and does not occur in equilibrium, and the ensuing implications for emission levels.

4.1 Price Competition, Licensing and Patenting Decisions

We begin with the third stage of the game and analyze the firms' pricing strategy for any combination of investment levels of both firms in the first stage of the game. Recall that firms choose from two levels of investment: 0 and I_P. As in Section 3, three possible cases may arise: CASE I: neither firm invests; CASE II: one firm invests and the other does not; CASE III: both firms invest.

CASE I:

If none of the firms invest ($I_1 = I_2 = 0$), the demand for each firm is given by (8). Both firms set their prices at marginal cost, $p^* = p_1^* = p_2^* = c + \tau\gamma_H$ to sell the dirty product, and, thus, their payoffs are zero.

CASE II:

If firm i decides to invest I_P, while firm j does not ($I_j = 0$), where $i, j = 1, 2$ and $i \neq j$, once firm i discovers the innovation, it must decide whether to obtain a patent or not in the second stage of the game. If it does patent its innovation and pays the patenting cost C_P, then firm i decides whether to license its innovation to firm j or not. We analyze each of these sub-cases separately. For clarity, we label the case where firm i decides not to license as CASE II(a), and the case where firm i decides to offer a license as CASE II(b).

CASE II(a): If firm i decides **not to license**, its payoff is $D_i(p_{iP}, p_j; \gamma_P)(p_{iP} - c - \tau\gamma_P)$, and the payoff of firm j is $D_j(p_{iP}, p_j; \gamma_H)(p_j - c - \tau\gamma_H)$ where $D_i(p_{iP}, p_j; \gamma_P)$ is firm i's demand given by (5), and $D_j(p_{iP}, p_j; \gamma_H)$ is firm j's demand given by (6). Thus, firms i and j choose the prices p_{iP} and p_j that maximize their payoffs. If the conditions $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$ are satisfied, the equilibrium prices are (see Appendix A3 for the details of the calculation) given by:

$$p_{iP}^{*G} = \frac{1}{4\gamma_H - \gamma_P}[(2\gamma_H + \gamma_P)c + 2\Delta v + 3\tau\gamma_H\gamma_P - \gamma_P\Delta(\underline{G} - \frac{1-\lambda}{\lambda})], \tag{14}$$

and

$$p_j^{*G} = \frac{1}{4\gamma_H - \gamma_P}[3\gamma_H c + v\Delta + \gamma_H\tau(2\gamma_H + \gamma_P) - 2\Delta\gamma_H(\underline{G} - \frac{1-\lambda}{\lambda})]. \tag{15}$$

Furthermore, we show that $p_{iP}^{*G} > p_j^{*G}$ if $\tau < \tau_E(\lambda)$, where

$$\tau_E(\lambda) \equiv \frac{v-c}{2\gamma_H} + \frac{2\gamma_H - \gamma_P}{2\gamma_H}(\underline{G} - \frac{1-\lambda}{\lambda}), \tag{16}$$

which is increasing with λ. As λ increases, both equilibrium prices (14) and (15) decrease, and as c, v and τ increase, both prices increase. We have that $\underline{G} < \widetilde{G}_1$ (in which case both firms have some demand from the green-conscious consumers) if $\tau < \tau_L(\lambda)$ where

$$\tau_L(\lambda) \equiv \frac{v-c}{2\gamma_H} - \underline{G} - \frac{2\gamma_H - \gamma_P}{2\gamma_H}\frac{1-\lambda}{\lambda}. \tag{17}$$

We check that $\tau_L(\lambda) < \tau_E(\lambda)$ so that when both firms have some demand from green-conscious consumers, their equilibrium prices always satisfy $p_{iP}^G > p_j^G$. Some green-conscious consumers will not buy any product if $\widetilde{G}_2 < \overline{G}$ or, equivalently, if $\tau > \tau_C(\lambda)$ with

$$\tau_C(\lambda) \equiv \frac{2\gamma_H + \gamma_P}{\gamma_H\gamma_P}\frac{v-c}{3} - \underline{G} - \frac{\gamma_H - \gamma_P}{3\gamma_H}\frac{1-\lambda}{\lambda} - \frac{4\gamma_H - \gamma_P}{3\gamma_H}, \tag{18}$$

as the product is not clean enough for them or the price is too high for a product that is not clean enough. On the other hand, if $\tau \leq \tau_C(\lambda)$, all the green-conscious consumers buy either a dirty product (consumers with a low G) or a cleaner product (with a large G). Depending on the parameter values, we could have either that $\tau_L(\lambda) > \tau_C(\lambda)$ or $\tau_L(\lambda) < \tau_C(\lambda)$. However, as long as $\tau < \tau_L(\lambda)$, both firms produce different products, as both green and non-green-conscious consumers will buy the products. If $\tau_C(\lambda) > \tau_L(\lambda)$, for any $\tau < \tau_L(\lambda)$, all the green-conscious consumers buy one unit of good. However, if $\tau_C(\lambda) < \tau_L(\lambda)$, for $\tau \in [\tau_C(\lambda), \tau_L(\lambda)]$, some green-conscious consumers do not buy any product, and for $\tau < \tau_C(\lambda)$, all green-conscious consumers buy one unit of the product. The implications of the above discussion are summarized in the following Lemma. Note that in Lemma 3, p_{iP}^{*GC} and p_j^{*GC} are specified in Appendix A3, where the subscript C is for covered market, p_{iP}^{*G} and p_j^{*G} as defined by (14) and (15), and the equilibrium profits are presented in the proof.

Lemma 3 *For different ranges of the emission tax, τ, the equilibrium features the following:*

(i) For $\tau < \min\{\tau_C(\lambda), \tau_L(\lambda)\}$, all the green-conscious consumers buy one of the (dirty or cleaner) products, the firms produce differentiated products where the equilibrium prices p_{iP}^{*GC} and p_j^{*GC} are such that $p_{iP}^{*GC} > p_j^{*GC}$, and the equilibrium profits are $\Pi_{iP}^{*GC} > 0$ and $\Pi_j^{*GC} > 0$.

(ii) For $\tau \in [\tau_C(\lambda), \tau_L(\lambda)]$, some green-conscious consumers do not buy any product, the firms produce differentiated products where the equilibrium prices are given by p_{iP}^{*G} and p_j^{*G}, and the equilibrium profits are $\Pi_{iP}^{*G} > 0$ and $\Pi_j^{*G} > 0$.

(iii) For $\tau \geq \tau_L(\lambda)$, firm i, producing the cleaner good, sets its price at $p_{iP}^* = c + \tau\gamma_H - \varepsilon$, so that it gets all the demand, and firm j does not get any demand such that he equilibrium profits are $\Pi_{iP}^{\prime G} > 0$ and $\Pi_j^{*G} = 0$.

Proof: First, when $\tau \in [\tau_C(\lambda), \tau_L(\lambda)]$, evaluated at prices p_{iP}^{*G} and p_j^{*G} as defined by (14) and (15), the demands for the cleaner and dirty products are, respectively,

$$D_i(p_{iP}^{*G}, p_j^{*G}; \gamma_P) = \frac{\lambda}{4\gamma_H - \gamma_P} \frac{\gamma_H}{\gamma_P} [2(v - c) - \gamma_P(\tau + \underline{G} - \frac{1 - \lambda}{\lambda})], \tag{19}$$

and

$$D_j(p_{iP}^{*G}, p_j^{*G}; \gamma_H) = \frac{\lambda}{4\gamma_H - \gamma_P} [v - c - 2\gamma_H(\tau + \underline{G} - \frac{1 - \lambda}{\lambda})]. \tag{20}$$

Both demands are strictly positive in equilibrium, and some of the green-conscious consumers do not buy any product. Both demands decrease as τ increases. However, the demand for the dirty product decreases at a faster rate than the demand for the cleaner product. Therefore, in the absence of a license, if $\tau \in [\tau_C(\lambda), \tau_L(\lambda)]$, the equilibrium profits for firms i and j are

$$\Pi_{iP}^{*G} = \lambda \frac{\gamma_H}{\gamma_P} \frac{\Delta}{(4\gamma_H - \gamma_P)^2} [2(v - c) - \gamma_P(\tau + \underline{G} - \frac{1 - \lambda}{\lambda})]^2, \tag{21}$$

and

$$\Pi_j^{*G} = \lambda \frac{\Delta}{(4\gamma_H - \gamma_P)^2} [v - c - 2\gamma_H(\tau + \underline{G} - \frac{1 - \lambda}{\lambda})]^2. \tag{22}$$

This proves Lemma 3(ii).

Second, if $\tau < \min\{\tau_C(\lambda), \tau_L(\lambda)\}$, all the green-conscious consumers buy one of the (dirty or cleaner) products, and both firms are producing different products where the equilibrium prices p_{iP}^{*GC} and p_j^{*GC} are such that $p_{iP}^{*GC} > p_j^{*GC}$ (see Appendix A3). The equilibrium profits for firms i and j are

$$\Pi_{iP}^{*GC} = \lambda\frac{\Delta}{9}[\tau + \underline{G} + 2 + \frac{1-\lambda}{\lambda}]^2, \tag{23}$$

and

$$\Pi_j^{*GC} = \lambda\frac{\Delta}{9}[1 - 2\tau - \underline{G} + 2\frac{1-\lambda}{\lambda}]^2. \tag{24}$$

This proves Lemma 3(i).

Third, we determine the equilibrium prices if $\tau \geq \tau_L(\lambda)$, in which case firm j does not have any demand from the green-conscious consumers. Firm i, producing the cleaner good, sets its price at $p_{iP}^* = c + \tau\gamma_H - \varepsilon$, so that it gets all the demand, and firm j does not get any demand. Thus, if $\tau \geq \tau_L(\lambda)$, the equilibrium payoffs for firm i is[23]

$$\Pi_{iP}^{\prime G} = \lambda\frac{\Delta}{\gamma_P}[v - c - \tau\gamma_H - \gamma_P(\underline{G} - \frac{1-\lambda}{\lambda})]\tau, \tag{25}$$

and firm j gets 0. This proves Lemma 3(iii). ■

CASE II(b): We now consider the case where firm i decides to offer **a licence** (r, F) such that, if accepted, firm j must pay a royalty fee r and a fixed fee F. For any given r and F, firm j accepts the license if its payoff is at least as much as the payoff it will get from refusing the license, which is Π_j^{*G} as defined by (22) or Π_j^{*GC} as defined by (24) if $\tau < \tau_L(\lambda)$, and 0 if $\tau \geq \tau_L(\lambda)$. This yields the following Lemma.

Lemma 4 *(i) For $\tau < \tau_L(\lambda)$, there is no licensing. Both firms compete with different products, and they obtain the payoffs Π_{iP}^{*G} and Π_j^{*G} as defined by (21) and (22) or Π_{iP}^{*GC} and Π_j^{*GC} as defined by (23) and (24).*

(ii) For $\tau \geq \tau_L(\lambda)$, firm i always licenses as $\Pi_{iP}^{*L} > \Pi_{iP}^{\prime G}$ is always satisfied where Π_{iP}^{*L} and $\Pi_{iP}^{\prime G}$ are defined by (28) and (25), respectively.

[23] We omit ε in the profit function of firm i.

Proof: If firm i offers a license that is accepted by firm j, both firms could produce the cleaner product, in which case each firm faces demand (8). Firm i obtains $D_i(p_{iP}, p_j)(p_{iP} - c - \tau\gamma_P) + rD_j(p_{iP}, p_j) + F$ and firm j gets $D_j(p_{iP}, p_j)(p_j - c - r - \tau\gamma_P) - F$. As firms compete with the same cleaner product, we have Bertrand competition in which both firms set their prices at the marginal cost of firm j, namely, $p^{*L} = c + \tau\gamma_P + r$, and both firms share the demand (firm i has no incentive to reduce its price even further to capture all the demand as it can get the same payoff with a license). Thus, if both firms produce the cleaner product, firm i gets $\Pi^L_{iP} + F$, and firm j gets $-F$, where Π^L_{iP} will be defined later.

However, firm j could decide to accept the license (r, F) and produce the dirty product, in which case it will only have to pay F, and will not have to pay r as it is not producing the cleaner product. If firm j decides not to produce the cleaner product, both firms compete with different products, and we obtain the previous results: if $\tau < \tau_L(\lambda)$, the equilibrium profits for firms i and j are $\Pi^{*G}_{iP} + F$ and $\Pi^{*G}_{j} - F$ where Π^{*G}_{iP} and Π^{*G}_{j} are given by (21) and (22) if $\tau > \tau_C(\lambda)$, and $\Pi^{*GC}_{iP} + F$ and $\Pi^{*GC}_{j} - F$ where Π^{*GC}_{iP} and Π^{*GC}_{j} are given by (23) and (24) if $\tau < \tau_C(\lambda)$. If $\tau \geq \tau_L(\lambda)$, the equilibrium profits for firms i and j are $\Pi'^{G}_{iP} + F$ and $-F$ where Π'^{G}_{iP} is given by (25).

Therefore, if $\tau < \tau_L(\lambda)$, firm j always prefers to produce the dirty product, even if it accepted the license, as $\Pi^{*G}_{jP} - F > 0 - F$ and $\Pi^{*GC}_{jP} - F > 0 - F$. Thus, unless $F < 0$, firm j will never accept a license as $\Pi^{*G}_{jP} > \Pi^{*G}_{jP} - F$, and $\Pi^{*GC}_{jP} > \Pi^{*GC}_{jP} - F$ and, in equilibrium, there is no licensing if $\tau < \tau_L(\lambda)$. Even if firm i were to choose $F < 0$ (which means that firm i would have to pay firm j to use its license), firm j would still prefer to produce the dirty product, and therefore firm i has no incentive to pay firm j to use its license.

If $\tau \geq \tau_L(\lambda)$, firm j is indifferent between producing the cleaner product and the dirty product once the license has been accepted, as it will get a payoff of zero in both cases. We assume that firm j always produces the cleaner good when indifferent. Therefore, firm j accepts the license if $-F \geq 0$. Both firms have the same demand,

$$D(p^{*L}) = \frac{1}{2}\frac{\lambda}{\gamma_P}(v - p^{*L} - \gamma_P \underline{G} + \gamma_P \frac{1-\lambda}{\lambda}),$$

where $p^{*L} = c + \tau\gamma_P + r$ and firm i obtains $D(p^{*L})(p^{*L} - c - \tau\gamma_P + r) + F = D(p^{*L})r + F$.

Therefore, firm i chooses (r, F) solution of

$$\underset{r,F}{Max}\frac{1}{\gamma_P}\lambda[v - c - r - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})]r + F,$$

such that $-F \geq 0$ and $c + \tau\gamma_P + r \leq v - \gamma_P\underline{G}$. The latter condition is to ensure that both green and non-green-conscious consumers buy the cleaner good. If the price is too high for green-conscious consumers to buy, only non-green-conscious consumers will buy, in which case the condition $c + \tau\gamma_P + r \leq v$ must be satisfied (see Appendix A3). Therefore, firm i chooses $F^* = 0$ and

$$r^* = \frac{1}{2}[v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})], \tag{26}$$

as long as $\tau \leq \tau_{LNG}(\lambda) \equiv (v - c)/\gamma_P - 2\underline{G} - 2(1-\lambda)/\lambda$. If $\tau > \tau_{LNG}(\lambda)$, only non-green-conscious consumers buy at price $p^{*L} = v$ (see Appendix A3). Evaluated at r^*, each firm faces the same demand

$$D^L = \frac{1}{4}\frac{\lambda}{\gamma_P}(v - c - \tau\gamma_P - \gamma_P\underline{G} + \gamma_P\frac{1-\lambda}{\lambda}). \tag{27}$$

Therefore, firm i gets

$$\Pi_{iP}^{*L} = \lambda\frac{1}{4\gamma_P}[v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})]^2, \tag{28}$$

while firm j gets 0. To summarize, if $\tau_L(\lambda) \leq \tau < \tau_{LNG}(\lambda)$, firm i offers the license $(r^*, F^* = 0)$ where r^* is defined by (26) that firm j accepts. Firm j produces the cleaner product, but firm i captures all the payoff of firm j through licensing. ■

Note that if $\tau \geq \tau_{LNG}(\lambda)$, only non-green-conscious consumers buy the cleaner good, and the case is identical to the one presented in Section 3. In what follows, we do not discuss this case.

CASE III:

Lastly, we consider the case where both firms decide to invest I_P, and they both decide to apply for a patent, in which case one firm gets the patent with a probability of 1/2. If firm i obtains the patent, the analyses without and with licensing are the ones that have been developed above for $i = 1, 2$. Thus, if $\tau < \tau_L(\lambda)$, there is no licensing, and once the innovation has been discovered, the expected payoff for each firm i for $i = 1, 2$, is

$$\frac{1}{2}(\Pi_{iP}^{*G} - C_P) + \frac{1}{2}\Pi_i^{*G}, \tag{29}$$

where Π_{iP}^{*G} is the equilibrium payoff that firm i gets if it obtains the patent and is defined by (21), and Π_i^{*G} is the payoff it gets if it does not obtain the patent and is defined by (22). On the other hand, if $\tau \geq \tau_L(\lambda)$, there is licensing, and firm i gets $(\Pi_{iP}^L - C_P)/2$.

Figure 3 illustrates Lemma 4, where the horizontal axis represents the fraction of green-conscious consumers, and the vertical axis represents the emission tax τ. Our discussion of Cases I-III reveals that an important threshold of τ for our analysis is $\tau_L(\lambda)$, as given by (17). Therefore, Figure 3 illustrates $\tau_L(\lambda)$ as a function of λ. To simplify, we represent the case where $\tau_C(\lambda = 1) \leq 0$.[24]

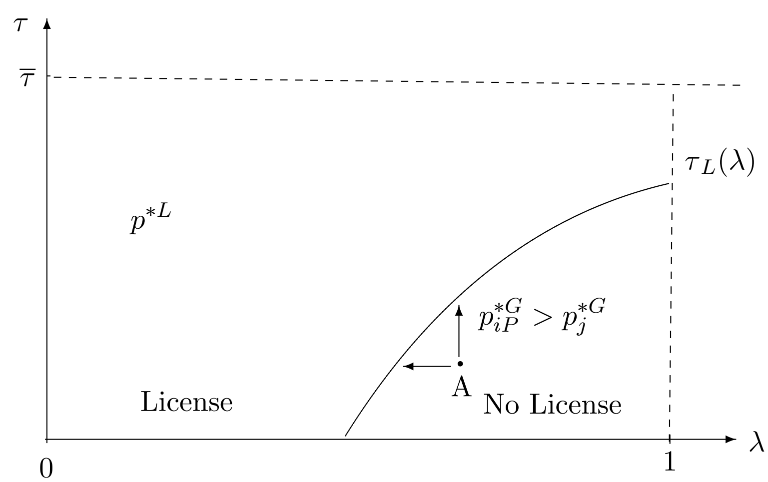

Figure 3: Licensing with green and non-green-conscious consumers

In the region above $\tau_L(\lambda)$ in Figure 3, i.e., for $\tau \geq \tau_L(\lambda)$, in equilibrium, the patentholder offers a license to its competitor who accepts it, and both firms set identical prices at p^{*L}. In the region below $\tau_L(\lambda)$ in Figure 3, for $\tau < \tau_L(\lambda)$, in equilibrium, there is no licensing, and both firms compete with differentiated goods (one dirty product and one cleaner product) with different prices, $p_{iP}^{*G} > p_j^{*G}$.

As shown by Figure 3, whether licensing occurs in equilibrium depends on λ and τ. Recall that the proportion of green-conscious consumers λ reflects the degree of heterogeneity across consumers. Therefore, if λ is sufficiently small, product differentiation is not a profit-maximizing strategy, and we obtain an equilibrium with licensing where both firms produce

[24] If $\tau_C(\lambda = 1) > 0$, prices would be different in the No License area, but the intuition would be similar.

the same (cleaner) product. Now if λ is larger, starting at the point A in Figure 3, let us consider what happens as λ decreases. For a given τ, a decrease in λ leads to an increase in both equilibrium prices, even though not at the same pace. Therefore, the difference in prices decreases as λ decreases, and so does $\widetilde{G}_1$ (See Appendix A3 for the detail of the calculation). As λ decreases further, $\widetilde{G}_1 = \underline{G}$ so that firm i gets all the demand from the green-conscious consumers. But, then firm i has an incentive to reduce its price below its competitor to capture all the non-green-conscious consumers so that the dirty product has no demand anymore. This explains why we move from an equilibrium without licensing at A to one with licensing as λ decreases sufficiently.

Now let us turn to the role played by τ in determining whether licensing occurs. Starting from point A, for a given λ, a sufficient increase in τ results in switching from an equilibrium with no licensing to one with licensing. This is because, as τ increases, while the marginal costs faced by both firms increase, it increases at a faster rate for the firm producing the dirty product. At high taxes, when the marginal costs of the firms are sufficiently asymmetric, it becomes profitable for the patent holder, firm i, to price just below the marginal cost of the dirty product to capture the whole market. In this case, whether it accepts the license or not, firm j obtains a payoff of zero. We assume that, when indifferent between not producing and producing the cleaner product via licensing, firm j chooses the latter. Thus, while an increase in τ increases both equilibrium prices, at the same time $\widetilde{G}_1$ decreases. Eventually further increases in τ leads to $\widetilde{G}_1 = \underline{G}$ so that firm i gets all the demand from the green-conscious consumers. Together, these effects imply that when τ is sufficiently low, such as a point A in Figure 3, it is profitable to differentiate products which is why there is no licensing. However, as τ increases to a level beyond $\tau_L(\lambda)$, it becomes unprofitable to differentiate products. Thus, we obtain licensing in equilibrium such that both firms produce the cleaner product.

After deciding whether or not to invest, the innovator must also decide whether to patent the innovation. If $\tau < \tau_L(\lambda)$, firm i applies for a patent only if $\Pi_{1P}^{*G} - C_P \geq 0$, (or if $\Pi_{1P}^{*GC} - C_P \geq 0$) and if $\tau \geq \tau_L(\lambda)$, if $\Pi_{iP}^{*L} - C_P \geq 0$. We summarize these findings in the following Lemma.

Lemma 5 *If firm i innovates, it applies for a patent if* *(i)* $C_P \leq \Pi_{iP}^{*GC}$ *if* $\tau < \min\{\tau_C(\lambda), \tau_L(\lambda)\}$;

(ii) $C_P \leq \Pi_{iP}^{*G}$ *if* $\tau_C(\lambda) < \tau < \tau_L(\lambda)$;

(iii) $C_P \leq \Pi_{iP}^{*L}$ *if* $\tau \geq \tau_L(\lambda)$.

Therefore, firm i applies for a patent only if the patenting cost is not too large. However, how large is this threshold level of patenting cost depends on the level of emission tax.

4.2 Investment Decisions

We now turn to the firms' investment decisions in the presence of green and non-green-conscious consumers. We start with $\tau < \tau_L(\lambda)$, in which case, there is **no license**. The best response of firm i for $i = 1, 2$ with $i \neq j$ to $I_j = 0$ is

$$BR_i(I_j = 0) = \begin{cases} I_P & \text{if } \Pi_{iP}^{*G} - C_P - I_P \geq 0 \Leftrightarrow \tau \leq \tau_1^{NL}(\lambda, C_P) \\ 0 & \text{if } \Pi_{iP}^{*G} - C_P - I_P < 0 \Leftrightarrow \tau > \tau_1^{NL}(\lambda, C_P) \end{cases}$$

where $\tau_1^{NL}(\lambda, C_P)$ is defined in Appendix A3. The best response of firm i to $I_j = I_P$ is

$$BR_i(I_j = I_P) = \begin{cases} I_P & \text{if } \frac{1}{2}(\Pi_{iP}^{*G} - C_P) + \frac{1}{2}\Pi_i^{*G} - I_P \geq \Pi_i^{*G} \Leftrightarrow \tau \leq \tau_2^{NL}(\lambda, C_P) \\ 0 & \text{if } \frac{1}{2}(\Pi_{iP}^{*G} - C_P) + \frac{1}{2}\Pi_i^{*G} - I_P < \Pi_i^{*G} \Leftrightarrow \tau > \tau_2^{NL}(\lambda, C_P) \end{cases}$$

where $\tau_2^{NL}(\lambda, C_P)$ is defined in Appendix A3, with $\tau_1^{NL}(\lambda, C_P) > \tau_2^{NL}(\lambda, C_P)$. We show that $\tau_1^{NL}(\lambda, C_P)$ and $\tau_2^{NL}(\lambda, C_P)$ are both increasing in λ for the range of parameters considered, and they are both decreasing in C_P. Therefore, if $\tau \leq \tau_2^{NL}(\lambda, C_P)$, the best response of firm i to any investment $I_j \in \{0, I_P\}$ by firm j is $BR_i(I_j) = I_P$. If $\tau \in [\tau_2^{NL}(\lambda, C_P), \tau_1^{NL}(\lambda, C_P)[$, the best response of firm i to any investment I_j by firm j is

$$BR_i(I_j) = \begin{cases} I_P & \text{if } I_j = 0 \\ 0 & \text{if } I_j = I_P \end{cases}$$

Lastly, if $\tau \geq \tau_1^{NL}(\lambda, C_P)$, the best response of firm i to any investment $I_j \in \{0, I_P\}$ by firm j is $BR_i(I_j) = 0$.

Thus, for $\tau \geq \tau_1^{NL}(\lambda, C_P)$, the unique Nash equilibrium is $(I_1^*, I_2^*) = (0, 0)$, that is, none of the firms invests. For $\tau \in [\tau_2^{NL}(\lambda, C_P), \tau_1^{NL}(\lambda, C_P)[$, there exist two Nash equilibria $(I_1^*, I_2^*) =$

$(0, I_P)$ and $(I_1^*, I_2^*) = (I_P, 0)$ such that one firm does not invest while the other invests I_P. For $\tau \leq \tau_2^{NL}(\lambda, C_P)$, there exists a unique Nash equilibrium $(I_1^*, I_2^*) = (I_P, I_P)$ such that both firms invest I_P.

If $\tau \geq \tau_L(\lambda)$, the innovator **licenses** the patented innovation. The best response of firm i for $i = 1, 2$ $i \neq j$ to $I_j = 0$ is

$$BR_i(I_j = 0) = \begin{cases} I_P & \text{if } \Pi_{iP}^{*L} - C_P - I_P \geq 0 \Leftrightarrow \tau \leq \tau_1^L(\lambda, C_P) \\ 0 & \text{if } \Pi_{iP}^{*L} - C_P - I_P < 0 \Leftrightarrow \tau > \tau_1^L(\lambda, C_P) \end{cases}$$

where $\tau_1^L(\lambda, C_P)$ is defined in Appendix A3. The best response of firm i to $I_j = I_P$ is

$$BR_i(I_j = I_P) = \begin{cases} I_P & \text{if } \Pi_{iP}^{*L} - C_P - 2I_P \geq 0 \Leftrightarrow \tau \leq \tau_2^L(\lambda, C_P) \\ 0 & \text{if } \Pi_{iP}^{*L} - C_P - 2I_P < 0 \Leftrightarrow \tau > \tau_2^L(\lambda, C_P) \end{cases}$$

where $\tau_2^L(\lambda, C_P)$ is defined in Appendix A3 with $\tau_1^L(\lambda, C_P) > \tau_2^L(\lambda, C_P)$. We show that $\tau_1^L(\lambda, C_P)$ and $\tau_2^L(\lambda, C_P)$ are both decreasing in C_P.

Thus, for $\tau \geq \tau_1^L(\lambda, C_P)$, the unique Nash equilibrium is $(I_1^*, I_2^*) = (0, 0)$, that is, none of the firms invests. For $\tau \in [\tau_2^L(\lambda, C_P), \tau_1^L(\lambda, C_P)[$, there exist two Nash equilibria $(I_1^*, I_2^*) = (0, I_P)$ and $(I_1^*, I_2^*) = (I_P, 0)$ such that one firm does not invest while the other invests I_P. For $\tau \leq \tau_2^L(\lambda, C_P)$, the unique Nash equilibrium is $(I_1^*, I_2^*) = (I_P, I_P)$ such that both firms invest I_P.

Figures 4a and 4b represent the different areas where firms invest in equilibrium in graphs where the horizontal axis represents the fraction of green-conscious consumers λ, and the vertical axis represents the emission tax τ. Figure 4a illustrates the different areas for low values of the patenting cost C_P, while Figure 4b represents the different areas for large values of the patenting cost C_P.

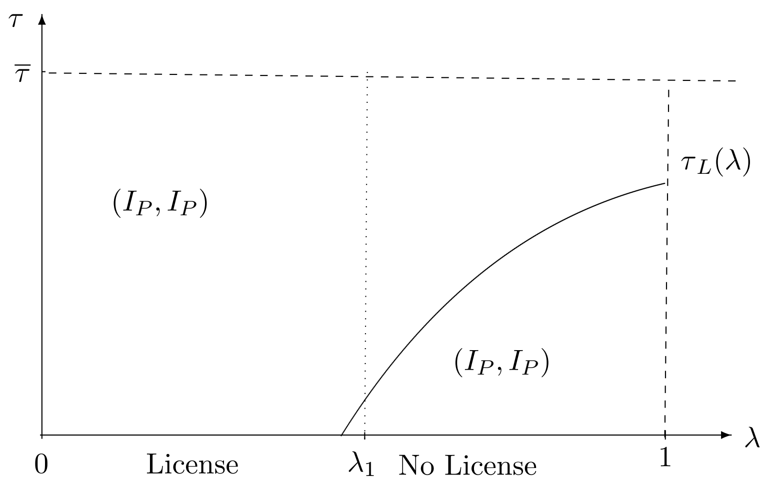

Figure 4a: Investments with green and non-green–conscious consumers for low values of C_P

In Figure 4a, as the patenting cost C_P is small, both firms always invest, no matter whether licensing occurs or not. For a given fraction of green-conscious consumers, let's say at λ_1 in Figure 4a, as the emission tax τ increases, both firms invest even though there is a shift of equilibrium regime from no licensing to licensing.

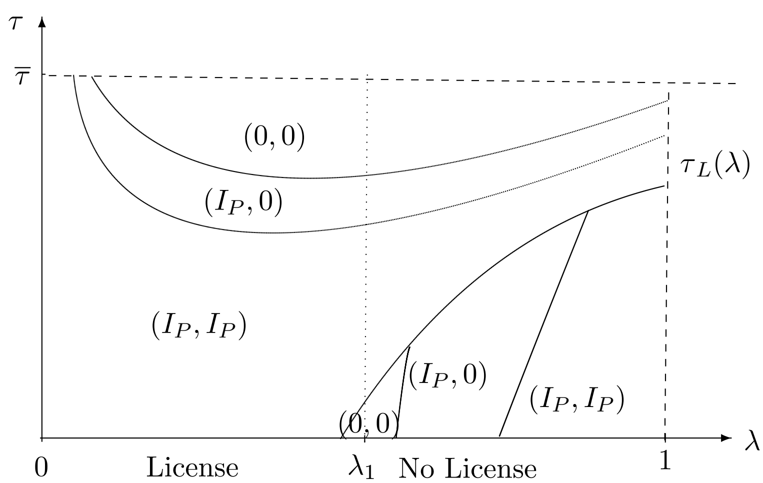

Figure 4b: Investments with green and non-green-conscious
for intermediate values of C_P

In Figure 4b, for larger values of C_P, we have all three possibilities (both firms invest, one of them invests, none of them invests) in both cases of licensing and not licensing. Here again, for a given value of λ at λ_1, as the emission tax τ increases, there is a shift from no licensing and no investment to licensing where both firms invest. As the emission tax τ increases further, only one firm invests and, eventually, none of them invests again.

To compare these findings with the non-green-conscious consumers only case, in Figure 5, we represent these different areas where firms invest in equilibrium in a graph where the horizontal axis represents the emission tax τ, and the vertical axis represents the patenting cost C_P when the fraction of green-conscious consumers is given at λ_1. In this representation, we take the fraction of consumers as given to understand how patenting cost and emission tax interact.

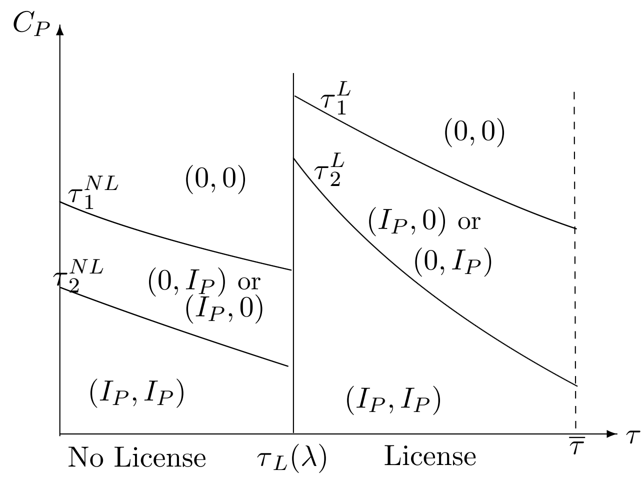

Figure 5: Equilibrium investments with green and non-green-conscious consumers

In Figure 5, for $\tau < \tau_L(\lambda)$, there is no licensing in equilibrium, whereas, for $\tau \geq \tau_L(\lambda)$, licensing occurs in equilibrium. For low values of C_P, both firms invest. As C_P increases, firms invest less. For intermediate values of C_P, as the emission tax τ increases, we go from no investment to investment as we switch from no licensing to licensing. For very high levels of C_P, as the emission tax τ increases, we go from no licensing and no investment to licensing and some investment to no investment.

The Subgame Perfect Nash Equilibria are presented in the following Propositions. They follow from Lemmas 3 - 4, and from the best response functions presented above.

Proposition 4 *For a given C_P, and for $\tau < \tau_L(\lambda)$, the Subgame Perfect Nash Equilibrium is such that there is* ***no license****, and* *(i) if* $\tau > \tau_1^{NL}(\lambda, C_P)$, *firms do not invest* $(I_1^*, I_2^*) = (0, 0)$, *they both produce the dirty product, and set prices at* $p^* = c + \tau\gamma_H$;

(ii) if $\tau \in [\tau_2^{NL}(\lambda, C_P), \tau_1^{NL}(\lambda, C_P)[$, *firm* i *invests while firm* j *does not* $(I_i^*, I_j^*) = (I_P, 0)$ *for* $i, j = 1, 2$ *and* $i \neq j$, *firm* i *produces the cleaner product at price* p_{iP}^{*G} *and firm* j *produces the dirty product at price* p_j^{*G} *where* $p_{iP}^{*G} > p_j^{*G}$;

(iii) if $\tau < \tau_2^{NL}(\lambda, C_P)$, *both firms invest* $(I_1^*, I_2^*) = (I_P, I_P)$, *but only one of them gets a patent and produces the cleaner product at price* p_{iP}^{*G} *while the other produces the dirty product at price* p_j^{*G} *where* $p_{iP}^{*G} > p_j^{*G}$.

Proposition 5 *For a given* C_P, *and for* $\tau \geq \tau_L(\lambda)$, *the Subgame Perfect Nash Equilibrium is such that* ***licensing*** *occurs, and*

(i) if $\tau > \tau_1^{L}(\lambda, C_P)$, *firms do not invest* $(I_1^*, I_2^*) = (0, 0)$, *they both produce the dirty product, and set prices at* $p^* = c + \tau\gamma_H$;

(ii) if $\tau \in [\tau_2^{L}(\lambda, C_P), \tau_1^{L}(\lambda, C_P)[$, *firm* i *invests while firm* j *does not* $(I_i^*, I_j^*) = (I_P, 0)$ *for* $i, j = 1, 2$ *and* $i \neq j$, *firm* i *licenses its technology to firm* j *and both firms produce the cleaner product at price* $p^{*L} = c + \tau\gamma_P + r^*$;

(iii) if $\tau < \tau_2^{L}(\lambda, C_P)$, *both firms invest* $(I_1^*, I_2^*) = (I_P, I_P)$, *the firm that gets a patent licenses it to the other firm, and both firms produce the cleaner product at price* $p^{*L} = c + \tau\gamma_P + r^*$.

To summarize, for low levels of the emission tax and the patenting cost, both firms invest. For a given level of the emission tax, as the patenting cost increases, only one firm has an incentive to invest, while the other does not. Eventually, as the patenting cost becomes too large, none of the firms invest.

If the patenting cost, C_P, is sufficiently low, both firms invest. At higher levels of C_P, the investment decisions depend on the emission tax. For intermediate values of the patenting cost, as the emission tax τ increases, investment at first increases due to a change from a no licensing regime to a licensing regime as one or both firms begin investing. At sufficiently high levels of the patenting cost, this initial increase in investment is followed by a decrease in investment as the emission tax increases further. This is because at such a high level of emission tax, the profits generated in the last stage in the product market are insufficient to outweigh the fixed cost of C_P. Thus, for high levels of the emission tax and the patenting cost, neither firm invests even within the licensing regime.

4.3 Equilibrium Emission Levels

In this section, we present the total emission level in equilibrium (see Appendix A3 for the details of the calculations). For clarity, we first present the scenario where C_P is sufficiently large, as depicted by Proposition 6 and Figure 6a, followed by the scenario where C_P is sufficiently small, as depicted by Proposition 7 and Figure 6b.

When C_P is sufficiently large, for low levels of the emission tax τ, $\tau < \tau_L(\lambda)$, none of the firms invest, and therefore the total emission is $2\gamma_H D(c+\tau\gamma_H, c+\tau\gamma_H; \gamma_H)$ where $D(c+\tau\gamma_H, c+\tau\gamma_H; \gamma_H)$ is defined by (8). If $\tau \geq \tau_L(\lambda)$, initially, either both firms invest or only one of them invests, which leads to a total emission level given by $2\gamma_P D^L$ where D^L is defined by (27). For even larger levels of τ, such that $\tau > \tau_1^L(\lambda, C_P)$, none of the firms invest, which leads to the emission level $2\gamma_H D(c+\tau\gamma_H, c+\tau\gamma_H; \gamma_H)$. We summarize these findings in the following Proposition.

Proposition 6 *For sufficiently large values of C_P, in equilibrium, the total emission level is given by:*

$$e_G^* = \begin{cases} e_H^* = \lambda(v - c - \gamma_H(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \text{if } \tau < \tau_L(\lambda) \\ e_L^* = \lambda\frac{1}{2}(v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \text{if } \tau_L(\lambda) \leq \tau < \tau_1^L(\lambda, C_P) \\ e_H^* & \tau \geq \tau_1^L(\lambda, C_P) \end{cases} \tag{30}$$

We denote by e_H^* the equilibrium emission when only the dirty product is produced, and e_L^* when the cleaner product is produced and there is a license. We represent the emission levels as a function of τ in Figure 6a for a high level of C_P and an intermediate value of λ.

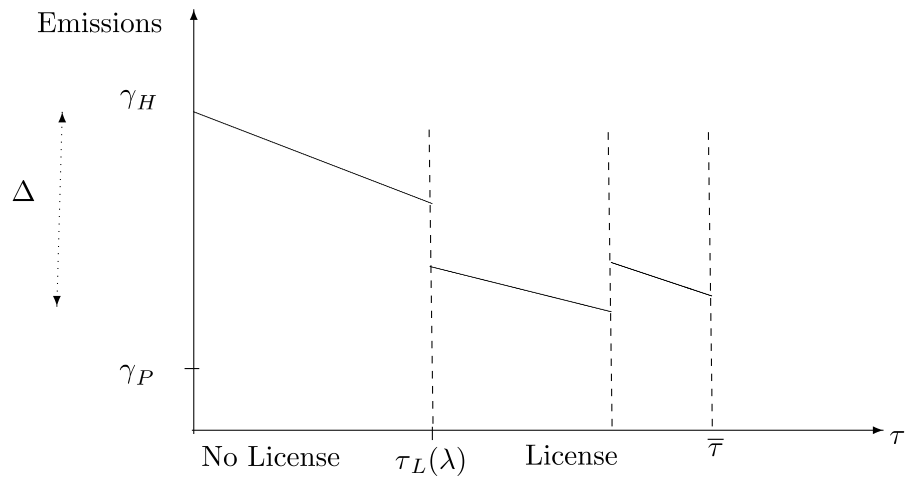

Figure 6a: Equilibrium emission with green and non-green-conscious consumers for large C_P

We note that, in contrast to the benchmark case without green-conscious consumers, in the case with green-conscious consumers, both e^*_H and e^*_L are decreasing functions of τ, as shown in Figure 6a-6b. This is because equilibrium prices are increasing in τ, causing demand to be decreasing in τ, unlike in the case without green-conscious consumers.

In Figure 6a, starting with a low value of τ, while the emission level decreases as the emission tax increases, the emission level is relatively high (as firms do not invest), as given by e^*_H. As τ increases, there is a discrete jump downward to e^*_L as firms invest in green innovation and licensing occurs. Thus, by endogenizing the investment and licensing decisions of firms, our model can capture a novel effect of increasing emission taxes. However, as the emission tax increases further, there is a jump upward, back to e^*_H, as firms do not invest anymore. There is a paradox: as the emission tax increases, the tax bill is too large for investment to be profitable, leading to a higher emission level. This implies that policy makers would be most effective at reducing emissions by choosing an intermediate level of emission tax which is large enough to induce licensing and, at the same time, small enough to avoid the paradoxical result.

Next we turn to the scenario where C_P is sufficiently low. In this case, both firms invest no matter whether licensing occurs. If $\tau < \tau_L(\lambda)$, there is no license in equilibrium, and if firm i produces the cleaner product and firm j the dirty product, the total emission is $\gamma_P D_i(p^{*G}_{iP}, p^{*G}_j; \gamma_P) + \gamma_H D_j(p^{*G}_{iP}, p^{*G}_j; \gamma_H)$ where $D_i(p^{*G}_{iP}, p^{*G}_j; \gamma_P)$ and $D_j(p^{*G}_{iP}, p^{*G}_j; \gamma_H)$ are defined by (19) and (20), respectively.

If $\tau \geq \tau_L(\lambda)$, there is licensing in equilibrium, and each firm produces the cleaner product and gets the demand D^L as defined by (27). The emission is thus $2\gamma_P D^L$. We summarize these findings in the following Proposition.

Proposition 7 *For sufficiently small values of C_P, in equilibrium, the total emission level is given by:*

$$e_G^* = \begin{cases} e_{NL}^* = \lambda \frac{\gamma_H}{4\gamma_H - \gamma_P}(3(v-c) - (2\gamma_H + \gamma_P)(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \text{if } \tau < \tau_L(\lambda) \\ e_L^* = \lambda \frac{1}{2}(v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \text{if } \tau \geq \tau_L(\lambda) \end{cases} \tag{31}$$

We denote by e_{NL}^* the equilibrium emission when there is a no license. We represent the emission levels as a function of τ in Figure 6b for a low level of C_P and an intermediate value of λ.

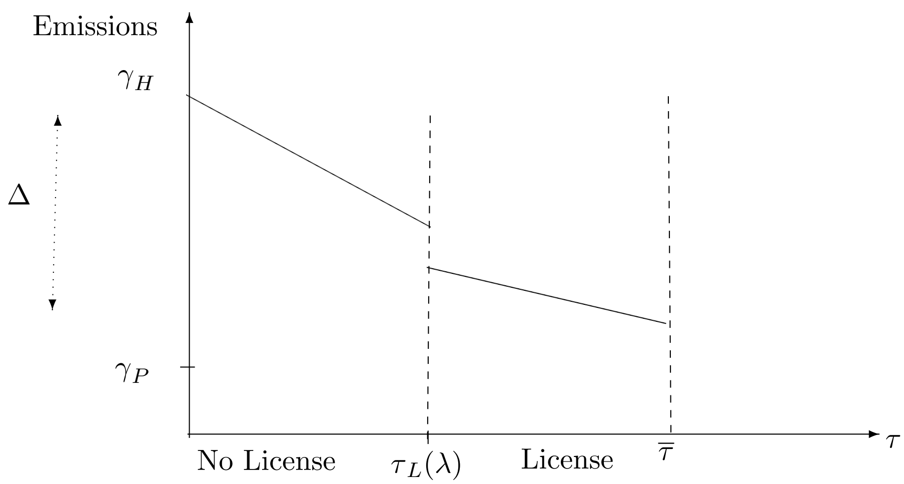

Figure 6b: Equilibrium emission with green and non-green-conscious consumers for low C_P

As the emission tax increases, the total emission is reduced, fulfilling the goal of having an emission tax. As we switch from the no licensing regime to the licensing regime, there is a discrete jump downward in the emission level. We note that, at sufficiently low level of C_P, the paradoxical result, whereby increasing emission tax increases the emission level, is circumvented.

For intermediate values of C_P, if $\tau < \tau_L(\lambda)$, only one firm invests. The total emission levels are identical to the previous case. Propositions (6) and (7) together imply that a decrease in the patenting cost restores firms' incentive to invest, thereby mitigating or avoiding the paradoxical effect of increasing emission taxes.

We note that the cutoff value $\tau_L(\lambda)$ increases with λ, which yields the following Corollary.

Corollary 3 *The presence of green-conscious consumers hinders the effectiveness of the emission tax rate in reducing the emission level.*

Corollary 3 arises because, as there are more green-conscious consumers, it becomes less likely that there will be licensing in equilibrium. On the other hand, as there are fewer green-conscious consumers, licensing becomes more likely. Thus, the discrete fall in the emission level, as illustrated in Figures 6a-6b, occurs at a higher emission tax in the presence of more green-conscious consumers. This explains Corollary 3, which is a direct implication of Propositions (6) and (7).

We also note that making the patentability requirement stricter, that is, lowering γ_P, decreases $\tau_L(\lambda)$, and increases $\tau_1^L(\lambda, C_P)$, which leads to lower emission levels for a greater range of τ. Reducing C_P does not affect $\tau_L(\lambda)$ but is shown to increase $\tau_1^L(\lambda, C_P)$ (See Appendix A3).

We summarize our results about the effects of a reduction of the patenting cost and a more stringent patentability requirement in the following Proposition.

Proposition 8 *With green and non-green-conscious consumers, i.e.,* $0 < \lambda \leq 1$*,*

(i) reducing patenting cost C_P *leads to an increase in* $\tau_1^L(\lambda, C_P)$*, thereby reducing the emission level for a larger range of the emission tax. As* C_P *is reduced even further, the emission level for low values of the emission tax starts decreasing as well;*

(ii) making the patentability requirement more stringent by reducing the level of γ_P *leads to a decrease in* $\tau_1(\lambda)$ *and an increase in* $\tau_1^L(\lambda, C_P)$*, thereby increasing the range of values of the emission tax for which the emission is lower. The emission level for low values of the emission tax starts decreasing as well for small values of* C_P*.*

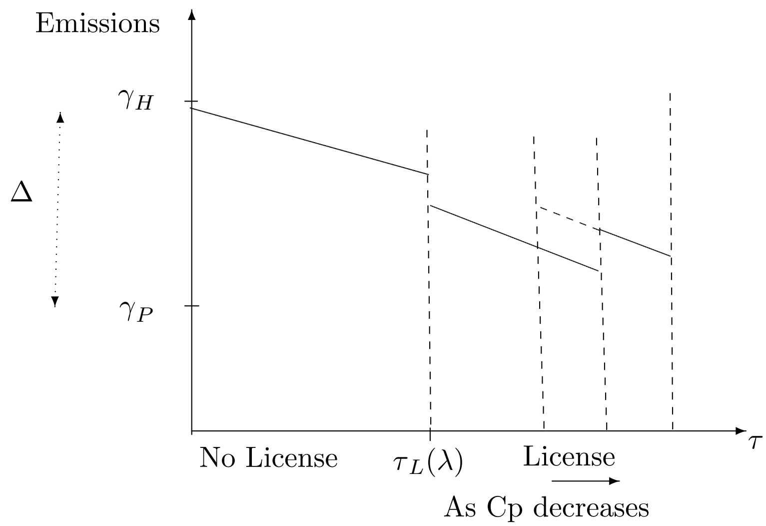

Figure 6c: Equilibrium emission for large C_P as C_P decreases

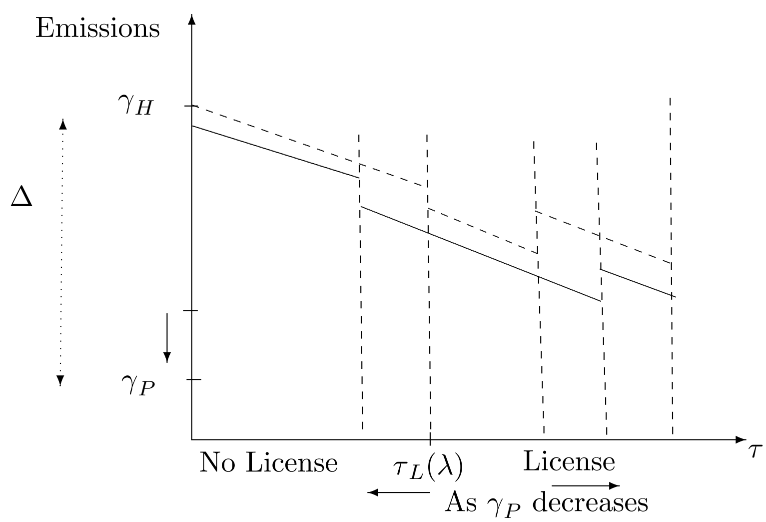

Figure 6d: Equilibrium emission for large C_P as γ_P decreases

From Proposition (8), it follows that, in the presence of green-conscious consumers, lowering γ_P is more effective in terms of reducing emissions. While reducing C_P only increases the right threshold of τ, $\tau_1^L(\lambda, C_P)$, lowering γ_P decreases the left threshold of τ, $\tau_L(\lambda)$, as well as increases $\tau_1^L(\lambda, C_P)$. Moreover, lowering γ_P decreases the emission level at any given τ unlike the case for reducing C_P, as shown by Figure 6c.

When C_P is reduced sufficiently, the paradoxical result may disappear completely. Any further reductions of C_P leave the emission level unaffected. However, reducing γ_P, even in the absence of the paradox, leads to further reductions in the emission level in line with Figure 6d.

Proposition (8) allows us to understand the impact of the different patent policies on the emission levels. Overall, a policy maker may use either instrument (reducing C_P or making the patentability requirement more stringent) or some combination of both to mitigate the paradoxical result associated with increasing the emission tax. Thus, both these instruments work in conjunction with the emission tax to allow the emission tax to function more smoothly.

5 Total Welfare

In this section, we consider the impact of the emission tax and patent policies on total welfare. We first consider the benchmark case with only non-green-conscious consumers ($\lambda = 0$), before studying the case with green and non-green- conscious consumers ($0 < \lambda \leq 1$).

5.1 Non-green-conscious Consumers

In the case without green-conscious consumers, in the absence of innovation, in equilibrium, both firms sell the dirty product γ_H at price $c + \tau\gamma_H$. The demand from consumers is 1, and the pollution damage to each consumer is $P(e) = \gamma_H$. Thus, the consumer surplus is $(v - c - \tau\gamma_H - \gamma_H)$, each firm gets a null payoff, the tax bill is $\tau\gamma_H$, and the total welfare is

$$W^H_{NG} = v - c - \gamma_H,$$

which does not depend on the tax τ.

If firm i innovates and licensing occurs (as it is the case in equilibrium), since the equilibrium price is v, all consumers choose to buy and get a null surplus $(v - v)$ from buying the product. In addition, each consumer faces pollution damage, $P(e) = \gamma_P$. Firm i gets $v - c - \tau\gamma_P - C_P - I_P$, firm j has a null payoff, and the tax bill is $\tau\gamma_P$. Thus, the total welfare is given by

$$W^P_{NG} = v - c - C_P - I_P - \gamma_P,$$

which is also independent of the tax τ.

From a social viewpoint, having only one firm investing always makes society better off, as it reduces duplication of investment. Thus, we consider the case in which only one firm invests when determining the social incentive to invest.

The difference in total welfare with and without innovation is thus $W_{NG}^P - W_{NG}^H = \Delta - C_P - I_P$. As long as the reduction in emission-output ratio, Δ, is larger than the patenting cost and cost of investment, as given by I_P, green innovation enhances total welfare. That is, it is socially efficient to invest in green innovation as long as $C_P \leq \Delta - I_P$. At the firm level, at least one firm invests if $C_P \leq v - c - \tau\gamma_P - I_P$. These conditions are simultaneously satisfied if

$$\tau = \tau_W \equiv \frac{(v - c - \Delta)}{\gamma_P}.$$

Figure 7 shows the two conditions, along with the threshold, τ_W.

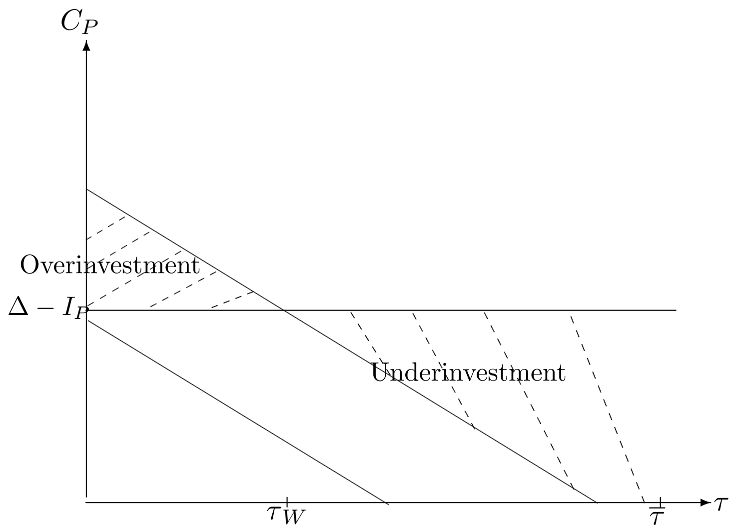

Figure 7: Over or Underinvestment with Non-Green Consumers

Figure 7 illustrates the overinvestment and underinvestment areas in a graph where the horizontal axis represents the emission tax and the vertical axis the patenting cost. In Figure 7, the region above $(\Delta - I_P)$ represents the combinations of C_P and τ for which it is not socially desirable to invest, while the region below $(\Delta - I_P)$ represents those for which it is socially desirable to invest. Also, the region above $(v - c - \tau\gamma_P - I_P)$ represents the combinations of C_P and τ for which it is not profitable to invest, while the region below $(v - c - \tau\gamma_P - I_P)$ represents those for which it is profitable to invest. Therefore, when $(\Delta - I_P) < C_P < (v - c - \tau\gamma_P - I_P)$, there is overinvestment relative to the socially optimal level since, in this region, firms find it profitable to invest even though it is not socially desirable to do so. As shown by Figure 7, this region of overinvestment occurs for $\tau < \tau_W$. When $(v - c - \tau\gamma_P - I_P) < C_P < (\Delta - I_P)$, there is underinvestment relative to the socially optimal level since, in this region, investment is unprofitable even though it is socially desirable. As shown by Figure 7, this region of underinvestment occurs for $\tau > \tau_W$.

We summarize these results in the following Proposition.

Proposition 9 *With only non-green-conscious consumers, i.e.,* $\lambda = 0$,

(i) *if* $\tau \in [0, \tau_W)$, *there is overinvestment for* $C_P \in [\Delta - I_P, v - c - \tau\gamma_P - I_P]$,

(ii) *if* $\tau \in (\tau_W, \bar{\tau}]$, *there is underinvestment for* $C_P \in [v - c - \tau\gamma_P - I_P, \Delta - I_P]$.

For Proposition 9(i), we note that $\tau_W > 0$ if $\Delta < (v - c)$. This implies that overinvestment only occurs when the innovative step, Δ, is not that large. For Proposition 9(ii), recall that we must have $\tau < \bar{\tau}$ in order to satisfy assumption (9). It can be shown that $\tau_W < \bar{\tau}$ if $\gamma_P \underline{G}/\Delta + (v - c)/\gamma_H < 1$. If this latter condition is not satisfied, there is always overinvestment as $\tau < \tau_W$ is satisfied for all $\tau < \bar{\tau}$.

Typically, higher emission taxes are expected to induce greater investment in green innovations. However, in this model, increasing the tax τ beyond τ_W leads to underinvestment since investment becomes unprofitable due to the increasing tax bill. We also note that for larger values of C_P, it is most likely that there is overinvestment as it is too costly from a social viewpoint for patenting to occur. This explains why reducing the patenting cost below $(\Delta - I_P)$ ensures that overinvestment does not occur. Finally, we note that the threshold, τ_W, depends on γ_P as follows:

$$\frac{\partial \tau_W}{\partial \gamma_P} = \frac{\gamma_H - (v - c)}{\gamma_P^2}.$$

Thus, $\partial\tau_W/\partial\gamma_P > 0$ if $\gamma_H > (v - c)$, and $\partial\tau_W/\partial\gamma_P < 0$ if $\gamma_H < (v - c)$. That is, in cases where the inventive step is large (as reflected by a higher γ_H), making the patentability requirement stricter by decreasing γ_P decreases τ_W, causing the region of overinvestment to shrink and that of underinvestment to increase. In cases where the inventive step is small (as reflected by a lower γ_H), making the patentability requirement stricter by decreasing γ_P increases τ_W, causing the region of overinvestment to increase and that of underinvestment to shrink.

5.2 Green and Non-green-conscious Consumers

We now consider the case with green and non-green-conscious consumers ($0 < \lambda \leq 1$). In the absence of innovation, in equilibrium, both firms sell the dirty product γ_H at price $c + \tau\gamma_H$. The demand is $\lambda(\widetilde{G}_3 - \underline{G}) + (1-\lambda)$, and the pollution damage to each consumer is $P(e) = \gamma_H(\lambda(\widetilde{G}_3 - \underline{G}) + (1-\lambda))$ where $\widetilde{G}_3$ is defined by (7). The consumer surplus (see Appendix A4 for calculations) is

$$CS_H = \frac{\lambda}{\gamma_H}\frac{(v-c-\tau\gamma_H)^2}{2} - \lambda(v-c-\tau\gamma_H)(\underline{G} - \frac{1}{\lambda}) + \lambda\frac{\gamma_H}{2}\underline{G}^2 + \lambda\gamma_H\underline{G} - \gamma_H(1-\lambda).$$

Each firm gets a null payoff, the tax bill is τe_H^*, and the total welfare is thus $W_G^H = CS_H + \tau e_H^*$, where e_H^* is defined by (30). As the tax τ increases, so does the total welfare. In fact, as the tax increases, the price increases, consumption decreases, and pollution decreases as well.

If firm i innovates and **licensing** occurs (as it is the case in equilibrium for $\tau \geq \tau_L(\lambda)$), firm i gets Π_{iP}^{*L} as defined by (28), firm j gets 0, and the pollution damage to each consumer is $e_L^* = \gamma_P 2D^L$ with D^L defined by (27). The consumers surplus is

$$CS_P^L = \frac{\lambda}{2\gamma_P}(v-p^L)^2 - \lambda(v-p^L)(\underline{G} - \frac{1-\lambda}{\lambda}) - \lambda\frac{\gamma_P}{2}\underline{G}^2 - e_L^*.$$

The total welfare is thus $W_G^L = CS_P^L + \tau e_L^* + \Pi_{iP}^{*L} - C_P - I_P$. It is socially efficient to invest in green innovation if $W_G^L - W_G^H > 0$ or, equivalently, if $\Phi_L(\tau) + \Pi_{iP}^{*L} - (C_P + I_P) > 0$, where $\Phi_L(\tau) = CS_P^L - CS_H - \tau(e_H^* - e_L^*)$. To evaluate whether there is underinvestment or overinvestment, we compare the social incentive to invest, $\Phi_L(\tau) + \Pi_{iP}^{*L} - (C_P + I_P) > 0$, with the private incentive, $\Pi_{iP}^{*L} - (C_P + I_P) > 0$. Thus, it is socially efficient to invest if $C_P < \Phi_L(\tau) + \Pi_{iP}^{*L} - I_P \equiv \tau_S^L$, while at the firm level, at least one firm will invest if $C_P < \Pi_{iP}^{*L} - I_P \equiv \tau_1^L$. Therefore, if $\Phi_L(\tau) > 0$, for some values of C_P, there might be underinvestment, while if $\Phi_L(\tau) < 0$, there might be overinvestment as τ_S^L can be above or below τ_1^L. As represented on the right part in Figure 8, for large enough λ, we have that for large values of τ, there is overinvestment, while for smaller values of τ there is underinvestment.

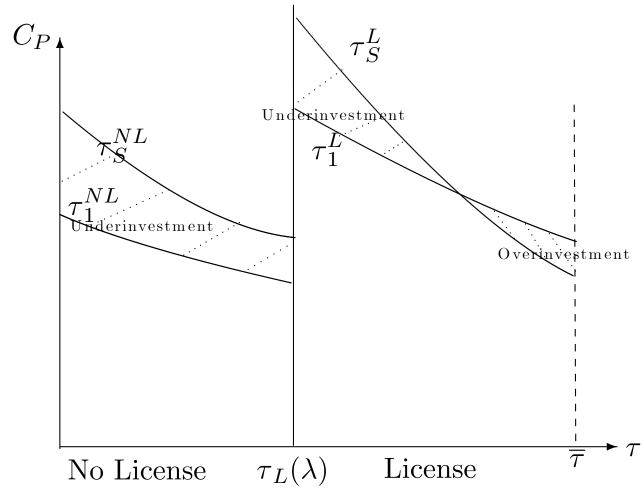

Figure 8: Over or underinvestment with green and non-green-conscious consumers

If firm i innovates and there is **no licensing** (as it is the case in equilibrium for $\tau < \tau_L(\lambda)$), firm i gets Π_{iP}^{*G} as defined by (21) and firm j gets Π_j^{*G} as defined by (22) when the market is not covered, and the consumers surplus is CS_P^{NL} (see Appendix A4). The total welfare is thus $W_G^{NL} = CS_P^{NL} + \Pi_{iP}^{*G} - C_P - I_P + \Pi_j^{*G} + \tau e_{NL}^*$ where e_{NL}^* is the pollution level without licensing as defined by (31). It is socially efficient to invest in green innovation if $W_G^{NL} - W_G^H > 0$ or, equivalently, if $\Phi_{NL}(\tau) + \Pi_{iP}^{*G} + \Pi_j^{*G} - (C_P + I_P) > 0$, where $\Phi_{NL}(\tau) = CS_P^{NL} - CS_H - \tau(e_H^* - e_{NL}^*)$. Thus, it is socially efficient to invest if $C_P < \Phi_{NL}(\tau) + \Pi_{iP}^{*G} + \Pi_j^{*G} - I_P \equiv \tau_S^{NL}$, while at the firm level, one firm will invest if $C_P < \Pi_{iP}^{*G} - I_P \equiv \tau_1^{NL}$. We show that for large enough λ, $\Phi_{NL}(\tau) + \Pi_j^{*G} > 0$, in which case $\tau_S^{NL} > \tau_1^{NL}$. Thus, for intermediate values of C_P, $C_P \in [\Pi_{iP}^{*G} - I_P, \Phi_{NL}(\tau) + \Pi_{iP}^{*G} + \Pi_j^{*G} - I_P]$, there is underinvestment as represented on the left side in Figure 8.

Comparing Figures 7 and 8, we find that the effect of increasing emission tax on investment is quite different with green-conscious consumers than in their absence. While tax increases lead to a switch from overinvestment to underinvestment in the absence of green-conscious consumers, they have the reverse effect in the presence of green-conscious consumers. This is driven by the switch from no-licensing to the licensing regime as emission tax increases in the presence of green-conscious consumers, whereas in their absence licensing occurs at all tax levels. Recall that with green-conscious consumers, for $\tau < \tau_L(\lambda)$, there is no licensing. Since at least one of the firms produces the dirty good, pollution damage is greater such that welfare could be increased by investing more and having more firms produce the cleaner good. Therefore, we have underinvestment for low tax rates. At tax rates sufficiently higher than $\tau_L(\lambda)$, on the other hand, there is licensing such that both firms are producing the cleaner good. While licensing reduces pollution, the product price rises to v, the monopoly price, thereby reducing consumer surplus. For sufficiently high tax rates, the latter effect outweighs the reduction in pollution damage such that we have overinvestment relative to the socially desirable level. We summarize these findings in the following Proposition.

Proposition 10 *With a sufficient large fraction of green-conscious consumers, $\lambda >> 0$,*

(i) if $\tau < \tau_L(\lambda)$ (no licensing), there is underinvestment for $C_P \in [\Pi_{iP}^{*G} - I_P, \Phi_{NL}(\tau) + \Pi_{iP}^{*G} + \Pi_j^{*G} - I_P]$,

(ii) if $\tau > \tau_L(\lambda)$ (licensing), there is underinvestment for lower values of τ for $C_P \in [\Pi_{iP}^{*L} - I_P, \Phi_L(\tau) + \Pi_{iP}^{*L} - I_P]$, and then overinvestment for larger values of τ for $C_P \in [\Phi_L(\tau) + \Pi_{iP}^{*L} - I_P, \Pi_{iP}^{*L} - I_P]$.

6 Conclusion

In this paper, we analyze how patent policy instruments work in conjunction with emission taxes to impact firms' investment and licensing decisions and emission levels. Our framework allows for strategic behaviour of firms within a duopolistic setting and for heterogeneity across consumers in terms of the degree to which they care about the environment. These key factors play a significant role in determining policy implications.

Given a product's market, the production of which causes pollution, we model the implementation of a cleaner technology as a reduction of the emission per unit of output ratio. We assume that the product is vertically differentiated in terms of its emission-output ratio, with green-conscious consumers preferring (to varying degrees) products with the lower emission-output ratio. Within this setting, we endogenize firms' investment, patenting, and licensing decisions for green innovations. After undertaking these decisions, firms engage in price competition.

A key finding of the paper is that the greater the proportion of green-conscious consumers, the less likely firms are to engage in licensing the green innovation. While in the absence of green-conscious consumers, there is always licensing and implementation of the green innovation by all firms, for high proportions of green-conscious consumers firms would rather differentiate their products by using production technologies with different emission-output ratios. For this reason, higher proportions of green-conscious consumers is associated with higher levels of emissions such that policy makers may consider implementing technology standards to force licensing when this proportion is sufficiently high. An alternative means of inducing licensing in equilibrium is to increase the emission tax beyond a certain threshold. This has the desirable effect of causing emissions to fall discretely as we move from an equilibrium without licensing to one with licensing.

We find that there exists a second threshold level of the tax beyond which increasing the emission tax leads to increasing the emission level. This paradox can be mitigated by decreasing patenting costs. Making the patentability requirement stricter also achieves this in the presence of green consumers, with the added benefit of yielding a lower level of emissions as long as the tax is below the threshold, compared to reducing patenting costs. However, we also show that in the absence of green consumers the impact of tightening the patentability requirement is ambiguous.

Regardless of the presence or absence of green consumers, we generate a paradoxical result whereby increasing the emission tax beyond a certain threshold leads to a higher emission level. This paradox may be mitigated by reducing patenting costs and, under specific conditions, by tightening the patentability requirements.

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Appendix

A1. Demand functions

From the utility functions (3) and (4), we derive the demand function for each firm. If $\gamma_1 = \gamma_2 = \gamma$, and if $\underline{G} < \widetilde{G}_3 < \overline{G}$, the demand function for firm i, for $i, j = 1, 2$ and $i \neq j$ is

$$D_i(p_i, p_j; \gamma) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2}\lambda(\frac{v-p_i}{\gamma} - \underline{G}) + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda(\frac{v-p_i}{\gamma} - \underline{G}) + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases}$$

If $\underline{G} < \overline{G} < \widetilde{G}_3$, all the green-conscious consumers buy the product and, thus, the demand for firm i, for $i, j = 1, 2$ and $i \neq j$ is

$$D_i(p_i, p_j; \gamma) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2}\lambda + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases}$$

Lastly, if $\widetilde{G}_3 < \underline{G} < \overline{G}$, there is no demand from the green-conscious consumers, and the demand for each firm i, for $i, j = 1, 2$ and $i \neq j$ is

$$D_i(p_i, p_j; \gamma) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases}$$

If $\gamma_i = \gamma_P < \gamma_j = \gamma_H$, such that firm i has the cleaner product, and if $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$ the demand functions are

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda(\frac{v-p_i}{\gamma_P} - \frac{p_i-p_j}{\Delta}) & \text{if } p_i > p_j \\ \lambda(\frac{v-p_i}{\gamma_P} - \underline{G}) + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda(\frac{v-p_i}{\gamma_P} - \underline{G}) + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} \lambda(\frac{p_i-p_j}{\Delta} - \underline{G}) + (1-\lambda) & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases}$$

If $\underline{G} < \widetilde{G}_1 < \overline{G} < \widetilde{G}_2$, the demands are

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda(\overline{G} - \frac{p_i - p_j}{\Delta}) & \text{if } p_i > p_j \\ \lambda + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} \lambda(\frac{p_i - p_j}{\Delta} - \underline{G}) + (1-\lambda) & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases}$$

If $\widetilde{G}_1 < \underline{G} < \widetilde{G}_2 < \overline{G}$, the demands are

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda(\frac{v - p_i}{\gamma_P} - \underline{G}) & \text{if } p_i > p_j \\ \lambda(\frac{v - p_i}{\gamma_P} - \underline{G}) + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda(\frac{v - p_i}{\gamma_P} - \underline{G}) + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} (1-\lambda) & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases}$$

If $\widetilde{G}_1 < \underline{G} < \overline{G} < \widetilde{G}_2$, the demands are

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda & \text{if } p_i > p_j \\ \lambda + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} (1-\lambda) & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases}$$

These demand functions are summarized in (5), (6), and (8).

A2. Non-green-conscious consumers

License case: Proof of the Nash equilibrium

Consider the case where firm j does not invest and firm i invests I_P, applies for a patent and offers a license to firm j. If firm j accepts it, both firms produce the cleaner good γ_P. Firm i offers a license (r, F) where r is the per-unit royalty rate, and F is a fixed fee. For any license (r, F) accepted by firm j, firm i obtains

$$D_i(p_{iP}, p_j)(p_{iP} - c - \tau\gamma_P) + rD_j(p_{iP}, p_j) + F,$$

and firm j obtains

$$D_j(p_{iP}, p_j)(p_j - c - \tau\gamma_P - r) - F,$$

where

$$D_i(p_{iP}, p_j) = \begin{cases} 1 & \text{if} \quad p_{iP} < p_j \text{ and } p_{iP} \leq v \\ \frac{1}{2} & \text{if} \quad p_{iP} = p_j \leq v \\ 0 & \text{if} \quad p_{iP} > \min\{p_j, v\} \end{cases}$$

and

$$D_j(p_{iP}, p_j) = \begin{cases} 1 & \text{if} \quad p_j < p_{iP} \text{ and } p_j \leq v \\ \frac{1}{2} & \text{if} \quad p_j = p_{iP} \leq v \\ 0 & \text{if} \quad p_j > \min\{p_{iP}, v\} \end{cases}$$

If $p_{iP} > p_j = v$, firm i does not get any demand and thus obtains a payoff $r + F$, while firm j gets $(v - c - \tau\gamma_P - r) - F$, which cannot be an equilibrium as firm i will reduce its price p_{iP}. If $p_{iP} = p_j = v$, firm i obtains $(v - c - \tau\gamma_P + r)/2 + F$ and firm j gets $(v - c - \tau\gamma_P - r)/2 - F$, which is not an equilibrium either, as each firm could reduce its price and capture the entire demand. Indeed, if firm i reduces its price to $p_{iP} < p_j$, it will obtain $(p_{iP} - c - \tau\gamma_P + r) + F$. Firm j will also reduce its price. Therefore, the unique equilibrium is for each firm to choose $p^* = p^*_{iP} = p^*_j = c + \tau\gamma_P + r$, so that both firms produce the cleaner good with firm i getting $r + F$ and firm j gets $-F$.

Firm i chooses (r, F) that maximize $r + F$ such that $-F \geq 0$ and $p^* = c + \tau\gamma_P + r \leq v$. Therefore, the equilibrium royalty is $r^* = v - c - \tau\gamma_P$ and $F^* = 0$, so that in equilibrium firm i obtains $v - c - \tau\gamma_P$ while firm j gets 0 even if both firms produce the cleaner good.

Emission levels

We consider the impact of a change in γ_P on the emission levels as represented in Figure 2c. Consider the threshold (13) which depends on γ_P

$$\tau_1(\gamma_P) = \frac{v - c - C_P - I_P(\gamma_P)}{\gamma_P}.$$

Its derivative with respect to γ_P is negative if

$$\frac{\partial \tau_1(\gamma_P)}{\gamma_P} = \frac{-I'_P(\gamma_P)}{\gamma_P} - \frac{(v - c - C_P - I_P(\gamma_P))}{(\gamma_P)^2} < 0,$$

or, equivalently, if $\tau_1(\gamma_P) > -I'_P(\gamma_P)$. Thus. for any $\gamma_P \in [0, \gamma_H]$, as long as $\tau_1(\gamma_P) > -I'_P(\gamma_P)$, a decrease in γ_P will induce an increase in $\tau_1(\gamma_P)$.

A3. Green and non-green-conscious consumers

CASE II(a): case without license

The best response of firm i to any price p_j set by firm j is

$$p_{iP}(p_j) = \frac{1}{2}(c + \tau\gamma_P + v\frac{\Delta}{\gamma_H} + \frac{\gamma_P}{\gamma_H}p_j),$$

and the best response function of firm j to any price p_{iP} set by firm i is

$$p_j(p_{iP}) = \frac{1}{2}(c + \tau\gamma_H - \Delta\underline{G} + \tfrac{1-\lambda}{\lambda}\Delta + p_{iP}).$$

Demanded quantities must be positive at the equilibrium prices. Therefore, if $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$, the equilibrium prices are (p_{iP}^{*G}, p_j^{*G}) as defined by (14) and (15) where

$$p_{iP}^{*G} = \frac{1}{4\gamma_H - \gamma_P}[c(2\gamma_H + \gamma_P) + 2v\Delta + 3\tau\gamma_H\gamma_P - \gamma_P\Delta\underline{G} + \gamma_P\Delta\frac{1-\lambda}{\lambda}],$$

and

$$p_j^{*G} = \frac{1}{4\gamma_H - \gamma_P}[3\gamma_H c + v\Delta + \gamma_H\tau(2\gamma_H + \gamma_P) - 2\Delta\gamma_H\underline{G} + 2\Delta\gamma_H\frac{1-\lambda}{\lambda}].$$

At these equilibrium prices, the demand for firm i is

$$D_i(p_{iP}^{*G}, p_j^{*G}; \gamma_P) = \lambda(\widetilde{G}_2 - \widetilde{G}_1) = \frac{\lambda}{4\gamma_H - \gamma_P}\frac{\gamma_H}{\gamma_P}(2(v-c) - \gamma_P\tau - \gamma_P\underline{G} + \gamma_P\frac{1-\lambda}{\lambda}),$$

and the demand for firm j is

$$D_j(p_{iP}^{*G}, p_j^{*G}; \gamma_H) = \lambda(\widetilde{G}_1 - \underline{G}) + (1-\lambda) = \frac{\lambda}{4\gamma_H - \gamma_P}(v - c - 2\tau\gamma_H - 2\underline{G}\gamma_H + 2\gamma_H\frac{1-\lambda}{\lambda}),$$

where $\widetilde{G}_1 = (p_{iP}^{*G} - p_j^{*G})/\Delta$, and $\widetilde{G}_2 = (v - p_{iP}^{*G})/\gamma_P$. We show that $D_i(p_{iP}^{*G}, p_j^{*G}; \gamma_P) \geq 0$ if $\tau < 2(v-c)/\gamma_P - \underline{G} + (1-\lambda)/\lambda$, which is always satisfied as

$$\frac{(v-c)}{2\gamma_H} - \underline{G} < 2\frac{(v-c)}{\gamma_P} - \underline{G} + \frac{1-\lambda}{\lambda}.$$

We show that $D_j(p_{iP}^{*G}, p_j^{*G}; \gamma_H) \geq 0$ if $\tau < (v-c)/2\gamma_H - \underline{G} + (1-\lambda)/\lambda$, which is always satisfied, as

$$\frac{(v-c)}{2\gamma_H} - \underline{G} \leq \frac{v-c}{2\gamma_H} - \underline{G} + \frac{1-\lambda}{\lambda}.$$

Furthermore, we show that $p_{iP}^{*G} > p_j^{*G}$ if $\tau < \tau_E(\lambda)$ where

$$\tau_E(\lambda) \equiv \frac{v-c}{2\gamma_H} + \frac{2\gamma_H - \gamma_P}{2\gamma_H}(\underline{G} - \frac{1-\lambda}{\lambda}).$$

The cutoff $\tau_E(\lambda)$ is increasing and concave as $\partial\tau_E(\lambda)/\partial\lambda = (2\gamma_H - \gamma_P)/2\lambda^2\gamma_H > 0$, and $\partial^2\tau_E(\lambda)/\partial\lambda^2 = -(2\gamma_H - \gamma_P)/\lambda^3\gamma_H < 0$.

Lastly, we need to verify that at prices p_{iP}^{*G} and p_j^{*G}, we have $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$. We check that $\underline{G} < \widetilde{G}_1$ if $\tau < \tau_L(\lambda)$ with

$$\tau_L(\lambda) \equiv \frac{v-c}{2\gamma_H} - \underline{G} - \frac{2\gamma_H - \gamma_P}{2\gamma_H}\frac{1-\lambda}{\lambda} < \tau_E(\lambda),$$

and $\widetilde{G}_2 < \overline{G}$ if $\tau > \tau_C(\lambda)$ where

$$\tau_C(\lambda) \equiv \frac{2\gamma_H + \gamma_P}{\gamma_H\gamma_P}\frac{v-c}{3} - \underline{G} - \frac{\gamma_H - \gamma_P}{3\gamma_H}\frac{1-\lambda}{\lambda} - \frac{4\gamma_H - \gamma_P}{3\gamma_H}.$$

We define λ_L the value of λ such that $\tau_L(\lambda) = 0$ as

$$\lambda_L = \frac{2\gamma_H - \gamma_P}{v - c - 2\gamma_H\underline{G} + 2\gamma_H - \gamma_P} < 1,$$

as $(v-c)/2\gamma_H - \underline{G} > 0$. We show that $\tau_L(\lambda)$ and $\tau_C(\lambda)$ are increasing and concave as

$$\frac{\partial\tau_L(\lambda)}{\partial\lambda} = \frac{2\gamma_H - \gamma_P}{2\gamma_H}\frac{1}{\lambda^2} > 0, \text{ and } \frac{\partial\tau_C(\lambda)}{\partial\lambda} = \frac{\gamma_H - \gamma_P}{3\gamma_H}\frac{1}{\lambda^2} > 0,$$

and

$$\frac{\partial^2\tau_E(\lambda)}{\partial\lambda^2} = -\frac{2\gamma_H - \gamma_P}{2\gamma_H}\frac{1}{\lambda^3} < 0, \text{ and } \frac{\partial^2\tau_E(\lambda)}{\partial\lambda^2} = -\frac{\gamma_H - \gamma_P}{3\gamma_H}\frac{1}{\lambda^3} < 0.$$

We further show that $\tau_L(\lambda) < \tau_E(\lambda)$ as $\underline{G}(4\gamma_H - \gamma_P)/2\gamma_H > 0$. Depending on the parameter values, we can have that $\tau_L(\lambda) > \tau_C(\lambda)$ or $\tau_L(\lambda) < \tau_C(\lambda)$. However, as long as there are some values of λ for which $\tau_L(\lambda) > \tau_C(\lambda)$, our findings hold.

We also show that $\partial\tau_L(\lambda)/\partial\gamma_P > 0$.

We calculate that

$$\frac{\partial p_{iP}^{*G}}{\partial\lambda} = -\frac{\gamma_P\Delta}{4\gamma_H - \gamma_P}\frac{1}{\lambda^2} < 0,$$

$$\frac{\partial p_{j}^{*G}}{\partial\lambda} = -\frac{2\gamma_H\Delta}{4\gamma_H - \gamma_P}\frac{1}{\lambda^2} < 0,$$

and finally

$$\frac{\partial(p_{iP}^{*G} - p_j^{*G})}{\partial\lambda} = \frac{(2\gamma_H - \gamma_P)\Delta}{4\gamma_H - \gamma_P}\frac{1}{\lambda^2} > 0.$$

If $\tau < \min\{\tau_C(\lambda), \tau_L(\lambda)\}$ ($\underline{G} < \widetilde{G}_1 < \overline{G} < \widetilde{G}_2$) all green-conscious consumers buy one of the (dirty or cleaner) goods, and both firms are producing different products, and thus the green-conscious consumers market is covered. Firm i chooses p_{iP} that solves

$$\underset{p_{iP}}{Max}\lambda(\overline{G} - \frac{p_{iP} - p_j}{\Delta})(p_{iP} - c - \gamma_P\tau),$$

and firm j chooses p_j that solves

$$\underset{p_j}{Max}(\lambda(\frac{p_{iP} - p_j}{\Delta} - \underline{G}) + (1-\lambda))(p_j - c - \gamma_H\tau).$$

The first order conditions give

$$\begin{aligned} p_{iP}(p_j) &= \frac{1}{2}(c + \gamma_P\tau + \Delta\overline{G} + p_j), \\ p_j(p_{iP}) &= \frac{1}{2}(c + \gamma_H\tau + p_{iP} - \Delta\underline{G} + \Delta\frac{(1-\lambda)}{\lambda}). \end{aligned}$$

Solving for the equilibrium prices give

$$p_{iP}^{*GC} = c + \frac{1}{3}[\tau(2\gamma_P + \gamma_H) + \Delta\underline{G} + 2\Delta + \Delta\frac{1-\lambda}{\lambda}],$$

and

$$p_{j}^{*GC} = c + \frac{1}{3}[\tau(\gamma_P + 2\gamma_H) - \Delta\underline{G} + \Delta + 2\Delta\frac{1-\lambda}{\lambda}],$$

where, $p_{iP}^{*GC} > p_j^{*GC}$ if $\tau < 2 + \underline{G} - \frac{1}{\lambda}$. Evaluated at these prices, the demands for both firms are

$$D_i(p_i^{*GC}, p_j^{*GC}; \gamma_P) = \lambda\frac{1}{3}[\underline{G} + \tau + 2 + \frac{1-\lambda}{\lambda}],$$

and

$$D_j(p_{iP}^{*GC}, p_j^{*GC}; \gamma_H) = \frac{1}{3}\lambda(1 - \tau - \underline{G} + 2\frac{1-\lambda}{\lambda}).$$

We show that $D_j(.) > 0$ if $\tau < -\underline{G} - 1 + 2\frac{1}{\lambda}$. The equilibrium payoffs are

$$\Pi_{iP}^{*GC} = \lambda\frac{\Delta}{9}[\tau + \underline{G} + 2 + \frac{1-\lambda}{\lambda}]^2,$$

and

$$\Pi_{j}^{*GC} = \lambda\frac{\Delta}{9}[1 - 2\tau - \underline{G} + 2\frac{1-\lambda}{\lambda}]^2.$$

CASE II(b): Case with license

In the licensing case, both firms produce the cleaner good and choose the equilibrium price $p^{*L} = c + \tau\gamma_P + r$. Thus, firm i will choose (r, F) solution of

$$\underset{r,F}{Max}\frac{1}{\gamma_P}\lambda[v - c - r - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})]r + F,$$

such that $-F \geq 0$ and $p^{*L} \leq v - \gamma_P\underline{G}$ to ensure that both green and non-green-conscious consumers buy. Therefore, firm i chooses $F^* = 0$ and

$$r^* = \frac{1}{2}[v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})],$$

as long as $\tau \leq (v-c)\gamma_P - \underline{G} - (1-\lambda)/\lambda$. Evaluated at r^*, each firm faces the same demand

$$D^L = \frac{1}{4}\frac{\lambda}{\gamma_P}(v - c - \tau\gamma_P - \gamma_P\underline{G} + \gamma_P\frac{1-\lambda}{\lambda}),$$

and the equilibrium price is thus

$$p^{*L} = \frac{1}{2}(v + c + \gamma_P(\tau - \underline{G} + \frac{1-\lambda}{\lambda})). \tag{32}$$

The equilibrium profit of firm i is

$$\Pi_{iP}^{*L} = \lambda\frac{1}{4\gamma_P}[v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})]^2,$$

and firm j gets 0.

However, if $\tau > (v-c)\gamma_P - \underline{G} - (1-\lambda)/\lambda$, only non-green-conscious consumers buy at price $p^{*L} = c + \tau\gamma_P + r$ with the royalty rate $r^{**} = v - c - \tau\gamma_P$ as the condition $c + \tau\gamma_P + r \leq v$ must be satisfied. Evaluated at r^{**}, each firm faces the same demand $D = (1-\lambda)/2$, and thus firm i's equilibrium payoff is

$$\Pi_{iP}^{**L} = (1-\lambda)(v - c - \tau\gamma_P).$$

Firm i will choose r^* rather than r^{**} as long as $\Pi_{iP}^{*L} > \Pi_{iP}^{**L}$. A necessary condition to have $\Pi_{iP}^{*L} > \Pi_{iP}^{**L}$ is that

$$\tau < \tau_{LNG}(\lambda) = \frac{v-c}{\gamma_P} - 2(\underline{G} + \frac{1-\lambda}{\lambda}) < \frac{v-c}{\gamma_P} - \underline{G} - \frac{1-\lambda}{\lambda}.$$

Thus, as long as $\tau \leq (v-c)\gamma_P - 2(\underline{G} + (1-\lambda)/\lambda)$, firm i will set the royalty rate at r^* and serve both green and non-green-conscious consumers. However, for $\tau > (v-c)\gamma_P - 2(\underline{G} + (1-\lambda)/\lambda)$, firm i will serve only non-green-conscious consumers.

Investment decisions

We calculate the different thresholds $\tau_1^{NL}(\lambda, C_P)$, $\tau_2^{NL}(\lambda, C_P)$, $\tau_1^{L}(\lambda, C_P)$, and $\tau_2^{L}(\lambda, C_P)$ for $\tau \in [\tau_C, \tau_L]$.

If there is **no license** in equilibrium, $\tau_1^{NL}(\lambda, C_P)$ is defined as the value of τ for which $\Pi_{iP}^{*G} - C_P - I_P = 0$ where Π_{iP}^{*G} is (21). Thus,

$$\tau_1^{NL}(\lambda, C_P) = 2\frac{(v-c)}{\gamma_P} - \underline{G} + \frac{1-\lambda}{\lambda} - \frac{1}{\lambda^{\frac{1}{2}}}(4\gamma_H - \gamma_P)(\frac{C_P + I_P}{\Delta\gamma_H\gamma_P})^{\frac{1}{2}},$$

where $\partial\tau_1^{NL}(\lambda, C_P)/\partial C_P < 0$, and

$$\frac{\partial\tau_1^{NL}(\lambda, C_P)}{\partial\lambda} = -\frac{1}{\lambda^2} + \frac{1}{2}(4\gamma_H - \gamma_P)(\frac{C_P + I_P}{\Delta\gamma_H\gamma_P})^{\frac{1}{2}}\frac{1}{\lambda^{\frac{3}{2}}},$$

$$\frac{\partial^2\tau_1^{NL}(\lambda, C_P)}{\partial\lambda^2} = \frac{2}{\lambda^3} - \frac{3}{4}(4\gamma_H - \gamma_P)(\frac{C_P + I_P}{\Delta\gamma_H\gamma_P})^{\frac{1}{2}}\frac{1}{\lambda^{\frac{5}{2}}}.$$

We show that $\partial\tau_1^{NL}(\lambda, C_P)/\partial\lambda > 0$ if $\lambda > 4\Delta\gamma_H\gamma_P/(4\gamma_H - \gamma_P)^2(C_P + I_P)$. There exists a value of λ such that $\tau_1^{NL}(\lambda, C_P) = \tau_L(\lambda)$. Let's denote $\overline{\lambda}(C_P)$ this value, which depends on C_P. Furthermore, there exists a value of C_P such that $\tau_1^{NL}(\overline{\lambda}(C_P), C_P) = \tau_L(\overline{\lambda}(C_P)) = 0$ Let's denote that value $\overline{C}_P$. As $C_P > \overline{C}_P$ increases, the function $\tau_1^{NL}(\lambda, C_P)$ shifts to the right as $\tau_1^{NL}(\lambda, C_P)$ decreases.

Similarly, $\tau_2^{NL}(\lambda, C_P)$ is defined as the value of τ for which $\Pi_{iP}^{*G} - \Pi_i^{*G} = C_P + 2I_P$ where Π_{iP}^{*G} is (21) and Π_i^{*G} is (22). We calculate that

$$\Pi_{iP}^{*G} - \Pi_i^{*G} = (4\gamma_H - \gamma_P)(\frac{(v-c)^2}{\gamma_P} - \gamma_H(\tau + \underline{G} - \frac{1-\lambda}{\lambda})^2),$$

and, therefore, $\Pi_{iP}^{*G} - \Pi_i^{*G} - (C_P + 2I_P) > 0$ if $\tau < \tau_2^{NL}(\lambda, C_P)$ with

$$\tau_2^{NL}(\lambda, C_P) \equiv (\frac{(v-c)^2}{\gamma_P\gamma_H} - \frac{(4\gamma_H - \gamma_P)}{\lambda\Delta\gamma_H}(C_P + 2I_P))^{\frac{1}{2}} - \underline{G} + \frac{1-\lambda}{\lambda}.$$

We have that $\tau_2^{NL}(\lambda, C_P) < \tau_1^{NL}(\lambda, C_P)$, as $\tau_1^{NL}(\lambda, C_P)$ is the value of τ for which $\Pi_{iP}^{*G} - C_P - I_P = 0$, and $\tau_2^{NL}(\lambda, C_P)$ is the value of τ for which $\Pi_{iP}^{*G} - C_P - I_P - (\Pi_i^{*G} + I_P) = 0$.

We find that

$$\frac{\partial \tau_2^{NL}(\lambda, C_P)}{\partial C_P} = -\frac{(4\gamma_H - \gamma_P)}{\lambda \Delta \gamma_H} \frac{1}{2} (\frac{(v-c)^2}{\gamma_P \gamma_H} - \frac{(4\gamma_H - \gamma_P)}{\lambda \Delta \gamma_H} (C_P + 2I_P))^{-\frac{1}{2}} < 0,$$

and

$$\frac{\partial \tau_2^{NL}(\lambda, C_P)}{\partial \lambda} = \frac{1}{2} \frac{(4\gamma_H - \gamma_P)(C_P + 2I_P)}{\lambda^2 \Delta \gamma_H} (\frac{(v-c)^2}{\gamma_P \gamma_H} - \frac{(4\gamma_H - \gamma_P)}{\lambda \Delta \gamma_H} (C_P + 2I_P))^{-\frac{1}{2}} - \frac{1}{\lambda}.$$

There is a value of C_P relatively large such that $\tau_2^{NL}(\lambda = 1, C_P) = 0$. As C_P decreases, the function $\tau_2^{NL}(\lambda, C_P)$ shifts to the left.

If there is **a license**, in equilibrium, $\tau_1^L(\lambda, C_P)$ is defined as the value of τ for which $\Pi_{iP}^{*L} - C_P - I_P = 0$ where Π_{iP}^{*L} is defined by (28). We find that $\Pi_{iP}^{*L} - C_P - I_P > 0$ if and only if $\tau \leq \tau_1^L(\lambda, C_P)$ with

$$\tau_1^L(\lambda, C_P) \equiv \frac{v-c}{\gamma_P} - \underline{G} + \frac{1-\lambda}{\lambda} - 2\frac{1}{\lambda^{\frac{1}{2}}} (\frac{C_P + I_P}{\gamma_P})^{\frac{1}{2}}.$$

We calculate that $\partial \tau_1^L(\lambda, C_P)/\partial C_P = -((C_P + I_P)/\lambda \gamma_P)^{-\frac{1}{2}} < 0$, and

$$\frac{\partial \tau_1^L(\lambda, C_P)}{\partial \lambda} = -\frac{1}{\lambda^2} + \frac{1}{\lambda^2} (\frac{C_P + I_P}{\lambda \gamma_P})^{-\frac{1}{2}}.$$

We have that $\partial \tau_1^L(\lambda, C_P)/\partial \lambda < 0$ for $\lambda < (C_P + I_P)/\gamma_P$, and $\partial \tau_1^L(\lambda, C_P)/\partial \lambda > 0$ for $\lambda < (C_P + I_P)/\gamma_P$. Thus, $\tau_1^L(\lambda, C_P)$ initially decreases and then increases. We also calculate that

$$\lim_{\lambda \to 0} \tau_1^L(\lambda, C_P) = +\infty,$$

and

$$\tau_1^L(\lambda = 1, C_P) \equiv \frac{v-c}{\gamma_P} - \underline{G} - 2(\frac{C_P + I_P}{\gamma_P})^{\frac{1}{2}} > 0.$$

We also calculate that

$$\frac{\partial \tau_1^L(\lambda, C_P)}{\partial \gamma_P} = -\frac{(v-c)}{\gamma_P^2} + \frac{1}{\gamma_P^{\frac{3}{2}}} (\frac{C_P + I_P}{\lambda})^{\frac{1}{2}}.$$

Thus, $\partial \tau_1^L(\lambda, C_P)/\partial \gamma_P < 0$ as long as $\gamma_P < (v-c)^2 \lambda/(C_P + I_P)$.

If there is a license in equilibrium, $\tau_2^L(\lambda, C_P)$ is defined as the value of τ for which $\Pi_{iP}^{*L} - C_P - 2I_P = 0$ where Π_{iP}^{*L} is defined by (28). Thus, $\Pi_{iP}^{*L} - C_P - 2I_P \geq 0$ if and only if $\tau \leq \tau_2^L(\lambda, C_P)$ where

$$\tau_2^L(\lambda, C_P) \equiv \frac{v-c}{\gamma_P} - \underline{G} + \frac{1-\lambda}{\lambda} - 2(\frac{C_P + 2I_P}{\lambda\gamma_P})^{\frac{1}{2}} < \tau_1^L(\lambda, C_P).$$

We have that $\partial\tau_2^L(\lambda, C_P)/\partial\lambda < 0$ for $\lambda < (C_P + 2I_P)/\gamma_P$, and $\partial\tau_2^L(\lambda, C_P)/\partial\lambda > 0$ for $\lambda < (C_P + 2I_P)/\gamma_P$, where $(C_P + 2I_P)/\gamma_P > (C_P + I_P)/\gamma_P$.

We show that $\tau_2^L(\lambda, C_P) < \tau_1^L(\lambda, C_P)$ as $\tau_1^L(\lambda, C_P)$ is the value of τ for which $\Pi_{iP}^{*L} - C_P - I_P = 0$ and $\tau_2^L(\lambda, C_P)$ is the value of τ for which $\Pi_{iP}^{*L} - C_P - 2I_P = 0$. There is a value of λ, $\underline{\lambda}$, such that $\tau_1^{NL}(\underline{\lambda}, C_P) = \tau_2^L(\underline{\lambda}, C_P)$ with $\underline{\lambda} > 0$ and $\underline{\lambda} < 1$. For any $\lambda < \underline{\lambda}$ we have $\tau_1^{NL} < \tau_2^L$.

We show that $\Pi_{iP}^{*L} > \Pi_{iP}^{\prime G}$ where Π_{iP}^{*L} is (28) and $\Pi_{iP}^{\prime G}$ is (25). We show that $\Pi_{iP}^{*L} > \Pi_{iP}^{\prime G}$ is equivalent to having $(v - c - \gamma_P\underline{G} + \gamma_P(1-\lambda)/\lambda - \tau(2\gamma_H - \gamma_P))^2 > 0$, which is always satisfied.

We calculate the different thresholds $\tau_{1C}^{NL}(\lambda, C_P)$, $\tau_{2C}^{NL}(\lambda, C_P)$ for $\tau < \tau_C$, when the market is covered where Π_{iP}^{*GC} is defined by (23) and Π_j^{*GC} is defined by (24) and we obtain similar findings. We also need to determine $\tau_{NG}^L(\lambda, C_P)$ which is defined as the value of τ for which $\Pi_{iP}^{**L} - C_P - I_P = 0$, where $\Pi_{iP}^{**L} = (1-\lambda)(v - c - \tau\gamma_P)$. As long as $\tau < \tau_{NG}^L(\lambda, C_P)$, one firm will invest.

Equilibrium emission levels

First, we consider the case of a sufficiently large patenting cost. For $\tau < \tau_L(\lambda)$, there is no licensing in equilibrium, and none of the firms invest. For $\tau_L(\lambda) \leq \tau < \tau_1^L(\lambda, C_P)$, there is licensing in equilibrium and both firms (or one of them) invest, while for $\tau > \tau_1^L(\lambda, C_P)$, none of the firms invest. Thus, the emission levels are

$$e_G^* = \begin{cases} e_H^* = \lambda(v - c - \gamma_H(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \text{if } \tau < \tau_L(\lambda) \\ e_L^* = \lambda\frac{1}{2}(v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \text{if } \tau_L(\lambda) \leq \tau < \tau_1^L(\lambda, C_P) \\ e_H^* & \tau \geq \tau_1^L(\lambda, C_P) \end{cases}$$

We show that both levels of emission are decreasing with τ as $\partial e_H^*/\partial\tau = -\lambda\gamma_H < 0$, and $\partial e_L^*/\partial\tau = -\lambda\gamma_P/2 < 0$. Furthermore, we have $\partial e_H^*/\partial\tau < \partial e_L^*/\partial\tau$. Evaluated at $\tau_L(\lambda)$ as defined by (17), we show that $e_H^* > e_L^*$ as

$$(v-c)\frac{\gamma_P}{2\gamma_H} + \frac{1-\lambda}{\lambda}(4\gamma_H - \gamma_P)\frac{2\gamma_H - \gamma_P}{2\gamma_H} > 0$$

is always satisfied. We also show that $\partial e_L^*/\partial\gamma_P = -\lambda\frac{1}{2}(\tau + \underline{G} - \frac{1-\lambda}{\lambda}) < 0$.

Second, we consider the case of a sufficiently low patenting cost. For $\tau < \tau_L(\lambda)$, there is no licensing in equilibrium, and both firms invest, while for $\tau > \tau_L(\lambda)$, there is licensing in equilibrium, and both firms invest as well. The levels of emission are

$$e_G^* = \begin{cases} e_{NL}^* = \lambda\frac{\gamma_H}{4\gamma_H-\gamma_P}(3(v-c) - (2\gamma_H+\gamma_P)(\tau+\underline{G}-\frac{1-\lambda}{\lambda})) & \text{if } \tau < \tau_L(\lambda) \\ e_L^* = \lambda\frac{1}{2}(v-c-\gamma_P(\tau+\underline{G}-\frac{1-\lambda}{\lambda})) & \text{if } \tau \geq \tau_L(\lambda) \end{cases}$$

We show that both levels of emission are decreasing with τ as $\partial e_L^*/\partial\tau < 0$ and $\partial e_{NL}^*/\partial\tau = -\lambda\gamma_H(2\gamma_H+\gamma_P)/(4\gamma_H-\gamma_P) < 0$. Furthermore, we have $\partial e_{NL}^*/\partial\tau < \partial e_L^*/\partial\tau$ as $(2\gamma_H)^2 - 2\gamma_P\gamma_H + (\gamma_P)^2 > 0$. Evaluated at $\tau_L(\lambda)$ as defined by (17), we show that $e_{NL}^* > e_L^*$ as

$$(v-c)\frac{\gamma_P}{2\gamma_H} + \frac{1-\lambda}{\lambda}\frac{4(\gamma_H)^2-2\gamma_H\gamma_P+(\gamma_P)^2}{2\gamma_H} > 0$$

is always satisfied. We also show that

$$\frac{\partial e_{NL}^*}{\partial\gamma_P} = 3\frac{\lambda\gamma_H}{(\gamma_P-4\gamma_H)^2}((v-c) - 2\gamma_H(\underline{G}+\tau-\frac{1-\lambda}{\lambda})) > 0$$

if $\tau < (v-c)/2\gamma_H - \underline{G} + (1-\lambda)/\lambda$, which is always satisfied as $(v-c)/2\gamma_H - \underline{G} + (1-\lambda)/\lambda > \tau_L(\lambda)$, and we have e_{NL}^* for values of $\tau < \tau_L(\lambda)$. Furthermore, we show that $e_{NL}^* < e_H^*$ if $\tau < (v-c)/2\gamma_H - \underline{G} + (1-\lambda)/\lambda$, which is always satisfied for $\tau < \tau_L(\lambda)$. This proves the second part of Proposition 8 (i) and (ii).

A4. Total welfare

Green and non-green-conscious consumers

In the absence of innovation, in equilibrium, both firms sell the dirty product γ_H at price $c+\tau\gamma_H$. The demand is $\lambda(\widetilde{G}_3 - \underline{G}) + (1-\lambda)$ and the pollution damage to each consumer is $P(e) = \gamma_H(\lambda(\widetilde{G}_3 - \underline{G}) + (1-\lambda))$ where

$$\widetilde{G}_3 = \frac{v-c-\tau\gamma_H}{\gamma_H}.$$

The consumer surplus is

$$\begin{aligned} CS_H &= \lambda\int_{\underline{G}}^{\widetilde{G}_3}(v-\gamma_H G-(c+\tau\gamma_H)-P(e))f(G)dG + \lambda\int_{\widetilde{G}_3}^{\overline{G}}(-P(e))f(G)dG \\ &\quad +(1-\lambda)(v-(c+\tau\gamma_H)-P(e)) \end{aligned}$$

or, equivalently,

$$CS_H = \frac{\lambda}{\gamma_H} \frac{(v-c-\tau\gamma_H)^2}{2} - \lambda(v-c-\tau\gamma_H)(\underline{G} - \frac{1}{\lambda}) + \lambda\frac{\gamma_H}{2}\underline{G}^2 + \lambda\gamma_H\underline{G} - \gamma_H(1-\lambda).$$

We calculate that

$$\frac{\partial CS_H}{\partial \tau} = -\lambda(v-c-\tau\gamma_H) + \lambda\gamma_H(\underline{G} - \frac{1-2\lambda}{\lambda}).$$

Thus, $\partial CS_H/\partial\tau > 0$ if $\tau > \overline{\tau}_H = (v-c)/\gamma_H - (\underline{G} - (1-2\lambda)/\lambda)$. Per assumption (9), we know that $\tau < \overline{\tau} = (v-c)/\gamma_H - \underline{G}$. Therefore, if $\overline{\tau}_H > \overline{\tau}$, then $\partial CS_H/\partial\tau < 0$. We find that $\overline{\tau}_H > \overline{\tau}$ if $\lambda < 1/3$. Thus, for values of λ not too large ($\lambda < 1/3$), we have that as τ increases, the consumer surplus decreases ($\partial CS_H/\partial\tau < 0$). However, for larger values of λ ($\lambda > 1/3$), initially CS_H decreases with τ and then it increases.

Each firm gets a null payoff, the tax bill is τe, and the total welfare is thus $W_G^H = CS_H + \tau e_H^*$ or, equivalently,

$$W_G^H = \frac{\lambda}{\gamma_H} \frac{(v-c-\tau\gamma_H)^2}{2} - \lambda(v-c-\tau\gamma_H)(\underline{G} - \tau - \tfrac{1-2\lambda}{\lambda}) + \lambda\frac{\gamma_H}{2}\underline{G}^2 + (1-\tau)\lambda\gamma_H\underline{G} - \gamma_H(1-\tau)(1-\lambda).$$

We show that $\partial W_G^H/\partial\tau = \lambda\gamma_H(1-\tau)$, and $\partial^2 W_G^H/\partial\tau^2 = -\lambda\gamma_H < 0$.

If firm i innovates and licenses to firm j (as it is the case in equilibrium for $\tau \geq \tau_L(\lambda)$), firm i gets Π_{iP}^{*L} as defined by (28), firm j gets 0, and $P(e) = e_L^* = \gamma_P 2D^L$ with D^L defined by (27). The consumers surplus is

$$\begin{aligned} CS_P^L &= \lambda \int_{\underline{G}}^{\widetilde{G}_3} (v - p^{*L} - \gamma_P G - P(e)) f(G) dG + \lambda \int_{\widetilde{G}_3}^{\overline{G}} (-P(e)) f(G) dG + (1-\lambda)(v - p^{*L} - P(e)) \\ &= \frac{\lambda}{2\gamma_P}(v - p^{*L})^2 - \lambda(v - p^{*L})(\underline{G} - \frac{1-\lambda}{\lambda}) + \lambda\frac{\gamma_P}{2}\underline{G}^2 - e_L^*. \end{aligned}$$

We calculate that

$$\frac{\partial CS_P^L}{\partial\tau} = -\frac{\lambda}{4}(v - c - \gamma_P\tau - \gamma_P(\underline{G} - \frac{1+\lambda}{\lambda})).$$

We have that $\partial CS_P^L/\partial\tau > 0$ if $\tau > \overline{\tau}_{LP} = (v-c)/\gamma_P - (\underline{G} - (1+\lambda)/\lambda)$. Using assumption (9), we find that $\overline{\tau}_{LP} > \overline{\tau}$ if $\lambda < 1/3$. Thus, for values of λ not too large ($\lambda < 1/3$), as τ increases, the consumer surplus decreases ($\partial CS_P^L/\partial\tau < 0$). However, for $\lambda > 1/3$, initially CS_P^L decreases with τ and then it increases. We also find that $\overline{\tau}_H < \overline{\tau}_{LP}$, so that, for $\lambda > 1/3$, there are values of τ for which CS_H increases while CS_P^L decreases.

The total welfare is thus $W_G^L = CS_P^L + \tau e_L + \Pi_{iP}^{*L} - C_P - I_P$ or, equivalently,

$$W_G^L = \frac{\lambda}{2\gamma_P}(v-p^L)^2 - \lambda(v-p^L)(\underline{G} - \frac{1-\lambda}{\lambda}) + \lambda\frac{\gamma_P}{2}\underline{G}^2 + \frac{4\gamma_P}{\lambda}(D^L)^2 - e_L(1-\tau).$$

We calculate that

$$\frac{\partial W_G^L}{\partial \tau} = \frac{3\lambda}{16}(v-c) + \frac{3\lambda}{16}\gamma_P(\underline{G} - \frac{1-\lambda}{\lambda}) - \frac{11\lambda}{16}\gamma_P\tau + \frac{\lambda}{2}\gamma_P.$$

Thus, $\partial W_G^L/\partial\tau > 0$ if

$$\tau < \frac{3}{11}\frac{v-c}{\gamma_P} + \frac{3}{11}(\underline{G} - \frac{1-\lambda}{\lambda}) + \frac{8}{11},$$

and $\partial^2 W_G^L/\partial\tau^2 = -11\lambda\gamma_P/16 < 0$.

We show that $W_G^L - W_G^H > 0$ if $CS_P^L - CS_H - \tau(e_H - e_L) + \Pi_{iP}^{*L} - (C_P + I_P) > 0$. Let's define $\Phi_L(\tau) = CS_P^L - CS_H - \tau(e_H - e_L)$. To evaluate whether there will be underinvestment or overinvestment, we compare the social incentive to invest, $\Phi_L(\tau) + \Pi_{iP}^{*L} - (C_P + I_P) > 0$, with the private incentive, $\Pi_{iP}^{*L} - (C_P + I_P) > 0$. Therefore, if $\Phi_L(\tau) > 0$, for some values of C_P there might be underinvestment, while if $\Phi_L(\tau) < 0$, there might be overinvestment.

Let's study $\Phi_L(\tau)$. We have

$$\frac{\partial \Phi_L(\tau)}{\partial\tau} = \frac{\partial CS_P^L}{\partial\tau} - \frac{\partial CS_H}{\partial\tau} - (e_H - e_L) - \tau\frac{\partial(e_H - e_L)}{\partial\tau},$$

which is equivalent to having

$$\frac{\partial \Phi_L(\tau)}{\partial\tau} = \frac{1}{4}\lambda[(v-c) - \tau\gamma_P - \gamma_P(\underline{G} - \frac{1-\lambda}{\lambda}) + 2(2\gamma_H - \gamma_P)(\tau - 1)],$$

and $\partial^2\Phi_L(\tau)/\partial\tau^2 = \lambda(4\gamma_H - 3\gamma_P) > 0$. Thus, $\Phi_L(\tau)$ is a convex function. There exists a value of τ, $\underline{\tau}_L$, such that $\partial\Phi_L(\tau)/\partial\tau = 0$. Thus, $\partial\Phi_L(\tau)/\partial\tau < 0$ for $\tau < \underline{\tau}_L$, and there exists a value of τ for which $\Phi_L(\tau) = 0$.

If firm i innovates and there is no licensing (as it is the case in equilibrium for $\tau < \tau_L(\lambda)$), firm i gets Π_{iP}^{*G} as defined by (21) and firm j gets Π_j^{*G} as defined by (22). The consumers surplus is

$$CS_P^{NL} = \lambda\int_{\underline{G}}^{\widetilde{G}_3}(v - p_j^{*GC} - \gamma_H G)f(G)dG + \lambda\int_{\widetilde{G}_3}^{\overline{G}}(v - p_i^{*GC} - \gamma_P G)f(G)dG + (1-\lambda)(v - p_j^{*GC}) - P(e).$$

The total welfare is thus $W_G^{NL} = CS_P^{NL} + \Pi_{iP}^{*G} - C_P - I_P + \Pi_j^{*G} + \tau e_{NL}^*$ where

$$e_{NL}^* = \lambda\frac{\gamma_H}{4\gamma_H - \gamma_P}(3(v-c) - (2\gamma_H + \gamma_P)(\tau + \underline{G} - \frac{1-\lambda}{\lambda})).$$

Let's define $\Phi_{NL}(\tau) = CS_P^{NL} - CS_H - \tau(e_H^* - e_{NL}^*)$. We compare the social incentive to invest, $\Phi_{NL}(\tau) + \Pi_{iP}^{*G} + \Pi_j^{*G} - I_P - C_P > 0$, with the private incentive, $\Pi_{iP}^{*G} - I_P - C_P > 0$. Therefore, if $\Phi_{NL}(\tau) + \Pi_j^{*G} > 0$, there is underinvestment. We have that $\Pi_j^{*G} > 0$ and we show that $\Phi_{NL}(\tau) > 0$ when τ is not too large, which correspond to the case where there is no licensing.

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