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Testing for State-Dependent Predictive Ability

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Testing for State-Dependent Predictive Ability

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Abstract

This paper proposes a new test for comparing the out-of-sample forecasting performance of two competing models for situations in which the predictive content may be state-dependent (for example, expansion and recession states or low and high volatility states). To apply this test the econometrician is not required to observe when the underlying states shift. The test is simple to implement and accommodates several different cases of interest. An out-of-sample forecasting exercise for US output growth using real-time data illustrates the improvement of this test over previous approaches to perform forecast comparison.

Keywords: Forecast Evaluation, Testing, Regime Switching, Structural Change

JEL Codes: C22, C53

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1 Introduction

A well-established empirical fact in the macroeconomic forecasting literature is that predictability is unstable over time. For example, many individual indicators exhibit significant out-of-sample predictive content for output growth and inflation but only sporadically. This result has been documented in Stock and Watson (2003), Giacomini and Rossi (2010), Rossi and Sekhposyan (2010), Rossi (2013), and Granziera and Sekhposyan (2014), among others. A recent literature has established a new (but related) empirical fact: predictability varies across economic states. For example, Dotsey et al. (2015) report that out-of-sample Phillips curve forecasts of the inflation rate tend to be more accurate, relative to a benchmark model, during economic recessions but less accurate during expansions (see also Gibbs and Vasnev, 2017). Similarly, Chauvet and Potter (2013) find that most output growth forecasting models exhibit a similar performance during economic expansions but one model performs significantly better during recessions. Evidence of state-dependent predictability has also been documented in the empirical finance literature. For example, work by Rapach et al. (2010), Henkel et al. (2011), and Dangl and Halling (2012) shows stronger evidence for out-of-sample stock return predictability during economic recessions than during expansions. Likewise, Gargano et al. (2016) find that the degree of predictability of bond excess returns rises during recessions, while Gargano and Timmermann (2014) find similar results for commodity prices.

The most common approach to test for state-dependent predictive ability relies on exogenously provided shift dates and the results on conditional predictive ability of Giacomini and White (2006). For example, the literature cited above uses peak and trough dates determined by the Business Cycle Dating Committee of the National

Bureau of Economic Research (NBER) to identify recession and expansion periods in the US economy. The relative performance of the two forecasting models is then evaluated using a test of unconditional predictive ability (Diebold and Mariano, 1995; West, 1996; Clark and West, 2006; Giacomini and White, 2006) applied to recession and expansion periods separately (see, for example, Chauvet and Potter, 2013). But this testing strategy is not feasible if the econometrician is not able to observe when the underlying states shift (for example, low and high volatility states).¹ In addition, existing methods for evaluating relative forecasting performance in unstable environments (Giacomini and Rossi, 2010; Martins and Perron, 2016) are inadequate when one of the states is constrained to just a few observations. This is the case of recessions in the US economy which tend to be short-lived, typically lasting less than four quarters. For example, the Fluctuation test of Giacomini and Rossi (2010) is based on a rolling average of loss differences and, as a result, it works best when the relative performance of the two models changes smoothly over time.

In this paper I propose a new test for comparing the out-of-sample forecasting performance of two competing models for situations in which the predictive content may be state-dependent. The main advantage of this test is that the econometrician is not required to observe when the underlying states shift. Similar to Giacomini and Rossi (2010) and Martins and Perron (2016), I use the framework of Giacomini and White (2006) to treat the sequence of out-of-sample loss differences as observed data. Next, forecast loss differences are modeled using a Markov-switching mean plus noise process which can be used to test for state-dependent predictive ability. The test is simple to implement and accommodates several different cases that can be of interest. For example, two competing models (or predictors) can exhibit equal predictive ability

¹I am assuming here that the NBER dating provides a reasonably accurate approximation to the underlying recession and expansion states which are, of course, not observable.

in one state, whereas one of the models is more accurate in the other. Alternatively, one model could be more accurate in the first state while the other model could be more accurate in the second. In addition, there can be differences in the degree of persistence or expected duration of the two states. For example, both states could be very persistent or one state could be very persistent (expansions) while the other state is constrained to shorter periods (recessions). Finally, the test can also accommodate a permanent structural break (one level shift) in the loss differences, as the one-time reversal test of Giacomini and Rossi (2010).

In the next section I use an illustrative example to show that current tests of equal predictive ability (Giacomini and White, 2006; Giacomini and Rossi, 2010) will struggle to reject the null hypothesis if the superior performance of one of the competing models is constrained to very short periods (just a few observations), even if observed repeatedly. Next, I present an (asymptotic) heteroskedasticity and autocorrelation consistent (HAC) test of the null hypothesis of equal and constant predictive ability against the alternative of state-dependent predictive ability. In section 3 I investigate the small-sample properties of the tests using Monte Carlo simulations. The results show that the HAC estimators of the covariance matrix are accurate and yield true confidence levels that are close to the nominal level. Size and power properties of the tests are evaluated in two situations: (i) allowing for unequal but constant relative forecasting performance of the two competing models; (ii) allowing for different relative forecasting performance of the two models in different states (the business cycle). In section 4 I use an out-of-sample forecasting exercise for US output growth to illustrate the usefulness of the proposed test over previous approaches to forecast comparison. Finally, section 5 concludes.

2 State-Dependent Predictive Ability

In this section I propose a new test for comparing the out-of-sample forecasting performance of two competing models for situations in which the predictive content may be state-dependent. The literature on testing for equal predictive ability is substantial. The focus of this paper is on finite-sample predictive ability (Giacomini and White, 2006) and unstable environments (Giacomini and Rossi, 2010). See Clark and McCracken (2011), Giacomini (2011), and Rossi (2013) for detailed surveys of the literature.

2.1 Environment

The objective is to compare sequences of h -step ahead out-of-sample forecasts for the variable y_t obtained from two competing models. The models are characterized by parameters δ and γ , respectively. It is assumed that a sample of size T has been divided into an in-sample portion of size R and an out-of-sample portion of size P . For a given loss function $L(\cdot)$, we have a sequence of P out-of-sample forecast loss differences $\{\Delta L_t(\hat{\delta}_{t-h,R}, \hat{\gamma}_{t-h,R})\}_{t=R+h}^T$ with $\Delta L_t(\hat{\delta}_{t-h,R}, \hat{\gamma}_{t-h,R}) = L^{(1)}(y_t, \hat{\delta}_{t-h,R}) - L^{(2)}(y_t, \hat{\gamma}_{t-h,R})$. As in Giacomini and White (2006), the parameters of the models are estimated using a fixed scheme (that is, estimated once using observations $1, \dots, R$) or a rolling scheme (that is, re-estimated at each $t = R+h, \dots, T$ using observations $t-h-R+1, \dots, t-h$). We are interested in testing the null hypothesis of equal predictive ability

$$H_0^{(1)} : E \left[\Delta L_t(\hat{\delta}_{t-h,R}, \hat{\gamma}_{t-h,R}) \right] = 0 \text{ for all } t = R+h, \dots, T, \quad (1)$$

which is tested against the alternative hypothesis that one of the models exhibits superior predictive ability (that is, $E[\Delta L_t(\hat{\delta}_{t-h,R}, \hat{\gamma}_{t-h,R})] \neq 0$). Giacomini and White (2006) note that since the null hypothesis $H_0^{(1)}$ is stated in terms of the estimated

parameters $\hat{\delta}_{t-h,R}$ and $\hat{\gamma}_{t-h,R}$, not the population parameters δ and γ , this is a test of predictive ability in the finite sample (that is, given models, estimation windows, estimation procedures, etc.).

In this context, Giacomini and White (2006) propose an asymptotic test of equal (unconditional) predictive ability that can be applied to both nested and non-nested models. The statistic is a t-test of the hypothesis that $\mu = 0$ in the simple linear regression $\Delta L_t = \mu + u_t$ for $t = R + h, \dots, T$ and is given by

$$GW = \hat{\sigma}_P^{-1} P^{1/2} \overline{\Delta L}_P, \quad (2)$$

where $\overline{\Delta L}_P = P^{-1} \sum_{t=R+h}^T \Delta L_t$ and $\hat{\sigma}_P^2$ is the sample variance of ΔL_t if $h = 1$ or a HAC estimator of the long-run variance if $h > 1$. A common choice for the latter is the kernel-based estimator

$$\hat{\sigma}_P^2 = \sum_{j=-(q-1)}^{q-1} k(j/q) P^{-1} \sum_{t=R+h}^T \Delta L_t^* \Delta L_{t-j}^* \quad (3)$$

where $k(\cdot)$ is a kernel weight function, for example the Bartlett kernel of Newey and West (1987), q is a bandwidth that grows with P , and $\Delta L_t^* = \Delta L_t - \overline{\Delta L}_P$ (see, for example, Andrews, 1991). Under (1), $GW \xrightarrow{d} N(0, 1)$ as $P \rightarrow \infty$ and the null hypothesis of equal unconditional predictive ability is rejected at the 5% level if $|GW| > 1.96$. As a result, the test of equal unconditional predictive ability of Giacomini and White (2006) coincides with the test proposed in Diebold and Mariano (1995).²

In related work, Giacomini and Rossi (2010) propose asymptotic tests of the joint hypothesis of equal and constant performance of the two models, $H_0^{(1)}$, against the

²In addition to this test of equal unconditional predictive ability, Giacomini and White (2006) also propose a test of conditional predictive ability. The intuition is that the econometrician could use available information to predict which of the competing models will be more accurate at a given time. In this case, the statistic is a Wald test of the hypothesis that $\beta = 0$ in the regression $\Delta L_t = \beta' X_{t-h} + u_t$, where X_{t-h} includes a constant and other variables (for example, a business cycle indicator).

alternative of local predictive ability. In the regression with time-varying parameters $\Delta L_t = \mu_t + u_t$, they propose tests of the null hypothesis that $\mu_t = 0$ for all t against different specifications of the alternative hypothesis. For example, the Fluctuation test is based on a rolling average of loss differences and the statistic is given by

$$\text{Fluct}_{t,m} = \hat{\sigma}_P^{-1} m^{-1/2} \sum_{j=t-m/2}^{t+m/2+1} \Delta L_j. \quad (4)$$

The statistic is computed for $t = R + h + m/2, \dots, T - m/2 + 1$, with m the window size and $\hat{\sigma}_P^2$ a HAC estimator of the long-run variance of ΔL_t , for example the kernel-based estimator (3). The test rejects when $\max_t |\text{Fluct}_{t,m}| > k_\alpha$ with the critical value k_α obtained by simulation. In addition, Giacomini and Rossi (2010) and Martins and Perron (2016) consider tests based on structural break models that can accommodate one or more sudden breaks in relative forecasting performance.

2.2 An illustrative example

A substantial recent literature on macroeconomic forecasting employs the tests of equal predictive ability described above to evaluate the predictive content of different indicators or models (see Rossi, 2013, for an extensive survey of this literature). Here I use a simple simulated example to illustrate how these tests can fail to uncover the superior performance of one of the two competing models when the predictive ability is state-dependent.

Consider a data generating process (DGP) given by

$$\begin{aligned} y_t &= -\beta s_t + \sigma_\varepsilon \varepsilon_t, \\ x_t &= \delta s_t + \sigma_\nu \nu_t, \end{aligned}$$

where ε_t and ν_t are i.i.d. $N(0,1)$ and s_t is a state variable identifying, for example,

recession ($s_t = 1$) and expansion ($s_t = 0$) periods. The econometrician does not observe s_t but rather x_t , which is correlated with s_t . For example, if $x_t \in [0, 1]$ for all t , then x_{t+1} could be a predicted probability of recession for quarter $t + 1$. I consider two competing forecasting models: (i) $y_{t+1} = u_{1t+1}$ and (ii) $y_{t+1} = \gamma x_{t+1} + u_{2t+1}$. As a result, the time- t one-step ahead forecasts of y_{t+1} are

$$\begin{aligned}\hat{f}_{t,R}^1 &= 0, \\ \hat{f}_{t,R}^2 &= \hat{\gamma}_{t,R} x_{t+1},\end{aligned}$$

where $\hat{\gamma}_{t,R}$ is the in-sample rolling estimate of γ based on the last R observations and x_{t+1} is assumed known at time t . Note that during expansions both models yield forecasts that are on average 0. On the other hand, during recessions $\hat{f}_{t,R}^2$ will be (on average) closer to the actual value y_{t+1} . As a result, while the two models should exhibit similar predictive content during expansions, model (ii) should yield more accurate forecasts during recessions. For this exercise, the variable s_t is the actual quarterly time series of NBER recession dates for the sample 1960Q1-2015Q4 (that is, 224 quarters), $\beta = 2$, $\delta = 1$, and $\sigma_\varepsilon = \sigma_\nu = .5$. Based on this set-up, I simulate *one sample* of y_t and x_t of size $T = 224$ and generate the sequence of one-step ahead forecasts of y_{t+1} with $R = 100$ and $P = 124$.³

Figure 1 shows the simulated time series (actual) and one-step ahead out-of-sample forecasts from the two competing models (top), as well as the forecast loss differences computed using a quadratic loss function (bottom). As we can observe, loss differences in recessions are generally large and positive suggesting that forecasts from model (ii) are more accurate during these periods. In expansions, on the other hand, loss differences are typically smaller and on average negative. Descriptive statistics and

³The random numbers were generated setting the MATLAB seed to `rng(1234)`.

tests of equal predictive ability are reported in Table 1. For the full hold-out sample (OOS), the average loss difference is close to 0 and positively autocorrelated. In this case, the GW and $\text{Fluct}_{t,m}$ tests fail to reject the null hypothesis of equal predictive ability and we conclude that both models exhibit similar predictive content over the full hold-out sample. Recently, researchers have used peak and trough dates determined by the NBER to evaluate the performance of the two forecasting models during recession and expansion periods separately. In this case, using the GW test we reject the null hypothesis of equal predictive ability for both sub-periods and conclude that model (ii) is more accurate during recessions ($\text{GW} > 0$) but less accurate during expansions ($\text{GW} < 0$).⁴

[FIGURE 1 ABOUT HERE]

[TABLE 1 ABOUT HERE]

In sum, this exercise illustrates the following issues. First, the GW and $\text{Fluct}_{t,m}$ tests may have difficulties rejecting the null hypothesis of equal predictive ability if the superior performance of one of the two competing models is constrained to short periods as in the case of recessions (just a few observations, even if observed repeatedly). This inability to reject extends to other tests of unconditional predictive ability (for example, West, 1996). Second, state-dependent predictive content can be uncovered using tests of unconditional predictive ability and exogenously provided shift dates if the econometrician is able to observe when the underlying states shift. In many cases,

⁴This approach is used in Rapach et al. (2010), Henkel et al. (2011), Dangel and Halling (2012), Chauvet and Potter (2013), Gargano and Timmermann (2014), Gargano et al. (2016), Dotsey et al. (2015), and Gibbs and Vasnev (2017) to test for state-dependent predictive ability over the business cycle and is equivalent to implementing the test of conditional predictive ability of Giacomini and White (2006).

however, such a testing strategy would not be feasible since the underlying states may not be observed by the econometrician.

2.3 State-dependent loss differences

To test for state-dependent predictive ability without exogenously provided shift dates, the out-of-sample forecast loss differences can be modeled using the Markov-switching mean plus noise regression

$$\Delta L_t(\hat{\delta}_{t-h,R}, \hat{\gamma}_{t-h,R}) = \mu_{s_t} + \sigma_{s_t} u_t, \quad (5)$$

where $s_t (= 0, 1)$ is an unobserved two-state first-order Markov process with transition probabilities

$$\text{Prob}(s_t = j \mid s_{t-1} = i) = p_{ij} \quad i, j = 0, 1, \quad (6)$$

and u_t is an unobservable moving-average (MA) process with zero mean and non-zero autocorrelations up to lag $h - 1$. Note that (5) can accommodate several cases of interest. For example, the two models can exhibit equal predictive ability in one regime ($\mu_0 = 0$), whereas one of the models is more accurate in the other regime ($\mu_1 \neq 0$). Alternatively, one model could be more accurate in the first regime ($\mu_0 > 0$) while the other model could be more accurate in the second regime ($\mu_1 < 0$). In addition, there can be differences in the degree of persistence or expected duration of the two regimes. For example, both regimes could be very persistent (both p_{00} and p_{11} are large) or one regime could be very persistent while the other regime is constrained to shorter periods (p_{00} large and p_{11} small). Finally, (5) can also accommodate a permanent structural break in the relative forecasting performance ($p_{11} = 1$ and the second regime is absorbing), as the one-time reversal test of Giacomini and Rossi (2010).

Based on (5), several hypotheses of interest can be formulated. For example, $H_0^{(1)}$ is the null hypothesis of *equal and constant predictive ability* of the two forecasting models considered in Giacomini and Rossi (2010) and implies testing $\mu_0 = \mu_1 = 0$.⁵ In contrast, Martins and Perron (2016) suggest testing the null hypothesis of *constant predictive ability* given by

$$H_0^{(2)} : E \left[\Delta L_t(\hat{\delta}_{t-h,R}, \hat{\gamma}_{t-h,R}) \right] = c \text{ for all } t = R + h, \dots, T, \quad (7)$$

for some constant c against the alternative of changing or, in this case, state-dependent predictive ability. $H_0^{(2)}$ implies testing $\mu_0 = \mu_1$ in (5).⁶

2.4 Asymptotic predictive ability tests

An important result of Giacomini and White (2006) is that under their framework we can treat the sequence of P out-of-sample loss differences $\{\Delta L_t(\hat{\delta}_{t-h,R}, \hat{\gamma}_{t-h,R})\}_{t=R+h}^T$ as observed data. For this result to hold, however, the estimation sample size R must remain finite as the out-of-sample size P grows to infinity. As a result, the parameters of the forecasting models ($\hat{\delta}$ and $\hat{\gamma}$) are usually estimated using a rolling scheme with a finite estimation window R , while a recursive scheme with an expanding estimation window cannot be used.

In this context, the regime-switching regression (5) can be estimated by maximum likelihood (ML) following Hamilton (1989, 1990) or Kim (1994). Equation (5) is characterized by six parameters $\theta = (\mu_0, \mu_1, \sigma_0, \sigma_1, p_{00}, p_{11})'$ and under standard

⁵It is worth noting that the null hypothesis being tested is $\mu_0 = \mu_1 = 0$ and $\sigma_0 \neq \sigma_1$. There are two reasons for not imposing $\sigma_0 = \sigma_1$. First, under $\mu_0 = \mu_1 = 0$ and $\sigma_0 = \sigma_1$, some parameters are only identified under the alternative hypothesis and the test becomes non-standard. Second, allowing for different variances across states appears to help with the identification of the regimes. See Table 1, Figure 1, and the empirical example below.

⁶In addition, Martins and Perron (2016) suggest the following testing strategy. If $H_0^{(2)}$ is rejected, conclude that the relative forecasting ability of the two models is not constant. If $H_0^{(2)}$ is not rejected, apply the GW test of unequal (but constant) predictive ability.

asymptotic normality arguments the asymptotic distribution of the ML estimator $\hat{\theta}$ is $\sqrt{P}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, \Omega)$ as $P \rightarrow \infty$. Linear hypotheses of the form $R_0\theta = 0$ can be tested by constructing the Wald test statistic

$$\text{Wald} = P(R_0\hat{\theta})'(R_0\hat{\Omega}R_0')^{-1}(R_0\hat{\theta}), \quad (8)$$

where R_0 is a $r \times 6$ matrix with r is the number of restrictions, and $\hat{\Omega}$ is a consistent estimator of Ω . Under the null hypothesis, $\text{Wald} \xrightarrow{d} \chi^2(r)$ as $P \rightarrow \infty$. Wald statistics for $H_0^{(1)}$ and $H_0^{(2)}$, SD-Wald hereafter, can easily be constructed by appropriately selecting the matrix of restrictions R_0 . For example, to test the null hypothesis of equal and constant predictive ability $H_0^{(1)}$, $R_0 = (I_2, 0_2, 0_2)$ with I_2 an identity matrix and 0_2 a matrix of zeros both of dimension 2×2 . As a result, SD-Wald $\overset{a}{\sim} \chi^2(2)$ and $H_0^{(1)}$ is rejected at the 5% level if SD-Wald > 5.99 . Similarly, to test the null hypothesis of constant predictive ability $H_0^{(2)}$, $R_0 = (1, -1, 0, 0, 0, 0)$ and the SD-Wald statistic is $P(\hat{\mu}_0 - \hat{\mu}_1)^2/(\hat{\omega}_{11} + \hat{\omega}_{22} - 2\hat{\omega}_{21}) \overset{a}{\sim} \chi^2(1)$, where $\hat{\omega}_{ij}$ is the (i, j) element of $\hat{\Omega}$. As a result, $H_0^{(2)}$ is rejected at the 5% level if SD-Wald > 3.84 . The rest of this paper will focus on testing the null hypothesis $H_0^{(1)}$.

Construction of the SD-Wald statistic requires a consistent estimator of Ω . Based on the information matrix condition, two estimators are typically used. The first estimator of the covariance matrix is minus the inverse of the average Hessian evaluated at the ML estimates

$$\hat{\Omega}^H = - \left[\frac{1}{P} \sum_{t=R+h}^T H_t(\hat{\theta}) \right]^{-1}, \quad (9)$$

where $H_t(\theta)$ is the Hessian for observation t and $\sum_{t=R+h}^T H_t(\hat{\theta})$ is usually obtained numerically as in Hamilton (1989). The second estimator of the covariance matrix is the inverse of the average outer-product of the score vector evaluated at the ML

estimates

$$\hat{\Omega}^{OP} = \left[\frac{1}{P} \sum_{t=R+h}^T g_t(\hat{\theta}) g_t(\hat{\theta})' \right]^{-1}, \quad (10)$$

where $g_t(\theta)$ is the score vector for observation t as in Hamilton (1996). For multi-step forecasts (that is, $h > 1$), however, the loss differences are autocorrelated up to lag $h - 1$ and (5) is misspecified.⁷ In this case, the information matrix condition does not hold, (9) and (10) are not valid, and a HAC estimator of the covariance matrix is required (Diebold and Mariano, 1995). In addition, if there are instabilities (for example, state-dependency), the forecast loss differences are also autocorrelated under the null hypothesis of linearity (see the illustrative example above) and, as a result, a HAC estimator is required even if $h = 1$ (Morley and Rabah, 2014; Martins and Perron, 2016).

An alternative estimator of the covariance matrix that remains valid when the information matrix condition does not hold is the ‘sandwich’ estimator

$$\hat{\Omega}^S = \left[\frac{1}{P} \sum_{t=R+h}^T H_t(\hat{\theta}) \right]^{-1} \hat{\Sigma}(\hat{\theta}) \left[\frac{1}{P} \sum_{t=R+h}^T H_t(\hat{\theta}) \right]^{-1}, \quad (11)$$

where Σ is the long-run variance of the score vector (see, for example, Hayashi, 2000). A HAC estimate of Σ can be obtained using the kernel-based estimator

$$\hat{\Sigma}(\hat{\theta}) = \sum_{j=-(q-1)}^{q-1} k(j/q) P^{-1} \sum_{t=R+h}^T g_t(\hat{\theta}) g_{t-j}(\hat{\theta})', \quad (12)$$

where $k(\cdot)$ is a kernel weight function, q is a bandwidth that grows with P , and $g_t(\theta)$ is the score vector for observation t (see, for example, Andrews, 1991). For serially uncorrelated scores $q = 1$ and (11) is the same estimator suggested in Hamilton (1996) to calculate robust standard errors.

⁷For $h > 1$, (5) is estimated by quasi-ML ignoring the MA($h - 1$) structure.

3 Monte Carlo Evidence

In this section I evaluate the properties of the tests of equal predictive ability (GW, $\text{Fluct}_{t,m}$, and SD-Wald) using Monte Carlo simulation. First, I analyze the performance of the quasi-ML estimator of θ and the HAC estimators of the covariance matrix Ω . In particular I evaluate the true confidence level of 95% confidence intervals for μ_0 constructed under the asymptotic theory discussed above. Next, I evaluate the size and power of the three tests under different DGPs.

3.1 HAC estimators and asymptotic confidence intervals

To evaluate the finite sample properties of the HAC estimators of the covariance matrix (discussed in section 2.4), sequences of loss differences ΔL_t are generated for the same parameterization of (5) considered in Hamilton (1996): $\mu_0 = -2$, $\mu_1 = 2$, $\sigma_0^2 = \sigma_1^2 = 1$, and $p_{00} = p_{11} = 0.8$. In this case, however, the error term is the MA(1) process

$$u_t = (1 + \theta^2)^{-1/2}(1 + \theta L)\varepsilon_t, \tag{13}$$

where ε_t is i.i.d. $N(0,1)$. Multiplying by $(1 + \theta^2)^{-1/2}$ normalizes the unconditional variance of u_t to 1. I consider sample sizes $T = 50, 100, 250$ and parameter values $\theta = 0, 0.5, 0.9$. For each sample, quasi-ML estimates are obtained using the EM algorithm described in Hamilton (1990). Next, the covariance matrix of $\hat{\theta}$ is estimated using the Hessian (H) estimator (9), the outer-product (OP) estimator (10), and the ‘sandwich’ estimator (11) implemented using three common kernels: the Bartlett kernel (NW), the Parzen kernel (PK), and the quadratic spectral kernel (QS). The results reported below are based on 1,000 replications, with the same set of random errors used across values of θ .

Table 2 reports the finite sample properties of the ML estimates of μ_0 . The bias of the estimator is calculated as the average value of $(\hat{\mu}_0^i - \mu_0)$ across the 1,000 replications (that is, $i = 1, \dots, 1000$). The actual variation (SD) is the square root of the average value of $(\hat{\mu}_0^i - \mu_0)^2$. Table 2 also summarizes the average standard errors and the true confidence levels of the nominal 95% confidence intervals using each of the five estimators of the covariance matrix. For $T = 50$ and $\theta = 0$, estimates of μ_0 are slightly biased and the average standard errors from all five estimators slightly understate the actual variation of $\hat{\mu}_0$. As a result, true confidence levels are just below the nominal level (between 0.920 and 0.936). As θ is increased from 0 to 0.5 to 0.9 the actual variation of $\hat{\mu}_0$ increases while the average standard errors using the H and OP estimators remain mostly constant and, as a result, true confidence levels drop substantially. This is expected as these commonly used estimators assume i.i.d errors. In contrast, average standard errors from the three HAC estimators increase and true confidence levels remain (somewhat) closer to the nominal level. When the sample size is increased from 50 to 100 we observe a similar pattern but with improved confidence levels. For $T = 250$, the difference between the average standard errors from any of the three HAC estimators and the actual variation of $\hat{\mu}_0$ almost disappears and true confidence levels are very close to the nominal 95% level.

[TABLE 2 ABOUT HERE]

In sum, when the model is correctly specified and no residual autocorrelation is present ($\theta = 0$), the H and OP estimators of the covariance matrix work well and generate average standard errors that accurately measure the actual variation of $\hat{\mu}_0$ and true confidence levels that are close to the nominal level. On the other hand, when autocorrelation is present ($\theta > 0$), the H and OP estimators understate the actual

variation of $\hat{\mu}_0$ and true confidence levels can be substantially below the nominal level. In contrast, the three HAC estimators of the covariance matrix are more accurate (on average) and true confidence levels are closer to the nominal level.

3.2 Size properties

To evaluate the empirical size of the tests of the null hypothesis of equal predictive ability, I consider three DGPs. The first DGP considers the case of a fundamental-based model and a random walk benchmark for first differences (with no state-dependency) and it is the same set up as in Giacomini and Rossi (2010) and Martins and Perron (2016). DGP-1 is given by

$$y_t = \beta x_t + \sigma_\varepsilon \varepsilon_t, \tag{14}$$

$$x_t = \phi x_{t-1} + \sigma_\nu \nu_t, \tag{15}$$

where ε_t and ν_t are i.i.d. $N(0, 1)$, with $\phi = .5$ and $\sigma_\varepsilon = \sigma_\nu = 1$. I consider one-step ahead out-of-sample forecasts from two models: (1) $y_{t+1} = u_{1t+1}$ and (2) $y_{t+1} = \beta x_{t+1} + u_{2t+1}$. As a result, the time- t one-step ahead out-of-sample forecasts are

$$\hat{f}_{t,R}^1 = 0,$$

$$\hat{f}_{t,R}^2 = \hat{\beta}_{t,R} x_{t+1},$$

where $\hat{\beta}_{t,R}$ is the in-sample estimate of β based on the last R observations and x_{t+1} is known at time t .

Under the null hypothesis $H_0^{(1)}$ the forecasting models are equally accurate in the finite sample. Setting $\beta = 0$, however, implies that the competing models are equally accurate in the population and, as a result, the smaller model (model 1) would be

preferred in the finite sample. To obtain properly sized tests, I follow Giacomini and White (2006) and Giacomini and Rossi (2010) and select β such that the two models have equal expected mean-squared errors

$$E \left[\left(y_{t+1} - \hat{f}_{t,R}^1 \right)^2 \right] = E \left[\left(y_{t+1} - \hat{f}_{t,R}^2 \right)^2 \right]. \quad (16)$$

Since $E[(y_{t+1} - \hat{f}_{t,R}^1)^2] = \sigma_\varepsilon^2 + \beta^2 x_{t+1}^2$ and $E[(y_{t+1} - \hat{f}_{t,R}^2)^2] = \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \frac{x_{t+1}^2}{\sum_{j=t-R+1}^t x_j^2}$, we have that $\beta^0 = \sqrt{\sigma_\varepsilon^2 / \sum_{j=t-R+1}^t x_j^2}$. Given that $\sigma_\varepsilon = \sigma_\nu = 1$ and $\phi = 0.5$, after some simplifications we have $\sum_{j=t-R+1}^t x_j^2 \approx R4/3$ and the value of β that satisfies (16) is

$$\beta^0 \approx 1/\sqrt{R4/3}. \quad (17)$$

The second DGP considers the case of state-dependent predictive ability and is the same set up as the illustrative example of section 2.2 but with a randomly generated state variable. DGP-2 is given by

$$y_t = -\beta s_t + \sigma_\varepsilon \varepsilon_t, \quad (18)$$

$$x_t = \delta s_t + \sigma_\nu \nu_t, \quad (19)$$

where ε_t and ν_t are i.i.d. $N(0,1)$, s_t ($= 0,1$) is an unobserved two-state first-order Markov process with transition probabilities $\text{Prob}(s_t = j | s_{t-1} = i) = p_{ij}$ for $i, j = 0, 1$. The specification is completed by assuming $\delta = 1$, $\sigma_\varepsilon = \sigma_\nu = 0.5$, and $p_{00} = p_{11} = 0.8$. As before, the two competing models are (1) $y_{t+1} = u_{1t+1}$ and (2) $y_{t+1} = \gamma x_{t+1} + u_{2t+1}$, and the time- t one-step ahead out-of-sample forecasts are

$$\hat{f}_{t,R}^1 = 0,$$

$$\hat{f}_{t,R}^2 = \hat{\gamma}_{t,R} x_{t+1},$$

where $\hat{\gamma}_{t,R}$ is the in-sample rolling estimate of γ based on the last R observations and

x_{t+1} is known at time t .

Again, to obtain properly sized tests, I select β such that the two models have equal expected mean-squared errors. Since $E[(y_{t+1} - \hat{f}_{t,R}^1)^2] = \sigma_\varepsilon^2 + \beta^2 S_{t+1}^2$ and $E[(y_{t+1} - \hat{f}_{t,R}^2)^2] = \sigma_\varepsilon^2 + \delta^2 S_{t+1}^2 \frac{\sigma_\varepsilon^2 + (\beta/\delta)^2 \sigma_\nu^2}{\sum_{j=t-R+1}^t x_j^2}$, after some simplifications we have that $\sum_{j=t-R+1}^t x_j^2 \approx R(\delta^2 p_1 + \sigma_\nu^2)$, $p_1 = \frac{1-p_{00}}{2-p_{00}-p_{11}}$, and the value of β that satisfies (16) is

$$\beta^0 = \sqrt{\delta^2 \left[1 - \frac{\sigma_\nu^2}{R(\delta^2 p_1 + \sigma_\nu^2)} \right]^{-1} \frac{\sigma_\varepsilon^2}{R(\delta^2 p_1 + \sigma_\nu^2)}}. \quad (20)$$

Finally, the third DGP also considers the case of state-dependent predictive ability but, in this case, the state variable s_t is the actual quarterly time series of NBER recession dates for the sample 1960Q1-2015Q4 (224 quarters) and equals 1 if the observation corresponds to a recession. DGP-3 is also given by (18), (19), and (20) with the unconditional probability of recession given by $p_1 = T^{-1} \sum_{t=1}^T s_t$.

Table 3 reports rejection frequencies of the GW, $\text{Fluct}_{t,m}$, and SD-Wald tests based on 1,000 replications. The three tests are constructed using the HAC estimator of the covariance matrix given by (11) and (12) and the Bartlett kernel of Newey and West (1987). The $\text{Fluct}_{t,m}$ test is constructed using a window size m of $.3P$. The GW test exhibits small size distortions under DGP-1 and DGP-2 but is oversized under DGP-3. The $\text{Fluct}_{t,m}$ test exhibits small size distortions under all three DGPs. The SD-Wald test also exhibits small size distortions under all three DGPs as long as the hold-out sample P is sufficiently large (at least 74 observations). The test is oversized when $P = 50$ under DGP-1 and DGP-2.

[TABLE 3 ABOUT HERE]

3.3 Power properties

Next, I evaluate the empirical power of the tests under the three DGPs discussed in section 3.2. To obtain the power curves, rejection frequencies are computed under the alternative hypothesis where data is generated assuming $\beta = \beta^0 + \beta^+$ with β^0 given by (17) or (20) and $\beta^+ > 0$. Under the Giacomini and White (2006) asymptotic framework, the estimation sample size R must remain finite as the out-of-sample size P grows to infinity. As a result, the parameters of the forecasting models need to be estimated using a rolling scheme. In practice, however, researchers appear to favor a recursive scheme with an expanding estimation window. So, in addition to results obtained using rolling estimation, I also report power curves using recursive estimation. As before, the three tests are constructed using the HAC estimator of the covariance matrix and the Bartlett kernel. All the results are based on 1,000 replications.

DGP-1 is given by (14), (15), and (17) and under the alternative hypothesis implies a case of unequal performance of the forecasting models that is constant over time (that is, no time variation or state-dependency). In this set up, model 2 is unconditionally more accurate on average. Figure 2 reports power curves for the three tests as β^+ is increased from 0 to 1 (with .1 increments) and $R = P = 100$. When the relative performance of the models is constant over time the SD-Wald test has power that is comparable to the $\text{Fluct}_{t,m}$ test with $m = .3P$ but lower than the GW test. Higher power of the GW test is expected as this is the set up for which the GW test is proposed (see, Giacomini and White, 2006). Rolling and recursive estimation generate similar results.

[FIGURE 2 ABOUT HERE]

DGP-2 and DGP-3 are given by (18), (19), and (20) and under the alternative

hypothesis both DGPs imply a case where the relative performance of the models varies across states. In this set up, model 1 (the smaller model) is on average more accurate when $s_t = 0$ (for example, economic expansions) but less accurate when $s_t = 1$ (for example, economic recessions). Figure 3 reports power curves for the three tests as β^+ is increased from 0 to 1 (with .1 increments) for $p_{00} = p_{11} = 0.8$ and $R = P = 100$. Relative to the results under unequal but constant performance (Figure 2), the power curves of the GW and $\text{Fluct}_{t,m}$ tests shift down. In contrast, the SD-Wald test exhibits an improvement in power.

[FIGURE 3 ABOUT HERE]

Recessions in the US economy tend to be short-lived, typically lasting less than four quarters, while expansions are more persistent. Therefore, to account for this difference in persistence, Figure 4 reports power curves for the three tests with $p_{00} = 0.9$, $p_{11} = 0.75$ and $R = P = 100$. This DGP implies economic expansions ($s_t = 0$) with an expected duration of ten quarters and economic recessions ($s_t = 1$) with an expected duration of four quarters.⁸ In this case, the GW and $\text{Fluct}_{t,m}$ tests exhibit a substantial deterioration in power (power curves shift down). In contrast, while the SD-Wald test also exhibits a reduction in power, the test is still able to frequently reject the null hypothesis of equal predictive ability. Similarly, Figure 5 reports power curves for the three tests but, in this case, s_t is the actual time series of NBER recession dates for the period 1960Q1 to 2015Q4, with $R = 100$ and $P = 124$. The power curves show that the GW and $\text{Fluct}_{t,m}$ tests are now unable to reject the null hypothesis when the superior performance of one of the two competing models is constrained to very short periods as in the case of recessions in the US economy. As a result, the GW and

⁸The expected duration (in quarters) of regime j is computed as $1/(1 - p_{jj})$ for $j = 0, 1$.

Fluct $_{t,m}$ tests exhibit no power. In contrast, the SD-Wald test is still able to detect the change in relative performance over the business cycle and exhibits superior power.

[FIGURE 4 ABOUT HERE]

[FIGURE 5 ABOUT HERE]

In sum, the simulation results suggest that the SD-Wald test has relatively more power when the superior performance of one of the two competing forecasting models is constrained to short periods as in the case of recessions in the US economy. In contrast, the GW and Fluct $_{t,m}$ tests can exhibit no power at all, generally failing to reject the (incorrect) hypothesis of equal predictive ability. When the relative performance of the models is constant over time, the SD-Wald test has power that is comparable to the Fluct $_{t,m}$ test (with $m = .3P$) but lower than the GW test. Overall, the tests exhibit more power using recursive estimation than rolling estimation.

4 Empirical Example: Forecasting Output

Chauvet and Potter (2013) survey the recent literature on real-time forecasting of US output growth and evaluate the accuracy of forecasts obtained from many different models (linear and nonlinear models, reduced-form and structural models, survey based forecasts, etc.). Among other results, they document that most output growth forecasting models exhibit a similar performance during economic expansions but *one model performs significantly better during recessions*. In this section I use this result to illustrate the improvement of the proposed test over previous approaches to perform forecast comparisons.

4.1 Forecasting models and data

The out-of-sample forecasting exercise is implemented as follows. The first forecasting model, the benchmark, is a simple linear autoregressive (AR) model for output growth. Let y_t be 400 times the log difference of quarterly real GDP, then the AR(k) model for output growth is given by

$$y_{t+1} = \alpha + \sum_{i=0}^{k-1} \beta_i y_{t-i} + \varepsilon_{t+1}. \quad (21)$$

Chauvet and Potter (2013) find that this model is the most accurate during expansions (or at least as good as any of the other models considered). For recessions, on the other hand, they find that the AR model augmented with an estimated real activity (dynamic) factor and an estimated probability of recession (AR-DF) exhibits the best performance. As a result, the alternative forecasting model is given by

$$y_{t+1} = \alpha + \sum_{i=0}^{k-1} \beta_i y_{t-i} + \delta \hat{g}_t + \gamma \hat{p}_t + \varepsilon_{t+1}, \quad (22)$$

where \hat{g}_t is the estimated real activity factor and \hat{p}_t is the estimated probability of recession.

Following Chauvet and Potter (2013), I estimate the factor and probabilities of recession from a set of four monthly real activity indicators previously used in Stock and Watson (1991), Diebold and Rudebusch (1996), Chauvet (1998), Chauvet and Piger (2008), Camacho et al. (2015), and Fossati (2015, 2016), among many others. This panel includes industrial production, real manufacturing sales, real personal income less transfer payments, and employment. It is assumed that the series in the panel have a factor structure of the form

$$x_{it} = \lambda_i g_t + e_{it}, \quad (23)$$

where $i = 1, \dots, 4$, $t = 1, \dots, T$, g_t is an unobserved common factor, λ_i is the factor loading, and e_{it} is the idiosyncratic error. The dynamics of the common factor are driven by $\phi(L)g_t = \eta_t$ with $\eta_t \sim \text{i.i.d.}N(0, 1)$, while the dynamics of the idiosyncratic errors are driven by $\psi_i(L)e_{it} = \nu_{it}$ with $\nu_{it} \sim \text{i.i.d.}N(0, \sigma_i^2)$ for $i = 1, \dots, 4$. Identification is achieved by assuming that all shocks are independent and the specification is completed with all autoregressive processes including two lags. Next, I use a Markov-switching model to generate recession probabilities directly from the dynamic factor as in Diebold and Rudebusch (1996) and Camacho et al. (2015). Assume that the factor \hat{g}_t switches between expansion and contraction regimes following a mean plus noise specification given by

$$\hat{g}_t = \mu_{s_t} + \epsilon_t, \tag{24}$$

where s_t is defined such that $s_t = 0$ during expansions and $s_t = 1$ during recessions, and $\epsilon_t \sim \text{i.i.d.}N(0, \sigma_\epsilon^2)$. As usual, s_t is an unobserved two-state first-order Markov process with transition probabilities given by $\text{Prob}(s_t = j | s_{t-1} = i) = p_{ij}$, with $i, j = 0, 1$. The models can be estimated by maximum likelihood following Hamilton (1989, 1990) and Kim and Nelson (1999).

Finally, we need to discuss the timing of the forecasting exercise. First, the dynamic factor model is estimated recursively, using (standardized) monthly real-time vintage data. In order to account for publication delay and important revisions that are usually observed in the first and the second release (see Chauvet and Piger, 2008), the four indicators are lagged two months. Next, the recession probabilities are estimated by fitting the Markov-switching model to the estimated dynamic factor. The dynamic factor and probabilities of recession are estimated for the period 1960:1 to 2010:12 using monthly vintage data from Camacho et al. (2015). The AR models are estimated recursively using real-time data, a rolling window of 80 quarters, and with the lag

order (p) selected using the Akaike information criterion. I use real-time quarterly data obtained from the Federal Reserve Bank of Philadelphia’s Real Time Data Set for Macroeconomists. The initial estimation sample is from 1967:Q1 to 1986:Q4, the first one-quarter-ahead forecast is for 1987:Q1, and the last forecast corresponds to 2010:Q4 (that is, $R = 80$ and $P = 96$). As an example, in late March 1987 the econometrician would have access to the February release of real GDP with data up to December 1986. The monthly real activity indicators would be available up to January 1987 (a two-month publication delay). These series are used to estimate the models described above and to generate a one-quarter-ahead forecast for 1987:Q1, effectively a nowcast.⁹

4.2 Results

Figure 6 shows the actual time series of real-time real GDP growth (the second release) and the one-quarter-ahead out-of-sample forecasts from the two competing models (top), as well as the forecast loss differences computed using a quadratic loss function for the hold-out sample 1987Q1-2010Q4 (bottom). As we can observe, both forecasts track the actual time series reasonably well during expansions and, as a result, loss differences are typically small and close to zero. In contrast, loss differences in recessions are generally large and positive suggesting that forecasts from the AR-DF model are more accurate during these periods. Descriptive statistics and tests of equal predictive

⁹This exercise is similar but not exactly the same as the one in Chauvet and Potter (2013). The main differences are: (1) Chauvet and Potter (2013) estimate the dynamic factor and recession probabilities simultaneously following Chauvet (1998) while I follow the two-step approach of Diebold and Rudebusch (1996). See also Camacho et al. (2015). (2) Chauvet and Potter (2013) estimate the AR models using a recursive scheme while I estimate the models using a rolling scheme with a fixed window of 80 quarters. (3) The hold-out sample in this paper is 1987:Q1-2010Q4 and includes the last three US recessions. In contrast, Chauvet and Potter (2013) use the sample 1992:Q1-2010Q4 and include only the last two recessions. As a result, there are some differences in the results reported.

ability are reported in Table 4. For the full hold-out sample (OOS), the average loss difference is greater than zero and positively autocorrelated. In this case, the GW test fails to reject the null hypothesis of equal predictive ability. As a result, we conclude that both models exhibit similar predictive content over the full hold-out sample. Chauvet and Potter (2013) then use peak and trough dates determined by the NBER to evaluate the performance of the two forecasting models during recession (OOS₁) and expansion (OOS₀) periods separately. In recessions, the GW test rejects the null hypothesis of equal predictive ability and we conclude that the AR-DF model is more accurate (GW > 0). In expansions, however, we fail to reject the null hypothesis of equal predictive ability and conclude that both models exhibit a similar performance (GW ≈ 0).

[FIGURE 6 ABOUT HERE]

[TABLE 4 ABOUT HERE]

Next, we turn our attention to the Fluctuation test of Giacomini and Rossi (2010). The $\max_t |\text{Fluct}_{t,m}|$ test statistics are reported in Table 4 and show that with a window size set to $m = .3P$ the test fails to reject the null hypothesis of equal predictive ability over the full hold-out sample. Given that recessions in the US are typically short-lived events, a smaller window size may be more appropriate. For example, setting the window size to $m = .1P$ we find that the test rejects the null hypothesis at the 5% level. Figure 7 (top) reports the time series of $\text{Fluct}_{t,m}$ test statistics and the corresponding two-sided critical values at the 5% level. Positive values of the test statistic indicate that the AR-DF model is more accurate. The results suggest that most of the time the two models exhibit a similar predictive ability ($\text{Fluct}_{t,m} \approx 0$), but

there is some evidence that the AR-DF model is better during the 2007-09 recession ($\text{Fluct}_{t,m} > 0$).

[FIGURE 7 ABOUT HERE]

Finally, the results reported in Table 4 show that the SD-Wald test rejects the null hypothesis of equal predictive ability. The parameter estimates (standard errors in parentheses) of the test equation (5) are: $\hat{\mu}_0 = 24.72$ (6.10), $\hat{\mu}_1 = -0.06$ (0.42), $\hat{\sigma}_0^2 = 129.13$ (57.90), $\hat{\sigma}_1^2 = 9.37$ (1.70), $\hat{p}_{00} = 0.48$ (0.25), and $\hat{p}_{11} = 0.98$ (0.02). The results suggest the presence of a regime in which the AR-DF model is more accurate ($\hat{\mu}_0 > 0$) and that this regime has an expected duration of (about) two quarters. In the other regime, the two models exhibit a similar performance ($\hat{\mu}_1 \approx 0$). The estimated state probabilities reported in Figure 7 (bottom) show that the AR-DF model performed better than the AR model during the 1990-91 and 2007-09 recessions, but not during the 2001 recession.

In sum, in this exercise I show that tests of the null hypothesis of equal predictive ability based on averages (unconditional or rolling) are inadequate when predictability is constrained to just a few observations. For example, the $\text{Fluct}_{t,m}$ test only rejects the null hypothesis during the longest recession in the sample and only if a very small window is used ($m = .1P$). But using such a small window typically yields oversized tests (Giacomini and Rossi, 2010). Using NBER peak and through dates and a test of unconditional predictive ability as in Chauvet and Potter (2013) I find that the AR-DF model performs significantly better during recessions. The SD-Wald test proposed in this paper is able to uncover this result without making assumptions about the state of the economy. In addition, we learn that the rejection of the null hypothesis for recessions is driven by two recessions (the 1990-91 and 2007-09 recessions) and just four observations.

5 Conclusion

In the macroeconomic forecasting literature, predictability has been shown to be both unstable over time and state-dependent. Tests of the null hypothesis of equal predictive ability based on averages are inadequate when predictability is constrained to just a few observations because the superior accuracy of a model can be averaged out. As a result, two models may appear to be equally accurate when they are not. This paper proposed a new test for comparing the out-of-sample forecasting performance of two competing models for situations in which the predictive content may be state-dependent. The main improvement over previous approaches to conditional forecast comparison is that the econometrician is not required to observe when the underlying states shift and, as a result, the test can be applied in situations in which the states are not observed (for example, expansion and recession states, low and high volatility states, etc.). I illustrate these results by analyzing the real-time performance of two forecasting models for US output growth discussed in Chauvet and Potter (2013).

The results discussed in this paper have implications for applied macroeconomic forecasting. For example, Gibbs and Vasnev (2017) show that if one model is found to be more accurate in one state but the other model is more accurate in the other state, a forecast combination strategy that weighs forecasts based on the predicted probability of being in each state (forward-looking weights) yields more accurate predictions of the inflation rate. The methods proposed in this paper may uncover states in which one model exhibits superior predictive ability and these results could be used to improve the overall accuracy of macroeconomic forecasts.

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Table 1: Loss difference statistics for a simulated time series

	OOS	OOS ₀	OOS ₁
Observations	124	110	14
Average	0.092	-0.179	2.217
Standard Dev.	0.969	0.492	1.169
AR(1)	0.407**	-0.137	-0.005
GW	1.056	-3.809**	7.096**
GW(HAC)	0.683		
Fluctuation(HAC, $m = .1P$)	2.704		
Fluctuation(HAC, $m = .3P$)	2.568		

Notes: Significance of the AR(1) coefficients is tested based on the asymptotic result $\sqrt{T}\hat{\rho} \xrightarrow{d} N(0, 1)$. HAC tests constructed using the Bartlett kernel of Newey and West (1987). The 5% (10%) critical value is 1.96 (1.645) for a two-sided GW test, 3.012 (2.766) for a two-sided Fluctuation test with a rolling window size of $m = .3P$, and 3.393 (3.170) for $m = .1P$. ** (*) denotes rejection of the null hypothesis at the 5% (10%) level.

Table 2: Small sample properties of the ML estimates of μ_0

	θ	Bias	SD	H	OP	NW	PK	QS
$T = 50$	0	0.008	0.256	0.223	0.235	0.247	0.243	0.245
				0.920	0.929	0.936	0.920	0.928
	0.5	0.007	0.314	0.219	0.231	0.266	0.275	0.276
				0.830	0.841	0.875	0.886	0.887
	0.9	0.025	0.345	0.226	0.228	0.291	0.301	0.302
				0.798	0.810	0.867	0.868	0.871
$T = 100$	0	0.006	0.159	0.154	0.156	0.163	0.162	0.163
				0.933	0.938	0.946	0.942	0.945
	0.5	0.015	0.220	0.161	0.156	0.202	0.210	0.210
				0.846	0.838	0.896	0.909	0.910
	0.9	0.010	0.233	0.163	0.157	0.211	0.219	0.220
				0.824	0.817	0.904	0.908	0.913
$T = 250$	0	-0.004	0.096	0.098	0.099	0.101	0.100	0.101
				0.962	0.959	0.963	0.961	0.963
	0.5	0.006	0.128	0.101	0.098	0.120	0.125	0.125
				0.869	0.856	0.925	0.939	0.937
	0.9	-0.001	0.135	0.102	0.098	0.129	0.134	0.134
				0.874	0.856	0.932	0.942	0.943

Notes: The true model is $\Delta L_t(\hat{\delta}_{t-h,R}, \hat{\gamma}_{t-h,R}) = \mu_{s_t} + \sigma_{s_t} u_t$ with $s_t = 0, 1$, $\mu_0 = -2$, $\mu_1 = 2$, $\sigma_0^2 = \sigma_1^2 = 1$, and $p_{00} = p_{11} = 0.8$. The error term is an MA(1) process $u_t = (1 + \theta^2)^{-1/2}(1 + \theta L)\varepsilon_t$ where $\varepsilon_t \sim \text{i.i.d.} N(0, 1)$. The covariance matrix estimators are: Hessian (H), outer-product (OP), Bartlett (NW), Parzen (PK), and quadratic spectral kernel (QS). Bias is the average value of $(\hat{\mu}_0^i - \mu_0)$. SD is the square root of the average value of $(\hat{\mu}_0^i - \mu_0)^2$.

Table 3: Empirical size under quadratic loss

R	P	GW	Fluct	SD-W	GW	Fluct	SD-W
		DGP-1			DGP-2		
50	50	0.044	0.043	0.084	0.053	0.056	0.093
	100	0.055	0.048	0.044	0.057	0.040	0.045
	250	0.031	0.037	0.034	0.076	0.044	0.055
100	50	0.059	0.043	0.103	0.047	0.053	0.097
	100	0.038	0.045	0.053	0.067	0.062	0.049
	250	0.038	0.051	0.030	0.064	0.030	0.046
250	50	0.054	0.052	0.118	0.075	0.068	0.114
	100	0.061	0.065	0.056	0.052	0.063	0.052
	250	0.047	0.071	0.041	0.073	0.062	0.045
		DGP-3					
50	174				0.104	0.053	0.056
100	124				0.081	0.061	0.051
150	74				0.058	0.065	0.062

Notes: The $\text{Fluct}_{t,m}$ test constructed using a window size of $.3P$. Nominal size 0.05.

Table 4: Loss difference statistics for real GDP growth

	OOS	OOS ₀	OOS ₁
Observations	96	82	14
Average	1.032	-0.159	8.005
Standard Dev.	6.388	3.019	13.422
AR(1)	0.279**	-0.136	0.121
GW	1.582	-0.477	2.232**
GW(HAC)	1.360		
Fluctuation(HAC, $m = .1P$)	3.517**		
Fluctuation(HAC, $m = .3P$)	2.232		
SD-Wald(HAC)	16.484**		

Notes: Significance of the AR(1) coefficients is tested based on the asymptotic result $\sqrt{T}\hat{\rho}_1 \xrightarrow{d} N(0, 1)$. HAC tests constructed using the Bartlett kernel of Newey and West (1987). The 5% (10%) critical value is 1.96 (1.645) for a two-sided GW test, 3.012 (2.766) for a two-sided Fluctuation test with a rolling window size of $m = .3P$, and 3.393 (3.170) for $m = .1P$. The 5% (10%) critical value for a SD-Wald test is 5.99 (4.61). ** (*) denotes rejection of the null hypothesis at the 5% (10%) level.

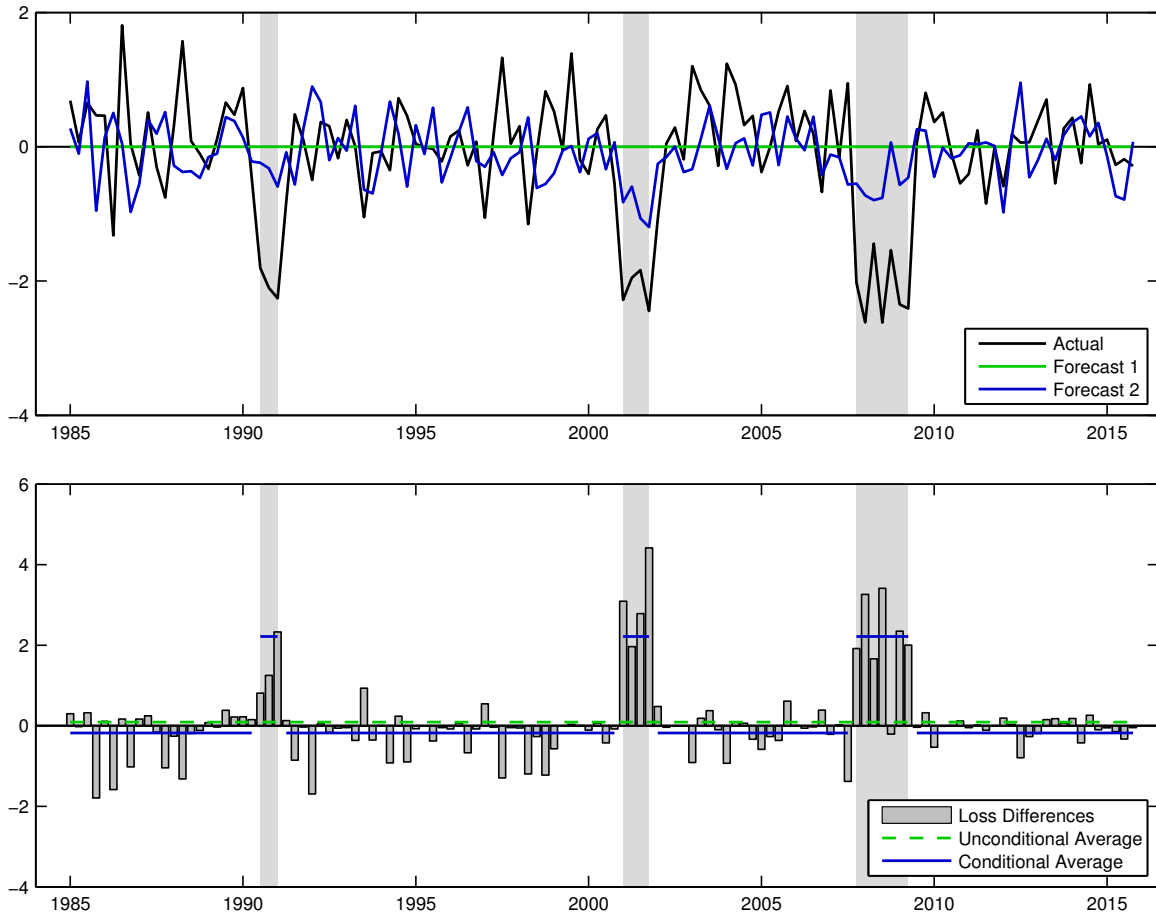


Figure 1: Shows the simulated time series and forecasts from the two competing models (top) and loss differences with sample averages (bottom). The two forecasting models are: (i) $\hat{f}_{t,R}^1 = 0$ and (ii) $\hat{f}_{t,R}^2 = \hat{\gamma}_{t,R}x_{t+1}$. Shaded areas denote NBER recessions.

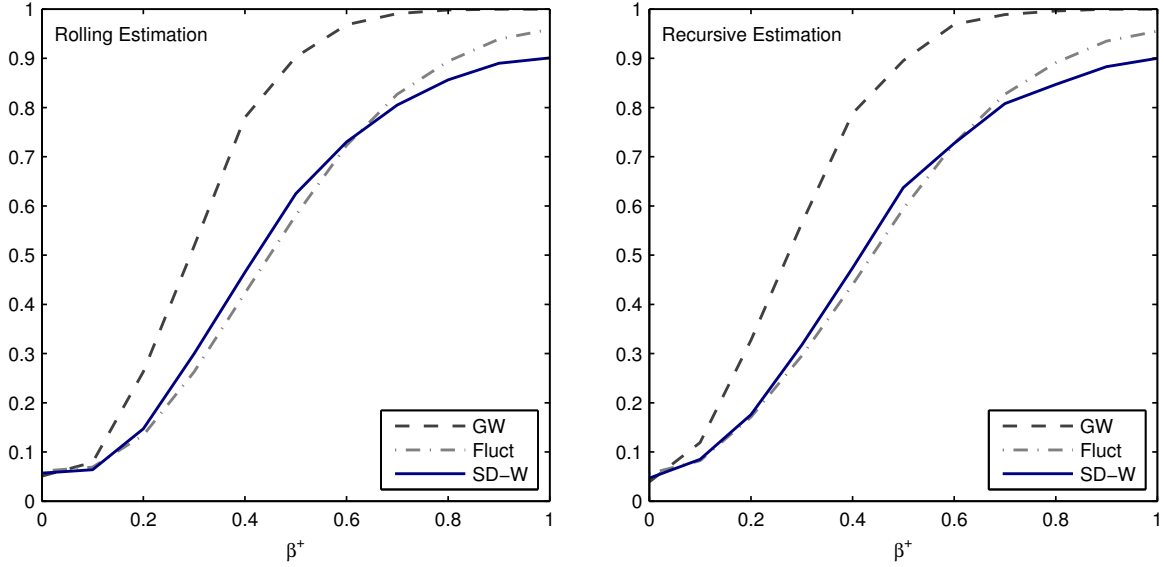


Figure 2: Empirical power (rejection frequencies) of the GW, $\text{Fluct}_{t,m}$ ($m = .3P$), and SD-Wald tests in the case of unequal but constant relative forecasting performance of the two models with $R = P = 100$.

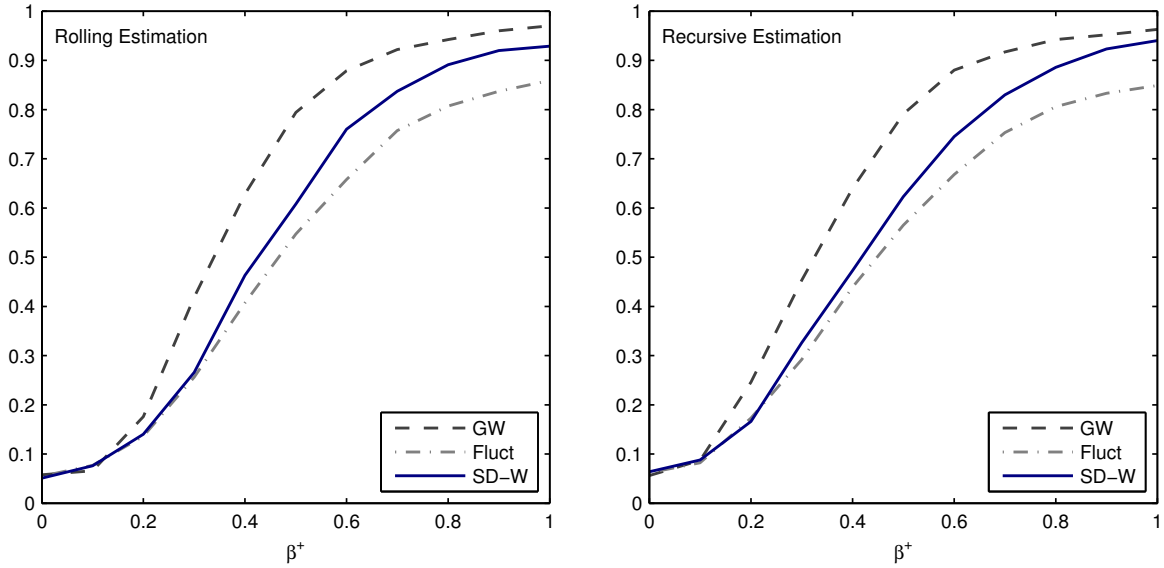


Figure 3: Empirical power (rejection frequencies) of the GW, $\text{Fluct}_{t,m}$ ($m = .3P$), and SD-Wald tests in the case of different relative forecasting performance of the two models in different states with $p_{00} = p_{11} = 0.8$ and $R = P = 100$.

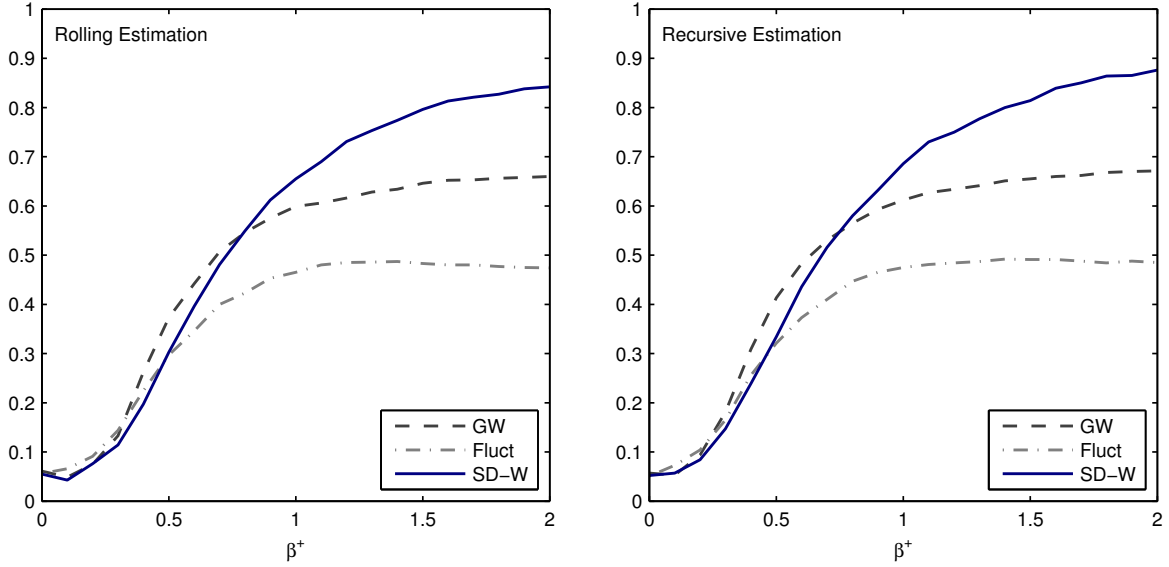


Figure 4: Empirical power (rejection frequencies) of the GW, $\text{Fluct}_{t,m}$ ($m = .3P$), and SD-Wald tests in the case of different relative forecasting performance of the two models in different states with $p_{00} = 0.90$, $p_{11} = 0.75$, and $R = P = 100$.

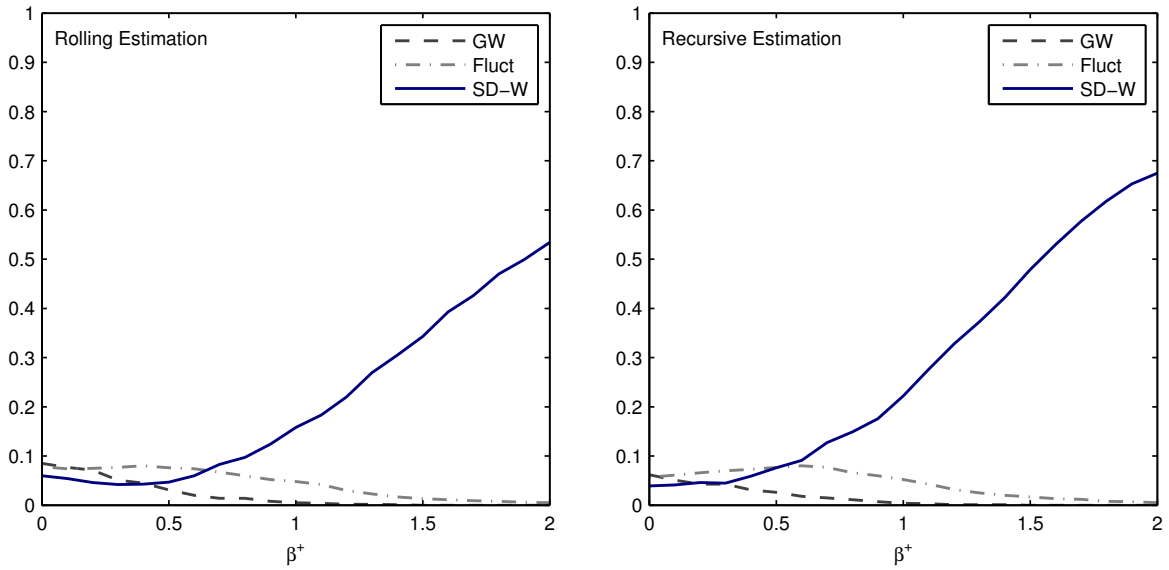


Figure 5: Empirical power (rejection frequencies) of the GW, $\text{Fluct}_{t,m}$ ($m = .3P$), and SD-Wald tests in the case of different relative forecasting performance of the two models in different states with s_t the actual time series of NBER recession dates for the period 1960Q1 to 2015Q4, $R = 100$, and $P = 124$.

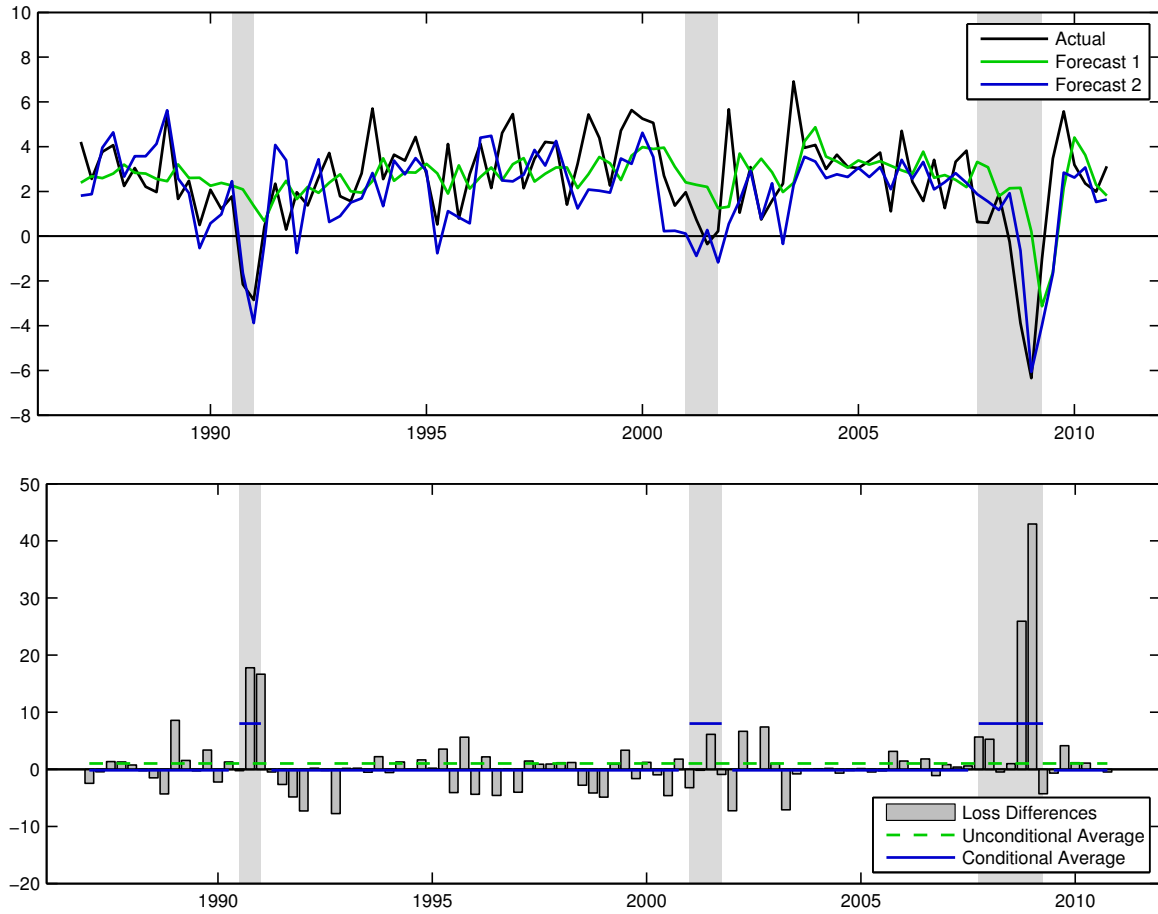


Figure 6: Shows real-time GDP growth and forecasts from the two competing models (top) and loss differences with sample averages (bottom). The two forecasting models are: (i) $\hat{f}_{t,R}^1 = \text{AR}$ and (ii) $\hat{f}_{t,R}^2 = \text{AR-DF}$. Shaded areas denote NBER recessions.

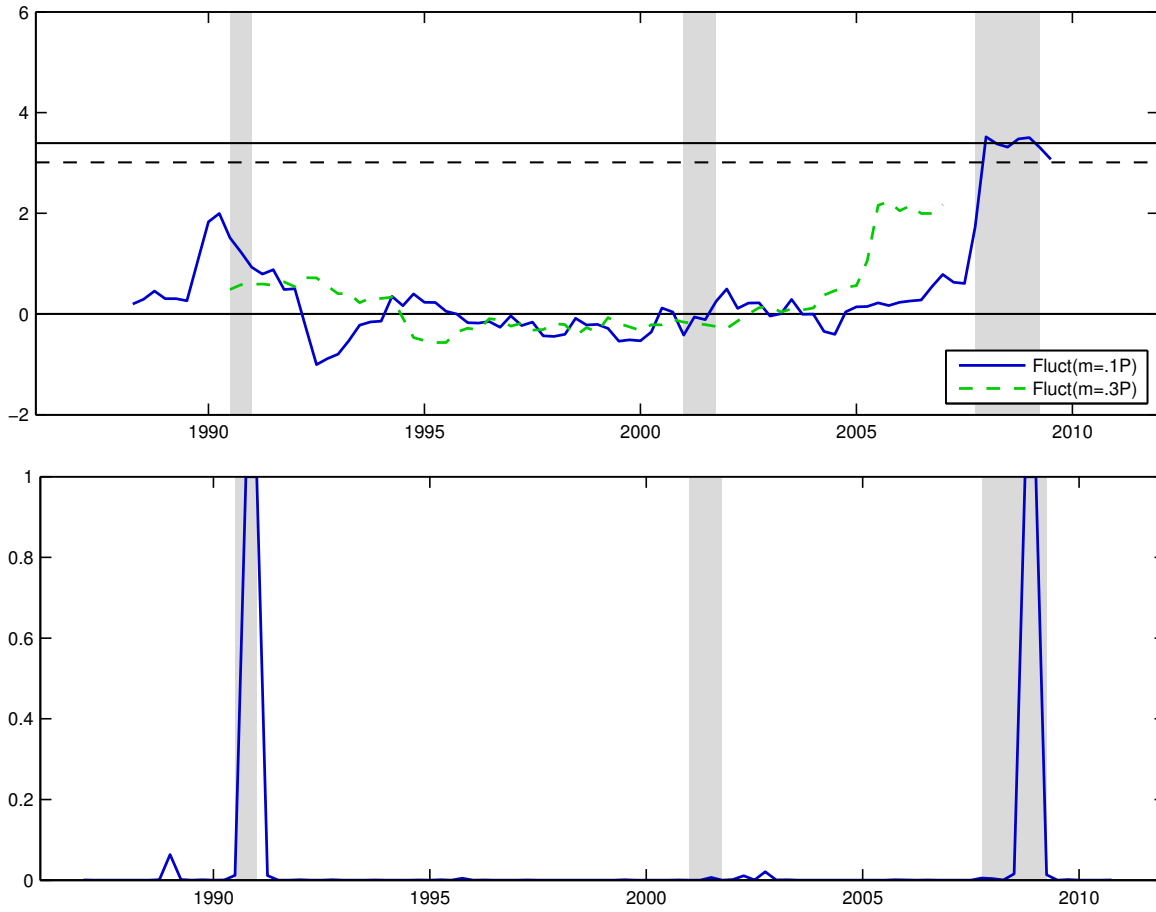


Figure 7: Shows the $\text{Fluct}_{t,m}$ test statistics for $m = .1P$ and $m = .3P$ with the corresponding critical values (top) and the state probabilities from the SD-Wald test (bottom). Shaded areas denote NBER recessions.

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