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The Mean-Variance Core of Cryptocurrencies: When More is Not Better

Valentina Galvani
University of Alberta

Vita Faychuk
Gustavus Adolphus College

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Valentina Galvani [*] and Vita Faychuk [†]

Abstract

We explore the existence of a mean-variance core subset of cryptocurrencies that subsumes the risk-reward of the broader market. The analysis considers both the perspective of long-short and long-only investors. The results indicate that most cryptocurrencies are redundant from the standpoint of both types of investors, with the exception of Bitcoin, which consistently improves the Sharpe ratio of even broad cryptocurrency portfolios. We show that the core can be often identified ex-ante as the cryptocurrencies attracting the highest levels of investors' attention.

Keywords: Sharpe Ratio; Cryptocurrencies; Bitcoin; Short-Selling; Spanning Tests. JEL: G11, G12, G14, G40

[*]Department of Economics, University of Alberta, vgalvani@ualberta.ca
[†]Gustavus Adolphus College, vfaychuk@gustavus.edu

Introduction

A large number of cryptocurrencies (henceforth, CCs) available for investment begets the question of whether some might be redundant. Investors benefit from pruning redundant assets as it minimizes portfolio monitoring efforts by narrowing the focus of their limited attention to fewer CCs.

This study discusses the existence of a minimal subset of CCs (i.e., a core) that subsumes the broader market. We consider scenarios with and without short sale restrictions to gain insights that are relevant to either long-only or long-short investors. This approach allows capturing heterogeneity in market participants' trading patterns.[1] Our discussion provides critical guidance for institutional and retail investors who have an interest in optimizing the risk-return trade-off.

To conduct our analysis, we use the Mean-Variance (MV) framework, which offers the advantage of abstracting away from the intricate choice of the proper asset pricing model (e.g., Subrahmanyam, 2010; Fama and French, 2017). Specifically, we rely on the regression-based tests for the equality of the Sharpe ratio ((e.g., Jobson and Korkie, 1989; DeRoon and Nijman, 2001) which are evaluated for the cases in which investors face short-selling restrictions or not.

Our results show that the core consists of only three CCs in over 90% of rolling samples. In over 75% of samples only two CCs suffice to yield the Sharpe ratio offered by broader portfolios of CCs. When short-selling is not allowed, the corresponding percentages are even higher. These findings indicate that many of the CCs are redundant from the perspective of the risk-reward trade-off and that short-selling is not essential to the

[1] For instance, retail investors very seldom sell short (e.g., Barber and Odean, 2008).

existence of a core.

We find that Bitcoin (BTC) is part of the minimal set in virtually all the instances a core is identified, indicating that the oldest cryptocurrency remains an essential constituent of CC investing. The relevance of Bitcoin is explained by a risk-return trade-off that is superior to those offered by portfolios of other CCs. No other CC shows the same ability to consistently increase the Sharpe ratio of broad portfolios of cryptocurrencies. We find that this distinctive advantage of BTC is in part due to investors' ability to sell short. For long-only traders, the distinctiveness of BTC's Sharpe ratio is less pervasive. Still, long-only investors with holdings in only two or three CCs would consistently benefit from including BTC in their portfolio.

Our study contributes to the line of research investigating diversification and portfolio optimization within the CC market(e.g., Brauneis and Mestel, 2019; Liu, 2019; Platanakis and Urquhart, 2019).[2] Prior research in this area generally supports the conclusion that diversification across CCs is advantageous to investors. This study expands this line of research by applying formal testing of diversification gains and considering short-selling restrictions.[3] Our findings highlight that diversification benefits in the cryptocurrency universe are limited.

We experiment with identifying a core ex-ante based on investors' attention, as measured by trading volume (e.g., Barber and Odean, 2008). The conjecture is that information flows faster for CCs attracting high levels of attention. Such CCs are more likely to form a core than cryptocurrencies that are slower to react to news. Consistent with high levels of investors' attention yielding an informational advantage, the frequency with

[2]See also the reviews (e.g., Bariviera and Merediz-Solà, 2021) and (e.g., Corbet et al., 2019)

[3]From a methodological perspective, the study most similar to ours is Briere et al. (2015) who evaluate the opportunity of including BTC into portfolios of traditional financial assets in the MV framework.

which the most traded CCs form a core is about twice as large as that documented for randomly picked cryptocurrencies.

1 Methodology and Data

To identify the core (i.e., a minimal subset of CCs that subsumes the broader market), we rely on the test for spanning (e.g., Jobson and Korkie, 1984, 1989; Huberman and Kandel, 1987; DeRoon and Nijman, 2001). Specifically, we study the relationship between the MV-frontier generated by K benchmark cryptocurrencies and that associated with an expanded set of CCs with N additional returns. These tests build on the following multivariate regression model:

$$r_{Nt} = \alpha + \beta r_{Kt} + \epsilon_t$$

where r_{Nt} and r_{Kt} are $N \times 1$ and $K \times 1$-vectors of excess returns on the N test and K benchmark assets, respectively.[4]

Testing the joint linear restrictions $\alpha = 0_N$ evaluates whether the Sharpe ratio of the tangency portfolio of the MV frontier generated by K assets is statistically indistinguishable from that of the frontier generated by an investment possibility set with $K + N$ assets. Under the null, expanding the benchmark set of K assets with additional N assets does not significantly improve the Sharpe ratio, and the K assets constitute a core.

In this study, we explore the existence of a core with as few as two or three CCs (i.e., $K = 2, 3$).[5] To provide an analogous assessment when short-selling is restricted, we rely

[4]The disturbances ϵ_t satisfy $E[\epsilon_t] = 0_N$.

[5]Following the literature, a Wald test is employed to evaluate the joint linear restrictions on the intercepts. This test rejects the null too often when based on its asymptotic distribution (e.g., Gibbons et al., 1989). In the context of this study, rejecting the null too often would work against the existence of the core. In this sense, relying on the Wald test biases the results against finding a core.

on a modified test for spanning proposed in DeRoon and Nijman (2001).[6]

Kan and Zhou (2001) show that joint tests of the intercepts of a multivariate regression have low power when the number of restrictions (N) is much larger than the number of explanatory variables (K). We thus restrict the investment possibility set to the ten most traded CCs (i.e., $K + N \leq 10$). We argue that such restriction does not weaken the scope of our assessment as the vast majority of trading activities involve fewer than ten CCs throughout our sample period. In untabulated results, we find that expanding the pool to 20 CCs yields qualitatively identical conclusions. The same pool of securities is employed to evaluate the increase in the Sharpe ratio for individual CCs (i.e., $N = 1$) with respect to an increasingly broad set of returns (i.e., $K = 3, 5, 9$).

We incorporate time-variability in the constituents of the core by using rolling samples of 52 weeks spaced by one week to account for the fact that the investment opportunity set for CCs varies over time. In each rolling sample, we test the null hypothesis of equality of the Sharpe ratio for all the C^K_{N+K} combinations of K cryptocurrencies out of the $N + K = 10$ most traded (by volume) cryptocurrencies. For instance, to assess the existence of a 3-CC core there are 120 tests to be evaluated in each of the 299 rolling subsamples.

For this study, we use weekly returns and volumes that we compute from daily trading data collected from Coinmarketcap.com from Sept 17,2014 to June 8, 2021.[7]

2 Empirical Results

[6]The asymptotic distribution of this test is a mixture of χ^2, and, thus, the inference is based on its empirical bootstrap distribution.

[7]We rely on weekly returns as a compromise between precision and bias since the statistical quality of our regression-based assessment of portfolio diversification improves with data stability.

Table 1: Existence of a Core

	Core %	BTC	BCH	ETC	ETH	LTC	XRP
			Panel A: Short-Selling Allowed				
K=2	72.2	71.2	30.43	32.8	44.5	44.82	48.5
K=3	92.3	92.3	52.2	50.2	77.6	83.27	91.3
			Panel B: Short-Selling Not Allowed				
K=2	85.2	84.6	42.8	56.2	53.5	52.2	59.5
K=3	92.3	90	58.2	70	79.2	89.6	84.3

The table summarizes the results of tests for the existence of a core of dimension $K = 2, 3$. The second column reports the percentage of one-year rolling samples for which at least one core exists. The remaining columns report the percentage of combinations in which individual cryptocurrencies are included in a core, on average over the rolling windows. Panel A refers to the classical framework with short sales being allowed, whereas for the results in Panel B short-selling is restricted. For each choice of K the number of tests evaluated is $\binom{J}{K}$ where J is the number of CCs with trading volume falling in the top 10% or ten CCs, whichever is the lowest. The CCs listed are Bitcoin (BTC), Bitcoin Cash (BCH), Ethereum Classic (ETC), Ethereum (ETH), Litecoin (LTC), and Ripple(XRP). The sample period is from Sept 17, 2014, to June 8, 2021.

Column two in Table 1 reports that a two-CC core can be identified in 72.2% of the rolling samples and a three-CC core in 93% of the subsamples when short-selling is allowed. The corresponding percentages are about 85% and 93%, if short-selling is restricted.[8] Such pervasive existence of the core constitutes evidence of the limits of diversification within the CC market.

Table 1 also reports the average percentage of the combinations that include a specific CC and form a core.[9] We find that for $K = 2$ the only cryptocurrency appearing in more than half (i.e., 71%) of instances in which a core exists is Bitcoin. For $K = 3$, Ripple and Bitcoin appear in the core in more than 90% of the instances. The other CCs that are often constituents of a core are Ethereum Classic, Bitcoin Cash, Ethereum, and Litecoin. These results identify the best candidates for a minimal set of CCs to subsume the returns of the

[8] We refer to the 5% significance level. The rolling samples include CCs with at least six months of observations.

[9] In each rolling sample, there are C^{K-1}_{N+K-1} combinations that potentially form a core that includes a specific CC. The reported percentage is an average over the subsamples. To simplify the exposition of the results, we report only the CCs that, on average, enter the core in at least half of these C^{K-1}_{N+K-1} combinations, for some choice of K.

broader CC market. Removing the availability of taking short positions reinforces these conclusions.

The high frequency with which Bitcoin is included in the core suggests that some of the CCs offer a particularly high compensation on risk and, therefore, including them into a portfolio is bound to improve the overall Sharpe ratio. We explore this possibility by testing the significance of the change in Sharpe ratio when a specific CC is added to an increasingly broader set of K cryptocurrencies. It is more difficult to improve the Sharpe ratio by adding another cryptocurrency to an efficient portfolio that already includes a large number of other CCs. Hence, a broader benchmark portfolio makes an individual cryptocurency less likely to stand out in terms of return on risk. For brevity, Table 2 reports the results of these tests only for $K = 3, 5, 9$.

Table 2: Individual Cryptocurrencies

	Panel A: Short-Selling Allowed					
	BTC	BCH	ETC	ETH	LTC	XRP
$K = 3$	96.7	23.4	37.9	37.9	27	50
$K = 5$	92	25.5	16.1	28.5	34	38.2
$K = 9$	79.3	35.1	13.5	20.4	14.6	36.1
	Panel B: Short-Selling Not Allowed					
	BTC	BCH	ETC	ETH	LTC	XRP
$K = 3$	99.4	18.8	46.1	55.4	70.1	67.8
$K = 5$	39	2.12	2.9	21.4	10.4	6.1
$K = 9$	33.4	2.34	5.02	20.1	14.4	7.35

The table shows the average (over the one-year rolling samples) percentage of portfolios of $K = 3, 5, 9$ CCs for which adding a specific CC does not significantly increase the Sharpe ratio. For each choice of K, the number of tests performed is $\binom{J-1}{K}$ where J is the number of CCs with trading volume falling in the top 10% or ten CC, whichever is the lowest. Panels A and B refer to the tests when short sales are allowed or restricted, respectively. The CCs listed are Bitcoin (BTC), Bitcoin Cash (BCH), Ethereum Classic (ETC), Ethereum (ETH), Litecoin (LTC), and Ripple(XRP). The sample period is from Sept 17, 2014, to June 8, 2021.

When short-selling is allowed (Panel A), Bitcoin statistically improves the Sharpe ratio of more than 90% of portfolios that include up to five CCs. Even when we focus on

portfolios of the nine most traded cryptocurrencies, adding BTC improves the achievable Sharpe ratio in 79% of the instances. Equivalently, even combining nine CCs fails to subsume the risk-reward offered by BTC. No other CC offers such pervasive Sharpe ratio improvements.[10] Bitcoin's superior ability to improve the Sharpe ratio of increasingly broader portfolios of CCs is confirmed when short-selling is not allowed (Panel B). Contrasting Panels A and B, however, shows that the enhancement of the Sharpe ratio offered by a specific cryptocurrency over a portfolio including more than three CCs is mostly driven by short-selling. For instance, restricting short-selling reduces drastically the probability that BTC improves the Sharpe ratio of 5-CC portfolios, dropping from 92 to 39% of the cases.

The results presented so far begets the question of whether the pervasive existence of a core is driven by the high risk-reward of BTC. We then re-examine the existence of the core after excluding Bitcoin from the sample. In untabulated results, we find that a core of two and three CCs can be identified in about 76% and 94% of the subsamples, respectively, when short sales are allowed. These percentages are very close to the ones identified for the samples that include BTC. While the composition of these cores is time-varying, the oldest cryptocurrencies are the CCs that appear most often in the core throughout the sample period. The implication is that the existence of a combination of two or three CCs that subsumes the broader investment possibility set does not depend on BTC alone.

[10] Ripple is a very distant runner up, as three CCs already suffice to subsume its risk-reward in about 50% of the instances. In untabulated results, for the CCs not included in the table, we find that the percentages are generally lower than 10% for $K = 3, 5, 9$, with the only exception being DASH at 43% for combinations of three returns.

2.1 Market State

The results discussed so far indicate that in the vast majority of subsamples at least one combination of K cryptocurrencies forms a core. However, not all combinations are equally likely to constitute a core. For instance, Table 3 shows that if we pick any three CCs randomly, on average, the probability that those three form a core is less than 30% and 15%, when short-selling is allowed or restricted, respectively.[11] The difference between these average frequencies implies that randomly picked CCs are more likely to form a core when short-selling is allowed. The implication is that the diversification benefits stemming from including additional CCs into a portfolio are expected to be smaller for long-short compared than for long-only investors. Hence, long-only investors may benefit from a broader portfolio of CCs than long-short investors.

Table 3: Market State and Real-time Core

	ALL	UP	DOWN
Panel A: Short-Selling Allowed			
$K = 2$	17.4	23.8	7.7
$K = 3$	28	35.1	17.2
$Real - time \;\; K = 2$	45.1	57.5	25.4
$Real - time \;\; K = 3$	53.2	65.2	34.2
Panel B: Short-Selling Not Allowed			
$K = 2$	13.1	12	14.7
$K = 3$	15.7	14.6	17.5
$Real - time \;\; K = 2$	40.5	41.8	38.6
$Real - time \;\; K = 3$	40.6	41.3	39.5

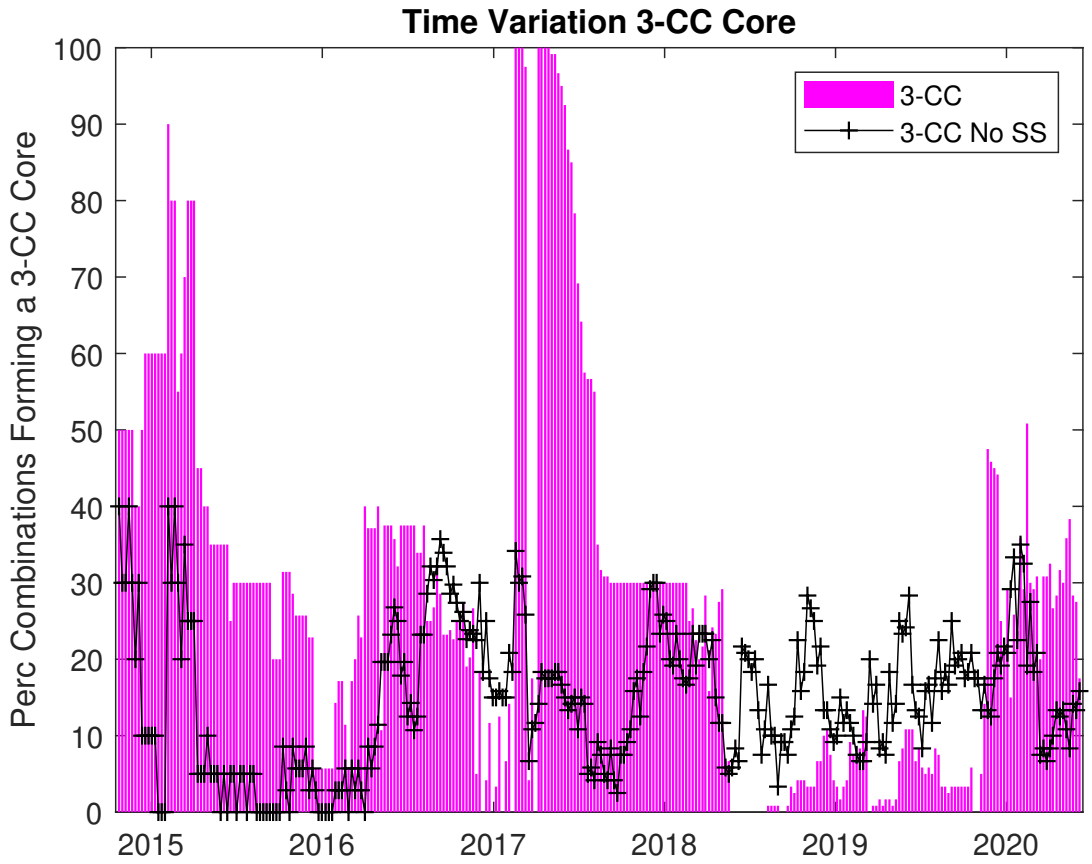
[11] As expected, randomly picking pairs of CCs, instead of triples, is even less likely to generate a core, at 17.4% and 13.1%, respectively.

The first two rows of results in Panel A show the average percentage of cases in which randomly chosen $K = 2, 3$ CCs form a core. The averages are taken separately for all subsamples and for UP and DOWN markets. The following rows report the percentage of rolling samples for which the two or three ($K = 2, 3$) most traded CCs (measured over the month preceding the beginning of the one-year sample) form a core, in aggregate and by market state. Panels A and B refer to the tests when short sales are and are not allowed, respectively. For $K = 2, 3$, there are 181 and 118 UP and DOWN market subsamples, respectively. For real-time series, there are 181 UP and 114 DOWN market states as we use four weeks of trading data to identify the real-time potential core. The sample period is from Sept 17, 2014, to June 8, 2021.

The averages reported in column two of Table 3, however, belie large time variations, as shown in Figure 1 which plots the probability that three randomly selected CCs constitute a core. We find that part of this time variation is explained by the market state, at least when short-selling is allowed. Table 3 shows that three randomly selected CCs are almost twice as likely to form a core following market upturns than market downturns.[12] When short-selling is not allowed, this market effect almost disappears. The implication is that it is the ability to short-sell following a buoyant market that increases the chances for a CC to be part of a core.

[12]The market phases are captured by the sign of the cumulative return on an equally weighted portfolio of all the CCs in the market, calculated over the six months preceding the start of the specific subsample. There are 181 UP and 118 DOWN market states.

Figure 1: Percentage of 3-CC Combinations Forming a Core



The figure displays the percentage of the 3-CC combinations forming a core plotted on the first week of each one-year window when short-selling is allowed or restricted. The sample period is from Sept 17, 2014, to June 8, 2021.

2.2 Real-time Cores

Our analysis shows that a few CCs can often deliver the risk-rewards of a broader set of cryptos. The composition of the core varies over time, which makes its practical use limited from the perspective of real-time investors. We investigate the existence of an ex-ante core that is identified based on investors' attention, as measured by trading volume (e.g., Barber and Odean, 2008). If information flows faster for CCs attracting the highest levels of attention, these cryptocurrencies are more likely to form a core, as the subsume the risk-reward offered by CCs that are slower to react to news.

For each rolling window, we pick the two or three CCs that are most traded in the month (i.e., four weeks) preceding the start of the subsample and evaluate whether they form a core. As reported in Panel A of Table 3, we find that these CCs form a two- and three-CC core, respectively, in 45% and 53% of the rolling windows, when short-selling is allowed.[13] These frequencies are about double those found for randomly selected CCs, as shown in the same panel. This result is confirmed when short-selling is not allowed (Panel B).

We find that, following market expansions, long-and-short investors can concentrate on the three or even two of the CCs that have been most traded in the past while incurring a less than 57.5% chance of forgoing high risk-rewards. In contrast, investors are better off with broader portfolios following market contractions. We find that when short-selling is not allowed, such market effect vanishes. The implication is that it is short-selling that yields superior risk-returns for the most (past) traded CCs following market gains. This conclusion echoes the finding for potential cores of randomly selected CCs discussed

[13]These results are stable for alternative parameterizations, such as relying on a three- or six-month window to identify the most traded strategies and referring to the trading volume of the expanding sample.

above.

3 Conclusions

The number of CCs available for trade has exploded over recent years. This increase makes it appear as if the investment opportunities in this market have effectively expanded. However, this study provides evidence that from the standpoint of MV investors, most CCs are redundant, especially following periods in which the cryptocurrency market is booming. Individual CCs might still be valuable when they cater to the needs of niche investors, for example, in terms of privacy protection or types of allowed transactions. However, from a pure investment perspective, agents are unlikely to benefit from expanding their CC portfolios beyond a handful of the most established cryptocurrencies. Additionally, our results suggest that long-only investors may benefit from a broad portfolio of CCs while long-and-short traders might limit their investment to a few CCs. Lastly, investor attention appears to have predictive power for the core constituents, especially following market expansions.

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