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Employing Gain-Sharing Regulation to Promote Forward Contracting in the Electricity Sector

by

David P. Brown* and David E. M. Sappington**

Abstract

We examine the reductions in electricity procurement costs that can be secured when gain-sharing regulation is employed to induce a regulated load serving entity (LSE) to undertake forward contracting despite associated political risk. We identify arguably plausible conditions under which a modest degree of gain sharing can induce an LSE to undertake forward contracting that substantially reduces the LSE's procurement costs, to the benefit of retail consumers.

Keywords: forward contracting; incentive regulation; gain sharing

JEL Codes: L51, L94, Q28, Q40

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1 Introduction

It is well known that forward contracting can motivate electricity generators to compete more aggressively and thereby reduce the expected wholesale price of electricity (e.g., Allaz and Vila, 1993; Bushnell et al., 2008). However, it has also been noted that regulated load serving entities (LSEs) – large buyers of electricity that deliver electricity to retail customers – often are reluctant to engage in forward contracting. The reluctance arises in part because forward contracting can expose LSEs to significant risk of financial penalties with limited potential for financial rewards. In practice, LSEs seldom are awarded a portion of the gains from forward contracting, but often are held financially responsible for the “overpayments” they are perceived to have made when the cost of electricity secured via forward contract exceeds the wholesale price of electricity (Costello, 2012; Cicchetti, 2017; Gettings, 2017).

The present research investigates two elements of this problem. First, we examine the magnitude of the problem that can arise in restructured electricity markets by analyzing the amount by which electricity procurement costs increase when LSEs decline to implement the levels of forward contracting preferred by generators. Second, we examine how gain-sharing regulation can be employed to avoid these elevated procurement costs. Specifically, we examine the share of realized gains from forward contracting an LSE must anticipate before it will undertake forward contracting in settings where the LSE faces significant risk of financial penalties should the forward price of electricity ultimately exceed the realized wholesale price.

Employing data from electricity markets in Alberta, Canada to calibrate our model, we find that forward contracting can reduce electricity procurement costs substantially (e.g., more than 25%) under arguably plausible conditions. We also find that relatively modest gain sharing (often less than 10%) can induce LSEs to agree to the levels of forward contracting preferred by generators even in the presence of substantial risk of penalties for perceived overpayments. Only modest gain sharing is required because the amount by which forward contracting reduces the expected wholesale price of electricity often substantially exceeds likely differences between the forward price and the wholesale price of electricity.

A well-established literature (e.g., Allaz and Vila, 1993; Bushnell, 2007; Bushnell et al., 2008; Holmberg, 2011) examines many elements of forward contracting, including the role it can play in reducing equilibrium wholesale prices.¹ Although we contribute some

¹These studies focus on strategic forward contracting by risk neutral generators. A related literature (e.g.,

new analytic findings to this literature, this contribution is modest because our findings reflect standard principles. Our primary contribution is to examine how the risk of financial penalties associated with forward contracting in restructured electricity markets can render LSEs reluctant to undertake such contracting, and to demonstrate how standard (and relatively modest) forms of gain-sharing regulation can overcome this reluctance, thereby facilitating the realization of the substantial gains that forward contracting can engender.²

The analysis proceeds as follows. Section 2 describes the key elements of our model. Section 3 characterizes equilibrium outcomes both in the presence of forward contracting and in its absence. Section 4 examines the potential gains from forward contracting in a setting designed to reflect conditions that prevail in Alberta’s electricity sector. Section 5 considers the risk of financial penalties that forward contracting can impose on LSEs and examines the extent of gain sharing that will motivate an LSE to undertake forward contracting despite the risk it entails.³ Section 6 provides concluding observations. The Appendix provides the proofs of all formal conclusions.

2 Model Elements

Electricity is consumed by two groups of customers in our model: retail customers and commercial and industrial customers. For simplicity, we assume the stochastic demand of retail customers is perfectly price inelastic. Realized retail demand is $\bar{Q} + \eta$, where $\bar{Q} > 0$ is expected retail demand and η is the realization of a random variable. The electricity

Bessembinder and Lemmon, 2002; van Eijkel et al., 2016; Brown and Sappington, 2022b) analyzes forward contracting by risk averse generators. Although forward contracting often reduces equilibrium wholesale prices, it does not always do so. In particular, forward contracts that are not observable to third parties may not reduce wholesale prices (Hughes and Kao, 1997). Capacity constraints also can limit the impact of forward contracting on wholesale prices (Murphy and Smeers, 2010; Ferreira, 2014). Furthermore, forward contracts can facilitate collusion among electricity suppliers (Liski and Montero, 2006), and can sometimes reduce the intensity of competition when firms compete by setting prices or supply functions (Mahenc and Salanie, 2004; Holmberg and Willems, 2015).

²Other potential impediments to forward contracting exist. For example, ceilings on the price of electricity in short-term wholesale markets can reduce incentives to procure electricity via long-term contract (e.g., Wolak, 2021; Mays and Jenkins, 2022). The prospect of customer migration to competing LSEs might also render an LSE reluctant to procure electricity via long-term contract. We abstract from these and other potential impediments to forward contracting to focus on the impediment imposed by asymmetric rewards and penalties for forward contracting.

³Asymmetric rewards and penalties for forward contracting prevail in other sectors, including the natural gas sector (Costello, 2012). Although the basic message of our analysis applies more broadly, our estimate of the losses induced by these asymmetric policies is specific to the illustrative electricity setting we consider.

demanded by retail customers is delivered by a single, regulated LSE.⁴ The LSE is required to deliver all the electricity its retail customers demand in return for the compensation specified by the regulator.

Commercial and industrial customers purchase electricity in the wholesale market at unit price w . Their demand for electricity is $Q^I(w) = a^I - b^I w$, where a^I and b^I are positive constants. Aggregate (i.e., retail plus industrial)⁵ demand for electricity at unit price w is:

$$Q(\cdot) = a^I - b^I w + \bar{Q} + \eta. \quad (1)$$

The corresponding inverse demand curve is:

$$w(\cdot) = a + \varepsilon - bQ \quad \text{where} \quad a = \frac{a^I + \bar{Q}}{b^I}, \quad \varepsilon = \frac{\eta}{b^I}, \quad \text{and} \quad b = \frac{1}{b^I}. \quad (2)$$

The random demand parameter $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$ has zero mean.

Electricity is supplied by $n > 1$ symmetric generators, labeled $G_1, \dots, G_n$. Each generator's cost of producing q units of output is $C(q) = c_0 q + \frac{1}{2} c q^2$. $c_0 = C'(0) > 0$ can be viewed as a generator's base-load marginal cost. $c = C''(q) \geq 0$ is the rate at which a generator's marginal cost increases with its output. We allow c to be strictly positive because, in practice, a generator typically brings less efficient units online as its output increases above a baseload level.

The load serving entity (L) can purchase electricity in the wholesale market and/or procure electricity via forward contract. A forward contract between L and generator G_i obligates G_i to ensure that L can purchase a unit of electricity in the wholesale market at net price p^f . This net price is the difference between the prevailing wholesale price, w , and the compensation that G_i delivers to L , which is $w - p^f$.⁶ Thus, when G_i and L sign F_i forward contracts and the realized wholesale price is $w > p^f$, G_i must pay L the amount $F_i [w - p^f]$ to ensure that L 's net cost of securing the F_i units of electricity is $F_i [w - (w - p^f)] = p^f F_i$.⁷ For expositional ease, the ensuing discussion will refer to p^f as

⁴Competition to serve retail customers prevails in some jurisdictions. We abstract from such competition to focus on the effects of asymmetric rewards and penalties for forward contracting that regulated LSEs often face in restructured electricity markets. Thus, our analysis is most relevant for electricity markets like those in California, parts of Texas, and portions of the Midcontinental Independent System Operator (MISO) region, which are characterized by limited retail competition.

⁵For expositional ease, commercial and industrial customers sometimes will be referred to simply as "industrial customers."

⁶Thus, we consider fixed-price, fixed-quantity financial forward contracts that are settled at the prevailing wholesale price. Such contracts are commonly employed in electricity markets.

⁷If G_i and L sign F_i forward contracts and $w < p^f$, then L pays G_i the amount $F_i [p^f - w]$. This payment

the *price* of a forward contract.⁸

The number of forward contracts that L signs with each generator is observed publicly. Furthermore, forward markets are efficient, so p^f is equal to the expected wholesale price of electricity, $E\{w\}$.⁹ The realized wholesale price, $w(\varepsilon)$, is the price that equates demand and supply in the wholesale market.

When wholesale price w prevails, Gi 's profit if it signs F_i forward contracts with L and produces q_i units of output is:

$$\pi_i^G = w[q_i - F_i] + p^f F_i - c_0 q_i - \frac{1}{2} c q_i^2. \quad (3)$$

Equation (3) reflects the fact that Gi sells $q_i - F_i$ units of output in the wholesale market at unit price w and effectively sells F_i units of output via forward contract at unit price p^f .

L 's corresponding profit when realized retail demand is $\bar{Q} + b^I \varepsilon$ is:

$$\pi^B = R(\varepsilon) - w \left[\bar{Q} + b^I \varepsilon - \sum_{i=1}^n F_i \right] - p^f \sum_{i=1}^n F_i - K - \Phi(\cdot), \quad (4)$$

where $R(\varepsilon)$ denotes L 's revenue when ε is realized,¹⁰ K denotes the (fixed) cost that L incurs in addition to electricity procurement costs, and $\Phi(\cdot)$ reflects any penalty L incurs because of its forward contracting. L may face such a penalty if it agrees to purchase electricity via forward contract at price p^f and this price ultimately exceeds the realized wholesale price, w .¹¹ Equation (4) reflects the fact that L purchases $\bar{Q} + b^I \varepsilon - \sum_{i=1}^n F_i$ units of electricity at unit price w in the wholesale market and procures $\sum_{i=1}^n F_i$ units of electricity at unit price p^f via forward contract.

The timing in the model is as follows. After the regulator specifies $R(\varepsilon)$, each generator chooses (simultaneously and noncooperatively) the number of forward contracts it will offer to L . L then decides whether to accept or reject the forward contracts it is offered, after

ensures that L 's net cost of securing the F_i units of electricity is $w F_i + F_i [p^f - w] = p^f F_i$.

⁸We follow much of the literature by analyzing these fixed-price forward contracts. In practice, some LSEs sign load-following forward contracts, which allow an LSE to secure a specified *fraction* of its realized demand at a fixed unit price (e.g., Brown and Sappington, 2023).

⁹Holmberg and Willems (2015) observe that forward markets will be efficient when risk-neutral, non-strategic actors (e.g., financial traders) with rational expectations ensure the price of a forward contract reflects its expected value. The authors explain why the assumption often constitutes a reasonable caricature of electricity forward markets and observe that the assumption is common in the literature.

¹⁰For example, the regulator might specify a unit price, r , that retail customers must pay for electricity. In this case, $R(\varepsilon) = r [\bar{Q} + b^I \varepsilon]$.

¹¹ $\Phi(\cdot)$ is considered in detail in Section 5.

which the realization of ε is observed publicly. Next, the generators choose their outputs, simultaneously and noncooperatively. After the wholesale price is determined, industrial customers decide how much electricity to purchase in the wholesale market. Finally, the terms of all forward contracts are fulfilled, L purchases the amount of electricity required to satisfy the realized demand of its retail customers, and any penalties that L incurs due to its forward contracting are determined.

3 Equilibrium Outcomes under Forward Contracting

We now characterize equilibrium outcomes under forward contracting, i.e., when L accepts the forward contracts offered by generators. We employ backward induction to do so. To begin, Lemma 1 characterizes Gi 's equilibrium output (q_i) when it signs F_i forward contracts with L , when all other generators sign a total of $F_{-i} \equiv \sum_{\substack{j=1 \\ j \neq i}}^n F_j$ forward contracts with L , and when realized retail demand is $\bar{Q} + b^I \varepsilon$. Lemma 1 also characterizes the corresponding equilibrium price of a forward contract (p^f) and the equilibrium wholesale price ($w(\varepsilon)$).

Lemma 1. *In equilibrium under forward contracting, given ε and $F_1, \dots, F_n$:*

$$\begin{aligned} w(\varepsilon) &= \frac{b+c}{D} \left[(b+c)(a+\varepsilon) + nb c_0 - b^2 \sum_{i=1}^n F_i \right], \\ p^f &= E\{w(\varepsilon)\} = \frac{b+c}{D} \left[a(b+c) + nb c_0 - b^2 \sum_{i=1}^n F_i \right], \text{ and} \\ q_i(\varepsilon) &= \frac{[b+c][a+\varepsilon-c_0] + b[bn+c]F_i - b^2 F_{-i}}{D} \text{ for } i = 1, \dots, n, \\ \text{where } D &\equiv b^2[n+1] + c[b(n+2)+c] > 0. \end{aligned} \tag{5}$$

Two features of Lemma 1 warrant emphasis. First, Gi 's equilibrium output increases as the number of forward contracts it signs with L increases (i.e., $\frac{\partial q_i(\varepsilon)}{\partial F_i} = \frac{b[bn+c]}{D} > 0$). This is the case because as F_i increases, more of Gi 's output is sold at the forward price p^f and less is sold at the wholesale price, w . This reduced exposure to w implies that Gi 's profit declines more slowly as w declines, which induces Gi to increase its output (despite the associated reduction in w). In this sense, forward contracting endows a generator with a credible commitment to compete relatively aggressively in the wholesale market (Allaz

and Vila, 1993). Second, the equilibrium wholesale price declines as the level of forward contracting increases (i.e., $\frac{\partial w(\varepsilon)}{\partial F_i} = -\frac{b^2[b+c]}{D} < 0$). The lower wholesale price reflects the increased output induced by the increased level of forward contracting.

Anticipating the equilibrium outcomes identified in Lemma 1, the generators choose their profit-maximizing levels of forward contracting, simultaneously and noncooperatively. Lemma 2 specifies the level of forward contracting each generator undertakes in a symmetric equilibrium. The lemma reflects the maintained assumption that the expected aggregate demand for electricity is strictly positive when it is priced at the level of base-load marginal cost (i.e., $a^I + \bar{Q} > b^I c_0$).

Lemma 2. *At a symmetric equilibrium under forward contracting, for $i = 1, \dots, n$:*

$$F_i = \frac{b^2 [b+c] [n-1] [a^I + \bar{Q} - b^I c_0]}{[bn+c] [2b(b+c) + c(bn+c)] + b^3 [n-1]^2}$$

$$\Rightarrow \frac{\partial F_i}{\partial a^I} > 0, \quad \frac{\partial F_i}{\partial \bar{Q}} > 0, \quad \frac{\partial F_i}{\partial b^I} < 0, \quad \frac{\partial F_i}{\partial c_0} < 0, \text{ and } \frac{\partial F_i}{\partial c} < 0. \quad (6)$$

The signs of the derivatives reported in Lemma 2 reflect the fact that forward contracting provides a generator with a credible commitment to compete relatively aggressively in the wholesale market (Allaz and Vila, 1993).¹² This commitment becomes more valuable as potential industry profit increases due to an increase in demand (a^I or $\bar{Q}$) or a reduction in production cost (c_0 or c). The commitment also becomes more valuable as the wholesale price increases due to a reduction in the sensitivity of industrial demand to price (b^I). Consequently, generators increase their forward contracting as a^I or $\bar{Q}$ increases and as c_0 , c , or b^I declines.

A generator's equilibrium level of forward contracting changes non-monotonically as the number of generators (n) changes. As Corollary 1 reports, this is the case even when $c = 0$.¹³ The level of forward contracting that each generator undertakes (F_i) is the same when $n = 2$ and when $n = 3$. F_i declines as n increases above 3. The reduced level of forward contracting reflects in part the reduced output that each firm produces under Cournot competition as

¹²Lemma 2 implies that $F_i = 0$ if $n = 1$. The increased output induced by forward contracting reduces the wholesale price, which reduces a generator's profit, *ceteris paribus*. Therefore, in the absence of any strategic competitive benefit from forward contracting, a monopoly generator refrains from forward contracting.

¹³As demonstrated in the proof of Corollary 1, $\frac{\partial F_i}{\partial n} \geq 0 \Leftrightarrow n \leq 1 + \sqrt{2}$ when $c = 0$.

the number of competitors increases. Corollary 1 also implies that the rate at which F_i declines as n increases is relatively modest in the sense that the aggregate level of forward contracting increases as the number of generators increases.¹⁴

Corollary 1. *Suppose $c = 0$. Then at a symmetric equilibrium under forward contracting, for $i = 1, \dots, n$, $F_i = \frac{n-1}{n^2+1} [a^I + \bar{Q} - b^I c_0] \Rightarrow F_i = \frac{1}{5} [a^I + \bar{Q} - b^I c_0]$ when $n = 2$ and when $n = 3$. Furthermore, $\frac{\partial F_i}{\partial n} < 0$ for all $n \geq 3$, and $\frac{\partial}{\partial n} \left(\sum_{j=1}^n F_j \right) > 0$ for all $n \geq 2$.*

Having characterized equilibrium outcomes under forward contracting, we next characterize the potential gains from such contracting.

4 Estimating the Gains from Forward Contracting

The increased output induced by forward contracting reduces the expected wholesale price.¹⁵ The lower wholesale price, in turn, reduces L 's expected cost of serving its retail customers. To determine the magnitude of this cost reduction, let $w(\varepsilon, F)$ denote the equilibrium wholesale price when realized retail demand is $\bar{Q} + b^I \varepsilon$ after L signs $F = \sum_{i=1}^n F_i$ forward contracts with the n generators. Then the amount by which L 's expected cost declines when it signs F forward contracts rather than refraining from forward contracting is:

$$\begin{aligned} \Delta^{LC}(F) &= E \{ w(\varepsilon, 0) [\bar{Q} + b^I \varepsilon] \} - (E \{ w(\varepsilon, F) [\bar{Q} + b^I \varepsilon] \} + [p^f - E \{ w(\varepsilon, F) \}] F) \\ &= E \{ [w(\varepsilon, 0) - w(\varepsilon, F)] [\bar{Q} + b^I \varepsilon] \}. \end{aligned} \quad (7)$$

Proposition 1 reports that this reduction in L 's expected cost increases linearly with the aggregate level of forward contracting.

Proposition 1. *Suppose L accepts the equilibrium levels of forward contracting (F_i) identified in Lemma 2. Then, relative to the setting with no forward contracting, L 's expected cost of serving its retail customers declines by $\Delta^{LC}(F) = \frac{b^2[b+c]\bar{Q}}{D} F$, where $F \equiv \sum_{i=1}^n F_i$.*

¹⁴The rate at which F_i declines with n is relatively modest in part because the value of a commitment to compete aggressively increases as the number of rivals increases (van Eijkel et al., 2016).

¹⁵LSE's recognize that forward contracting reduces wholesale prices in practice (Anderson et al., 2007, p. 3099).

To interpret the magnitude of the expected cost reduction identified in Proposition 1,¹⁶ it is helpful to consider values of model parameters that might prevail in plausible settings. To this end, we employ data from the electricity sector in Alberta, Canada in 2020, in part because these data are publicly available and in part because Alberta’s wholesale electricity market is moderately concentrated and entails relatively limited regulation to restrict generators’ supply decisions. We employ these data to identify arguably plausible initial values for the parameters in our model. We then consider substantial variation in these initial parameter values to inform a more complete assessment of the practical importance of the cost reduction identified in Proposition 1.¹⁷

The initial “baseline” values for model parameters informed by data from Alberta’s electricity sector are summarized in Table 1.

n	b^I	a^I	$\overline{Q}$	c_0	c	$\underline{\varepsilon}$	$\overline{\varepsilon}$	σ_ε
4	55.6945	10,487.05	1,615	15	0.008	−8.08	8.08	2.558

Table 1. Baseline Parameter Values.

The first entry in Table 1 ($n = 4$) reflects the four large strategic generators of electricity in Alberta: Transalta, Heartland, ENMAX, and Capital Power.¹⁸ The second entry ($b^I = 55.6945$) is set to ensure the price elasticity of industrial demand is -0.33 when w is \$46.72, the average wholesale price of electricity in Alberta in 2020.¹⁹ This elasticity reflects empirical estimates in the literature, e.g., Labandeira et al. (2017).

The third and fourth entries in Table 1 ($a^I = 10,487.05$ and $\overline{Q} = 1,615$) ensure: (i) the aggregate demand for electricity (as specified in equation (1)) is 9,500 MWhs when $w = \$46.72$ and $b^I = 55.6945$; and (ii) expected residential demand ($\overline{Q}$) is 17% of this aggregate demand. The 9,500 MWhs and 17% statistics reflect the corresponding statistics

¹⁶The analysis in this section abstracts from L ’s potential reluctance to engage in forward contracting. This reluctance is considered in Section 5.

¹⁷The fact that we employ data from Alberta to establish initial model parameters is not meant to suggest that our model captures all relevant aspects of Alberta’s electricity sector. For instance, our model does not capture the regulations that are imposed on forward contracting activities in Alberta (Brown et al., 2022).

¹⁸Suncor and the Balancing Pool also supply substantial amounts of electricity in Alberta. (Alberta Market Surveillance Administrator, 2020). However, Suncor only operates must-run cogeneration units. Furthermore, the Balancing Pool is a government agency that bids electricity at or near its marginal cost.

¹⁹Alberta Electric System Operator (<http://ets.aeso.ca>).

in Alberta in 2020 (Leach et al., 2020).²⁰

The fifth entry in Table 1 ($c_0 = 15$) reflects the marginal cost of generating electricity with a base-load combined cycle gas turbine unit in Alberta in 2020. The sixth entry ($c = 0.008$) is set to ensure that if the electricity market were perfectly competitive, the equilibrium wholesale price would be \$35/MWh. This price reflects the marginal cost of generating electricity with a peaker gas unit in Alberta in 2020. Part B of the Appendix explains the derivation of these baseline parameters in greater detail.

To express the expected cost reduction identified in Proposition 1 as a function of expected cost in the absence of forward contracting, it is necessary to specify a density function for η (and thus for $\varepsilon = \frac{\eta}{b^I}$).²¹ Figure 1 plots the histogram of the difference between the day-ahead forecast and the amount of electricity actually purchased in Alberta for each hour in 2020.²² The maximum difference in this sample is 644 (MWhs), the minimum difference is -450 , and the standard deviation is 142.48. These data inform our initial assumption that η has a truncated normal density on $[-450, 450]$ with standard deviation 142.48, so ε has a truncated normal density on $[-8.08, 8.08]$ with standard deviation $\sigma_\varepsilon = 2.558$ in the baseline parameterization where $b^I = 55.6945$.²³

[Figure 1 About Here]

In this baseline parameterization, the expected wholesale price is 68.52 and L 's equilibrium expected variable cost in the absence of forward contracting is 110,691.75. (See the first row of data in Table 2.) In the presence of forward contracting, the expected wholesale price is 47.41 and L 's expected variable cost declines to 76,597.90. Thus, forward contracting reduces L 's equilibrium expected variable cost by 30.80% ($= \frac{34,093.85}{110,691.75}$) in the baseline parameterization.

²⁰Residential customers account for substantially more than 17% of aggregate demand in many electricity markets. The ensuing analysis examines the changes that arise when residential customers account for more than 17% of aggregate demand.

²¹Equation (4) and Lemmas 1 and 2 imply that L 's equilibrium expected cost varies with $E\{\varepsilon^2\}$, and thus with the density of ε . In contrast, equation (7) and Lemmas 1 and 2 imply that the equilibrium value of $\Delta^{LC}(\sum_{i=1}^n F_i)$ reflects the maintained normalization that $E\{\varepsilon\} = 0$, but does not vary with any other properties of the density of ε .

²²The data that underlie Figure 1 are drawn from the Alberta Electric System Operator's wholesale market data (<http://ets.aeso.ca>).

²³These data reflect day-ahead uncertainty. More pronounced uncertainty may arise when forward contracts are signed further in advance of trading. As explained in the next footnote, our key conclusions are largely insensitive to variations in $\underline{\varepsilon}$, $\bar{\varepsilon}$, and σ_ε .

Table 2 reports how this illustrative cost reduction and other key outcomes change as model parameters change. The first column in the table identifies the single parameter that is being changed from its level in Table 1. All other parameter values are held constant at the levels specified in Table 1. The second column ($E\{w\}_{F=0}$) records the expected equilibrium wholesale price in the absence of forward contracting. The fifth column ($E\{w\}$) presents the corresponding expected wholesale price in the presence of the forward contracting identified in Lemma 2. The third column ($E\{LCost\}_{F=0}$) specifies L 's equilibrium expected variable cost in the absence of forward contracting. The fourth column (F) presents the aggregate level of forward contracting identified in Lemma 2 ($F = \sum_{i=1}^n F_i$). The sixth column ($E\{Q\}$) records the expected equilibrium output in the presence of this forward contracting. The last column ($\% \Delta^{LC}$) presents the percentage reduction in L 's equilibrium expected variable cost when the forward contracting identified in Lemma 2 is undertaken, rather than no forward contracting.

The first row of data in Table 2 records outcomes under the baseline parameterization. The second and third rows present outcomes when the number of generators is reduced and increased by 2, respectively. The fourth and fifth rows of data specify outcomes when expected retail demand ($\bar{Q}$) declines and increases by 25%, respectively. The remaining rows in Table 2 present the outcomes that arise when corresponding 25% reductions and increases are implemented for a^I (the intercept of the demand curve of industrial customers), b^I (the sensitivity of industrial demand to price), c_0 (the base-load marginal cost), and c (the rate at which a generator's marginal cost increases with its output).²⁴

Table 2 indicates that forward contracting continues to induce a substantial reduction in L 's expected variable cost as parameter values diverge from their levels in Table 1.²⁵ The smallest percentage reduction in L 's expected variable cost in Table 2 arises when there are only two generators. The value of a credible commitment to compete aggressively is relatively small in the presence of a single rival. Consequently, the ratio of equilibrium

²⁴The effects of changes in $\bar{\varepsilon}$ and σ_ε are not presented in Table 2 because changes in these parameters do not alter equilibrium levels of $E\{w\}$, F , $E\{Q\}$, or $\% \Delta^{LC}$. Even substantial changes in other elements of the distribution of ε have little impact on the extent to which equilibrium forward contracting reduces L 's expected variable cost. To illustrate, forward contracting reduces L 's equilibrium expected variable cost by 30.72% when all parameters other than σ_ε are as specified in Table 1 and ε has a uniform density rather than a truncated normal density.

²⁵The same conclusion holds when more substantial (i.e., 50%) changes in baseline parameters are considered. See Table A1 in Part C of the Appendix.

forward contracting to total expected output is relatively small in the presence of duopoly generator competition. The resulting relatively modest reduction in the expected wholesale price limits the extent to which forward contracting reduces L 's expected variable cost.²⁶

Baseline Change	$E\{w\}_{F=0}$	$E\{LCost\}_{F=0}$	F	$E\{w\}$	$E\{Q\}$	$\% \Delta^{LC}$
None	68.52	110,691.75	6,394.75	47.41	9,461.79	30.80
$n = 2$	99.64	160,979.87	3,050.45	83.72	7,439.61	15.98
$n = 6$	54.13	87,444.42	7,879.19	35.11	10,146.59	35.12
$\bar{Q} = 1,211.25$	66.60	80,705.11	6,165.59	46.25	9,122.72	30.55
$\bar{Q} = 2,018.75$	70.44	142,227.03	6,623.91	48.57	9,800.86	31.04
$a^I = 7,865.29$	56.06	90,579.42	4,906.68	39.87	7,260.02	28.88
$a^I = 13,108.81$	80.97	130,804.07	7,882.82	54.95	11,663.56	32.13
$b^I = 41.771$	83.52	134,917.26	6,861.06	52.70	9,990.84	36.89
$b^I = 69.619$	59.33	95,855.97	5,966.94	43.88	9,047.02	26.03
$c_0 = 11.25$	65.76	106,237.68	6,513.29	44.26	9,637.19	32.69
$c_0 = 18.75$	71.28	115,145.81	6,276.21	50.56	9,286.39	29.06
$c = 0.006$	65.60	105,974.22	6,719.94	42.98	9,708.44	34.47
$c = 0.010$	71.68	115,800.06	6,056.60	52.11	9,199.86	27.29

Table 2. Effects of Changing Baseline Parameter Values

Forward contracting induces the largest percentage reduction in L 's expected variable cost in Table 2 when aggregate demand for electricity is relatively insensitive to price (i.e., when $b^I = 41.771$). Relatively inelastic demand promotes a relatively high wholesale price, which increases the value of a credible commitment to expand output aggressively. Consequently, generators increase their forward contracting as b^I declines, inducing a relatively pronounced reduction in L 's expected variable cost.

The last four rows in Table 2 illustrate the finding in Lemma 2 that generators undertake more forward contracting (which induces a larger percentage reduction in L 's expected

²⁶Although the wholesale price reductions induced by forward contracting in Table 2 are substantial, they are considerably smaller than the corresponding reductions estimated by Bushnell et al. (2018). The authors do not model forward contracting explicitly. Rather, they estimate the effects of “vertical arrangements” that take the form of agreements to supply electricity to retail customers at prespecified prices. The relatively pronounced percentage reductions in wholesale prices due to vertical arrangements that Bushnell et al. (2018, Table 3) identify reflect in part the relatively price-inelastic demand functions that arise in their study, which employs data from New England and PJM.

variable cost) as the generators' production costs (c_0 or c) decline. As costs decline, the value of a credible commitment to expand output aggressively increases, which promotes expanded forward contracting and a relatively pronounced reduction in L 's expected variable cost.

The fraction of the aggregate demand for electricity accounted for by residential consumers is lower in Alberta than in many other jurisdictions. For example, residential consumers accounted for approximately 39.3% of total electricity consumption in the United States in 2020.²⁷ It is readily verified that if: (i) a^I , $\bar{Q}$, and b^I are set to ensure the price elasticity of industrial demand is -0.33 when $w = \$46.72$; (ii) the ratio of expected residential demand to aggregate expected demand is 0.393 when $w = \$46.72$; and (iii) the values of c_0 , c , $\underline{\varepsilon}$, $\bar{\varepsilon}$, and σ_ε are as specified in Table 1, then the equilibrium forward contracting identified in Lemma 2 reduces L 's expected variable cost below its level in the absence of forward contracting by 36.76%.²⁸ This more pronounced cost reduction arises because as residential consumption accounts for a larger fraction of total consumption, aggregate demand becomes less sensitive to price. This reduced sensitivity promotes increased forward contracting and a more pronounced reduction in L 's expected variable cost, for the reasons explained above.

5 Motivating Forward Contracting

The analysis in Section 4 suggests that forward contracting has the potential to reduce electricity procurement costs considerably in arguably plausible settings. However, in practice, load serving entities (LSEs) may have limited incentive to engage in forward contracting. For example, if an LSE operates under cost of service regulation (COSR) that sets authorized revenue to reflect expected or realized cost, the LSE anticipates no strict financial gains from forward contracting that reduces the expected wholesale price of electricity.

This incentive problem is compounded if the LSE faces some (political or regulatory) risk that it will be penalized if the forward price it agrees to pay for electricity (p^f) exceeds the realized wholesale price (w). A forward price that reflects the expected wholesale price often will exceed the realized wholesale price. Consequently, if the penalties an LSE incurs when p^f exceeds w are not offset by financial rewards when p^f is less than w , an LSE that

²⁷United States Energy Information Administration (2021).

²⁸See Part B of the Appendix for additional details. It can also be shown that the equilibrium level of forward contracting in this setting is 6,460.35, which reduces the expected wholesale price from 80.99 to 51.21. Expected output in the presence of this level of forward contracting is 9,317.09. This forward contracting reduces L 's expected variable cost from 302,403.18 to 191,230.00.

operates under COSR and faces such asymmetric financial risk will strictly prefer to avoid forward contracting.

Such considerations likely have suppressed forward contracting in practice. To increase forward contracting, regulators sometimes dictate a minimum level of forward contracting an LSE must undertake.²⁹ However, because LSEs typically have more expertise than regulators in managing risk, regulators often afford LSEs considerable discretion in the amount of forward contracting they undertake. Regulators also often decline to specify precisely how an LSE’s compensation will vary with the perceived efficacy of its forward contracting (Gettings, 2008; Costello, 2012; Ryan and Lieberman, 2012). Gettings (2008, p. 5) observes that regulators seldom implement policies that “reward judgment informed by risk expertise, and by not rewarding that expert judgment, it tends to suppress it.” Gettings (2008, p. 5) further observes that “Where there is no explicit agreement on the treatment of uncertain outcomes, the natural response of utilities often will be, and has been to minimize the activities that give rise to that uncertainty.”³⁰

Cicchetti (2017, pp. 10, 16) observes that utilities “are reluctant to engage in [forward contracting and other] hedging strategies because ... [they] receive no special benefit or reward when hedging programs result in gains, but ... [may] be exposed to risks of non-recovery when hedging programs result in losses.” Costello (2012, p. iv) further observes that “utilities consider long-term contracts to carry an unfavorable reward-risk imbalance. ... [U]tilities generally receive no profits from long-term contracts but risk cost disallowances from an after-the fact review.” Facing the prospect of limited rewards for effective hedging of risk but substantial penalties for perceived “overpayments” for electricity, LSEs often undertake only the minimum mandated level of risk hedging (Gettings, 2008, 2017; Cicchetti, 2017).

Awarding LSEs a share (s) of the reduction in procurement cost secured from forward contracting can help overcome their reluctance to undertake forward contracting (Gettings, 2008; Cicchetti, 2017). The share required to fully overcome this reluctance ($\hat{s}$) depends on the potential penalties an LSE faces when the price of a forward contract it signs ultimately exceeds the prevailing wholesale price. Such penalties might arise, for example, when elected

²⁹For example, retail electricity suppliers in Australia are required to secure via forward contract a specified fraction of the maximum demand for electricity they anticipate (Australian Energy Regulator, 2019).

³⁰Wheelwright (2010) documents explicit regulatory concern with insufficient forward contracting in the presence of rising wholesale prices.

officials, acting in response to complaints from their constituents about high utility bills despite prevailing low wholesale prices of electricity, demand that utilities be held financially responsible for perceived mismanagement.

To develop some understanding of potentially plausible values of $\hat{s}$, let $x \equiv [p^f - w] F$ denote the “overpayment” L is perceived to have made when, after it signs F forward contracts at unit price p^f , the realized wholesale price is w . Also, let $\Phi(x)$ denote the penalty imposed on L when (perceived) overpayment x occurs, and suppose:

$$\Phi(x) = \phi(x) m x$$

$$\text{where } \phi(x) = \begin{cases} 0 & \text{if } x \leq x_0 \\ \tilde{\phi}(x) & \text{if } x > x_0. \end{cases} \quad (8)$$

In equation (8), $m > 0$ is a constant that might be viewed as the fraction of any identified overpayment for which L is held financially responsible. x_0 is the smallest value of x for which L might face a financial penalty. x_0 might be strictly positive if, for example, political pressure to penalize L only arises if L 's perceived overpayment is sufficiently large. $\tilde{\phi}(x)$ in equation (8) is a nondecreasing function of x , with $\tilde{\phi}(x) > 0$ for all $x > x_0$. $\tilde{\phi}(\cdot)$ might be viewed as the probability that political pressure leads to a formal investigation of the magnitude of any overpayment L might have made. Such pressure is particularly likely to arise when, as presumed here, the retail price of electricity is set to allow L to recover any cost it incurs that is not explicitly challenged during such formal investigations.

Lemma 1 and equation (8) imply that L incurs a positive penalty in this setting if and only if:

$$x = [p^f - w(\varepsilon)] F > x_0 \Leftrightarrow -\frac{[b+c]^2 \varepsilon}{D} F > x_0 \Leftrightarrow \varepsilon < -\frac{D x_0}{F [b+c]^2} \equiv \hat{\varepsilon}. \quad (9)$$

Equations (8) and (9) imply that L 's corresponding expected penalty is:

$$\begin{aligned} E \{ \Phi([p^f - w(\varepsilon)] F) \} &= E \left\{ m \tilde{\phi}(x) x \right\}_{\varepsilon \leq \hat{\varepsilon}} \\ &= m E \left\{ \tilde{\phi}\left(-\frac{[b+c]^2 \varepsilon}{D} F\right) \left[-\frac{(b+c)^2 \varepsilon}{D} F\right] \right\}_{\varepsilon \leq \hat{\varepsilon}} \\ &= -m F \frac{[b+c]^2}{D} E \left\{ \varepsilon \tilde{\phi}\left(-\frac{[b+c]^2 F}{D} \varepsilon\right) \right\}_{\varepsilon \leq \hat{\varepsilon}}. \end{aligned} \quad (10)$$

In the special case where $\tilde{\phi}(x) = \alpha$ (where $\alpha > 0$ is a constant), equation (10) implies:

$$E \{ \Phi([p^f - w(\varepsilon)] F) \} = - \frac{\alpha m [b + c]^2 F}{D} E \{ \varepsilon \}_{\varepsilon \leq \hat{\varepsilon}}. \quad (11)$$

To illustrate the magnitude of expected penalties in the baseline parameterization, initially consider a *stringent penalty regime* in which L is held fully responsible for all perceived overpayments. Thus, in this regime, $\alpha = 1$ (so an investigation of L 's forward contracting activities is always undertaken), $x_0 = 0$ (so the LSE is penalized whenever p^f exceed w), and $m = 1$ (so the LSE is held financially responsible for the entire perceived overpayment, $[p^f - w] F$). It can be verified that $\hat{s} = 1.97\%$ in the stringent penalty regime when the baseline parameter values in Table 1 prevail and ε has the identified truncated normal density. Thus, in this setting, L will accept the forward contracts the generators offer in equilibrium whenever it is promised at least 1.97% of the cost reduction induced by the forward contracting.

L will accept the forward contracts the generators offer in equilibrium in return for a smaller share of the associated cost reduction when it faces less stringent penalties for perceived overpayments. To illustrate, consider the (arguably more realistic) *moderate penalty regime* in which L is held responsible for one-half of any relatively pronounced perceived overpayment. Formally, in this regime, $\alpha = 1$ (so an investigation of L 's forward contracting activities is always undertaken), x_0 is set to ensure $\hat{\varepsilon} = -\sigma_\varepsilon$ (so the LSE is penalized only if the realization of ε is at least one standard deviation below its expected value, resulting in a relatively large value of $p^f - w$), and $m = \frac{1}{2}$ (so the LSE is held financially responsible for one-half of the (sufficiently large) perceived overpayment, $[p^f - w] F$). It can be verified that $\hat{s} = 0.6\%$ in the moderate penalty regime when the parameter values in Table 1 prevail and ε has the specified truncated normal density.

These relatively small values of $\hat{s}$ arise in part because the reduction in the expected wholesale price engendered by forward contracting is large relative to potential deviations from the reduced expected price. To illustrate, under the baseline parameterization, the forward contracting identified in Lemma 2 reduces the expected wholesale price from 68.52 to 47.41, whereas the maximum difference between p^f and w is 8.08.³¹

The identified relatively small values of $\hat{s}$ also reflect in part the relatively limited

³¹Furthermore, perceived unit overpayments only pertain to the fraction of total purchases secured via forward contracts. In contrast, a reduction in w and thus $p^f = E\{w\}$ affects all equilibrium purchases.

likelihood of the largest perceived overpayments. Larger values of $\hat{s}$ will arise if the extreme realizations of ε are more likely than they are when ε has a truncated normal density, *ceteris paribus*.³² To illustrate, consider the relatively extreme (and arguably unrealistic) setting in which ε has a uniform density, so the largest and smallest ε realizations are just as likely as intermediate realizations. In this setting, when all parameters other than σ_ε are as specified in Table 1, $\hat{s}$ is 4.51% in the moderate penalty regime.

Comparable values of $\hat{s}$ arise as parameter values diverge from those specified in Table 1. To illustrate, consider the setting where ε has a truncated normal density and the moderate penalty regime prevails. The parameter variations considered in Table 2 give rise to values of $\hat{s}$ that vary between 0.41% and 0.80% in this setting. The corresponding values of $\hat{s}$ vary between 0.25% and 1.19% in this setting when the more pronounced parameter variations considered in Table A1 in the Appendix prevail.

Of course, gain sharing is not the only instrument a regulator can employ to encourage LSEs to undertake forward contracting. For example, a regulator might implement *ex ante* reviews of intended forward contracting activities, during which the regulator can explicitly approve (or decline to approve) any forward contract an LSE proposes to sign. Explicit *ex ante* regulatory approval of an LSE's forward contracting activities may reduce the risk that *ex post* concerns on the part of politicians or consumer groups about perceived overpayments will induce the regulator to penalize the LSE for engaging in forward contracting that, at the time it was undertaken, served the best interests of retail (and industrial) customers.³³

6 Conclusions

We have illustrated the reductions in electricity procurement costs that can arise when a load serving entity (LSE) is motivated to implement the levels of forward contracting preferred by generators. We found that these cost reductions can be substantial under arguably plausible conditions. We also noted that LSEs may be reluctant to undertake forward contracting if they are forced to bear some or all of the associated downside risk while enjoying little or none of the associated gains. We found that relatively modest sharing

³²Larger values of $\hat{s}$ also will arise if additional impediments to forward contracting prevail. For example, an LSE might incur unmeasured (transactions) costs in designing and implementing forward contracts.

³³In the context of the simple model considered here, an *ex ante* review of intended forward contracting activities could reduce the values of m and/or α . Gettings (2017) and Cicchetti (2017) discuss elements of *ex ante* reviews of this type. Encinosa and Sappington (1995) examine the optimal design of *ex ante* and *ex post* regulatory reviews in a different context.

of gains from forward contracting with LSEs often can motivate them to undertake forward contracting, even when they bear considerable downside risk.

Our analysis was designed to illustrate the potential gains that forward contracting might provide in the relatively simple settings on which the literature focuses. Our consideration of nonlinear costs generalizes one special feature (namely, constant marginal cost) that is commonly adopted for analytic ease.³⁴ Other generalizations remain to be considered, including risk aversion on the part of generators and/or LSEs. The literature suggests that risk averse generators often will undertake more forward contracting than risk neutral generators (Hughes and Kao, 1997; van Eijkel et al., 2016; Brown and Sappington, 2022b). Consequently, the gains from forward contracting might be expected to increase as generator risk aversion increases. The effects of LSE risk aversion may be more nuanced in part because forward contracting can increase the variance of an LSE’s profit (Brown and Sappington, 2022b). Thus, increased risk aversion might render an LSE less likely to undertake forward contracting, particularly if such contracting exposes the LSE to considerable risk of penalties for perceived overpayments.

We have focused on the effects of targeted gain-sharing policies to counteract political/regulatory risk of asymmetric penalties from forward contracting. To do so, we have abstracted from different policies that might promote both forward contracting and other activities that can reduce electricity procurement costs. These other policies include price cap regulation and earnings sharing regulation, for example (Sappington and Weisman, 2010).³⁵ Future research might analyze the optimal design of targeted and broad incentives to reduce all operating costs, including procurement costs.

Future research might also consider generator asymmetries and alternative forms of generator competition. For example, generators might bid supply functions rather than choose output levels (e.g., Green, 1999; Fabra et al., 2006). Although generators often bid supply functions in practice, Bushnell et al. (2008) find that after accounting for generator forward commitments, observed outcomes in wholesale electricity markets are predicted reasonably well by models of Cournot competition.³⁶

³⁴We are not the only authors to consider nonlinear production costs. See Green (1999) and Anderson and Hu (2008), for example.

³⁵Implementing gain sharing through common reward structures of this sort could conceivably help to establish the credibility and sustainability of the reward structures.

³⁶Willems et al. (2009) report that Cournot models and models with supply functions explain similar fractions of observed price variation in the German electricity sector.

Other elements of real-world settings that might be modeled formally include price-sensitive retail demand, nonlinear industrial demand, vertically-integrated LSEs,³⁷ and negotiated quantities and prices of forward contracts.^{38,39} These considerations may change some of the quantitative conclusions drawn above.⁴⁰ However, they seem unlikely to change the primary qualitative conclusions that forward contracting has significant potential to reduce electricity procurement costs and that modest gain sharing can render LSEs receptive to forward contracting in plausible settings.

³⁷See, for example, Boom and Buehler (2020) and Brown and Sappington (2022a).

³⁸Models of negotiation might allow the LSE to implement some, but not all, of the forward contracting preferred by generators. The models might also account for the fact that LSEs sometimes prefer more extensive forward contracting than generators (Brown and Sappington, 2022b). In such cases, asymmetric rewards and penalties that eliminate forward contracting can raise procurement costs even more than they raise these costs in the settings we have analyzed.

³⁹Negotiated prices can differ from the expected wholesale price. To illustrate, in Powell (1993)'s model, competition among generators compels them to set the price of a forward contract below the expected wholesale price. In Anderson and Hu (2008)'s model, a buyer raises the price of a forward contract above the expected wholesale price to ensure generator participation. Anderson et al. (2007) report that the price of a forward contract often exceeds the average wholesale price of electricity in Australia.

⁴⁰Future research might also model formally the (political) forces that limit a regulator's ability to insulate the regulated firm against penalties when the wholesale price falls (substantially) below the forward price of electricity.

Appendix

Part A of this Appendix presents the proofs of the formal conclusions in the text. Part B provides additional explanation of the baseline parameterization discussed in Section 4. Part C supplements the findings recorded in Table 2 by considering more pronounced variation in model parameters.

A. Proofs of Formal Conclusions in the Text.

Proof of Lemma 1. (3) implies that when ε is realized, Gi 's problem is:

$$\underset{q_i \geq 0}{\text{Maximize}} \quad \pi_i^G(\varepsilon) = w(\varepsilon) [q_i - F_i] + p^f F_i - c_0 q_i - \frac{1}{2} c (q_i)^2. \quad (12)$$

(2) and (12) imply that the necessary conditions for an interior maximum include:

$$\frac{\partial \pi_i^G(\varepsilon)}{\partial q_i} = w(\varepsilon) + [q_i - F_i] \frac{\partial w(\cdot)}{\partial Q} - c_0 - c q_i = 0. \quad (13)$$

Define $Q_{-i} \equiv \sum_{\substack{j=1 \\ j \neq i}}^n q_j$. Then (2) and (13) imply that Gi 's profit-maximizing choice of

$q_i > 0$ is determined by:

$$\begin{aligned} a + \varepsilon - b [q_i + Q_{-i}] - b [q_i - F_i] - c_0 - c q_i &= 0 \\ \Rightarrow [2b + c] q_i &= a + \varepsilon - b Q_{-i} + b F_i - c_0 \\ \Rightarrow q_i &= \frac{1}{2b + c} [a + \varepsilon - c_0 + b F_i] - \frac{b}{2b + c} Q_{-i}. \end{aligned} \quad (14)$$

(14) implies that in equilibrium:

$$\begin{aligned} Q_{-i} &= \sum_{\substack{j=1 \\ j \neq i}}^n \left(\frac{1}{2b + c} [a + \varepsilon - c_0 + b F_j] - \frac{b}{2b + c} Q_{-j} \right) \\ &= \frac{n-1}{2b + c} [a + \varepsilon - c_0] + \frac{b}{2b + c} \sum_{\substack{j=1 \\ j \neq i}}^n F_j - \frac{b}{2b + c} \sum_{\substack{j=1 \\ j \neq i}}^n Q_{-j} \\ &= \frac{n-1}{2b + c} [a + \varepsilon - c_0] + \frac{b}{2b + c} F_{-i} \\ &\quad - \frac{b}{2b + c} [Q_{-1} + \dots + Q_{-(i-1)} + Q_{-(i+1)} + \dots + Q_{-n}] \\ &= \frac{n-1}{2b + c} [a + \varepsilon - c_0] + \frac{b}{2b + c} F_{-i} - \frac{b}{2b + c} [(n-1) q_i + (n-2) Q_{-i}]. \end{aligned} \quad (15)$$

(15) implies:

$$\begin{aligned}
Q_{-i} \left[1 + \frac{b(n-2)}{2b+c} \right] &= \frac{n-1}{2b+c} [a + \varepsilon - c_0] + \frac{b}{2b+c} F_{-i} - \left[\frac{b(n-1)}{2b+c} \right] q_i \\
\Rightarrow Q_{-i} \left[\frac{bn+c}{2b+c} \right] &= \frac{n-1}{2b+c} [a + \varepsilon - c_0] + \frac{b}{2b+c} F_{-i} - \left[\frac{b(n-1)}{2b+c} \right] q_i \\
\Rightarrow Q_{-i} &= \frac{n-1}{bn+c} [a + \varepsilon - c_0] + \frac{b}{bn+c} F_{-i} - \left[\frac{b(n-1)}{bn+c} \right] q_i. \tag{16}
\end{aligned}$$

(14) and (16) imply that in equilibrium:

$$\begin{aligned}
q_i &= \frac{1}{2b+c} [a + \varepsilon - c_0 + b F_i] \\
&\quad - \frac{b}{[2b+c][bn+c]} [(n-1)(a + \varepsilon - c_0) + b F_{-i} - b(n-1) q_i] \\
\Rightarrow q_i \left[1 - \frac{b^2(n-1)}{(2b+c)(bn+c)} \right] &= \frac{bn+c-b[n-1]}{[2b+c][bn+c]} [a + \varepsilon - c_0] \\
&\quad + \frac{b}{2b+c} F_i - \frac{b^2}{[2b+c][bn+c]} F_{-i} \\
\Rightarrow q_i \left[\frac{(2b+c)(bn+c) - b^2(n-1)}{(2b+c)(bn+c)} \right] &= \frac{b+c}{[2b+c][bn+c]} [a + \varepsilon - c_0] \\
&\quad + \frac{b[bn+c]}{[2b+c][bn+c]} F_i - \frac{b^2}{[2b+c][bn+c]} F_{-i}. \tag{17}
\end{aligned}$$

Observe that:

$$\begin{aligned}
[2b+c][bn+c] - b^2[n-1] &= 2b^2n + 2bc + bcn + c^2 - b^2n + b^2 \\
&= b^2n + 2bc + bcn + b^2 + c^2 = b^2[n+1] + c[b(n+2) + c]. \tag{18}
\end{aligned}$$

(17) and (18) imply:

$$q_i(\varepsilon) = \frac{[b+c][a + \varepsilon - c_0] + b[bn+c] F_i - b^2 F_{-i}}{b^2[n+1] + c[b(n+2) + c]}. \tag{19}$$

Because $\sum_{i=1}^n F_{-i} = [n-1] \sum_{i=1}^n F_i$:

$$\sum_{i=1}^n ([bn+c] F_i - b F_{-i}) = [bn+c] \sum_{i=1}^n F_i - b[n-1] \sum_{i=1}^n F_i = [b+c] \sum_{i=1}^n F_i. \tag{20}$$

(19) and (20) imply that in equilibrium:

$$Q(\varepsilon) = \sum_{i=1}^n q_i = \frac{n[b+c][a+\varepsilon-c_0] + b[b+c] \sum_{i=1}^n F_i}{b^2[n+1] + c[b(n+2)+c]}. \quad (21)$$

(2) and (21) imply:

$$w(\varepsilon) = a + \varepsilon - \frac{nb[b+c][a+\varepsilon-c_0] + b^2[b+c] \sum_{i=1}^n F_i}{b^2[n+1] + c[b(n+2)+c]}. \quad (22)$$

Observe that:

$$\begin{aligned} & b^2[n+1] + c[b(n+2)+c] - nb[b+c] \\ &= b^2n + b^2 + bc n + 2bc + c^2 - b^2n - bc n \\ &= b^2 + 2bc + c^2 = [b+c]^2. \end{aligned} \quad (23)$$

(22) and (23) imply:

$$\begin{aligned} w(\varepsilon) &= \frac{[b+c]^2[a+\varepsilon] + nb[b+c]c_0 - b^2[b+c] \sum_{i=1}^n F_i}{b^2[n+1] + c[b(n+2)+c]} \\ &= \frac{[b+c] \left[(b+c)(a+\varepsilon) + nb c_0 - b^2 \sum_{i=1}^n F_i \right]}{b^2[n+1] + c[b(n+2)+c]} \\ \Rightarrow p^f = E\{w(\varepsilon)\} &= \frac{[b+c] \left[(b+c)(a + E\{\varepsilon\}) + nb c_0 - b^2 \sum_{i=1}^n F_i \right]}{b^2[n+1] + c[b(n+2)+c]}. \quad \blacksquare \quad (24) \end{aligned}$$

Proof of Lemma 2. (3) implies that because $p^f = E\{w(\varepsilon)\}$:

$$\begin{aligned} E\{\pi_i^G(\varepsilon)\} &= E\left\{ w(\varepsilon) q_i(\varepsilon) - c_0 q_i(\varepsilon) - \frac{c}{2} [q_i(\varepsilon)]^2 + [p^f - w(\varepsilon)] F_i \right\} \\ &= E\left\{ w(\varepsilon) q_i(\varepsilon) - c_0 q_i(\varepsilon) - \frac{c}{2} [q_i(\varepsilon)]^2 \right\}. \end{aligned} \quad (25)$$

Lemma 1 implies:

$$\begin{aligned} w(\varepsilon) q_i(\varepsilon) &= \frac{b+c}{D^2} \left\{ [b+c][a+\varepsilon-c_0] + b[b n + c] F_i - b^2 F_{-i} \right\} \\ &\quad \cdot \left[(b+c)(a+\varepsilon) + nb c_0 - b^2 \sum_{i=1}^n F_i \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{b+c}{D^2} \left\{ [b+c]^2 [a+\varepsilon] [a+\varepsilon-c_0] + n b c_0 [b+c] [a+\varepsilon-c_0] \right. \\
&\quad - b^2 [b+c] [a+\varepsilon-c_0] \sum_{i=1}^n F_i + b [b+c] [b n + c] [a+\varepsilon] F_i \\
&\quad + b^2 n [b n + c] c_0 F_i - b^3 [b n + c] F_i \sum_{i=1}^n F_i \\
&\quad \left. - b^2 [b+c] [a+\varepsilon] F_{-i} - b^3 n c_0 F_{-i} + b^4 F_{-i} \sum_{i=1}^n F_i \right\}. \tag{26}
\end{aligned}$$

(26) implies that when $E\{\varepsilon\} = 0$:

$$\begin{aligned}
E\{w(\varepsilon) q_i(\varepsilon)\} &= \frac{b+c}{D^2} \left\{ [b+c]^2 a [a-c_0] + [b+c]^2 E\{\varepsilon^2\} + n b c_0 [b+c] [a-c_0] \right. \\
&\quad - b^2 [b+c] [a-c_0] \sum_{i=1}^n F_i + a b [b+c] [b n + c] F_i \\
&\quad + b^2 n [b n + c] c_0 F_i - b^3 [b n + c] F_i \sum_{i=1}^n F_i \\
&\quad \left. - a b^2 [b+c] F_{-i} - b^3 n c_0 F_{-i} + b^4 F_{-i} \sum_{i=1}^n F_i \right\} \\
\Rightarrow \frac{\partial E\{w(\varepsilon) q_i(\varepsilon)\}}{\partial F_i} &= \frac{b+c}{D^2} \left\{ - b^2 [b+c] [a-c_0] + a b [b+c] [b n + c] \right. \\
&\quad \left. + b^2 n [b n + c] c_0 - b^3 [b n + c] \left[F_i + \sum_{i=1}^n F_i \right] + b^4 F_{-i} \right\}. \tag{27}
\end{aligned}$$

Lemma 1 implies:

$$\begin{aligned}
-c_0 E\{q_i(\varepsilon)\} &= -\frac{c_0}{D} E\{[b+c] [a+\varepsilon-c_0] + b [b n + c] F_i - b^2 F_{-i}\} \\
\Rightarrow \frac{-c_0 \partial E\{q_i(\varepsilon)\}}{\partial F_i} &= -\frac{c_0}{D} b [b n + c]. \tag{28}
\end{aligned}$$

Lemma 1 also implies:

$$\begin{aligned}
-\frac{c}{2} E\{[q_i(\varepsilon)]^2\} &= -\frac{c}{2 D^2} E\left\{ ([b+c] [a+\varepsilon-c_0] + b [b n + c] F_i - b^2 F_{-i}) \right. \\
&\quad \left. \cdot ([b+c] [a+\varepsilon-c_0] + b [b n + c] F_i - b^2 F_{-i}) \right\}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{c}{2D^2} E \left\{ [b+c]^2 [a+\varepsilon-c_0]^2 + 2b[bn+c][b+c][a+\varepsilon-c_0]F_i \right. \\
&\quad \left. - 2b^2[b+c][a+\varepsilon-c_0]F_{-i} - 2b^3[bn+c]F_i F_{-i} \right. \\
&\quad \left. + b^2[bn+c]^2(F_i)^2 + b^4(F_{-i})^2 \right\} \\
&= -\frac{c}{2D^2} \left\{ [b+c]^2[a-c_0]^2 + [b+c]^2 E\{\varepsilon^2\} \right. \\
&\quad \left. + 2b[bn+c][b+c][a-c_0]F_i - 2b^2[b+c][a-c_0]F_{-i} \right. \\
&\quad \left. - 2b^3[bn+c]F_i F_{-i} + b^2[bn+c]^2(F_i)^2 + b^4(F_{-i})^2 \right\} \\
&\Rightarrow -\frac{c}{2} \frac{\partial E\{[q_i(\varepsilon)]^2\}}{\partial F_i} = -\frac{c}{2D^2} \left\{ 2b[bn+c][b+c][a-c_0] \right. \\
&\quad \left. - 2b^3[bn+c]F_{-i} + 2b^2[bn+c]^2 F_i \right\}. \quad (29)
\end{aligned}$$

(25), (27), (28), and (29) imply:

$$\begin{aligned}
\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i} &= \frac{b+c}{D^2} \left\{ -b^2[b+c][a-c_0] + ab[b+c][bn+c] \right. \\
&\quad \left. + b^2n[bn+c]c_0 - b^3[bn+c][F_i + \sum_{i=1}^n F_i] + b^4F_{-i} \right\} - \frac{c_0}{D} b[bn+c] \\
&\quad - \frac{c}{2D^2} \left\{ 2b[bn+c][b+c][a-c_0] - 2b^3[bn+c]F_{-i} + 2b^2[bn+c]^2 F_i \right\}. \quad (30)
\end{aligned}$$

(30) implies that Gi 's profit-maximizing choice of F_i is determined by:

$$\begin{aligned}
&[b+c] \left\{ -b^2[b+c][a-c_0] + ab[b+c][bn+c] \right. \\
&\quad \left. + b^2n[bn+c]c_0 - b^3[bn+c][F_i + \sum_{i=1}^n F_i] + b^4F_{-i} \right\} \\
&\quad - c_0 b[bn+c][b^2(n+1) + c(b[n+2] + c)] \\
&\quad - \frac{c}{2} \left\{ 2b[bn+c][b+c][a-c_0] - 2b^3[bn+c]F_{-i} + 2b^2[bn+c]^2 F_i \right\} = 0 \\
&\Leftrightarrow [b+c] \left\{ -b^2[b+c][a-c_0] + ab[b+c][bn+c] \right.
\end{aligned}$$

$$\begin{aligned}
& + b^2 n [b n + c] c_0 - b^3 [b n + c] F_{-i} + b^4 F_{-i} \Big\} \\
& - c_0 b [b n + c] [b^2 (n + 1) + c (b [n + 2] + c)] \\
& - \frac{c}{2} \left\{ 2 b [b n + c] [b + c] [a - c_0] - 2 b^3 [b n + c] F_{-i} \right\} \\
& = [b n + c] b^2 [2 b (b + c) + c (b n + c)] F_i \\
\Leftrightarrow X F_i &= Y - b^5 [n - 1] F_{-i}, \tag{31}
\end{aligned}$$

where $X \equiv b^2 [b n + c] [2 b (b + c) + c (b n + c)]$ and

$$\begin{aligned}
Y &\equiv [b + c] \left\{ -b^2 [b + c] [a - c_0] + a b [b + c] [b n + c] + b^2 n [b n + c] c_0 \right\} \\
& - c_0 b [b n + c] [b^2 (n + 1) + c (b [n + 2] + c)] \\
& - b c [b n + c] [b + c] [a - c_0]. \tag{32}
\end{aligned}$$

The coefficient on F_{-i} in (31) reflects the fact that:

$$\begin{aligned}
& [b + c] [b^4 - b^3 (b n + c)] + b^3 c [b n + c] \\
& = -b^3 [b (n - 1) + c] [b + c] + b^3 c [b n + c] \\
& = b^3 \{ c [b n + c] - [b + c] [b (n - 1) + c] \} \\
& = b^3 \{ b c n + c^2 - b^2 [n - 1] - b c - b c [n - 1] - c^2 \} \\
& = b^3 \{ b c n - b^2 n + b^2 - b c - b c n + b c \} = -b^5 [n - 1].
\end{aligned}$$

(32) implies that $Y = y_a a + y_0 c_0$ where:

$$\begin{aligned}
y_a &= b [b + c] \{ [b + c] [b n + c] - b [b + c] - c [b n + c] \} \\
&= b [b + c] \{ b [b n + c] - b [b + c] \} \\
&= b^2 [b + c] [b n + c - (b + c)] = b^3 [b + c] [n - 1]; \\
y_0 &= b^2 [b + c]^2 + b^2 n [b + c] [b n + c] + b c [b + c] [b n + c] \\
& - b^3 [b n + c] [n + 1] - b^2 c [b n + c] [n + 2] - b c^2 [b n + c] \\
&= b^2 [b + c]^2 + b [b n + c] [b n (b + c) + c (b + c) - b^2 (n + 1) - b c (n + 2) - c^2] \\
&= b^2 [b + c]^2 + b [b n + c] [b^2 n + b c n + b c + c^2 - b^2 n - b^2 - b c n - 2 b c - c^2]
\end{aligned}$$

$$\begin{aligned}
&= b^2 [b+c]^2 + b [bn+c] [-bc-b^2] = b^2 [b+c]^2 - b^2 [bn+c] [b+c] \\
&= b^2 [b+c] [b+c-bn-c] = -b^3 [b+c] [n-1].
\end{aligned} \tag{33}$$

(33) implies:

$$Y = b^3 [b+c] [n-1] [a-c_0]. \tag{34}$$

(2), (31), (32), and (34) imply that at a symmetric equilibrium:

$$\begin{aligned}
X F_i &= Y - b^5 [n-1]^2 F_i \Leftrightarrow [X + b^5 (n-1)^2] F_i = Y \\
\Leftrightarrow F_i &= \frac{Y}{X + b^5 [n-1]^2} = \frac{b^3 [b+c] [n-1] [a-c_0]}{b^2 [bn+c] [2b(b+c) + c(bn+c)] + b^5 [n-1]^2} \\
&= \frac{b [b+c] [n-1] \left[\frac{a^I + \bar{Q}}{b^I} - c_0 \right]}{[bn+c] [2b(b+c) + c(bn+c)] + b^3 [n-1]^2} \\
&= \frac{b^2 [b+c] [n-1] [a^I + \bar{Q} - b^I c_0]}{[bn+c] [2b(b+c) + c(bn+c)] + b^3 [n-1]^2}.
\end{aligned} \tag{35}$$

It is apparent from (35) that $\frac{\partial F_i}{\partial a^I} > 0$, $\frac{\partial F_i}{\partial \bar{Q}} > 0$, and $\frac{\partial F_i}{\partial c_0} < 0$. Furthermore:

$$\begin{aligned}
\frac{\partial F_i}{\partial c} &\stackrel{s}{=} [bn+c] [2b(b+c) + c(bn+c)] + b^3 [n-1]^2 \\
&\quad - [b+c] \{ [bn+c] [2b+bn+2c] + 2b[b+c] + c[bn+c] \} \\
&= 2b[b+c] [bn+c] + c[bn+c]^2 + b^3 [n-1]^2 - 2b[b+c] [bn+c] \\
&\quad - bn[b+c] [bn+c] - 2c[b+c] [bn+c] - 2b[b+c]^2 - c[b+c] [bn+c] \\
&= c[bn+c]^2 + b^3 [n-1]^2 - bn[b+c] [bn+c] - 3c[b+c] [bn+c] - 2b[b+c]^2 \\
&= b^3 [n-1]^2 - 2b[b+c]^2 + [bn+c] \{ c[bn+c] - bn[b+c] - 3c[b+c] \} \\
&= b^3 [n-1]^2 - 2b[b+c]^2 + [bn+c] [bcn+c^2 - b^2n - bcn - 3bc - 3c^2] \\
&= b^3 [n-1]^2 - 2b[b+c]^2 - [bn+c] [b^2n + 3bc + 2c^2] \\
&= b^3 [n^2 - 2n + 1] - 2b [b^2 + 2bc + c^2] - bn [b^2n + 3bc + 2c^2] \\
&\quad - c [b^2n + 3bc + 2c^2] \\
&= b^3 n^2 - 2b^3 n + b^3 - 2b^3 - 4b^2 c - 2bc^2 - b^3 n^2 - 3b^2 cn - 2bc^2 n \\
&\quad - b^2 cn - 3bc^2 - 2c^3
\end{aligned}$$

$$= -2b^3n - b^3 - 4b^2c - 2bc^2 - 3b^2cn - 2bc^2n - b^2cn - 3bc^2 - 2c^3 < 0.$$

Because $b^I = \frac{1}{b}$, (35) implies:

$$\begin{aligned} \frac{\partial F_i}{\partial b} &\stackrel{s}{=} \left\{ [bn + c][2b(b + c) + c(bn + c)] + b^3[n - 1]^2 \right\} \\ &\quad \cdot \left\{ 2b[b + c][n - 1][a^I + \overline{Q} - b^I c_0] + b^2[n - 1][a^I + \overline{Q} - b^I c_0] \right. \\ &\quad \left. + \left[\frac{c_0}{b^2} \right] b^2[b + c][n - 1] \right\} \\ &\quad - \left\{ b^2[b + c][n - 1][a^I + \overline{Q} - b^I c_0] \right\} \\ &\quad \cdot \left\{ n[2b(b + c) + c(bn + c)] + [bn + c][4b + 2c + cn] + 3b^2[n - 1]^2 \right\} \\ &= \left\{ 2b[b + c][bn + c] + c[bn + c]^2 + b^3[n - 1]^2 \right\} \\ &\quad \cdot \left\{ 2b[b + c][n - 1][a^I + \overline{Q} - b^I c_0] + b^2[n - 1][a^I + \overline{Q} - b^I c_0] \right. \\ &\quad \left. + c_0[b + c][n - 1] \right\} \\ &\quad - \left\{ b^2[b + c][n - 1][a^I + \overline{Q} - b^I c_0] \right\} \\ &\quad \cdot \left\{ 2bn[b + c] + [bn + c][4b + 2c + 2cn] + 3b^2[n - 1]^2 \right\} \\ &= 4b^2[bn + c][b + c]^2[n - 1][a^I + \overline{Q} - b^I c_0] \\ &\quad + 2b^3[bn + c][b + c][n - 1][a^I + \overline{Q} - b^I c_0] \\ &\quad + 2bc_0[bn + c][b + c]^2[n - 1] \\ &\quad + 2bc[bn + c]^2[b + c][n - 1][a^I + \overline{Q} - b^I c_0] \\ &\quad + b^2c[bn + c]^2[n - 1][a^I + \overline{Q} - b^I c_0] \\ &\quad + cc_0[bn + c]^2[b + c][n - 1] \\ &\quad + 2b^4[b + c][n - 1]^3[a^I + \overline{Q} - b^I c_0] \\ &\quad + b^5[n - 1]^3[a^I + \overline{Q} - b^I c_0] + b^3c_0[b + c][n - 1]^3 \\ &\quad - 2b^3n[b + c]^2[n - 1][a^I + \overline{Q} - b^I c_0] \end{aligned}$$

$$\begin{aligned}
& -b^2[4b+2c+2cn][bn+c][b+c][n-1][a^I+\overline{Q}-b^Ic_0] \\
& -3b^4[b+c][n-1]^3[a^I+\overline{Q}-b^Ic_0] \\
= & [a^I+\overline{Q}-b^Ic_0][n-1] \\
& \cdot \left\{ 4b^2[bn+c][b+c]^2 + 2b^3[bn+c][b+c] + 2bc[bn+c]^2[b+c] \right. \\
& + b^2c[bn+c]^2 + 2b^4[b+c][n-1]^2 + b^5[n-1]^2 - 2b^3n[b+c]^2 \\
& \left. - b^2[4b+2c+2cn][bn+c][b+c] - 3b^4[b+c][n-1]^2 \right\} \\
& + 2bc_0[bn+c][b+c]^2[n-1] + cc_0[bn+c]^2[b+c][n-1] \\
& + b^3c_0[b+c][n-1]^3 > 0. \tag{36}
\end{aligned}$$

The inequality in (36) implies that $\frac{\partial F_i}{\partial b^I} < 0$ because $b^I = \frac{1}{b}$. The inequality holds because $a^I + \overline{Q} - b^Ic_0 > 0$, $n > 1$, and:

$$\begin{aligned}
& 4b^2[bn+c][b+c]^2 + 2b^3[bn+c][b+c] \\
& + 2bc[bn+c]^2[b+c] + b^2c[bn+c]^2 \\
& + 2b^4[b+c][n-1]^2 + b^5[n-1]^2 - 2b^3n[b+c]^2 \\
& - 4b^3[bn+c][b+c] - 2cb^2[bn+c][b+c] \\
& - 2cnb^2[bn+c][b+c] - 3b^4[b+c][n-1]^2 \\
= & [b+c] \left\{ 2b^3[bn+c] + 2bc[bn+c]^2 + 2b^4[n-1]^2 \right. \\
& \left. - 4b^3[bn+c] - 2cb^2[bn+c] - 2cnb^2[bn+c] - 3b^4[n-1]^2 \right\} \\
& + [b+c]^2[4b^2(bn+c) - 2b^3n] + b^2c[bn+c]^2 + b^5[n-1]^2 \\
= & [b+c] \left\{ 2bc[bn+c]^2 - 2b^3[bn+c] - 2cb^2[bn+c] \right. \\
& \left. - 2cnb^2[bn+c] - b^4[n-1]^2 \right\} \\
& + [b+c]^2[2b^3n + 4b^2c] + b^2c[bn+c]^2 + b^5[n-1]^2 \\
= & [b+c] \left\{ 2bc[b^2n^2 + 2bnc + c^2] - 2b^4n - 2b^3c - 2cb^3n - 2c^2b^2 \right.
\end{aligned}$$

$$\begin{aligned}
& - 2cn^2b^3 - 2c^2nb^2 - b^4[n^2 - 2n + 1] \Big\} \\
& + [b+c]^2[2b^3n + 4b^2c] + b^2c[bn+c]^2 + b^5[n-1]^2 \\
= & [b+c] \Big\{ 2b^3cn^2 + 4b^2c^2n + 2bc^3 - 2b^4n - 2b^3c - 2cb^3n - 2c^2b^2 \\
& - 2cn^2b^3 - 2c^2nb^2 - b^4n^2 + 2nb^4 - b^4 \Big\} \\
& + [b+c]^2[2b^3n + 4b^2c] + b^2c[bn+c]^2 + b^5[n-1]^2 \\
= & [b+c][2b^2c^2n + 2bc^3 - 2b^3c - 2cb^3n - 2c^2b^2 - b^4n^2 - b^4] \\
& + [b^2 + 2bc + c^2][2b^3n + 4b^2c] + b^2c[bn+c]^2 + b^5[n-1]^2 \\
= & 2b^3c^2n + 2b^2c^3 - 2b^4c - 2cb^4n - 2c^2b^3 - b^5n^2 - b^5 \\
& + 2b^2c^3n + 2bc^4 - 2b^3c^2 - 2c^2b^3n - 2c^3b^2 - b^4n^2c - b^4c \\
& + 2b^5n + 4b^4c + 4b^4nc + 8b^3c^2 + 2b^3nc^2 + 4b^2c^3 \\
& + b^2c[b^2n^2 + 2bnc + c^2] + b^5[n^2 - 2n + 1] \\
= & 2b^3c^2n + 2b^2c^3 - 2b^4c - 2cb^4n - 2c^2b^3 - b^5n^2 - b^5 \\
& + 2b^2c^3n + 2bc^4 - 2b^3c^2 - 2c^2b^3n - 2c^3b^2 - b^4n^2c - b^4c \\
& + 2b^5n + 4b^4c + 4b^4nc + 8b^3c^2 + 2b^3nc^2 + 4b^2c^3 \\
& + b^4n^2c + 2b^3nc^2 + b^2c^3 + b^5n^2 - 2b^5n + b^5 \\
= & b^3c^2[2n - 2 - 2 - 2n + 8 + 2n + 2n] + b^2c^3[2 + 2n - 2 + 4] \\
& + b^4c[-2 - 2n - n^2 - 1 + 4 + 4n + n^2] \\
& + b^5[-n^2 - 1 + 2n + n^2 - 2n + 1] + 2bc^4 + b^2c^3 \\
= & b^3c^2[4n + 4] + b^2c^3[2n + 4] + b^4c[2n + 1] + 2bc^4 + b^2c^3 > 0. \blacksquare
\end{aligned}$$

Proof of Corollary 1. (35) implies:

$$\begin{aligned}
\lim_{c \rightarrow 0} F_i &= \frac{b^3[n-1][a^I + \overline{Q} - b^I c_0]}{bn[2b^2] + b^3[n-1]^2} = \frac{[n-1][a^I + \overline{Q} - b^I c_0]}{2n + [n-1]^2} \\
&= \frac{[n-1][a^I + \overline{Q} - b^I c_0]}{2n + n^2 - 2n + 1} = \frac{n-1}{n^2+1} [a^I + \overline{Q} - b^I c_0].
\end{aligned} \tag{37}$$

Observe that:

$$\frac{2-1}{(2)^2+1} = \frac{1}{5} \quad \text{and} \quad \frac{3-1}{(3)^2+1} = \frac{2}{10} = \frac{1}{5}.$$

Therefore, (37) implies that if $c = 0$, then $F_i = \frac{1}{5} [a^I + \bar{Q} - b^I c_0]$ when $n = 2$ and when $n = 3$.

(37) also implies that when $c = 0$:

$$\begin{aligned} \frac{\partial F_i}{\partial n} &\stackrel{s}{=} n^2 + 1 - 2n[n-1] = -n^2 + 2n + 1 \\ \Rightarrow \frac{\partial F_i}{\partial n} &\begin{matrix} \geq \\ \leq \end{matrix} 0 \Leftrightarrow g(n) \equiv n^2 - 2n - 1 \begin{matrix} \leq \\ \geq \end{matrix} 0. \end{aligned} \quad (38)$$

The roots of the equation $g(n) = 0$ are:

$$n = \frac{1}{2} [2 \pm \sqrt{4+4}] = 1 \pm \sqrt{2}. \quad (39)$$

(38) and (39) imply:

$$g(n) \begin{cases} < 0 & \text{for } n \in (1, 1 + \sqrt{2}) \\ > 0 & \text{for } n > 1 + \sqrt{2}. \end{cases} \quad (40)$$

(38) and (40) imply that when $c = 0$:

$$\frac{\partial F_i}{\partial n} \begin{cases} > 0 & \text{for } n \in (1, 1 + \sqrt{2}) \\ = 0 & \text{for } n = 1 + \sqrt{2} \\ < 0 & \text{for } n > 1 + \sqrt{2}. \end{cases} \Rightarrow \frac{\partial F_i}{\partial n} < 0 \text{ for all } n \geq 3.$$

(37) further implies that when $c = 0$:

$$\begin{aligned} \frac{\partial}{\partial n} \left(\sum_{j=1}^n F_j \right) &\stackrel{s}{=} \frac{\partial}{\partial n} \left(\frac{n[n-1]}{n^2+1} \right) \stackrel{s}{=} [n^2+1][2n-1] - 2n^2[n-1] \\ &= 2n^3 + 2n - n^2 - 1 - 2n^3 + 2n^2 = n^2 + 2n - 1 > 0 \text{ for all } n \geq 2. \end{aligned} \quad (41)$$

The strict inequality in (41) holds because $n^2 + 2n - 1$ is a strictly increasing function of n for all $n \geq 0$ that is strictly positive at $n = 2$. ■

Proof of Proposition 1. Lemma 1 implies that when $E\{\varepsilon\} = 0$:

$$E\{w(\varepsilon, 0)\varepsilon\} = E\{w(\varepsilon, F)\varepsilon\} = \frac{[b+c]^2 E\{\varepsilon^2\}}{D}. \quad (42)$$

(42) and Lemma 1 imply that when $E\{\varepsilon\} = 0$ and $p^f = E\{w(\varepsilon, F)\}$, the difference between B 's expected total cost in the absence of forward contracting (F) and in its presence

is:

$$\begin{aligned}
& E \{ w(\varepsilon, 0) [\bar{Q} + b^I \varepsilon] \} - (E \{ w(\varepsilon, F) [\bar{Q} + b^I \varepsilon] \} + [p^f - E \{ w(\varepsilon, F) \}] F) \\
&= E \{ w(\varepsilon, 0) [\bar{Q} + b^I \varepsilon] \} - E \{ w(\varepsilon, F) [\bar{Q} + b^I \varepsilon] \} \\
&= \bar{Q} [E \{ w(\varepsilon, 0) \} - E \{ w(\varepsilon, F) \}] + b^I [E \{ w(\varepsilon, 0) \varepsilon \} - E \{ w(\varepsilon, F) \varepsilon \}] \\
&= \bar{Q} [E \{ w(\varepsilon, 0) \} - E \{ w(\varepsilon, F) \}] = \frac{b^2 [b + c] \bar{Q}}{D} \sum_{i=1}^n F_i. \blacksquare
\end{aligned}$$

B. Elements of the Baseline Parameterization.

As noted in Section 4, the value of b^I is set to ensure that the price elasticity of industrial demand is -0.33 when $w = \$46.72$. The average hourly sales of electricity in Alberta in 2020 was approximately 9,500 MWhs, of which 83% (7,885 MWhs) was sold to commercial and industrial consumers, and 17% (1,615 MWhs) was sold to residential consumers (Leach et al., 2020). Therefore, using (1), the value of b^I in the baseline parameterization is determined by:

$$\begin{aligned}
\frac{\partial Q^I}{\partial w} \frac{w}{Q^I} &= -0.33 \Leftrightarrow [-b^I] \frac{46.72}{7,885} = -0.33 \\
\Leftrightarrow b^I &= [0.33] \frac{7,885}{46.72} \approx 55.6945.
\end{aligned} \tag{43}$$

The entries for a^I and $\bar{Q}$ in Table 1 now follow from (1) because:

$$a^I - b^I w + \bar{Q} = 9,500 \Leftrightarrow a^I + \bar{Q} = 9,500 + [55.6945][46.72] \approx 12,102.05. \tag{44}$$

Because 17% of electricity sales in Alberta in 2020 were delivered to retail consumers, $\bar{Q} = 0.17[9,500] = 1,615$. Therefore, (44) implies that $a^I = 12,102.05 - 1,615 = 10,487.05$.

The baseline value of $c_0 = 15$ reflects the estimated marginal cost of generating electricity with a base-load combined cycle gas turbine unit in Alberta in 2020. This estimate reflects in part the fact that the average price of natural gas was \$2.11 (GJ/MWh) in Alberta in 2020.⁴¹ Furthermore, combined cycle natural gas turbine units that typically operate as base-load generators in Alberta have heat rates of approximately 6.5 GJ/MWh (Alberta Electric System Operator, 2021). These statistics imply that the relevant marginal fuel cost is approximately \$13.715 ($= 2.11 \times 6.5$). Additional variable operating and maintenance (O&M) costs for units of this type are approximately \$2.55 MWh (United States Energy

⁴¹Alberta Natural Gas Exchange (<https://www.theice.com/ngx/overview>).

Information Administration [EIA], 2020, Table 2, Case 8). Brown et al. (2018) estimate that these units secure a credit for environmental compliance of approximately \$1.16/MWh. Together, these fuel costs, O&M costs, and environmental compliance credits amount to an estimated marginal cost of \$15.105/MWh ($= 13.715 + 2.55 - 1.16$), which we round to \$15/MWh.

The value of c is set to ensure that if the electricity market were perfectly competitive, the equilibrium wholesale price would be \$35/MWh. This price reflects the marginal cost of generating electricity with a peaker coal-to-gas turbine unit in Alberta in 2020. The heat rate of such units (which typically operate during periods of mid-to-high demand) is approximately 12.5 GJ/MWh (Alberta Electric System Operator, 2021). Variable O&M costs for these units are approximately \$4.70/MWh (EIA, 2020, Table 2, Case 5). Brown et al. (2018) estimate the environmental compliance cost for these units to be approximately \$4.96/MWh. Together, these fuel, O&M, and environmental compliance costs amount to an estimated marginal cost of \$36.035/MWh ($= 12.5 \times 2.11 + 4.70 + 4.96$), which we round to \$35/MWh.

Under perfect competition, generator Gi with cost function $C(q) = c_0 q + \frac{1}{2} c q^2$ takes the wholesale price (w) as given and increases its output to the point where:

$$w = c_0 + c q \Rightarrow q_i^S(w) = \frac{1}{c} [w - c_0]. \quad (45)$$

The market supply function, $Q^S(w)$, is the horizontal sum of the generators' supply functions. Therefore, (45) implies:

$$Q^S(w) = \sum_{i=1}^n q_i^S(w) = \frac{n}{c} [w - c_0]. \quad (46)$$

The perfectly competitive wholesale price (w^c) is the price that equates expected demand and perfectly competitive market supply. From (1) and (46):

$$\begin{aligned} Q^S(w^c) = E\{Q(\cdot)\} &\Leftrightarrow \frac{n}{c} [w^c - c_0] = a^I - b^I w^c + \bar{Q} \\ \Leftrightarrow \left[b^I + \frac{n}{c} \right] w^c &= a^I + \bar{Q} + \frac{n}{c} c_0 \Leftrightarrow w^c = \frac{c}{b^I c + n} \left[a^I + \bar{Q} + \frac{n}{c} c_0 \right]. \end{aligned} \quad (47)$$

(47) implies that, given the identified values for b^I , n , a^I , $\bar{Q}$, and c_0 , the value of c in the baseline parameterization is determined by:

$$\frac{c}{55.6945 c + 4} \left[12,102.05 + \frac{4}{c} (15) \right] = 35 \Rightarrow c \approx 0.008.$$

Section 4 also considers the setting in which residential demand accounts for 39.3% of aggregate demand. As in (44), when aggregate market demand is 9,500 and $w = 46.72$:

$$a^I + \bar{Q} = 9,500 + b^I [46.72]. \quad (48)$$

Furthermore, as in (43), when the price elasticity of industrial demand is -0.33 at $w = 46.72$:

$$\frac{\partial Q^I}{\partial w} \frac{w}{Q^I} = -0.33 \Leftrightarrow b^I = 0.33 \left[\frac{a^I - b^I (46.72)}{46.72} \right]. \quad (49)$$

When the ratio of expected residential demand to expected aggregate demand is 0.393:

$$\frac{\bar{Q}}{a^I - b^I [46.72] + \bar{Q}} = 0.393. \quad (50)$$

The values of a^I , b^I , and $\bar{Q}$ that solve (48) – (50) are $a^I = 7,669.445$, $b^I = 40.731$, and $\bar{Q} = 3,733.50$. When these parameter values prevail along with the values for n , c_0 , c , $\underline{\varepsilon}$, $\bar{\varepsilon}$, and σ_ε specified in Table 1, the equilibrium forward contracting identified in Lemma 2 reduces L 's expected variable cost below its level in the absence of forward contracting by 36.76%.

C. Additional Comparative Static Analyses.

Table A1 presents equilibrium outcomes corresponding to those in Table 2 as the values of $\bar{Q}$, a^I , b^I , c_0 , and c decline and increase by 50% below and above their values in the baseline parameterization. The entries in Table A1 provide further evidence that forward contracting continues to induce a substantial reduction in L 's expected variable cost as parameter values diverge substantially from their levels in Table 1.

Baseline Change	$E\{w\}_{F=0}$	$E\{LCost\}_{F=0}$	F	$E\{w\}$	$E\{Q\}$	$\% \Delta^{LC}$
None	68.52	110,691.75	6,394.75	47.41	9,461.79	30.80
$\bar{Q} = 807.5$	64.68	52,267.11	5,936.43	45.08	8,783.64	30.28
$\bar{Q} = 2,442.5$	72.35	173,310.96	6,853.08	49.73	10,139.93	31.26
$a^I = 5,243.525$	43.61	70,467.10	3,418.61	32.32	5,058.24	25.87
$a^I = 15,730.575$	93.42	150,916.40	9,370.89	62.49	13,865.33	33.11
$b^I = 27.8475$	113.03	182,573.34	7,370.66	62.32	10,366.65	44.86
$b^I = 83.5425$	53.06	85,730.73	5,573.57	41.27	8,654.43	22.21
$c_0 = 7.5$	63.00	101,783.62	6,631.84	41.11	9,812.58	34.74
$c_0 = 22.5$	74.03	119,599.88	6,157.67	53.71	9,110.99	27.45
$c = 0.004$	62.36	100,746.95	7,095.37	37.97	9,987.38	39.10
$c = 0.012$	74.54	120,423.35	5,762.57	56.29	8,967.20	24.48

Table A1. Effects of Additional Changes in Baseline Parameter Values

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