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Financially-Constrained Lawyers: An Economic Theory of Legal Disputes

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Financially-Constrained Lawyers: An Economic Theory of Legal Disputes*

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Abstract

Financial constraints reduce the lawyer's ability to file lawsuits and bring cases to trial. As a result, access to justice for victims, pretrial bargaining, and potential injurers' precaution might be affected. We study civil litigation using a model that allows for asymmetric information, financially-constrained lawyers, third-party lawyer lending, and a continuum of plaintiff's types. We contribute to the economic analysis of law by generalizing seminal models of litigation (Bebchuk, 1984, 1988; Katz, 1990), offering the first formal definition of access to justice, and presenting comprehensive social welfare analysis of relevant public policy. We provide complete equilibrium characterization and identify necessary conditions for the existence of the mixed- and pure-strategy PBE. The mixed-strategy equilibrium arises in a state of the world characterized by lawyers facing strong financial constraints. Access to justice is denied to some victims under the mixed-strategy equilibrium. We then study the social welfare effects of policies aimed at relaxing lawyers' financial constraints, and identify a necessary and sufficient condition for a welfare-enhancing effect.

KEYWORDS: Civil Litigation; Access to Justice; Social Welfare; Financially-Constrained Lawyers; Asymmetric Information; Perfect Bayesian Equilibrium; Deterrence; Lawsuits; Settlement; Litigation; Third-Party Lawyer Lending Industry; Third-Party Litigation Funding

JEL Categories: K41, C70, D82

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1 Introduction

The U.S. tort system provides \$172 billion in gross compensation to plaintiffs each year. Litigation expenses, which are generally covered by personal injury lawyers on behalf of their clients, represent \$5.2 billion of this compensation (Engstrom, 2014). The average cost of taking a medical malpractice claim to trial is \$97,000 (Shepherd, 2014). Expenses on expert witnesses in the \$50,000-\$100,000 range are not uncommon (Trautner, 2009). As cases become more complex and hence, more expensive, lawyers might experience financial constraints (Engstrom, 2014; Garber, 2010). Financial constraints weaken the lawyers' ability to file lawsuits and bring cases to trial. As a result, access to justice for victims is compromised. Importantly, by affecting the pool of filed cases, lawyers' financial constraints might also influence pretrial bargaining outcomes and potential injurers' precaution. Hence, a comprehensive analysis of civil litigation should consider the financial constraints that lawyers face. Policy debate, however, has been centered only on the effects of lawyers' financial constraints on access to justice. Previous theoretical work on legal disputes has simply abstracted from lawyers' financial constraints. Our paper aims to fill these gaps. We present the first strategic model of civil litigation in an environment characterized by asymmetric information, financially-constrained lawyers, and a continuum of plaintiff's types. Our framework generalizes seminal economic models of litigation (Bebchuk, 1984, 1988; Katz, 1990).

Traditionally, financially-constrained lawyers have relied on fellow lawyers' contributions and bank loans. In the late 1990s, a third-party lawyer's lending industry emerged.[1] Lawyer lenders such as *Counsel Financial*, among others, started funding activities. These lawyer lending institutions specialize in providing recourse loans (non-contingent loans) to cover the expenses associated with particular legal cases (Garber, 2010). According to legal commentators, "[R]ecourse lawyer lending ... is making a significant mark ... [and has] remarkable potential for future growth" (Engstrom, 2014; p. 397).[2] In contrast to traditional banks, these lenders do not require lawyers'

[1] The third-party litigation funding industry, and the demand for these services by plaintiffs' lawyers and clients, have been amply debated by policy analysts (see for instance the RAND Institute for Civil Justice policy reports by Garber, 2010, and McGovern, et al., 2010), studied by legal scholars (see for instance, Molot, 2010, 2009; Rodak, 20016; Steinitz, 2012; and the fifteen articles included in the *DePaul Law Review* symposium volume on litigation and law firm finance, 2014 (see Landsman et al. in the References section)), and discussed in the media (see for instance, *The Economist*, April 6th 2013; the article by Glater published at *The New York Times*, June 2 2009; the article by Frankel published at *The American Lawyer,* February 13 2006; and, the article by Carter published at The American Bar Association's *ABA Journal*, October 2004). However, there are no previous formal models of civil litigation involving financially-constrained lawyers and lawyer's lending institutions.

[2] Due to the absence of a central repository of data, it is impossible to estimate the exact amount of loans associated with the lawyer lending industry. Engstrom (2014) provides some evidence of the scope of the industry.

personal assets as collaterals. Instead, the loans are secured by the law firm's assets, including future fees (Garber, 2010).[3] The loans involve significantly larger sums and the interest charged is higher than the traditional bank's interest (around 15-20 percent per year). Our theoretical framework also captures the role of the lawyer lending industry on legal disputes.

We model the interaction between a defendant, a plaintiff, and a plaintiff's lawyer as a sequential game of incomplete information. Our framework allows for asymmetric information, financially-constrained lawyers, third-party lawyer lending, and a continuum of plaintiff's types. The source of information asymmetry is the damage level of the plaintiff's case, which is unknown by the defendant. Our original framework depicts legal disputes from cradle to grave. Specifically, our model allows for endogenous care-taking (precaution), endogenous filing, and endogenous out-of-court settlement decisions. We model the lawyers' financial constraints by incorporating real-world characteristics of the third-party lawyer lending industry. We present the first formal definition of "Access to Justice" and incorporate this concept in the social welfare analysis of relevant public policy.

We provide complete characterization of the two mutually-exclusive perfect Bayesian equilibria (PBE): Mixed- and pure-strategy equilibria. In the mixed-strategy equilibrium, a lawyer with a low-damage case mixes between accepting the case and filing a lawsuit and not accepting the case, and a lawyer with a high-damage case always accepts the case and files a lawsuit; the defendant mixes between making a zero offer and a positive offer. In the pure-strategy equilibrium, lawyers with all types of cases always accept the cases and file lawsuits; and, the defendant always makes a positive offer. Across equilibria, accidents and pre-trial bargaining disagreement are observed. Access to justice is denied to some victims only under the mixed-strategy equilibrium.

We identify necessary conditions for the existence of each PBE. In particular, our analysis demonstrates that the pure-strategy equilibrium arises in a state of the world characterized by lawyers facing mild financial constraints. In contrast, the mixed-strategy equilibrium arises in a state of the world characterized by lawyers facing strong financial constraints.The intuition is as follows. When the level of lawyers' financial constraints is high, then low-damage plaintiffs (i.e., plaintiffs with cases that cannot proceed to trial due to the limited financial resources of

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An important lawyer lender, "*Counsel Financial*, apparently had 'more than $200 million' in loans outstanding as of 2010" (p. 397). Lawyer lenders operate in almost all U.S. states. For instance, *Advocate Capital* funds law firms in forty states. Public records for the state of New York indicate that 250 law firms borrowed from lawyer lending providers during the period 2000–2010.

[3]Loans are secured by the estimated value of a firm's total portfolio of cases. Lenders generally require access to the firm's entire docket of cases and assets.

their lawyers and/or high litigation costs) are relatively common. The defendant will reduce his expected litigation loss by mixing between a zero offer and a positive offer (instead of making a unique positive offer) because low-damage plaintiffs who receive a zero offer will drop their cases. In contrast, a unique positive offer will be accepted by all low-damage plaintiffs.

We then use our model to study the effects of policies aimed at relaxing lawyers' financial constraints by lowering the costs associated with legal disputes. The interest charged to lawyers by third party lenders and the costs associated with expert witnesses' fees hired by plaintiff's lawyers are examples of such costs.[4] Given that these policies do not affect equilibrium strategies and outcomes under the pure-strategy PBE, we focus our analysis on the mixed-strategy PBE. Our findings suggest that cost-reducing policies increase filing of low-damage cases. The uninformed defendant reacts by reducing his positive out-of-court settlement offer, and the likelihood of trial increases. Higher expected litigation costs for the defendant are observed, which increase his care-taking incentives, and lower the probability of an accident. Our findings also suggest that these policies improve access to justice by allowing additional victims to file lawsuits and by reducing the number of victims forced to drop their lawsuits.

Finally, we present social welfare analysis of cost-reducing policies and identify a necessary and sufficient condition for a positive welfare effect of these policies. We demonstrate that, in a state of the world characterized by strong society's concerns about preserving citizens' right of access to justice, a relaxation of the lawyers' financial constraints always increase social welfare. The intuition is as follows. As a result of relaxing the lawyers' financial constraints, many additional victims will get access to justice. In addition, potential injurers, initially underdeterred, will have more incentives to spend resources on accident prevention to avoid costly litigation. This will contribute to the alignment of privately- and socially-optimal probabilities of accidents. These positive welfare effects more than offset the higher social costs associated with more filing and litigation. Hence, social welfare will be improved.

Strong policy implications are derived from our work. Our complete social welfare analysis of cost-reducing policies provides clear policy recommendations. Importantly, this analysis applies to other policies aimed at alleviating lawyers' constraints. For instance, legal commentators are currently proposing the implementation of a cost-shifting policy that allows lawyers to transfer part of the financial costs of loans to their clients (Engstrom, 2014). Proponents claim that, by

[4]Policies devoted to strengthening competition in the lawyer lending industry might reduce the costs of third-party loans. Similarly, policies designed to increase efficiency of legal procedures and reduce unpredictability of the legal system by capping punitive damages (Landeo et al., 2007b) might lower the plaintiffs' costs associated with litigation.

improving access to justice, this policy will be welfare-enhancing. We contribute to this debate by underscoring the importance of assessing public policies in terms of their overall effect on social welfare, and by identifying a necessary and sufficient condition for a positive welfare effect of a reduction in lawyers' constraints. Our analysis suggests that cost-shifting policies, which also alleviate lawyers' constraints, should be implemented when preserving access to justice for victims is a fundamental goal of society.

Several important methodological contributions to the economic analysis of law are derived from our work. We generalize seminal models of civil litigation by endogenizing the decisions associated with every step of a legal dispute. Our environment allows for asymmetric information, financially-constrained lawyers, third-party lawyer lending, and a continuum of plaintiff's types. Importantly, we offer the first formal characterization of Access to Justice, and provide a comprehensive welfare analysis of relevant public policies.

The closest to our work are Bebchuk (1984, 1988) and Katz (1990). Bebchuk (1984) studies settlement and litigation. The paper focuses on a pure-strategy PBE with a unique positive settlement offer. This equilibrium resembles the equilibrium strategies of the litigation stage that correspond to our pure-strategy PBE. Bebchuk (1988) extends Bebchuk (1984) by including a filing stage and allowing for frivolous lawsuits. In this environment, filing costs are equal to zero. Then, all potential plaintiffs have an incentive to file a lawsuit. The paper focuses on a pure-strategy PBE where all potential plaintiffs file a lawsuit, and all plaintiffs receive a positive out-of-court settlement offer. An important limitation of Bebchuk (1984, 1988) is that, unrealistically, all victims get access to justice in these environments. Importantly, a care-taking stage is not included in these model. Hence, these frameworks do not allow for a complete welfare analysis of relevant public policies.

Katz (1990) extends Bebchuk (1988) by relaxing the assumption of costless filing. Binary-type (meritorious and frivolous plaintiff's types) and continuum-type models are studied. Our work differs from Katz (1990) in several fundamental aspects. First, our model generalizes Katz (1990) by including a care-taking stage, by introducing the plaintiff's lawyer as a third player, and by allowing for lawyer's financial constraints and third-party lawyer lending. The introduction of a financially-constrained plaintiff's lawyer as a third player and third-party lawyer lending permits us to study the effect of public policies aimed at relaxing lawyers' financial constraints. Second, although the sequence of moves of the continuum-type model analyzed in Katz (1990) is similar to the sequence of moves of the filing and litigation stages in our model, the focus of Katz's (1990) analysis of the continuum-type model is on the pure-strategy PBE where a unique positive out-of-court settlement offer is always made. Hence, a comprehensive analysis of access to justice cannot be performed. In contrast to this paper, we formally characterize access to justice, and perform comparative-statics analysis of the effect of cost-reducing policies on access to justice. Third, we provide a comprehensive social welfare analysis of the effect of policies aimed at alleviating lawyers' financial constraints and identify a necessary and sufficient condition for a welfare-enhancing effect of these policies. Importantly, by endogenizing the care-taking decisions, our model permits us to assess not only the direct welfare effect of public policies that operates through the filing and litigation decisions but also the indirect welfare effect that operates through the defendant's care-taking decisions. Hence, the first and most important contribution of our paper is to present an original model of civil litigation that allows for a comprehensive social welfare analysis of relevant public policies.[5]

Our paper also contributes to the literature on third-party litigation funding. As discussed in Garber (2010), third-party litigation funding includes three types of financing: Recourse loans to plaintiffs' lawyers, non-recourse loans to individual plaintiffs, and investments in commercial lawsuits (business against business lawsuits). Previous theoretical work on third-party litigation funding has been focused on the plaintiff's lending segment of the industry. Daughety and Reinganum (2014) study the design of third-party loans to plaintiffs using dynamic incomplete information models with the plaintiff as the first mover.[6] Their findings suggest that the third-party institution affects only pre-trial bargaining outcomes.[7] Analysis of the effects of the third-party institution on filing is not included. We provide several contributions to the literature on third-party litigation funding. First, our work contributes to a better understanding of the third-party litigation funding institution by presenting the first formal analysis of legal disputes in an environment that allows for third-party lawyer lending institutions. Second, our work incorporates care-taking and filing decisions into the analysis, and demonstrates that changes in the features of the third-party litigation institution might affect not only settlement but also care-taking in-

[5]For additional seminal theoretical work on civil litigation, see Shavell (1982), Png (1983), Rosenberg and Shavell (1985), Reinganum and Wilde (1986) and Nalebuff (1987). These models are focused on the litigation stage. See Landeo et al. (2007a, b; 2006) and Png (1987) for seminal civil litigation models that allow for endogenous care-taking decisions. In contrast to our work, all these models do not allow for constraints at the litigation stage (i.e., they assume that the party who makes the decision at the litigation stage has sufficient funds to cover the litigation costs) and do not study access to justice. See Landeo (forthcoming) and Daughety and Reinganum (2012) for recent surveys of theoretical and experimental work on civil litigation.

[6]See also Deffains and Desrieux (2015), Avraham and Wickelgren (2014), Demougin and Maultzsch (2014) and Hylton (2012). These papers are focused on the plaintiff's lending segment of the third-party litigation funding industry, and only study the litigation stage.

[7]In this framework, the defendant's care taking incentives are not affected by the third-party institution.

centives and filing decisions. In particular, we show that in a state of the world characterized by strong society's concern about access to justice by victims, policies aimed at reducing the interest rate charged on third-party loans induce more low-damage cases to be filed, increase the likelihood of trial, increase the defendant's care-taking incentives, and increase social welfare.

The rest of the article is organized as follows. Section Two presents the setup of the model and provides the formal definition of Access to Justice. Section Three outlines the equilibrium analysis. Section Four discusses the effect of a cost-reducing policy on equilibrium outcomes and access to justice. Section Five provides formal characterization of the social welfare loss function and presents an analysis of deterrence. Section Six presents the social welfare analysis of a cost-reducing policy. Section Seven provides concluding remarks. The Appendix includes formal proofs of the mixed- and pure-strategy PBE of our general model, and proofs related to deterrence and the social welfare analysis of a cost-reducing policy.

2 Model Setup

This section describes the game stages, the "Lawyer's Constraint" component, and the "No-Access to Justice" component. It also introduces the notation.

2.1 Game Stages

We model the interaction between a potential defendant,[8] a potential plaintiff, and a potential plaintiff's lawyer as a sequential game of incomplete information. The source of information asymmetry is the damage level of the potential plaintiff's case A, which is unknown by the defendant. The stages of the game are as follows.

Care-Taking Stage

In the first stage, the potential defendant decides his level of care, which determines the probability of accident λ. The cost of care is denoted by $K(\lambda)$. We assume that all potential defendants have the same cost of care, which is common knowledge. We also assume that $K(\lambda)$ is a continuous and differentiable function defined on the interval $[0,1]$ with with $K(1)=0$, $\frac{\partial K(\lambda)}{\partial \lambda}<0$, $\lim_{\lambda \to 0^+} \frac{\partial K(\lambda)}{\partial \lambda} = -\infty$, $\lim_{\lambda \to 1^-} \frac{\partial K(\lambda)}{\partial \lambda}=0$; and, that $\frac{\partial K(\lambda)}{\partial \lambda}$ is a continuous and differentiable function with $\frac{\partial K^2(\lambda)}{\partial \lambda^2}>0$. The potential defendant's optimal level of care, i.e., the optimal λ, is the one that minimizes

[8] We use the terms "potential defendant," "potential injurer," and "defendant" interchangeably.

the defendant's expected total loss $L_D(\lambda) = K(\lambda) + \lambda l_D$, where l_D is the defendant's expected litigation loss. We take the expected litigation loss as parametric in order to describe L_D, but ultimately l_D will be derived as the continuation value of the litigation stage, and hence, it will reflect the equilibrium outcomes at the litigation stage. We assume that accident occurrence is common knowledge. If an accident occurs, we assume that there is a mass 1 of potential plaintiffs. Nature determines the damage level A and informs this to the potential plaintiff only. We assume that the damage levels A are distributed according to the probability density function $g(A)$ and the cumulative distribution function $G(A)$, with support $(0, \bar{A}]$. We also assume that $g(A)$ is a continuous and differentiable function. The functions are common knowledge.[9]

Filing Stage

If an accident occurs, the second stage starts. We assume that the mass of potential lawyers is greater than or equal to 1, i.e., it is sufficiently high to serve all potential plaintiffs. The potential plaintiff and his potential lawyer meet.[10] The lawyer, who perfectly observes the potential plaintiff's type (damage level), decides whether to take the case and file a lawsuit. We denote the mass of filed cases as ζ. The lawyer is hired under a contingency-fee compensation. Under this scheme, the lawyer receives gross payment equal to a share γ of the plaintiff's gross recovery (award at trial or out-of-court settlement amount).[11] The lawyer's share γ is an exogenous constant, known by all parties.[12] The lawyer's costs of taking the case and filing a lawsuit are denoted by f, common knowledge. Following empirical regularities, we assume that the plaintiff is financially constrained. Then, the plaintiff's lawyer pays f.

Litigation Stage

If a lawsuit is filed, the third stage starts. The uninformed defendant makes a take-it-or-leave-it out-of-court settlement offer to the plaintiff. Zero and positive offers are possible. $S > 0$ denotes a positive offer. β denotes the probability that a defendant makes a zero offer. The plaintiff then

[9]For simplicity, we abstract from non-meritorious potential plaintiffs (i.e., frivolous cases). Our equilibrium analysis and comparative statics are robust to the presence of the threat of frivolous lawsuits.

[10]For simplicity, our framework abstracts from searching costs.

[11]A typical contingency fee involves a lawyer's share equal to 33-45% of any eventual recovery by his client. See *Rule 1.5(d) of the Model Rules of Professional Conduct* and statutory rules enacted by U.S. states. See Miceli (1994) for seminal formal work on contingency fees.

[12]Our main qualitative findings hold in an environment that allows for different values of γ in case of trial and out-of-court settlement. See Emons (2007).

decides whether to accept or reject the defendant's offer. If an offer $S > 0$ is accepted by the plaintiff, then the defendant transfers the settlement amount to the plaintiff, and the game ends. The plaintiff gets $(1-\gamma)S$ and his lawyer gets a net payoff of $\gamma S - f$. Acceptance of a zero offer implies that the plaintiff drops the case. If an out-of-court settlement offer is rejected, the case proceeds to costly trial. We denote the mass of cases that proceed to trial as ρ. Both litigants incur litigation costs: C_P denotes the plaintiff's litigation cost, which is paid by his lawyer; and, C_D denotes the defendant's litigation cost. We assume that the court perfectly observes the plaintiff's type A. When a case goes to trial, the court orders the defendant to pay A to the plaintiff.[13] The plaintiff gets $(1-\gamma)A$ at trial and his lawyer gets a net payoff of $\gamma A - C_P - f$.

2.2 Lawyer's Constraint Component

This section describes the lawyer's constraint component of our model. We denote the amount of the lawyer's own funds as $x > f$. We assume that the lawyer is financially constrained. His own funds x are insufficient to bring a case to trial, i.e., $x < f + C_P$. We also assume that there are available third-party lawyer lenders that can lend money to the lawyer to allow him to bring a case to trial, and that the third-party lawyer lending industry is not perfectly competitive.[14] To be able to bring a case to trial, the financially-constrained lawyer needs to borrow $C_P + f - x$ at a net interest rate r. We denote $\tilde{A}$ as the damage threshold at which the lawyer's net payoff when the case proceed to trial and his net payoff when the case is dropped (i.e., when a zero offer is accepted) are the same: $\gamma\tilde{A} - f - C_P - (C_P + f - x)r = -f$. Then, cases with $A < \tilde{A}$ will not proceed to trial. Hence, $\tilde{A}$ represents the lawyer's constraint component, i.e., the lawyer's inability to bring a case to trial.

Definition 1. *The lawyer's constraint component $\tilde{A}$ is defined as follows.*

$$\tilde{A} \equiv \frac{C_P}{\gamma} + \frac{(C_P + f - x)r}{\gamma}.$$

The lawyer's constraint component encompasses two terms: (1) The litigation-cost constraint $LC \equiv \frac{C_P}{\gamma}$, which denotes the lawyer's constraint associated with costly litigation; and, (2) the

[13] We assume that the court applies a strict liability rule. Under this rule, the injurer has to bear the costs of the accident regardless of the extent of his precaution. Strict liability is common in tort law cases. Then, this assumption is empirically relevant. Our tractable model can be easily extended to accommodate the court's application of a negligence rule in an environment where the defendant's level of care is common knowledge.

[14] The specialized nature of the service provided by these lenders might act as a barrier to entry. As a result, lenders might have market power.

financial constraint $FC \equiv \frac{(C_P+f-x)r}{\gamma}$, which denotes the lawyer's constraint associated with the financial costs of third-party loans (due to the lawyer's limited own funds). Then, $\tilde{A} \equiv LC+FC$.[15] It is worth noting that our model provides a more accurate characterization of the constraints confronted by litigants in real-world settings than previous models of litigation. In fact, seminal models of litigation abstract from the role of the plaintiff's lawyer, and implicitly assume that the plaintiff's own funds are sufficient to cover the costs associated with litigation. Then, in those models, the plaintiff's inability to bring a case to trial is determined by C_P only. Importantly, our approach to incorporate financially-constrained lawyers to a model of litigation is aligned with empirical regularities. In fact, in his descriptive analysis of the lawyer-lending industry, Garber (2010) states: "Plaintiffs' lawyers pursuing personal-injury claims typically work on a contingent-fee basis, may have insufficient hourly work to provide steady streams of revenue, and incur out-of-pocket expenses to pursue their clients' claims" (p. 23). Our model captures these features.

In our framework, potential plaintiffs with $A \geq \tilde{A}$ are called "high-damage cases," and potential plaintiffs with $0 < A < \tilde{A}$ are called "low-damage cases." Remember that a measure 1 represents the potential plaintiffs. Then, $G(\tilde{A})$ represents the mass of low-damage potential plaintiffs, and $1 - G(\tilde{A})$ represents the mass of high-damage potential plaintiffs. We denote the mass of low-damage cases that are filed as ν. We assume that the lawyer's constraint component $\tilde{A}$ is common knowledge. In particular, the plaintiff knows that the lawyer's constraint allows only high-damage cases ($A \geq \tilde{A}$) to proceed to trial, and takes this information into consideration when deciding whether to accept a settlement offer. Under this assumption, the plaintiff holds the settlement decision authority but her decision is conditioned by her lawyer's constraints. Then, this assumption might be interpreted as a partial delegation of settlement authority to the lawyer. According to the *Restatement (Third) of Law Governing Lawyers* (1988, Sec. 33.1), the plaintiff can validly make partial or full delegation to his lawyer. Importantly, delegation of settlement authority is commonly observed in real-world settings (Miller, 1987).[16]

[15] Even if the third-party litigation funding industry is perfectly competitive ($r = 0$) or if the lawyer's own funds are sufficient to cover the costs associated with litigation ($x \geq C_P + f$), when $A < \tilde{A}$, the lawyer will still face a litigation-cost constraint. We thank a referee for pointing this out.

[16] See Dana and Spier (1990) for a litigation model involving full delegation of authority to the lawyer regarding the decision to drop the case. See Choi (2003) for a litigation model involving plaintiff's endogenous decision of delegation of settlement authority to the lawyer.

2.3 No-Access to Justice Component

This section formally defines the "No-Access to Justice" component of our model. The "No-Access to Justice" component η represents the inability of meritorious (potential) plaintiffs to get access to justice. To the best of our knowledge, ours represents the first formal definition of "No-Access to Justice."

Definition 2. *No-Access to Justice η is defined as the sum of two terms.*

(1) The mass of meritorious cases that are not filed.

(2) The mass of meritorious cases that receive a zero offer and cannot proceed to trial.

Intuitively, the first term represents the inability of lawyers with meritorious cases to file a lawsuit. The second term represents the inability of meritorious plaintiffs to get compensation for the inflicted injury through a positive out-of-court settlement transfer or an award at trial. The specific elements included in η depend on the equilibrium outcomes.

3 Equilibrium Analysis

Our analysis demonstrates that the lawyers' financial constraints affect the equilibrium structure and permeates the decisions of all the parties involved in a legal dispute. We completely characterize the two-mutually exclusive PBE of the model: Pure- and mixed-strategy PBE. In the pure-strategy PBE, the defendant always make a positive offer and all victims file lawsuits. In the mixed-strategy equilibrium, the defendant randomizes between a positive and a zero offer, and not all victims are able to file a lawsuit. We show that, across equilibria, accidents occur and bargaining disagreement at the litigation stage is observed. Access to justice is compromised only in case of the mixed-strategy PBE. In real-world settings, access to justice is denied to some victims. Then, the mixed-strategy PBE provides a better description of the actual state of affairs in civil litigation.

We provide necessary conditions for the existence of the two PBE. The pure-strategy PBE arises in a state of the world characterized by lawyers facing mild financial constraints; and the mixed-strategy PBE arises in a state of the world characterized by lawyers facing strong financial constraints. The intuition is as follows. When the level of lawyers' financial constraints is high, then low-damage plaintiffs (i.e., plaintiffs with cases that cannot proceed to trial due to the limited financial resources of their lawyers and/or high litigation costs) are relatively common and high-damage plaintiffs are relatively rare. The defendant will reduce his expected litigation loss by mixing between a zero offer and a positive offer because low-damage plaintiffs who receive a zero offer will drop their cases and only high-damage plaintiffs will reject a zero offer and proceed to trial; a unique positive offer, in contrast, will be accepted by all low-damage plaintiffs. Hence, when the level of lawyers' financial constraints is high, then the defendant will have a strong incentive to mix between a zero offer and a positive offer. In contrast to previous theoretical models of litigation (Bebchuk, 1984, 1988; Katz, 1990), our work shows that the plaintiff's litigation costs influence the lawyer's constraints, and hence, might affect not only the behavior of the litigants at the litigation stage but also the structure of the equilibrium.

The next sections outline the main steps in the construction of the mixed- and pure-strategy PBE. Formal proofs are presented in the Appendix.

3.1 Mixed-Strategy PBE

In a state of the world characterized by strong lawyers' financial constraints, the mixed-strategy PBE emerges. In the mixed-strategy equilibrium, the defendant mixes between making a zero offer and a positive offer; and, lawyers with high-damage cases always file a lawsuit and lawyers with low-damage cases mix between accepting the case and filing a lawsuit and not accepting the case. Due to asymmetric information, some high-damage plaintiffs receive an insufficiently high offer (i.e., an offer below their actual harm from the accident) and proceed to costly trial; and, some low-damage plaintiffs receive a generous offer (an offer above their actual harm from the accident) and settle out-of-court, and some low-damage plaintiffs receive a zero offer and need to drop their cases. Given that only a subset of low-damage cases are filed, and some low-damage cases are dropped, then some low-damage victims do not get access to justice.

Equilibrium Strategies

Step 1: Potential Composition of the Set of Equilibrium Offers

First, we analyze the potential composition of the set of equilibrium offers and demonstrate that this set must include a zero offer and at least one positive offer $S \in [\tilde{A}, \bar{A}]$ (see Lemma 1 and Claim 1 in the Appendix). We start with the elimination of the strictly dominated offers $S > \bar{A}$ and offers $S \in (0, \tilde{A})$. We then demonstrate that a zero offer must be made in equilibrium. If a zero offer is not made in equilibrium, then a positive offer $S \in [\tilde{A}, \bar{A}]$ must be made. Assume that the filing costs are low enough ($f < \gamma\tilde{A}$). Then, all low- and high-damage cases will be filed. As shown in the proof of Lemma 1, the defendant's expected litigation loss from making the positive offer will be greater than his expected litigation loss from making a zero offer. Contradiction follows. Hence, a zero offer must be made in equilibrium, and the probability of making a zero offer should be greater than zero ($\beta > 0$).

Finally, we show that a zero offer cannot be the only equilibrium offer. If the defendant always makes a zero offer, then only high-damage cases with types $A \geq \hat{A} > \tilde{A}$ will be filed.[17] These types of plaintiffs will always reject a zero offer. Then, the defendant will be better off by deviating from a zero offer to a positive offer slightly higher than $\hat{A}$. Hence, a zero offer cannot be the only equilibrium offer. Given our previous findings, in addition to a zero offer, there must be at least one positive offer $S \in [\tilde{A}, \bar{A}]$ in the set of equilibrium offers.

Step 2: Potential Composition of the Equilibrium Mass of Filed Cases ζ

Next, we study the potential composition of the equilibrium mass of filed cases ζ, and show that at least some low-damage cases and all high-damage cases must be filed in equilibrium (see Lemma 2 in the Appendix). We start with the evaluation of filing of low-damage cases. If no low-damage cases are filed in equilibrium (i.e., if $\nu = 0$), then a mixed strategy involving a zero offer and at least one positive offer $S \in [\tilde{A}, \bar{A}]$ is not an equilibrium strategy. As we show in the proof of Lemma 2, the defendant will be better off by deviating from the mixed-strategy involving a zero offer and a positive offer $S \in [\tilde{A}, \bar{A}]$ to a pure-strategy involving a positive offer greater than $\tilde{A}$. The intuition is as follows. If low-damage cases are not filed, then a zero offer does not provide any benefits for the defendant: A zero offer offer will always be rejected by high-damages plaintiffs, and hence, no cases will be dropped. The defendant will economize on litigation costs by deviating to a pure strategy with a positive offer greater than $\tilde{A}$. We conclude that at least some low-damage cases must be filed.

We proceed with the study of filing of high-damage cases. Given that at least some low-damage cases are filed in equilibrium, it must be the case that at least some lawyers with low-damage cases will get non-negative expected payoffs: $\gamma[\beta(0) + (1-\beta)S)] - f \geq 0$, where S and β represent a positive offer and the probability of getting a zero offer, respectively. Remember also that the equilibrium probability that the defendant makes a zero offer is positive ($\beta > 0$). Consider now the case of a lawyer with a high-damage case $A \geq \tilde{A}$. The expected payoff for the lawyer is $\beta[\gamma A - C_P - (C_P + f - x)r] + (1-\beta)\gamma S - f$. This expression is non-negative for $A = \tilde{A}$ and positive for $A > \tilde{A}$. As a consequence, all high-damage cases will be filed. Hence, the equilibrium

[17]Note that $\hat{A} \equiv \frac{C_P + f + (C_P + f - x)r}{\gamma}$ refers to the damage threshold at which the lawyer's net payoff is zero when a high-damage case is solved at trial.

mass of cases that are filed $\zeta = \nu + \int_{\tilde{A}}^{\bar{A}} g(A)dA$, where $\nu > 0$ represents the mass of low-damage cases that are filed.

Step 3: Equilibrium S and ν

Now, we are in the position to analyze the equilibrium positive settlement offer S and the equilibrium mass of low-damage cases that are filed ν (see Lemmas 3 and 4 in the Appendix). We will show that equilibrium S and ν exist and are unique.[18] Remember that lawyers with high-damage cases always file lawsuits, and some lawyers with low-damage cases also file lawsuits; and, that there are at least two equilibrium offers, a zero offer and a positive $S \in [\tilde{A}, \bar{A}]$. This implies that both a zero offer and the positive offer S must minimize the defendant's expected litigation loss l_D, and that the defendant must be indifferent between these two offers. The defendant's indifference condition is given by:

$$\nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}} (A + C_D)g(A)dA = \int_{\tilde{A}}^{\bar{A}} (A + C_D)g(A)dA. \tag{1}$$

The left- and right-hand sides of equation (1) represent the defendant's expected litigation loss l_D from making an offer $S > 0$ and a zero offer, respectively. Then, the first-order optimality condition is:

$$\frac{\partial}{\partial S}\left[\nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}} (A + C_D)g(A)dA\right] = 0.$$

This last equation simplifies to:

$$\nu + G(S) - G(\tilde{A}) = C_D g(S). \tag{2}$$

Equation (2) indicates that the marginal cost of raising S (represented by the mass of plaintiffs who accept S; left-hand side of the equation) equals the marginal benefit of raising S (represented by the savings in litigation cost C_D as fewer cases go to trial; right-hand side of the equation). As demonstrated in Lemma 3, the system of equations (1)–(2) has a unique solution, S and ν. When, $g(A) - C_D \frac{\partial g(A)}{\partial A} > 0$ for any $A \in [\tilde{A}, \bar{A}]$, the second-order optimality condition is satisfied. Finally, when $\int_{\tilde{A}}^{\bar{A}} Ag(A)dA + C_D[1 - G(\tilde{A}) - \bar{A}g(\bar{A})] > 0$, the optimal positive offer S is interior.

Figure 1 presents a graphical representation of the optimal decision of the defendant at the litigation stage. The defendant's expected litigation loss function l_D is constructed by keeping the

[18] To simplify notation, we also refer to the *equilibrium* positive offer and mass of low-damage cases that are filed as S and ν, respectively, in this section and the rest of the paper.

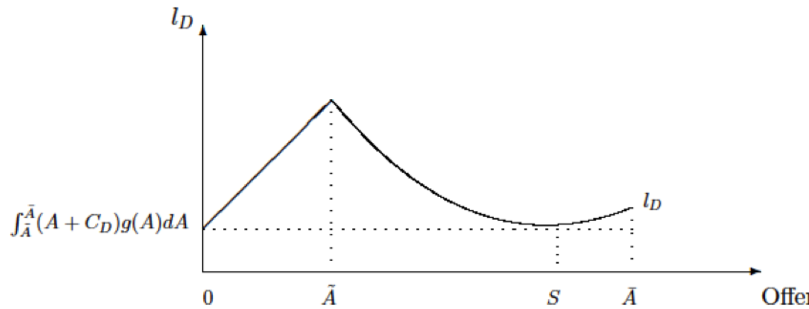

Figure 1: Defendant's Equilibrium Mixed-Strategy at the Litigation Stage

equilibrium mass of filed cases $\zeta = \nu + \int_{\tilde{A}}^{\bar{A}} g(A)dA$ constant.

$$l_D = \begin{cases} \int_{\tilde{A}}^{\bar{A}}(A+C_D)g(A)dA & \text{if zero offer} \\ \nu S + \int_{\tilde{A}}^{\bar{A}}(A+C_D)g(A)dA & \text{if } S \in (0,\tilde{A}) \\ \nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}}(A+C_D)g(A)dA & \text{if } S \in [\tilde{A},\bar{A}] \end{cases}$$

When $S \in (0, \tilde{A})$, l_D is linear and upward-sloping. When $S \in [\tilde{A}, \bar{A}]$, l_D is strictly convex and achieves a unique interior minimum. The defendant's expected loss function l_D attains the same value at a zero offer and at the optimal positive offer S. Hence, in equilibrium, the defendant mixes between a zero offer and a positive (interior) offer $S \in (\tilde{A}, \bar{A})$.

Step 4: Composition of Equilibrium ν and β

Next, we study the composition of equilibrium mass of low-damage cases that are filed ν and the equilibrium probability that the defendant makes a zero offer β (see Claim 2 and Lemma 5 in the Appendix). Remember that at least some low-damages cases are filed in equilibrium. Claim 2 in the Appendix demonstrates that the total mass of low-damage cases $G(\tilde{A})$ is greater than the equilibrium mass of low-damage cases that are filed ν. Hence, not all low-damage cases are filed in equilibrium. Intuitively, the defendant's randomization between a zero offer and a positive offer S induces a lawyer with a low-damage case to randomize between filing and not filing a lawsuit. Although low-damage cases cannot proceed to trial, some of these cases might still receive a positive out-of-court settlement offer due to asymmetric information.

A lawyer with an average low-damage case who randomizes between filing and not filing a lawsuit must be indifferent between these two strategies.[19] In other words, his expected net payoff

[19]In principle, the probability of filing for a low-damage plaintiff may depend on the specific A. Then, the expression "average low-damage client" is used here.

from filing and not filing should be the same: $0 = \gamma[\beta(0) + (1-\beta)S] - f$. Using the lawyer's indifference condition, we can compute the equilibrium probability that a defendant makes a zero offer β: $\beta = 1 - \frac{f}{\gamma S}$.

Step 5: Equilibrium $\boldsymbol{\lambda}$

Finally, we assess the optimal decision of the defendant at the care-taking stage. Given the equilibrium outcomes at the filing and litigation stages, the defendant's optimal probability of an accident is $\lambda_D = \arg\min\{K(\lambda) + \lambda l_D\}$, where $l_D = \int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA$ represents the defendant's expected litigation loss in equilibrium. The proof of Lemma 6 in the Appendix demonstrates that for any positive value of l_D, the function $K(\lambda) + \lambda l_D$ has a unique interior minimum $\lambda_D \in (0, 1)$.

Proposition 1 characterizes the mixed-strategy PBE under conditions (3)–(6).

$$f < \gamma \tilde{A}. \tag{3}$$

$$\min_{\tau \in [\tilde{A}, \bar{A}]} \left\{ [G(\tau)]\tau - \int_{\tilde{A}}^{\tau} (A + C_D) g(A) dA \right\} > 0. \tag{4}$$

For any $A \in [\tilde{A}, \bar{A}]$,

$$g(A) - C_D \frac{\partial g(A)}{\partial A} > 0. \tag{5}$$

$$\int_{\tilde{A}}^{\bar{A}} A g(A) dA + C_D[1 - G(\tilde{A}) - \bar{A} g(\bar{A})] > 0. \tag{6}$$

Condition (3) implies that the lawyer's net payoff $(\gamma \tilde{A} - f)$ is positive when an offer greater than or equal to $\tilde{A}$ is made in equilibrium. Then, this is a sufficient condition for low-damage cases to be filed if such an offer is made in equilibrium.

Condition (4) implies that, for any positive offer $\tau \in [\tilde{A}, \bar{A}]$, the defendant's expected loss from making this positive offer, $G(\tau)\tau + \int_{\tau}^{\bar{A}} (A + C_D) g(A) dA$, is strictly greater than his expected loss from making a zero offer, $\int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA$. This condition ensures that a zero offer is in the set of equilibrium offers, and hence, it is a necessary condition for the existence of the mixed-strategy PBE. Intuitively, when the lawyers' financial constraint are strong enough (i.e., when $\tilde{A}$ is high enough and hence, $\int_{\tilde{A}}^{\tau} (A + C_D) g(A) dA$ is low enough), then the mixed-strategy PBE emerges.

Condition (5) implies that the second derivative of the defendant's expected loss function l_D, $g(A) - C_D \frac{\partial g(A)}{\partial A}$, is greater than zero , i.e., l_D is strictly convex in the interval $[\tilde{A}, \bar{A}]$. Then, this condition ensures the uniqueness of the equilibrium positive settlement offer S. Condition (6) is a necessary condition for the existence of the mixed-strategy PBE with an equilibrium positive offer in the interval $(\tilde{A}, \bar{A})$, i.e., an interior positive offer.[20]

Proposition 1. *Assume conditions (3)-(6) hold. The following strategy profile, together with the defendant's beliefs, characterize the mixed-strategy perfect Bayesian equilibrium.*

(1) The defendant chooses a probability of accident $\lambda_D = \arg\min \left\{K(\lambda) + \lambda \int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA\right\}$. *If a lawsuit is filed, the defendant mixes between proposing a zero offer with probability* $\beta = 1 - \frac{f}{\gamma S}$*; and, proposing an offer* $S \in (\tilde{A}, \bar{A})$*, implicitly defined by equations (1) and (2), with the complementary probability.*

(2) A high-damage case is always filed by the plaintiff's lawyer; an average low-damage case is filed with probability $\frac{\nu}{G(\tilde{A})}$.

(3) A high-damage plaintiff always rejects a zero offer and accepts an offer $S > 0$ *if and only if* $A \leq S$*; a low-damage plaintiff always accepts a non-negative offer.*

(4) The defendant's equilibrium beliefs are as follows. When the defendant observes a lawsuit, he believes that $P(0 < A < \tilde{A}) = \frac{\nu}{\nu + 1 - G(\tilde{A})}$, $P(\tilde{A} \leq A \leq y) = \frac{G(y) - G(\tilde{A})}{\nu + 1 - G(\tilde{A})}$ *for any* $y \in [\tilde{A}, \bar{A}]$.[21]

Equilibrium No-Access to Justice η: Formal Characterization

We now formally characterize the No-Access to Justice component η under the mixed-strategy PBE. Given Definition 2 and the equilibrium outcomes:

$$\eta = \left[\int_0^{\tilde{A}} g(A) dA - \nu \right] + \nu\beta. \tag{7}$$

Intuitively, η represents the inability of true victims to get access to justice. It takes into account (1) the mass of low-damage potential plaintiffs who cannot file a lawsuit, $\int_0^{\tilde{A}} g(A) dA - \nu$;[22] and, (2) the mass of low-damage plaintiffs who file a lawsuit but receive a zero offer, and hence, need to drop their cases, $\nu\beta$. It is worth noting that ours is the first formal characterization of the access to justice concept in the literature.

Table 1 summarizes the equilibrium outcomes and no-access to justice component. Our previous analysis suggests that the lawyer's constraint $\tilde{A}$ affects equilibrium strategies and outcomes

[20] Formal proofs of the mixed- and pure-strategy PBE with corner solutions (i.e., where the equilibrium positive settlement offer is equal to the highest possible plaintiff's type $\bar{A}$) are presented in the Supplementary Material.

[21] See Lemma 7 in the Appendix for a detailed analysis of the defendant's equilibrium beliefs.

[22] Remember that $\int_0^{\tilde{A}} g(A) dA - \nu = 1 - \zeta = 1 - \left[\int_{\tilde{A}}^{\bar{A}} g(A) dA + \nu \right]$, where ζ is the mass of filed cases.

Table 1: Equilibrium Outcomes and No-Access to Justice Component (Mixed-Strategy PBE)

Probability of Zero Offer	$\beta = 1 - \frac{f}{\gamma S}$
Mass of Cases that Proceed to Trial	$\rho = \beta[1 - G(\tilde{A})] + (1-\beta)[1 - G(S)]$
Probability of Filing	$\zeta = \nu + \int_{\tilde{A}}^{\bar{A}} g(A)dA$
Probability of Accident	$\lambda_D = \arg\min\{K(\lambda) + \lambda l_D\}$
Defendant's Expected Litigation Loss	$l_D = \int_{\tilde{A}}^{\bar{A}} (A + C_D)g(A)dA$
No-Access to Justice Component	$\eta = \left[\int_0^{\tilde{A}} g(A)dA - \nu\right] + \nu\beta$

Notes: Probability of filing conditional on accident occurrence; other outcomes conditional on accident occurrence and filing.

under the mixed-strategy PBE. Importantly, the mixed-strategy PBE and the characterization of the η component are aligned with the actual state of affairs in civil litigation. In real-word settings, lawyers confront financial constraints. As a result, only some low-damage victims get legal representation and file a lawsuit. Financial constraints also preclude lawyers to bring cases associated with low-damage clients to trial. Then, when defendants refuse to make a settlement offer, then lawyers are forced to drop the lawsuit. As a result, some victims get deprived of their civil right to access to justice.

3.2 Pure-Strategy PBE

When the sign of condition (4) is reversed, i.e., when $G(\tau)\tau - \int_{\tilde{A}}^{\tau}(A + C_D)g(A)dA < 0$ for some $\tau \in [\tilde{A}, \bar{A}]$, then the pure-strategy PBE emerges. Intuitively, in a state of the world characterized by mild lawyers' financial constraints, then low-damage plaintiffs are relatively rare and high-damage plaintiffs are relatively common. The defendant will reduce his expected litigation loss by always making a positive offer because high-damage plaintiffs will always reject a zero offer and proceed to costly trial. Hence, when the level of lawyers' financial constraints is mild, then the defendant will have a strong incentive to always make a positive offer.

In the pure-strategy equilibrium, lawyers with all types of cases cases file lawsuits. Due to asymmetric information, some high-damage plaintiffs receive an insufficiently high offer (i.e., an offer lower than the actual harm from the accident) and proceed to costly trial, and all low-damage plaintiffs receive a generous offer (i.e., an offer greater than the actual harm from the accident) and settle out-of-court. Although attorneys with low-damage cases cannot proceed to trial due to their financial constraints, their expectations about a positive out-of-court settlement offer induce them to file a lawsuit. Given that all low-damage cases are filed, and all filed low-damage cases receive a positive offer, all victims get access to justice.

Equilibrium Strategies

First, we analyze the potential composition of the set of equilibrium settlement offers and the potential composition of the equilibrium mass of filed cases. The proof of Lemma 8 shows that the set of equilibrium offers might only involve pure strategies with positive offers $S \in [\tilde{A}, \bar{A}]$. By condition (3), lawyers with high- and low-damage cases will get strictly positive expected payoffs. Then, all cases will be filed. Hence, the equilibrium mass of low-damage cases that are filed will be equal to the mass of low-damage cases, $\nu = G(\tilde{A})$, and the equilibrium mass of filed cases will be equal to the mass of potential plaintiffs, $\zeta = 1$.

We now demonstrate that the equilibrium settlement offer S exists and is unique. The defendant's expected litigation loss $l_D(S) = G(S)S + \int_S^{\bar{A}}(A + C_D)g(A)dA$. The first term, $G(S)S$, represents the loss associated with the acceptance of the out-of-court settlement offer S by low- and high-damage plaintiffs. The second term, $\int_S^{\bar{A}}(A + C_D)g(A)dA$, represents the loss associated with high-damage cases that proceed to trial.[23] Minimization of $l_D(S)$ yields the first-order condition:

$$G(S) = C_D g(S). \tag{8}$$

When $g(S) - C_D \frac{\partial g(S)}{\partial S} > 0$ for all $S \in [\tilde{A}, \bar{A}]$, the second-order optimality is satisfied. Then, the function is strictly convex and has a unique minimum. When $1 - C_D g(\bar{A}) > 0$, the optimal offer S is interior (i.e., $S \in (\tilde{A}, \bar{A})$).

Finally, we assess the optimal decision of the defendant at the care-taking stage (see Lemma 9 in the Appendix). By applying the logic used in case of the mixed-strategy PBE, we show that the defendant's equilibrium probability of an accident $\lambda_D = \arg\min K(\lambda) + \lambda l_D$ exists, is unique, and is decreasing in l_D.

Table 2 summarizes the equilibrium outcomes and no-access to justice component under the pure-strategy PBE. Proposition 2 characterizes the pure-strategy PBE under conditions (3), (4′), (5) and (6′). Conditions (4′) and (6′) are as follows.

$$\min_{\tau \in [\tilde{A}, \bar{A}]} \left\{ G(\tau)\tau - \int_{\tilde{A}}^{\tau} (A + C_D)g(A)dA \right\} < 0. \tag{4'}$$

[23] The net award at trial also determines the likelihoods of out-court-settlement and trial Bebchuk (1984). Note, however, that neither filing nor care-taking precautions are endogenous in Bebchuk (1984).

Table 2: Equilibrium Outcomes and No-Access to Justice Component (Pure-Strategy PBE)

Probability of a Zero Offer	$\beta = 0$
Mass of Cases that Proceed to Trial	$\rho = [1 - G(S)]$
Probability of Filing	$\zeta = 1$
Probability of Accident	$\lambda_D = \arg\min\{K(\lambda) + \lambda l_D\}$
Defendant's Expected Litigation Loss	$l_D = SG(S) + \int_S^{\bar{A}} (A + C_D) g(A) dA$
No-Access to Justice Component	$\eta = 0$

Notes: Probability of filing conditional on accident occurrence; other outcomes conditional on accident occurrence and filing.

$$1 - C_D g(\bar{A}) > 0. \tag{6$'$}$$

Condition (4′) implies that for some positive offer $\tau \in [\tilde{A}, \bar{A}]$, the defendant's expected loss from making this offer, $G(\tau)\tau + \int_\tau^{\bar{A}} (A + C_D) g(A) dA$, is strictly lower than his expected loss from making a zero offer, $\int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA$. This condition ensures that a zero offer is not made in equilibrium. Then, this is a necessary condition for the existence of the pure-strategy PBE. Condition (6′) is a necessary condition for the existence of the pure-strategy PBE with an equilibrium positive offer in the interval $(\tilde{A}, \bar{A})$, i.e., an interior offer, i.e., an offer $S \in (\tilde{A}, \bar{A})$.

Proposition 2. *Assume that conditions (3), (4′), (5) and (6′) hold. The following strategy profile, together with the defendant's beliefs, characterize the pure-strategy perfect Bayesian equilibrium.*

(1) The defendant chooses a probability of accident $\lambda_D = \arg\min \big\{K(\lambda) + \lambda l_D(S)\big\}$*, where* $l_D(S) = G(S)S + \int_S^{\bar{A}} (A + C_D) g(A) dA$*. If a lawsuit is filed, the defendant always proposes an offer* $S \in (\tilde{A}, \bar{A})$*, implicitly defined by* $G(S) = C_D g(S)$*.*

(2) All cases are filed by the plaintiff's lawyer.

(3) A high-damage plaintiff rejects an offer $S \in (\tilde{A}, \bar{A})$ *if and only if* $A \geq S$*; a low-damage plaintiff always accepts* $S \in (\tilde{A}, \bar{A})$*.*

(4) The defendant's equilibrium beliefs are as follows. When the defendant observes a lawsuit, he believes that $P(0 < A \leq y) = G(y)$*.*[24]

3.3 An Illustration: Uniform Distribution of Damages

A simple example using a uniform distribution of damages illustrates the model's results. Suppose that the plaintiff's damage types A are uniformly distributed on the interval $(0, \bar{A}]$. Suppose also

[24]See Lemma 10 in the Appendix for a detailed analysis of the defendant's equilibrium beliefs.

that the injurer's cost of care function is $K(\lambda) = B[1 - \sqrt{1-(\lambda-1)^2}]$, where $B > 0$ is the cost of care when $\lambda = 0$. In this example, conditions (4) and (4′) become $\tilde{A} > C_D(\sqrt{2}-1)$ and $\tilde{A} < C_D(\sqrt{2}-1)$, respectively.

Intuitively, in a state of the world where the the lawyers confront high constraints, i.e., where $\tilde{A} > C_D(\sqrt{2}-1)$, the mixed-strategy PBE emerges. Using equation (1) and (2), it can be demonstrated that the defendant's equilibrium positive settlement offer is $S = \sqrt{\tilde{A}^2 + 2\tilde{A}C_D}$, which will be offered with probability $1-\beta = \frac{f}{\gamma S}$, and the mass of low-damage cases that are filed $\nu = \frac{\tilde{A}+C_D-S}{\bar{A}}$. Importantly, the no-access to justice component $\eta = \frac{\tilde{A}}{\bar{A}} - \nu + \nu\beta > 0$. Hence, some low-damage plaintiffs will be deprived of their right of access to justice under the mixed-strategy PBE.

In contrast, in a state of the world where the the lawyers confront mild constraints, i.e., where $\tilde{A} < C_D(\sqrt{2}-1)$, the pure-strategy PBE emerges. Using equation (8), it can be shown that the equilibrium settlement offer is $S = C_D$, which is always offered by the defendant, i.e, it is offered with probability $1-\beta = 1$. As a result, in addition to all high-damages cases, all low-damage cases are filed, i.e., the mass of low-damage cases that are filed is equal to the mass of low-damage plaintiffs $\nu = \frac{\tilde{A}}{\bar{A}} = G(\tilde{A})$. Due to the absence of a zero offer in equilibrium and the filing of all low-damage cases, all victims will get access to justice.

Our previous analysis demonstrates that the lawyers' constraints represented by $\tilde{A}$ do not affect the equilibrium strategies and outcomes under the pure-strategy PBE.[25] Hence, $\tilde{A}$ does not affect social welfare. Importantly, all victims get access to justice under the pure-strategy PBE. Given that our main goal is to assess the effect of policies aimed at relaxing the lawyers' constraints on equilibrium outcomes, access to justice and social welfare, the focus of the next sections will be on the mixed-strategy PBE.

4 Effect of a Cost-Reducing Policy on Equilibrium Outcomes and Access to Justice

This section studies the effect of a cost-reducing policy on the equilibrium outcomes and access to justice under the mixed-strategy PBE. For instance, consider the effect of a policy aimed at reducing the lawyer's financial cost of loans r. Given the definition of $\tilde{A}$, a reduction in r lowers $\tilde{A} \equiv \frac{C_P}{\gamma} + \frac{(C_P+f-x)r}{\gamma}$. Importantly, a reduction in r also lowers $FC \equiv \frac{(C_P+f-x)r}{\gamma}$. Note also that

[25] In fact, the only exogenous parameter that affects equilibrium strategies and outcomes under the pure-strategy PBE is the defendant's litigation cost C_D.

the lawyer's constraint $\tilde{A}$ decreases (increases) if and only if the lawyer's financial constraint FC decreases (increases). Then, it is appropriate to perform comparative-statics analysis of cost-reducing policies aimed at lowering the lawyer's financial constraint through the analysis of the effects of changes in $\tilde{A}$.[26] Formal proofs are presented in the Appendix.

4.1 Effect on Equilibrium Outcomes

Proposition 3 summarizes the comparative statics results of a reduction in $\tilde{A}$.

Proposition 3. *A reduction in $\tilde{A}$: (1) increases the expected litigation loss of the defendant l_D; (2) reduces the probability of accident λ_D; (3) reduces the positive out-of-court settlement offer S; (4) increases the probability of a positive offer $(1-\beta)$; (5) increases the mass of filed cases ζ; and (6) increases the probability of trial ρ if $C_D < \tilde{A}$.*[27]

Intuitively, a relaxation of the lawyers' financial constraints increases the mass of filed cases ζ by inducing more cases with low A to be filed. Due to the asymmetry of information between the plaintiff and the defendant, the higher likelihood of confronting plaintiffs with low A induces the defendant to lower the positive settlement offer S. Given the inverse relationship between the positive settlement offer and the probability of making that offer $(1-\beta) = \frac{f}{\gamma S}$, the probability of making a positive offer increases. Although a positive offer is more frequently made, this offer is lower. As a result, the likelihood of trial ρ and hence, the expected litigation costs, increase. As a consequence of the negative impact of the higher likelihoods of filing and trial (and higher litigation costs), which are not offset by the positive effect of the lower positive settlement offer, the defendant's expected loss l_D increases and hence, his expenses on care also increase. This latter effect reduces the likelihood of accident λ_D.[28]

4.2 Effect on No-Access to Justice

The effect of a reduction in $\tilde{A}$ on No-Access to Justice η are outlined in Proposition 4.

[26]This analysis applies to the other four exogenous parameters encompassed in FC: C_P, f, γ, and x.

[27]Intuitively, if the defendant's trial costs C_D are low enough, then the decrease in S due to a reduction in $\tilde{A}$ will be strong enough to increase the probability of trial. This is a sufficient (but not necessary) condition.

[28]Consider a binary-type version of our model. Assume that $A_L < \tilde{A} < A_H$, where A_L and A_H represent the low and high type, respectively. It is simple to show that the equilibrium outcomes do not depend on C_P, x and r. Then, comparative statics analysis with respect to the determinants of the lawyers' constraints cannot be performed using a binary-type model. These results apply to any discrete-type version of our model.

Proposition 4. *A reduction in $\tilde{A}$ reduces No-Access to Justice η.*

A relaxation of the lawyers' financial constraints increases access to justice by victims by affecting η in two ways. First, a reduction in $\tilde{A}$ induces more cases with low A to be filed. Specifically, a lower $\tilde{A}$ increases the mass of high-damage cases $1-G(\tilde{A})$: Some initially low-damage cases become high-damage cases. Remember that high-damage cases are always filed. Second, a reduction in $\tilde{A}$ reduces the mass of low-damage cases that are dropped. Specifically, a lower $\tilde{A}$ reduces the likelihood of a zero offer β.

5 Social Welfare and Deterrence

This section characterizes the social welfare loss function and presents an analysis of deterrence for the mixed-strategy PBE.

5.1 Social Welfare Loss Function

The social welfare loss function SWL includes the social welfare loss associated with the resources devoted to accident prevention $K(\lambda)$, and the unconditional social welfare loss from an accident λl_W. The social welfare loss from an accident l_W includes the social welfare loss associated with the expected harm from an accident $H = \int_0^{\bar{A}} Ag(A)dA$, and the social welfare loss associated with the resources devoted to filing, ζf, and litigation, $\rho(C_P + C_D)$, conditional on accident occurrence. Second, it includes the social welfare loss associated with the expected infringement of citizens' right of access to justice, $\theta\eta$, where $\theta \geq 0$ represents the society's concern regarding the citizens' right of access to justice.[29] Preserving the right of access to justice is an increasing concern for the U.S. and the international community. The U.S. Department of Justice established the *Access to Justice Initiative* (ATJ) in March 2010. Similarly, the United Nations considered for the first time including Access to Justice as part of its *Sustainable Development Goals* (Goal 16) in 2015.

Definition 3. *The social welfare loss function SWL is defined as follows.*

$$SWL(\lambda) = K(\lambda) + \lambda l_W, \tag{9}$$

where

$$l_W = H + \zeta f + \rho(C_P + C_D) + \theta\eta. \tag{10}$$

[29] The higher θ, the higher the social welfare loss associated with no-access to justice by victims. See Kornhauser (2015, 2003) for a discussion on economic analysis of law and welfare analysis of public policies. See also Chang (2000).

5.2 Analysis of Deterrence

This section presents an analysis of deterrence. We first formally define undeterrence and overdeterrence. We then identify the main components that affect deterrence. Finally, we present a numerical example to illustrate the deterrence components.

Definition 4. *The defendant's deterrence level is defined as follows.*

(1) Underdeterrence: $l_W - l_D > 0$.

(2) Overdeterrence: $l_W - l_D < 0$.

Let $\lambda_W = \arg\min \left\{ K(\lambda) + \lambda l_W \right\}$ represent the socially-optimal probability of accidents. By Lemma 6 in the Appendix, $\lambda_W < \lambda_D$ if and only if $l_W > l_D$, and $\lambda_W > \lambda_D$ if and only if $l_W < l_D$. Intuitively, underdeterrence involves private expenses on accident prevention below the socially optimal. As a result, the likelihood of accident is above the socially optimal. Conversely, overdeterrence involves private expenses on accident prevention above the socially optimal. Hence the likelihood of accident is below the socially optimal.

Deterrence Components

The defendant's expected litigation loss l_D, given by each side of equation (1), can be also expressed as:

$$l_D = (1-\beta)\left[\nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}} (A + C_D)g(A)dA)\right] + $$
$$+\beta\left[\int_{\tilde{A}}^{\bar{A}} (A + C_D)g(A)dA\right].$$

Then, $l_W - l_D$ is given by:

$$l_W - l_D =$$
$$= [H + \zeta f + \rho(C_P + C_D) + \theta\eta] -$$
$$-\left\{(1-\beta)\left[\nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}} (A + C_D)g(A)dA)\right] + \beta\left[\int_{\tilde{A}}^{\bar{A}} (A + C_D)g(A)dA\right]\right\}.$$

Rearranging terms:

$$l_W - l_D =$$
$$= \left\{\rho C_D - \left[\beta \int_{\tilde{A}}^{S} g(A)dA + (1-\beta)\int_{S}^{\bar{A}} g(A)dA\right] C_D\right\} +$$
$$+(\zeta f + \rho C_P + \theta\eta) +$$

$$+\Big\{H-\Big\{(1-\beta)\Big[\nu S+\int_{\tilde{A}}^{S} Sg(A)dA+\int_{S}^{\bar{A}} Ag(A)dA)\Big]+\beta\Big[\int_{\tilde{A}}^{\bar{A}} Ag(A)dA\Big]\Big\}\Big\}.$$

Consider the first term (in curly brackets, first line). Note that ρC_D represents the social loss associated with resources devoted to defendant's litigation costs;[30] and, $\big[\beta\int_{\tilde{A}}^{S} g(A)dA+(1-\beta)\int_{S}^{\bar{A}} g(A)dA\big]C_D$ represents the defendant's expected loss associated with litigation costs paid at trial when zero and positive offers are rejected by high-damage plaintiffs. The social and private losses associated with the defendant's litigation costs should be equal. Then, this first term is equal to zero. Hence,

$$l_W - l_D =$$

$$= (\zeta f+\rho C_P+\theta\eta)+$$

$$+\Big\{H-\Big\{(1-\beta)\Big[\nu S+\int_{\tilde{A}}^{S} Sg(A)dA+\int_{S}^{\bar{A}} Ag(A)dA)\Big]+\beta\Big[\int_{\tilde{A}}^{\bar{A}} Ag(A)dA\Big]\Big\}\Big\}. \qquad (11)$$

The first term (in parentheses, first line) includes the social loss associated with resources devoted to filing and plaintiff's litigation costs, and the social loss associated with the infringement of victims' right of access to justice. The defendant does not consider these three items. Then, this term is greater than zero.

The second term (in curly brackets, second line) includes the social loss associated with the expected harm from an accident H experienced by all victims (harm from an accident related to low-damage cases that are filed, low-damage cases that are not filed, and high-damage cases that are always filed) minus the expected compensation paid by the defendant (in the form of an out-of-court settlement amount or an award at trial). The second term might be greater or lower than zero.

The second term is positive when the harm caused by an accident is undercompensated, i.e., when H is greater than the expected compensation paid by the defendant. This occurs when undercompensation (to low-damage plaintiffs who receive a zero offer and are forced to drop their cases and to low-damage victims who cannot file a lawsuit) more than offsets overcompensation (to high- and low-damage plaintiffs who receive a positive offer greater than their actual damages). Conversely, the second term is negative when the harm caused by an accident is overcompensated, i.e., when the expected compensation paid by the defendant is greater than H. This occurs when overcompensation (to low- and high-damage plaintiffs who receive a positive offer greater than their actual damages) more than offsets undercompensation (to low-damage plaintiffs who receive a zero offer and are forced to drop their cases and low-damage victims who cannot file a lawsuit).

[30]Remember that the equilibrium mass of cases that proceed to trial is $\rho=\beta\int_{\tilde{A}}^{S} g(A)dA+(1-\beta)\int_{S}^{\bar{A}} g(A)dA$.

It is worth noting that infringement of some victims' right of access to justice will occur even in case of overcompensation. This result is aligned with real-world environments.

Hence, underdeterrence ($l_W - l_D > 0$) or overdeterrence ($l_W - l_D < 0$) might occur in our environment.[31] Proposition 5 establishes a sufficient condition for underdeterrence.

Proposition 5. *There exists $\bar{f} > 0$ such that, for any $f < \bar{f}$, $l_W - l_D > 0$.*

The intuition is as follows. Remember that the probability that the defendant makes a positive offer is $(1 - \beta) = \frac{f}{\gamma S}$. Overcompensation to low-damage plaintiffs and to high-damage plaintiffs with $A \in [\tilde{A}, S)$ might occur only when the defendant makes a positive out-of-court settlement offer.[32] Specifically, overcompensation to low-damage plaintiffs occurs with probability $(1-\beta)\frac{\nu}{G(\tilde{A})}$, and overcompensation to high-damage plaintiffs with $A \in [\tilde{A}, S)$ occurs with probability $(1 - \beta)$. Then, low filing costs f imply a low probability of overcompensation, and hence, a low probability of overdeterrence. As we show in the Appendix, when f is low enough ($f < \bar{f}$), then the social loss from an accident l_W is greater than the defendant's private expected loss from litigation l_D. Hence, the defendant is underdeterred.

It is worth noting that, even if the defendant is overdeterred, when the concern of society about access to justice is strong enough (i.e., when θ is large enough), the overall welfare effect of a cost-reducing policy will be always positive. (See Section 6.3 for details.)

A Numerical Example of Deterrence

We present a numerical example to illustrate the deterrence components. We use the uniform-distribution model presented in Section 3.3. We adopt the following set of exogenous parameters: $\{C_P, C_D, \gamma, f, x, \theta, \bar{A}, B, r\} = \{200, 500, .33, 70, 100, 750, 1200, 1000, .18\}$. The model conditions hold under the chosen parameter set. The MATLAB software is used to construct the numerical example.

The equilibrium outcomes under this numerical example are as follows. First, the social loss associated with resources devoted to filing and plaintiff's litigation costs and the social loss associated with the infringement of victims' right of access to justice are equal to 530 (first term of

[31] When f is sufficiently low, which is likely to occur in real-world settings, then $\beta = 1 - \frac{f}{\gamma S}$ is sufficiently close to unity. As a result, $\{(1 - \beta)[\nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}} Ag(A)dA)] + \beta[\int_{\tilde{A}}^{\bar{A}} Ag(A)dA]\}$ is sufficiently close to $[\int_{\tilde{A}}^{\bar{A}} Ag(A)dA]$. Given that $H = \int_0^{\bar{A}} Ag(A)dA$, then the second term of equation (11) (in curly brackets, second line) will approach $\int_0^{\tilde{A}} Ag(A)dA > 0$. Hence, we might expect that $l_W - l_D > 0$. In other words, we might expect that underdeterrence will occur.

[32] Remember that high-damage plaintiffs with $A \in (S, \bar{A}]$ always reject an offer S and proceed to trial.

equation (11)). Note that these costs are not considered by the defendant. Second, the expected harm from an accident is 600 and the expected compensation paid by the defendant is only 428. In other words, the harm caused by an accident is undercompensated. The undercompensated harm is equal to $600 - 428 = 172$ (second term of equation (11)). Then, $l_W - l_D = 530 + 172 = 702$. Hence, in this numerical example, the defendant is underdeterred.

6 Effect of a Cost-Reducing Policy on Social Welfare

We analyze the social welfare effect of a cost-reducing policy for the mixed-strategy PBE. Formal proofs are presented in the Appendix.

We show that a cost-reducing policy affects social welfare in several ways. First, a cost-reducing policy positively affects social welfare by increasing access to justice, and negatively affects social welfare by increasing the social costs associated with higher probabilities of filing and trial. In addition, a cost-reducing policy affects social welfare by increasing the defendant's care-taking incentives, and hence, by reducing the probability of accidents. This effect is positive if the defendant is initially underdeterred and hence the policy contributes to the alignment of the private and social probabilities of accident. We provide a necessary and sufficient condition for a positive overall welfare effect of a cost-reducing policy: When when society's concern of preserving access to justice for true victims is strong enough, then a cost-reducing policy is always welfare improving.

We start our analysis by decomposing the overall welfare effect into two components, indirect and direct effects. The indirect effect captures the impact of a cost-reducing policy on the potential injurer's care-taking incentives and hence, on the privately optimal probability of accidents λ_D. The direct effect captures the impact of a cost-reducing policy on the social loss from an accident l_W. Then, the indirect and direct effects are computed for a given l_W and a given λ_D, respectively.

Let $SWL(\lambda_D) = K(\lambda_D) + \lambda_D l_W$ be the social welfare loss function evaluated at the privately-optimal probability of an accident λ_D. The overall welfare effect (OE) of a reduction in $\tilde{A}$ is given by:

$$\frac{dSWL(\lambda_D)}{d\tilde{A}} = \frac{\partial SWL(\lambda_D)}{\partial \lambda_D}\frac{\partial \lambda_D}{\partial \tilde{A}} + \frac{\partial SWL(\lambda_D)}{\partial \tilde{A}}.$$

The first term in the right-hand side of the equation represents the indirect effect (IE) while the second term represents the direct effect (DE).

6.1 Direct Effect (DE)

The direct effect can be computed by explicit differentiation:

$$\frac{\partial SWL(\lambda_D)}{\partial \tilde{A}} = \lambda_D f \frac{\partial \zeta}{\partial \tilde{A}} + \lambda_D (C_P + C_D) \frac{\partial \rho}{\partial \tilde{A}} + \lambda_D \theta \frac{\partial \eta}{\partial \tilde{A}}. \tag{12}$$

The direct effect includes two negative welfare effects: (1) The effect of larger filing costs due to higher probability of filing (first term in f; $\frac{\partial \zeta}{\partial \tilde{A}} < 0$, by Proposition 3); and, (2) the effect of larger litigation costs due to higher probability of trial (second term in $C_P + C_D$; $\frac{\partial \rho}{\partial \tilde{A}} < 0$, by Proposition 3). It also includes a positive welfare effect: The effect on the No-Access to Justice component η. By Proposition 4, $\frac{\partial \eta}{\partial \tilde{A}} > 0$, i.e, a reduction in $\tilde{A}$ lowers No-Access to Justice η. This positive effect offsets the negative effects if, for instance, the concern of society about preserving the citizens' right of access to justice is strong enough (i.e., if the value of θ is large enough). In this case, the direct effect of a cost-reducing policy will be welfare improving.[33]

6.2 Indirect Effect (IE)

To compute the indirect effect $\frac{\partial SWL(\lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial \tilde{A}}$, first we take into account that at the point of the defendant's optimum, λ_D, the first-order optimality condition implies:

$$\frac{\partial K(\lambda_D)}{\partial \lambda_D} = -l_D.$$

Differentiating the first-order optimality condition with respect to $\tilde{A}$ yields:

$$\frac{\partial \lambda_D}{\partial \tilde{A}} = -\frac{\frac{\partial l_D}{\partial \tilde{A}}}{\frac{\partial^2 K(\lambda_D)}{\partial \lambda_D^2}} = \frac{(\tilde{A} + C_D) g(\tilde{A})}{\frac{\partial^2 K(\lambda_D)}{\partial \lambda_D^2}}.^{34}$$

Second, we consider that:

$$\frac{\partial SWL(\lambda_D)}{\partial \lambda_D} = \left[\frac{\partial K(\lambda_D)}{\partial \lambda_D} + l_W \right] = -l_D + l_W.$$

Then, the indirect effect can be written as:

$$\frac{\partial SWL(\lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial \tilde{A}} = (l_W - l_D) \frac{\partial \lambda_D}{\partial \tilde{A}},$$

where $(l_W - l_D)$ is given by equation (11). Hence, then the indirect effect can be written as:

$$\frac{\partial SWL(\lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial \tilde{A}} =$$

[33] When $\theta = 0$, the direct effect will be always welfare reducing.

[34] By Proposition 3, $\frac{\partial \lambda_D}{\partial \tilde{A}} > 0$.

$$= \Big\{ \big(\zeta f + \rho C_P + \theta\eta \big) +$$

$$+ \Bigg\{ H - \Bigg\{ (1-\beta) \Bigg[\nu S + \int_{\tilde{A}}^{S} S g(A) dA + \int_{S}^{\bar{A}} A g(A) dA) \Bigg] + \beta \Bigg[\int_{\tilde{A}}^{\bar{A}} A g(A) dA \Bigg] \Bigg\} \Bigg\} \Bigg\} \frac{\partial \lambda_D}{\partial \tilde{A}}. \tag{13}$$

The indirect effect operates through a change in the defendant's care-taking incentives, i.e., through a change in l_D which affects the probability of accidents λ_D ($\frac{\partial \lambda_D}{\partial \tilde{A}}$ term). Note, however, that it is also influenced by the access to justice component ($\theta\eta$ term).

Given that $\frac{\partial \lambda_D}{\partial \tilde{A}} > 0$ (by Proposition 3), the sign of the indirect effect depends on the sign of the term $(l_W - l_D)$ (term multiplied by $\frac{\partial \lambda_D}{\partial \tilde{A}}$). That is, the sign of the indirect effect depends on the relationship between the social loss from an accident and the defendant's expected litigation loss. If the defendant is initially underdeterred ($l_D < l_W$ or $\lambda_D > \lambda_W$), then a reduction in λ_D will positively affect social welfare.[35] In other words, the indirect effect of a cost-reducing policy will be welfare enhancing by aligning the private and social probabilities of accidents. In contrast, if the defendant is initially overderdeterred ($l_W < l_D$ or $\lambda_D < \lambda_W$), then the indirect effect of a cost-reducing policy will be welfare reducing.

6.3 Overall Effect(OE)

The overall effect of a cost-reducing policy is:

$$\frac{dSWL(\lambda_D)}{d\tilde{A}} = \left[(l_W - l_D) \frac{\partial \lambda_D}{\partial \tilde{A}} \right] + \left[\lambda_D f \frac{\partial \zeta}{\partial \tilde{A}} + \lambda_D (C_P + C_D) \frac{\partial \rho}{\partial \tilde{A}} + \lambda_D \theta \frac{\partial \eta}{\partial \tilde{A}} \right], \tag{14}$$

where $(l_W - l_D)$ is given by equation (11). The first and second terms in brackets represent the indirect and direct effects of a cost-reducing policy, respectively.

It is worth noting that our framework generalizes seminal models of civil litigation (Bebchuk, 1984, 1988; Katz, 1990) by endogenizing the decisions associated with every step of a legal dispute. Then, our study provides detailed information of the effect of a cost-reducing policy on social welfare. As we discussed in the previous two sections, a cost-reducing policy affects social welfare through many individual forces, that although observed in real-world settings, produce in some cases opposite effects on social welfare. As result, the overall effect of a cost-reducing policy is generally ambiguous in our environment.

Proposition 6 provides a necessary and sufficient condition on θ for a positive overall effect of a cost-reducing policy. Intuitively, this condition describes the state of the world where cost-reducing policies will be always welfare improving.[36]

[35] In this case, a reduction in $\tilde{A}$ will reduce the social welfare loss, and hence, will increase social welfare.

[36] When $\theta = 0$, a cost-reducing policy will be welfare improving if the defendant is initially underdeterred and the positive indirect effect more than offsets the negative direct effect.

We start by isolating the terms in equation (14) that do not include η and the terms that include η. We first define Ω as:

$$\Omega \equiv$$

$$\equiv \Big\{ \big(\zeta f + \rho C_P \big) +$$

$$+ \Bigg\{ H - \Bigg\{ (1-\beta) \Bigg[\nu S + \int_{\tilde{A}}^{S} S g(A) dA + \int_{S}^{\bar{A}} A g(A) dA) \Bigg] + \beta \Bigg[\int_{\tilde{A}}^{\bar{A}} A g(A) dA \Bigg] \Bigg\} \Bigg\} \Bigg\} \frac{\partial \lambda_D}{\partial \tilde{A}} +$$

$$+ \Bigg[\lambda_D f \frac{\partial \zeta}{\partial \tilde{A}} + \lambda_D (C_P + C_D) \frac{\partial \rho}{\partial \tilde{A}} \Bigg],$$

where the first two lines (terms multiplied by $\frac{\partial \lambda_D}{\partial \tilde{A}}$) include the terms of the indirect effect that do not depend on η. The last line (in brackets) includes the terms of the direct effect that do not depend on η. Second, we define Λ as: $\Lambda \equiv \eta \frac{\partial \lambda_D}{\partial \tilde{A}} + \lambda_D \frac{\partial \eta}{\partial \tilde{A}}$. Then,

$$\Lambda \theta = \eta \frac{\partial \lambda_D}{\partial \tilde{A}} \theta + \lambda_D \frac{\partial \eta}{\partial \tilde{A}} \theta,$$

where the first component corresponds to the term of the indirect effect that depends on η, and the second component corresponds to the term of the direct effect that depends on η. Hence,

$$\frac{dSWL(\lambda_D)}{d\tilde{A}} = \Omega + \Lambda \theta.$$

Third, we define $\underline{\theta} \equiv \frac{-\Omega}{\Lambda}$.

Proposition 6. *A cost-reducing policy is welfare improving if and only $\theta > \underline{\theta}$.*

Intuitively, in a state of the world where society's concern about access to justice for victims is high enough ($\theta > \underline{\theta}$), the overall effect of a cost-reducing policy will be always welfare improving. Hence, cost-reducing policies might be recommended.

Finally note that, in this environment, the overall effect of a cost-reducing policy will be positive even if the defendant is initially overdeterred (i.e., even if $IE < 0$). The reason is that, when society's concern about preserving access to justice for victims is strong enough, the positive direct effect, driven by access to justice by more victims, will more than offset the negative effect associated with an increase in overdeterrence.

6.4 Effect of a Change in r on Social Welfare: A Numerical Example

We present a numerical example to illustrate the effect of a change in r (a specific cost-reducing policy) on social welfare.[37] We use the uniform-distribution model presented in Section 3.3 and

[37] The MATLAB software is used to construct this numerical example.

Table 3: Effect of a Change in r on Social Welfare when $\theta > \underline{\theta}$

	$\Delta r = .28 - .18$	$\Delta r = .18 - .08$
Direct Effect (DE) - Main Components	$DE > 0$	$DE > 0$
$\Delta\zeta$ (Probability of Filing)	+10.2%	+9.4%
$\Delta\frac{\varrho}{\zeta}$ (Probability of Trial)	+2.5%	+1.9%
$\Delta\eta$ (No-Access to Justice)	−7.3%	−7.9%
•$\Delta\nu$ (Mass of Low-Damage Cases that Are Filed)	+4.7%	+5.0%
•$\Delta\beta$ (Probability of Zero Offer)	−1.2%	−1.3%
Indirect Effect (IE) - Main Components	$IE > 0$	$IE > 0$
Δl_D (Defendant's Expected Litigation Loss)	+9.5%	+8.3%
$\Delta\lambda_D$ (Probability of Accidents)	−6.6%	−6.3%
Overall Effect (OE)	$OE > 0$	$OE > 0$
ΔSWL (Social Welfare Loss)	−3.2%	−2.8%

consider a state of the world characterized by $\theta > \underline{\theta}$. The parameter set adopted is as follows: $\{C_P, C_D, \gamma, f, x, \theta, \bar{A}, B\} = \{200, 500, .33, 70, 100, 750, 1200, 1000\}$. Three values of r are studied: $r \in \{.08, .18., .28\}$.[38]

Table 3 summarizes our findings. In this example, the direct and indirect welfare effects of a reduction in r are positive. The effect of changes in r on the main components associated with the direct and indirect welfare effects are reported. Our findings are consistent across changes in r. Consider, for instance, a reduction in r from 18% to 8%. This cost-reducing policy lowers social welfare loss by 3%. The result might be explained by the positive indirect effect of the policy that operates through the increase in the care-taking incentives for an initially underdeterred potential injurer (i.e., an 8% increase in l_D; as result, a 6% reduction in the probability of accident λ_D). The reduction in the social welfare loss can be also explained by the positive direct effect of the cost-reducing policy that operates through an 8% reduction in the mass of low-damage victims that do not get access to justice. The improvement in access to justice is driven by a 5% increase in the mass of low-damage victims who file lawsuits. It is also driven by the reduction in the probability of a zero offer by 1%, i.e., by the reduction in the likelihood that low-damage victims who file lawsuits will be forced to drop their cases.

Our results provide important policy implications. Specifically, our findings suggest that in a state of the world characterized by strong society's concern about access to justice by victims,

[38]The model conditions and the condition $\theta > \underline{\theta}$ hold under the chosen exogenous parameters. In this numerical example, SWL is a monotonically increasing function of r. The value of r that minimizes SWL is $r = .08$. The values for $\underline{\theta}$ are $\underline{\theta} \in \{192, 158, 125\}$, for $r \in \{.08, .18, .28\}$, respectively.

the negative direct effect of cost-reducing policies explained by the higher probabilities of filing and trial is more than offset by the positive direct effect of access to justice by more victims and the positive indirect effect of higher care-taking incentives for potential injurers initially underdeterred. As a result, the overall effect of cost-reducing policies is welfare increasing. Importantly, our analysis underscores the alignment of cost-reducing policies with society's goal of preserving citizens' right of access to justice.

7 Summary and Conclusions

This article presents a comprehensive economic analysis of civil litigation in the presence of financially-constrained lawyers. The decisions at all the stages associated with a legal dispute are endogenized. Our tractable framework allows for asymmetric information, financially-constrained lawyers, third-party lawyer lending institutions, and a continuum of plaintiff's types. We completely characterize the equilibria of the model. Our work provides important methodological contributions to the economic analysis of law by generalizing seminal economic models of civil litigation, providing the first formal definition of access to justice, and offering a complete welfare analysis of relevant public policies.

Our paper demonstrates that the state of the world regarding the financial constraints faced by lawyers determines whether the mixed- or the pure-strategy equilibrium emerges. In particular, the mixed-strategy equilibrium occurs under strong financial constraints. Importantly, we show that the financial constraints faced by lawyers permeate every decision made by the parties involved in a legal dispute. Accidents and pre-trial bargaining disagreement are observed in equilibrium, and access to justice is denied to some victims under the mixed-strategy equilibrium.

Our work provides significant lessons for policy makers. First, our welfare analysis of policies aimed at alleviating lawyers' financial constraints demonstrates that, in a state of the world characterized by high society's concerns about preserving the citizens' right of access to justice, these policies are always welfare improving. Moreover, our findings indicate that even if the potential injurers are initially overdeterred, these policies will enhance social welfare. Hence, cost-reducing policies might be recommended. Second, our study suggests that a comprehensive assessment of public policies associated with legal disputes should consider the effects of these policies on care-taking incentives for potential injurers, in addition to their effects on filing and litigation. Third, our analysis underscores the importance of incorporating the concept of access to justice to the study of the social welfare effect of relevant public policies.

In future work, we plan to extend our model to study the optimal design of contracts between the plaintiff's lawyer and the third-party lender. The new framework will include the third-party lender as a fourth player, and will allow for uncertainty about the outcome at trial in the form of court errors. Recourse and non-recourse loans, as well as other contract terms, will be evaluated. Filing and access to justice, pretrial bargaining, and care-taking incentives for potential injurers will be assessed in this environment. These, and other extensions, remain fruitful areas for future research.

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Appendix: Mixed- and Pure-Strategy PBE

Define $\hat{A} \equiv \frac{C_P+f+(C_P+f-x)r}{\gamma}$.[1]

Claim 1. *Let $\check{A} \equiv \inf\{A_{filed}\}$, $\hat{A} > \check{A} \geq \tilde{A}$, and that defendant makes a zero offer or an offer $S \in (0, \tilde{A})$. Then, $n(A) = 1$ for $A > \check{A}$.*

Proof. By definition of $\hat{A}$, a case with $A \geq \hat{A}$ is always filed. Remember that $n(A) \in [0,1]$ is the probability that a type A will be filed, for $A \in [\tilde{A}, \hat{A}]$. By assumption, $\hat{A} > \check{A} \geq \tilde{A}$. Then, there are two possible options: (1) Type $\check{A}$ is filed, and (2) type $\check{A}$ is not filed. Let b be the probability that a zero offer or an offer $S_1 \in (0, \check{A})$ is made by the defendant; and $(1-b)$ be the probability that an offer $S_2 \in [\check{A}, \bar{A}]$ is made by the defendant.[2] By assumption, the defendant makes a zero offer or an offer $S \in (0, \tilde{A})$; then, $b > 0$. A zero offer or an offer $S_1 \in (0, \check{A})$ will be always rejected by a plaintiff with type $\check{A}$. Consider option (1). Given that an attorney with a case $\check{A}$ files a lawsuit by assumption, then it must be the case that his expected payoff is non-negative: $b[\gamma\check{A} - C_P - (C_P + f - x)r] + (1-b)\gamma S_2 - f \geq 0$. Then, the expected payoff for an attorney with a case $A > \check{A}$: $b[\gamma A - C_P - (C_P + f - x)r] + (1-b)\gamma S_2 - f > 0$. An attorney with a case $A > \check{A}$ will always file a lawsuit. Hence, $n(A) = 1$. Now consider option (2). Given that $\check{A} \equiv \inf\{A_{filed}\}$, in any neighborhood of $\check{A}$, there exists $A_1 > \check{A}$ such that A_1 is filed. Then, all $A > A_1$ are also filed. The neighborhood of $\check{A}$ can be infinitely small. Then, all $A > \check{A}$ are filed. Hence, $n(A) = 1$. ■

1 Mixed-Strategy PBE – Proofs

Assume that conditions (3)–(6) hold.

Claim 2. *Suppose that condition (4) holds. Suppose that the defendant mixes between a zero offer and a positive offer $S \in [\tilde{A}, \bar{A}]$. Then, $G(\tilde{A}) > \nu$.*

Proof. By assumption, the defendant mixes between a zero offer and a positive offer $S \in [\tilde{A}, \bar{A}]$. Then, he must be indifferent between these two offers: $\int_{\tilde{A}}^{\bar{A}}(A + C_D)g(A)dA = \nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}}(A + C_D)g(A)dA$, where the left- and right-hand sides correspond to the defendant's expected litigation loss when he makes a zero offer and an offer $S \in [\tilde{A}, \bar{A}]$, respectively. By condition (4), for any $S \in [\tilde{A}, \bar{A}]$, including the positive offer that satisfies the indifference condition above: $G(S)S > \int_{\tilde{A}}^{S}(A+C_D)g(A)dA$. Then, $G(\tilde{A})S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}}(A + C_D)g(A)dA > \nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}}(A + C_D)g(A)dA$, for the positive offer that satisfies the indifference condition above. Hence, $G(\tilde{A}) > \nu$. ■

Lemma 1. *The set of equilibrium offers must include a zero offer and at least one positive offer $S \in [\tilde{A}, \bar{A}]$.*

Proof. First, it is simple to show that offers greater than $\bar{A}$ are strictly dominated by an offer equal to $\bar{A}$. Second, we will demonstrate that offers $S \in (0, \tilde{A})$ are not in the set of equilibrium offers.[3] Suppose not. An offer $S \in (0, \tilde{A})$ is an equilibrium offer. By definition of $\hat{A}$, a case with $A \geq \hat{A}$ is always filed. Remember also that $n(A) \in [0,1]$ is the probability that a type A will be filed, for $A \in [\tilde{A}, \hat{A}]$. Let the mass of low-damage cases that are filed be $\nu \in [0, G(\tilde{A})]$. Let $\epsilon > 0$ denote a small number. Consider the

[1] $\hat{A}$ refers to the damage threshold at which the lawyer's net payoff is zero when a high-damage case is solved at trial.

[2] An offer $S_2 \in [\check{A}, \bar{A}]$ must be considered to ensure that a lawyer with type $A < \hat{A}$ will file.

[3] S might be non-unique, i.e., multiple S might be made in equilibrium with positive probabilities.

case when $\nu \in (0, G(\tilde{A})]$. When the defendant makes an offer $S \in (0, \tilde{A})$, his expected litigation loss is $\nu S + \int_{\tilde{A}}^{\hat{A}}(A + C_D)n(A)g(A)dA + \int_{\hat{A}}^{\bar{A}}(A + C_D)g(A)dA$. The defendant is better off by deviating to a zero offer, which generates an expected litigation loss equal to $\int_{\tilde{A}}^{\hat{A}}(A+C_D)n(A)g(A)dA+\int_{\hat{A}}^{\bar{A}}(A+C_D)g(A)dA$. Contradiction follows. Consider now the case when $\nu = 0$. Let $\check{A} \equiv \inf\{A_{filed}\}$, where A_{filed} is the set of values of A that are filed and $\hat{A} \geq \check{A} \geq \tilde{A}$. Two options are possible: (1) $\check{A} = \hat{A}$ and (2) $\check{A} < \hat{A}$.[4] Under option (1), the defendant's expected litigation loss from offering $S \in (0, \tilde{A})$ and offering $\hat{A} + \epsilon$ are $\int_{\hat{A}}^{\hat{A}+\epsilon}(A + C_D)g(A)dA + \int_{\hat{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA$ and $\int_{\hat{A}}^{\hat{A}+\epsilon}(\hat{A} + \epsilon)g(A)dA + \int_{\hat{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA$, respectively. The defendant will be better off by deviating and offering $\hat{A} + \epsilon$: $\int_{\hat{A}}^{\hat{A}+\epsilon}(\hat{A} + \epsilon)g(A)dA < \int_{\hat{A}}^{\hat{A}+\epsilon}(A+C_D)g(A)dA$, when $C_D > \epsilon$. Contradiction follows. Under option (2), the defendant's expected litigation loss from offering $S \in (0, \tilde{A})$ will be $\int_{\check{A}}^{\check{A}+\epsilon}(A + C_D)n(A)g(A)dA + \int_{\check{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA$. His expected litigation loss from offering $\check{A} + \epsilon$ will be $\int_{\check{A}}^{\check{A}+\epsilon}(\check{A} + \epsilon)n(A)g(A)dA + \int_{\check{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA$. By Claim 1, $n(A) = 1$, for $A > \check{A}$. The defendant will be better off by deviating and offering $\check{A} + \epsilon$: $\int_{\check{A}}^{\check{A}+\epsilon}(\check{A} + \epsilon)g(A)dA < \int_{\check{A}}^{\check{A}+\epsilon}(A + C_D)g(A)dA$, when $C_D > \epsilon$. Contradiction follows. Hence, an offer $S \in (0, \tilde{A})$ cannot be in the set of equilibrium offers.

Third, we will demonstrate that a zero offer is in the set of equilibrium offers. Suppose not. There is not a zero offer in equilibrium. Then the defendant offers $S \in [\tilde{A}, \bar{A}]$.[5] By condition (3), all low- and high-damage cases will be filed. The defendant's expected litigation loss from offering S will be $G(S)S + \int_{S}^{\bar{A}}(A + C_D)g(A)dA$. By condition (4), $G(S)S + \int_{S}^{\bar{A}}(A + C_D)g(A)dA > \int_{\tilde{A}}^{\bar{A}}(A + C_D)g(A)dA$, where the right-hand side of the inequality represents the defendant's expected litigation loss from making a zero offer. Then, the defendant's is better off by deviating to a zero offer. Contradiction follows. Hence, a zero offer must be in the set of equilibrium offers. As a result, the probability that the defendant makes a zero offer $\beta > 0$. Fourth, we will show that a zero offer cannot be the only equilibrium offer. Suppose not. The defendant always makes a zero offer. Then, only cases with types $A \geq \hat{A}$ will be filed. These plaintiffs will always reject a zero offer. The defendant will be better off by deviating and offering $\hat{A} + \epsilon$: $\int_{\hat{A}}^{\hat{A}+\epsilon}(\hat{A} + \epsilon)g(A)dA + \int_{\hat{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA < \int_{\hat{A}}^{\hat{A}+\epsilon}(A + C_D)g(A)dA + \int_{\hat{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA$, when $C_D > \epsilon$. Contradiction follows. Hence, in addition to a zero offer, there must be at least one positive offer $S \in [\tilde{A}, \bar{A}]$ in the set of equilibrium offers. ■

Lemma 2. *There must be at least some low-damage cases that are filed in equilibrium. All high-damage cases are filed in equilibrium.*

Proof. First, we will demonstrate that there must be at least some low damage cases that are filed in equilibrium, $\nu > 0$. Suppose not. Low-damage cases are never filed. By definition of $\hat{A}$, $A \geq \hat{A}$ always files. By assumption, $\hat{A} \geq \check{A} \geq \tilde{A}$. Then, there are two possible options: (1) $\check{A} = \hat{A}$, and (2) $\check{A} < \hat{A}$. Consider option 1. The defendant will be better off by deviating from the mixed-strategy involving a zero offer and a positive offer $S \in [\tilde{A}, \bar{A}]$ to a pure-strategy involving an offer $\hat{A} + \epsilon$: $\int_{\hat{A}}^{\hat{A}+\epsilon}(\hat{A} + \epsilon)g(A)dA + \int_{\hat{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA < \int_{\hat{A}}^{\hat{A}+\epsilon}(A + C_D)g(A)dA + \int_{\hat{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA$, when $C_D > \epsilon$. Consider option 2. The defendant's expected litigation loss from the mixed-strategy involving a zero offer and a positive offer $S \in [\tilde{A}, \bar{A}]$ will be $\int_{\check{A}}^{\check{A}+\epsilon}(A + C_D)n(A)g(A)dA + \int_{\check{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA$. His expected litigation loss from offering $\check{A} + \epsilon$ will be $\int_{\check{A}}^{\check{A}+\epsilon}(\check{A} + \epsilon)n(A)g(A)dA + \int_{\check{A}+\epsilon}^{\bar{A}}(A + C_D)g(A)dA$. By

[4] In words, at least for some $A \in [\tilde{A}, \hat{A})$ are filed, $n(A) > 0$.

[5] S might be non-unique, i.e. multiple S might be made in equilibrium with positive probabilities.

Claim 1 in Appendix A, $n(A) = 1$, for $A > \breve{A}$. The defendant will be better off by deviating and offering $\breve{A} + \epsilon$: $\int_{\breve{A}}^{\breve{A}+\epsilon}(\breve{A} + \epsilon)g(A)dA < \int_{\breve{A}}^{\breve{A}+\epsilon}(A + C_D)g(A)dA$, when $C_D > \epsilon$. Then, a mixed-strategy with a zero offer and a positive offer $S \in [\tilde{A}, \hat{A}]$ is not in the set of equilibrium offers. Contradiction follows. Hence, $v > 0$.

Second, we will show that all high-damage cases are filed. We just demonstrated that at least some low-damage cases are filed in equilibrium. Then, it must be the case that at least some attorneys with low-damage cases will get non-negative expected payoffs: $\gamma[\beta(0) + (1 - \beta)S)] - f \geq 0$, where S and β represent a positive offer and the probability of getting a zero offer, respectively.[6] We also showed that $\beta > 0$ in equilibrium. Consider now the case of an attorney with $A \geq \tilde{A}$. The expected payoff for the attorney is $\beta[\gamma A - C_P - (C_P + f - x)r] + (1 - \beta)\gamma S - f$. This expression is non-negative for $A = \tilde{A}$ and positive for $A > \tilde{A}$. As a consequence, all high-damage cases will be filed. Hence, the equilibrium mass of cases that are filed $\zeta = \nu + \int_{\tilde{A}}^{\bar{A}} g(A)dA$, where $\nu > 0$. ■

Lemma 3. *The system of equations (1)–(2) has a unique solution (S, ν), where $S \in (\tilde{A}, \bar{A})$.*

Proof. Inserting equation (2) into (1) yields $\int_{\tilde{A}}^{S}(A + C_D - S)g(A)dA - S[C_D g(S) - G(S) + G(\tilde{A})] = 0$. After simplification, $\int_{\tilde{A}}^{S}(A + C_D - S)g(A)dA - S[C_D g(S) - G(S) + G(\tilde{A})] = \int_{\tilde{A}}^{S} Ag(A)dA + C_D[G(S) - G(\tilde{A}) - Sg(S)]$. Denote $\Phi(S) \equiv \int_{\tilde{A}}^{S} Ag(A)dA + C_D[G(S) - G(\tilde{A}) - Sg(S)]$. The proof proceeds in several steps: (1) $\Phi(S)$ is continuous on the interval $[\tilde{A}, \bar{A}]$; (2) $\frac{\partial \Phi(S)}{\partial S} > 0$; (3) $\Phi(\tilde{A}) < 0$; (4) $\Phi(\bar{A}) > 0$; (5) $\Phi(S) = 0$ has exactly one solution.

(1) Continuity of $\Phi(S)$ follows from the assumptions about $G(A)$ and $g(A)$ functions. (2) Differentiation and further algebraic transformations yield $\frac{\partial \Phi(S)}{\partial S} = S\left[g(S) - C_D \frac{\partial g(S)}{\partial S}\right] > 0$. The last inequality follows from condition (5). (3) $\Phi(\tilde{A}) = -\tilde{A}C_D g(\tilde{A}) < 0$. (4) By condition (4), $\Phi(\bar{A}) = \int_{\tilde{A}}^{\bar{A}} Ag(A)dA + C_D[1 - G(\tilde{A}) - \bar{A}g(\bar{A})] > 0$. (5) We have showed that $\Phi(S)$ is a strictly increasing and continuous function with $\Phi(\tilde{A}) < 0$ and $\Phi(\bar{A}) > 0$. Hence, there exists a unique $S \in (\tilde{A}, \bar{A})$ such that $\Phi(S) = 0$. By equation (2), existence and uniqueness of S implies existence and uniqueness of ν. ■

Lemma 4. *The equilibrium positive settlement offer $S \in (\tilde{A}, \bar{A})$ and the equilibrium mass of low-damage cases that are filed ν, which are implicitly defined by equations (1) and (2), exist and are unique.*

Proof. We have established that a zero offer and at least one positive offer are made in equilibrium. This implies that the positive equilibrium offer S must minimize the expected litigation loss of the defendant, and the defendant must be indifferent between offering the optimal positive offer S and a zero offer. The indifference condition is given by equation (1). The first-order optimality condition simplifies to equation (2). Lemma 3 demonstrates that the system of equations (1)–(2) has a unique solution, S and ν. By condition (5), the second-order optimality condition $g(A) > C_D \frac{\partial g(A)}{\partial A}$ is satisfied for all $A \in [\tilde{A}, \bar{A}]$. By Claim 2, $\nu < G(\tilde{A})$. ■

Lemma 5. *The equilibrium probability that the defendant makes a zero offer is $\beta = 1 - \frac{f}{\gamma S}$.*

Proof. An attorney with an average low-damage client mixes between filing and not filing.[7] Then, he must be indifferent between these two strategies: $f = \gamma[\beta(0) + (1 - \beta)S]$. This indifference condition allows us to compute the equilibrium β: $\beta = 1 - \frac{f}{\gamma S}$. ■

[6]The mixed-strategy equilibrium might involve a zero offer and multiple positive offers.

[7]In principle, the probability of filing for a low-damage plaintiff may depend on the specific A. Then, the expression "average low-damage client" is used here.

Lemma 6. *For any positive value of l, the function $K(\lambda)+\lambda l$ has a unique interior minimum, $\lambda^* \in (0,1)$, which is decreasing in l.*

Proof. By the assumptions on $K(\lambda)$, the first derivative of $K(\lambda)+\lambda l$ is continuous and strictly increasing, negative when λ approaches zero, and positive when λ approaches 1. Hence, $K(\lambda)+\lambda l$ has a unique interior minimum, λ^*. Totally differentiating the first derivative of $K(\lambda)+\lambda l$, $\frac{\partial^2 K(\lambda)}{\partial \lambda^2} d\lambda + dl = 0$. Hence, $\frac{\partial \lambda}{\partial l} = -\frac{1}{\frac{\partial^2 K(\lambda)}{\partial \lambda^2}} < 0$. ■

Lemma 7. *In equilibrium, the defendant's posterior beliefs are as follow: $P(0 < A < \tilde{A}|Filing) = \frac{\nu}{\nu+1-G(\tilde{A})}$, $P(\tilde{A} \leq A \leq y|Filing) = \frac{G(y)-G(\tilde{A})}{\nu+1-G(\tilde{A})}$ for any $y \in [\tilde{A}, \bar{A}]$.*

Proof. All high-damage cases and a subset of low-damage cases are filed. The mass of high-damage cases that are filed is equal to the total mass of high-damage cases, $1 - G(\tilde{A})$. ν includes only the subset of low-damage cases that are filed. Therefore, the total mass of filed cases is $1 - G(\tilde{A}) + \nu$. The posterior (conditional on filing) probability of an average low-damage case is $\frac{\nu}{1-G(\tilde{A})+\nu}$, while the posterior (conditional on filing) probability of an average high-damage case is $\frac{1-G(\tilde{A})}{1-G(\tilde{A})+\nu}$. Given that all high-damage cases are filed and the distribution $G(A)$ is known by the defendant, he computes the posterior probability for any range of high-damage cases as the product of conditional probability for this particular range (conditional on a case being a high-damage case) and the posterior probability of a high-damage case. Hence, for any $y \in (\tilde{A}, \bar{A}]$, $P(\tilde{A} < A \leq y|Filing) = \frac{G(y)-G(\tilde{A})}{1-G(\tilde{A})} \frac{1-G(\tilde{A})}{1-G(\tilde{A})+\nu} = \frac{G(y)-G(\tilde{A})}{1-G(\tilde{A})+\nu}$.
■

Proposition 3. *A reduction in $\tilde{A}$: (1) increases the expected litigation loss of the defendant l_D; (2) reduces the probability of an accident λ_D; (3) reduces the positive out-of-court settlement offer S; (4) reduces the probability of a zero offer β; (5) increases the mass of filed cases ζ; and (6) increases the probability of trial ρ if $C_D < \tilde{A}$.*

Proof.
(1) The defendant's expected litigation loss is $l_D = \int_{\tilde{A}}^{\bar{A}} (A+C_D) g(A) dA$. Then, $\frac{\partial l_D}{\partial \tilde{A}} = -(\tilde{A}+C_D) g(\tilde{A}) < 0$.
(2) By Lemma 6, an increase in the defendant's expected litigation loss, l_D, increases the spending on care and, therefore, reduces the probability of an accident.
(3) Totally differentiating equations (1) and (2) yields $d\nu = dS\left[C_D \frac{\partial g(S)}{\partial S} - g(S)\right]$ and $(S-\tilde{A}-C_D) g(\tilde{A}) d\tilde{A} = S d\nu$. From the last equation: $\frac{\partial \nu}{\partial \tilde{A}} = \frac{(S-\tilde{A}-C_D) g(\tilde{A})}{S}$ and $\frac{\partial S}{\partial \tilde{A}} = \frac{(\tilde{A}+C_D) g(\tilde{A})}{S\left[g(S) - C_D \frac{\partial g(S)}{\partial S}\right]}$. By condition (5), the last expression is greater than zero.
(4) The probability of a zero offer is: $\beta = 1 - \frac{f}{\gamma S}$. Then, $\frac{\partial \beta}{\partial \tilde{A}} = \frac{f}{\gamma S^2} \frac{\partial S}{\partial \tilde{A}} > 0$.
(5) Let $\zeta = \int_{\tilde{A}}^{\bar{A}} g(A) dA + \nu = 1 - G(\tilde{A}) + \nu$ represent the aggregate filing. Then, $\frac{\partial \zeta}{\partial \tilde{A}} = -g(\tilde{A}) + \frac{\partial \nu}{\partial \tilde{A}} = -\frac{(\tilde{A}+C_D) g(\tilde{A})}{S} < 0$.
(6) The probability of trial (conditional on accident occurrence) is: $\rho = \beta(1-G(\tilde{A})) + (1-\beta)(1-G(S)) = 1 - G(\tilde{A}) - \frac{f(G(S)-G(\tilde{A}))}{\gamma S}$. Then, $\frac{\partial \rho}{\partial \tilde{A}} = -g(\tilde{A}) + \frac{\partial S}{\partial \tilde{A}} \frac{f}{\gamma} \frac{[G(S)-G(\tilde{A})-g(S)S]}{S^2} < 0$. The last inequality holds because $G(S) - G(\tilde{A}) - g(S)S = C_D g(S) - \nu - g(S)S = (C_D - S) g(S) - \nu < (C_D - \tilde{A}) g(S) - \nu < 0$, when $C_D < \tilde{A}$.
■

Proposition 4. *A reduction in $\tilde{A}$ reduces No-Access to Justice η.*

Proof. $\eta = \left[\int_0^{\tilde{A}} g(A)dA - \nu\right] + \nu\beta$. Then, $\frac{\partial\eta}{\partial\tilde{A}} = g(\tilde{A}) - \frac{\partial\nu}{\partial\tilde{A}} + \frac{\partial\nu}{\partial\tilde{A}}\beta + \frac{\partial\beta}{\partial\tilde{A}}\nu = g(\tilde{A}) - \left[\frac{(S-\tilde{A}-C_D)}{S}\right]g(\tilde{A})\frac{f}{\gamma S} + \frac{\partial\beta}{\partial\tilde{A}}\nu > g(\tilde{A}) - g(\tilde{A})\left[1 - \frac{(\tilde{A}+C_D)}{S}\right] + \frac{\partial\beta}{\partial\tilde{A}}\nu > 0$. ■

Proposition 5. *There exists $\bar{f} > 0$ such that, for any $f < \bar{f}$, $l_W - l_D > 0$.*

Proof. Equation (11) can be rewritten as: $l_W - l_D = \left[\int_0^{\bar{A}} Ag(A)dA - \int_{\tilde{A}}^{\bar{A}} Ag(A)dA\right] + \left\{C_P[1-G(\tilde{A})] - \frac{f(C_P+C_D)}{\gamma S}[G(S) - G(\tilde{A})]\right\} + \left\{\left[\int_{\tilde{A}}^{\bar{A}} g(A)dA + \nu\right]f + \theta\left[\int_0^{\tilde{A}} g(A)dA - \nu + \nu\beta\right]\right\}$. The first term (brackets) and the third term (curly brackets) are positive. We will show that the second term (curly brackets) is positive for sufficiently small values of f. Four steps are included.
(1) $\tilde{A} = \frac{C_P+(f+C_P-x)r}{\gamma}$ is a continuous function of f.
(2) We have already proved that $\frac{\partial S}{\partial\tilde{A}} > 0$. If $S(\tilde{A})$ is a differentiable function, then it is also continuous in $\tilde{A}$.
(3) $S(\tilde{A}(f))$ is a continuous function of f. Therefore, $\Gamma(f) \equiv \left\{C_P[1-G(\tilde{A})] - \frac{f(C_P+C_D)}{\gamma S(\tilde{A}(f))}[G(S(\tilde{A}(f))) - G(\tilde{A})]\right\}$ is also a continuous function of f.
(4) $\lim_{f\to0}\Gamma(f) = \lim_{f\to0}\left\{C_P[1-G(\tilde{A})] - \frac{f(C_P+C_D)}{\gamma S(\tilde{A}(f))}[G(S(\tilde{A}(f))) - G(\tilde{A})]\right\} = C_P[1-G(\tilde{A})] > 0$. Hence by continuity of $\Gamma(f)$, there exists $\bar{f}$, such that for any $f < \bar{f}$, $\left\{C_P[1-G(\tilde{A})] - \frac{f(C_P+C_D)}{\gamma S}[G(S) - G(\tilde{A})]\right\} > 0$. ■

Proposition 6. *A cost-reducing policy is welfare improving if and only $\theta > \underline{\theta}$.*

Proof. Define $\underline{\theta} \equiv \frac{-\Omega}{\Lambda}$. Define $\Theta(\theta) \equiv \Omega + \Lambda\theta$. By assumption, $\theta \geq 0$. By propositions 3 and 4, $\Lambda > 0$. Hence, $\Theta(\theta) = \Omega + \Lambda\theta$ is an upward-sloping linear function in θ. There two possible cases. (1) $\Omega > 0$: $\underline{\theta} < 0$. Then, $\theta > \underline{\theta}$ and $\Theta(\theta) > 0$. Hence, $\Theta(\theta) > 0$ if and only if $\theta > \underline{\theta}$. (2) $\Omega \leq 0$: $\underline{\theta} \geq 0$. Then, $\Theta(\underline{\theta}) = 0$. Hence, $\Theta(\theta) > 0$ if and only if $\theta > \underline{\theta}$. ■

2 Pure-Strategy PBE – Proofs

Assume that conditions (3), (4′), (5) and (6′) hold.

Lemma 8. *All cases are filed in equilibrium; the equilibrium settlement offer $S \in (\tilde{A}, \bar{A})$, implicitly defined by $G(S) = C_D g(S)$, exists and is unique.*

Proof. First, it is simple to show that offers greater than $\bar{A}$ are strictly dominated by an offer equal to $\bar{A}$. Similarly, offers $S \in (0, \tilde{A})$ are not in the set of equilibrium offers. These proofs follow the logic applied in Lemma 1. Second, we will demonstrate that a zero offer cannot be an equilibrium offer in pure strategy. Suppose not. The defendant always makes a zero offer in equilibrium. Then, only cases with types $A \geq \hat{A}$ will be filed. When facing a plaintiff of a type $\hat{A} \leq A \leq \hat{A}+\epsilon$, the defendant's expected litigation loss will be lower by offering $\hat{A}+\epsilon$, where $\epsilon > 0$ (small number): $\int_{\hat{A}}^{\hat{A}+\epsilon}(\hat{A}+\epsilon)g(A)dA < \int_{\hat{A}}^{\hat{A}+\epsilon}(A+C_D)g(A)dA$, when $C_D > \epsilon$. Contradiction follows. Hence, a zero offer cannot be an equilibrium offer in pure strategy.

Third, we will show that a mixed-strategy with a zero offer and an offer $S \in [\tilde{A}, \bar{A}]$ is not in the set of equilibrium offers.[8] Suppose not. The defendant mixes between a zero offer and a positive offer

[8] S might be non-unique, i.e., multiple S might be made in equilibrium with positive probabilities.

$S \in [\tilde{A}, \bar{A}]$ in equilibrium. By condition (3) and following the logic applied in the proof of Lemma 2, we can establish that all cases with $A \geq \tilde{A}$ and some low-damage cases must be filed. Then, the defendant's expected litigation loss, which is the same under both offers, is: $\nu S + \int_{\tilde{A}}^{S} Sg(A)dA + \int_{S}^{\bar{A}} (A + C_D)g(A)dA = \int_{\tilde{A}}^{\bar{A}} (A + C_D)g(A)dA$. When the defendant offers a unique $S_1 \in [\tilde{A}, \bar{A}]$, all cases are filed. His expected litigation loss is $G(S_1)S_1 + \int_{S_1}^{\bar{A}} (A + C_D)g(A)dA$. By condition (4′), there exists $S_1 \in [\tilde{A}, \bar{A}]$ such that, $G(S_1)S_1 < \int_{\tilde{A}}^{S_1} (A + C_D)g(A)dA$. Then, $G(S_1)S_1 + \int_{S_1}^{\bar{A}} (A + C_D)g(A)dA < \int_{\tilde{A}}^{\bar{A}} (A + C_D)g(A)dA$, where the left- and right-hand sides represent the defendant's expected litigation loss when he makes an offer $S_1 \in [\tilde{A}, \bar{A}]$ and when he mixed between a zero offer and a positive offer $S \in [\tilde{A}, \bar{A}]$, respectively. The defendant is better off, and then, will always deviate to offer $S_1 \in [\tilde{A}, \bar{A}]$. Contradiction follows. Hence, a mixed strategy with a zero offer and a positive offer $S \in [\tilde{A}, \bar{A}]$ cannot be in set of equilibrium offers.

Fourth, from the previous analysis, we conclude that the set of equilibrium offers might only involve a pure strategy with an offer $S \in [\tilde{A}, \bar{A}]$ or a mixed strategy with multiple offers $S \in [\tilde{A}, \bar{A}]$. By condition (3), lawyers with high- and low-damage cases will get positive expected payoffs. Then, all cases will be filed. Hence, $\nu = G(\tilde{A})$ and the total filed cases $\zeta = 1$, in equilibrium.

Fifth, we will show that the equilibrium offer $S \in [\tilde{A}, \bar{A}]$ exists and is unique. Under both, the pure and mixed strategies, the defendant's expected litigation loss $l_D(S) = G(S)S + \int_{S}^{\bar{A}} (A + C_D)g(A)dA$, by condition (1). Minimization of $l_D(S)$ yields the first-order condition: $G(S) = C_D g(S)$. By condition (5), the second derivative $g(S) - C_D \frac{\partial g(S)}{\partial S} > 0$, for all $S \in [\tilde{A}, \bar{A}]$. Then, the function is strictly convex, and has a unique minimum. Hence, the set of equilibrium offers must involve a unique offer $S \in [\tilde{A}, \bar{A}]$. Fifth, we will demonstrate that the equilibrium offer is interior, $S \in (\tilde{A}, \bar{A})$. There are three possible options for a unique minimum of the function $l_D(S) = G(S)S + \int_{S}^{\bar{A}} (A + C_D)g(A)dA$. (1) The function $l_D(S)$ is strictly increasing on the interval $[\tilde{A}, \bar{A}]$; it achieves a unique corner minimum at $S = \tilde{A}$. (2) The function $l_D(S)$ is strictly decreasing on the interval $[\tilde{A}, \bar{A}]$; it achieves a corner minimum at $S = \bar{A}$. (3) The function $l_D(S)$ achieves a unique interior minimum on the interval $[\tilde{A}, \bar{A}]$; the value of S is implicitly defined by the first-order condition. Consider option (1). When $S = \tilde{A}$, all cases are filed, by condition (3). Then, the defendant is better off by making a zero offer: $G(\tilde{A})\tilde{A} + \int_{\tilde{A}}^{\bar{A}} (A + C_D)g(A)dA > \int_{\tilde{A}}^{\bar{A}} (A + C_D)g(A)dA$, where the right-hand side term corresponds to the defendant's expected litigation loss from making a zero offer. Contradiction follows. Hence, an offer $S = \tilde{A}$ cannot be an equilibrium offer. Consider options (2) and (3). The necessary and sufficient conditions for options (2) and (3) to occur are $\lim_{S \to \bar{A}^-} \frac{\partial l_D(S)}{\partial S} = 1 - C_D g(\bar{A}) \leq 0$, and $\lim_{S \to \bar{A}^-} \frac{\partial l_D(S)}{\partial S} = 1 - C_D g(\bar{A}) > 0$, respectively. By condition (6′), $\lim_{S \to \bar{A}^-} \frac{\partial l_D(S)}{\partial S} = 1 - C_D g(\bar{A}) > 0$. Hence, the function has an interior minimum $S \in (\tilde{A}, \bar{A})$. ■

Lemma 9. *The defendant's equilibrium probability of an accident* $\lambda_D = \arg\min K(\lambda) + \lambda l_D$ *exists, is unique, and is decreasing in* l_D.

Proof. Apply the logic used in the proof of Lemma 6. ■

Lemma 10. *In equilibrium, the defendant's prior and posterior beliefs are the same.*

Proof. Given that all cases are filed, the defendant cannot update his beliefs upon observing filing. Hence, his posterior and prior beliefs are the same. The defendant's equilibrium beliefs are as follows. For any $y \in (\tilde{A}, \bar{A}]$, $P(0 < A \leq y | Filing) = G(y)$. ■

[TO BE AVAILABLE ONLINE ONLY]

FINANCIALLY-CONSTRAINED LAWYERS: AN ECONOMIC THEORY OF LEGAL DISPUTES

SUPPLEMENTARY MATERIAL

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1 Introduction

This document presents supplementary material. Section 2 provides formal analysis of the additional equilibria of the general model: Mixed- and pure-strategy PBE with corner solution (i.e., where the equilibrium positive settlement offer is equal to $\bar{A}$, the maximum possible damage level). Section 3 provides formal analysis of a version of our model under a uniform distribution of damages and presents the MATLAB program used to compute the numerical examples of deterrence and social welfare discussed in the paper. Note that some conditions and propositions used in the general model directly apply to this document. If a condition or a proposition is similar to a condition presented in the general model, then the letter "A" or the letter "B" (Sections 2 and 3, respectively) is placed besides the number.

2 Mixed- and Pure-Strategy PBE with Corner Solution

2.1 Mixed-Strategy PBE with Corner Solution

Proposition 1A characterizes the mixed-strategy PBE with a corner solution under conditions (3)-(5) and (6A).

$$f < \gamma \tilde{A}. \tag{3}$$

$$\min_{\tau \in [\tilde{A}, \bar{A}]} \left\{ G(\tau)\tau - \int_{\tilde{A}}^{\tau} (A + C_D) g(A) dA \right\} > 0. \tag{4}$$

For any $A \in [\tilde{A}, \bar{A}]$,

$$g(A) - C_D \frac{\partial g(A)}{\partial A} > 0. \tag{5}$$

$$\int_{\tilde{A}}^{\bar{A}} A g(A) dA + C_D [1 - G(\tilde{A}) - \bar{A} g(\bar{A})] \leq 0. \tag{6A}$$

Proposition 1A. *Assume condition (3)-(5) and (6A) hold. The following strategy profile, together with the defendant's beliefs, characterize the mixed-strategy perfect Bayesian equilibrium with a corner solution.*

(1) The defendant chooses a probability of accident $\lambda_D = \arg\min\left\{K(\lambda) + \lambda \int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA\right\}$*. If a lawsuit is filed, the defendant mixes between proposing a zero offer with probability* $\beta = 1 - \frac{f}{\gamma S}$ *and proposing an offer* $S = \bar{A}$ *with the complementary probability.*

(2) A high-damage case is always filed by the plaintiff's lawyer; an average low-damage case is filed with probability $\frac{\nu}{G(\tilde{A})}$*.*

(3) A high-damage plaintiff always rejects a zero offer and accepts an offer $S = \bar{A}$ *only if* $A \leq \bar{A}$*; a low-damage plaintiff always accepts an offer* $S = \bar{A}$*.*

(4) The defendant's equilibrium beliefs are as follows. When the defendant observes a lawsuit, he believes that $P(0 < A < \tilde{A}) = \frac{\nu}{\nu + 1 - G(\tilde{A})}$*,* $P(\tilde{A} \leq A \leq y) = \frac{G(y) - G(\tilde{A})}{\nu + 1 - G(\tilde{A})}$ *for any* $y \in [\tilde{A}, \bar{A}]$*.*

Proof. The proof consists of several steps.

Step 1. We will first demonstrate that that offers greater than $\bar{A}$ and offer $S \in (0, \tilde{A})$ are not in the set of equilibrium offers. Second, we will show that a zero offer must be in the set of equilibrium offers. Third, we will demonstrate that a zero offer cannot be the only equilibrium offer: At least one strictly positive offer $S \in [\tilde{A}, \bar{A}]$ must be in the set of equilibrium offers, in addition to a zero offer. The proofs follow the logic applied in the proof of Lemma 1.

Step 2. We will show that there must be some low-damage cases that are filed ($\nu > 0$), and that all high-damage cases are filed. The proofs follow the logic applied in the proof of Lemma 2.

Step 3. We will demonstrate that the strictly positive equilibrium offer is $\bar{A}$. By condition (6A), the system (1)-(2) does not have an interior solution. By condition (5), the second derivative of $l_D(S)$, $g(S) - C_D \frac{\partial g(S)}{\partial S} > 0$, which is satisfied for all $S \in [\tilde{A}, \bar{A}]$. Then, there is a unique minimum. There are two potential options. (1) $l_D(S)$ is strictly increasing and the optimal strictly positive offer is $\tilde{A}$; (2) $l_D(S)$ is strictly decreasing and the optimal strictly positive offer is $\bar{A}$. Consider Option 1. Suppose that Option 1 holds. The defendant is indifferent between a zero offer and $\tilde{A}$. The indifference condition implies: $\int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA = \nu \tilde{A} + \int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA$. This condition holds only if $\nu = 0$. Contradiction follows. Consider Option 2 now. The defendant is indifferent between a zero offer and $\bar{A}$. The indifference condition implies: $[\nu + 1 - G(\tilde{A})]\bar{A} = \int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA$. Solving for ν yields: $\nu = \frac{\int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA}{\bar{A}} - [1 - G(\tilde{A})]$. By Claim 2 in the Appendix, $\nu < G(\tilde{A})$. Hence, $S = \bar{A}$ and $\nu = \frac{\int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA}{\bar{A}} - [1 - G(\tilde{A})]$ are the equilibrium strictly positive offer and the equilibrium mass of low-damage cases that are filed.

Step 4. The composition of ν determines the equilibrium probability that the defendant makes a zero offer β The proof follows the logic applied in the proof of Lemma 5.

Step 5. The defendant's optimal probability of an accident $\lambda_D = \arg\min\{K(\lambda) + \lambda l_D\} = \arg\min\{K(\lambda) + \lambda \int_{\tilde{A}}^{\bar{A}} (A + C_D) g(A) dA\}$. The proof follows the logic applied in the proof of Lemma 6.

Step 6. The equilibrium strategies of the average plaintiff and his lawyer and the equilibrium mass of filed cases determine the beliefs of the defendant. The proof follows the logic applied in Lemma 7. ■

2.2 Pure-Strategy PBE with Corner Solution

Proposition 2A characterizes the pure-strategy PBE with a corner solution under conditions (3), (4′), (5), and (6′A).

$$f < \gamma \tilde{A}. \tag{3}$$

$$\min_{\tau \in [\tilde{A}, \bar{A}]} \left\{ G(\tau)\tau - \int_{\tilde{A}}^{\tau} (A + C_D) g(A) dA \right\} < 0. \tag{4′}$$

For any $A \in [\tilde{A}, \bar{A}]$,

$$g(A) - C_D \frac{\partial g(A)}{\partial A} > 0. \tag{5}$$

$$1 - C_D g(\bar{A}) \leq 0. \tag{6$'$A}$$

Proposition 2A. *Assume conditions (3), (4′), (5), and (6′A) hold. The following strategy profile, together with the defendant's beliefs, characterize the pure-strategy perfect Bayesian equilibrium with a corner solution.*

(1) The defendant chooses a probability of accident $\lambda_D = \arg\min\{K(\lambda) + \lambda l_D(S)\}$*, where* $l_D(S) = G(S)S + \int_S^{\bar{A}}(A + C_D)g(A)dA$*. If a lawsuit is filed, the defendant proposes an offer* $S = \bar{A}$ *with certainty.*

(2) All cases are filed by the plaintiff's lawyer.

(3) A plaintiff always accepts an offer $S = \bar{A}$*.*

(4) The defendant's equilibrium beliefs are as follows. When the defendant observes a lawsuit, he believes that $P(0 < A \leq y) = G(y)$*.*

Proof. The proof consists of several steps.

Step 1. We will first demonstrate that offers greater than $\bar{A}$ and offer $S \in (0, \tilde{A})$ are not in the set of equilibrium offers. Second, we will show that a zero offer cannot be an equilibrium offer. Third, we will demonstrate that a mixed-strategy with a zero offer and $S \in [\tilde{A}, \hat{A}]$ cannot be in the set of equilibrium offers. The proofs follow the logic applied in the proof of Lemma 1.

Step 2. From the previous analysis, we conclude that the set of equilibrium offers might only involve a pure strategy with a unique offer $S \in [\tilde{A}, \bar{A}]$ or a mixed strategy with multiple offers $S \in [\tilde{A}, \bar{A}]$. By condition (3), lawyers with high- and low-damage cases will get positive expected payoffs. Then, all cases will be filed. Hence, $\nu = G(\tilde{A})$ and the total filed cases $\zeta = 1$, in equilibrium.

Step 3. We will first show that the equilibrium offer $S \in [\tilde{A}, \bar{A}]$ exists and is unique. Under both, the pure and the mixed strategies, the defendant's expected litigation loss $l_D(S) = G(S)S + \int_S^{\bar{A}}(A + C_D)g(A)dA$, by equation (1). Minimization of $l_D(S)$ yields the first-order condition: $G(S) = C_D g(S)$. By condition (5), the second derivative $g(S) - C_D \frac{\partial g(S)}{\partial S} > 0$, for all $S \in [\tilde{A}, \bar{A}]$. Then, the function is strictly convex and has a unique minimum. Hence, the set of equilibrium offers must involve a unique offer $S \in [\tilde{A}, \bar{A}]$. Second, we will demonstrate that the equilibrium offer is corner, $S = \bar{A}$. There are three possible options for a unique minimum of the function $l_D(S)$. (1) The function $l_D(S)$ is strictly increasing on the interval $[\tilde{A}, \bar{A}]$; it achieves a unique corner minimum at $S = \tilde{A}$. (2) The function $l_D(S)$ is strictly decreasing on the interval $[\tilde{A}, \bar{A}]$; it achieves a corner minimum at $S = \bar{A}$. (3) The function $l_D(S)$ achieves a unique interior minimum on the interval $(\tilde{A}, \bar{A})$; the value of S is implicitly defined by the first-order condition. Consider option (1). When $S = \tilde{A}$, all cases are filed, by condition (3). Then, the defendant is better off by making a zero offer: $G(\tilde{A})\tilde{A} + \int_{\tilde{A}}^{\bar{A}}(A + C_D)g(A)dA > \int_{\tilde{A}}^{\bar{A}}(A + C_D)g(A)dA$, where the right-hand side term corresponds to the defendant's expected litigation loss from making a zero offer. Contradiction follows. Hence, an offer $S = \tilde{A}$ cannot be an equilibrium offer. Consider options (2) and (3). The necessary and sufficient conditions for options (2) and (3) to occur are $\lim_{S \to \bar{A}^-} \frac{\partial l_D(S)}{\partial S} = 1 - C_D g(\bar{A}) \leq 0$, and $\lim_{S \to \bar{A}^-} \frac{\partial l_D(S)}{\partial S} = 1 - C_D g(\bar{A}) > 0$, respectively. By condition (6′A), $\lim_{S \to \bar{A}^-} \frac{\partial l_D(S)}{\partial S} = 1 - C_D g(\bar{A}) \leq 0$. Hence, the function has a corner minimum at $S = \bar{A}$.

Step 4. The defendant's optimal probability of an accident $\lambda_D = \arg\min\{K(\lambda) + \lambda l_D(S)\}$. The proof follows the logic applied in the proof of Lemma 6.

Step 5. We will show that, in equilibrium, the defendant's posterior and prior beliefs are the same. The proof follows the logic applied in the proof of Lemma 10. ■

3 A Model with Uniform Distribution of Damages

This section refers to a version of our model under a uniform distribution of damages. We first present formal analysis of the mixed- and pure-strategy PBE. We then present the MATLAB program related to the numerical analysis of deterrence and the numerical analysis of the effect of a change in r on social welfare, which are discussed in the main text of the paper.

Assume A is uniformly distributed on the interval $(0, \bar{A}]$. Then, $G(A) = \frac{A}{\bar{A}}$ and $g(A) = \frac{1}{\bar{A}}$. Assume also that $K(\lambda)$ has the following functional form: $K(\lambda) = B\big[1 - \sqrt{1-(\lambda-1)^2}\big]$. This function satisfies the assumptions stated in the paper: $K'(\lambda) = -\frac{B(1-\lambda)}{\sqrt{1-(\lambda-1)^2}} < 0$, $K''(\lambda) = B[1-(\lambda-1)^2]^{-3/2} > 0$, $\lim_{\lambda\to 0+} K'(\lambda) = +\infty$, $\lim_{\lambda \to 1-} K'(\lambda) = 0$. The other assumptions of our general model hold.

3.1 Mixed-Strategy PBE

Technical Conditions

$$f < \gamma \tilde{A}. \tag{3}$$

$$\min_{\tau \in [\tilde{A}, \bar{A}]} \{(\tau - C_D)^2 + \tilde{A}^2 + 2\tilde{A}C_D - C_D^2\} > 0. \tag{4B}$$

$$\frac{\bar{A}^2}{2} - \tilde{A}^2/2 - C_D\tilde{A} > 0. \tag{6B}$$

Condition 5 holds trivially under a uniform distribution of damages. Condition (4B) implies $\tilde{A} > C_D(\sqrt{2}-1)$. Intuitively, the mixed-strategy PBE arises in a state of the world is characterized by a high level of lawyers' constraints.

Equilibrium Strategies and Outcomes

Equilibrium S and ν are jointly determined by the system of equations (1B)-(2B):

$$\nu S + \int_{\tilde{A}}^{S} S\frac{1}{\bar{A}}dA + \int_{S}^{\bar{A}} (A + C_D)\frac{1}{\bar{A}}dA = \int_{\tilde{A}}^{\bar{A}} (A + C_D)\frac{1}{\bar{A}}dA \tag{1B}$$

$$\nu + \frac{S}{\bar{A}} - \frac{\tilde{A}}{\bar{A}} = \frac{C_D}{\bar{A}}. \tag{2B}$$

The system of equations (1B)-(2B) has an explicit solution. The equilibrium positive offer S and the equilibrium mass of low-damage cases that are filed ν are:

$$S = \sqrt{\tilde{A}^2 + 2\tilde{A}C_D}$$

and

$$\nu = \frac{\tilde{A} + C_D - S}{\bar{A}} = \frac{\tilde{A} + C_D - \sqrt{\tilde{A}^2 + 2\tilde{A}C_D}}{\bar{A}}.$$

The additional equilibrium outcomes are as follows.

Mass of filed cases: $\zeta = 1 - G(\tilde{A}) + \nu = 1 + \left(\frac{C_D - S}{\bar{A}}\right)$.

Probability of a zero offer: $\beta = 1 - \frac{f}{\gamma S}$.

Mass of cases that proceed to trial: $\rho = 1 - \frac{\tilde{A}}{\bar{A}} - \frac{f}{\gamma \bar{A}} + \frac{f\tilde{A}}{\gamma \bar{A} S}$.

Defendant's expected litigation loss: $l_D = \frac{0.5\bar{A}^2 + C_D\bar{A} - \frac{\tilde{A}^2}{2} - C_D\tilde{A}}{\bar{A}}$.

No-access-to-justice component: $\eta = \frac{\tilde{A}}{\bar{A}} + \frac{f}{\gamma \bar{A}} - \left(\frac{\tilde{A}+C_D}{\bar{A}}\right)\frac{f}{\gamma S}$.

Probability of accident:

$$\lambda_D = 1 - \frac{l_D}{\sqrt{B^2 + l_D^2}}.$$

Social Welfare and Deterrence

Social Welfare Loss Function

$$SWL = K(\lambda) + \lambda l_W,$$

where

$$l_W = H + \zeta f + \rho(C_P + C_D) + \theta\eta =$$

$$= \frac{\bar{A}}{2} + \left(1 + \frac{C_D}{\bar{A}} - \frac{S}{\bar{A}}\right)f + \left(1 - \frac{\tilde{A}}{\bar{A}} - \frac{f}{\gamma \bar{A}} + \frac{f\tilde{A}}{\gamma \bar{A} S}\right)(C_P + C_D) + \theta\left(\frac{\tilde{A}}{\bar{A}} + \frac{f}{\gamma \bar{A}} - \frac{f(\tilde{A} + C_D)}{\gamma \bar{A} S}\right).$$

Deterrence Components

The defendant's expected litigation loss l_D, given by each side of equation (1B), can be expressed as:

$$l_D = (1-\beta)\left[\nu S + \int_{\tilde{A}}^{S} S\frac{1}{\bar{A}}dA + \int_{S}^{\bar{A}}(A + C_D)\frac{1}{\bar{A}}dA)\right] +$$

$$+\beta\left[\int_{\tilde{A}}^{\bar{A}}(A + C_D)\frac{1}{\bar{A}}dA\right] =$$

$$= (1-\beta)\left[\frac{C_D S}{\bar{A}} + \frac{1}{\bar{A}}\left(\frac{\bar{A}^2}{2} + C_D\bar{A} - \frac{S^2}{2} - C_D S\right)\right] + \frac{\beta}{\bar{A}}\left[\frac{\bar{A}^2}{2} + C_D\bar{A} - \frac{\tilde{A}^2}{2} - C_D\tilde{A}\right].$$

Then, $l_W - l_D$ is given by:

$$l_W - l_D =$$

$$= \left\{\left(1 + \frac{C_D - S}{\bar{A}}\right)f + \left[\beta\left(1 - \frac{\tilde{A}}{\bar{A}}\right) + (1-\beta)\left(1 - \frac{S}{\bar{A}}\right)\right]C_P + \theta\left(\frac{\tilde{A}}{\bar{A}} - \nu + \nu\beta\right)\right\} +$$

$$+\left\{H - \left\{(1-\beta)\left[\frac{C_D S}{\bar{A}} + \frac{1}{\bar{A}}\left(\frac{\bar{A}^2}{2} - \frac{S^2}{2}\right)\right] + \frac{\beta}{\bar{A}}\left[\frac{\bar{A}^2}{2} - \frac{\tilde{A}^2}{2}\right]\right\}\right\}.$$

Effects of a Cost-Reducing Policy on Social Welfare

Direct Effect (DE)

$$\frac{\partial SWL(\lambda_D)}{\partial \tilde{A}} = \lambda_D f\frac{\partial \zeta}{\partial \tilde{A}} + \lambda_D(C_P + C_D)\frac{\partial \rho}{\partial \tilde{A}} + \lambda_D\theta\frac{\partial \eta}{\partial \tilde{A}} =$$

$$= \lambda_D\left(f\frac{\partial \zeta}{\partial \tilde{A}} + (C_P + C_D)\frac{\partial \rho}{\partial \tilde{A}} + \theta\frac{\partial \eta}{\partial \tilde{A}}\right) =$$

$$= \lambda_D \left\{ f \frac{\tilde{A} + C_D}{\bar{A} S} + (C_P + C_D) \frac{1}{\bar{A}} \left(1 - \frac{f \tilde{A} C_D}{\gamma S^3} \right) + \theta \left(\frac{1}{\bar{A}} + \frac{f C_D^2}{\gamma \bar{A} S^3} \right) \right\}.$$

Indirect Effect (IE)

$$\frac{\partial SWL(\lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial \tilde{A}} = (l_W - l_D) \frac{\partial \lambda_D}{\partial \tilde{A}} =$$
$$= \{ (\zeta f + \rho C_P + \theta \eta) +$$
$$+ \left\{ H - \left\{ (1-\beta) \left[\frac{C_D S}{\bar{A}} + \frac{1}{\bar{A}} \left(\frac{\bar{A}^2}{2} - \frac{S^2}{2} \right) \right] + \frac{\beta}{\bar{A}} \left[\frac{\bar{A}^2}{2} - \frac{\tilde{A}^2}{2} \right] \right\} \right\} \Bigg\} \left\{ \frac{(\tilde{A} + C_D)[1 - (1-\lambda_D)^2]^{3/2}}{\bar{A} B} \right\}.$$

Overall Effect (OE)

The overall effect of a cost-reducing policy is:

$$\frac{dSWL(\lambda_D)}{d\tilde{A}} = \left[(l_W - l_D) \frac{\partial \lambda_D}{\partial \tilde{A}} \right] + \left[\lambda_D f \frac{\partial \zeta}{\partial \tilde{A}} + \lambda_D (C_P + C_D) \frac{\partial \rho}{\partial \tilde{A}} + \lambda_D \theta \frac{\partial \eta}{\partial \tilde{A}} \right].$$

The overall effect can be written as:

$$\frac{dSWL(\lambda_D)}{d\tilde{A}} = \Omega + \Lambda \theta$$

where:

$$\Omega =$$
$$\left\{ \left(1 + \frac{C_D - S}{\bar{A}} \right) f + \left[\beta \left(1 - \frac{\tilde{A}}{\bar{A}} \right) + (1-\beta) \left(1 - \frac{S}{\bar{A}} \right) \right] C_P \right\} +$$
$$+ \left\{ H - \left\{ (1-\beta) \left[\frac{C_D S}{\bar{A}} + \frac{1}{\bar{A}} \left(\frac{\bar{A}^2}{2} - \frac{S^2}{2} \right) \right] + \frac{\beta}{\bar{A}} \left[\frac{\bar{A}^2}{2} - \frac{\tilde{A}^2}{2} \right] \right\} \right\} \left\{ \frac{(\tilde{A} + C_D)[1 - (1-\lambda_D)^2]^{3/2}}{\bar{A} B} \right\} +$$
$$+ \lambda_D \left\{ - \frac{f}{\bar{A}} \left[\frac{(\tilde{A} + C_D)}{S} \right] + (C_P + C_D) \frac{1}{\bar{A}} \left[1 - \frac{f \tilde{A} C_D}{\gamma S^3} \right] \right\}$$

and

$$\Lambda = \left(\frac{\tilde{A}}{\bar{A}} - \nu + \nu \beta \right) \left\{ \frac{(\tilde{A} + C_D)[1 - (1-\lambda_D)^2]^{3/2}}{\bar{A} B} \right\} + \lambda_D \left[\frac{1}{\bar{A}} + \frac{f C_D^2}{\gamma \bar{A} S^3} \right].$$

The threshold $\underline{\theta}$ is defined as follows: $\underline{\theta} \equiv \frac{-\Omega}{\Lambda}$.

3.2 Pure-Strategy PBE

Technical Conditions

$$f < \gamma \tilde{A} \tag{3}$$

$$\min_{\tau \in [\tilde{A}, \bar{A}]} \{ (\tau - C_D)^2 + \tilde{A}^2 + 2\tilde{A} C_D - C_D^2 \} < 0 \tag{4'B}$$

$$\bar{A} > C_D. \tag{6'B}$$

Condition 5 holds trivially under a uniform distribution of damages. Condition $(4'B)$ implies $\tilde{A} < C_D(\sqrt{2} - 1)$. Intuitively, the pure-strategy PBE arises in a state of the world characterized by a low level of lawyer's constraint.

Equilibrium Strategies and Outcomes

Equilibrium S:

$$\frac{S}{\bar{A}} = \frac{C_D}{\bar{A}}. \tag{8B}$$

Then,

$$S = C_D.$$

The additional equilibrium outcomes are as follows.

Mass of filed cases: $\zeta = 1$.
Mass of low-damage cases that are filed: $\nu = \frac{\tilde{A}}{\bar{A}}$.
Mass of cases that proceed to trial: $\rho = \frac{\bar{A}-C_D}{\bar{A}}$.
Defendant's expected litigation loss : $l_D = C_D + 0.5\bar{A} - \frac{C_D^2}{2\bar{A}}$.
Probability of accident:

$$\lambda_D = 1 - \frac{l_D}{\sqrt{B^2 + l_D^2}}.$$

3.3 Numerical Examples of Deterrence and Social Welfare

In Section 5.2 of the paper, we present a numerical example to illustrate the deterrence components. In Section 6.4 of the paper, we present a numerical example to illustrate the effect of a change in r (a specific cost-reducing policy) on social welfare. We use the uniform-distribution model presented in Section 3.3. The parameter set adopted is as follows: $\{C_P, C_D, \gamma, f, x, \theta, \bar{A}, B\} = \{200, 500, .33, 70, 100, 750, 1200, 1000\}$. One value of r, $r = .18$, is studied in the numerical example of deterrence. Three values of r are studied in the numerical example of social welfare: $r \in \{.08, .18., .28\}$.[1] The MATLAB software is used to construct the numerical examples. Figure 1 presents the MATLAB program.

[1]The model conditions and the condition $\theta > \underline{\theta}$ hold under the chosen exogenous parameters. In this numerical example, SWL is a monotonically increasing function of r. The value of r that minimizes SWL is $r = .08$. The values for $\underline{\theta}$ are $\underline{\theta} \in \{192, 158, 125\}$, for $r \in \{.08, .18, .28\}$, respectively.

```
%Numerical Example on Deterrence and Numerical Example on the Effect of a Change in r on Social Wlefare

%Exogenous variables

Cp=200; Cd=500; f=70; x = 100; gamma = 0.33;Abar = 1200; B = 1000; theta=750;
for i = 1:1:3
  r(i) = -0.02+ i*0.10;

  %Conditions of the model

  Atilde(i)=(Cp+(Cp+f-x)*r(i))/gamma;
  if Cd < (sqrt(2)+1)*Atilde(i)
    if f < gamma*Atilde(i)
      if Cd  - (Abar^2 - Atilde(i)^2)/(2*Atilde(i)) < 0
       %Computation of endogenous variables
        S(i) = sqrt((Atilde(i))^2 + 2*Atilde(i)*Cd);
        beta(i)=1- f/(gamma*S(i));
        nu(i) = (Atilde(i)+Cd-S(i))/Abar;
        rho(i) = beta(i)*(1 - Atilde(i)/Abar)+(1-beta(i))*(1-S(i)/Abar);
        eta(i) = Atilde(i)/Abar + f/(gamma*Abar) - (f*(Atilde(i)+Cd))/(gamma*S(i)*Abar);
        zeta(i) = 1+Cd/Abar - S(i)/Abar;
        probtrial(i) = rho(i)/zeta(i);
        ld(i) = (0.5*Abar^2 + Cd*Abar - 0.5*(Atilde(i))^2 - Cd*Atilde(i))/Abar;
        lambdad(i) = 1 - ld(i)/sqrt(B^2+(ld(i))^2);
        lw(i) = Abar/2 + zeta(i)*f + rho(i)*(Cp+Cd) + theta*eta(i);
        lambdaw(i) = 1 - lw(i)/sqrt(B^2+(lw(i))^2);
        SWL(i) = B*(1-B/sqrt(B^2 + (ld(i))^2)) + lambdad(i)*lw(i);
        deterrence(i) = lw(i)-ld(i);
        zeta_f(i) = zeta(i)*f;
        rho_Cp(i) = rho(i)*Cp;
        theta_eta(i) = theta*eta(i);
        expected_compensation_def(i) = (1-beta(i))*(nu(i) + S(i)/Abar - Atilde(i)/Abar)*S(i) +
       (beta(i)/Abar)*(Abar^2/2- (Atilde(i))^2/2) +((1-beta(i))/Abar)*(Abar^2/2 - (S(i))^2/2);
        H = Abar/2;
        derivative_rho_Atilde(i) = 1/Abar*(-1+(f*Atilde(i)*Cd)/(gamma*(S(i))^3));
        derivative_zeta_Atilde(i) = -(Atilde(i)+Cd)/(Abar*S(i));
        derivative_eta_Atilde(i) = 1/Abar*(1+(f*Cd^2)/(gamma*(S(i))^3));
        K_prime_prime(i) = B*(1- (1-lambdad(i))^2)^(-1.5);
        Omega(i)= (0.5*Abar+ zeta(i)*f + rho(i)*(Cd+Cp)-
        ld(i))*(Atilde(i)+Cd)/(Abar*K_prime_prime(i))+lambdad(i)*((Cp+Cd)*derivative_rho_Atilde(i)-
        (Atilde(i)+Cd)*f/(Abar*S(i)));
        Lambda(i) = eta(i) *(Atilde(i)+Cd)/(Abar*K_prime_prime(i))+lambdad(i)*derivative_eta_Atilde(i);
        theta_under(i) =  - Omega(i)/Lambda(i);
      end
    end
  end
end

%Computation of percentage changes when r decreases

for i = 2:1:3
  delta_zeta(i-1) = - 100*(zeta(i)-zeta(i-1))/zeta(i);
  delta_probtrial(i-1) = -100*(probtrial(i)-probtrial(i-1))/probtrial(i);
  delta_eta(i-1) = - 100*(eta(i)-eta(i-1))/eta(i);
  delta_nu(i-1) = - 100*(nu(i)-nu(i-1))/nu(i);
  delta_beta(i-1) = - 100*(beta(i)-beta(i-1))/beta(i);
  delta_ld(i-1) = -100*(ld(i)-ld(i-1))/ld(i);
  delta_lambda(i-1) = -100*(lambdad(i)-lambdad(i-1))/lambdad(i);
  delta_SWL(i-1) = - 100*(SWL(i)-SWL(i-1))/SWL(i);
end
```

Figure 1: MATLAB Program

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