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Financial Frictions, Equity Constraints, and Average Firm Size Across Countries

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Abstract

We document new evidence that low equity in financially under-developed economies is associated with lower productivity investment, a smaller employment share of large firms, and smaller average firm size within sectors. We present a tractable model with heterogeneous entrepreneurs that face equity constraints that limit investment at entry. The model can be solved analytically, making clear predictions for the impact of equity constraints on outcomes of interest consistent with the evidence we document. The model can account for one-fifth to one-third of the variance in observed average firm size and TFP across countries, all substantial relative to the literature.

JEL: O1, O14, O41, O43.

Key Words: financial development, equity, firm size, investment, aggregate productivity.

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1 Introduction

The link between financial development and economic growth has long been thought to be an important part of any explanation for why some countries are drastically poorer and less productive than others. Poorer countries are characterized by low use of formal financial institutions, lower investment, more but smaller firms, and a low share of the labor force employed in large firms. In this paper we study the impact of financial frictions on average firm size and aggregate productivity in a model where constrained firms must invest in productivity early in their life cycle.

Our contribution is two-fold. First, we document that financial frictions (proxied by equity/GDP ratios) are associated with lower aggregate productivity, lower investment in productivity, less employment at large firms, and smaller firms (on average) in both manufacturing and services. Second, we develop a tractable quantitative model in which exogenous limits on equity generate outcomes consistent with the evidence we document. We calibrate our model to match several moments from U.S. aggregate and firm-level data, and show that empirically reasonable differences in equity limits can account for 10-20% of the log-variance in average firm size observed across countries, 10% of the log-variance in the share of employment at large firms, and a full one third of the log-variance in aggregate total factor productivity (TFP). Importantly, the quantitative relationships generated by the model between average firm size and equity ratios, as well as between TFP and equity ratios, are almost identical to the corresponding relationships in the data.

Our model extends the heterogeneous firm model of Hopenhayn (1992) by incorporating a costly productivity decision at entry, where entrepreneurs must finance their productivity investment by selling claims to future profits. We characterize the financial development of an economy by the maximum fraction of future profits that can be sold. Potential entrepreneurs base entry decisions on the equity constraint they face, as well as their own entrepreneurial ability and fixed cost of operation, implying a threshold ability characterizing entry that depends endogenously on the equity constraint. In the model, a lower maximum fraction of future profits that can be sold results in a lower level of firm-level productivity. Lower investment at entry *increases* the value of entry (given a number of firms), relative to the wage, resulting in more entry by entrepreneurs with relatively low ability. With decreasing returns to scale with respect to variable productive inputs, this increased entry by itself results in higher aggregate TFP. But this is more than offset by the lower investment in productivity. As a result, a more binding equity constraint leads to a substantially lower average firm size and aggregate TFP in equilibrium.

In focusing on how equity constraints impact aggregate TFP and average firm size when firms make investments at entry, this paper is most closely related to Buera et al. (2011) and Midrigan and Xu (2014). Buera et al. use a two-sector model with sectors characterized by larger or smaller fixed costs of operation to show that financial constraints (modelled as collateral constraints) can have a differential impact on average firm size across sectors, with average size dropping in the service sector (where firms face low fixed costs) but increasing in manufacturing (characterized by large fixed costs). We provide evidence that financial constraints (whether proxied by equity ratios or other measures used in the literature) are in fact associated with a larger average firm size in both manufacturing and services. Indeed, the relationship between average firm size and financial constraints is almost quantitatively identical across these sectors in the data, an outcome that our model can rationalize.

Midrigan and Xu (2014) model an economy where entrepreneurs can produce using a traditional technology or pay a fixed cost to adopt a modern (more productive) technology. As in our paper, financial frictions in their model can generate a large impact on aggregate TFP. But whereas TFP drops in our model because of lower productivity investment by each entering firm (an intensive margin), in Midrigan and Xu the large impact on TFP is due to fewer firms adopting modern technology (an extensive margin). In their model, financial constraints make entry into the modern sector effectively more costly, and as a result reduce both aggregate investment in productivity and the number of modern firms in equilibrium. In contrast, our model suggests financial constraints lead to *more* (though on average, smaller) firms, consistent with our data. Midrigan and Xu assume an exogenous total number of firms (traditional and modern), and so do not generate implications for the total number of firms. But we provide evidence suggesting that the number of employer firms (firms with employees, as a fraction of total sectoral employment), is not significantly related to the extent of financial constraints across countries. If anything, less financially-developed economies seem to have *more* employer firms.[1] Midrigan and Xu do not map 'traditional' firms to firms without employees, indeed they allow for traditional firms to hire. But to the extent that employers (in contrast to firms without employees) can proxy for modern firms, this evidence can be interpreted as suggesting the intensive margin is important in accounting for investment at entry in the presence of financial frictions. In comparison to Midrigan and Xu, our model with endogenous entry and investment at entry generates a similar impact on aggregate TFP from financial constraints without relying on their extensive margin. To be clear, allowing for endogenous firm entry is not necessary in our model to generate a large impact on TFP. In our model, the impact on TFP from equity constraints would be even larger without endogenous entry. But the higher entry induced by equity constraints is consistent with the evidence we document.

We keep our model very simple, and so neglect some of the richness captured by workhorse models in the literature (e.g. Buera et al., 2011; Midrigan and Xu, 2014; Moll, 2014), for two reasons. First, to make clear the intuition for how financial constraints can generate the outcomes we highlight above when firms make costly investments at entry. Second, to make clear that our framework is complementary to these models in the literature. The mechanism we highlight here can be thought of as operating in addition to other financial frictions like collateral constraints. We focus on the impact of financial frictions on the decisions of newly formed firms. Recent evidence suggests that decisions made during the early stages of firm formation are significant determinants of firm performance over its life cycle. For example, Haltiwanger et al. (2013) find firms in the U.S. grow substantially more in their first year

[1] It is still the case in the data that employer firms are a lower fraction of all firms in less financially-developed economies.

of life relative to later years. Moreira (2017) shows the investment and scale decisions of entrants depend importantly on the state of the economy at entry, and that these decisions have large and persistent effects over the life of firms. In a cross-country context, Bento and Restuccia (2017) show the impact of misallocation working through investment decisions at entry is much larger than than the impact working through firms' decisions over their life cycle.

Focusing on decisions made early in a firm's life cycle is particularly important in the context of financial frictions. Midrigan and Xu (2014) and Moll (2014), among others, find the aggregate impact of low financial development is dampened by an entrepreneur's incentive to self-finance in order to overcome collateral and other financial constraints over the life cycle of a firm, especially when firm-level productivity is persistent. We show that even in an environment where firm-level productivity is constant after entry, financial constraints have large effects on firm entry and investment, in addition to aggregate TFP. In our model, we do not allow for self-financing to undo the impact of equity constraints. If this assumption were relaxed, there are two potential mechanisms through which self-financing might matter. First, workers might save before starting a firm, in order to make a larger investment in productivity at entry. Although this seems like a plausible strategy in richer economies, we note that less financially-developed economies also tend to be very poor and unproductive. We do not explicitly model nonhomothetic preferences, but we appeal to them as a reason why saving before entry is not likely to be a quantitatively important margin, at least in poorer countries. Second, if firms continue to invest in productivity over their life-cycle, they might choose to make larger investments (financed through operating profits) when equity constraints are more binding, in order to compensate for low initial investment. We show (in an appendix) that allowing for life-cycle investment can potentially lead to higher compensating investment over the life-cycle (depending on how life-cycle investment is modeled), but that the quantitative impact of equity constraints on our outcomes of interest (including average size and aggregate TFP) hardly changes.

Our paper contributes to a recent literature exploring the potential determinants of average firm size across sectors and countries. Hsieh and Klenow (2014) and Bento and Restuccia (2017, 2021) show how average size can be lower in economies characterized with a high degree of misallocation and document evidence consistent with this mechanism. Bento (2020) shows that average firm size is lower in economies with high barriers to competition. Workhorse models studying financial frictions can in principle generate a positive relationship between average firm size and financial development, qualitatively consistent with the empirical relationships we document. But calibrated models almost universally predict the opposite relationship. One important exception is Buera et al. (2011), referred to above, who develop a two-sector model that suggests financial development should lead to smaller firms in manufacturing but larger firms in services. Both our model and evidence suggest financial development should lead to larger firms in all sectors. In particular we show that across countries with data for average firm size in both manufacturing and services, the elasticity of average size with respect to several proxies for financial development (including equity ratios) is both positive and nearly identical in both sectors. Poschke (2018) goes beyond average firm size, documenting systematic relationships between development and the shape of the firm size distribution. In particular, he provides evidence that the share of employment at large firms is higher in richer countries. We document a similar analogous relationship between financial development (characterized by low equity ratios) and the employment share of large firms. Our model generates a relationship that is qualitatively consistent with this evidence.

The paper proceeds as follows. In the next section, we document new evidence of the relationships between financial development, equity ratios, investment in R&D, and firm size distributions across sectors. We present our model in Section 3, and then calibrate it to match relevant moments from U.S. data. We use the model to quantify the impact of equity constraints on several outcomes in Sections 4. We conclude in Section 5.

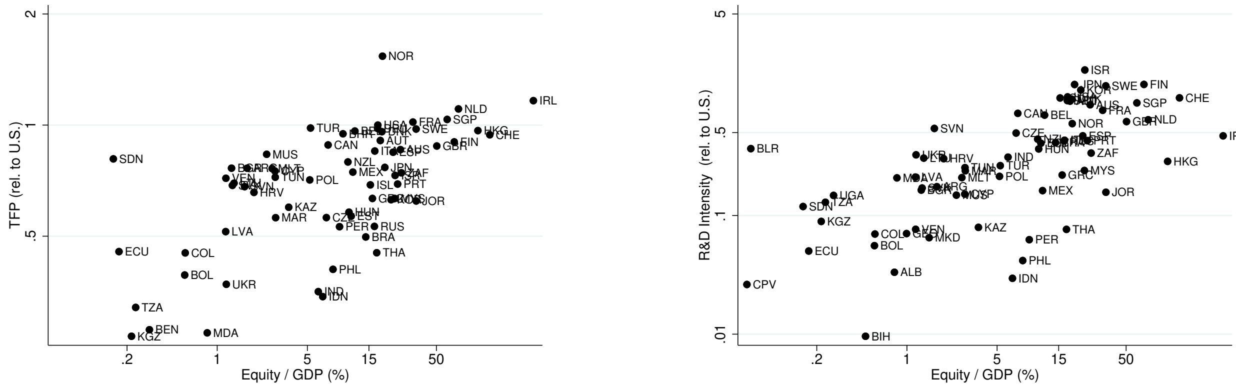
2 Data/Facts

We describe our data and document relationships between total equity relative to GDP, average firm size in manufacturing and services, R&D expenditure relative to GDP, the share of employment in large firms, and aggregate total factor productivity (TFP) across countries. Data on equity is from the Global Financial Development Database (GFDD) based on Beck et al. (2000, 2009), and measures average gross portfolio equity liabilities relative to GDP averaged over 2002-2007. We use this measure of equity to proxy financial market development across countries, consistent with the equity measure used in Midrigan and Xu (2014). While we focus on equity given the framework that follows, we note that equity is highly correlated with the aggregate quantity of bank deposits relative to GDP and the quantity of external finance to GDP, two widely-used proxies for financial development in the literature. In many models, these ratios are inversely related to the extent of financial frictions. For our measures of average firm size, we use data from Bento and Restuccia (2017, 2021). This data reports the average number of persons engaged per establishment in the service sector for 127 economies and in the manufacturing sector for 134 economies.[2] These data are meant to be representative of all persons engaged (employees, owners, unpaid family workers, etc...) and all establishments (formal and informal) in each sector. Measures of average size in manufacturing reflect an average over available data from each country in the 2000s, while average size in services reflects data from 2007 (or the closest year with data available). We use the share of employment at large firms from Poschke (2018), which measures the share of aggregate private sector employment at firms with at least 250 employees.[3] R&D data is from UNESCO and accounts for all R&D expenditure in an economy, reported relative to GDP. TFP data is from the Penn World Tables v9.0 (Feenstra et al., 2015).

[2] An establishment is a physical location where economic activity takes place, while a firm is a collection of at least one establishment under common ownership. We use 'firm' and 'establishment' interchangeably for ease of exposition. The data include measures of both the number of firms and the number of establishments for only a small number of countries, but we note that at least across these countries, differences in the average firm size across countries are almost identical to differences in average establishment size.

[3] Poschke's (2018) data comes from the World Bank's World Development Indicators and Amadeus (from Bureau Van Dijk).

Figure 1: Aggregate TFP, R&D Intensity, and Equity Ratios



(a) Aggregate TFP (b) R&D Intensity

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.14 (0.02) and (b) 0.38 (0.06), based on 66 and 71 observations.

Figure 1a shows how aggregate TFP is related to equity ratios (equity relative to GDP) across countries. Clearly, economies with lower equity ratios tend to be much less productive. Figure 1b illustrates a strong positive relationship between equity ratios and R&D intensity (aggregate R&D expenditure over GDP). To the extent that R&D intensity can proxy for overall investment in productivity, this points to a potentially important source of observed productivity differences across countries.

Figure 2 illustrates the relationship between average establishment size and equity, both in manufacturing and in services. Economies with lower equity ratios, and hence less access to finance, are associated with smaller establishments on average, both in manufacturing and services. For example in the U.S., which has an equity ratio of 18 percent, the average size of establishments in manufacturing is 22 and in services is 5. In India, with an equity ratio of 6 percent, average sizes in manufacturing and services are 3.1 and 1.7. Although Figure 2 suggests equity ratios are associated with larger differences in average size in services, relative to manufacturing, this is largely the result of including different samples of countries.[4] We acknowledge that similar relationships between financial development and average size

[4]When we regress the ratio of average size in manufacturing to average size in services (for countries with size data for both sectors) on equity ratios, we find a very small and statistically insignificant relationship. This suggests a similar relationship between equity ratios and average establishment size across sectors.

across sectors is in conflict with the evidence in Buera et al. (2011), who compare average establishment size across sectors in the U.S. and Mexico. They report that sectors with lower average size in the U.S have even lower average size in Mexico, while sectors with high average size in the U.S. have even higher average size in Mexico. In our standardized data (which, importantly, includes establishments with no employees), average size in Mexico is lower than in the U.S., both in manufacturing (9 vs. 22) and in services (2.5 vs. 5). More importantly, this pattern holds across countries with varying levels of financial development.[5]

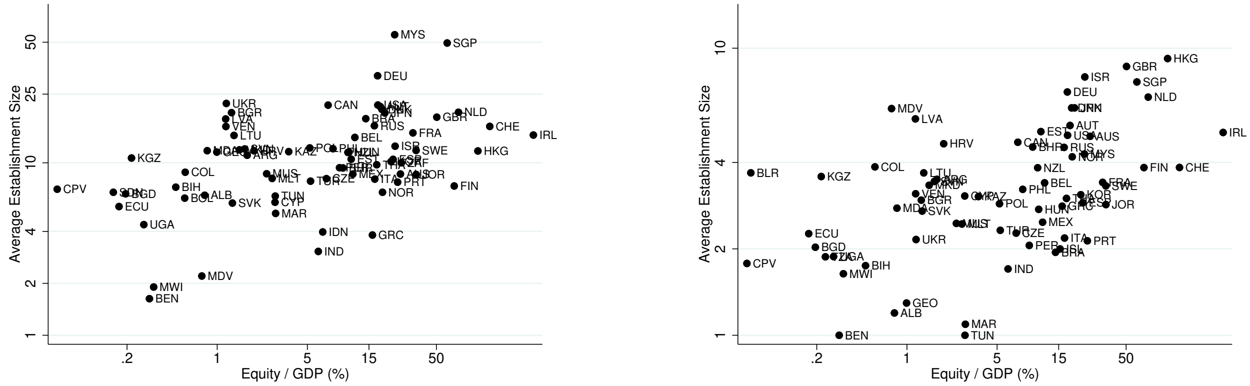
Figure 3a shows how the share of employment in large firms is related to equity ratios. The number of countries with both employment share and equity ratio data is limited, but there is a clear negative relationship. Economies with high equity ratios have a larger share of employment in large firms.

Figure 3b illustrates how the number of employers (firms with employees) per person engaged in the service sector is related to equity ratios. We do not have separate data on employers and nonemployers (firms without formal employees), so our measure of employers here is constructed by subtracting the number of self-employed entrepreneurs without employees (taken from the International Labour Organization's Employment Statistics) from the total number of establishments in services (Bento and Restuccia, 2021).[6] The large impact on TFP from financial constraints implied by the model in Midrigan and Xu (2014) depends on a mechanism that predicts a larger number of modern firms in economies with fewer constraints. To the extent that 'modern' firms can be proxied by employers (rather than nonemployers), this prediction is not supported by our data. Figure 3b suggests that the number of employers (normalized by the size of the service sector) is not statistically different across economies with varying levels of financial development. If anything, the number

[5]Two features of the data in Buera et al. (2011) make our comparison less straightforward. First, they consider only employer firms (firms with paid employees). When we do the same, we still find a larger average establishment size in U.S. manufacturing and services. Second, Buera et al. use data from the 1990s, while Figure 2 reflects more recent data. It could be the case that these relationships have changed over time.

[6]We note that the number of self-employed entrepreneurs without employees does not map exactly to the number of nonemployer firms, for two reasons. First, an entrepreneur may have multiple firms. Second, a firm may have multiple owners.

Figure 2: Establishment Size and Equity Ratios



(a) Manufacturing

(b) Services

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.16 (0.04) and (b) 0.13 (0.03), based on 73 and 72 observations.

of employers is slightly lower in more financially-developed countries.[7]

Finally, we again note that equity ratios are highly positively correlated with two widely-used proxies for financial development. Countries with high equity ratios have high levels of external finance and bank deposits, relative to GDP. In Appendix A we show each of the relationships illustrated by Figures 1 through 3 are robust to replacing equity ratios with either of these two proxies for financial development. Across countries, financial development (however measured) is associated with higher aggregate TFP, higher investment in productivity, and larger establishments in both manufacturing and services.

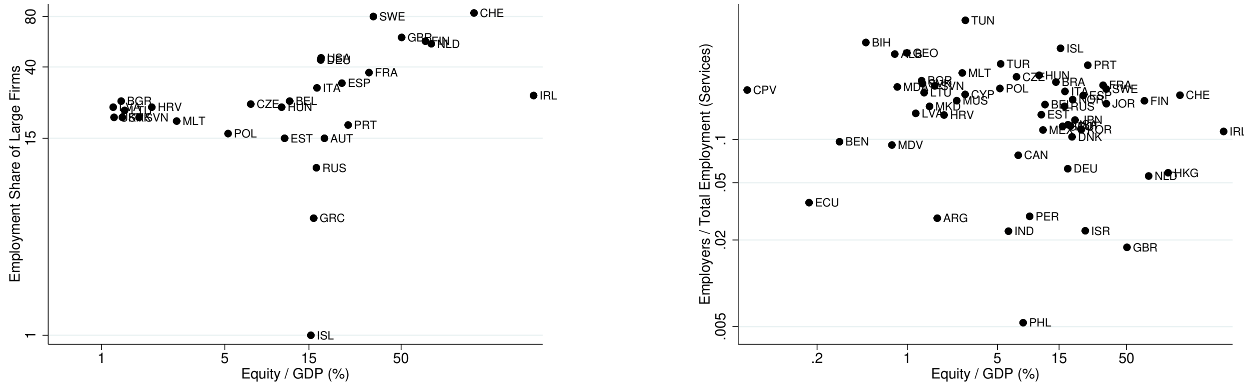
3 Model

We now present a one-sector model that highlights a mechanism through which financial development can affect average firm size and generate the patterns just documented.[8] We consider an infinite horizon setting where workers can choose to become entrepreneurs. Work-

[7] Although not illustrated here, the same conclusions can be drawn from the manufacturing sector across countries.

[8] We extend our model to a two-sector model with manufacturing and service sectors in Appendix B, and show that all of our results hold within and across sectors.

Figure 3: Employment Share of Large Firms, Employer Firms, and Equity Ratios



(a) Employment Share of Large Firms

(b) Employer Firms per Person (Services)

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.16 (0.06) and (b) -0.09 (0.08), based on 29 and 53 observations.

ers are heterogeneous with respect to their entrepreneurial ability, and so their decision of whether to start a firm depends in part on this ability. Upon choosing to become an entrepreneur, they must make a costly decision about their firm's productivity. The cost of this investment is financed by selling shares of future operating profits (equity) and we characterize the extent of financial development by the fraction of future profits that can be sold.

3.1 Environment

We consider a setting with a continuum of consumer-workers of measure L. People can choose to start firms, and are heterogeneous with respect to their initial ability to run a firm A_0. We assume the distribution of ability is described by a Pareto distribution with pdf $f(A_0) = \kappa A_0^{-\kappa-1}$, and that this ability is known before choosing to start a firm. We assume entrepreneurs continue to earn a wage, consistent with Davis et al. (2009).[9] Before

[9]Most firms in the U.S. have no paid employees, and many owners of these firms maintain employment at other firms. One can interpret our assumption as implying that owners of firms with less than one unit of optimal labor fulfill all labor requirements themselves, spending the rest of their time working at other firms, while owners of more productive firms spend all of their time working for their own firms and pay themselves a wage in addition to any profits.

producing, entrants make an initial investment to improve their productivity, after which we assume productivity is fixed over the life-cycle.[10] Given an entrepreneur's ability A_0, the cost of raising productivity to A in sector i is;

$$w \cdot c_A A_0 (A/A_0)^{\phi}, \quad c_A > 0, \;\; \phi > 1.$$

The above investment cost is in terms of the final good, but increasing with the wage.[11] Further, it is proportional to an entrant's initial ability, following Atkeson and Burstein (2010).[12] Entrants must finance this investment by selling shares of future net operating profits (revenue net of variable costs), but can not sell more than a fraction χ of these profits. We only consider equilibria where χ is at least weakly binding.

Once entrants decide on their productivity, they produce a homogenous product each period according to;

$$y = (Az)^{1-\alpha} \ell^{\alpha},$$

where $\alpha \in (0, 1)$, $A^{1-\alpha}$ is firm-specific productivity, $z^{1-\alpha}$ is a productivity term common to all firms, and ℓ is the quantity of labor used by a firm.[13]

Producers must incur a fixed operating cost equal to $w \cdot c_p$, which is again specified in terms of the final good and increasing in the wage. There is an exogenous probability of producer exit represented by $0 < \lambda < 1$, and we assume that exiting entrepreneurs retain their initial entrepreneurial ability and can choose to start a new firm.[14] We denote the total

[10] In Section 3.5 we discuss how our results are affected if we allow firms to continue investing in productivity over their lives.

[11] Making this cost scale up with the wage ensures that (A/A_0) is independent of exogenous productivity z.

[12] This ensures both the marginal benefit and marginal cost of productivity investment is increasing in an entrepreneur's ability. This serves to remove any heterogeneity in the productivity increase across firms.

[13] Models featuring financial frictions typically focus on frictions that limit access to capital which distorts optimal firm size. We focus on financial frictions affecting productivity investment at entry. Including financial frictions on capital that distort firm size over its life-cycle would generate an impact on TFP differences through misallocation and also generate an impact on capital accumulation and aggregate output., as in much of the literature.

[14] We could instead reinterpret 'ability' as the productivity of an entrepreneurial idea, which would then be redrawn after exit. Given the rest of our specified environment, our choice of interpretation is innocuous.

number of producers by N, and so average firm size is therefore;

$$size = \frac{L}{N}. \tag{1}$$

Finally, we assume an exogenous real interest rate r.

3.2 Firm-Level Production

A producer with productivity A chooses labor to maximize operating profits in each period, resulting in the following optimal labor, output, and net operating profits (excluding fixed operating costs);

$$\ell(A) = Az\left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}}. \tag{2}$$

$$y(A) = Az\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}. \tag{3}$$

$$\pi(A) = Az(1-\alpha)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}. \tag{4}$$

3.3 Productivity Investment

We first derive an entrant's optimal choice of productivity A, given entrepreneurial ability A_0, in a frictionless setting when the constraint on financing (χ) is not binding. An unconstrained entrant chooses A to maximize the following;

$$\frac{\pi(A) - w \cdot c_p}{1-\rho} - w \cdot c_A A_0 (A/A_0)^{\phi},$$

$$\rho \equiv \frac{1-\lambda}{1+r},$$

where $\pi(A)$ is defined above. The solution to this problem is;

$$wc_A A_0 (A/A_0)^{\phi} = \frac{\pi(A_0) \cdot (A/A_0)}{\phi(1-\rho)}. \tag{5}$$

If χ is binding, then constrained investment is equal to a share χ of the present discounted value of future net operating profits and characterized in the following way;

$$wc_A A_0 (A/A_0)^{\phi} = \chi \frac{\pi(A_0) \cdot (A/A_0)}{1-\rho}. \tag{6}$$

Note that in either case, investment spending is rising in A_0 but the optimal productivity increase (A/A_0) is independent of A_0. In addition, constrained investment approaches optimal investment in the frictionless case as $\chi \rightarrow 1/\phi$.

This implies the value of an entrant (henceforth assumed constrained), given A_0, is;

$$V(A_0) = \frac{(1-\chi)\pi(A_0) \cdot (A/A_0) - w \cdot c_p}{1-\rho} \tag{7}$$

with (A/A_0) given by equation (6).

3.4 Equilibrium conditions

We now derive the equilibrium conditions of the model. Using equation (2), labor-market clearing implies;

$$L = N\mathbb{E}(A)z \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}},$$

where N denotes the number of firms. $\mathbb{E}(A)$ refers to the average A across producers, equal to;

$$\mathbb{E}(A) = \mathbb{E}(A_0 | A_0 > A_0^*)(A/A_0), \tag{8}$$

where A_0^* is the threshold entrepreneurial ability above which a person chooses to become an entrepreneur. Combining the above expressions with equation (4), operating profits for a producer with initial productivity draw A_0 can now be expressed as;

$$\pi(A_0) \cdot (A/A_0) = w \left(\frac{1-\alpha}{\alpha}\right) \frac{A_0 \cdot (L/N)}{\mathbb{E}(A_0 | A_0 > A_0^*)}. \tag{9}$$

To characterize the threshold A_0^* we first note that operating profits are monotonically increasing in A_0. Given the fixed operating cost, we can therefore find A_0^* by noting that the marginal entrant must be indifferent between entering and not;

$$wc_p = (1-\chi)\pi(A_0^*)(A/A_0),$$

or

$$c_p = (1-\chi)\left(\frac{1-\alpha}{\alpha}\right)\frac{A_0^* \cdot (L/N)}{\mathbb{E}(A_0|A_0 > A_0^*)}. \tag{10}$$

With our assumed Pareto distribution for A_0, we know that N/L is equal to $1 - F(A_0^*) = (A_0^*)^{-\kappa}$ and $\mathbb{E}(A_0|A_0 > A_0^*) = A_0^* \cdot \kappa/(\kappa-1)$. We can therefore characterize A_0^* and average firm size as follows;

$$(A_0^*)^\kappa = \frac{L}{N} = \frac{c_p}{1-\chi}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\kappa}{\kappa-1}\right) \tag{11}$$

As χ increases towards $1/\phi$, average firm size and average firm-level productivity both increase. Intuitively, an increase in χ increases the fraction of revenue invested in productivity which raises firm-level productivity. This raises the wage, as higher productivity raises the demand for labor, but a higher wage increases operating costs and profits proportionally, leaving the incentive to enter for a marginal entrant unchanged. But the fact that a higher fraction of revenue is spent on investment by itself implies a lower value of entry and therefore less entry and a higher threshold A_0^*.

Combining equations (6), (9), and (11), we can characterize (A/A_0) in the following way;

$$(A/A_0)^\phi = \left(\frac{\chi}{1-\chi}\right)\frac{c_p}{c_A(1-\rho)A_0^*} = \frac{\chi}{c_A(1-\rho)}\left(\frac{c_p}{1-\chi}\right)^{\frac{\kappa-1}{\kappa}}\Psi^{1/\kappa}, \tag{12}$$

$$\Psi \equiv \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\kappa-1}{\kappa}\right).$$

Similarly to average firm size, (A/A_0) is increasing in both χ and c_p. Higher χ and c_p both decrease the number of firms, thereby increasing market share for those firms that still enter and increasing the absolute quantity of investment per firm. To see how the relationship between (A/A_0) and χ translates to that between aggregate investment over output and χ, we combine equations (3), (4), and (6), taking into account both that investment over output is constant across entrants, and entrants are a fraction λ of all firms;

$$inv.\ ratio = \frac{\lambda N w c_A A_0 (A/A_0)^{\phi}}{Y} = \chi \frac{\lambda(1-\alpha)}{1-\rho}. \tag{13}$$

Clearly, the aggregate investment ratio increases proportionately with χ.

To see how the share of employment in large firms is affected by χ, note that labor-market clearing combined with equation (2) imply firm-level employment in equilibrium as a function of firm-level productivity is equal to;

$$\ell(A) = \frac{A}{\mathbb{E}(A)} \cdot \frac{L}{N} = \frac{A_0}{\mathbb{E}(A_0|A_0 > A_0^*)} \cdot \frac{L}{N}, \tag{14}$$

where the last equality holds because (A/A_0) is constant across firms. In Section 2 we showed that lower equity ratios are associated with a lower share of employment in firms with at least 250 employees. Here we can use the above expression to derive the threshold ability A_0^{250} at which a firm hires 250 employees, relative to A_0^*;

$$\frac{A_0^{250}}{A_0^*} = \frac{250}{L/N}\left(\frac{\mathbb{E}(A_0|A_0 > A_0^*)}{A_0^*}\right) = \frac{250}{(A_0^*)^{\kappa}}\left(\frac{\kappa}{\kappa-1}\right). \tag{15}$$

From the above equation, an increase in average firm size (higher A_0^*) due to higher χ lowers the ability (relative to A_0^*) at which a firm chooses to hire 250 employees. This implies the fraction of firms described as 'large' will increase, as shown by the following expression;

$$\frac{large\ firms}{all\ firms} = 1 - F(A_0^{250}|A_0 > A_0^*) = \left(\frac{A_0^*}{A_0^{250}}\right)^{\kappa}.$$

While higher c_p and higher χ have similar qualitative impacts on average firm size and firm-level productivity, their impacts on aggregate total factor productivity (TFP) differ.

Note that TFP, as measured in the data, is the ratio of total output to a Cobb-Douglas function of inputs. In our context, this is simply TFP $= Y/L$. Summing output across firms using labor-market clearing and equations (3) and (10–12), TFP can be characterized in the following way;

$$\left(\frac{Y}{L}\right)^{\frac{1}{1-\alpha}} = TFP^{\frac{1}{1-\alpha}} = \frac{z(A/A_0)}{(A_0^*)^{\kappa-1}}\left(\frac{\kappa}{\kappa-1}\right) = z\left(\frac{\chi}{c_A(1-\rho)}\right)^{\frac{1}{\phi}}\left(\frac{1-\chi}{c_p}\right)^{\frac{(\kappa-1)(\phi-1)}{\phi\kappa}} \cdot \Omega, \quad (16)$$

$$\Omega \equiv \left(\frac{\kappa}{\kappa-1}\right)^{\frac{\phi-1}{\phi\kappa}}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{\phi(\kappa-1)+1}{\phi\kappa}}.$$

Above we can see that a higher c_p lowers TFP. While a higher c_p results in higher average firm-level productivity (both through selection and by encouraging investment), the negative impact from fewer (larger) firms due to decreasing returns to scale in production more than offsets this. A higher χ also results in both fewer (and larger) firms and higher firm-level productivity through the same mechansim. But because a higher χ also directly increases investment in productivity, a higher χ on net raises TFP. To see this from equation (16), note that TFP is increasing in χ (when χ is binding) if the following holds;

$$\frac{\partial}{\partial\chi}\chi(1-\chi)^{\frac{(\kappa-1)(\phi-1)}{\kappa}} > 0, \quad \forall\chi \in (0, 1/\phi],$$

or

$$\chi(\phi-1) + \kappa(1-\chi\phi) > 0.$$

The above condition holds for all $\phi > 1$, $\chi \in (0, 1/\phi]$.

3.5 Remarks

In our model we have made simplifying assumptions in order to present a simple and tractable framework that clearly highlights the impact of limited access to finance on investment, average firm size and TFP. We now focus on two of these assumptions that are perhaps nonstandard in the literature, specifically that entrepreneurs cannot self-finance productivity investment before entry and that a firm cannot offset the impact of equity constraints by self-financing productivity improvements after entry over the life of a firm.

Our assumption that entrants cannot self-finance investment at entry is driven by a large micro-development literature emphasizing subsistence consumption constraints and nonhomethic preferences, especially prevalent among poor households and exacerbated in countries where access to finance is lacking. In particular, low wages together with subsistence requirements can dissuade potential entrepreneurs from accumulating savings before entry in order to increase entry investment. This point is made in Banerjee et al. (2019) and Balboni et al. (2021), who focus on India and Bangladesh and find that investment costs at entry are a severe barrier for poor households to start a business and escape a poverty trap. In light of this evidence, we think it is quite plausible to assume that entrepreneurs cannot invest at entry by self-financing, especially in places where access to finance is limited.

Regarding investment in productivity over a firm's life-cycle, several papers in the literature have shown that entrepreneurs/firms can self-finance investment by saving out of profit over the life-cycle which can dampen the impacts of limited access to finance on TFP, especially when productivity is highly persistent as is in our setting (Moll, 2014; Midrigan and Xu, 2014). We address this in the Appendix C where we consider a simplified extension of our setting where firms can raise productivity by investing out of operating profits over the life-cycle of a firm. We show that whether equity constraints result in more or less investment in life-cycle productivity growth depends on how the cost of investment is specified. More importantly, we show that allowing for life-cycle investment (regardless of how the cost function is specified) only marginally changes the quantitative impacts of equity constraints on average firm size and aggregate TFP. This result is driven by the very low elasticity of firm-level productivity with respect to life-cycle investment suggested by the empirical literature, as in for example Bontempi and Mairesse (2015).

4 Quantitative Analysis

In section 3 we show that allowing for endogenous entry and investment at entry generates a qualitative relationship between financial development and average firm size that is consistent with the data. We now use a simple exercise to quantify the importance of these mechanisms in a model with financial frictions, both in terms of accounting for the observed quantitative relationship between average size and financial development, and in terms of generating a substantial impact on TFP. We begin by discussing the calibration of the model and how key parameters are identified from the data. We then quantify the impact of equity constraints on our variables of interest, and evaluate how well our calibrated model can account for observed differences in outcomes across countries characterized by large differences in financial development.

4.1 Calibration

There are only 4 parameter values we need for our exercises: the input elasticity of firm output α; the convexity parameter in the investment cost function ϕ; the Pareto scale parameter κ for the distribution of entrepreneurial ability; and a value for χ in our benchmark economy. As our goal here is to quantify the factor change in outcomes due to a change in χ, we can ignore all other parameters since they do not interact with χ. We follow the literature in setting $\alpha = 0.8$, and assume $\chi = 1/\phi$ in our benchmark economy (so it is essentially unconstrained). The 2 remaining parameters are chosen to match relevant moments in the U.S. economy as we describe next.

In our framework, ϕ is the elasticity of investment in productivity at entry with respect to output. We set $\phi = 1.39$ targeting the same moment in Bento and Restuccia (2017), who model entry investment in a similar way. The Pareto parameter κ determines the shape of the firm-size distribution. Equation (14) shows that relative firm-level employment is proportional to ability. We therefore choose a value for κ in the following way. We note that the employment share of manufacturing firms with at least 500 employees in the U.S. is 11 percent lower than the employment share of firms with at least 250 employees. Since the ability A_0 corresponding to 500 employees is double that corresponding to 250 employees, we choose a value for κ such that the employment share of firms with at least 500 employees relative to that of firms with at least 250 employees is equal to 0.89:

$$\left(\frac{A_0^{500}}{A_0^{250}}\right)^{1-\kappa} = 2^{1-\kappa} = 0.89.$$

We obtain $\kappa = 1.17$. Note that the above strategy for choosing κ targets the shape of the firm-size distribution without requiring any assumption about the average size of firms in our benchmark economy, since equation (11) implies c_p could be adjusted to obtain any average size for any given value κ .

4.2 Results

We now evaluate the quantitative importance of differences in financial development stemming from a lower χ in less-developed financial markets. To this end, we adjust χ, holding all other parameters fixed, and evaluate its impact on investment, the firm-size distirbution, and TFP. We emphasize that we do not think countries with varying levels of financial development differ only due to differences in χ. Rather, the exercise serves to isolate the impact of this mechanism on outcomes of interest and thus allows us to evaluate the quantitative relevance of our primary mechanism in the cross-country data. Note that our approach here is similar in spirit to that taken in much of the quantitative literature.[15]

Table 1 shows how average firm size, the employment share of large firms, investment relative to GDP, and aggregate TFP change as we decrease χ from 0.72 ($= 1/\phi$) in our benchmark economy to 0.01. Noting that differences in χ correspond to proportional differ-

[15]For instance, workhorse models of financial frictions adjust a parameter that affects collateral requirements that are tied to financial market development (e.g. Buera et al., 2011; Midrigan and Xu, 2014; Moll, 2014).

ences in equity ratios in the model, this range captures most of the cross-country variation in equity ratios illustrated in Section 2. All outcomes are calculated using equations (8), (11), (13), (15), and (16), and are reported relative to the benchmark economy.

Table 1: Model Results Across Values of χ: $\alpha = 0.8$

χ	Average Firm Size	Emp. Share Large Firms	Average Firm-Level Productivity	Investment Ratio	TFP
0.72 (bench.)	1.00	1.00	1.00	1.00	1.00
0.60	0.70	0.66	0.62	0.83	0.98
0.50	0.56	0.51	0.44	0.69	0.95
0.20	0.35	0.29	0.15	0.28	0.84
0.10	0.31	0.26	0.08	0.14	0.76
0.01	0.28	0.23	0.01	0.01	0.55

The are several points highlighted in the table. First, differences in equity ratios can account for more than a three-fold difference in average firm size across economies, with large decreases in average firm size for even small decreases in χ, relative to the benchmark. For instance, average firm size is 30 percent lower in the $\chi = 0.6$ economy, and 65 percent lower in the $\chi = 0.2$ economy. In this regard, while much of the quantitative work on financial frictions abstract from average firm size or predicts that higher financial market development should reduce average size, our model captures the relationship in the data that average firm size rises with financial market development. The model also predicts large decreases in the employment share of large firms, where for an economy with $\chi = 0.2$ the share of large firms is 71 percent lower than in the benchmark economy.

Lower equity ratios also lower the share of output going to investment, lowering both average firm-level and aggregate productivity (TFP). For instance, in the $\chi = 0.2$ economy, investment at entry is 72 percent lower, and as a consequence, average firm-level productivity falls by 85 percent. The impact on TFP is smaller and partially offset by the drop in average firm size, since the production technology features decreasing returns to labor. Nevertheless, TFP falls by 16 percent when $\chi = 0.2$, and by 45 percent in the $\chi = 0.01$ economy. Our framework can therefore rationalize the large differences in investment across countries, as well as a significant portion of the large differences in TFP. Importantly, the impact of equity ratios on TFP is large relative to what is found in much of the existing literature on financial frictions.

In the above exercise we assume $\alpha = 0.8$, a relatively high value that implies changes in firm-level productivity translate to relatively small changes in TFP. If we instead assume $\alpha = 2/3$, as in Hsieh and Klenow (2009, 2014) for example, the model generates a much larger impact on TFP. For example when χ decreases from the benchmark level to 0.1, TFP drops by 37 percent (compared to 24 percent in Table 1).

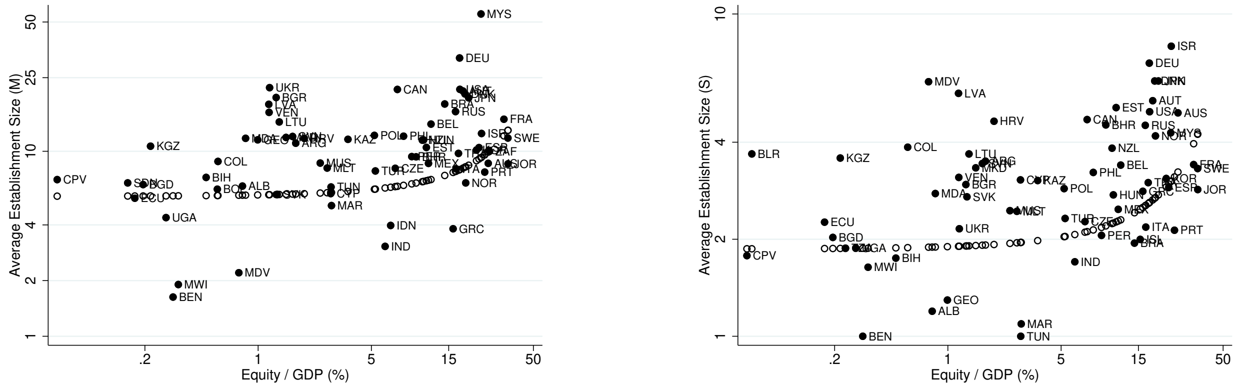
4.3 Cross-Country Exercise

The results in Section 4.2 show the quantitative implications of financial frictions on firm size, investment spending, and TFP due to differences in equity ratios. We now evaluate more closely the country-specific predictions of the model against the data. To this end, we map equity ratios in the data to χ in the model as follows. We treat countries that have equity ratios and average firm size above the U.S. as our benchmark ($\chi = 1/\phi$), and use the average values of average firm size and employment share of large firms in these countries as our benchmark values.[16] We assign each country a value of χ equal to its equity ratio relative to the benchmark ratio (equal to 255 percent), multiplied by $1/\phi$. For example the U.S. has an equity ratio of 17.6 percent, which translates to $\chi = 0.28$.

We can now look at how differences in χ across countries, keeping all other parameters fixed, can account for variation in firm size, investment, and TFP observed in the data (Figures 4 through 6). Specifically, we compare the model counterparts to the figures in Section 2. Figure 4 plots the cross-country relationships between average firm size and equity, both model-generated and from the data, for manufacturing and services. There

[16] An alternative is to treat the U.S. as our benchmark, but this reduces the country sample by over 10 percent since several countries have equity ratios that exceed the U.S. value. Because relative differences in χ would otherwise remain unchanged, this would simply shift up the model-generated outcomes in Figures 4 and 5a.

Figure 4: Establishment Size



(a) Manufacturing

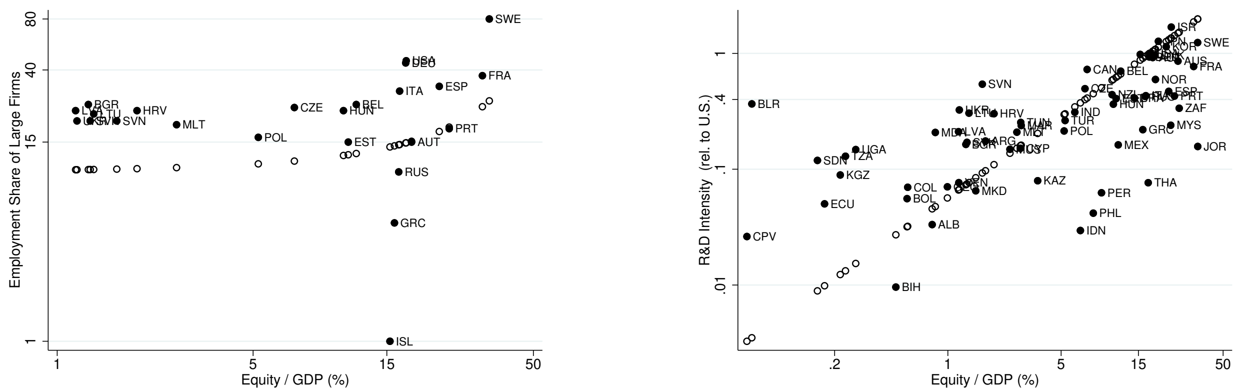
(b) Services

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. Solid circles represent data, hollow circles represent model-generated outcomes. The coefficients (standard errors) from OLS regressions are (a) Data: 0.15 (0.04); Model: 0.10 (0.01) and (b) Data: 0.11 (0.03); Model: 0.10 (0.01).

is a strong correlation between the model prediction for average size and the data in both sectors. Comparing the variation in logged outcomes from the model to that in the data, Figure 4 suggests variation in equity ratios can account for 12 percent of the cross-country variation in average size in manufacturing and 20 percent in services. More strikingly, OLS-estimated elasticities of average size with respect to equity using the data are very close to those estimated using model-generated outcomes (when elasticities are estimated using observations from the same economies).

Figure 5 compares model-generated outcomes with the data for the employment share of large firms and the productivity investment ratio (R&D in the data). The model captures 10 percent of the observed cross-country variation in the large firm employment share, but does poorly in matching OLS-estimated elasticities due to outliers in the data. That differences in financial development alone can account for a reasonable portion of the variation in firm size is notable, given that we abstract from differences in labor-market institutions which can have a first-order impact on average size and the employment share of large firms. In our model, differences in this employment share are solely due to changes in the number of low-productivity firms (rather than a misallocation of employment across a given set of

Figure 5: Employment Share of Large Firms and R&D Intensity



(a) Employment Share of Large Firms

(b) R&D Intensity

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. Solid circles represent data, hollow circles represent model-generated outcomes. The coefficients (standard errors) from OLS regressions are (a) Data: 0.01 (0.12); Model: 0.19 (0.03) and (b) Data: 0.40 (0.07); Model: 1 (proportional to equity in model).

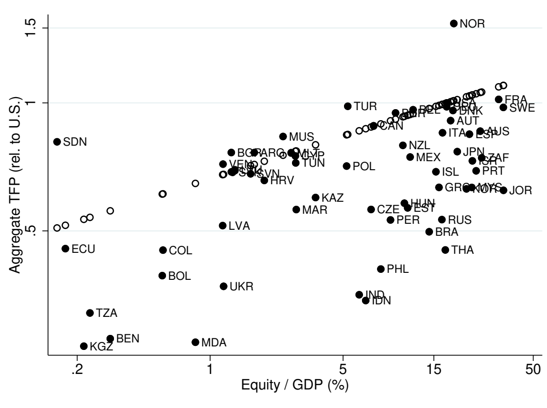
firms). Turning to Figure 5b, we note that the productivity investment ratio generated by the model is proportional to the equity ratio (equation 13). Variation in this ratio is more than double the variation in R&D intensity from the data, and the OLS-estimated elasticity is also much lower in the data. Nevertheless, the model-generated investment ratio is highly correlated with R&D in the data, with a correlation coefficient of 0.62.

Figure 6 illustrates how differences in equity ratios map to aggregate TFP in the model, compared to the data. Here there is again a strong correlation between the model and the data, with variation in equity ratios alone accounting for 34 percent of the cross-country variation in TFP. As with average firm size, the implied elasticity of TFP with respect to equity in the data is very close to that generated by the model.

5 Conclusion

There is now a large literature that studies the importance of financial frictions for understanding cross-country income and productivity differences. We contribute to this literature by documenting that financial development is associated with high equity ratios, high invest-

Figure 6: Aggregate TFP



Notes: All variables are shown in log scale. See the text for the definition of variables and sources. Solid circles represent data, hollow circles represent model-generated outcomes. The coefficients (standard errors) from OLS regressions are Data: 0.13 (0.03); Model: 0.14 (0.00).

ment in R&D, and large average firm size in all sectors. To account for these facts, we present a tractable model with heterogeneous entrepreneurs who invest in the productivity of their firms at entry, where investment is constrained by the share of future profits that can be sold as equity. The model is tractable and makes analytical predictions for the impact of financial development on investment, firm size, and productivity that is consistent with the evidence we document. Calibrating the model, we find the quantitative impact of equity constraints is large relative to the existing literature. Specifically, differences in equity ratios alone can account for a significant fraction of the cross-country variation in productivity investment, the employment share of large firms, and average firm size, and can account for almost a two-fold difference in TFP.

Much of the work that examines the macro implications of financial frictions focus on entrepreneurial differences in access to capital due to collateral constraints, which affect both selection into entrepreneurship and the ultimate scale of firms. While these models imply an inefficient allocation of resources across firms, their quantitative impacts depend on the persistence of firm-level productivity, which affects an entrepreneur's incentive to self-finance. Our model complements the quantitative findings of this literature by highlighting the importance that equity constraints can have on start-up investment and entry. We show that the impact of financial frictions on investment at entry can be substantial even when incumbent entrepreneurs can self-finance investment over time, and serves to amplify the quantitative impacts found in the literature.

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Appendix

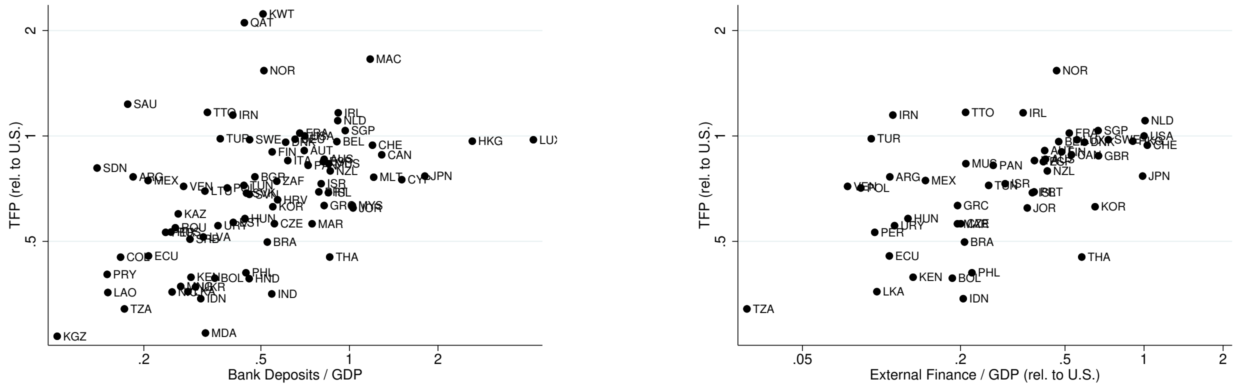
A Other Measures of Financial Development

Here we show the cross-country relationships between equity ratios and other outcomes of interest are robust to replacing equity ratios with two commonly-used proxies for financial development – external finance and bank deposits, both measured relative to GDP. As in Section 2, the elasticity of average firm size with respect to each proxy seems to differ across sectors in Figures 9 and 10. But the ratio of average size in manufacturing relative to that in services has no statistically significant relationship to either proxy (as is the case with respect to equity ratios), suggesting that the differences between manufacturing and services in Figures 9 and 10 are simply due to different samples of countries. While the employment share of large firms is higher in economies with more external finance, Figure 11a shows no systematic relationship between the large firm employment share and bank deposits. As is the case in Section 2, there is no statistically significant relationship between financial development and the number of employer firms per person in the service sector in Figure 12. We do note that in manufacturing (not shown here), the number of employers has a significantly negative relationship with both bank deposits and external finance.

B Allowing for Multiple Sectors

Here we extend the model developed in Section 3 to allow for two sectors - manufacturing (M) and services (S). We assume these sectors differ with respect to operating costs $c_{p,i}$, investment costs $c_{A,i}$, and exogenous productivity (common across firms within a sector) z_i, $i \in \{M, S\}$. We make the simplifying assumption that some fraction of the population γ can potentially operate a manufacturing firm, while the rest of population can potentially operate a service-sector firm. Each person knows their entrepreneurial ability, which is drawn from

Figure 7: Aggregate TFP, Bank Deposits, and External Finance



(a) Bank Deposits / GDP

(b) External Finance / GDP

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.36 (0.06) and (b) 0.28 (0.04), based on 86 and 54 observations.

a Pareto distribution as in Section 3.[17] Whether a potential entrepreneur decides to start a firm again depends in part on their ability.

Aggregate output in the economy is produced by a representative final-good firm that uses output produced in the manufacturing and service sector;

$$Y = Y_M^{\eta} Y_S^{1-\eta}, \quad \eta \in (0, 1).$$

Profit maximization for the final-good firm implies;

$$P_M Y_M = \eta Y, \quad P_S Y_S = (1 - \eta) Y,$$

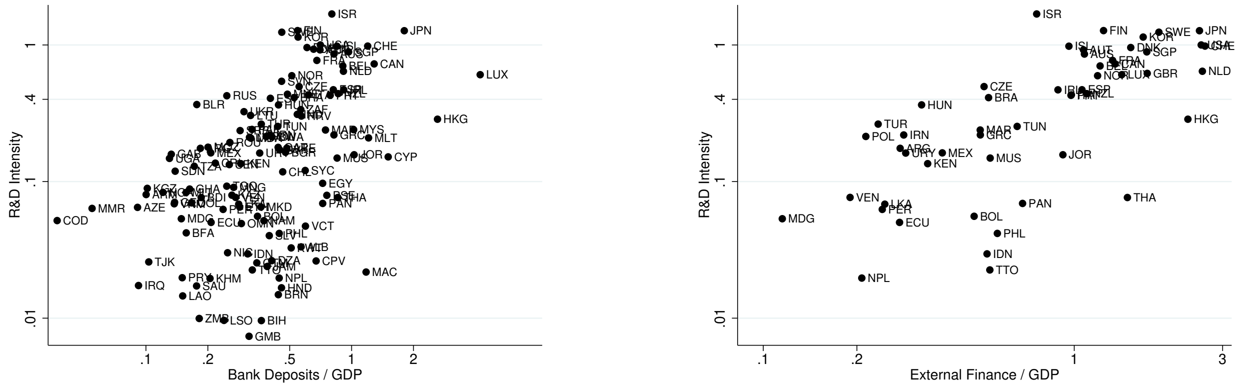
and we continue to assume the aggregate good serves as the numéraire.

Producers in each sector maximize operating profits in each period, resulting in the following analogues to equations (2) through (4);

$$\ell_i(A) = A z_i P_i^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{1}{1-\alpha}}. \tag{17}$$

[17]This is a simplified version of how Buera et al. (2011) model sector-specific abilities.

Figure 8: R&D Intensity, Bank Deposits, and External Finance



(a) Bank Deposits / GDP

(b) External Finance / GDP

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.86 (0.14) and (b) 0.94 (0.13), based on 100 and 53 observations.

$$y_i(A) = Az_i P_i^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}. \tag{18}$$

$$\pi_i(A) = Az_i(1-\alpha) P_i^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}. \tag{19}$$

Labor-market clearing within each sector implies;

$$L_i = N_i \mathbb{E}(A_i) z_i P_i^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}},$$

where L_i and N_i denote the supply of labor and the number of producers in sector i. $\mathbb{E}(A_i)$ refers to the average A across producers in sector i, equal to;

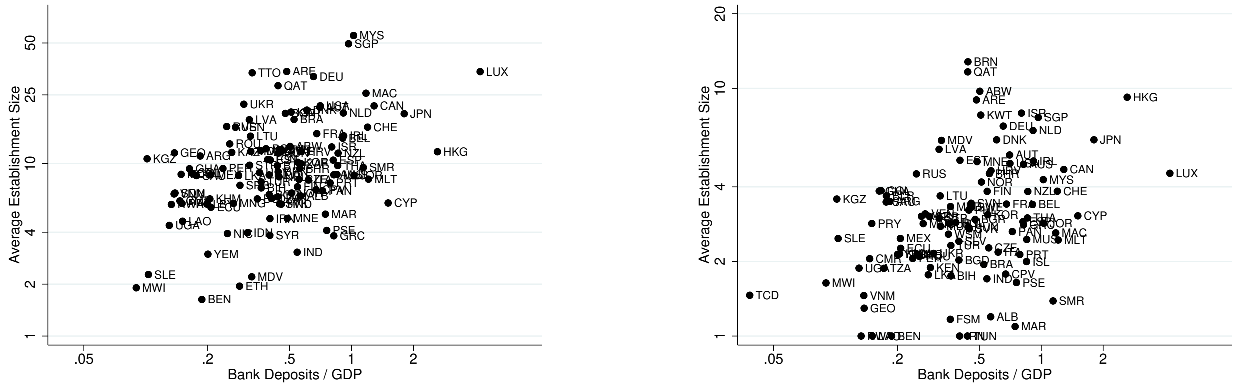
$$\mathbb{E}(A_i) = \mathbb{E}(A_0 | A_0 > A^*_{0,i})(A/A_0)_i = A^*_{0,i}\left(\frac{\kappa}{\kappa-1}\right)(A/A_0)_i,$$

where A_0^* and (A/A_0) are now sector-specific.

Given the fixed operating cost in each sector, we again find $A^*_{0,i}$ by using the condition that the marginal entrant in each sector must be indifferent between entering and not;

$$c_{p,i} = (1-\chi)\left(\frac{1-\alpha}{\alpha}\right)\frac{A^*_{0,i}\cdot(L/N)_i}{\mathbb{E}(A_0|A_0 > A^*_{0,i})}. \tag{20}$$

Figure 9: Establishment Size and Bank Deposits / GDP



(a) Manufacturing

(b) Services

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.41 (0.08) and (b) 0.28 (0.06), based on 112 and 105 observations.

Average size in each sector L_i/N_i can now be characterized as follows;

$$\frac{L_i}{N_i} = \frac{c_{p,i}}{1-\chi}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\kappa}{\kappa-1}\right), \tag{21}$$

which we note is identical (but for sector-specific $c_{p,i}$) to average size in Section 3. Our expression for sector-specific $A^*_{0,i}$ is calculated in a similar way to that in Section 3, but must take into account the fraction of the population that can potentially operate firms in each sector;

$$(A^*_{0,M})^{\kappa}\left(\frac{\eta}{\gamma}\right) = \frac{L_M}{N_M} = \frac{c_{p,M}}{1-\chi}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\kappa}{\kappa-1}\right), \tag{22}$$

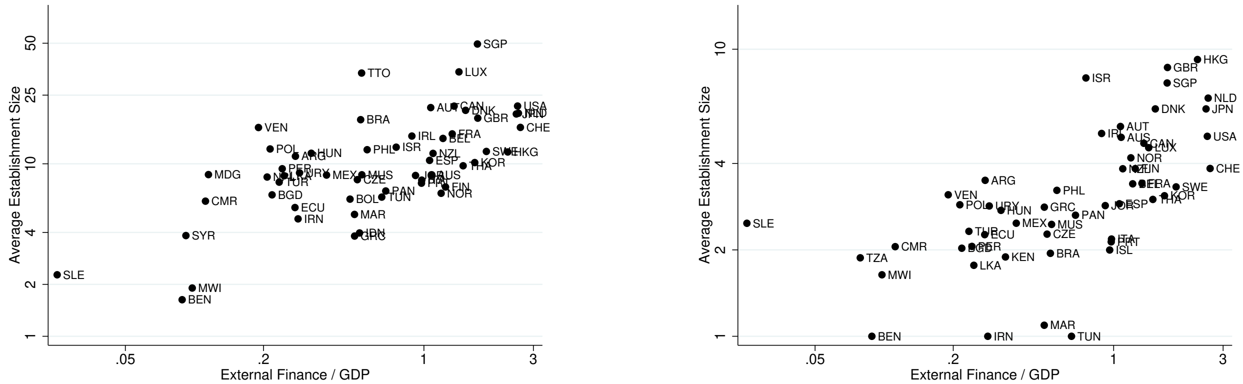
$$(A^*_{0,S})^{\kappa}\left(\frac{1-\eta}{1-\gamma}\right) = \frac{L_S}{N_S} = \frac{c_{p,S}}{1-\chi}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\kappa}{\kappa-1}\right). \tag{23}$$

Optimal $(A/A_0)_i$ is the same as in Section 3, but adjusted for sector-specific variables;

$$(A/A_0)^{\phi}_i = \left(\frac{\chi}{1-\chi}\right)\frac{c_{p,i}}{c_{A,i}(1-\rho)A^*_{0,i}}. \tag{24}$$

It should be clear from equations (20) through (24) that the impacts of χ on average size, A^*_0, and (A/A_0) are identical across sectors and identical to the impacts in Section 3. As a

Figure 10: Establishment Size and External Finance / GDP



(a) Manufacturing

(b) Services

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.42 (0.06) and (b) 0.33 (0.06), based on 56 and 53 observations.

result, it is clear that the impact of χ on aggregate TFP is also identical.

C Allowing for Life-Cycle Growth

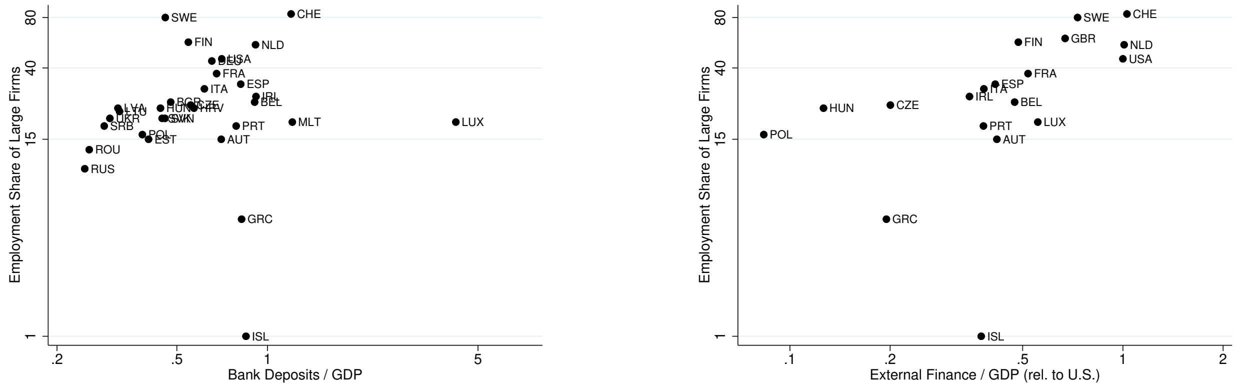
Here we extend the model developed in Section 3 to allow for endogenous productivity growth over the life-cycle of a firm. We now change notation slightly to let $\hat{A}_0$ denote the ability of an entrepreneur, A_0 to denote productivity after investment at entry, and A to denote productivity for producers after entry. After entry-investment, and in each period thereafter, each firm chooses how much to increase its productivity for the subsequent period and finances the required investment out of current profit. We use x to denote the factor increase in productivity chosen by a firm, which incurs the following investment cost (in terms of goods);

$$wc_X \Omega x^{\theta}, \tag{25}$$

where $c_X > 0$, $\theta > 1$, and $x \geq 1$. We consider two potential functions for Ω:

$$\text{Case 1: } \Omega \equiv A/\mathbb{E}(A), \qquad \text{Case 2: } \Omega \equiv A,$$

Figure 11: Employment Share of Large Firms, Bank Deposits, and External Finance



(a) Bank Deposits / GDP

(b) External Finance / GDP

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.10 (0.20) and (b) 0.72 (0.24), based on 31 and 19 observations.

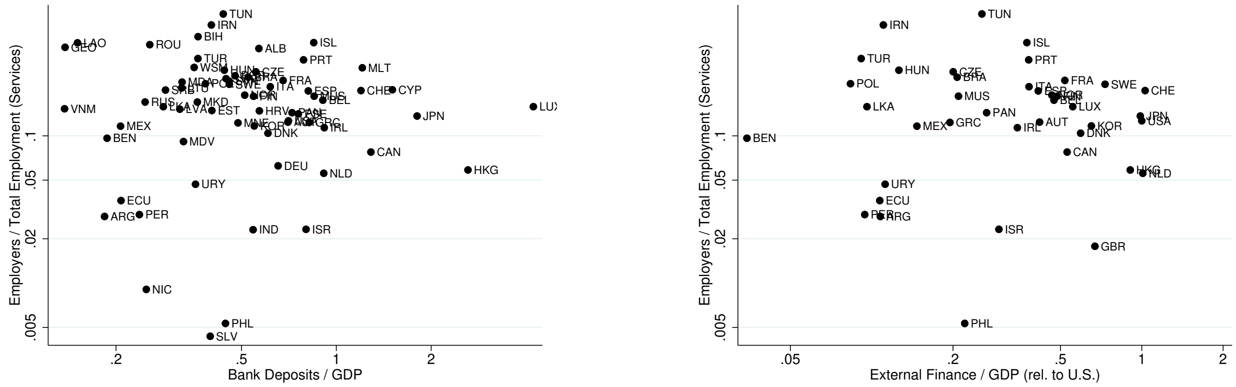
where A denotes a firm's current productivity, $x \cdot A$ is a firm's next-period productivity, and $\mathbb{E}(A)$ denotes the average productivity across all firms. In Case 1 the investment cost is increasing in a firm's productivity relative to all other firms, while in Case 2 the cost is not directly related to the productivity of other firms. Note our specification of the cost of improving productivity (in either case) is a deterministic version of that used in Atkeson and Burstein (2010). Assuming a firm can somehow finance this investment, it chooses x to solve the following problem each period after production (but before knowing whether it will survive into the following period);

$$\max_{x} \quad -wc_X\Omega x^{\theta} - x \cdot wc_X\Omega(x')^{\theta}\left(\frac{1-\lambda}{1+r-x'(1-\lambda)}\right)$$

$$+x \cdot \pi(A)\left(\frac{1-\lambda}{1+r-x'(1-\lambda)}\right),$$

where x' denotes future choices of x and the large bracketed terms reflect the fact that both future expected discounted productivity investments and operating profits scale up with a firm's current choice of x. The solution to this problem can be characterized in the following way, taking into account that the firm will choose the same x each period in a stationary

Figure 12: Employer Firms per Person (Services), Bank Deposits, and External Finance



(a) Bank Deposits / GDP

(b) External Finance / GDP

Notes: All variables are shown in log scale. See the text for the definition of variables and sources. The coefficients (standard errors) from OLS regressions are (a) 0.05 (0.16) and (b) 0.03 (0.16), based on 66 and 40 observations.

equilibrium;

$$wc_X\Omega x^{\theta} = \pi(A)\frac{x(1-\lambda)}{\theta(1+r)-(\theta-1)x(1-\lambda)}. \tag{26}$$

For an optimal x to exist, we require parameter values to be such that firms choose $x < \left(\frac{\theta}{\theta-1}\right)\left(\frac{1+r}{1-\lambda}\right)$. We note two implications of equation (26). First, optimal productivity growth x is independent of current productivity. Second, if optimal x exists, then optimal investment is always less than $\pi(A)$.

Before we go on, we now make an important simplifying assumption – that firms must invest optimally in x in every period. Without this assumption, low-productivity entrepreneurs might enter and invest less until they grow large enough to invest optimally. As a result, firm-level productivity growth would be a function of A, for all A under some threshold. As our focus here is on the possibility of investment over the life-cycle undoing the impact of χ, we simply assume that all firms must invest optimally.

We can now characterize the value of an incumbent firm with productivity A as;

$$V(A) = \frac{\left(\pi(A) - wc_X\Omega x^{\theta}\right)(1+r)}{1+r-x(1-\lambda)} - \frac{wc_p}{1-\rho} = \frac{\pi(A)\theta(1+r)}{\theta(1+r)-(\theta-1)x(1-\lambda)} - \frac{wc_p}{1-\rho}.$$

Entrants continue to be constrained in financing initial productivity investment at entry by selling a fraction χ of life-time net operating profits (revenue net of all variable costs), so constrained investment can be characterized in the following way (replacing equation 6);

$$wc_A\hat{A}_0(A_0/\hat{A}_0)^{\phi} = \chi\frac{\pi(\hat{A}_0)(A_0/\hat{A}_0)\theta(1+r)}{\theta(1+r)-(\theta-1)x(1-\lambda)}, \tag{27}$$

and the value of an entrant, given $\hat{A}_0$, is (replacing equation 7);

$$V(\hat{A}_0) = (1-\chi)\frac{\pi(\hat{A}_0)(A_0/\hat{A}_0)\theta(1+r)}{\theta(1+r)-(\theta-1)x(1-\lambda)} - \frac{wc_p}{1-\rho}, \tag{28}$$

with x given by (26) and $(A_0/\hat{A}_0)$ given by (27).

Using labor market clearing, operating profits are now characterized by;

$$\pi(A) = w\left(\frac{1-\alpha}{\alpha}\right)\frac{A\cdot(L/N)}{\mathbb{E}(A)},$$

$$\mathbb{E}(A) = \mathbb{E}(\hat{A}_0|\hat{A}_0 > \hat{A}_0^*)(A_0/\hat{A}_0)\left(\frac{\lambda}{1-x(1-\lambda)}\right).$$

To solve for $\hat{A}_0^*$, we use the fact that all firms invest optimally in x and must self-finance this investment. Given that $\pi(A)$ is increasing in A, this implies that for the marginal entrepreneur the first-period proceeds from production net of all costs must be exactly zero;

$$(1-\chi)\frac{\pi(\hat{A}_0^*)(A_0/\hat{A}_0)\theta\left[1+r-x(1-\lambda)\right]}{\theta(1+r)-(\theta-1)x(1-\lambda)} - wc_p = 0.$$

Combining this condition with our expression for operating profits above, we can characterize average size and $\hat{A}_0^*$ in the following way;

$$(A_0^*)^{\kappa} = \frac{L}{N} = \frac{c_p}{1-\chi}\left(\frac{\theta(1+r)-(\theta-1)x(1-\lambda)}{[1+r-x(1-\lambda)][1-x(1-\lambda)]}\right)\psi \tag{29}$$

$$\psi \equiv \frac{\alpha\lambda\kappa}{\theta(1-\alpha)(\kappa-1)}.$$

Optimal initial productivity can be expressed as;

$$(A_0/\hat{A}_0)^{\phi} = \frac{c_p}{c_A \hat{A}_0^*} \left(\frac{\chi}{1-\chi}\right) \left(\frac{1+r}{1+r-x(1-\lambda)}\right). \tag{30}$$

Life-cycle productivity growth x depends on the investment cost function as follows;

$$\text{Case 1:} \quad x^{\theta-1} = \frac{c_p(1-\lambda)\lambda\kappa}{(1-\chi)c_X\theta(\kappa-1)[1+r-x(1-\lambda)][1-x(1-\lambda)]} \tag{31}$$

$$\text{Case 2:} \quad x^{\theta-1} = \frac{c_p(1-\lambda)}{(1-\chi)\hat{A}_0^*(A_0/\hat{A}_0)c_X\theta[1+r-x(1-\lambda)]} \tag{32}$$

Equations (29) and (30) make clear that the impacts of χ on average size and initial productivity interacts with the impact on life-cycle productivity growth x. The impact of χ on x, in turn, depends both on the investment cost function and on θ, the elasticity of investment with respect to x. We use a value for θ to match the elasticity of output with respect to intangible investment (equal to θ^{-1} in the model) estimated by Bontempi and Mairesse (2015). Building on a large literature estimating R&D elasticities, Bontempi and Mairesse estimate a corresponding elasticity for a wider range of intangible capital investments equal to 0.009.[18] This translates to $\theta = 111$. For this value of θ an increase in χ results in higher life-cycle productivity growth under Case 1, and lower growth under Case 2. Why the difference? Under Case 1, the cost of life-cycle productivity investment is increasing in firm-level productivity *relative to* average productivity across firms. As such, an increase in overall productivity brought about by a higher χ does not impact the cost life-cycle productivity growth. Under Case 2, a higher (A/A_0) increases the cost of productivity growth enough that firms choose a lower growth rate of productivity when χ is higher. While the relationship between χ and x under Case 1 is more consistent with our intuition (that financial frictions tend to lower firm growth rates), we discuss outcomes under both cases.

In Table 2 we show how average size, x, and TFP depend on χ under both cases, as well

[18] The value 0.009 is taken from Bontempi and Mairesse's Table 7d, where total intangible investment is assumed to be a CES function of investments in separate types of intangible capital.

Table 2: Extended Model Results Across Values of χ

χ	Average Firm Size			Life-Cycle Prod. Growth (%)		TFP		
	(B)	(1)	(2)	(1)	(2)	(B)	(1)	(2)
0.01	1.42	1.12	2.72	3.48	7.85	0.55	0.53	0.58
0.10	1.56	1.25	1.99	3.59	6.29	0.76	0.74	0.78
0.20	1.75	1.44	2.04	3.73	5.83	0.84	0.82	0.85
0.50	2.81	2.50	2.91	4.29	5.22	0.95	0.94	0.96
0.60	3.51	3.25	3.57	4.56	5.11	0.98	0.97	0.98
0.72 (U.S.)	5.00	5.00	5.00	5.00	5.00	1.00	1.00	1.00

as under the benchmark model (B) for comparison. Under Case 1, where x is increasing in χ, both average size and TFP increase faster with χ than in the benchmark model. Under Case 2, the lower growth rates associated with higher χ tend to dampen the impact of χ on average size. This is the case because the productivity of entrants is closer to that of incumbents, all else equal, increasing their market share and value of entry (Atkeson and Burstein, 2010). At very low levels of χ, this effect dominates and a higher χ actually *decreases* average size.

The impact of χ on aggregate TFP differs when we account for life-cycle productivity investment, but Table 2 shows it does not differ by much. When χ is decreased from the benchmark U.S. value to a very low 0.01, TFP still drops by 42-47%, very close to the 45% calculated in our model without life-cycle investment in Section 3.

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