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Country-Based Investing with Exchange Rate and Reserve Currency

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Abstract

This study examines how style investing impact correlations in a small and large economy, with exchange rate risk, and a reserve currency. The results show that style investing increases correlations in both economies, but more so in the smaller market. The impact of style investing on either country's correlations depends nonlinearly on the volatility of the exchange rate and the strength of the reserve currency effect. Higher levels of risk aversion amplify the impact of style investing on correlations. Imprecise signals and country preferences increase correlation distortions. The results have risk management implications for portfolio diversification.

Keywords: Style investing; International Markets; Portfolio Diversification; Return Correlations; International Markets.

JEL Classification Codes: G1, G11, G12.

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Introduction

The empirical literature has provided evidence consistent with investors' tendency to follow styles in designing their portfolio (e.g., Kumar (2009)). Styles are securities groupings that are not necessarily based on firm characteristics related to fundamental risk factors (Merton et al. (1973)).[1] Strong demand for categorization of securities in the financial market is also evident by a large number of mutual funds and exchange-traded funds (ETFs) following specific investment styles or index investing.

A large body of evidence shows that prices might move together for reasons that appear to be unrelated to fundamentals, with shocks to investors' demand explaining substantial price comovements across securities. Style investing, especially in the form of index investing, is one of the causes of these demand shocks (e.g., Greenwood and Thesmar (2011), Anton and Polk (2014)). Hence, style investing might increase within-style correlations.[2]

An interest in evaluating the effect of style investing on correlations for markets of different sizes, with exchange rate risk, is prompted by the strong surge in country-based investing that has marked the last two decades (e.g., Israel and Maloney (2014), Ben-David et al. (2017)). This increased popularity was facilitated by the availability of country or region-focused ETFs.[3] Style investing based on recognized risk premia (e.g., strategies long in value equities and short in growth equities), can also result in country-based investing (e.g., Israel and Maloney (2014)). The reason is that, typically, countries are assigned to a style (e.g., value countries) based on aggregated measures.

Country-based investing has resulted in the shuffling of large amounts of wealth among country- or region-based portfolios tracking popular market indexes (e.g., the S&P 500 index, MSCI indexes.). Consistently, many

[1]Country-based investment styles might also arise from categorization due to limitations in attention span (e.g., Peng and Xiong (2006)) or in response to both limited cognition and limited data (Al-Najjar and Pai (2014)).

[2]Cross-country variation in within-country correlation has been linked to a variety of explanations, ranging from institutional differences (Morck et al. (2000)), degrees of capital market openness (Li et al. (2004)), lack of transparency at the firm level (Jin and Myers (2006)), and limits to arbitrage (e.g., Bris et al. (2007)), and correlated beliefs (David and Simonovska (2016)).

[3]Early contributions (e.g., Bekaert and Urias (1999)) already noted that without low-cost investing vehicles able to replicate country indexes, investing in emerging was unlikely to offer significant diversification benefits. According to Miffre (2007), international country ETFs offer such investment opportunity.

country ETFs are on the list of Top 20 funds, by traded volume. As of the end of 2019, the largest ETFs cover US securities in terms of assets under management, but runner-up ETFs focus on geographic areas covering several non-US markets.[4] Popular choices among investors have been country-based ETF for Brazil, Japan, China, Taiwan, India, Hong Kong, Mexico, Germany and South Korea, where the number of securities grouped by these ETFs varies significantly across countries and regions. Economies smaller than the ones just mentioned are often bundled into regional indexes by international style investors. The index weights on each country are fairly constant over time. Hence, demand shocks for a region-based international fund transmit rather rigidly to smaller markets (Jotikasthira et al. (2012), Brooks and Del Negro (2005)).[5]

The theoretical model of Barberis and Shleifer (2003) predicts that the correlation between securities grouped into the same category should rise above the level implied by fundamentals, due to the demand pressure of style investors. Their analysis, however, abstracts from key features of international investing, namely the exchange rate and differences in market size. Consistently, the vast majority of the empirical studies related to the insights yielded in Barberis and Shleifer (2003) focuses on index membership changes for same-currency same-country equity indexes.[6]

Motivated by the rise of country-based international investing, this study extends the model of Barberis and Shleifer (2003) in several directions. First, we account for exchange trade risk, which is modeled by a risk factor capturing currency hedging costs. These costs are correlated with country-specific risk factors, which is intuitive. Second, the exchange rate risk factor is correlated to a global risk factor to model a reserve currency effect, where the

[4]Source: Morningstar.

[5]Barberis and Shleifer (2003) focus on securities groups of the same numerosity, an approach that is suitable to the evaluation of the effect of style investing across asset groupings of similar size, like, for example, US equities and US (liquid) corporate bonds. However, assuming that styles include a similar number of assets is limiting, especially when analyzing the implication of style investing for international financial markets. As mentioned, they also do not consider the exchange rate.

[6]Excess comovement has been documented for S&P500 index additions and deletions (Vijh (1994), Barberis et al. (2005)), for changes in S&P500 value and growth indexes (Boyer (2011)), for changes in the Nikkei 225 index (Greenwood and Sosner (2007)), for changes in UK, Japanse, and other national indexes (Mase (2008), Greenwood (2007), Claessens and Yafeh (2013)), among others. Wahal and Yavuz (2013) do not find support for style investing increasing correlations.

currency of one country is perceived as a safe asset by international investors. Jointly with the effects of the exchange rate and the existence of a reserve currency, this study examines the impact of style investing for security groups including a different number of assets. Last, this study explores the possibility that style investors display irrational preferences for specific countries. Within-country asset correlations matter, as it has been long recognized that investors overinvest in domestic stocks and other domestic assets (e.g., Chan et al. (2005), Ardalan (2019)).

This study confirms the baseline result of Barberis and Shleifer (2003) that for security groups of equal numerosity, and with no currency risk, style investing increases (distorts) return correlations in each country beyond the correlation levels implied by fundamentals. These correlation increases are the same for the two security grouping, as long as the markets have the same size.

Allowing for a different number of securities in each style immediately yields the additional insight that style investing increases correlations more for the small than the large economy, even in the absence of exchange rate risk (e.g., with a credible peg). This result has practical risk management implications, especially for the investors operating in small economies.

At its core, domestic risk diversification depends on within-country correlations. Hence, an increase in the correlations among the securities of a country matters for the perspective of portfolio management, as highly correlated within-country returns decrease the scope for domestic portfolio diversification. This study shows that this detrimental effect is particularly strong for small countries' investors. Put differently, the results indicate that for the portion of their portfolio that is allocated in the domestic market, which is usually large, investors operating in smaller economies can rely on fewer and more correlated assets to diversify risk than investors active in broader economies.

Further, this study shows that the correlation increases caused by style investing are stronger in the smaller economy when the exchange rate risk is low and when the reserve currency effect is weak or absent. This finding might provide at least a partial explanation for the result that emerging markets tend to exhibit within-country higher degrees of comovements than developed ones in equity markets (e.g., David and Simonovska (2016)), even when the domestic government manages exchange rate (e.g., through an imperfect currency peg), and thus exchange rate risk is low.[7] Moreover, we show that higher disparities in the number of securities increase correlation distortions more for the smaller economy than for the larger one, independent of the level of currency risk and the reserve currency effect.

Another conclusion of this study is that an increase in risk aversion heightens correlation distortions in both the large and small economies. Hence, during financial crises, or following losses, when risk propensity decreases (e.g., Campbell and Cochrane (1999)), domestic returns tend to become more correlated, due to style investing. The unfortunate implication is that country-based investing makes domestic portfolios diversification more challenging, especially during downturns, that is when it is needed the most.

Following Barberis and Shleifer (2003), in the first part of this study we assume that style investors shift their resources between countries based on the past relative performance of the indexes of the small and large economies. Further, these agents are assumed to hold agnostic views in terms of preference for the assets of the small and large economies, which is a consequence of assuming that style investors are not affiliated with either country. Admittedly, these are rather strong assumptions. It is indeed plausible that style investors' demand for country-based portfolios might shift in response to shocks that are not captured by past returns. Surveys of mutual fund investors show that investor recollections of past performance are consistently biased (e.g., Berk and Green (2004)).[8] The implication is that investors might implicitly express irrational preferences over investment portfolios, possibly to justify past investment patters.[9]

In view of these considerations, this study also considers the possibility that style investors receive a signal on the past realizations of the global risk factor and express country-preferences. Under this scenario, the conclusions are similar to those drawn for the case without the signal. An additional insight, however, is that a more imprecise signal yields higher correlation distortions in both large and small countries. The implication is that the

[7]David and Simonovska (2016) argue that the within-country excess comovement is due to commonality of beliefs of informed investors, which they gauge by analyst forecasts. The insight of this study is that the activities of uninformed traders also increase within-country correlations, due to country-based style investing.

[8]Mutual fund investors are typically considered unsophisticated (e.g., "dumb money", in Akbas et al. (2015)), in terms of pricing abilities.

[9]For instance, Milesi-Ferretti and Tille (2011) showed that in the aftermath of the 2007-2008 financial crisis, domestic investors shifted their wealth into the US equity market.

effect of country-based style investing might be amplified when style investors imprecisely extrapolate the state of the global economy and hold country preferences.

While this study focuses on international investing, the results yield implications also for markets with securities that are valued in the same currency, like industry-based domestic investing. In this context, the finding is that investment grouping will result in returns being over-correlated, and to a larger extent for the less populated group. Hence, industry-based style investing is predicted to have different effects on within-industry asset correlations, depending on the number of investment opportunities in each industry. In particular, industry-based domestic investing makes within-industry diversification harder, particularly for the less populated industry group. Consistently, the empirical results presented in Chan et al. (2007) shows a negative relationship between the average number of companies in each industry (using several GICS classifications) and the within-industry pair-wise correlation of equity excess returns. This study argues that this negative relationship is due, at least partially, to the effect of style investing, and thus offers additional insights on the workings of the industry effect.

1 Model Overview

There are two investor types. The first type (fundamental traders) allocates resources across securities based on fundamentals, irrespective of country affiliation. Fundamental valuations are modeled by a factor model, including risk factors for global market risk, country-specific risk, security-specific idiosyncratic risk, and currency risk. Fundamental traders aggregate these risks into prices with imperfect knowledge of the process determining securities' cash-flows. The second investor type (style investors) bundles the assets of each country together into country-specific fixed-weight portfolios (e.g., country-based indexes). These investors switch between country-based portfolios based on the past relative average performance of the country indexes. This behavioral assumption is grounded in empirical evidence. Early empirical findings on style investing show that fund flows tend to focus on investment funds with high past returns (e.g., Froot et al. (2001), Bergstresser and Poterba (2002), and Sapp and Tiwari (2004)). Further, patterns in individual investors activities suggest that market participants often grow enthusiastic about certain stock categories and allocate their resources to the associated investment funds, in expectation of a continuation of these showings (e.g., Kumar (2009)).[10] Further, style investors evaluate past performance in the reserve currency, without explicitly taking into account currency risk, which is consistent with the behavior of unsophisticated market participants.

The economy comprises two countries, with a different number of securities (i.e., different market sizes) and different currencies. Cash-flows are modeled in a familiar way (e.g., Hong and Stein (1999)), that is as claims to a security-specific liquidating dividend.

To fix ideas, in the ensuing presentation of the model and of the results, one can identify the large economy holding the reserve currency with the US and the small economy with an emerging market. Securities can be thought of as equities listed in the respective domestic exchanges, and styles as dollar-denominated country-based equity ETFs, one for the US and one for the emerging market.

1.1 Assets

There are two asset classes or groups which are indexed by X and Y, respectively.[11] Class X represents the asset pool of a country with a broad financial market, counting n_1 securities. Class Y includes the n_2 securities, with $n_2 < n_1$, of the smaller financial market of country Y. In both countries, assets are in fixed supply. The payoff of the generic risky assets i of country X or Y is a claim to a single principal $D_{i,T}$ payable at the end of the economy T, and it is expressed in the currency of country X, which is the reserve currency. The time-t payoff of asset i is described by the following sum:

$$D_{i,t} = D_{i,0} + \varepsilon_{i,1} + \ldots + \varepsilon_{i,t}$$

where $D_{i,0}$ and the cash-flow shock $\varepsilon_{i,t}$ are announced at time 0 and time t respectively. All cash-flows shocks are expressed in the currency of country X. For each security i, the cash-flow shocks follow a linear factor model, and are determined by the realizations of a factor f_{Gt} summarizing global macroeconomic conditions, two country-specific factors, f_{Xt}, and f_{Yt}, and

[10] For example, Cooper et al. (2005) find that mutual funds that change their name to associate themselves with a style that has performed exceptionally well in the recent past receive significantly increased inflows.

[11] This study focuses on styles or investment groups for which membership is known at the time securities are issued, and it is time-invariant.

by security-specific idiosyncratic shocks f_{it}. The effects of these factors are combined by time-invariant weights, summing up to 1, for simplicity. The exchange rate risk factor f_{Et} affects only the cash-flow shocks of securities of country Y (as cash-flows are expressed in the currency of country X). One can think of f_{Et} as capturing the cost of hedging currency risk. For each t the cash-flow shocks take the following form:

$$\varepsilon_{it} = \sqrt{G} f_{Gt} + \sqrt{S} f_{Xt} + \sqrt{I} f_{it} \qquad \text{for } i \text{ in } X \tag{1}$$

$$\varepsilon_{ht} = \sqrt{G} f_{Gt} + \sqrt{S} f_{Yt} + \sqrt{I} f_{ht} + f_{Et} \qquad \text{for } h \text{ in } Y \tag{2}$$

All factors have zero-mean and unit variance, with the exception of f_{Et}, which has variance e, and zero mean. All factors are serially independent and identically distributed (iid). With the exception of the exchange rate factor f_{Et}, all factors are independent from each other. It is assumed that f_{Et} is negatively correlated with f_{Xt}. The intuition is that a favorable (unfavorable) shock to economy X, i.e., a positive (negative) realization of f_{Xt}, implies an appreciation (a depreciation) of country X currency. This appreciation (depreciation) is associated with a lower (higher) realization for the exchange rate factor f_{Et}, which decreases (increases) the cash-flows offered by investments in country Y, which are expressed in the currency of country X. Alternatively, one can think of this effect as an increase cost of hedging the exchange rate risk of country Y. An analog reasoning entails a positive correlation between f_{Yt} and f_{Et}. A positive shock to the economy of country Y increases the value of the country's currency and thus yields higher cash-flow, in terms of the currency of country X. For simplicity, the correlation between the exchange rate and the country-specific risk factors are equal, in absolute value. Hence, after some harmless scaling, we have:

$$cov(f_{Xt}, f_{Et}) = -\frac{\theta}{2\sqrt{S}} < 0$$

$$cov(f_{Yt}, f_{Et}) = \frac{\theta}{2\sqrt{S}} > 0$$

A further assumption is that when the word economy is hit by an unfavorable shock, the reserve currency, (i.e., the currency of country X) appreciates with respect to the currency of country Y, so the value of the cash-flows of country Y decreases, when expressed in the reserve currency. The effect can be thought as an increase in the cost of exchange risk hedging in country Y, which lowers the value (in the reserve currency) of country Y assets. When the global shock is positive, it is the currency of country Y that appreciates on the reserve currency, yielding higher cash-flows. Formally,

$$cov(f_{Gt}, f_{Et}) = \frac{\delta}{2\sqrt{G}} > 0$$

where the scaling factor simplifies calculations, without implying any loss of generality. Finally, firm-specific idiosyncratic risk is independent from the exchange rate, for all firms, so that

$$cov(f_{it}, f_{Et}) = 0 \text{ for all } i \in X \cup Y$$

Under these assumptions, the covariance Σ of the cash-flow shocks is:

$$cov\left(\varepsilon_{it}, \varepsilon_{jt}\right) = \begin{cases} 1 & \text{for } i = j \in X \\ G + S = 1 - I & \text{for } i \neq j \text{ and } i, j \in X \\ 1 + e + \theta + \delta & \text{for } i = j \in Y \\ G + S + e + \theta + \delta & \text{for } i \neq j \text{ and } i, j \in Y \\ G + \frac{\delta - \theta}{2} & \text{for } i \in X \text{ and } j \in Y \end{cases} \tag{3}$$

Before introducing investors into this economy, let's identify some notation. The price of a security i at time t is $P_{i,t}$ and it is expressed in the currency of country X. Price changes between $t-1$ and t are denoted by:

$$\Delta P_{i,t} = P_{i,t} - P_{i,t-1}$$

For simplicity, price changes are referred to as returns. The time-t returns of the equally weighted index of the securities in countries X and Y are:

$$\Delta P_{X,t} = \frac{\sum_{i \in X} \Delta P_{i,t}}{n_1} \tag{4}$$

$$\Delta P_{Y,t} = \frac{\sum_{j \in Y} \Delta P_{j,t}}{n_2}. \tag{5}$$

1.2 Switchers

The empirical literature recognizes individual investors often extrapolate from past performance when choosing investment funds based on securities groupings. Hence, as also done in Barberis and Shleifer (2003), I assume that a group of investors (switchers) allocates funds to country X and Y on the basis of past relative performance. Namely, switchers modify their holdings in the securities of each country on the basis of their average past *relative* performance. For instance, if the past average return is higher for securities in country X than country Y, switchers will sell some holdings in Y and use the proceeds to fund long positions in class X. To abstract from portfolio optimization issues, switchers invest uniformly in all the securities of each country.[12]

Denote with αN_{Xt} and αN_{Yt} the aggregate demand of switchers for each asset class, where the parameter $\alpha > 0$ is a scalar summarizing the incidence of style investing in the global economy and allows investigating how the prevalence of style investing influences return correlations. Switchers' aggregate demand for assets $i \in X$ and $h \in Y$ are:

$$N_{it}^{S\alpha} \equiv \frac{\alpha N_{Xt}}{n_1} \text{ and } N_{ht}^{S\alpha} \equiv \frac{\alpha N_{Yt}}{n_2} \tag{6}$$

where

$$N_{it}^{S\alpha} = \frac{\alpha}{n_1}\left[A_X + \sum_{k=1}^{\mathbf{t-1}} w^{t-k}\left(\frac{\Delta P_{Xt-k} - \Delta P_{Yt-k}}{2}\right)\right] \text{ for } i \text{ in } X \tag{7}$$

$$N_{ht}^{S\alpha} = \frac{\alpha}{n_2}\left[A_Y + \sum_{k=1}^{\mathbf{t-1}} w^{t-k}\left(\frac{\Delta P_{Yt-k} - \Delta P_{Xt-k}}{2}\right)\right] \text{ for } h \text{ in } Y \tag{8}$$

where the parameter $0 < w < 1$ gives the weights on past realizations. The constants A_X and A_Y can be interpreted as the long-run average of the holdings in each asset class. As in Barberis and Shleifer (2003), it is assumed that switchers have sufficient funds to support their asset allocations. The $(t+1)$-time changes in switchers' portfolio for countries X and Y are defined by

$$\Delta N_{Xt+1}^{S\alpha} = N_{Xt+1}^{S\alpha} - N_{Xt}^{S\alpha}$$
$$\Delta N_{Yt+1}^{S\alpha} = N_{Yt+1}^{S\alpha} - N_{Yt}^{S\alpha}$$

[12]The portfolios selected by switchers are country-based equally weighted indexes. Focusing on different time-variant weighting schemes would complicate the exposition, but would not affect the overall results.

Note that $\Delta N^{S\alpha}_{Xt+1}$ and $\Delta N^{S\alpha}_{Yt+1}$ depend on prices up to time t, and that, by construction, we have

$$\Delta N^{S\alpha}_{Xt+1} = -\Delta N^{S\alpha}_{Yt+1}$$

1.3 Fundamental Traders

Fundamental traders have a time-invariant exponential utility (CARA), and choose the portfolio N_t^F in the $n_1 + n_2$ securities:

$$\max_{N_t^F} E_t^F \left[-\exp\left(-\gamma \left(W_t + N_t^F \left(P_{t+1} - P_t \right) \right) \right) \right]$$

where $\gamma > 0$ is the risk aversion, W_t is wealth at time t, and P_t is the vector of prices for the $n_1 + n_2$ securities.[13] The superscript F refers to the information set of fundamental traders, so that the expectation E_t^F is the time-t conditional expectation of fundamental traders. At time t, fundamental traders assume normally distributed conditional returns, with return variance matrix V_t defined by:

$$V_t = var^F \left(P_{t+1} - P_t \right) = var^F \left(\Delta P_{t+1} \right)$$

which implies that their holding for each asset i satisfies:

$$N_t^F = \frac{V_t^{-1}}{\gamma} \left(E_t^F \left(P_{t+1} \right) - P_t \right) \tag{9}$$

Securities are in fixed supply Q, so that:

$$P_{i,t} = E^F \left(P_{it+1} \right) - \gamma V_t \left(N_t^F \right)$$

where:

$$N_t^F = Q - N_t^{S\alpha}$$

and $N_t^{S\alpha}$ is the vector of switchers' holdings of the $n_1 + n_2$ securities. Fundamental traders base their expectations on the final dividends D_T for the $n_1 + n_2$ assets. In vector notation:

$$E_{T-1}^F \left(P_T \right) = E_{T-1}^F \left(D_T \right) = D_{T-1}$$

and prices can be obtained by backward substitution:

$$P_t = D_t - \gamma V_t \left(Q - N_t^{S\alpha} \right) - E_t^F \left(\sum_{k=1}^{T-t-1} \gamma V_{t+k} \left(Q - N_{t+k}^{S\alpha} \right) \right) \tag{10}$$

[13] All vectors and matrices relative to the $n_1 + n_2$ securities are indexed with the n_1 securities of X followed by the n_2 securities of Y, always listed in the same order.

1.4 Partially Savant Fundamental Traders

To avoid price-run dynamics due to strategic feedback (e.g., De Long et al. (1990)), I assume that fundamental traders are not sufficiently sophisticated to figure out that switchers determine their allocations solely on the basis of past average returns.[14] In fact, fundamental traders assimilate switchers' trading activities to zero-mean supply shocks around the constant level $\overline{N}^{S\alpha}$. In expectations:

$$E_t^F\left(N_{t+k}^{S\alpha}\right) = \overline{N}^{S\alpha}$$

Fundamental traders also assume a time-invariant variance matrix, the nature of which will be detailed later in this section. Hence,

$$E_t^F\left(V_{t+k}\right) = V \text{ for } k > 1.$$

Due to these two assumptions, the prices P_t displayed in (10) simplify to:

$$P_t = D_t - \gamma V\left(Q - N_t^{S\alpha}\right) - (T - 1 - t)\,\gamma V\left(Q - \overline{N}^{S\alpha}\right)$$

Dropping non stochastic terms, yields:

$$P_t = D_t + \gamma V N_{t+1}^{S\alpha} \tag{11}$$

and, in terms of returns, entails:

$$\Delta P_{t+1} = P_{t+1} - P_t = \varepsilon_{t+1} + \gamma\left(V N_{t+1}^{S\alpha} - V_t N_t^{S\alpha}\right)$$

Conditioning on time t, we obtain an expression for the returns between t and $t+1$ for the assets in the economy.

$$\Delta P_{t+1} = \varepsilon_{t+1} + \gamma V \Delta N_{t+1}^{S\alpha} \tag{12}$$

The return dynamics assumed by fundamental traders, displayed hereafter, describes how the exchange risk and the reserve currency may affect the return variance, as perceived by fundamental traders. I assume that fundamental traders are aware of the iid risk factors driving cash flows. They

[14] Arbitrageurs could play-up the price of a given asset class to gain on the return predictability induced by switchers demand pressure, and thus amplify switchers' effect on prices. Excluding this strategic behavior thus yields a conservative assessment of the effect of style investing on return correlations.

are, however, unable to correctly asses the impact of these factors, as well as the magnitude of the correlations between the exchange risk factor and the global and country-specific risk factors.[15] An explanation for this latter assumption is that the shocks in demand caused by switchers confound the effects of the exchange rate on returns, to the eyes of fundamental traders. Fundamental traders do not have access to the true time-invariant weights of the cash-flow generating process.[16] Fundamental traders assume the following return generating process:

$$\Delta P^F_{it+1} = \sqrt{G^F} f_{G,t+1} + \sqrt{S^F} f_{X,t+1} + \sqrt{I^F} f_{i,t+1} \qquad \text{for } i \text{ in } X \tag{13}$$

$$\Delta P^F_{ht+1} = \sqrt{G^F} f_{G,t+1} + \sqrt{S^F} f_{Y,t+1} + \sqrt{I^F} f_{h,t+1} + f_{Et+1} \quad \text{for } h \text{ in } Y \tag{14}$$

where:

$$\begin{aligned} cov^F(f_{X,t+1}, f_{Et+1}) &= -\frac{\theta^F}{2\sqrt{S^F}} < 0 \\ cov^F(f_{Y,t+1}, f_{Et+1}) &= \frac{\theta^F}{2\sqrt{S^F}} > 0 \\ cov^F(f_{G,t+1},, f_{Et+1}) &= \frac{\delta^F}{2\sqrt{G^F}} > 0 \\ var\left(f^E_t\right) &= e^F > 0 \end{aligned}$$

Following the process in equations (13) and (14), fundamental traders' variance is:

$$cov^F\left(\Delta P^F_{it+1}, \Delta P^F_{ht+1}\right) = \begin{cases} G^F + S^F + I^F = 1 & \text{for } i = h \in X \\ G^F + S^F & \text{for } i \neq h \text{ and } i, h \in X \\ 1 + e^F + \delta^F + \theta^F & \text{for } i = h \in Y \\ G^F + S^F + e^F + \delta^F + \theta^F & \text{for } i \neq h \text{ and } i, h \in Y \\ G^F + \frac{\delta^F - \theta^F}{2} & \text{for } i \in X \text{ and } h \in Y \end{cases} \tag{15}$$

[15]Fundamental investors recognize the independence of firm-level idiosyncratic risk from all other sources of risk.

[16]The assumption of constant weights for fundamental investors can be relaxed, assuming that fundamental traders observe the true weights G, S, I with independently distributed measurement error. This additional assumption however does not yield particular insights on the role of style investing, and it is therefore omitted.

We note that the within-class covariance of asset returns in Y is larger than its analog in class X, due to the variability of the exchange rate (as prices in country Y are expressed in the currency of class X). In contrast, the effects of the exchange rate and of the reserve currency on the correlations across asset classes is nuanced. The direct effect of (uncorrelated) country-specific shocks on the exchange rate (governed by the parameter θ^F) is to decrease the correlation across classes. For instance, a negative shock to country X weakens domestic returns, but also increases the returns of country Y, due to the relative appreciation of the currency of country Y, as these returns are expressed in the currency of country X (i.e., the exchange rate innovation, f_{Et+1}, increases in equation 14). The indirect effect is a pull to decrease on the correlation across classes. The reserve currency effect impacts cross-asset correlations in the opposite fashion. A negative shock to the global economy has the direct effect of weakening the returns for both countries. However, the global unfavorable shock also results in an appreciation of the reserve currency, which is perceived as a safe asset by fundamental traders. In relative terms, the negative global shock thus causes a depreciation of country Y currency (i.e., the exchange rate innovation, f_{Et+1}, declines), an effect captured by the parameter δ^F. This depreciation further decreases country Y returns, as these are expressed in the reserve currency. Hence, the reserve currency effect strengthens the correlation between the asset classes.[17]

The return dynamics assumed for fundamental traders, displayed in equations (13) and (14), are not essential to the results discussed in this study. Put differently, the return variance matrix assumed by fundamental traders can be derived from alternative return dynamics. The specification of these processes is meant only to formalize how the exchange risk and the reserve currency affect the return variance, as perceived by fundamental traders. Hence, we can generalize the variance matrix assumed by fundamental traders dis-

[17]A positive shock to the global economy has the direct effect of strengthening returns in both countries, but the currency of country Y appreciates (i.e., the exchange rate innovation, f_{Et+1}, increases). This appreciation further increases country Y returns, as these are expressed in the reserve currency.

played in (15) and rely on a more general form, as follows.

$$V = cov_{t+1}^{F}\left(\Delta P_{it+1}, \Delta P_{jt+1}\right) = \begin{cases} \sigma^2 & \text{for } i = j \in X \\ \sigma^2\rho_1 & \text{for } i \neq j \text{ and } i, j \in X \\ \sigma^2 + \kappa & \text{for } i = j \in Y \\ \sigma^2\rho_1 + \kappa & \text{for } i \neq j \text{ and } i, j \in Y \\ \sigma^2\rho_2 - \lambda & \text{for } i \in X \text{ and } j \in Y \end{cases} \tag{16}$$

with the parameters ρ_1 and ρ_2 satisfying:

$$1 > \rho_1 > \rho_2 > 0 \tag{17}$$

and $\kappa > 0$. The equivalence between the two matrices is guaranteed by the following chain of identities:

$$1 = \sigma^2,\ G^F + S^F = \sigma^2\rho_1\ ,\ G^F = \sigma^2\rho_2\ ,\ \kappa = e^F + \delta^F + \theta^F\ ,\ 2\lambda = \theta^F - \delta^F \tag{18}$$

In the form proposed in (16), the return variance V is sufficiently general to include more general return dynamics than the one specified in equations (13) and (14). The structure of the variance V displayed in (16) is intentionally chosen to be similar to its analog in Barberis and Shleifer (2003), but for the inclusion of parameters κ and λ, which capture the effect of exchange rate risk and the assumption that country X has the safe-haven currency.[18] Note that the parameter λ in V contributes positively to the covariance between the asset classes when $\lambda < 0$, that is when fundamental traders assess that the reserve currency effect (governed by δ^F) on the interest rate is weaker than the direct effect of country-specific shocks (governed by θ^F).

2 Equilibrium Prices

The following proposition identifies the equilibrium returns for the $n_1 + n_2$ assets in countries X and Y. The proof capitalizes on the rigidity of the shifts of switchers' demand, which entails that the portfolio changes $\Delta N_{Xt+1}^{S\alpha}$ and $\Delta N_{Yt+1}^{S\alpha}$ are equal in magnitude, but opposite in sign.

[18] This variation, besides easing notation and offering a more general framework to this study's results, facilitates the comparison with Barberis and Shleifer (2003). In particular note that when κ and λ are zero the variance matrix reduces to the one proposed in Barberis and Shleifer (2003), who, however, do not formalize the return generating process of fundamental traders.

Proposition 1 *Given the expression of prices in* (11)*, and the variance* V *displayed in* (16)*, the equilibrium returns are:*

$$\Delta P_{it+1} = \varepsilon_{it+1} + \gamma \Delta N^{S\alpha}_{Xt+1} A_1 \qquad \textit{for } i \textit{ in } X \tag{19}$$

$$\Delta P_{ht+1} = \varepsilon_{ht+1} + \gamma \Delta N^{S\alpha}_{Yt+1} A_2 \qquad \textit{for } h \textit{ in } Y \tag{20}$$

where:

$$n_1 A_1 = \sigma^2 \left(1 + (\rho_1 - \rho_2) n_1 - \rho_1\right) + n_1 \lambda \tag{21}$$

$$n_2 A_2 = \sigma^2 \left(1 + n_2 (\rho_1 - \rho_2) - \rho_1\right) + n_2 (\kappa + \lambda) \tag{22}$$

Referring to the return variance matrix expressed in terms of the returns generating factors assumed by fundamental traders, displayed in (15), the equilibrium prices satisfy the same analytical expression, but with the parameters A_1 and A_2 satisfying the following equations:

$$n_1 A_1 = I^F + n_1 \left(\frac{\theta^F - \delta^F}{2}\right) + n_1 S^F \tag{23}$$

$$n_2 A_2 = n_2 I^F + e^F + S^F + \frac{3\theta^F + \delta^F}{2} \tag{24}$$

3 Correlation Gaps

Deviations from the return correlation levels implied by fundamentals (i.e., by cash-flow shocks), are named correlation gaps. Henceforth they are denoted by Gap_{n_1} and Gap_{n_2} for country X and Y, respectively, where n_1 is the number of securities in X and n_2 is the number of securities in Y. Formally:

$$Gap_{n_1}(i,j) = corr\left(\Delta P_{it+1}, \Delta P_{jt+1}\right) - corr\left(\varepsilon_{it+1}, \varepsilon_{jt+1}\right) \quad \text{for any } i,j \in X \tag{25}$$

$$Gap_{n_2}(h,k) = corr\left(\Delta P_{ht+1}, \Delta P_{kt+1}\right) - corr\left(\varepsilon_{ht+1}, \varepsilon_{kt+1}\right) \quad \text{for any } h,k \in Y \tag{26}$$

Barberis and Shleifer (2003) analyze the effect of style investing when securities are partitioned into two equally numerous styles, and in the absence of currency risk. They show that switchers' activities cause returns to be more correlated than the level implied by their fundamentals, within each style. The following proposition confirms their result in the current framework and, in addition, offers the closed form solutions of the correlation gaps, which are unavailable in the framework of Barberis and Shleifer (2003).

Proposition 2 *Within each country, asset returns are more correlated than the underlying cash-flows: for any i, j in X and h, k in Y, we have:*

$$Gap_{n_1}(i,j) = \frac{I\gamma^2 v\alpha^2 A_1^2}{1+\gamma^2 v\alpha^2 A_1^2} > 0 \tag{27}$$

$$Gap_{n_2}(h,k) = \frac{v\alpha^2\gamma^2 A_2^2 I}{(\theta+\delta+e+1)\left(v\alpha^2\gamma^2 A_2^2+\theta+\delta+e+1\right)} > 0 \tag{28}$$

where:

$$v = var\left(\Delta N_{Xt+1}^S\right) \tag{29}$$

and A_1 and A_2 are defined in (21) *and* (22)*, respectively.*

The correlation gaps identified in Proposition 2 are measures of correlation distortion at a given moment in time, say time t, for correlations in the period between t and $t+1$. In the following sections we will perform several assessments of the effect of changes in the exogenous parameters on the correlation gaps and derived quantities. These evaluations are performed conditionally on the equilibrium levels reached at time t, and for small changes. In particular, we note that shocks in exogenous variables do not affect the demand levels of switchers at time $t+1$ (e.g., variables N_{Xt+1}^S, and N_{Yt+1}^S, and thus v), as these are determined by prices up to time t. For example, the effect of a stronger presence of switchers (i.e., an increase in the parameter α), as examined in the next proposition, is assumed to take place after time-t prices have been determined. Hence, the change in α does affect the returns ΔP_{t+1}, through the $(t+1)$-prices, namely P_{t+1}, but not through P_t. In particular, an increase in α does not affect switchers' portfolio rebalancing, namely ΔN_{Xt+1}^S and ΔN_{Yt+1}^S, as well as their second moment v, as these quantities are determined by the sequence of prices from $t=0$ to t.[19] Further, we shall assume that fundamental traders do not modify the variance

[19]To further clarify, it is the unscaled (by α) switchers' demand that is unaffected by changes in α. An increase in α increases the scaled aggregate demands $N_{it+1}^{S\alpha}$ and $N_{jt+1}^{S\alpha}$ for $i \in X$ and $j \in Y$, respectively. For ease of notation, the variance v is not indexed on time.

structure V defined in (16) for small changes in the exogenous parameters, conditional on the equilibrium levels reached at time t. This brief discussion is summarized by the following remark.

Remark 3 *Small changes in the exogenous parameters γ, α, and in the cash-flow parameters e, θ, and δ, occurring at time t, do not affect switchers' demand changes ΔN^S_{Xt+1} and ΔN^S_{Yt+1}, as well their variance v. Further, it is assumed that these small changes do not affect the variance structure V assumed by the fundamental traders, which is defined in* (16).

3.1 Risk Aversion and the Incidence of Style Investing

The next proposition outlines that an increase in the participation of style investors worsen the correlation distortions within both countries, as expected. The same proposition shows that the gaps also increase with risk aversion. The intuition resides in role of fundamental investors as a counterbalance to the trades of style investors. As highlighted in equation (9), fundamental traders lean against the demand shocks of style investors more aggressively when realized prices deviate more strongly from expectations. Higher levels of risk aversion reduces fundamental traders' holdings of risky assets, thus leaving an highlighted role for switchers' demand in determining prices.

Proposition 4 *The country-specific correlation distortions Gap_{n_1} and Gap_{n_2} increase in the incidence of style investing and in risk aversion, that is:*

$$\frac{\partial Gap_{n_1}}{\partial \alpha} > 0 \text{ and } \frac{\partial Gap_{n_2}}{\partial \alpha} > 0$$
$$\frac{\partial Gap_{n_1}}{\partial \gamma} > 0 \text{ and } \frac{\partial Gap_{n_2}}{\partial \gamma} > 0$$

From equations (27) and (28), we gather that if there are no switchers (i.e., if $\alpha = 0$) both correlation gaps are zero, as expected. Proposition 4 shows that as more style investors enter the market, correlation gaps increase. Hence, the gaps gauge the distorting effect on correlations caused by style investing. Further, these correlation distortions increase when the risk aversion of non-style investors rises. Campbell and Cochrane (1999) have argued that risk aversion increases during downturns, following losses. Under this adverse scenario, the model predicts that the distortion caused by style investing increases, making returns more correlated within each country.

4 Spread of Correlation Gaps

As shown in Proposition 2, the correlation distortions are not equal across the two countries. We can thus define the spread between the country-specific correlation gaps to assess whether style investing has a different effect on return correlations in countries X and$Y$. The spread between the correlation gaps is denoted by ΔGap, and is defined as follows:

$$\Delta Gap = Gap_{n_2} - Gap_{n_1} \tag{30}$$

The next two sections examine whether the correlation distortions caused by style investing are stronger in either the smaller or larger economy. This amounts to evaluating the sign of the correlation distortion spread ΔGap defined in (30). To simplify the exposition, we can identify X as the country with the highest number of securities, so that $n_1 > n_2$. We commence the exposition with the special case of no exchange rate risk, to define a backdrop against which the effect of currency risk can be better understood.

4.1 Correlation Gaps Spread with No Exchange risk

This section examines whether correlation distortions are stronger in either the smaller or larger economy when there is no currency risk. This theoretical framework applies to differently sized financial markets using the same currency (e.g., to contrast equity markets in the European Union), to style groups within the same country (e.g., industries), and when country Y engages in a credible peg.

If there is no exchange risk, then the parameters e, δ and θ are zero.[20] In this case, the parameters A_1 and A_2, displayed in equations (21) and (22), simplify to the following expressions:

$$\overline{A}_1 = \frac{\sigma^2 \left(1 + n_1 \left(\rho_1 - \rho_2\right) - \rho_1\right)}{n_1} \tag{31}$$

$$\overline{A}_2 = \frac{\sigma^2 \left(1 + n_2 \left(\rho_1 - \rho_2\right) - \rho_1\right)}{n_2} \tag{32}$$

[20]In the proof of Proposition 5, it is assumed that fundamental traders acknowledge the absence of exchange rate risk and modify accordingly the variance structure identified in (16), by setting λ and κ to zero. The results of Proposition 5 obtain also for the case in which fundamental traders fail to recognize the absence of exchange rate risk, as shown in the proof of Proposition 7.

while the correlation gaps of countries X and Y are:

$$\overline{Gap}_{n_1} = \frac{I\gamma^2 v\alpha^2 \overline{A}_1^2}{v\alpha^2\gamma^2\overline{A}_1^2 + 1} > 0 \tag{33}$$

$$\overline{Gap}_{n_2} = \frac{I\gamma^2 v\alpha^2 \overline{A}_2^2}{1 + \gamma^2 v\alpha^2\overline{A}_2^2} > 0 \tag{34}$$

The following proposition shows that in the absence of currency risk the disparity in the number of securities in the two countries suffices to differentiate the effect of style investing on correlations. This is intuitive: as switchers shift their wealth between the two countries lumping all the securities of each country into the same group, the shock to individual asset demand due to switchers' activities are bound to be stronger for the country with fewer securities, that is for country Y. The higher is the number of securities in the large market, relatively to that of the smaller economy, the lower is the spread in the correlation gaps, as the correlation distortions in the large markets become smaller.

Proposition 5 *If there is no exchange rate risk then $\overline{Gap}_{n_2}$ is larger than $\overline{Gap}_{n_1}$, that is:*

$$\Delta\overline{Gap} = \overline{Gap}_{n_2} - \overline{Gap}_{n_1} > 0$$

Further, if $n_1 = n_2$ then $\Delta\overline{Gap} = 0$, and if $n_1 = sn_2$ with $s > 0$, then

$$\frac{\partial\Delta\overline{Gap}}{\partial s} > 0$$

and

$$\frac{\partial\overline{Gap}_{n_1}}{\partial n_1} < 0$$

$$\frac{\partial\overline{Gap}_{n_2}}{\partial n_2} < 0$$

This result indicates that when style investing involves two groups of assets with different numerosity, correlation distortions arise at different rates, with the distortions being stronger in the asset group with the lowest number of securities.[21] Indeed, the sheer difference in size between two asset markets

[21] The scenario of no exchange rate risk and equal number of securities (i.e., $n_1 = n_2$) is the framework examined by BS. Consistently with their results, this study shows that the correlation distortion is the same for all the securities in the two asset groups.

suffices to create a difference in the effect of style investing. For example, industry-investing within a country causes a stronger increase in correlations among the securities belonging to the less numerous industry sector.

4.2 Correlation Gap Spread with Exchange risk

Consider now the case in which there is exchange rate risk. It is convenient, for ease of notation, to summarize all the facets of the effect of the exchange rate risk by a unique variable:

$$z = \theta + \delta + e \tag{35}$$

Naturally, z is large when any of the parameters θ, δ and e is large. Note, however, that when e is zero, as it is the case for a credible peg, then it will be assumed that z is zero as well.

According to Propositions 5 a difference in the numerosity of the asset groups suffices to generate more correlation distortion in country Y, even in the absence of exchange risk. The next proposition shows that in each country a larger number of securities weakens the effect of style investing also when there is exchange rate risk. The intuition is that style investing distorts correlations to a lesser extent when the impact of the trades of switchers is spread over a larger number of assets.

Proposition 6 *For $z \geq 0$, and $n_1 > n_2$, then the correlation gap in X decreases in the number of securities in X, and the correlation gap in Y decreases in the number of securities in Y. That is:*

$$\frac{\partial Gap_{n_1}}{\partial n_1} < 0$$
$$\frac{\partial Gap_{n_2}}{\partial n_2} < 0$$

Further, the correlation distortion spread between countries X and Y increases in n_1 and declines in n_2. If $n_1 = sn_2$ with $s > 1$, then

$$\frac{\partial \Delta Gap}{\partial s} > 0$$

The next proposition shows that for low levels of exchange rate risk the correlation distortions are larger in the smaller economy Y, a result that is consistent with the case with no exchange rate risk (i.e., with Proposition 5). The intuition is that the currency risk is too low to counterbalance the effect of the disparity in the number of securities between the two countries. When the currency risk is more substantial, the fundamental correlations of assets in country Y increase to eventually reduce the correlations gaps in country Y turning them lower than those of country X, despite the disparity in the number of securities.[22] Stronger levels of currency risk are required to cause this latter effect, when the disparity in the number of securities between countries is larger.

Proposition 7 *Let $z \geq 0$ and $n_1 > n_2$. Low (high) levels of exchange rate risk make the correlation distortions stronger (weaker) in the small economy than in the large economy. In particular*

$$\begin{aligned} &\Delta Gap > 0 \text{ for } 0 \leq z < z_0 \\ &\Delta Gap < 0 \text{ for } z > z_0 \\ &\Delta Gap = 0 \text{ for } z = z_0 \end{aligned}$$

where $z_0 > 0$ and

$$z_0 = \frac{A_2}{2A_1}\sqrt{4v\alpha^2\gamma^2 A_1^2 + v^2\alpha^4\gamma^4 A_1^2 A_2^2 + 4} - 1 - v\alpha^2\gamma^2 A_2^2 \tag{36}$$

with

$$\frac{\partial z_0}{\partial s} > 0$$

where $n_1 = sn_2$. The threshold z_0 is minimized when $n_1 = n_2$.

Next, we can examine the effect of small changes in currency risk, as summarized by the variable z, over the correlation gaps and the gap spread.[23] Note that for small changes in z fundamental traders do not modify their estimation of the return variance matrix, so there is no effect on the correlation gap of country X. The next result indicates that, on the margin, a

[22] When z increases the fundamental correlations of the cash-flows of country Y, which are defined in (3), increase as well. Hence, in view of the defintion of the correlation gap, displayed in (26), an increase in z has the potential of decreasing the correlations gap of country Y.

[23] See Remark 3.

higher exchange rate risk reduces the impact of style investing in the smaller country Y. The intuition is that the fundamental correlations of country Y cash-flows, against which country Y correlations distortions are evaluated, increase with exchange rate risk. This increase in the baseline correlation levels lowers the correlation distortion of country Y.

Proposition 8 *The higher the effect of the exchange rate risk, the lower is the spread in the correlation distortions across the two counties, as the correlation distortion in Y declines but it remains unchanged in X.*

$$\frac{\partial Gap_{n_1}}{\partial z} = 0$$
$$\frac{\partial Gap_{n_2}}{\partial z} = \frac{\partial \Delta Gap}{\partial z} < 0$$

5 Signal and Country Preferences

Up to this point, style investors have been assumed to shift wealth between countries solely on the basis of past country-specific average returns. In particular, switchers do not take into consideration any aspect of the global economy in allocating their resources. Admittedly, this is a rather strong assumption. Further, switchers appear to hold agnostic views in terms of preference for the assets of countries X and Y. Both switchers and fundamental traders are global investors and are not characterized by a country affiliation, so this assumption is not groundless. However, switchers' demand for country-based portfolios might shift in response to shocks that are not driven by past returns. For example, surveys of mutual fund investors show that investor recollections of past performance are consistently biased (e.g., Berk and Green (2004)). In view of these considerations, this section presents an extension of the model augmented with a signal for switchers and country-based preferences.

In each period, switchers receive a signal that is correlated with the past realizations of the global economy, and extrapolate from this signal a general view on the future state of the world economy. Style investors act upon these views by expressing their preferences over countries by raising or decreasing the amount of wealth allocated to the assets of countries X and Y. For ease of exposition, it is assumed that style investors consider assets in country X akin to a reserve market, so that an unfavorable signal boosts their holding in the securities of X, whereas the effect of a positive signal makes them more willing to invest in country Y. The case in which investors express the opposite preference patterns yields identical conclusions (as it is evident from the proof of the next proposition) and is therefore omitted.

The signal received by switchers, denoted by BB_t, takes the form of incremental shocks, with

$$BB_t = B_0 + \ldots + B_t$$

where B_t are zero-mean identically distributed and serially uncorrelated shocks, with variance p, and where B_t is correlated with f_{Gt-1}, but uncorrelated with all the remaining sources of risk, for any t. Recalling the demand for assets in countries X and Y defined in (6), switchers' demand for the assets in the two countries, with the signal, takes the form:

$$\begin{aligned}
\widetilde{N}_{it}^{S\alpha} &\equiv \frac{\alpha\left(N_{Xt} - BB_t\right)}{n_1} \text{ for } i \in X \\
\widetilde{N}_{ht}^{S\alpha} &\equiv \frac{\alpha\left(N_{Yt} + BB_t\right)}{n_1} \text{ for } h \in Y
\end{aligned}$$

Fundamental traders are aware that the global risk factor realizations are serially uncorrelated, so they have no use for the signal on the lagged state of the global economy. Again, these investors are not sophisticated enough to figure out the effect of the signal on switchers and thus continue to treat style investors as supply shocks, as they did for the case without signal. Since nothing changes from the perspective of fundamental traders, in view of Proposition 1, equilibrium returns take the following form:

$$\Delta P_{it+1} = \varepsilon_{it+1} + \alpha\gamma\left(\Delta N_{Xt+1}^{S} - B_{t+1}\right) A_1 \text{ for } i \text{ in } X \tag{37}$$

$$\Delta P_{ht+1} = \varepsilon_{ht+1} + \alpha\gamma\left(\Delta N_{Yt+1}^{S} + B_{t+1}\right) A_2 \text{ for } h \text{ in } Y \tag{38}$$

where the constants A_1 and A_2 have been defined in (21) and (22).

The correlations gaps are defined as in (25) and (26), and the next proposition shows their expression and sign.

Proposition 9 *Within each country, asset returns are more correlated than the underlying cash-flows: in both county X and Y, and, for any i, j in X*

and h, k in Y, it is:

$$Gap_{n_1}^{B} = \frac{I\gamma^2\alpha^2 A_1^2 (v+p)}{1+\gamma^2\alpha^2 A_1^2 (v+p)} > 0$$
$$Gap_{n_2}^{B} = \frac{\alpha^2\gamma^2 A_2^2 I (v+p)}{(\theta+\delta+e+1)(\alpha^2\gamma^2 A_2^2 (v+p)+\theta+\delta+e+1)} > 0$$

The proposition shows that the signal only adds to the fluctuations of switchers' demand (captured by the parameter v). Note that when $p = 0$ (i.e., with no signal), then the correlation gaps are those of Proposition 2. In particular, substituting $v + p$ to v in the proofs of the results reported in the previous sections yields analogous conclusions for the case with the signal. When p increases, switchers shift more of their wealth between the two countries, in response to a more volatile signal, which contributes to the distortion of return correlations. The next proposition clarifies this point.

Proposition 10 *A less precise signal increases correlation distortions.*

$$\frac{\partial Gap_{n_1}^{B}}{\partial p} > 0$$
$$\frac{\partial Gap_{n_2}^{B}}{\partial p} > 0$$

In the aftermath of the 2007-2008 financial crisis, wealth flew into the US equity market (e.g., Milesi-Ferretti and Tille (2011)). This shift is then consistent with country-based style investing coupled with country preferences. The model predicts an increase in the cross-section of within-country asset correlation for the US equity market.

6 Conclusions

This study argues that, in the presence of country-based style investing, return correlations increase from the levels implied by fundamentals. Different market sizes suffice to yield uneven increases in return correlations, with smaller countries experiencing a more marked increase. The exchange rate risk and the existence of a reserve currency have a nonlinear impact on the correlation distortions caused by style investing. For low levels of exchange risk, the effect of style investing is to increase correlations more in small economies. For higher levels of exchange rate risk, the results are more nuanced, and might depend on how fundamental investors account for the switch to a high-currency risk regime.

Correlation increases more markedly when risk aversion rises, thus during downturns. Under this adverse scenario, this study predicts that the distortion caused by style investing increases, making returns more correlated within each country.

In view of the strong evidence of home bias, which affects investors of both large and small economies, the results imply that style investing reduces the potential for domestic risk diversification, but that this effect might be stronger in either economy, depending on the exchange rate risk. Lastly, investors with country preferences exacerbate correlation distortions, when they imprecisely extrapolate from the past realizations of global risk.

Chen et al. (2016) note that any empirical assessment of the correlation distortion associated with style investing requires controlling for changes in fundamental return drivers, which might be, however, imprecisely measured. Thus this study's results offer a way to overcome this hurdle. From the perspective of empirical research, it is distinctively advantageous that this study yields predictions for the response of correlations to both market size and exchange rate. Market size varies at a much lower frequency than the exchange rate. Based on this study's theoretical results, we can identify subsamples for which relevant fundamentals either do not vary or vary only for one country, with clear predictions for the impact of these changes on correlations. Insights on the workings of style investing can then be gathered by contrasting the realized correlations across subsamples.

7 Appendix A: Proof of Propositions no Signal

Proof of Proposition 1 The variance matrix V is a block matrix, defined by expression 29, so that:

$$V = \begin{pmatrix} A & B \\ B' & C \end{pmatrix}$$

with

$$A_{n_1\times n_1}=\begin{pmatrix}\sigma^2 & \sigma^2\rho_1 & \cdots & \sigma^2\rho_1 & \sigma^2\rho_1\\ \sigma^2\rho_1 & \sigma^2 & \cdots & \sigma^2\rho_1 & \sigma^2\rho_1\\ \vdots & \vdots & \vdots & & \vdots\\ \sigma^2\rho_1 & \sigma^2\rho_1 & \cdots & \sigma^2 & \sigma^2\rho_1\\ \sigma^2\rho_1 & \sigma^2\rho_1 & \cdots & \sigma^2\rho_1 & \sigma^2\end{pmatrix}$$

$$C_{n_1\times n_2}=\begin{pmatrix}\sigma^2+\kappa & \sigma^2\rho_1+\kappa & \cdots & \sigma^2\rho_1+\kappa\\ \vdots & \ddots & \vdots & \vdots\\ \sigma^2\rho_1+\kappa & \sigma^2\rho_1+\kappa & \sigma^2+\kappa & \sigma^2\rho_1+\kappa\\ \sigma^2\rho_1+\kappa & \sigma^2\rho_1+\kappa & \sigma^2\rho_1+\kappa & \sigma^2+\kappa\end{pmatrix}$$

and B is a $n_1 \times n_2$ matrix with all the elements equal to $\sigma^2\rho_2-\lambda$. Recalling (11), for the return ΔP_{it+1} with $i \in X$ we have:

$$\Delta P_{it+1}=\varepsilon_{it+1}+\gamma R_i \Delta N^{S\alpha}_{t+1}$$

where R_i is raw i of V, with $i \leq n_1$. Recalling that that $\Delta N^{S\alpha}_{Xt+1}=-\Delta N^{S\alpha}_{Yt+1}$ we have:

$$\begin{aligned}R_i\Delta N^{S\alpha}_{t+1} &= \frac{\Delta N^{S\alpha}_{Xt+1}}{n_1}\left(\sigma^2+(n_1-1)\,\sigma^2\rho_1\right)+\frac{\Delta N^{S\alpha}_{Yt+1}}{n_2}\left(n_2\left(\sigma^2\rho_2-\lambda\right)\right)\\ &= \frac{\Delta N^{S\alpha}_{Xt+1}}{n_1}\left(\sigma^2+(n_1-1)\,\sigma^2\rho_1\right)-\Delta N^{S\alpha}_{Xt+1}\left(\sigma^2\rho_2-\lambda\right)\\ &= \Delta N^{S\alpha}_{Xt+1}\left(\frac{\sigma^2\left(1+(\rho_1-\rho_2)\,n_1-\rho_1\right)+n_1\lambda}{n_1}\right)\end{aligned}$$

which yields the expression (19). For row j with $j > n_1$ we have instead:

$$\begin{aligned}R_j\Delta N^{S\alpha}_{t+1} &= \frac{\Delta N^{S\alpha}_{Xt+1}}{n_1}\left(n_1\left(\sigma^2\rho_2-\lambda\right)\right)+\frac{\Delta N^{S\alpha}_{Yt+1}}{n_2}\left((n_2-1)\left(\sigma^2\rho_1+\kappa\right)+\sigma^2+\kappa\right)\\ &= \Delta N^{S\alpha}_{Yt+1}\left(-\left(\sigma^2\rho_2-\lambda\right)+\frac{(n_2-1)\,(\sigma^2\rho_1+\kappa)+\sigma^2+\kappa}{n_2}\right)\\ &= \Delta N^{S\alpha}_{Yt+1}\frac{\sigma^2\left(1+n_2\,(\rho_1-\rho_2)-\rho_1\right)+n_2\,(\kappa+\lambda)}{n_2}\end{aligned}$$

which obtains expression (20).

Referring to the equivalences displayed in (18), for country X we have:

$$\begin{aligned}
&\sigma^2\left(1+(\rho_1-\rho_2)\,n_1-\rho_1\right)+n_1\lambda \\
&=G^F+S^F+I^F-n_1\left(\frac{\delta^F-\theta^F}{2}\right)-\left(G^F+S^F\right)+n_1\left(G^F+S^F\right)-G^F n_1 \\
&=I^F+n_1\left(\frac{\theta^F-\delta^F}{2}\right)+n_1 S^F
\end{aligned}$$

which yields expression (23). For country Y, we have:

$$\begin{aligned}
&\sigma^2\left(1+n_2\left(\rho_1-\rho_2\right)-\rho_1\right)+n_2\left(\kappa+\lambda\right) \\
&=G^F+S^F+I^F+\left(e^F+\delta^F+\theta^F\right)n_2-\frac{\left(\delta^F-\theta^F\right)}{2}n_2 \\
&-\left(G^F+S^F\right)+\left(G^F+S^F\right)n_2-G^F n_2 \\
&=I^F+\left(e^F+\frac{3\theta^F+\delta^F}{2}+S^F\right)n_2
\end{aligned}$$

which is expression (24).

Proof of Proposition 2. Note that the change in the position of switchers $\Delta N^{S\alpha}_{Xt+1}$ and $\Delta N^{S\alpha}_{Yt+1}$depend on past prices (up to time t), and they are therefore uncorrelated with current (i.e., time $t+1$) cash-flow shocks. Note also that:

$$var(\Delta N^{S\alpha}_{Xt+1})=var(\Delta N^{S\alpha}_{Yt+1})=\alpha^2 v$$

Note that v, A_1 and A_2 are time-varying and depend on past price and cash-flow realizations, up to time t included. The risk aversion coefficient γ and the participation of switchers to the economy α are time invariant. Recalling the cash-flow equation (3), and the return equations (19) and (20), we find that, conditioning on time t, the variances of the returns take this form:

$$\begin{aligned}
var\left(\Delta P_{i,t+1}\right)&=1+\gamma^2 A_1^2\alpha^2 v \text{ for } i\in X \\
var\left(\Delta P_{h,t+1}\right)&=1+e+\theta+\delta+\gamma^2 A_2^2\alpha^2 v \text{ for } h\in Y
\end{aligned}\tag{39}$$

Using the expression of the returns in (19), we calculate the return correlation for assets i and j in X. Hence,

$$corr\left(\Delta P_{it+1},\Delta P_{jt+1}\right)=\frac{G+S+\gamma^2\alpha^2 v A_1^2}{1+\gamma^2 A_1^2\alpha^2 v}.$$

and thus the correlation gap in X is:

$$Gap_{n_1} = \frac{G + S + \gamma^2\alpha^2 v A_1^2}{1 + \gamma^2\alpha^2 A_1^2 v} - (M + S).$$

A bit of algebra yields equation (27). Next, for $h, k \in Y$ we have:

$$corr\left(\Delta P_{ht+1}, \Delta P_{kt+1}\right) = \frac{G + S + e + \theta + \delta + \gamma^2\alpha^2 v A_2^2}{1 + e + \theta + \delta + \gamma^2\alpha^2 A_2^2 v}.$$

and the correlation gap for asset class Y is:

$$\begin{aligned} Gap_{n_2} &= \frac{G + S + e + \theta + \delta + \gamma^2\alpha^2 v A_2^2}{1 + e + \theta + \delta + \gamma^2\alpha^2 A_2^2 v} - \frac{G + S + e + \theta + \delta}{1 + e + \theta + \delta} \\ &= \frac{v\alpha^2\gamma^2 A_2^2 I}{(\theta + \delta + e + 1)\left(v\alpha^2\gamma^2 A_2^2 + \theta + \delta + e + 1\right)} > 0 \end{aligned}$$

which yields the expressions in (28).

Proof of Proposition 4 Taking derivatives of the correlation gaps, for country X we have

$$\frac{\partial Gap_{n_1}}{\partial \alpha} = \frac{2v\alpha\gamma^2 I A_1^2}{\left(v\alpha^2\gamma^2 A_1^2 + 1\right)^2} > 0 \tag{40}$$

$$\frac{\partial Gap_{n_1}}{\partial \gamma} = \frac{2v\alpha^2\gamma I A_1^2}{\left(v\alpha^2\gamma^2 A_1^2 + 1\right)^2} > 0 \tag{41}$$

and for class Y

$$\frac{\partial Gap_{n_2}}{\partial \alpha} = \frac{2v\alpha\gamma^2 I A_2^2}{\left(v\alpha^2\gamma^2 A_2^2 + \theta + \delta + e + 1\right)^2} > 0 \tag{42}$$

$$\frac{\partial Gap_{n_2}}{\partial \gamma} = \frac{2v\alpha^2\gamma I A_2^2}{\left(v\alpha^2\gamma^2 A_2^2 + \theta + \delta + e + 1\right)^2} > 0 \tag{43}$$

The next Lemma simplifies the exposition.

Lemma 11 *Let $n_1 > n_2$, and let the parameters ρ_1 and ρ_2 satisfy the restrictions displayed in* (17). *Given the expressions of $\overline{A}_1$ and $\overline{A}_2$ displayed in* (31) *and* (32), *respectively, then.*

$$\overline{A}_2^2 > \overline{A}_1^2$$

If, additionally, $n_1 = n_2$ *then* $\overline{A}_2$ *is equal to* $\overline{A}_1$. *Further, let* A_1 *and* A_2 *be defined as in* (21) *and* (22), *respectively, then*

$$A_2 > A_1 \tag{44}$$

Further, if $n_1 = n_2$ *then*

$$\overline{A}_2 = \overline{A}_1$$
$$A_2 - A_1 = \kappa$$

Proof of Lemma. We have:

$$\overline{A}_2^2 - \overline{A}_1^2 = \frac{\sigma^4}{n_1^2 n_2^2} (n_1 - n_2)(1 - \rho_1)(n_1(1 - \rho_1) + n_2(1 - \rho_1) + 2n_1 n_2(\rho_1 - \rho_2)) > 0$$

Note that if, additionally, $n_1 = n_2$ then $\overline{A}_2$ is equal to $\overline{A}_1$. Next, the difference between A_1 and A_2 is:

$$A_1 - A_2 = -\frac{1}{n_1 n_2} \left(\left(\sigma^2 n_1 - \sigma^2 n_2 \right)(1 - \rho_1) + \kappa n_1 n_2 \right) < 0$$

If $n_1 = n_2$, then

$$A_1 - A_2 = -\frac{1}{n_1 n_2} \kappa n_1 n_2 = -\kappa.$$

Proof of Proposition 5. With $z = 0$, the difference ΔGap between the correlation gaps of countries Y and X, which is defined in (30), relies on the correlation gaps displayed in (33) and (34). After some calculations we obtain:

$$\Delta \overline{Gap} = Iv\alpha^2\gamma^2 \frac{\left(v\alpha^2\gamma^2\overline{A}_1^2 + v\alpha^2\gamma^2\overline{A}_2^2 + 1 \right)\left(\overline{A}_2^2 - \overline{A}_1^2 \right)}{\left(v\alpha^2\gamma^2\overline{A}_1^2 + 1 \right)\left(v\alpha^2\gamma^2\overline{A}_2^2 + 1 \right)} \tag{45}$$

Given Lemma 11, the ΔGap is positive. If, additionally, $n_1 = n_2$ then then ΔGap is zero, as $\overline{A}_2^2 = \overline{A}_1^2$ in equation (45). When $n_1 = sn_2$ then

$$\frac{\partial \overline{Gap}}{\partial s} = -\frac{\partial \overline{Gap}_{n_1}}{\partial \overline{A}_1} \frac{\partial \overline{A}_1}{\partial n_1} \frac{\partial n_1}{\partial s} = -\left(\frac{2v\alpha^2\gamma^2 I \overline{A}_1}{\left(v\alpha^2\gamma^2\overline{A}_1^2 + 1 \right)^2} \right)\left(\frac{\sigma^2}{n_1^2}(\rho_1 - 1) \right) n_2 > 0$$

as $\rho_1 < 1$. Hence, $\Delta\overline{Gap}$ increases in s when s increases. Analogously, we could have proved that $\overline{Gap}_{n_1}$ and $\overline{Gap}_{n_2}$ are both declining in the parameters n_1 and n_2, respectively. The steps are identical to those outlined in the next proof, for the case with exchange risk.

Proof of Proposition 6. Recalling the expression of the gaps Gap_{n_1} and Gap_{n_2} displayed in (27) and (28), and the expressions of A_1 and A_2 displayed in (21) and (22), we note that n_1 and n_2 appear in the expressions of Gap_{n_1} and Gap_{n_2} only through A_1 and A_2, respectively. Hence,

$$\frac{\partial Gap_{n_1}}{\partial n_1} = \frac{\partial Gap_{n_1}}{\partial A_1}\frac{\partial A_1}{\partial n_1} < 0$$
$$\frac{\partial Gap_{n_2}}{\partial n_2} = \frac{\partial Gap_{n_2}}{\partial A_2}\frac{\partial A_2}{\partial n_2} < 0$$

As

$$\frac{\partial Gap_{n_1}}{\partial A_1} = \frac{2v\alpha^2\gamma^2 I A_1}{\left(v\alpha^2\gamma^2 A_1^2 + 1\right)^2} > 0$$
$$\frac{\partial A_1}{\partial n_1} = \frac{\sigma^2}{n_1^2}\left(\rho_1 - 1\right) < 0$$
$$\frac{\partial \Delta Gap_{n_2}}{\partial A_2} = \frac{2v\alpha^2\gamma^2 I A_2}{\left(v\alpha^2\gamma^2 A_2^2 + z + 1\right)^2} > 0$$
$$\frac{\partial A_2}{\partial n_2} = \frac{\sigma^2}{n_2^2}\left(\rho_1 - 1\right) < 0$$

due to the restriction $\rho_1 < 1$ displayed in (17). If there is no exchange rate risk, the proof goes through after imposing z, κ and λ equal to zero.

Proof of Proposition 7 Based on equations (27), and (28), we have:

$$\Delta Gap\left(z\right) = I\gamma^2 v\alpha^2\left(\frac{A_2^2}{\left(z+1\right)\left(v\alpha^2\gamma^2 A_2^2 + z + 1\right)} - \frac{A_1^2}{1 + \gamma^2 v\alpha^2 A_1^2}\right) \tag{46}$$
$$= \frac{-v\alpha^2\gamma^2 I\left(z^2 A_1^2 + vz\alpha^2\gamma^2 A_1^2 A_2^2 + 2zA_1^2 + A_1^2 - A_2^2\right)}{\left(v\alpha^2\gamma^2 A_1^2 + 1\right)\left(z+1\right)\left(v\alpha^2\gamma^2 A_2^2 + z + 1\right)} \tag{47}$$

Note that if e is zero, then z is zero then

$$A_1\left(e = 0\right) = \frac{\sigma^2\left(1 + \left(\rho_1 - \rho_2\right)n_1 - \rho_1\right)}{n_1}$$
$$A_2\left(e = 0\right) = \frac{\sigma^2\left(1 + \left(\rho_1 - \rho_2\right)n_2 - \rho_1\right)}{n_2}$$

so that the $\Delta Gap\,(0)$ is to $\Delta\overline{Gap}$, the spread in the case of no exchange risk, displayed in equation 45.

Let $z > 0$ so that the ΔGap is defined by equation (46). Then ΔGap is positive as long as

$$\eta\,(z) = A_1^2 z^2 + \left(v\alpha^2\gamma^2 A_1^2 A_2^2 + 2A_1^2\right) z + \left(A_1^2 - A_2^2\right) < 0 \tag{48}$$

The roots of the associated polynomial in z are

$$z_0 = -\frac{1}{2A_1}\left(2A_1 - A_2\sqrt{4v\alpha^2\gamma^2 A_1^2 + v^2\alpha^4\gamma^4 A_1^2 A_2^2 + 4} + v\alpha^2\gamma^2 A_1 A_2^2\right)$$
$$z_1 = -\frac{1}{2A_1}\left(2A_1 + A_2\sqrt{4v\alpha^2\gamma^2 A_1^2 + v^2\alpha^4\gamma^4 A_1^2 A_2^2 + 4} + v\alpha^2\gamma^2 A_1 A_2^2\right)$$

Note that $z_1 < 0$ and $z_0 > 0$ when

$$2A_1 + v\alpha^2\gamma^2 A_1 A_2^2 - A_2\sqrt{4v\alpha^2\gamma^2 A_1^2 + v^2\alpha^4\gamma^4 A_1^2 A_2^2 + 4} < 0 \tag{49}$$

Since

$$\left(2A_1 + v\alpha^2\gamma^2 A_1 A_2^2\right)^2 - \left(A_2\sqrt{4v\alpha^2\gamma^2 A_1^2 + v^2\alpha^4\gamma^4 A_1^2 A_2^2 + 4}\right)^2 = 4\left(A_1^2 - A_2^2\right) < 0$$

by Lemma 44, then inequality (49) is satisfied and $z_0 > 0$.

By the same lemma, if $n_1 = n_2$, we have $A_2 = A_1 + \kappa$ so that

$$A_1^2 - A_2^2 = A_1^2 - (A_1 + \kappa)^2 < 0$$

and thus $z_0 > 0$ also in this case. Further, assuming $n_1 = sn_2$ we have:

$$\frac{\partial z_0}{\partial s} = \frac{\partial z_0}{\partial A_1}\frac{\partial A_1}{\partial s} = \frac{-2A_2}{A_1^2\sqrt{v^2\alpha^4\gamma^4 A_1^2 A_2^2 + 4v\alpha^2\gamma^2 A_1^2 + 4}}\left(\frac{1}{s^2}\frac{\sigma^2}{n_2}\left(\rho_1 - 1\right)\right) > 0$$

due to the condition $\rho_1 < 1$.

Proof of Proposition 8 Note that , as switchers' determine their allocations on the basis of time-t prices, a change in the exchange rate risk occurring after time t does not affect $\Delta N_{t+1}^{S\alpha}$, which is switchers demand change between t and $t+1$. Further, it assumed that fundamental traders do not modify their assessment of the covariance structure V due to the change in z. Taking derivatives of the expressions in (27) and (28) and recalling the definition of ΔGap, we have

$$\frac{\partial Gap_{n_1}}{\partial z} = 0$$
$$\frac{\partial Gap_{n_2}}{\partial z} = \frac{\partial \Delta Gap}{\partial z} = -\frac{A_2^2\left(v\alpha^2\gamma^2 A_2^2 + 2z + 2\right)}{(z+1)^2\left(v\alpha^2\gamma^2 A_2^2 + z + 1\right)^2} < 0$$

8 Appendix D: Proof of Propositions With Signal

Proof of Proposition 9. The signal increment B_{t+1} is correlated with f_{Gt}, the global risk factor realization at time t. The implication is that the signal is correlated with prices in t and thus it is correlated with the component of switchers' demand that does not include the signal, namely N^S_{Xt+1} and N^S_{Yt+1}, which are obtained by updating equations (7) and (8), respectively. From these equation, some algebra obtains the following expressions:

$$\Delta N^S_{Xt+1} = \frac{\Delta P_{Xt} - \Delta P_{Yt}}{2} - (1-\theta)\sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{Xt-k} - \Delta P_{Yt-k}}{2}\right)$$
$$\Delta N^S_{Yt+1} = \frac{\Delta P_{Yt} - \Delta P_{Xt}}{2} - (1-\theta)\sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{Yt-k} - \Delta P_{Xt-k}}{2}\right)$$

where ΔP_{Xt} and ΔP_{Yt} are the equally weighted averages of the return in each asset class, at time t defined in (4) and (5) The signal B_{t+1} is correlated to f_{Gt} but not with f_{Gt-k} for $k = 1, \ldots, t$, and so:

$$cov\left(\Delta N^S_{Xt+1}, B_{t+1}\right) = cov\left(\frac{\Delta P_{Xt} - \Delta P_{Yt}}{2}, B_{t+1}\right) = cov\left(\sum_{i=1}^{n_1}\frac{\Delta P_{it}}{2} - \sum_{k=1}^{n_2}\frac{\Delta P_{kt}}{2}, B_{t+1}\right) \tag{50}$$

where

$$\frac{\Delta P_{Xt} - \Delta P_{Yt}}{2} = \sum_{i=1}^{n_1}\frac{\Delta P_{it}}{2} - \sum_{k=1}^{n_2}\frac{\Delta P_{kt}}{2}$$

Note that ΔP_{it} and ΔP_{kt} are returns on a securities in X and Y, respectively. Using the return expressions in equations (37) and (38), we obtain:

$$\begin{aligned}
&\frac{\Delta P_{Xt}-\Delta P_{Yt}}{2} \qquad (51)\\
&=\frac{1}{2}\sum_{i=1}^{n_1}\frac{\varepsilon_{it}+\alpha\gamma A_1\left(\Delta N_{Xt}^S-B_t\right)}{n_1}-\frac{1}{2}\sum_{k=1}^{n_2}\frac{\varepsilon_{kt}+\alpha\gamma A_2\left(\Delta N_{Yt}^S+B_t\right)}{n_2}\\
&=\sum_{i=1}^{n_1}\frac{\varepsilon_{it}+\alpha\gamma A_1\left(\gamma\Delta N_{Xt}^S-B_t\right)}{2n_1}-\sum_{k=1}^{n_2}\frac{\varepsilon_{kt}+\alpha\gamma A_2\left(-\Delta N_{Xt}^S+B_t\right)}{2n_2}\\
&=\sum_{i=1}^{n_1}\frac{\varepsilon_{it}}{2n_1}-\sum_{k=1}^{n_2}\frac{\varepsilon_{kt}}{2n_2}+\frac{n_1\alpha\gamma A_1\left(\Delta N_{Xt}^S-B_t\right)-n_2\alpha\gamma A_2\left(-\Delta N_{Xt}^S+B_t\right)}{2}\\
&=\sum_{i=1}^{n_1}\frac{\varepsilon_{it}}{2n_1}-\sum_{k=1}^{n_2}\frac{\varepsilon_{kt}}{2n_2}+\alpha\gamma\frac{n_1A_1\left(\Delta N_{Xt}^S-B_t\right)-n_2A_2\left(-\Delta N_{Xt}^S+B_t\right)}{2}\\
&=\sum_{i=1}^{n_1}\frac{\varepsilon_{it}}{2n_1}-\sum_{k=1}^{n_2}\frac{\varepsilon_{kt}}{2n_2}+\frac{\alpha\gamma}{2}\left(\Delta N_{Xt}^S-B_t\right)\left(n_1A_1-n_2A_2\right) \qquad (52)
\end{aligned}$$

Recalling the definition of the cash-flows shocks in equation (1), we have

$$\begin{aligned}
2\left(\sum_{i=1}^{n_1}\frac{\varepsilon_{it}}{2n_1}-\sum_{k=1}^{n_2}\frac{\varepsilon_{kt}}{2n_2}\right)&=\frac{n_1\left(\sqrt{G}f_{Gt}+\sqrt{S}f_{Xt}\right)}{n_1}-\frac{n_2\left(\sqrt{G}f_{Gt}+\sqrt{S}f_{Yt}\right)}{n_2}\\
&+\sum_{i=1}^{n_1}\sqrt{I}f_{it}-\sum_{j=1}^{n_2}\sqrt{I}f_{jt}\\
&=\left(\sqrt{G}f_{Gt}+\sqrt{S}f_{Xt}\right)-\left(\sqrt{G}f_{Gt}+\sqrt{S}f_{Yt}\right)\\
&+\sum_{i=1}^{n_1}\sqrt{I}f_{it}-\sum_{j=1}^{n_2}\sqrt{I}f_{jt}\\
&=\sqrt{S}\left(f_{Xt}-f_{Yt}\right)+\sum_{i=1}^{n_1}\sqrt{I}f_{it}-\sum_{j=1}^{n_2}\sqrt{I}f_{jt}
\end{aligned}$$

Hence,

$$
\begin{aligned}
&\frac{\Delta P_{Xt}-\Delta P_{Yt}}{2} \\
&=\sqrt{S}\left(f_{Xt}-f_{Yt}\right)++\sum_{i=1}^{n_1}\sqrt{I}f_{it}-\sum_{j=1}^{n_2}\sqrt{I}f_{jt} \\
&+\frac{\alpha\gamma}{2}\left(\Delta N_{Xt}^{S}-B_t\right)\left(n_1A_1-n_2A_2\right)
\end{aligned}
$$

Recalling expression (50), and since B_{t+1} is uncorrelated with all time-t variables (with the exception of f_{Gt}), including ΔN_{Xt}^{S} which is determined on prices preceding t, we have:

$$
cov\left(\Delta N_{Xt+1}^{S}, B_{t+1}\right)=0 \tag{53}
$$

By a similar argument, but for minor modifications, the signal B_{t+1} correlation with ΔN_{Yt+1}^{S} is also zero. Using again the expression of the returns in (37) for securities in the asset class X, we can then calculate the covariance of returns for assets i and j in X, where $i \neq j$.

$$
\begin{aligned}
&cov(\Delta P_{it+1}, \Delta P_{jt+1}) \\
&=cov\left(\left(\varepsilon_{it+1}+\alpha\gamma A_1\left(\Delta N_{Xt+1}^{S}-B_{t+1}\right)\right),\left(\varepsilon_{jt+1}+\alpha\gamma A_1\left(\Delta N_{Xt+1}^{S}-B_{t+1}\right)\right)\right) \\
&=cov\left(\varepsilon_{it+1},\varepsilon_{jt+1}\right)+\alpha\gamma A_1 cov\left(\varepsilon_{it+1},\Delta N_{Xt+1}^{S}\right) \\
&-\alpha\gamma A_1 cov\left(\varepsilon_{it+1},B_{t+1}\right)+\alpha\gamma A_1 cov\left(\Delta N_{Xt+1}^{S},\varepsilon_{jt+1}\right)+ \\
&+\gamma^2\alpha^2A_1^2 cov\left(\Delta N_{Xt+1}^{S},\Delta N_{Xt+1}^{S}\right)-\gamma^2\alpha^2A_1^2 cov\left(\Delta N_{Xt+1}^{S},B_{t+1}\right) \\
&-\gamma\alpha A_1 cov\left(\varepsilon_{jt+1},B_{t+t}\right)-\gamma^2\alpha^2A_1^2 cov\left(\Delta N_{Xt+1}^{S},B_{t+t}\right) \\
&+\gamma^2\alpha^2A_1^2 cov\left(B_{t+t},B_{t+t}\right).
\end{aligned}
$$

Note now that since ΔN_{Xt+1}^{S} (not $\Delta\widetilde{N}_{Xt+1}^{S}$) is determined on the basis of

prices up to time t, then cash-flow shocks at $t+1$, that is ε_{it+1} for $i \in X \cup Y$, are uncorrelated with ΔN_{Xt+1}^{S}, and, for later use, with ΔN_{Yt+1}^{S}. The shocks ε_{it+1} are also uncorrelated with the signal B_{t+1}. Hence, for security i in X we have:

$$
cov(\Delta P_{it+1}, \Delta P_{jt+1})=G+S+A_1^2\gamma^2\alpha^2v-2A_1^2\gamma^2\alpha^2 cov\left(\Delta N_{Xt+1}^{S},B_{t+t}\right)+\gamma^2\alpha^2A_1^2p
$$

and given equation (53), then

$$cov(\Delta P_{it+1}, \Delta P_{jt+1}) = (1 - I) + \gamma^2 \alpha^2 A_1^2 (v + p)$$

The time $t + 1$ variance of the return of assets $i \in X$ is

$$\begin{aligned} var(\Delta P_{it+1}) &= var(\varepsilon_{it+1}) + \gamma^2 \alpha^2 A_1^2 v + \gamma^2 \alpha^2 A_1^2 var(B_{t+1}) \\ &= G + S + I + \gamma^2 \alpha^2 A_1^2 (v + p) \\ &= 1 + \gamma^2 \alpha^2 A_1^2 (v + p). \end{aligned}$$

The correlation gap with the signal, following the definition displayed in equation (25), is then

$$\begin{aligned} Gap_{n_1}^B &= \frac{(1 - I) + \gamma^2 \alpha^2 A_1^2 (v + p)}{1 + \gamma^2 \alpha^2 A_1^2 (v + p)} - (1 - I) \\ &= \frac{\alpha^2 \gamma^2 I A_1^2 (p + v)}{p \alpha^2 \gamma^2 A_1^2 + v \alpha^2 \gamma^2 A_1^2 + 1} > 0 \end{aligned}$$

When $h, k \in Y$ we have that B_{t+1} and ΔN_{Yt+1}^S are uncorrelated, as $\Delta N_{Xt+1}^S = -\Delta N_{Xt+1}^S$. Hence, the correlation across returns is readily calculated as:

$$corr(\Delta P_{ht+1}, \Delta P_{kt+1}) = \frac{G + S + z + \gamma^2 \alpha^2 A_2^2 (v + p)}{1 + z + \gamma^2 \alpha^2 A_1^2 (v + p)}.$$

and the correlation gap for asset class Y is:

$$\begin{aligned} Gap_{n_2}^B &= \frac{G + S + z + \gamma^2 \alpha^2 A_2^2 (v + p)}{1 + z + \gamma^2 \alpha^2 A_2^2 (v + p)} - \frac{G + S + z}{1 + z} \\ &= \frac{\alpha^2 \gamma^2 A_2^2 I (v + p)}{(z + 1)((v + p) \alpha^2 \gamma^2 A_2^2 + z + 1)} > 0 \end{aligned}$$

Proof of Proposition 10. Taking derivatives, we have:

$$\begin{aligned} \frac{\partial Gap_{n_1}^B}{\partial p} &= \frac{\alpha^2 \gamma^2 I A_1^2}{((p + v) \alpha^2 \gamma^2 A_1^2 + 1)^2} > 0 \\ \frac{\partial Gap_{n_2}^B}{\partial p} &= \frac{\alpha^2 \gamma^2 I A_2^2}{(z + (p + v) + 1)^2} > 0 \end{aligned}$$

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