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Policies to Motivate the
Efficient Implementation of
Distributed Energy Resources**

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Employing Simple Cost-Sharing Policies to Motivate the Efficient Implementation of Distributed Energy Resources

by

David P. Brown* and David E. M. Sappington**

Abstract

We consider the optimal design of simple cost-sharing policies to motivate electricity distribution utilities to manage the costs of distributed energy resource (DER) projects. The optimal share of realized cost savings (s) that is awarded to the utility takes a particularly simple form in certain settings. More generally, s can vary with the prevailing environment in subtle and sometimes counterintuitive ways. For instance, s may increase as cost savings become less onerous for the utility to secure and as the utility becomes more averse to risk. Gains from affording the utility a choice among cost-sharing policies typically are minimal.

Keywords: distributed energy resources, procurement, regulation

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1 Introduction

Distributed energy resources (DERs) are becoming increasingly popular as alternatives to traditional investment in electricity distribution networks. Common forms of DERs include demand-side management programs to reduce electricity consumption, solar generation of electricity at dispersed locations, and temporary storage of electricity at remote locations to ensure its availability during periods of peak demand.¹ DERs typically are not turnkey operations, and a utility’s diligence in pursuing the least-cost implementation of a DER project can substantially influence its ultimate cost.² Consequently, the incentives that utilities face to control the costs of DER projects can determine the extent to which DERs serve as cost-effective alternatives to traditional network investment.

It is well known that simple cost-sharing policies can motivate utilities to limit the cost of DER projects while securing a portion of the cost savings for customers.³ Under a simple cost-sharing policy, the utility receives as increased profit a constant fraction of the difference between a benchmark level of cost and the realized cost of a DER project. The benchmark cost can be the projected cost of the DER project or the cost of the traditional investment in network infrastructure that the DER project can replace, for example. To illustrate, Central Hudson Gas and Electric Company operates under a cost-sharing program in New York State that awards the utility 30% of the difference between the estimated cost of traditional network investment and the realized cost of specified alternative DER projects (Rivera Linares, 2016).⁴

¹Sullivan (2014), St. John (2016), National Grid (2017), Northeast Energy Efficiency Partnerships (2017), and RevConnect (2017) describe specific DER projects that have been implemented in practice. The California ISO recently cancelled 18 network investment projects, thereby reducing costs by an estimated \$2.6 billion. CAISO (2018) reports that the cost savings were “strongly influenced by energy efficiency programs and increasing levels of residential, rooftop solar generation.”

²Electric Power Research Institute (2013) reviews some of the many subtleties involved in managing the costs and enhancing the performance of DERs.

³See Lowry and Woolf (2016) and Perez-Arriaga and Knittel (2016), for example.

⁴A different sharing rate was proposed in New York City where the utility, Consolidated Edison, planned to replace traditional network investment with a DER project known as the Brooklyn-Queens Demand-side Management (BQDM) Program. The original proposal was for Consolidated Edison (“the Company”) to “retain a 50% share of the annual net savings realized by customers due to the BQDM Program and other investments made by the Company, calculated as the difference between the annual carrying cost of the

Although the potential value of cost-sharing policies is widely recognized, relatively little is known about the optimal design of such policies. The purpose of this research is to analyze the design of simple cost-sharing policies that best serve electricity customers by motivating a utility to identify promising DER projects and implement these projects at minimum cost. We consider settings in which vital customer needs can be satisfied either through traditional network investment or through implementation of a DER project. The utility and the regulator are both well informed about the cost of traditional network investment. In contrast, the utility is better informed than the regulator about the difficulty of managing the cost of a DER project.⁵ We focus on the optimal design of simple cost-sharing policies under which the utility is awarded a fraction $s \in [0, 1]$ of the cost reduction that is achieved under the DER project.

We demonstrate that the optimal value of s takes a particularly simple form when the utility always undertakes the DER project and cannot be penalized severely if it fails to secure a cost reduction. In this case, the optimal s varies inversely with the utility’s ability to secure a cost reduction. More generally, analytic characterizations of the optimal policy can be complex. However, numerical solutions permit additional conclusions. Relatively straightforward conclusions include the following four.

First, the optimal s often declines as the utility’s information advantage over the regulator increases. Although the reduction in s increases expected project cost by diminishing the utility’s incentive to manage costs, it reduces the rent that the utility secures from its superior knowledge of the prevailing environment. Second, the optimal s often declines as the maximum potential cost reduction under the DER project increases. A more pronounced

[traditional network investment] as originally anticipated by the Company and the total annual collections for the BQDM Program and all other related investments” (New York Public Service Commission (NYPSC), 2014). The NYPSC ultimately chose not to implement the proposed cost-sharing policy (NYPSC, 2014, pp. 21-22). The Office of Gas and Electricity Markets (Ofgem) employs cost sharing in the form of an “efficiency incentive rate” in its RIIO program for electricity distribution companies in the UK. The rate varies between 40% and 50% of the difference between projected and realized cost (Ofgem, 2010).

⁵Such information asymmetry is common in practice. See, for instance, Joskow (2014), California Public Utilities Commission (CPUC) (2016a), and Jenkins and Perez-Arriaga (2017).

potential cost reduction itself enhances the utility’s incentive to increase its cost-containment effort, so s can be reduced (in order to secure more of the realized cost savings for consumers) without diminishing the utility’s effort unduly. Third, the optimal s often declines as the utility becomes more averse to risk, and so requires a larger expected payment to compensate for any specified level of payment variation. Fourth, the optimal s can vary considerably as industry conditions change. Consequently, expected procurement cost can increase considerably if s reflects a simple rule of thumb rather than careful consideration of the prevailing environment.

We also derive several less apparent findings, including the following five. First, the optimal s often increases as cost reduction becomes less onerous for the utility to secure. The increased incentive reduces the expected cost of the DER project by complementing the firm’s enhanced ability to reduce project costs. Second, the maximum reduction in expected procurement cost that can be secured by employing a continuum of optional compensation structures rather than a single structure often is quite limited.⁶ Therefore, although more complex regulatory policies have the potential to benefit consumers, the benefits they deliver in practice may be limited.⁷ Third, expected procurement cost can decline as the highest possible cost under the DER project increases. The gain from a less favorable potential outcome arises because the higher maximum cost enhances the utility’s incentive to secure a cost reduction.

Fourth, the optimal s can increase as the utility becomes more averse to risk. This is the case because increased risk aversion effectively reduces the gain the utility perceives from a given financial reward for successful cost reduction, which can lead the utility to reduce its cost-containment effort. The regulator may optimally increase s to avoid excessive reduction

⁶The academic literature (e.g., Laffont and Tirole, 1993) emphasizes the merits of employing a continuum of optional compensation structures. Ofgem employs optional compensation structures in the UK electricity sector (Ofgem, 2009). Crouch (2006), Cossent and Gomez (2013), and Jenkins and Perez-Arriaga (2017) analyze the design of these structures.

⁷Although an optimally-designed continuum of compensation structures typically do not secure major reductions in procurement cost, they can reduce the utility’s profit substantially.

in this effort. Fifth, when the regulator has limited ability to penalize the utility for poor performance under the DER project, she may intentionally cede rent to the utility if it chooses to undertake traditional network investment. The increased rent helps to counteract the utility’s tendency to unduly favor the DER project over traditional network investment.

The basic forces at play in our model have been studied in other settings.⁸ We contribute to the literature in part by deriving simple analytic characterizations of the optimal regulatory policy in certain specified settings. In addition, we employ numerical solutions to characterize the optimal policy in a variety of more general settings.⁹ We also complement other studies by analyzing: (i) the potential gains from employing relatively sophisticated regulatory instruments (i.e., a continuum of optional cost-sharing arrangements); (ii) the optimal response to common contracting frictions (including utility risk aversion and limited liability); and (iii) the losses that arise when simple rules of thumb (e.g., equal sharing of realized cost savings) are employed in place of optimal cost-sharing policies.

The analysis proceeds as follows. Section 2 describes the key elements of the formal model that we analyze. Section 3 characterizes the optimal procurement policy when the DER project is always the most attractive option. Section 4 characterizes the optimal policy when traditional network investment and a DER project are both viable options. Section 5 examines the changes in the optimal policy that arise when the regulator can offer a menu of optional compensation structures, when the firm is risk averse, and when the maximum financial loss that the firm can incur is limited. Section 5 also explores the losses that arise when simple rules of thumb are employed rather than the optimal cost-sharing policy. Section 6 reviews the key findings and discusses additional extensions of the analysis. The Appendix provides the proofs of the formal conclusions in the text.

⁸For original contributions and reviews of related studies, see Schmalensee (1989), Laffont and Tirole (1993), Laffont and Martimort (1993), Armstrong and Sappington (2007), and Joskow (2014), for example.

⁹Brown and Sappington (2018a) derive analytic comparative static conclusions in a related, but distinct, model. Their analysis restricts the utility’s private information to be binary, does not consider the effects of risk aversion or limited liability, and does not characterize the losses that arise when the optimal cost-sharing policy is replaced by a simple rule of thumb (e.g., equal sharing of realized cost reductions).

2 The Model

We consider a setting where an electric utility (“the firm”) must satisfy the pressing needs of its customers. The firm can do so by undertaking either a core project or a non-core project. The core project, which entails traditional network investment, has known cost c_0 . The non-core project is an alternative to standard infrastructure investment. This project might entail demand-side management or remote temporary storage of electricity to ensure its local availability during periods of peak demand, for example. Because the non-core project involves non-traditional operations, its implementation cost is uncertain. For simplicity, the cost is assumed to be either low ($\underline{c}$) or high ($\bar{c}$). The expected cost of the non-core project is $c^e = p \underline{c} + [1 - p] \bar{c}$, where $p \in [0, 1]$ is the probability that the low cost $\underline{c}$ is achieved.

The firm can increase the “success probability” p through diligent oversight and management of the non-core project. $D(p)$ will denote the disutility the firm must incur to implement success probability p . For analytic tractability, we assume $D(p) = \frac{\delta}{\gamma} p^\gamma$, where $\gamma > 1$, $\delta \in [\underline{\delta}, \bar{\delta}]$, and $\underline{\delta} > 0$. Although this functional form is not completely general, it admits a wide range of functional relationships between p and $D(\cdot)$. It is readily verified that $D(p)$ declines as γ increases for given p .¹⁰ Therefore, it becomes less onerous for the firm to increase p as its productivity parameter, γ , increases. Because $\gamma > 1$, the firm’s disutility increases with p at an increasing rate, reflecting diminishing returns to effort designed to reduce the cost of the non-core project.

The regulator and the firm both know γ . However, to capture the limited information with which regulators typically are forced to operate, only the firm knows the realization of δ . The regulator views δ as the realization of a random variable with distribution $G(\delta)$ and density $g(\delta)$.

The realized cost of the non-core project, $c \in \{\underline{c}, \bar{c}\}$, is observed publicly. In contrast, p and $D(p)$ are not observed publicly, so the regulator cannot base payments to the firm

¹⁰ $\frac{\partial D}{\partial \gamma} = \frac{\delta}{\gamma} p^\gamma \ln p - \frac{\delta}{\gamma^2} p^\gamma < 0$ for $p \in (0, 1)$.

on these variables.¹¹ To motivate the firm to increase p , the regulator must promise the firm greater profit when the non-core project is implemented at low cost ($\underline{c}$) than when it is implemented at high cost ($\bar{c}$). $\underline{r}$ will denote the payment the regulator delivers to the firm when $c = \underline{c}$. $\bar{r}$ will denote the corresponding payment when $c = \bar{c}$.¹²

The firm's expected profit when it undertakes the non-core project and implements success probability p is:

$$\pi(p) = p[\underline{r} - \underline{c}] + [1 - p][\bar{r} - \bar{c}] - \frac{\delta}{\gamma} p^\gamma.$$

Let $p(\delta)$ denote the success probability that maximizes $\pi(p)$ when δ is realized. Then the firm's expected profit under the non-core project, given δ , is:

$$\pi_n(\delta) = p(\delta)[\underline{r} - \underline{c}] + [1 - p(\delta)][\bar{r} - \bar{c}] - \frac{\delta}{\gamma} [p(\delta)]^\gamma. \quad (1)$$

The firm's profit when it undertakes the core project is $r_0 - c_0$, where r_0 is the payment the regulator delivers to the firm when it undertakes the core project at known cost c_0 .¹³ The firm will only undertake a project if it anticipates nonnegative profit from doing so.¹⁴

The firm will implement the non-core project if and only if $\pi_n(\delta) \geq \max\{0, r_0 - c_0\}$. The firm's expected profit under the non-core project increases as it becomes less onerous for the firm to increase p , i.e., as δ declines.¹⁵ Therefore, the firm will undertake the non-core project if $\delta \in [\underline{\delta}, \delta_n]$ and will implement the core project if $\delta \in (\delta_n, \bar{\delta}]$ (and $r_0 \geq c_0$), where $\delta_n \leq \bar{\delta}$.¹⁶

¹¹The disutility that managers incur when working diligently to control costs typically is difficult, if not impossible, to measure with precision.

¹²This payment can be viewed as the sum of the revenue the firm secures directly from its customers and any additional compensation delivered by the regulator. We abstract from potential differences in project capital intensity and the associated variation in utility compensation that can arise when revenue is set to provide a fair return on the utility's rate base. See Costa et al. (2017) for an analysis of this issue.

¹³This cost can be viewed as including any (known) disutility associated with diligent project oversight. The analysis could readily be extended to admit uncertainty about the cost of the core project. The present approach constitutes a streamlined means to capture greater certainty and less pronounced information asymmetry regarding familiar, traditional investment in network infrastructure.

¹⁴The minimum (extranormal) expected profit that the firm will accept is normalized to zero for expositional convenience.

¹⁵See Lemma 2 in Appendix B.

¹⁶If $\delta_n = \bar{\delta}$, then the firm always implements the non-core project. If $\delta_n < \underline{\delta}$, then the firm always implements the core project.

The regulator seeks to minimize the expected payment required to induce the firm to satisfy its customers' needs by implementing either the core or the non-core project. The interaction between the regulator and the firm proceeds as follows. First, the firm learns the realization of δ . Second, the regulator sets payments r_0 , $\underline{r}$, and $\bar{r}$. Third, the firm selects the project it will implement. Fourth, the firm selects p if it undertakes the non-core project. Finally, the project cost is realized and the promised payment is delivered.

The regulator's formal problem in this setting, [P], is:

$$\text{Minimize}_{\underline{r}, \bar{r}, r_0} \int_{\underline{\delta}}^{\delta_n} \{ p(\delta) \underline{r} + [1 - p(\delta)] \bar{r} \} dG(\delta) + [1 - G(\delta_n)] r_0 \quad (2)$$

subject to: $\pi_n(\delta) \geq \text{maximum} \{0, r_0 - c_0\}$ for all $\delta \in [\underline{\delta}, \delta_n]$, and

$$r_0 - c_0 \geq \text{maximum} \{0, \pi_n(\delta)\} \text{ for all } \delta \in (\delta_n, \bar{\delta}]. \quad (3)$$

Expression (2) reflects expected procurement cost under the non-core project ($p(\delta) \underline{r} + [1 - p(\delta)] \bar{r}$) and under the core project (r_0). Expression (3) ensures that the firm will undertake the non-core project when $\delta \in [\underline{\delta}, \delta_n]$ and the core project when $\delta \in (\delta_n, \bar{\delta}]$.

Before proceeding to characterize the solution to [P], consider as a benchmark the hypothetical setting in which the regulator shares the firm's knowledge of δ and can observe the firm's choice of p . In this *full-information* setting, the regulator can calculate for each realization of δ the efficient total expected cost of the non-core project, $C^*(\delta) \equiv C(p^*(\delta), \delta)$, where:

$$C(p, \delta) = p \underline{c} + [1 - p] \bar{c} + \frac{\delta}{\gamma} p^\gamma \text{ and } p^*(\delta) \equiv \arg \min_{p \in [0,1]} C(p, \delta).$$

The regulator can then instruct the firm to undertake the non-core project if and only if $C^*(\delta) \leq c_0$. The regulator can pay the firm c_0 when it undertakes the core project and $C^*(\delta)$ when it undertakes the non-core project and implements success probability $p^*(\delta)$.¹⁷ This policy ensures that: (i) the project with the lowest expected total cost is always undertaken; (ii) the expected total cost of the non-core project is minimized whenever it is undertaken;

¹⁷The regulator can threaten to penalize the utility severely if it implements a success probability other than $p^*(\delta)$.

and (iii) the firm's expected profit is held to its minimum feasible level.

3 Setting Where the Non-Core Project is Always Implemented

In some jurisdictions, utilities are required to implement DERs rather than undertake additional investment in core capacity.¹⁸ Such a requirement can minimize procurement cost when the non-core project is always substantially less costly than the core project. We begin by characterizing the optimal regulatory policy when such a requirement prevails. We focus on s , the share of the “cost reduction” ($\bar{c} - \underline{c}$) that is effectively awarded to the firm when the low cost ($\underline{c}$) is realized under the non-core project. Formally:

$$\underline{r} - \underline{c} - (\bar{r} - \bar{c}) = s[\bar{c} - \underline{c}] \Rightarrow s = \frac{\underline{r} - \underline{c} - (\bar{r} - \bar{c})}{\bar{c} - \underline{c}}. \quad (4)$$

It is readily verified that s is also the share of the realized reduction in cost below c_0 that is awarded to the firm when it implements the non-core project.¹⁹

Let [PNC] denote the regulator's problem when she induces the firm to always implement the non-core project (so $\delta_n = \bar{\delta}$). Let [PNC]' denote the corresponding problem where, in addition, $\bar{r}$ is set exogenously at a sufficiently high level (e.g., $\bar{r} \geq \bar{c}$) that the requirement to ensure the firm's participation ($\pi_n(\delta) \geq 0$) is not constraining.²⁰

The optimal degree of cost-sharing takes a simple, intuitive form when increasing p is always sufficiently onerous for the firm that it sets $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$. This will be the case under the condition specified in Conclusion 1.

Conclusion 1. *Suppose $\underline{\delta} > \frac{1}{\gamma}[\bar{c} - \underline{c}]$. Then $s = \frac{1}{\gamma}$ at the solution to [PNC]'.*

Conclusion 1 considers a setting where the regulator is constrained to deliver a relatively

¹⁸California has adopted a pilot program that requires the state's major utilities to implement DER alternatives to traditional capital investment (CPUC, 2016b). Proposed legislation in the Commonwealth of Massachusetts (2017) would require utilities to consider “non-wire” alternatives before undertaking new investments in the electricity distribution grid.

¹⁹In this case, $\underline{r} = \underline{c} + s[c_0 - \underline{c}]$ and $\bar{r} = \bar{c} - s[\bar{c} - c_0]$, so $\underline{r} - \underline{c} - (\bar{r} - \bar{c}) = s[c_0 - \underline{c}] + s[\bar{c} - c_0] = s[\bar{c} - \underline{c}]$.

²⁰Such a requirement might reflect, for example, a desire to avoid financial distress for the utility and thereby ensure the ongoing, reliable delivery of electricity. This requirement is considered in more detail in Section 5.

generous profit to the firm even when it fails to implement the non-core project at low cost. In this event, the regulator optimally concedes the fraction $\frac{1}{\gamma}$ of the realized cost reduction to the firm to motivate it to enhance p . The inverse relationship between s and γ arises because as it becomes less onerous for the firm to increase p (i.e., as γ increases), less financial incentive is required to induce the firm to implement any specified level of p . Consequently, the regulator optimally reduces the fraction of the realized cost reduction that she awards to the firm (i.e., s declines) in order to secure more of the realized cost savings for consumers. Conclusion 1 indicates that when $\gamma = 2$, so the firm's cost of increasing p is quadratic, an equal sharing of realized cost savings ($s = \frac{1}{2}$) is optimal under the specified conditions.

Conclusion 2 characterizes the optimal sharing rule (s) when her choice of $\bar{r}$ does not face an additional exogenous constraint.

Conclusion 2. *Suppose $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$ at the solution to [PNC]. Then at this solution:*

$$\frac{s[\gamma - 1]}{s\gamma - 1} = \int_{\underline{\delta}}^{\bar{\delta}} \left(\frac{\bar{\delta}}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta). \quad (5)$$

Consequently: (i) $s > \frac{1}{\gamma}$; (ii) $\frac{ds}{d\underline{\delta}} > 0$; (iii) $\frac{ds}{d\bar{\delta}} < 0$; and (iv) $\frac{ds}{d\underline{c}} = \frac{ds}{d\bar{c}} = 0$.

When the regulator can set $\bar{r}$ freely, she increases s above $\frac{1}{\gamma}$ and reduces $\bar{r}$ to ensure the firm secures no rent when $\delta = \bar{\delta}$. The increase in s induces the firm to increase p , which reduces c^e (the expected value of c) and thereby reduces expected procurement cost when the regulator can adjust $\bar{r}$ to eliminate the firm's rent when $\delta = \bar{\delta}$.²¹

Conclusion 2 implies that as the information asymmetry the regulator faces ($\bar{\delta} - \underline{\delta}$) becomes more pronounced, she optimally reduces s in order to avoid ceding excessive rent to the firm when its personal cost of increasing p (i.e., δ) is relatively small. Conclusion 2 also implies that the optimal s does not change as $\bar{c} - \underline{c}$ changes. This is the case because the

²¹In essence, the ability to adjust $\bar{r}$ reduces the regulator's cost of increasing s . Consequently, the regulator employs her less costly instrument more intensively.

marginal benefit of increasing s (i.e., the corresponding reduction in c^e) and the marginal cost of increasing s (i.e., the associated rent that accrues to the firm) are both proportional to $\bar{c} - \underline{c}$.

Conclusion 3 extends the analysis to allow for the possibility that the firm might ensure low-cost implementation of the non-core project by setting $p(\delta) = 1$ for some δ realizations. It is straightforward to verify that $p(\delta)$ is non-increasing in δ , so if the firm sets $p(\tilde{\delta}) = 1$ for some $\tilde{\delta} \in (\underline{\delta}, \bar{\delta}]$, then it will set $p(\delta) = 1$ for all $\delta \in [\underline{\delta}, \tilde{\delta}]$.²² We define $\delta_1 \in [\underline{\delta}, \bar{\delta}]$ to be the largest realization of δ for which $p(\delta) = 1$ under the non-core project.

Conclusion 3. *If $\delta_1 < \bar{\delta}$ at the solution to [PNC], then at this solution:*

$$\begin{aligned} & [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} \left[\frac{s\gamma - 1}{s(\gamma - 1)} \right] \int_{\delta_1}^{\bar{\delta}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) \\ &= \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} - [\bar{c} - \underline{c}] G(\delta_1), \text{ and} \end{aligned} \quad (6)$$

$$\bar{r} = \bar{c} - \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma - 1}{\gamma} \right]. \quad (7)$$

Numerical solutions of equations (6) and (7) can be employed to further characterize the optimal regulatory policy when the firm always implements the non-core project. The numerical solutions are presented in Section 4, where we account for the possibility that the firm might implement the core project when δ is sufficiently large.

4 The Setting Where Either Project May be Implemented.

If $C^*(\delta)$, the efficient total expected cost of the non-core project, is sufficiently far below c_0 when $\delta = \underline{\delta}$ and sufficiently far above c_0 when $\delta = \bar{\delta}$, the regulator will induce the firm to undertake the non-core project when δ is sufficiently small ($\delta \leq \delta_n$) and implement the core project when δ is sufficiently large ($\delta > \delta_n$).²³ Conclusion 4 characterizes the cost sharing (s) that the regulator optimally implements in this setting.

²²See Lemma 1 in Appendix B.

²³See Lemma 2 in Appendix B.

Conclusion 4. *At the solution to [P]:*

$$\begin{aligned} & [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} \left[\frac{s\gamma - 1}{s(\gamma - 1)} \right] \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) \\ &= G(\delta_n) \left[\frac{\gamma(\gamma - 1)}{(\gamma - s)^2} \right] [\bar{c} - c_0] - [\bar{c} - \underline{c}] G(\delta_1); \end{aligned} \quad (8)$$

$$\bar{r} = \frac{\bar{c}\gamma[1 - s] + c_0[\gamma - 1]s}{\gamma - s}; \text{ and } r_0 = c_0. \quad (9)$$

Numerical solutions of equations (6) – (9) permit a characterization of the optimal regulatory policy. This characterization is facilitated by introducing a *baseline setting*, where $c_0 = 1,000$, $\underline{c} = 200$, $\bar{c} = 1,250$, $\gamma = 2$, $\underline{\delta} = 100$, and $\bar{\delta} = 2,205$. These parameters are informed by Consolidated Edison’s Brooklyn-Queens demand-side management project. This (non-core) project was initially forecast to cost \$200 million and deemed capable of replacing a \$1 billion traditional investment in network infrastructure (Sullivan, 2014). Reflecting these relative costs, we take $c_0 = 1,000$ to be the cost of the core investment and $\underline{c} = 200$ to be the minimum possible cost of the non-core project. Reflecting the possibility that the full potential cost-savings of the non-core project will not materialize (Picard, 2017), we let $\bar{c} = 1.25 c_0$, so the high-cost realization of the non-core project is 25% greater than the cost of the core project.

$\gamma = 2$ in the baseline setting, so the disutility the firm incurs to increase p is quadratic in p . δ is assumed to be distributed uniformly on $[\underline{\delta}, \bar{\delta}]$. The bounds on this distribution are set to reflect variation in potential efficient cost savings under the non-core project. Define $\underline{k} \equiv \frac{C^*(\underline{\delta})}{c_0}$ to be the ratio of $C^*(\delta)$ to c_0 when $\delta = \underline{\delta}$, and define $\bar{k} \equiv \frac{C^*(\bar{\delta})}{c_0}$ to be the corresponding ratio when $\delta = \bar{\delta}$. $\underline{k} = 0.25$ and $\bar{k} = 1$ in the baseline setting, so $C^*(\delta)$ is 25% of c_0 when $\delta = \underline{\delta}$, and equal to c_0 when $\delta = \bar{\delta}$.²⁴ These considerations imply $\underline{\delta} = 100$ and $\bar{\delta} = 2,205$ in the baseline setting.²⁵

²⁴Therefore, the efficient total expected cost of the non-core project is less than the cost of the core project for all $\delta < \bar{\delta}$.

²⁵It is readily verified that $p^*(\delta) = 1$ and $C^*(\delta) = \underline{c} + \frac{\delta}{2}$ for $\delta \leq \bar{c} - \underline{c}$ in the baseline setting. Therefore, $C^*(\underline{\delta}) = 0.25 c_0 \Leftrightarrow 200 + \frac{\underline{\delta}}{2} = 250 \Leftrightarrow \underline{\delta} = 100$. Furthermore, $p^*(\delta) = \frac{1}{\delta} [\bar{c} - \underline{c}]$ for $\delta > \bar{c} - \underline{c}$.

Key elements of the solution to [P] in the baseline setting are presented in the first row of data in Table 1. The table reports the optimal degree of cost sharing (s), the payment to the firm when $\underline{c}$ is realized ($\underline{r}$), the corresponding payment when $\bar{c}$ is realized ($\bar{r}$), the largest realization of δ for which $p(\delta) = 1$ under the non-core project (δ_1), the largest realization of δ for which the firm implements the non-core project (δ_n), the expected probability that $\underline{c}$ is realized under the non-core project ($E\{p\}$),²⁶ the firm's expected disutility from implementing success probability p ($E\{D\}$),²⁷ the firm's expected profit ($E\{\pi\}$),²⁸ and expected procurement cost ($E\{Cost\}$).²⁹ Subsequent rows in Table 1 provide the corresponding characterization of the solution to [P] when the parameter identified in the first column of the table is increased by 50%, holding all other parameters at their levels in the baseline setting.³⁰ Tables 1.1 and 1.2 in Appendix A further characterize how the optimal procurement policy changes as parameters vary from their levels in the baseline setting.

Tables 1, 1.1, and 1.2 inform several observations that warrant emphasis. First, in contrast to Conclusions 1 and 2, when $\gamma = 2$, the regulator sets $s < \frac{1}{2}$ when she induces the firm to ensure low-cost implementation of the core project (so $p(\delta) = 1$) for the smaller δ realizations.³¹ A reduction in s induces an undesirable reduction in p for all $\delta \in [\underline{\delta}, \delta_n]$ when $p(\delta) \in (0, 1)$ for all such δ . This undesirable effect of a reduction in s does not arise for the smaller δ realizations (i.e., for $\delta \in [\underline{\delta}, \delta_1]$) when $p(\delta) = 1$ for these realizations. As this cost

$$\text{Therefore, } C^*(\bar{\delta}) = c_0 \Leftrightarrow \frac{1,050}{\bar{\delta}}[200] + \frac{\bar{\delta}-1,050}{\bar{\delta}}[1,000] + \frac{\bar{\delta}}{2} \left(\frac{1,050}{\bar{\delta}} \right)^2 = 1,000 \Leftrightarrow \bar{\delta} = 2,205.$$

²⁶Formally, $E\{p\} = \int_{\underline{\delta}}^{\delta_n} p(\delta) dG(\delta)$, where $p(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_1]$.

²⁷Formally, $E\{D\} = \int_{\underline{\delta}}^{\delta_n} \frac{\delta}{\gamma} [p(\delta)]^\gamma dG(\delta)$, where $p(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_1]$.

²⁸Formally, $E\{\pi\} = \int_{\underline{\delta}}^{\delta_n} \{p(\delta)[\underline{r} - \underline{c}] + [1 - p(\delta)][\bar{r} - \bar{c}] - \frac{\delta}{\gamma} [p(\delta)]^\gamma\} dG(\delta) + [1 - G(\delta_n)][r_0 - c_0]$, where $p(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_1]$.

²⁹Formally, $E\{Cost\} = \int_{\underline{\delta}}^{\delta_n} \{p(\delta)\underline{r} + [1 - p(\delta)]\bar{r}\} dG(\delta) + [1 - G(\delta_n)]r_0$, where $p(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_1]$.

³⁰The value of r_0 is not reported in the table because it is always equal to $c_0 = 1,000$. The entries in Table 1 and all subsequent tables reflect rounding.

³¹Recall from Conclusion 2 that the regulator optimally sets $s > \frac{1}{\gamma}$ when she induces the firm to undertake the non-core project and set $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$.

of reducing s declines, the regulator often reduces s below $\frac{1}{\gamma}$ in order to secure for consumers a larger fraction of the realized cost reduction.

	s	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{D\}$	$E\{\pi\}$	$E\{Cost\}$
Baseline	0.463	611	1,175	487	1,570	0.45	92.8	75.6	865.9
$\gamma = 3$	0.524	567	1,067	550	2,205	0.74	119.2	103.3	698.1
$\bar{\delta} = 3,307.5$	0.463	611	1,175	487	1,570	0.30	60.9	49.6	912.0
$\underline{\delta} = 150$	0.478	624	1,171	502	1,605	0.46	99.2	73.8	871.9
$\bar{c} = 1,850$	0.377	628	1,672	631	980	0.38	87.8	69.8	879.7
$\underline{c} = 300$	0.463	664	1,175	440	1,284	0.39	71.0	56.0	901.7

Table 1. Elements of the Optimal Regulatory Policy.

Second, as it becomes less onerous for the firm to increase p (i.e., as γ increases), the regulator often enhances the firm's incentive to increase p by increasing s . The increase in s , coupled with the relative ease with which the firm can increase p , reduces expected procurement cost by inducing the firm to undertake the non-core project often and secure a low expected cost (c^e) when it does so. The firm's rent increases as γ increases, but expected procurement cost declines due to the reduction in c^e .³²

Third, as $\bar{\delta} - \underline{\delta}$ increases, the relevant information asymmetry becomes more pronounced. Consequently, the regulator often reduces s to limit the rent that accrues to the firm for the smaller δ realizations.³³ Despite the increased information asymmetry, expected procurement cost can decline if the increase in $\bar{\delta} - \underline{\delta}$ stems from a decline in $\underline{\delta}$, which reduces the firm's expected cost of enhancing p .

Fourth, as $\bar{c}$ increases, the firm's financial gain from enhancing p increases, *ceteris paribus*. Consequently, the regulator often reduces s in order to secure for consumers a

³²Recall from Conclusion 1 that when an exogenous restriction on $\bar{r}$ limits the regulator's ability to control the firm's rent, the regulator optimally reduces s (to limit excessive rent) as γ increases. When the regulator can adjust $\bar{r}$ to limit the firm's rent, she increases s as γ increases to secure a substantial reduction in c^e .

³³See Schmalensee (1989) and Jenkins and Perez-Arriaga (2017), for example, for similar conclusions.

larger fraction of the realized cost reduction, knowing that the reduction in s will not reduce p unduly. In isolation, the increase in $\bar{c}$ increases expected procurement cost by increasing the highest cost of the non-core project. However, expected procurement cost can decline as $\bar{c}$ increases because of the enhanced incentive to increase p that arises as $\bar{c} - \underline{c}$ increases.³⁴ This observation is recorded formally in Conclusion 5.

Conclusion 5. *Expected procurement cost can decline as $\bar{c}$ increases at the solution to [P].*

Numerical solutions to [P] indicate that expected procurement cost can decline as $\bar{c}$ increases when $\bar{c}$ is relatively high and the firm's cost of enhancing p (i.e., $\underline{\delta}$ or $\bar{\delta}$) is relatively low.³⁵ When this cost is low, the firm will increase p relatively rapidly as the financial gain it secures from implementing the non-core project at low cost ($\underline{c}$) increases. The relatively rapid increase in p as $\bar{c} - \underline{c}$ increases can lower expected procurement cost despite the increase in $\bar{c}$.

5 Extensions.

A. Multiple Optional Compensation Structures.

To this point, the regulator has been presumed to set a single compensation structure $(\underline{r}, \bar{r})$ to govern the firm's operations under the non-core project. In principle, the regulator could allow the firm to choose its preferred compensation structure from a carefully-designed set of such structures. Doing so might reduce expected procurement cost by tailoring the selected compensation structure more finely to the environment in which it is implemented.³⁶ We now characterize the regulator's optimal policy in this alternative setting and assess the reduction in procurement cost that the more general policy engenders.

Under the more general policy, the regulator links the project that will be pursued to the firm's report of δ . Let $\psi(\hat{\delta}) \in [0, 1]$ denote the probability that the firm is instructed

³⁴Expected procurement cost never declines as $\bar{c}$ increases in the full-information setting where the regulator can select p directly. However, when the regulator must motivate the utility to increase p through her choice of s , she can benefit from the enhanced motivation that an increase in $\bar{c}$ provides. This observation is an illustration of the Theory of the Second Best (Lipsey and Lancaster, 1956-7).

³⁵See the proof of Conclusion 5 in Appendix B.

³⁶See Laffont and Tirole (1993), for example.

to implement the non-core project when the firm reports $\delta = \widehat{\delta}$. Also let $r_0(\widehat{\delta})$ denote the payment the regulator delivers to the firm when it reports $\delta = \widehat{\delta}$ and implements the core project. $\underline{r}(\widehat{\delta})$ and $\bar{r}(\widehat{\delta})$ will denote the corresponding payments when the realized cost of the non-core project is $\underline{c}$ and $\bar{c}$, respectively. $s(\widehat{\delta}) \in (0, 1]$ is the share of the cost reduction $(\bar{c} - \underline{c})$ awarded to the firm when it reports $\delta = \widehat{\delta}$ and implements the non-core project, and the realized cost is $\underline{c}$. Formally:

$$\underline{r}(\widehat{\delta}) - \underline{c} - (\bar{r}(\widehat{\delta}) - \bar{c}) = s(\widehat{\delta}) [\bar{c} - \underline{c}] \Rightarrow s(\widehat{\delta}) = \frac{\underline{r}(\widehat{\delta}) - \underline{c} - (\bar{r}(\widehat{\delta}) - \bar{c})}{\bar{c} - \underline{c}}. \quad (10)$$

$p(\widehat{\delta}|\delta) \in [0, 1]$ is the success probability the firm implements under the non-core project after reporting its true disutility parameter, δ , to be $\widehat{\delta}$.

In this setting, the firm first observes $\delta \in [\underline{\delta}, \bar{\delta}]$ privately. Then the regulator sets her policy instruments $\{r_0(\widehat{\delta}), \underline{r}(\widehat{\delta}), \bar{r}(\widehat{\delta})\}$. Next, the firm chooses its preferred option, formally by reporting the realized δ . Then, if the firm undertakes the non-core project, it chooses p . Finally, the project cost is realized and the regulator delivers the promised payment to the firm.

The firm's expected profit when it reports the true disutility parameter, δ , to be $\widehat{\delta}$ in this setting is:

$$\begin{aligned} \Pi(\widehat{\delta}|\delta) = & \psi(\widehat{\delta}) \left\{ p(\widehat{\delta}|\delta) [\underline{r}(\widehat{\delta}) - \underline{c}] + [1 - p(\widehat{\delta}|\delta)] [\bar{r}(\widehat{\delta}) - \bar{c}] - \frac{\delta}{\gamma} [p(\widehat{\delta}|\delta)]^\gamma \right\} \\ & + [1 - \psi(\widehat{\delta})] [r_0(\widehat{\delta}) - c_0]. \end{aligned}$$

The regulator's problem, [PM], is to choose $r_0(\delta)$, $\underline{r}(\delta)$, $\bar{r}(\delta)$, and $\psi(\delta)$ to:³⁷

$$\text{Minimize } \int_{\underline{\delta}}^{\bar{\delta}} \{ \psi(\delta) (\bar{r}(\delta) + p(\delta) [\underline{r}(\delta) - \bar{r}(\delta)]) + [1 - \psi(\delta)] r_0(\delta) \} dG(\delta) \quad (11)$$

$$\text{subject to, for all } \delta, \widehat{\delta} \in [\underline{\delta}, \bar{\delta}]: \quad \Pi(\delta|\delta) \geq 0 \quad \text{and} \quad \Pi(\delta|\delta) \geq \Pi(\widehat{\delta}|\delta). \quad (12)$$

Conclusion 6 characterizes the optimal regulatory policy in this setting. The Conclusion refers to $h(\delta) \equiv \frac{\delta}{\delta + \frac{G(\delta)}{g(\delta)}}$ and the following definitions:

³⁷This formulation of the regulator's problem is without loss of generality (e.g., Myerson, 1979).

$$\delta_n \equiv \min \{ \tilde{\delta}_n, \bar{\delta} \} \text{ where } \frac{\tilde{\delta}_n}{h(\tilde{\delta}_n)} = \left[\frac{\gamma - 1}{\gamma} \right]^{\gamma-1} [\bar{c} - c_0]^{1-\gamma} [\bar{c} - \underline{c}]^\gamma, \text{ and}$$

$$z(\delta) \equiv \bar{c} + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta}^{\delta_n} \left(\frac{\tilde{\delta}_n}{h(\tilde{\delta}_n)} \right)^{-\frac{\gamma}{\gamma-1}} d\tilde{\delta} - \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma - 1}{\gamma} \right] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} [h(\delta)]^{\frac{\gamma}{\gamma-1}}.$$

Conclusion 6. Suppose $\frac{d}{d\delta} \left(\frac{G(\delta)}{g(\delta)} \right) \geq \frac{1}{\delta} \left[\frac{G(\delta)}{g(\delta)} \right]$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$ and $\bar{c} - \underline{c} < \gamma [c_0 - \underline{c}]$. Then at a solution to [PM], there exist $\delta_1, \delta_n \in [\underline{\delta}, \bar{\delta}]$ with $\delta_1 \leq \delta_n$ such that: (i) $\psi(\delta) = 1$, $s(\delta) = h(\delta_1)$, and $\bar{r}(\delta) = \bar{r}(\delta_1)$ for all $\delta \in [\underline{\delta}, \delta_1)$, where $\frac{\delta_1}{h(\delta_1)} = \bar{c} - \underline{c}$; (ii) $\psi(\delta) = 1$, $s(\delta) = h(\delta)$, and $\bar{r}(\delta) = z(\delta)$ for all $\delta \in [\delta_1, \delta_n]$; and (iii) $\psi(\delta) = 0$ and $r(\delta) = c_0$ for all $\delta \in (\delta_n, \bar{\delta}]$.

Conclusion 6 reports that, given δ_1 and δ_n , the optimal values of $s(\cdot)$ do not vary with $\underline{c}$ and $\bar{c}$ in this setting. This is the case because, as noted above, the marginal benefit and the marginal cost of increasing s are both proportional to $\bar{c} - \underline{c}$. Conclusion 6 also reports that, given δ_1 and δ_n , the optimal values of $s(\cdot)$ do not vary with γ . This conclusion reflects two countervailing effects. As γ increases, the surplus-maximizing level of p increases, so higher values of $s(\cdot)$ may be optimal to increase p . However, because it becomes less onerous for the firm to increase p as γ increases, any desired level of p can be induced with a smaller $s(\cdot)$. These forces that promote higher and lower values of s offset each other when the regulator can tailor the value of s to the realization of δ .³⁸

The key elements of the optimal regulatory policy can all vary with δ at the solution to [PM]. Table 2 reports the minimum, maximum, and average values for the specified variables, along with the corresponding standard deviations (Std. Dev.), in the baseline setting.

³⁸In contrast, when the regulator can only set a single s that prevails for all δ realizations, the countervailing forces do not exactly offset each other.

	s	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{D\}$	$E\{\pi\}$	$E\{Cost\}$
Mean	0.539	623	1,107	575	1,153	0.41	89.9	72.1	855.8
Std. Dev.	0.01	2.25	7.58	0	0	0.43	98.6	108.7	159.0
Minimum	0.5227	618	1,100	575	1,153	0.0	0.0	0.0	625.0
Maximum	0.5476	625	1,119	575	1,153	1.0	287.5	375.0	1,000.0

Table 2. Outcomes at the Solution to [PM].

The feature of Table 2 and its counterparts in Appendix A that warrants particular emphasis is the relatively small reduction in expected procurement cost that the regulator secures by implementing a continuum of compensation structures rather than a single structure.³⁹ The reduction is 1.2% (from 865.9 to 855.8) in the baseline setting⁴⁰ and at most 1.3% in the other settings considered in Table 2.1 in Appendix A. These modest reductions in expected procurement cost are accompanied by more substantial reductions in the firm's expected profit (4.6%, from 75.6 to 72.1 in the baseline setting). By tailoring the prevailing compensation structure to the realization of δ more finely, the regulator is able to reduce the firm's rent considerably. The rent reduction is achieved by reducing s for the higher δ realizations, thereby reducing the induced success probability and increasing c^e , which limits the reduction in expected procurement cost.

B. Limited Liability.

As noted in Section 3, a regulator may sometimes be compelled to limit the maximum financial loss that the firm sustains. Doing so may help to ensure the firm's financial integrity and thereby promote the ongoing supply of high quality service to customers, for example.

³⁹This conclusion is consistent with the corresponding conclusions in Reichelstein (1992), Bower (1993), Rogerson (2003), and Brown and Sappington (2018). Chu and Sappington (2007) identify settings where the gains from implementing a continuum of contracts can be substantial.

⁴⁰The small variation in s reported in Table 2 reflects in part the substantial set of δ 's ($\delta \in [100, 575]$) for which $p(\delta) = 1$ at the solution to [PM]. The variation in s typically will be more pronounced when $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$. (Observe from Conclusion 6 that $s(\underline{\delta}) = 1$ when $\delta_1 = \underline{\delta}$.) Table 2.2 in Appendix A characterizes the solution to [PM] in a setting where $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$. Despite the more pronounced variation in s at this solution, the optimal continuum of compensation structures reduces expected procurement cost below the level achieved with a single compensation structure by less than 0.7% (from 982.3 to 975.8).

We now consider how the optimal regulatory policy changes when $\bar{r} - \bar{c}$ cannot fall below a specified minimum level, m . Formally, the regulator's problem in this setting is $[Pm]$, which is problem $[P]$ with the additional constraint that $\bar{r} - \bar{c} \geq m$.

This limited liability constraint does not bind in the baseline setting when $m < -75.4$ (because $\bar{r} - \bar{c} \approx -75.4$ at the solution to $[P]$). Table 3 compares the optimal regulatory policy and associated outcomes when the limited liability constraint does not bind (so $m < -75.4$), when $m = 0$ (so the firm must always secure non-negative profit), and for intermediate values of m .

m	s	r_0	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{\pi\}$	$E\{Cost\}$
0	0.339	1,051	556	1,250	356	1,242	0.33	90.4	904.6
-20	0.339	1,031	536	1,230	356	1,242	0.33	70.4	893.7
-40	0.350	1,011	556	1,210	356	1,242	0.33	50.4	882.9
-60	0.387	1,000	546	1,190	406	1,377	0.38	52.2	882.9
< -75.4	0.463	1,000	611	1,175	487	1,570	0.45	75.6	865.9

Table 3. The Impact of Limited Liability Constraints.

Five elements of Table 3 and its counterparts in Appendix A merit emphasis. First, binding limited liability constraints increase expected procurement cost moderately (by 4.5% when $m = 0$) in the baseline setting. More pronounced increases can arise in other settings. For instance, when $\gamma = 3$, the regulator induces the firm to increase p in part by increasing s and reducing $\bar{r}$ in the absence of limited liability constraints. (See Table 1.) Therefore, the limited liability restrictions are more constraining, causing expected procurement cost to increase by 31.8%.⁴¹

Second, s generally declines in the presence of binding limited liability constraints. It is more costly for the regulator to increase s (in order to increase p) when her ability to reduce $\bar{r}$ accordingly is restricted. Consequently, the regulator optimally employs her more costly

⁴¹Table 1 and Table 3.1 (in Appendix A) report that when $\gamma = 3$, expected procurement cost increases from 698.1 when $\bar{r} - \bar{c}$ is unrestricted to 920.1 when $\bar{r} - \bar{c}$ must be nonnegative.

instrument less intensively by reducing s .⁴²

Third, binding limited liability constraints can either increase or reduce the firm's expected profit. To illustrate, the firm's expected profit in the baseline setting increases by 19.6% (from 75.6 to 90.4) when $m = 0$ but declines by 33.3% (from 75.6 to 50.4) when $m = -40$.⁴³ A stringent liability restriction can increase the firm's profit by guaranteeing the firm a relatively large financial payoff even when it does not successfully implement the non-core project at cost $\underline{c}$. However, the smaller s that the regulator implements can reduce the firm's profit when less stringent limited liability constraints are imposed.

Fourth, the regulator often induces the firm to implement the core project for a broader range of δ realizations as m increases.⁴⁴ The more extensive implementation of the core project reflects the increased expected procurement cost under the non-core project.

For emphasis, the fifth observation is recorded formally in Conclusion 7.

Conclusion 7. *The regulator may intentionally afford the firm rent under the core project (i.e., $r_0 > c_0$) at the solution to $[Pm]$.*

When $\bar{r} - \bar{c}$ is constrained to substantially exceed its optimal level, the firm is inclined to undertake the non-core project too often when s is set to induce the desired p without ceding excessive rent to the firm. The regulator counteracts this tendency in part by reducing s below the preferred level. However, to avoid an unduly large reduction in p , the regulator does not reduce s too drastically. This leaves the firm inclined to implement the non-core project more often than the regulator would like. To further offset this inclination, the regulator may increase r_0 above c_0 , intentionally delivering rent to the firm when it undertakes the core project. Forfeiting this rent under the core project is a costly means of reducing the firm's tendency to undertake the non-core project too often. However, this means can be

⁴²The reduction in s can be particularly pronounced when γ is relatively large. To illustrate, suppose $\gamma = 3$ and all other parameters are as specified in the baseline setting. Then $s = 0.524$ at the solution to $[P]$ (see Table 1) whereas $s = 0.165$ at the solution to $[P-m]$ when $m = 0$ (see Table 3.1 in Appendix A).

⁴³See Table 3.

⁴⁴Formally, δ_n often is smaller when m is larger. See Table 3.

less costly than reducing s excessively (thereby reducing p unduly).⁴⁵

C. Risk Aversion.

The analysis to this point has presumed the firm to be risk neutral. Because a firm (e.g., its shareholders and managers) may be averse to risk in practice, it is useful to determine how the optimal regulatory policy changes when the firm is averse to risk. To do so, let $u(w)$ denote the utility the firm derives from financial payoff w , where $u'(w) > 0$ and $u''(w) < 0$ for all w . Then, given δ , the firm's expected utility under the non-core project is:

$$U_n(\delta) \equiv p(\delta) u(\underline{r} - \underline{c}) + [1 - p(\delta)] u(\bar{r} - \bar{c}) - \frac{\delta}{\gamma} [p(\delta)]^\gamma. \quad (13)$$

The regulator's problem, [PR], in this setting is:⁴⁶

$$\text{Minimize}_{\underline{r}, \bar{r}, r_0} \int_{\underline{\delta}}^{\delta_n} \{ p(\delta) \underline{r} + [1 - p(\delta)] \bar{r} \} dG(\delta) + [1 - G(\delta_n)] r_0$$

subject to: $U_n(\delta) \geq \text{maximum} \{ 0, u(r_0 - c_0) \}$ for all $\delta \in [\underline{\delta}, \delta_n]$, and

$$u(r_0 - c_0) \geq \text{maximum} \{ 0, U_n(\delta) \} \text{ for all } \delta \in (\delta_n, \bar{\delta}].$$

Table 4 summarizes key elements of the solution to [PR] and the associated industry outcomes as the firm's aversion to risk changes in the baseline setting.⁴⁷ The table considers the case where $u(w) = \frac{1}{\alpha} w^\alpha$, so $\alpha \in (0, 1]$ represents the magnitude of the firm's constant relative risk aversion.

Four elements of Table 4 and its counterparts in Appendix A warrant emphasis. First, expected procurement cost increases as the firm becomes more averse to risk. This is the case because as the firm becomes more risk averse, it requires a higher expected payment to compensate for the payment variation it faces.

Second, holding the firm's aversion to risk (α) constant, the regulator generally imposes

⁴⁵In essence, when her ability to structure her primary "stick" (reducing $\bar{r}$) is impeded, the regulator employs a "carrot" ($r_0 > c_0$) to induce the utility to undertake the desired behavior.

⁴⁶The formulation of [PR] parallels the formulation of [P].

⁴⁷ $E\{U\} \equiv \int_{\underline{\delta}}^{\delta_n} U(\delta) dG(\delta) + [1 - G(\delta_n)] u(r_0 - c_0)$ in Table 4.

less payment variation on the firm (i.e., s declines) as γ increases.⁴⁸ This is the case because as γ increases, it becomes less onerous for the firm to increase p . Consequently, less payment variation is required to induce the firm to deliver any desired level of p .

α	s	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{D\}$	$E\{U\}$	$E\{Cost\}$
1.0	0.463	611	1,175	487	1,570	0.45	92.8	75.6	865.9
0.95	0.407	567	1,190	339	1,113	0.31	44.9	33.6	901.3
0.9	0.404	568	1,194	268	865	0.23	27.3	18.9	927.2
0.8	0.406	578	1,202	172	532	0.13	10.2	5.8	962.5
0.7	0.428	606	1,207	116	336	0.07	3.8	1.6	983.5
0.6	0.443	603	1,219	100	209	0.03	1.0	0.3	995.3

Table 4. Outcomes with a Risk Averse Firm.

Third, when γ is sufficiently high (so the firm's cost of increasing p is sufficiently low), the regulator often imposes less payment variation on the firm as it becomes more averse to risk, i.e., s declines as α declines.⁴⁹ This is the case because when it is not particularly onerous for the firm to increase p , the primary effect of increased risk aversion is the higher level of compensation required to induce the firm to accept any specified level of payment variation. Consequently, the regulator reduces the payment variation she imposes on the firm as it becomes more averse to risk.

Fourth, when γ is sufficiently low (so it is relatively onerous for the firm to increase p), a non-monotonic relationship between s and α can arise.⁵⁰ Specifically, when α is sufficiently close to 1 (so the firm is not too averse to risk), the regulator imposes less payment variation on the firm as it become more averse to risk, i.e., s declines as α declines. In contrast, for smaller values of α (so the firm exhibits more pronounced risk aversion), the regulator imposes more payment variation on the firm as it become more averse to risk, i.e., s increases

⁴⁸See Table 4.1 in Appendix A.

⁴⁹See Table 4.2 in Appendix A.

⁵⁰See Tables 4 and 4.2.

as α declines.

This non-monotonic relationship between s and α reflects two countervailing effects that arise as the firm becomes more averse to risk. A “compensation effect” arises because the firm requires greater compensation for any given payment variation it faces. This effect leads the regulator to reduce the payment variation (s) she imposes on the firm.⁵¹ A “motivation effect” arises because as the firm becomes more averse to risk, the increase in utility associated with success ($\underline{c}$) becomes less pronounced for any given s . Consequently, the firm may labor less diligently to ensure success for any given level of payment variation. This effect can lead the regulator to increase s in order to avoid an undue reduction in p .

The motivation effect tends to outweigh the compensation effect when the firm exhibits relatively pronounced aversion to risk (so α is considerably below 1) and cost-containment effort is relatively onerous for the firm to deliver (so γ is relatively small). Under these circumstances, the firm will reduce p substantially as it becomes more risk averse in the absence of any change in payment variation. The regulator optimally increases s to avoid an unduly large reduction in p .

D. Simple Sharing Policies.

In practice, regulators often have less information about the potential cost savings from DER projects than assumed above. Consequently, regulators may be inclined to employ simple rules of thumb when specifying how the firm’s profit will vary with realized cost when it implements a non-core project. To illustrate, a regulator may award the firm a specified fraction (s_t) of the measured cost reduction that is secured relative to the core project (so $\underline{r} = \underline{c} + s_t [c_0 - \underline{c}]$) and require the firm to bear the same fraction of any corresponding cost increase (so $\bar{r} = \bar{c} - s_t [\bar{c} - c_0]$).

Table 5 contrasts outcomes under such policies with outcomes under the optimal cost-sharing policy in the baseline setting. The first row of data in Table 5 reflects the sharing rule (s), the firm’s expected profit ($E\{\pi\}$) and expected procurement cost ($E\{Cost\}$) at

⁵¹In essence, the regulator reduces the use of an instrument that has become more costly to employ.

the solution to [P]. The subsequent rows present the corresponding data under the specified sharing policy (s_t). Table 5 also records the percentage change in the firm's expected profit ($\% \Delta_\pi$) and in expected procurement cost ($\% \Delta_C$) from implementing the identified sharing policy rather than the optimal policy.

s, s_t	$E\{\pi\}$	$\% \Delta_\pi$	$E\{Cost\}$	$\% \Delta_C$
0.463	75.65	—	865.90	—
0.2	6.60	— 912.8	948.53	9.5
0.3	19.06	— 748.1	919.80	6.2
0.4	38.02	— 49.7	904.09	4.4
0.5	63.49	— 16.1	901.39	4.1
0.6	95.47	26.2	911.69	5.3
0.7	133.95	77.1	935.01	8.0
0.8	178.93	136.5	971.33	12.2

Table 5. The Effects of Simple Sharing Policies.

Table 5 indicates that expected procurement cost increases moderately and the firm's expected profit varies substantially as the simple sharing rule (s_t) diverges from the optimal rule (s).⁵² When $s_t < s$, procurement cost increases because the firm implements a relatively small value of p . When $s_t > s$, the firm sets a relatively high value of p . However, expected procurement cost increases because consumers capture relatively little of any realized cost reduction.

The firm can incur negative profit under simple sharing policies when the high cost ($\bar{c}$) is realized under the non-core project. If financial losses are deemed to be unacceptable, a regulator might: (i) award the firm a fraction (s_m) of the cost savings secured under the non-core project (so $\underline{r} = \underline{c} + s_m [c_0 - \underline{c}]$); but (ii) not require the firm to bear any associated cost increase (so $\bar{r} = \bar{c}$). Table 6 compares the outcomes in this setting with the outcomes

⁵²This same pattern holds for a wide range of settings other than the baseline setting.

at the solution to $[Pm]$ when $m = 0$, where the regulator is constrained to set $\bar{r} = \bar{c}$ but can choose the sharing rule to minimize expected procurement cost.⁵³ (Recall Table 3.)

s, s_m	$E\{\pi\}$	$\%\Delta_\pi$	$E\{Cost\}$	$\%\Delta_C$
0.339	90.39	—	904.56	—
0.2	18.66	− 79.4	1,047.17	15.8
0.3	40.65	− 55.0	991.31	9.6
0.4	69.42	− 23.2	959.51	6.1
0.5	104.07	15.1	946.52	4.6
0.6	143.93	59.2	948.93	4.9
0.7	188.41	108.4	964.26	6.6
0.8	237.07	162.3	990.62	9.5

Table 6. The Effects of Simple Gain-Sharing Policies.

As Table 6 illustrates, protecting the firm against financial losses can limit the profit reduction that stringent rules of thumb (i.e., relatively low values of s_t and s_m) impose on the firm. Such protection can also increase the relatively large profit that generous rules of thumb can award the firm. Table 6 also illustrates that stringent rules of thumb and protection against financial losses together can cause expected procurement cost to exceed the cost of traditional infrastructure investment.⁵⁴

6 Conclusions.

We have analyzed the design of simple cost-sharing policies to motivate a utility to identify promising DER projects and implement the projects at minimum cost. We found that the optimal cost-sharing policy generally is sensitive to many elements of the environment in which it is implemented. Consequently, it typically is not possible to provide a concise,

⁵³Even when $s_m = s$, the outcomes in this setting do not coincide with the outcomes at the solution to $[Pm]$ with $m = 0$ because $\underline{r} = \underline{c} + s_m [c_0 - \underline{c}]$ in the former setting whereas $\underline{r} = \underline{c} + s [\bar{c} - \underline{c}]$ at the solution to $[Pm]$.

⁵⁴Observe that expected procurement cost is $1,047.17 > 1,000 = c_0$ when $s_m = 0.2$ in Table 6.

simple characterization of the optimal cost-sharing rule. However, our analysis generated some qualitative conclusions that may help regulators as they design cost-sharing policies to reflect the idiosyncratic environments in which they operate.

For instance, we found that the share of the realized cost reduction (s) that is optimally awarded to the utility often declines as the information asymmetry under which the regulator operates becomes more pronounced.⁵⁵ In contrast, and perhaps more surprisingly, the optimal s tends to increase as it becomes less onerous for the utility to manage the costs of a DER project. The increase in s complements the firm's increased ability to reduce costs, thereby reducing expected procurement cost.

We also found that when the expected cost of a DER project is well below the cost of traditional infrastructure investment (so the utility is optimally induced to always undertake the DER project) and when the risk-neutral utility has relatively limited ability to reduce the cost of the DER project (so $\gamma \in (1, 2]$), the optimal s often exceeds $\frac{1}{2}$. However, the optimal s can be less than $\frac{1}{2}$ more generally, and often is when the cost advantage of the DER project is less pronounced.

In addition, we demonstrated that the optimal cost-sharing policy varies with the regulator's ability to penalize the utility for failing to manage costs effectively. When this ability is limited, the optimal value of s can decline substantially, particularly when the utility is relatively adept at securing cost reductions. The optimal value of s also often declines as the utility becomes more averse to risk. However, s can increase as the utility's aversion to risk increases in order to counteract the tendency of increased risk aversion to diminish the firm's cost-reducing effort.

Because the optimal cost-sharing rule varies with many elements of the prevailing environment, implementing policies that reflect simple rules of thumb can cause procurement cost to increase considerably and can introduce particularly pronounced variation in the utility's

⁵⁵Expected procurement cost can increase considerably as the information asymmetry the regulator faces becomes more pronounced. Consequently, activities that reduce information asymmetry, including the development of sophisticated engineering models of electricity markets (e.g., Perez-Arriaga and Knittel, 2016; Jenkins and Perez-Arriaga, 2017), have the potential to reduce procurement costs substantially.

earnings.⁵⁶ Consequently, it is important for regulators to carefully tailor the reward structures they implement to the prevailing environment. However, regulators may not increase procurement cost substantially if they limit themselves to a single cost-sharing plan. This conclusion reflects our finding that allowing the utility to select a cost-sharing policy from a continuum of such policies often does not reduce procurement cost substantially below the level achieved with a single optimally-designed sharing policy.⁵⁷

Future research should expand our streamlined model to consider more general cost structures, more severe information asymmetries, and stochastic output under DER projects.⁵⁸ Future research should also consider intertemporal environments in which the regulator's information improves over time and is affected by observing the utility's performance. The utility may act strategically in such settings in order to influence the regulator's information and future regulatory policy.⁵⁹

⁵⁶Cost-sharing policies that reflect simple rules of thumb can cause expected procurement cost to rise above the cost of standard infrastructure investment. (Recall Table 6.)

⁵⁷However, the implementation of a continuum of cost-sharing policies can reduce the utility's profit significantly.

⁵⁸Future research should also consider the interaction between DER procurement and the operation of wholesale electricity markets, particularly in light of the recent mandate by the U.S. Federal Energy Regulatory Commission (FERC, 2018) to employ storage resources in wholesale markets.

⁵⁹See Laffont and Tirole (1993, chapter 9), for instance.

Appendix A

This Appendix presents additional numerical solutions.

Extensions of Table 1.

Tables 1.1 and 1.2 correspond to Table 1 in the text. These tables record elements of the solution to [P] as parameters vary from their baseline levels. The first column identifies the variation in the relevant parameter. All other parameter values are as specified in the baseline setting.

	s	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{D\}$	$E\{\pi\}$	$E\{Cost\}$
Baseline	0.463	611	1,175	487	1,570	0.45	92.8	75.6	865.9
$\bar{\delta} = 1,102.5$	0.535	618	1,107	561	1,103	0.84	182.2	145.4	687.5
$\bar{\delta} = 4,410$	0.463	611	1,175	487	1,570	0.22	45.3	36.9	934.5
$\underline{\delta} = 50$	0.449	599	1,178	471	1,535	0.45	86.3	77.6	859.8
$\underline{\delta} = 200$	0.493	636	1,168	518	1,639	0.46	105.5	72.0	878.0
$\bar{c} = 625$	0.547	420	613	232	2,205	0.31	34.1	26.0	552.8
$\bar{c} = 2,500$	0.306	633	2,229	703	913	0.37	88.3	70.1	877.9
$\underline{c} = 100$	0.464	558	1,174	534	1,885	0.53	118.0	98.6	823.8
$\underline{c} = 400$	0.462	718	1,175	393	1,027	0.32	52.3	39.6	931.4

Table 1.1. Elements of the Optimal Regulatory Policy when $\gamma = 2$.

	s	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{D\}$	$E\{\pi\}$	$E\{Cost\}$
Baseline	0.524	567	1,067	550	2,205	0.74	119.2	103.3	698.1
$\bar{\delta} = 1,102.5$	0.534	494	983	561	1,103	0.91	134.7	108.7	538.4
$\bar{\delta} = 4,410$	0.517	616	1,123	543	4,410	0.57	95.3	86.4	834.5
$\underline{\delta} = 50$	0.512	561	1,073	537	2,205	0.74	113.8	105.8	695.0
$\underline{\delta} = 200$	0.549	580	1,054	576	2,205	0.74	129.8	98.6	704.3
$\bar{c} = 625$	0.538	380	576	229	2,205	0.52	38.2	31.3	474.1
$\bar{c} = 2,500$	0.359	618	2,092	826	1,505	0.62	128.8	110.5	815.8
$\underline{c} = 100$	0.523	492	1,041	601	2,205	0.76	132.6	115.5	623.2
$\underline{c} = 400$	0.527	713	1,116	447	2,205	0.68	92.4	79.1	840.4

Table 1.2. Elements of the Optimal Regulatory Policy when $\gamma = 3$.

Extensions of Table 2.

Tables 2.1 and 2.2 correspond to Table 2 in the text. Table 2.1 compares expected procurement cost ($E\{Cost\}$) and the firm's expected profit ($E\{\pi\}$) at the solution to [P] (where the regulator offers a single reward structure, i.e., a "Single Contract") and at the solution to [PM] (where the regulator offers a continuum of reward structures, i.e., "Multiple Contracts"). The last column in the table reports $|\% \Delta_C|$, the percentage reduction in expected procurement cost from employing a continuum of reward structures rather than a single structure. The sixth column reports $|\% \Delta_\pi|$, the corresponding percentage reduction in the firm's expected profit. Table 2.1 allows model parameters to vary from their baseline levels. The first column identifies the variation in the relevant parameter. All other parameter values are as specified in the baseline setting.

	Single Contract		Multiple Contracts			
	$E\{\pi\}$	$E\{Cost\}$	$E\{\pi\}$	$E\{Cost\}$	$ \% \Delta_\pi $	$ \% \Delta_C $
Baseline	75.6	865.9	72.1	855.8	4.6	1.2
$\gamma = 3$	103.3	698.1	103.2	698.1	0.1	0.0
$\bar{\delta} = 2,645$	62.6	889.1	59.6	880.7	4.8	0.9
$\underline{\delta} = 200$	72.0	878.0	66.7	866.7	7.4	1.3
$\bar{c} = 1,350$	72.5	872.9	69.0	861.9	4.8	1.3
$\underline{c} = 300$	56.0	901.7	53.2	893.6	5.0	0.9

Table 2.1. The Effects of Offering a Menu of Reward Structures.

Table 2.2 provides the same data as in Table 2 for a setting where $c_0 = 1,000$, $\underline{c} = 500$, $\bar{c} = 1,100$, $\gamma = 2$, $\underline{\delta} = 600$, and $\bar{\delta} = 2,205$. In this setting, $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$ at the solution to [PM]. At the solution to [P] in this setting, expected procurement cost ($E\{Cost\}$) is 982.3 and the firm's expected profit ($E\{\pi\}$) is 16.9.

	s	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{D\}$	$E\{\pi\}$	$E\{Cost\}$
Mean	0.775	865	1,000	600	1,200	0.21	49.5	12.1	975.8
Std. Dev.	0.09	20.83	33.33	0	0	0.29	73.8	22.6	33.8
Minimum	0.6667	833	900	600	1,200	0.0	0.0	0.0	900.0
Maximum	1.0	900	1,033	600	1,200	1.0	300.0	100.0	1,000.0

Table 2.2. Outcomes at the Solution to [PM] when $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$.

Extensions of Table 3.

Tables 3.1 and 3.2 correspond to Table 3 in the text. These tables record elements of the solution to $[Pm]$ for the identified value of m as parameters vary from their baseline levels. The first column identifies the variation in the relevant parameter. All other parameter values are as specified in the baseline setting. The “—” in Table 3.1 indicates that the firm always implements the non-core project at the solution to $[Pm]$.

	s	r_0	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{\pi\}$	$E\{Cost\}$
Baseline	0.339	1,051	556	1,250	356	1,242	0.33	90.4	904.6
$\gamma = 3$	0.165	—	477	1,875	277	905	0.30	120.1	920.1
$\bar{\delta} = 3,307.5$	0.290	1,042	504	1,250	304	1,092	0.18	60.6	939.4
$\underline{\delta} = 150$	0.353	1,054	571	1,250	371	1,282	0.33	91.4	912.7
$\bar{c} = 1,875$	0.145	1,069	443	1,875	243	432	0.13	76.6	945.5
$\underline{c} = 300$	0.308	1,046	593	1,250	293	941	0.25	69.0	933.0

Table 3.1. Outcomes at the Solution to $[Pm]$ when $m = 0$.

	s	r_0	$\underline{r}$	$\bar{r}$	δ_1	δ_n	$E\{p\}$	$E\{\pi\}$	$E\{Cost\}$
Baseline	0.387	1,000	546	1,190	406	1,377	0.38	52.2	882.9
$\gamma = 3$	0.165	1,000	417	1,815	277	905	0.30	60.1	897.1
$\bar{\delta} = 3,307.5$	0.387	1,000	546	1,190	406	1,377	0.25	34.3	914.7
$\underline{\delta} = 150$	0.387	1,000	546	1,190	406	1,377	0.37	46.6	877.8
$\bar{c} = 1,875$	0.145	1,009	383	1,815	243	432	0.13	16.6	936.0
$\underline{c} = 300$	0.387	1,000	608	1,190	368	1,127	0.32	38.6	904.7

Table 3.2. Outcomes at the Solution to $[Pm]$ when $m = -60$.

Extensions of Table 4.

Tables 4.1 and 4.2 correspond to Table 4 in the text. Table 4.1 records elements of the solution to [PR] as γ varies for selected values of the firm's degree of relative risk aversion, α . All parameter values other than γ are as specified in the baseline setting.

α	γ	s	$E\{p\}$	$E\{U\}$	$E\{Cost\}$
0.9	1.5	0.622	0.29	36.7	930.3
	2.0	0.404	0.23	18.9	927.2
	2.5	0.340	0.35	23.3	891.1
	3.0	0.289	0.49	25.8	831.6
	4.0	0.244	0.59	20.7	707.7
0.8	1.5	0.613	0.14	10.1	963.9
	2.0	0.406	0.13	5.8	962.6
	2.5	0.334	0.20	7.8	940.5
	3.0	0.274	0.33	10.4	906.5
	4.0	0.213	0.50	10.7	787.2
0.7	1.5	0.630	0.05	1.3	992.7
	2.0	0.428	0.07	1.6	983.5
	2.5	0.335	0.11	2.5	970.4
	3.0	0.260	0.19	3.6	949.2
	4.0	0.186	0.42	7.8	863.6

Table 4.1. Outcomes as the Firm's Risk Aversion (α) and Ability (γ) Vary.

Table 4.2 records how the optimal extent of cost sharing (s) changes as the firm's degree of risk aversion (α) changes at the solution to [PR], for selected values of γ . All parameter values other than γ are as specified in the baseline setting. The “—” entry denotes a setting where the firm always undertakes the core project.

γ	α	s	γ	α	s	γ	α	s	γ	α	s
1.5	1.0	0.462	2.0	1.0	0.463	3.0	1.0	0.524	4.0	1.0	0.531
	0.95	0.670		0.95	0.407		0.95	0.304		0.95	0.264
	0.90	0.622		0.90	0.404		0.90	0.289		0.90	0.244
	0.80	0.613		0.80	0.406		0.80	0.274		0.80	0.213
	0.70	0.630		0.70	0.428		0.70	0.260		0.70	0.186
	0.60	—		0.60	0.443		0.60	0.234		0.60	0.162

Table 4.2. The Relationship Between α and s as γ Varies.

Appendix B

This Appendix presents the proofs of the formal conclusions in the text.

Lemma 1. *When δ is realized and the firm implements the non-core project, it implements success probability $p(\delta) = \min \{1, (\frac{s}{\delta} [\bar{c} - \underline{c}])^{\frac{1}{\gamma-1}}\}$, which is non-increasing in δ .*

Proof. When δ is realized and the firm implements the non-core project, it will act to:

$$\underset{p \in [0,1]}{\text{Maximize}} \quad \bar{r} - \bar{c} + p \Delta - \frac{\delta}{\gamma} p^\gamma,$$

where $\Delta \equiv \underline{r} - \underline{c} - (\bar{r} - \bar{c}) = s [\bar{c} - \underline{c}]$. When $p \in (0, 1)$, the firm's choice of p is determined by:

$$\Delta = \delta p^{\gamma-1} \Rightarrow s [\bar{c} - \underline{c}] = \delta p^{\gamma-1} \Rightarrow p(\delta) = \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}}. \quad (14)$$

The conclusion follows from (14) because $p(\cdot)$ cannot exceed 1. ■

Lemma 2. *Expected procurement cost is lower under the non-core project than under the core project for all $\delta \leq \delta_n(s)$, where:*

$$\delta_n(s) \equiv s [1 - s]^{\gamma-1} [\bar{c} - \underline{c}]^\gamma \left[\frac{1}{\bar{r} - r_0} \right]^{\gamma-1}. \quad (15)$$

Proof. (4) and (14) imply that the regulator's expected procurement cost when the firm undertakes the non-core project and $p(\delta) \in (0, 1)$ is:

$$\bar{r} + p(\delta) [\underline{r} - \bar{r}] = \bar{r} + s^{\frac{1}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{1}{\gamma-1}} [s - 1] [\bar{c} - \underline{c}] \leq r_0 \quad (16)$$

$$\Leftrightarrow s^{\frac{1}{\gamma-1}} [s - 1] \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \leq r_0 - \bar{r}. \quad (17)$$

if $r_0 \geq \bar{r}$, then the inequality in (17) holds because $s \in [0, 1]$ under an optimal procurement policy. This is the case because if $s < 0$, the firm will always set $p = 0$ when it undertakes the non-core project, which ensures $c = \bar{c}$, the same cost realization that is ensured when $s = 0$. Consequently, the regulator cannot secure a strict reduction in expected procurement cost by setting $s < 0$. If $s > 1$, then the firm will set $p(\delta) > p^*(\delta)$ for all δ for which it undertakes the non-core project. The regulator can reduce expected procurement cost by setting $s = 1$ and adjusting $\bar{r}$ to ensure the firm secures the same expected profit for the highest δ for which it undertakes the non-core project. Doing so ensures that: (i) the efficient total expected cost of the non-core project is achieved whenever it is undertaken; and (ii) the firm's expected profit is not increased and is not reduced below 0 for any δ realization.

If $r_0 < \bar{r}$, then (17) holds if:

$$\begin{aligned} \bar{r} - r_0 &\leq s^{\frac{1}{\gamma-1}} [1-s] \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \Leftrightarrow \delta^{\frac{1}{\gamma-1}} \leq s^{\frac{1}{\gamma-1}} [1-s] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left[\frac{1}{\bar{r} - r_0} \right] \\ \Leftrightarrow \delta &\leq s [1-s]^{\gamma-1} [\bar{c} - \underline{c}]^{\gamma} \left[\frac{1}{\bar{r} - r_0} \right]^{\gamma-1} \equiv \delta_n(s). \quad \blacksquare \end{aligned}$$

Lemma 3. $p(\delta) = 1$ for all $\delta \leq \delta_1(s) \equiv s[\bar{c} - \underline{c}]$ when δ is realized and the firm implements the non-core project.

Proof. (14) implies that $p(\delta_1(s)) = 1$. Therefore, the conclusion follows from (14) because:

$$p'(\delta) \stackrel{s}{=} \left[\frac{1}{\gamma-1} \right] \left[\frac{1}{\delta} \right]^{\frac{2-\gamma}{\gamma-1}} \left[-\frac{1}{\delta^2} \right] < 0. \quad \blacksquare$$

Lemma 4. The firm's expected profit when δ is realized, it implements the non-core project, and it sets $p(\delta) = \min \{ 1, \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} \}$ is:

$$\pi(\cdot) = \begin{cases} \bar{r} - \bar{c} + \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] & \text{if } \delta > \delta_1(s), \\ \bar{r} - \bar{c} + s[\bar{c} - \underline{c}] - \frac{\delta}{\gamma} & \text{if } \delta \leq \delta_1(s). \end{cases} \quad (18)$$

Proof. (4), (14), and Lemma 3 imply that the firm's expected profit under the specified conditions when $\delta > \delta_1(s)$ is:

$$\begin{aligned} \pi(\cdot) &= \bar{r} - \bar{c} + p(\delta) \Delta - \frac{\delta}{\gamma} [p(\delta)]^{\gamma} \\ &= \bar{r} - \bar{c} + \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} s [\bar{c} - \underline{c}] - \frac{\delta}{\gamma} \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{\gamma}{\gamma-1}} \\ &= \bar{r} - \bar{c} + \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right]. \end{aligned}$$

Lemma 3 implies that the firm's expected profit under the specified conditions when $\delta \leq \delta_1$ is:

$$\pi(\cdot) = \underline{r} - \underline{c} - \frac{\delta}{\gamma} [1]^{\gamma} = \bar{r} - \bar{c} + s[\bar{c} - \underline{c}] - \frac{\delta}{\gamma}. \quad \blacksquare$$

Lemma 5. The regulator's problem when she induces the firm to always implement the non-core project, [PNC], is:

$$\text{Maximize}_{\bar{r}, s} \quad - \int_{\underline{\delta}}^{\delta_1} \{ \underline{c} + \bar{r} - \bar{c} + s[\bar{c} - \underline{c}] \} dG(\delta)$$

$$\begin{aligned}
& - \int_{\delta_1}^{\bar{\delta}} \left\{ \bar{r} + \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}] \right\} dG(\delta) \\
& \text{subject to: } \pi(\bar{\delta}) = 0.
\end{aligned} \tag{19}$$

Proof. Lemmas 1 and 3 imply that if $s[\bar{c} - \underline{c}] \geq \underline{\delta}$, the regulator's problem is:

$$\begin{aligned}
& \underset{\bar{r}, s}{\text{Minimize}} \quad \int_{\underline{\delta}}^{\delta_1} \underline{r} dG(\delta) + \int_{\delta_1}^{\bar{\delta}} \{ \bar{r} + p(\delta) [\underline{r} - \bar{r}] \} dG(\delta) \\
& \text{subject to: } \pi(\delta) \geq 0 \quad \text{for all } \delta \in [\underline{\delta}, \bar{\delta}].
\end{aligned} \tag{20}$$

From (18):

$$\frac{\partial \pi(\cdot)}{\partial \delta} = \begin{cases} -\frac{1}{\gamma} \left(\frac{1}{\delta} \right)^{\frac{2-\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^2 [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{\gamma}{\gamma-1}} < 0 & \text{if } \delta \in (\delta_1, \bar{\delta}], \\ -\frac{1}{\gamma} < 0 & \text{if } \delta \in [\underline{\delta}, \delta_1]. \end{cases} \tag{21}$$

(21) implies that $\pi(\cdot)$ is monotonically decreasing in δ . Therefore, the constraint in (20) implies that $\pi(\bar{\delta}) = 0$ at the solution to the regulator's problem. Consequently, (4) and (14) imply that the regulator's problem is as specified in [PNC].

If $s[\bar{c} - \underline{c}] < \underline{\delta}$, then $p(\delta) \in (0, 1)$ at the solution to [PNC], so the regulator's problem is as specified in (19), with $\delta_1 = \underline{\delta}$. ■

Proof of Conclusion 1.

(16) implies that if $p \in (0, 1)$ at the solution to the regulator's problem in the identified setting, then the regulator's problem is to:

$$\underset{s}{\text{Maximize}} \quad - \int_{\underline{\delta}}^{\bar{\delta}} \left\{ \bar{r} + \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}] \right\} dG(\delta).$$

The necessary condition for a solution to this problem is:

$$\begin{aligned}
& \int_{\underline{\delta}}^{\bar{\delta}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left[(s)^{\frac{1}{\gamma-1}} + [s-1] \left(\frac{1}{\gamma-1} \right) (s)^{\frac{2-\gamma}{\gamma-1}} \right] dG(\delta) = 0 \\
& \Rightarrow (s)^{\frac{1}{\gamma-1}} + [s-1] \left(\frac{1}{\gamma-1} \right) (s)^{\frac{2-\gamma}{\gamma-1}} = 0 \Rightarrow s + [s-1] \left[\frac{1}{\gamma-1} \right] = 0 \\
& \Rightarrow -s[\gamma-1] = s-1 \Rightarrow s\gamma = 1 \Rightarrow s = \frac{1}{\gamma}.
\end{aligned}$$

Lemma 1 implies that when $s = \frac{1}{\gamma}$:

$$p(\delta) = \min \left\{ 1, \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} \right\} = \min \left\{ 1, \left(\frac{1}{\gamma\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} \right\}$$

$$= \left(\frac{1}{\gamma \delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} \in (0, 1).$$

This conclusion holds because:

$$\underline{\delta} > \frac{1}{\gamma} [\bar{c} - \underline{c}] \Rightarrow \frac{1}{\gamma \underline{\delta}} [\bar{c} - \underline{c}] < 1 \Rightarrow \frac{1}{\gamma \delta} [\bar{c} - \underline{c}] < 1 \text{ for all } \delta \in [\underline{\delta}, \bar{\delta}]. \quad \blacksquare$$

Proof of Conclusion 3.

Let λ_N denote the Lagrange multiplier associated with constraint (20). Then (4) and (14) imply that the Lagrangian function associated with [PNC] is:

$$\begin{aligned} \mathcal{L} = & - \int_{\underline{\delta}}^{\delta_1} \{ \underline{c} + \bar{r} - \bar{c} + s [\bar{c} - \underline{c}] \} dG(\delta) \\ & - \int_{\delta_1}^{\bar{\delta}} \left\{ \bar{r} + \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}] \right\} dG(\delta) + \lambda_N \pi(\bar{\delta}). \end{aligned} \quad (22)$$

The necessary conditions for a solution to [PNC] include:

$$\bar{r} : -G(\delta_1) - [1 - G(\delta_1)] + \lambda_N \frac{\partial \pi(\bar{\delta})}{\partial \bar{r}} = 0; \quad (23)$$

$$\begin{aligned} s : & \frac{\partial \delta_1}{\partial s} g(\delta_1) \left\{ \bar{r} + \left(\frac{s}{\delta_1} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}] - (\underline{c} + \bar{r} - \bar{c} + s [\bar{c} - \underline{c}]) \right\} \\ & + \lambda_N \frac{\partial \pi(\bar{\delta})}{\partial s} - [\bar{c} - \underline{c}] G(\delta_1) \\ & - \int_{\delta_1}^{\bar{\delta}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left[(s)^{\frac{1}{\gamma-1}} + [s-1] \left(\frac{1}{\gamma-1} \right) (s)^{\frac{2-\gamma}{\gamma-1}} \right] dG(\delta) = 0. \end{aligned} \quad (24)$$

Because $\delta_1 < \bar{\delta}$, (18) implies:

$$\frac{\partial \pi(\bar{\delta})}{\partial \bar{r}} = 1 \quad \text{and} \quad \frac{\partial \pi(\bar{\delta})}{\partial s} = \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{1}{\gamma-1}}. \quad (25)$$

(25) implies that (23) can be written as:

$$\lambda_N \frac{\partial \pi(\bar{\delta})}{\partial \bar{r}} = 1 \Leftrightarrow \lambda_N = 1. \quad (26)$$

Because $\delta_1(s) = s [\bar{c} - \underline{c}]$:

$$\begin{aligned} & \bar{r} + \left(\frac{s}{\delta_1} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}] - (\underline{c} + \bar{r} - \bar{c} + s [\bar{c} - \underline{c}]) \\ & = \bar{r} + [s-1] [\bar{c} - \underline{c}] - (\underline{c} + \bar{r} - \bar{c} + s [\bar{c} - \underline{c}]) \end{aligned}$$

$$= [s - 1][\bar{c} - \underline{c}] - (\underline{c} - \bar{c} + s[\bar{c} - \underline{c}]) = s[\bar{c} - \underline{c}] - s[\bar{c} - \underline{c}] = 0. \quad (27)$$

(25), (26), and (27) imply that (24) can be written as:

$$\begin{aligned} & \lambda_N \frac{\partial \pi(\bar{\delta})}{\partial s} - [\bar{c} - \underline{c}] G(\delta_1) \\ & - \int_{\delta_1}^{\bar{\delta}} \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left[(s)^{\frac{1}{\gamma-1}} + [s - 1] \left(\frac{1}{\gamma - 1} \right) (s)^{\frac{2-\gamma}{\gamma-1}} \right] dG(\delta) = 0 \\ \Leftrightarrow & \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} - [\bar{c} - \underline{c}] G(\delta_1) \\ & - [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} \left[1 + [s - 1] \left(\frac{1}{\gamma - 1} \right) (s)^{-1} \right] \int_{\delta_1}^{\bar{\delta}} \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} dG(\delta) = 0 \\ \Leftrightarrow & [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} \left[\frac{s(\gamma - 1) + s - 1}{s(\gamma - 1)} \right] \int_{\delta_1}^{\bar{\delta}} \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} dG(\delta) \\ & = \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} - [\bar{c} - \underline{c}] G(\delta_1), \end{aligned}$$

which implies that (6) holds.

Finally, observe that (18), (19), and (26) imply:

$$\bar{r} = \bar{c} - \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma - 1}{\gamma} \right]. \quad \blacksquare$$

Proof of Conclusion 2.

$\delta_1 = \underline{\delta}$ under the specified conditions. Therefore, $G(\delta_1) = 0$, so (6) implies:

$$\left[\frac{s\gamma - 1}{s(\gamma - 1)} \right] \int_{\underline{\delta}}^{\bar{\delta}} \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}} dG(\delta) = \left(\frac{1}{\bar{\delta}} \right)^{\frac{1}{\gamma-1}},$$

which implies that (5) holds.

Define $LHS \equiv \frac{s[\gamma-1]}{s\gamma-1}$ and $RHS \equiv \int_{\underline{\delta}}^{\bar{\delta}} \left(\frac{\bar{\delta}}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) > 0$. (5) implies that $LHS > 0$ because $RHS > 0$. Therefore, $s\gamma - 1 > 0$, which implies $s > \frac{1}{\gamma}$. Furthermore:

$$\frac{d}{ds}(LHS) \stackrel{s}{=} [s\gamma - 1][\gamma - 1] - \gamma s[\gamma - 1] = 1 - \gamma < 0; \quad (28)$$

$$\frac{d}{d\underline{c}}(LHS) = \frac{d}{d\bar{c}}(LHS) = \frac{d}{d\underline{\delta}}(LHS) = \frac{d}{d\bar{\delta}}(LHS) = 0; \quad (29)$$

$$\frac{d}{d\underline{\delta}}(RHS) = - \left(\frac{\bar{\delta}}{\underline{\delta}} \right)^{\frac{1}{\gamma-1}} g(\underline{\delta}) < 0; \quad (30)$$

$$\frac{d}{d\bar{\delta}}(RHS) = \frac{1}{\gamma-1} \int_{\underline{\delta}}^{\bar{\delta}} \left(\frac{\bar{\delta}}{\delta} \right)^{\frac{2-\gamma}{\gamma-1}} \frac{1}{\delta} dG(\delta) + g(\bar{\delta}) > 0; \quad (31)$$

$$\frac{d}{d\underline{c}}(RHS) = \frac{d}{d\bar{c}}(RHS) = \frac{d}{ds}(RHS) = 0. \quad (32)$$

(5), (28), (29), and (30) imply that $\frac{ds}{d\underline{\delta}} > 0$. (5), (28), (29), and (31) imply that $\frac{ds}{d\bar{\delta}} < 0$. (5), (28), (29), and (32) imply that $\frac{ds}{d\underline{c}} = \frac{ds}{d\bar{c}} = 0$. ■

Lemma 6. *The regulator's problem, [P], is:*

$$\begin{aligned} \underset{\bar{r}, r_0, s}{\text{Maximize}} \quad & - \int_{\underline{\delta}}^{\delta_1} \{ \underline{c} + \bar{r} - \bar{c} + s [\bar{c} - \underline{c}] \} dG(\delta) \\ & - \int_{\delta_1}^{\delta_n} \left\{ \bar{r} + \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}] \right\} dG(\delta) - [1 - G(\delta_n)] r_0 \\ \text{subject to} \quad & r_0 \geq c_0, \quad \text{and} \end{aligned} \quad (33)$$

$$\pi(\delta_n) = r_0 - c_0. \quad (34)$$

Proof. Lemmas 1 – 4 imply that if $s [\bar{c} - \underline{c}] \geq \underline{\delta}$, then the regulator's problem is:

$$\begin{aligned} \underset{\bar{r}, r_0, s}{\text{Minimize}} \quad & \int_{\underline{\delta}}^{\delta_1} \underline{r} dG(\delta) + \int_{\delta_1}^{\delta_n} \{ \bar{r} + p(\delta) [\underline{r} - \bar{r}] \} dG(\delta) + \int_{\delta_n}^{\bar{\delta}} r_0 dG(\delta) \\ \text{subject to:} \quad & r_0 \geq c_0, \\ & \pi(\delta) \geq r_0 - c_0 \quad \text{for all } \delta \leq \delta_n, \text{ and} \\ & r_0 - c_0 \geq \pi(\delta) \quad \text{for all } \delta > \delta_n. \end{aligned} \quad (35)$$

From (18):

$$\frac{\partial \pi(\cdot)}{\partial \delta} = \begin{cases} -\frac{1}{\gamma} \left(\frac{1}{\delta} \right)^{\frac{2-\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^2 [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} (s)^{\frac{\gamma}{\gamma-1}} < 0 & \text{if } \delta \in (\delta_1, \delta_n], \\ -\frac{1}{\gamma} < 0 & \text{if } \delta \in [\underline{\delta}, \delta_1]. \end{cases} \quad (36)$$

(36) implies that $\pi(\cdot)$ is monotonically decreasing in δ . Therefore, the constraints in (35)

imply that $r_0 - c_0 = \pi(\delta_n)$ at the solution to the regulator's problem. Consequently, (4) and (14) imply that the regulator's problem is as specified in [P].

If $s[\bar{c} - \underline{c}] < \underline{\delta}$, then $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \delta_n]$ at the solution to [P], so the regulator's problem is as specified, with $\delta_1 = \underline{\delta}$. ■

Proof of Conclusion 4.

Let λ_0 and λ_n denote the Lagrange multipliers associated with constraints (33) and (34), respectively. Then the Lagrangian function associated with [P] is:

$$\begin{aligned} \mathcal{L} = & - \int_{\underline{\delta}}^{\delta_1} \{ \underline{c} + \bar{r} - \bar{c} + s[\bar{c} - \underline{c}] \} dG(\delta) \\ & - \int_{\delta_1}^{\delta_n} \left\{ \bar{r} + \left(\frac{s}{\delta} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1][\bar{c} - \underline{c}] \right\} dG(\delta) \\ & - [1 - G(\delta_n)] r_0 + \lambda_0 [r_0 - c_0] + \lambda_n [r_0 - c_0 - \pi(\delta_n)]. \end{aligned} \quad (37)$$

$\delta_1 = \delta_1(s)$ and $\delta_n = \delta_n(\bar{r}, s, r_0)$, from Lemmas 2 and 3. Therefore, the necessary conditions for a solution to [P] include:

$$\begin{aligned} r_0 : & - \frac{\partial \delta_n}{\partial r_0} g(\delta_n) \left[\bar{r} + \left(\frac{s}{\delta_n} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1][\bar{c} - \underline{c}] - r_0 \right] - [1 - G(\delta_n)] \\ & + \lambda_0 + \lambda_n - \lambda_n \frac{\partial \pi(\delta_n)}{\partial r_0} = 0; \end{aligned} \quad (38)$$

$$\begin{aligned} \bar{r} : & - \frac{\partial \delta_n}{\partial \bar{r}} g(\delta_n) \left[\bar{r} + \left(\frac{s}{\delta_n} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1][\bar{c} - \underline{c}] - r_0 \right] - G(\delta_1) \\ & - [G(\delta_n) - G(\delta_1)] - \lambda_n \frac{\partial \pi(\delta_n)}{\partial \bar{r}} = 0; \end{aligned} \quad (39)$$

$$\begin{aligned} s : & \frac{\partial \delta_1}{\partial s} g(\delta_1) \left\{ \bar{r} + \left(\frac{s}{\delta_1} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1][\bar{c} - \underline{c}] - (\underline{c} + \bar{r} - \bar{c} + s[\bar{c} - \underline{c}]) \right\} \\ & - \frac{\partial \delta_n}{\partial s} g(\delta_n) \left[\bar{r} + \left(\frac{s}{\delta_n} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1][\bar{c} - \underline{c}] - r_0 \right] \\ & - \lambda_n \frac{\partial \pi(\delta_n)}{\partial s} - [\bar{c} - \underline{c}] G(\delta_1) \end{aligned}$$

$$- \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left[s^{\frac{1}{\gamma-1}} + [s-1] \left(\frac{1}{\gamma-1} \right) (s)^{\frac{2-\gamma}{\gamma-1}} \right] dG(\delta) = 0. \quad (40)$$

(15) implies:

$$\begin{aligned} \bar{r} + \left(\frac{s}{\delta_n} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}] - r_0 &= \bar{r} - r_0 + (\delta_n)^{-\frac{1}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \\ &= \bar{r} - r_0 + \left(s [1-s]^{\gamma-1} [\bar{c} - \underline{c}]^\gamma \left[\frac{1}{\bar{r} - r_0} \right]^{\gamma-1} \right)^{-\frac{1}{\gamma-1}} (s)^{\frac{1}{\gamma-1}} [s-1] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \\ &= \bar{r} - r_0 - s^{-\frac{1}{\gamma-1}} [1-s]^{-1} [\bar{c} - \underline{c}]^{-\frac{\gamma}{\gamma-1}} \left[\frac{1}{\bar{r} - r_0} \right]^{-1} (s)^{\frac{1}{\gamma-1}} [1-s] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \\ &= \bar{r} - r_0 - (\bar{r} - r_0) = 0. \end{aligned} \quad (41)$$

(27) and (41) imply that (38) – (40) can be written as:

$$\lambda_0 + \lambda_n - \lambda_n \frac{\partial \pi(\delta_n)}{\partial r_0} - [1 - G(\delta_n)] = 0; \quad (42)$$

$$- G(\delta_n) - \lambda_n \frac{\partial \pi(\delta_n)}{\partial \bar{r}} = 0; \quad (43)$$

$$\begin{aligned} &- \lambda_n \frac{\partial \pi(\delta_n)}{\partial s} - [\bar{c} - \underline{c}] G(\delta_1) \\ &- \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left[s^{\frac{1}{\gamma-1}} + [s-1] \left(\frac{1}{\gamma-1} \right) (s)^{\frac{2-\gamma}{\gamma-1}} \right] dG(\delta) = 0. \end{aligned} \quad (44)$$

(15) and (18) imply:

$$\begin{aligned} \pi(\delta_n) &= \bar{r} - \bar{c} + \left(\frac{1}{\delta_n} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} s^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] \\ &= \bar{r} - \bar{c} + \left(\frac{[\bar{r} - r_0]^{\gamma-1}}{s [1-s]^{\gamma-1} [\bar{c} - \underline{c}]^\gamma} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} s^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] \\ &= \bar{r} - \bar{c} + \left[\frac{\gamma-1}{\gamma} \right] [\bar{r} - r_0] \left[\frac{s}{1-s} \right]. \end{aligned} \quad (45)$$

From (45):

$$\frac{\partial \pi(\delta_n)}{\partial \bar{r}} = 1 + \left[\frac{\gamma-1}{\gamma} \right] \left[\frac{s}{1-s} \right]; \quad (46)$$

$$\frac{\partial \pi(\delta_n)}{\partial r_0} = - \left[\frac{\gamma-1}{\gamma} \right] \left[\frac{s}{1-s} \right]; \text{ and} \quad (47)$$

$$\frac{\partial \pi(\delta_n)}{\partial s} = \left[\frac{\gamma-1}{\gamma} \right] [\bar{r} - r_0] \left[\frac{1}{1-s} \right]^2. \quad (48)$$

Summing (42) and (43), using (46) – (48), provides:

$$\begin{aligned} 0 &= \lambda_0 + \lambda_n - \lambda_n \frac{\partial \pi(\delta_n)}{\partial r_0} - [1 - G(\delta_n)] - G(\delta_n) - \lambda_n \frac{\partial \pi(\delta_n)}{\partial \bar{r}} \\ &= \lambda_0 + \lambda_n \left[1 - \frac{\partial \pi(\delta_n)}{\partial \bar{r}} - \frac{\partial \pi(\delta_n)}{\partial r_0} \right] - 1 \\ &= \lambda_0 + \lambda_n \left\{ 1 - 1 - \left[\frac{\gamma-1}{\gamma} \right] \left[\frac{s}{1-s} \right] + \left[\frac{\gamma-1}{\gamma} \right] \left[\frac{s}{1-s} \right] \right\} - 1 \\ &= \lambda_0 - 1 \quad \Rightarrow \quad \lambda_0 = 1. \end{aligned}$$

Because $\lambda_0 = 1$, (33) implies that $r_0 = c_0$. Therefore, (34) and (45) imply:

$$\begin{aligned} \bar{r} - \bar{c} + \left[\frac{\gamma-1}{\gamma} \right] [\bar{r} - c_0] \left[\frac{s}{1-s} \right] &= 0 \\ \Leftrightarrow \bar{r} \left[1 + \left(\frac{\gamma-1}{\gamma} \right) \left(\frac{s}{1-s} \right) \right] &= \bar{c} + c_0 \left[\frac{\gamma-1}{\gamma} \right] \left[\frac{s}{1-s} \right] \\ \Leftrightarrow \bar{r} \left[\frac{\gamma(1-s) + s(\gamma-1)}{\gamma(1-s)} \right] &= \left[\frac{\bar{c}\gamma(1-s) + c_0 s(\gamma-1)}{\gamma(1-s)} \right] \\ \Leftrightarrow \bar{r} &= \frac{\bar{c}\gamma[1-s] + c_0[\gamma-1]s}{\gamma-s}. \end{aligned} \quad (49)$$

(46) implies that (43) can be written as:

$$\begin{aligned} -G(\delta_n) - \lambda_n \left[1 + \left(\frac{\gamma-1}{\gamma} \right) \left(\frac{s}{1-s} \right) \right] &= 0 \\ \Leftrightarrow \lambda_n \left[\frac{\gamma(1-s) + s(\gamma-1)}{\gamma(1-s)} \right] &= -G(\delta_n) \quad \Leftrightarrow \quad \lambda_n = -G(\delta_n) \left[\frac{\gamma(1-s)}{\gamma-s} \right]. \end{aligned} \quad (50)$$

(48), (49), and (50) imply that (44) can be written as:

$$\begin{aligned} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} s^{\frac{1}{\gamma-1}} \left[1 - \left(\frac{1-s}{s} \right) \left(\frac{1}{\gamma-1} \right) \right] \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) \\ = -\lambda_n \frac{\partial \pi(\delta_n)}{\partial s} - [\bar{c} - \underline{c}] G(\delta_1) \end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} s^{\frac{1}{\gamma-1}} \left[\frac{s\gamma - 1}{s(\gamma - 1)} \right] \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) \\
&= G(\delta_n) \left[\frac{\gamma(1-s)}{\gamma-s} \right] [\bar{r} - c_0] \left[\frac{\gamma-1}{\gamma} \right] \left[\frac{1}{1-s} \right]^2 - [\bar{c} - \underline{c}] G(\delta_1) \\
&= G(\delta_n) \left[\frac{\gamma-1}{\gamma} \right] \left[\frac{\bar{c}\gamma(1-s) + c_0(\gamma-1)s - c_0(\gamma-s)}{\gamma-s} \right] \left[\frac{\gamma}{\gamma-s} \right] \left[\frac{1}{1-s} \right] \\
&\quad - [\bar{c} - \underline{c}] G(\delta_1) \\
&= G(\delta_n) \left[\frac{\gamma-1}{\gamma} \right] \left[\frac{(\bar{c} - c_0)\gamma(1-s)}{\gamma-s} \right] \left[\frac{\gamma}{\gamma-s} \right] \left[\frac{1}{1-s} \right] - [\bar{c} - \underline{c}] G(\delta_1) \\
&= G(\delta_n) \left[\frac{\gamma(\gamma-1)}{(\gamma-s)^2} \right] [\bar{c} - c_0] - [\bar{c} - \underline{c}] G(\delta_1). \tag{51}
\end{aligned}$$

(51) implies that (8) holds. ■

Proof of Conclusion 5.

The following table identifies how expected procurement cost ($E\{Cost\}$) varies with $\bar{c}$ at the solution to [P] in two settings. In the first setting (recorded in the second column of the table), $\underline{\delta} = 50$. In the second setting (recorded in the third column of the table), $\bar{\delta} = 1,500$. In both settings, all parameters other than $\bar{c}$, $\underline{\delta}$, and $\bar{\delta}$ are as specified in the baseline setting.

	$E\{Cost\}$	
$\bar{c}$	$\underline{\delta} = 50$	$\bar{\delta} = 1,500$
1,250	845.11	791.34
1,750	860.37	819.12
2,250	859.57	817.81
2,750	857.73	815.18
3,250	856.11	812.92

In both settings, expected procurement cost declines as $\bar{c}$ increases when $\bar{c}$ is sufficiently large. ■

Proof of Conclusion 6.

When the firm's cost parameter is δ and the firm implements the non-core project under sharing rule $s(\hat{\delta}) \in (0, 1]$, the firm will:

$$\text{Maximize}_p \quad \bar{r}(\hat{\delta}) - \bar{c} + p \Delta(\hat{\delta}) - \frac{\delta}{\gamma} p^\gamma, \tag{52}$$

where $\Delta(\hat{\delta}) \equiv \underline{r}(\hat{\delta}) - \underline{c} - (\bar{r}(\hat{\delta}) - \bar{c}) = s(\hat{\delta})[\bar{c} - \underline{c}]$. (10) and (52) imply that if $p \in (0, 1)$, then:

$$\Delta(\widehat{\delta}) = \delta p^{\gamma-1} \Rightarrow s(\widehat{\delta})[\bar{c} - \underline{c}] = \delta p^{\gamma-1} \Rightarrow p(\widehat{\delta}|\delta) = \left(\frac{s(\widehat{\delta})}{\delta}[\bar{c} - \underline{c}]\right)^{\frac{1}{\gamma-1}}. \quad (53)$$

(53) implies that because $p(\cdot) \leq 1$:

$$p(\widehat{\delta}|\delta) = \min \left\{ 1, \left(\frac{s(\widehat{\delta})}{\delta}[\bar{c} - \underline{c}]\right)^{\frac{1}{\gamma-1}} \right\}. \quad (54)$$

(10), (52), and (54) imply that the firm's expected profit when its cost parameter is δ , it reports the parameter to be $\widehat{\delta}$, and it implements the non-core project is:

$$\begin{aligned} \Pi(\widehat{\delta}|\delta) &= \bar{r}(\widehat{\delta}) - \bar{c} + p(\widehat{\delta}|\delta) \Delta(\widehat{\delta}) - \frac{\delta}{\gamma} \left[p(\widehat{\delta}|\delta) \right]^\gamma \\ &= \bar{r}(\widehat{\delta}) - \bar{c} + \min \left\{ 1, \left(\frac{s(\widehat{\delta})}{\delta}[\bar{c} - \underline{c}]\right)^{\frac{1}{\gamma-1}} \right\} s(\widehat{\delta})[\bar{c} - \underline{c}] \\ &\quad - \frac{\delta}{\gamma} \left[\min \left\{ 1, \left(\frac{s(\widehat{\delta})}{\delta}[\bar{c} - \underline{c}]\right)^{\frac{1}{\gamma-1}} \right\} \right]^\gamma \\ &= \begin{cases} \bar{r}(\widehat{\delta}) - \bar{c} + \left(s(\widehat{\delta})[\bar{c} - \underline{c}] \right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] & \text{if } p(\widehat{\delta}|\delta) \in (0, 1) \\ \bar{r}(\widehat{\delta}) - \bar{c} + s(\widehat{\delta})[\bar{c} - \underline{c}] - \frac{\delta}{\gamma} & \text{if } p(\widehat{\delta}|\delta) = 1. \end{cases} \end{aligned} \quad (55)$$

Lemma 6A. $\psi(\delta)(s(\delta))^{\frac{\gamma}{\gamma-1}}$ is non-increasing in δ under any incentive compatible policy in which $p(\delta) \in (0, 1)$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$ for which the firm undertakes the non-core project.

Proof. Incentive compatibility requires $\Pi(\delta|\delta) \geq \Pi(\widehat{\delta}|\delta)$ and $\Pi(\widehat{\delta}|\widehat{\delta}) \geq \Pi(\delta|\widehat{\delta})$ for all $\delta, \widehat{\delta} \in [\underline{\delta}, \bar{\delta}]$. Therefore, (55) implies that if $p(\widehat{\delta}|\delta) \in (0, 1)$, then:

$$\begin{aligned} &\psi(\delta) \left\{ \bar{r}(\delta) - \bar{c} + (s(\delta)[\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] \right\} + [1 - \psi(\delta)][r_0(\delta) - c_0] \\ &\geq \psi(\widehat{\delta}) \left\{ \bar{r}(\widehat{\delta}) - \bar{c} + (s(\widehat{\delta})[\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\widehat{\delta}} \right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] \right\} + [1 - \psi(\widehat{\delta})][r_0(\widehat{\delta}) - c_0] \end{aligned}$$

and

$$\begin{aligned} &\psi(\delta) \left\{ \bar{r}(\delta) - \bar{c} + (s(\delta)[\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\widehat{\delta}} \right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] \right\} + [1 - \psi(\delta)][r_0(\delta) - c_0] \\ &\leq \psi(\widehat{\delta}) \left\{ \bar{r}(\widehat{\delta}) - \bar{c} + (s(\widehat{\delta})[\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\widehat{\delta}} \right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] \right\} + [1 - \psi(\widehat{\delta})][r_0(\widehat{\delta}) - c_0]. \end{aligned}$$

Subtracting the second of these inequalities from the first provides:

$$\begin{aligned}
& \psi(\delta) (s(\delta) [\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma}\right] - \psi(\delta) (s(\delta) [\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\hat{\delta}}\right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma}\right] \\
& \geq \psi(\hat{\delta}) (s(\hat{\delta}) [\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma}\right] - \psi(\hat{\delta}) (s(\hat{\delta}) [\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\hat{\delta}}\right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma}\right] \\
& \Rightarrow \psi(\delta) (s(\delta) [\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left[\left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} - \left(\frac{1}{\hat{\delta}}\right)^{\frac{1}{\gamma-1}} \right] \\
& \geq \psi(\hat{\delta}) (s(\hat{\delta}) [\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left[\left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} - \left(\frac{1}{\hat{\delta}}\right)^{\frac{1}{\gamma-1}} \right] \\
& \Rightarrow \left[\psi(\delta) (s(\delta))^{\frac{\gamma}{\gamma-1}} - \psi(\hat{\delta}) (s(\hat{\delta}))^{\frac{\gamma}{\gamma-1}} \right] \left[\left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} - \left(\frac{1}{\hat{\delta}}\right)^{\frac{1}{\gamma-1}} \right] \geq 0. \tag{56}
\end{aligned}$$

(56) implies $\psi(\delta) (s(\delta))^{\frac{\gamma}{\gamma-1}} \gtrless \psi(\hat{\delta}) (s(\hat{\delta}))^{\frac{\gamma}{\gamma-1}} \Leftrightarrow \hat{\delta} \gtrless \delta$. $\square$

Corollary. $s'(\delta) \leq 0$ for all δ for which the firm implements the non-core project under any incentive compatible policy for which $p(\delta) \in (0, 1)$ for all such δ .

Proof. The conclusion follows directly from Lemma 6A because $\psi(\delta) = \psi(\hat{\delta}) = 1$ for all $\delta, \hat{\delta}$ for which the firm implements the non-core project. $\square$

Lemma 6B. Suppose $p(\delta_b) = \psi(\delta_b) = 1$ for some $\delta_b \in (\underline{\delta}, \bar{\delta}]$ at the solution to [PM]. Then $p(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_b]$ for which $\psi(\delta) = 1$ at the solution to [PM].

Proof. Let π denote the firm's expected profit. (52) implies:

$$\frac{\partial \pi}{\partial p} = s[\bar{c} - \underline{c}] - \delta p^{\gamma-1}. \tag{57}$$

(57) implies $\frac{\partial}{\partial \delta} \left(\frac{\partial \pi}{\partial p} \right) = -p^{\gamma-1} < 0$. (57) and the envelope theorem imply $\frac{d}{ds} \left(\frac{\partial \pi}{\partial p} \right) = \bar{c} - \underline{c} > 0$ if $p \in (0, 1)$. Therefore, $p(\cdot)$ is a non-increasing function of δ and a non-decreasing function of s .

Lemma 6A implies that if $\psi(\delta_a) = 1$ for some $\delta_a \in [\underline{\delta}, \delta_b]$, then $s(\delta_a) \geq s(\delta_b)$ under any incentive compatible policy. Consequently:

$$p(\delta_a | \delta_a) \geq p(\delta_b | \delta_a) \geq p(\delta_b | \delta_b) = 1. \quad \square$$

Lemma 6C. Under an incentive compatible policy, $\psi(\delta)$ is non-increasing in δ for δ in an interval of $[\underline{\delta}, \bar{\delta}]$ in which $p(\delta) = 1$ for all δ in the interval.

Proof. Suppose $p(\delta) = 1$ for all $\delta \in [\delta_L, \delta_H] \subset [\underline{\delta}, \bar{\delta}]$. Then (55) implies that in any incentive compatible policy, for $\delta_a, \delta_b \in [\delta_L, \delta_H]$:

$$\begin{aligned} & \psi(\delta_a) \left\{ \bar{r}(\delta_a) - \bar{c} + s(\delta_a) [\bar{c} - \underline{c}] - \frac{\delta_a}{\gamma} \right\} + [1 - \psi(\delta_a)] [r_0(\delta_a) - c_0] \\ & \geq \psi(\delta_b) \left\{ \bar{r}(\delta_b) - \bar{c} + s(\delta_b) [\bar{c} - \underline{c}] - \frac{\delta_b}{\gamma} \right\} + [1 - \psi(\delta_b)] [r_0(\delta_b) - c_0] \end{aligned}$$

and

$$\begin{aligned} & \psi(\delta_a) \left\{ \bar{r}(\delta_a) - \bar{c} + s(\delta_a) [\bar{c} - \underline{c}] - \frac{\delta_b}{\gamma} \right\} + [1 - \psi(\delta_a)] [r_0(\delta_a) - c_0] \\ & \leq \psi(\delta_b) \left\{ \bar{r}(\delta_b) - \bar{c} + s(\delta_b) [\bar{c} - \underline{c}] - \frac{\delta_b}{\gamma} \right\} + [1 - \psi(\delta_b)] [r_0(\delta_b) - c_0]. \end{aligned}$$

Subtracting the second of these inequalities from the first provides:

$$\psi(\delta_a) \left[\frac{\delta_b - \delta_a}{\gamma} \right] \geq \psi(\delta_b) \left[\frac{\delta_b - \delta_a}{\gamma} \right] \Rightarrow [\psi(\delta_a) - \psi(\delta_b)] [\delta_b - \delta_a] \geq 0. \quad (58)$$

(58) implies that $\psi(\delta)$ is non-increasing in δ for $\delta \in [\delta_L, \delta_H]$. $\square$

Lemma 6B implies that at the solution to [PM]: (i) if there are δ realizations for which $p(\delta) = 1$, these realizations will be in a $[\underline{\delta}, \delta_1)$ interval; whereas (ii) $p(\delta) \in (0, 1)$ for $\delta \in [\delta_1, \bar{\delta}]$. Define $\pi(\delta) \equiv \Pi(\delta|\delta)$. Then (55) and the revelation principle imply that the rate at which the firm's expected profit varies with $\delta \in [\delta_1, \bar{\delta}]$ under an incentive compatible policy is:

$$\pi'(\delta) = -\frac{\psi(\delta)}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \leq 0. \quad (59)$$

Similarly, for $\delta \in [\underline{\delta}, \delta_1)$:

$$\pi'(\delta) = \frac{\partial \Pi(\hat{\delta}|\delta)}{\partial \delta} \Big|_{\hat{\delta}=\delta} = -\frac{\psi(\delta)}{\gamma} \leq 0. \quad (60)$$

(12) and (59) imply that $\pi(\bar{\delta}) = 0$ at the solution to [PM]. Therefore, for $\delta \in [\delta_1, \bar{\delta}]$:

$$\pi(\delta) = \pi(\bar{\delta}) - \int_{\delta}^{\bar{\delta}} \pi'(\delta_a) d\delta_a = \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta}^{\bar{\delta}} \psi(\delta_a) \left(\frac{s(\delta_a)}{\delta_a} \right)^{\frac{\gamma}{\gamma-1}} d\delta_a. \quad (61)$$

(61) and integration by parts imply that for $\delta \in [\delta_1, \bar{\delta}]$:

$$\int_{\delta_1}^{\bar{\delta}} \pi(\delta) dG(\delta) = \pi(\delta) G(\delta) \Big|_{\delta_1}^{\bar{\delta}} + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} G(\delta) d\delta$$

$$\begin{aligned}
&= -\pi(\delta_1) G(\delta_1) + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} G(\delta) d\delta \\
&= -G(\delta_1) \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} d\delta \\
&\quad + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} G(\delta) d\delta \\
&= \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} [G(\delta) - G(\delta_1)] d\delta. \tag{62}
\end{aligned}$$

(60) and (61) imply that for $\delta \in [\underline{\delta}, \delta_1)$:

$$\begin{aligned}
\pi(\delta) &= \pi(\delta_1) - \int_{\delta}^{\delta_1} \pi'(\delta_a) d\delta_a = \pi(\delta_1) + \frac{1}{\gamma} \int_{\delta}^{\delta_1} \psi(\delta) d\delta \\
&= \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} d\delta + \frac{1}{\gamma} \int_{\delta}^{\delta_1} \psi(\delta) d\delta. \tag{63}
\end{aligned}$$

(60), (61), (63), and integration by parts imply:

$$\begin{aligned}
\int_{\underline{\delta}}^{\delta_1} \pi(\delta) dG(\delta) &= \pi(\delta) G(\delta) \Big|_{\underline{\delta}}^{\delta_1} - \int_{\underline{\delta}}^{\delta_1} \left[-\frac{\psi(\delta)}{\gamma} \right] G(\delta) d\delta \\
&= G(\delta_1) \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} d\delta + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \psi(\delta) G(\delta) d\delta. \tag{64}
\end{aligned}$$

From (55):

$$\begin{aligned}
\pi(\delta) &= \psi(\delta) \left\{ \bar{r}(\delta) - \bar{c} + p(\delta) \Delta(\delta) - \frac{\delta}{\gamma} [p(\delta)]^\gamma \right\} + [1 - \psi(\delta)] [r_0(\delta) - c_0] \\
&= \psi(\delta) \left\{ \bar{r}(\delta) - \bar{c} + p(\delta) [\underline{r}(\delta) - \underline{c} - \bar{r}(\delta) + \bar{c}] - \frac{\delta}{\gamma} [p(\delta)]^\gamma \right\} \\
&\quad + [1 - \psi(\delta)] [r_0(\delta) - c_0] \\
&\Rightarrow \psi(\delta) \{ \bar{r}(\delta) + p(\delta) [\underline{r}(\delta) - \bar{r}(\delta)] \} + [1 - \psi(\delta)] r_0(\delta)
\end{aligned}$$

$$= \pi(\delta) + \psi(\delta) \left(\frac{\delta}{\gamma} [p(\delta)]^\gamma + \bar{c} - p(\delta) [\bar{c} - \underline{c}] \right) + [1 - \psi(\delta)] c_0. \quad (65)$$

From (11), the regulator seeks to minimize:

$$\begin{aligned} & \int_{\underline{\delta}}^{\bar{\delta}} \{ \psi(\delta) (\bar{r}(\delta) + p(\delta) [\underline{r}(\delta) - \bar{r}(\delta)]) + [1 - \psi(\delta)] r_0(\delta) \} dG(\delta) \\ &= \int_{\underline{\delta}}^{\bar{\delta}} \left\{ \pi(\delta) + \psi(\delta) \left(\frac{\delta}{\gamma} [p(\delta)]^\gamma + \bar{c} - p(\delta) [\bar{c} - \underline{c}] \right) + [1 - \psi(\delta)] c_0 \right\} dG(\delta) \\ &= \int_{\underline{\delta}}^{\bar{\delta}} \pi(\delta) dG(\delta) + \int_{\underline{\delta}}^{\bar{\delta}} \left\{ \psi(\delta) \left(\frac{\delta}{\gamma} [p(\delta)]^\gamma + \bar{c} - p(\delta) [\bar{c} - \underline{c}] \right) + [1 - \psi(\delta)] c_0 \right\} dG(\delta) \\ &= \int_{\underline{\delta}}^{\delta_1} \pi(\delta) dG(\delta) + \int_{\underline{\delta}}^{\delta_1} \left\{ \psi(\delta) \left[\frac{\delta}{\gamma} + \underline{c} \right] + [1 - \psi(\delta)] c_0 \right\} dG(\delta) \\ &\quad + \int_{\delta_1}^{\bar{\delta}} \pi(\delta) dG(\delta) + \int_{\delta_1}^{\bar{\delta}} \left\{ \psi(\delta) \left(\frac{\delta}{\gamma} [p(\delta)]^\gamma + \bar{c} - p(\delta) [\bar{c} - \underline{c}] \right) + [1 - \psi(\delta)] c_0 \right\} dG(\delta) \\ &= G(\delta_1) \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} d\delta + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \psi(\delta) G(\delta) d\delta \\ &\quad + \int_{\underline{\delta}}^{\delta_1} \left\{ \psi(\delta) \left[\frac{\delta}{\gamma} + \underline{c} \right] + [1 - \psi(\delta)] c_0 \right\} dG(\delta) \\ &\quad + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} [G(\delta) - G(\delta_1)] d\delta \\ &\quad + \int_{\delta_1}^{\bar{\delta}} \left\{ \psi(\delta) \left(\frac{\delta}{\gamma} [p(\delta)]^\gamma + \bar{c} - p(\delta) [\bar{c} - \underline{c}] \right) + [1 - \psi(\delta)] c_0 \right\} dG(\delta) \quad (66) \\ &= G(\delta_1) \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \frac{\psi(\delta)}{g(\delta)} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} dG(\delta) + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \psi(\delta) \frac{G(\delta)}{g(\delta)} dG(\delta) \end{aligned}$$

$$\begin{aligned}
& + \int_{\underline{\delta}}^{\delta_1} \psi(\delta) \left[\frac{\delta}{\gamma} + \underline{c} \right] dG(\delta) + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \left[\frac{G(\delta) - G(\delta_1)}{g(\delta)} \right] dG(\delta) \\
& + \int_{\delta_1}^{\bar{\delta}} \psi(\delta) \left(\frac{\delta}{\gamma} [p(\delta)]^\gamma + \bar{c} - p(\delta) [\bar{c} - \underline{c}] \right) dG(\delta) + c_0 \int_{\underline{\delta}}^{\bar{\delta}} [1 - \psi(\delta)] dG(\delta). \tag{67}
\end{aligned}$$

The first equality in (66) reflects (65). The fourth equality in (66) reflects (62) and (64).

Pointwise minimization of (67) with respect to $s(\delta)$ implies that for those $\delta \in [\delta_1, \bar{\delta}]$ for which $\psi(\delta) = 1$:

$$\begin{aligned}
& \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \frac{G(\delta_1)}{g(\delta)} \left[\frac{\gamma}{\gamma-1} \right] [s(\delta)]^{\frac{1}{\gamma-1}} + [\delta [p(\delta)]^{\gamma-1} - (\bar{c} - \underline{c})] \frac{dp(\delta)}{ds(\delta)} \\
& + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \left[\frac{G(\delta) - G(\delta_1)}{g(\delta)} \right] \left[\frac{\gamma}{\gamma-1} \right] [s(\delta)]^{\frac{1}{\gamma-1}} = 0 \\
\Rightarrow & \left[\delta \left(\frac{s(\delta)}{\delta} [\bar{c} - \underline{c}] \right) - (\bar{c} - \underline{c}) \right] \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{1}{\gamma-1}} \left[\frac{1}{\gamma-1} \right] [s(\delta)]^{\frac{1}{\gamma-1}-1} \\
& + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \frac{G(\delta)}{g(\delta)} \left[\frac{\gamma}{\gamma-1} \right] [s(\delta)]^{\frac{1}{\gamma-1}} = 0 \tag{68} \\
\Rightarrow & [s(\delta) - 1] [\bar{c} - \underline{c}] \left[\frac{1}{\gamma-1} \right] \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{1}{\gamma-1}} [s(\delta)]^{\frac{1}{\gamma-1}-1} \\
& + \left[\frac{1}{\gamma-1} \right] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \frac{G(\delta)}{g(\delta)} [s(\delta)]^{\frac{1}{\gamma-1}} = 0 \\
\Rightarrow & [s(\delta) - 1] \left(\frac{1}{\delta} \right)^{\frac{\gamma}{\gamma-1}-1} [s(\delta)]^{\frac{1}{\gamma-1}-1} + \left(\frac{1}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \frac{G(\delta)}{g(\delta)} [s(\delta)]^{\frac{1}{\gamma-1}} = 0 \\
\Rightarrow & [s(\delta) - 1] \left(\frac{1}{\delta} \right)^{-1} [s(\delta)]^{-1} + \frac{G(\delta)}{g(\delta)} = 0 \\
\Rightarrow & 1 - \frac{1}{s(\delta)} = -\frac{1}{\delta} \left[\frac{G(\delta)}{g(\delta)} \right] \Rightarrow \frac{1}{s(\delta)} = 1 + \frac{1}{\delta} \left[\frac{G(\delta)}{g(\delta)} \right] \\
\Rightarrow & s(\delta) = \frac{1}{1 + \frac{1}{\delta} \frac{G(\delta)}{g(\delta)}} = \frac{\delta}{\delta + \frac{G(\delta)}{g(\delta)}} = h(\delta). \tag{69}
\end{aligned}$$

(68) reflects (54). (69) implies:

$$s'(\delta) \stackrel{s}{=} \delta + \frac{G(\delta)}{g(\delta)} - \delta \left[1 + \frac{d}{d\delta} \left(\frac{G(\delta)}{g(\delta)} \right) \right] = \frac{G(\delta)}{g(\delta)} - \delta \frac{d}{d\delta} \left(\frac{G(\delta)}{g(\delta)} \right) \leq 0. \quad (70)$$

(70) implies that $\frac{s(\delta)}{\delta}$ is non-increasing in δ . Therefore, (54) and (69) imply:

$$\begin{aligned} \left(\frac{s(\delta_1)}{\delta_1} [\bar{c} - \underline{c}] \right)^{\frac{1}{\gamma-1}} &= 1 \Rightarrow s(\delta_1) = \frac{\delta_1}{\bar{c} - \underline{c}} \\ \Rightarrow h(\delta_1) &= \frac{\delta_1}{\bar{c} - \underline{c}} \Rightarrow \frac{\delta_1}{h(\delta_1)} = \bar{c} - \underline{c}. \end{aligned} \quad (71)$$

Pointwise minimization of (67) with respect to $\psi(\delta)$ implies that for $\delta \in [\delta_1, \bar{\delta}]$:

$$\psi(\delta) = \begin{cases} 1 & \text{if } H_1(\delta) < 0 \\ 0 & \text{if } H_1(\delta) > 0, \end{cases}$$

where:

$$\begin{aligned} H_1(\delta) &\equiv \frac{G(\delta_1)}{g(\delta)} \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} + \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \left[\frac{G(\delta) - G(\delta_1)}{g(\delta)} \right] \\ &\quad + \frac{\delta}{\gamma} [p(\delta)]^\gamma + \bar{c} - p(\delta) [\bar{c} - \underline{c}] - c_0 \\ &= \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \frac{G(\delta)}{g(\delta)} + \frac{\delta}{\gamma} [p(\delta)]^\gamma + \bar{c} - p(\delta) [\bar{c} - \underline{c}] - c_0. \end{aligned} \quad (72)$$

From (55):

$$\begin{aligned} \frac{\delta}{\gamma} [p(\delta)]^\gamma - p(\delta) [\bar{c} - \underline{c}] &= \frac{\delta}{\gamma} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} - \left(\frac{s(\delta)}{\delta} \right)^{\frac{1}{\gamma-1}} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \\ &= [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \left[\frac{\delta}{\gamma} - \left(\frac{s(\delta)}{\delta} \right)^{-1} \right] \\ &= [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \delta \left[\frac{1}{\gamma} - \frac{1}{s(\delta)} \right]. \end{aligned} \quad (73)$$

(72) and (73) imply:

$$\begin{aligned} H_1(\delta) &= \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \frac{G(\delta)}{g(\delta)} \\ &\quad + [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \delta \left[\frac{1}{\gamma} - \frac{1}{s(\delta)} \right] + \bar{c} - c_0 \end{aligned}$$

$$= [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{s(\delta)}{\delta} \right)^{\frac{\gamma}{\gamma-1}} \left[\frac{1}{\gamma} \left(\frac{G(\delta)}{g(\delta)} \right) + \frac{\delta}{\gamma} - \frac{\delta}{s(\delta)} \right] + \bar{c} - c_0. \quad (74)$$

(69) and (74) imply:

$$\begin{aligned} H_1(\delta) &= [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta + \frac{G(\delta)}{g(\delta)}} \right)^{\frac{\gamma}{\gamma-1}} \left[\frac{1}{\gamma} \left(\frac{G(\delta)}{g(\delta)} \right) + \frac{\delta}{\gamma} - \left(\delta + \frac{G(\delta)}{g(\delta)} \right) \right] + \bar{c} - c_0 \\ &= [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta + \frac{G(\delta)}{g(\delta)}} \right)^{\frac{\gamma}{\gamma-1}} \left[\delta + \frac{G(\delta)}{g(\delta)} \right] \left[\frac{1}{\gamma} - 1 \right] + \bar{c} - c_0 \\ &= \bar{c} - c_0 - \left[\frac{\gamma-1}{\gamma} \right] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta + \frac{G(\delta)-G(\delta_1)}{g(\delta)}} \right)^{\frac{\gamma}{\gamma-1}} \left[\delta + \frac{G(\delta)}{g(\delta)} \right] \\ &= \bar{c} - c_0 - \left[\frac{\gamma-1}{\gamma} \right] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta + \frac{G(\delta)}{g(\delta)}} \right)^{\frac{1}{\gamma-1}}. \end{aligned} \quad (75)$$

(75) implies that $H_1'(\delta) > 0$ because $\frac{d}{d\delta} \left(\frac{G(\delta)}{g(\delta)} \right) \geq 0$. Therefore, (72) and (75) imply: (i) $\psi(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_n]$; and (ii) $\psi(\delta) = 0$ for $\delta \in (\delta_n, \bar{\delta}]$, where $\delta_n \equiv \min \{ \tilde{\delta}_n, \bar{\delta} \}$ and $\tilde{\delta}_n$ is defined by:

$$\begin{aligned} H_1(\tilde{\delta}_n) = 0 &\Leftrightarrow \bar{c} - c_0 = \left[\frac{\gamma-1}{\gamma} \right] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\tilde{\delta}_n + \frac{G(\tilde{\delta}_n)}{g(\tilde{\delta}_n)}} \right)^{\frac{1}{\gamma-1}} \\ &\Leftrightarrow \left(\tilde{\delta}_n + \frac{G(\tilde{\delta}_n)}{g(\tilde{\delta}_n)} \right)^{\frac{1}{1-\gamma}} = [\bar{c} - c_0] \left[\frac{\gamma-1}{\gamma} \right]^{-1} [\bar{c} - \underline{c}]^{-\frac{\gamma}{\gamma-1}} \\ &\Leftrightarrow \frac{\tilde{\delta}_n}{h(\tilde{\delta}_n)} = \tilde{\delta}_n + \frac{G(\tilde{\delta}_n)}{g(\tilde{\delta}_n)} = \left[\frac{\gamma-1}{\gamma} \right]^{\gamma-1} [\bar{c} - c_0]^{1-\gamma} [\bar{c} - \underline{c}]^{\gamma}. \end{aligned} \quad (76)$$

(75) implies that $\psi(\delta) = 1$ for all $\delta \in [\delta_1, \bar{\delta}]$ if:

$$\bar{c} - c_0 < \left[\frac{\gamma-1}{\gamma} \right] [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \left[\frac{1}{\bar{\delta} + \frac{1}{g(\bar{\delta})}} \right]^{\frac{1}{\gamma-1}}. \quad (77)$$

(75) and (76) imply that $\tilde{\delta}_n < \bar{\delta}$ and $\psi(\delta) = 0$ for all $\delta \in (\tilde{\delta}_n, \bar{\delta}]$ if the inequality in (77) is reversed.

Pointwise minimization of (67) with respect to $\psi(\delta)$ implies that for $\delta \in [\underline{\delta}, \delta_1]$:

$$\psi(\delta) = \begin{cases} 1 & \text{if } H_2(\delta) < 0 \\ 0 & \text{if } H_2(\delta) > 0, \end{cases}$$

where:

$$H_2(\delta) = \frac{1}{\gamma} \left[\frac{G(\delta)}{g(\delta)} \right] + \frac{\delta}{\gamma} + \underline{c} - c_0 < 0 \Leftrightarrow \delta + \frac{G(\delta)}{g(\delta)} < \gamma [c_0 - \underline{c}]. \quad (78)$$

The inequality in (78) holds for all $\delta \in [\underline{\delta}, \delta_1)$ because:

$$\delta + \frac{G(\delta)}{g(\delta)} < \delta_1 + \frac{G(\delta_1)}{g(\delta_1)} = \bar{c} - \underline{c} < \gamma [c_0 - \underline{c}] \text{ for all } \delta \in [\underline{\delta}, \delta_1). \quad (79)$$

The first inequality in (79) holds because $\frac{d}{d\delta} \left(\frac{G(\delta)}{g(\delta)} \right) \geq 0$ for all $\delta \in [\underline{\delta}, \bar{\delta}]$, by assumption. The equality in (79) reflects (71). The last inequality in (79) holds by assumption. (78) implies that $\psi(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_1)$.

(12) and (55) imply that procurement cost is minimized by setting $r_0(\delta) = c_0$ for all $\delta \in (\delta_n, \bar{\delta}]$.

(55), (61), and (69) imply that for $\delta \in [\delta_1, \delta_n]$:

$$\begin{aligned} \pi_h(\delta) &= \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta}^{\delta_n} \left(\delta_a + \frac{G(\delta_a)}{g(\delta_a)} \right)^{-\frac{\gamma}{\gamma-1}} d\delta_a \\ &= \bar{r}(\delta) - \bar{c} + (s(\delta) [\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] \\ \Rightarrow \bar{r}(\delta) &= \bar{c} + \pi_h(\delta) - (s(\delta) [\bar{c} - \underline{c}])^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma} \right] = z(\delta). \end{aligned}$$

Because $\psi(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_1)$, (63) implies that for $\delta \in [\underline{\delta}, \delta_1)$:

$$\pi_l(\delta) = \frac{1}{\gamma} [\bar{c} - \underline{c}]^{\frac{\gamma}{\gamma-1}} \int_{\delta}^{\delta_1} \left(\delta + \frac{G(\delta)}{g(\delta)} \right)^{-\frac{\gamma}{\gamma-1}} d\delta + \frac{1}{\gamma} [\delta_1 - \delta]. \quad (80)$$

The expression in (67) does not vary with $s(\delta)$ for $\delta \in [\underline{\delta}, \delta_1)$. Furthermore, for each $\delta \in [\underline{\delta}, \delta_1)$, the firm will be indifferent among all $(\bar{r}(\delta), s(\delta))$ options in this interval under any incentive compatible policy. This is the case because incentive compatibility requires that for any $\delta_a, \delta_b \in [\underline{\delta}, \delta_1)$:

$$\begin{aligned} \bar{r}(\delta_a) - \bar{c} + s(\delta_a) [\bar{c} - \underline{c}] - \frac{\delta_a}{\gamma} &\geq \bar{r}(\delta_b) - \bar{c} + s(\delta_b) [\bar{c} - \underline{c}] - \frac{\delta_a}{\gamma} \\ \Rightarrow \bar{r}(\delta_a) - \bar{r}(\delta_b) &\geq [s(\delta_b) - s(\delta_a)] [\bar{c} - \underline{c}], \text{ and} \end{aligned} \quad (81)$$

$$\bar{r}(\delta_a) - \bar{c} + s(\delta_a) [\bar{c} - \underline{c}] - \frac{\delta_b}{\gamma} \leq \bar{r}(\delta_b) - \bar{c} + s(\delta_b) [\bar{c} - \underline{c}] - \frac{\delta_b}{\gamma}$$

$$\Rightarrow \bar{r}(\delta_a) - \bar{r}(\delta_b) \leq [s(\delta_b) - s(\delta_a)][\bar{c} - \underline{c}]. \quad (82)$$

(81) and (82) imply:

$$\begin{aligned} \bar{r}(\delta_a) - \bar{r}(\delta_b) &= [s(\delta_b) - s(\delta_a)][\bar{c} - \underline{c}] \\ \Rightarrow \bar{r}(\delta_a) - \bar{c} + s(\delta_a)[\bar{c} - \underline{c}] - \frac{\delta_a}{\gamma} &= \bar{r}(\delta_b) - \bar{c} + s(\delta_b)[\bar{c} - \underline{c}] - \frac{\delta_a}{\gamma}, \text{ and} \\ \bar{r}(\delta_b) - \bar{c} + s(\delta_b)[\bar{c} - \underline{c}] - \frac{\delta_b}{\gamma} &= \bar{r}(\delta_a) - \bar{c} + s(\delta_a)[\bar{c} - \underline{c}] - \frac{\delta_b}{\gamma}. \end{aligned}$$

It follows that for $\delta \in [\underline{\delta}, \delta_1)$:

$$\frac{\partial}{\partial \delta} \{ \bar{r}(\delta) - \bar{c} + s(\delta)[\bar{c} - \underline{c}] \} = \bar{r}'(\delta) + s'(\delta)[\bar{c} - \underline{c}] = 0. \quad (83)$$

Because $p(\delta) = 1$ for all $\delta \in [\underline{\delta}, \delta_1)$, procurement cost is $\bar{r}(\delta) + s(\delta)[\bar{c} - \underline{c}]$ for each $\delta \in [\underline{\delta}, \delta_1)$. Therefore, the rate at which procurement cost varies with δ for $\delta \in [\underline{\delta}, \delta_1)$ under any incentive compatible policy is:

$$\bar{r}'(\delta) + s'(\delta)[\bar{c} - \underline{c}] = 0. \quad (84)$$

The equality in (84) reflects (83). (84) implies that an optimal policy can entail $s(\delta) = s(\delta_1)$ and $\bar{r}(\delta) = \bar{r}(\delta_1)$ for all $\delta \in [\underline{\delta}, \delta_1)$. ■

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