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Collateralized Borrowing and Risk Taking at Low Interest Rates

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Abstract

Empirical evidence suggests financial intermediaries increase risky investments when interest rates are low. We develop a model consistent with this observation and ask whether the risks undertaken exceed the social optimum. Interest rate policy affects risk taking in the model through two opposing channels. First, low policy rates make riskier assets more attractive than safe bonds. Second, low policy rates reduce the amount of safe bonds available for collateralized borrowing in interbank markets. The calibrated model features excessive risk taking at the optimal policy. However, at low policy rates, collateral constraints tighten and risk taking doesn't exceed the social optimum.

Keywords: *Financial intermediation, risk taking, optimal interest rate policy.*

JEL-Codes: *E44, E52, G11, G18.*

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1 Introduction

The late-2000s global financial crisis has renewed interest in the determinants of portfolio investments into safe and risky assets by financial intermediaries. A view advanced in the aftermath of the crisis is that during extended periods of low interest rates financial intermediaries take on excessive risks. The idea that interest rate policy affects risk taking by intermediaries—also referred to as the *risk taking channel of monetary policy*, a term coined by Borio and Zhu (2012)—prompted a recent empirical literature. One main finding of this literature is a negative relationship between the level of interest rates and bank risk taking.[1] In light of this observation, it has been argued that central banks could have prevented the build-up of risk in the run-up to the recent financial crisis and the ensuing negative consequences for the macroeconomy by raising interest rates.[2]

An important caveat is that the empirical literature is silent about the optimality of risk taking by intermediaries. Financing riskier investments (i.e. with high variance and high expected return) when interest rates are low may well be socially optimal. Thus, assessing whether intermediaries' risk taking is excessive is key for determining whether monetary policy should actively aim to curtail such risks. We contribute to this debate by developing a quantitative model to measure if the risks undertaken by intermediaries when interest rates are low exceed the social optimum.

A main feature of our model is that financial intermediaries can alter their portfolio investments by using safe assets as collateral in interbank borrowing. Although collateralized borrowing is a primary margin of balance sheet adjustment for intermediaries (Adrian and Shin (2010)), it has not received a lot of attention in quantitative macro studies. The tight

[1] Dell'Ariccia, Laeven, and Marquez (2014) and de Nicolò, Dell'Ariccia, Laeven, and Valencia (2010) document a negative relationship between the real fed funds rate and the riskiness of U.S. banks' assets. Others use nominal interest rate data to establish a negative relationship to bank risk taking in different countries, e.g. Gambacorta (2009), Ioannidou, Ongena, and Peydró (2015), Jiménez, Ongena, Peydró, and Saurina (2014), Delis and Kouretas (2011) and Altunbas, Gambacorta, and Marques-Ibane (2014).

[2] For example, Taylor (2009) argues that monetary policy was low for too long in the run-up to the crisis. Borio and Zhu (2012) and Agur and Demertzis (2013) discuss "leaning against the wind", the idea that monetary policy should tighten as soon as financial risks build up.

empirical relationship between monetary policy rates and the cost of collateralized interbank borrowing (Bech, Klee, and Stebunovs (2012)), as well as the shortage of collateral and reductions in interbank borrowing observed in the recent crisis (Gorton (2010)) motivate us to model collateralized borrowing when examining intermediaries' risk taking incentives.

To conduct our analysis, we develop a dynamic model with persistent aggregate shocks and idiosyncratic uncertainty in which the monetary authority influences the real interest rate on safe bonds.[3] Each period, intermediaries with limited liability are funded through insured deposits and equity from households which they allocate to safe bonds and risky projects. The latter are investments in firms, whose returns are correlated with aggregate productivity.[4] The combination of limited liability and deposit insurance creates a moral hazard problem, which generates the potential for intermediaries to overinvest in risky projects.[5]

After the initial portfolio decision, intermediaries find out whether they hold high-risk projects, with high variance of returns, or low-risk projects, with low variance of returns. Given this information, intermediaries reoptimize their portfolios using collateralized borrowing in the interbank market. During an expansion, when aggregate productivity is expected to be high, intermediaries with high-risk projects—which we term high-risk intermediaries—trade their risk-free bonds to invest more into their risky projects. These projects are relatively attractive from a social point of view due to their high expected return, and are even more attractive for intermediaries because potential losses in the event of a contraction are avoided through limited liability (as in Allen and Gale (2000)). Low-risk intermediaries on the other side of the transaction accept bonds and reduce exposure to their risky projects, which have lower expected returns. In this framework, we define risk taking as excessive if

[3] We do not model the changes in nominal interest rates that are needed to deliver the real rates implemented in our model. Having the nominal interest rate as a policy instrument would enrich the policy insights by introducing additional trade-offs. For example, the monetary authority may choose to keep nominal interest rates low because the recovery of the economy from a recession is weak, or because inflation is falling (Bernanke (2010)). Analyzing these additional trade-offs is beyond the scope of this paper.

[4] In our model, the investment market is segmented in that households cannot invest directly in the risky projects of some firms and are forced to use intermediaries. This is similar to Gale (2004).

[5] We note that moral hazard leads to a failure of the Modigliani and Miller (1958) theorem, see Hellwig (1981) and Myers (2003).

investments in high-risk projects in the decentralized economy exceed the social optimum, defined as the solution to a social planner problem.

In the model, collateralized borrowing can be interpreted as repurchase agreements (repos).[6] Empirically, repos are largely collateralized using government bonds (Krishnamurthy, Nagel, and Orlov (2014)). Consistent with this evidence, intermediaries in our model use government bonds as collateral for borrowing. The implicit theoretical assumption is that government bonds are special because there is no information asymmetry about their value.[7]

Collateralized borrowing in our model is beneficial because it facilitates reallocation of resources between intermediaries in response to new information about the riskiness of their investments. However, borrowing against safe bonds also allows intermediaries to take advantage of their limited liability and to overinvest in risky projects. This is socially costly because intermediaries can go bankrupt, in which case, payments to depositors are guaranteed by government-funded deposit insurance. The monetary authority's role is to set interest rate policy so as to mitigate the moral hazard problem of intermediaries. This is achieved by making the collateral constraint of intermediaries bind.

The inclusion of collateralized borrowing acts as an opposing force on the propensity to take on risk by financial intermediaries. On the one hand, our model captures the standard portfolio choice result that a risk averse investor's optimal investment into risky assets is decreasing in the return to safe assets (Merton (1969), Samuelson (1969) and Fishburn and Porter (1976)).[8] On the other hand, at low interest rates, limited amounts of safe assets constrain collateralized interbank borrowing and ultimately result in reduced risk taking by intermediaries. We term the opposing channels through which interest rate policy influences risk taking by intermediaries as the *portfolio* and the *collateral channel*, respectively.

[6] A repo transaction is a sale of a security and a simultaneous agreement to repurchase the security at a future date. Repos are secured loans in which the borrower receives money against collateral.

[7] In the run-up to the recent financial crisis, some assets, such as asset-backed securities, used as collateral in the repo market were not truly safe (see Gorton (2010), Gorton and Metrick (2012), Krishnamurthy, Nagel, and Orlov (2014) and Hoerdahl and King (2008)). This type of collateral disappeared from the repo market as the crisis unfolded. Considering other types of collateral assets is an interesting extension of our model, that we leave for future work.

[8] This idea is also the basis of Rajan (2006), who discusses excessive risk in the financial sector.

To gain intuition about the qualitative trade-offs implied by the two channels, we examine a simplified version of our model with i.i.d. aggregate shocks. In this case, we can derive analytical results on risk taking. We show that if equity is sufficiently high, there exists an interest rate policy which implements the socially optimal investments as a competitive equilibrium. The intuition is that, with high enough equity, the moral hazard problem of intermediaries is reduced and intermediaries do not go bankrupt, as most of their liabilities are state-contingent.

If equity of financial intermediaries is relatively low (as observed in U.S. data), the equilibrium investments in risky projects no longer coincide with the social optimum. In this case, the collateral channel provides a safeguard against increased risk taking, especially at low interest rates. Namely, low policy rates lead intermediaries to purchase fewer safe bonds (*portfolio channel*), and thus have less collateral available for interbank borrowing (*collateral channel*). The collateral channel dominates, as scarce collateral constrains high-risk intermediaries who have the strongest incentives to overinvest in risky projects during expansions. Thus, low policy rates lower investments in risky projects in the competitive equilibrium and reduce risk taking during expansions.

A drawback of the assumption of i.i.d. aggregate shocks is that the model cannot match the negative relationship between interest rates and risky investments observed during extended periods of expansion. To be consistent with this empirical observation, we calibrate the model with persistent aggregate shocks to the U.S. economy and conduct numerical experiments. First, we solve for the optimal interest rate policy which maximizes households' welfare. Second, we consider upward or downward shifts in the interest rate schedule, to evaluate risk taking behavior when interest rates are either lower or higher than optimal.

Our numerical results are consistent with the empirical finding that intermediaries take on more risks when interest rates are low. Expansions in our decentralized economy are characterized by optimally declining interest rates and feature higher investments in risky projects. However, the socially optimal amount of investments in risky assets also rises in expansions. At the optimal interest rate policy, risky investments in the competitive equilibrium exceed the social planner's by about 5 percent, on average, but the associated welfare losses are small.[9] Moreover, as in the simple model, lower than optimal interest rates lead to reductions in risk taking by financial intermediaries.

Higher than optimal interest rates entail larger welfare losses in our environment compared to lower than optimal interest rates. At higher interest rates, intermediaries purchase more safe bonds (*portfolio channel*), which leads to a relaxation of their collateral constraint and allows for more borrowing in the interbank market (*collateral channel*). Our paper makes an important contribution by highlighting that relaxing collateral constraints increases risk taking with adverse effects for real activity and welfare. This insight is in contrast to Kiyotaki and Moore (1997) where shocks to credit-constrained firms are amplified and transmitted to output through changes in collateral values. In their framework, relaxing collateral constraints is beneficial.

Several papers in the literature build quantitative models to illustrate that financial frictions in interbank markets magnify downturns and lead to banking crises. Gertler and Kiyotaki (2010) and Gertler, Kiyotaki, and Queralto (2012) focus on various policies that can help mitigate a crisis; Boissay, Collard, and Smets (2016) emphasize that banking crises are due to excessive credit booms which trigger large declines in interest rates and interbank market freezes.[10] Similar to these papers, our model features borrowing constraints in the interbank market. Moreover, a shutdown in the interbank market occurs whenever interest rates are sufficiently low, just as in Boissay, Collard, and Smets (2016). Our contribution

[9] The average is taken over expansions and contractions.

[10] These papers augment quantitative macro models with financial amplification mechanisms à la Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). There is also a broad theoretical literature that examines related aspects of financial intermediation. For example, Dell'Ariccia, Laeven, and Marquez (2014) build a model to examine the link between interest rates and bank risk taking in an environment where leverage is either endogenous or exogenous. Drees, Eckwert, and Várdy (2013) argue that the impact of interest rates on risk taking depends on the source of risk. Challe, Mojon, and Ragot (2013) study risk taking when the financial system is opaque. Dubecq, Mojon, and Ragot (2015) study the interaction between capital regulation and risk. Stein (1998) examines the transmission mechanism of monetary policy in a model in which banks' portfolio choices respond to changes in the availability of financing via insured deposits. Diamond and Rajan (2009), Acharya and Naqvi (2012) and Agur and Demertzis (2013) examine the optimal policy when the monetary authority has a financial stability objective.

relative to these papers is to show that binding collateral constraints in the interbank market are desirable, as they limit excessive risk taking.

The rest of the paper is organized as follows. Section 2 presents the decentralized environment, the social planner's problem and the measurement of risk taking. Section 3 presents equilibrium properties of our full model, and results from the version of our model with i.i.d. aggregate shocks. Section 4 describes the quantitative analysis and Section 5 concludes.

2 Model Economy

The economy is populated by households, financial intermediaries, nonfinancial firms and a government. The rationale for the existence of intermediation is the same as in Gale (2004): households are excluded from directly investing in some of the risky assets available in the economy and are forced to use financial intermediaries.

Time is discrete and infinite. Each period, the economy is subject to an exogenous aggregate shock which affects the productivity of all firms. In addition, financial intermediaries are subject to idiosyncratic shocks which determine their type, $j \in \{h, l\}$. The aggregate shock $s_t \in \{\overline{s}, \underline{s}\}$ follows a first-order Markov process. The history of aggregate shocks up to time t is s^t. The idiosyncratic shock is i.i.d. across time and across financial intermediaries. A summary of the timing of events in our model is presented in Figure 1.

2.1 Financial Sector

We describe the financial sector first, as it comprises the innovative features of our model.

Financial intermediaries choose portfolios of safe and risky investments to maximize expected profits. Three features make the portfolio choices interesting. Intermediaries have *limited liability* and are partly funded through *insured deposits*.[11] In combination, these two

[11] Our analysis is focused on the risk taking incentives of deposit taking institutions. While risk taking incentives of other types of intermediaries have been analyzed in the literature (e.g. Chevalier and Ellison (1997) and Palomino and Prat (2003)), they are beyond the scope of this paper.

features create a moral hazard problem which makes risky investments attractive for intermediaries. Moreover, within each period, intermediaries can borrow or lend against collateral through an interbank market in order to change the scale of their risky investments after finding out their type. *Collateralized interbank borrowing* is the novel feature of our model.

There is a measure $1 - \pi_m$ of financial intermediaries who make two portfolio decisions each period. At the first stage, the type $j \in \{h, l\}$ and the aggregate shock $s_t \in \{\bar{s}, \underline{s}\}$ are unknown. Financial intermediaries are identical, so they receive the same amounts of deposits and equity from households and make the same portfolio investments into government bonds, $b(s^{t-1})$, and risky projects, $k(s^{t-1})$. The latter are investments into the production technologies of small firms and can be one of two types: high-risk projects with productivity $q_h(s_t)$ and low-risk projects with productivity $q_l(s_t)$. For simplicity, we do not model loans between financial intermediaries and the small firms, but rather assume that intermediaries operate their production technology directly.[12]

After the initial investment decisions, intermediaries acquire more information about the riskiness of their projects. With probability π_j, the project an intermediary previously invested into is of type $j \in \{h, l\}$. The probabilities, π_h and $\pi_l = 1 - \pi_h$, are time and state invariant and known. We refer to intermediaries at this second portfolio stage as being high-risk or low-risk, based on the type j of their risky projects. We assume that high-risk financial intermediaries are more productive during a good aggregate state ($s_t = \bar{s}$), and less productive during a bad state ($s_t = \underline{s}$), compared to low-risk financial intermediaries. Formally, $q_h(\bar{s}) > q_l(\bar{s}) \geq q_l(\underline{s}) > q_h(\underline{s})$. We also assume that it is not possible for intermediaries to trade contingent claims on their projects. However, once type j is known, but before the realization of the aggregate shock s_t, intermediaries may trade bonds in the interbank market in order to adjust the amount of resources invested into the risky projects. The resulting capital, $k_j(s^{t-1})$, is invested into the production technologies of the small firms. Here, $k_j(s^{t-1}) \equiv k(s^{t-1}) + \tilde{p}(s^{t-1})\tilde{b}_j(s^{t-1})$ where $k(s^{t-1})$ is the first stage portfolio

[12] Implicitly, we abstract from information problems à la Bernanke and Gertler (1989).

investment and $\tilde{b}_j(s^{t-1})$ are bonds traded at the interbank market price $\tilde{p}(s^{t-1})$.

The assumption regarding the timing of shocks is crucial for the existence of an interbank market in this model. In particular, if j and s_t were known at the beginning of each period, then resources from households would be allocated to intermediaries so as to equalize marginal rates of return, and there would be no need for an interbank market. The timing assumption which gives rise to the two stages of an intermediary's portfolio choice is meant to capture the idea that information about the riskiness of projects evolves over time. As a result, financial intermediaries adjust their portfolios, but may be constrained in their choices by the amount of bonds, $b(s^{t-1})$, available as collateral for interbank borrowing.

After the two portfolio decisions, the aggregate shock, s_t, realizes at the beginning of period t. Intermediaries choose labor demand, $l_j(s^t)$, and produce using technology $q_j(s_t)\left[k_j(s^{t-1})\right]^{\theta}\left[l_j(s^t)\right]^{1-\theta-\alpha}$, where parameters θ and α satisfy $1-\alpha-\theta \geq 0$ with $\alpha, \theta \in [0,1]$. If $\alpha > 0$ there is a fixed factor present in the production process, whose returns are paid to equityholders. As outlined in Section 4.1, the fixed factor α helps our model match the equity to total asset ratio of the U.S. financial sector. Following production, intermediaries unable to pay the promised rate of return to deposits declare bankruptcy.

We now describe in detail the stages of an intermediary's problem.

Portfolio Choice in the Bond Market

Financial intermediaries maximize expected profits. Since households own the financial intermediaries, profits at history s^t are valued at the households' marginal utility of consumption (weighted by the probability of history s^t), denoted $\lambda(s^t)$.

At the first stage of the portfolio decision, the type $j \in \{h, l\}$ and the aggregate shock $s_t \in \{\overline{s}, \underline{s}\}$ are unknown. A financial intermediary chooses deposit demand, $d(s^{t-1})$, safe bonds, $b(s^{t-1})$, and risky investments, $k(s^{t-1})$ to solve the problem $(P1)$. An intermediary takes as given the bonds traded in the interbank market, $\tilde{b}_j(s^{t-1})$, and the labor input, $l_j(s^t)$. Note that $\tilde{b}_j(s^{t-1})$ is chosen after the type, j, is realized, while $l_j(s^t)$ is chosen after the type, j, and the aggregate shock, s_t, are realized. In addition, an intermediary takes as given $\lambda(s^t)$, all prices and the amount of equity chosen by households, $z(s^{t-1})$.[13]

$$\max_{\{d(s^{t-1}),\, b(s^{t-1}),\, k(s^{t-1})\}} \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right) \tag{P1}$$

subject to:

$$z\left(s^{t-1}\right) + d\left(s^{t-1}\right) = k\left(s^{t-1}\right) + p\left(s^{t-1}\right) b\left(s^{t-1}\right) \tag{1}$$

$$V_j\left(s^t\right) = \max\left\{\begin{bmatrix} q_j(s_t)\left[k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right]^{\theta}\left[l_j\left(s^t\right)\right]^{1-\theta-\alpha} \\ +q_j(s_t)(1-\delta)\left[k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right] \\ +\left[b\left(s^{t-1}\right) - \tilde{b}_j\left(s^{t-1}\right)\right] - W_j\left(s^t\right) l_j\left(s^t\right) - R^d\left(s^{t-1}\right) d\left(s^{t-1}\right) \end{bmatrix}, 0\right\} \tag{2}$$

Here, $V_j(s^t)$ are profits for intermediary $j \in \{h,l\}$ at history s^t which are paid to equity holders, $p(s^{t-1})$ is the bond price, $\tilde{p}(s^{t-1})$ is the interbank market price, $W_j(s^t)$ is the wage rate paid by a financial intermediary of type j and $R^d(s^{t-1})$ is the return to deposits.

The balance sheet of an intermediary (equation (1)) shows that investments are funded through equity, $z(s^{t-1})$, and deposits, $d(s^{t-1})$. The main difference between these two forms of funding is that equity returns are contingent on the realization of the aggregate state in the period when they are paid, while returns to deposits are not (i.e. $V_j(s^t)$ depends on s_t, while $R^d(s^{t-1})$ does not). In addition, equity returns are bounded below by zero due to the limited liability of intermediaries (i.e. $V_j(s^t)$ cannot be negative as seen in equation (2)), while deposit returns are guaranteed by deposit insurance. The limited liability introduces an asymmetry in that it allows intermediaries to make investment decisions that bring profits in good aggregate states, while being shielded from losses in bad states.

In equation (2), the undepreciated capital stock of firms is adjusted by the productivity level, i.e. $q_j(s_t)$ multiplies $(1-\delta)\,k_j(s^{t-1})$ where δ is the depreciation rate, and $k_j(s^{t-1}) \equiv$

[13] Due to limited liability and deposit insurance, financial intermediaries prefer to be funded via deposits rather than equity. To avoid zero equity financing (which is not supported by U.S. data), we assume that equity is determined by households. Some alternative modelling choices—which we do not pursue in this paper—are to assume an agency problem (e.g. Holmstrom and Tirole (1997)) or to impose a financial capital regulation constraint (e.g. Van den Heuvel (2009)), both of which result in intermediaries holding equity.

$k\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)$. This allows for variation in the value of capital, similar to Merton (1973) and Gertler and Kiyotaki (2010). The idea is that while capital may not depreciate in a physical sense during contraction periods, it does so in an economic sense.[14]

Portfolio Adjustments via the Interbank Market

Once financial intermediaries find out their type $j \in \{h, l\}$, they may adjust the riskiness of their portfolios by trading bonds, $\tilde{b}_j\left(s^{t-1}\right)$, amongst themselves. Intermediaries choose $\tilde{b}_j\left(s^{t-1}\right)$ and, implicitly, $k_j\left(s^{t-1}\right) \equiv k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)$ to solve the problem $(P2)$. Intermediaries take as given the choices made at the first stage portfolio decision, $d\left(s^{t-1}\right)$, $b\left(s^{t-1}\right)$, $k\left(s^{t-1}\right)$. As before, intermediaries also take as given $l_j\left(s^t\right)$, $\lambda\left(s^t\right)$, all prices and equity, $z\left(s^{t-1}\right)$.

$$\max_{\left\{\tilde{b}_j\left(s^{t-1}\right),\ k_j\left(s^{t-1}\right)\right\}} \sum_{s^t|s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right) \tag{P2}$$
$$\text{subject to: } -\frac{k\left(s^{t-1}\right)}{\tilde{p}\left(s^{t-1}\right)} \leq \tilde{b}_j\left(s^{t-1}\right) \leq b\left(s^{t-1}\right)$$

where $V_j\left(s^t\right)$ is defined in equation (2). Inada conditions guarantee that $k_j\left(s^{t-1}\right) \equiv k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right) > 0$, and hence the only potentially binding constraint in problem $(P2)$ is $\tilde{b}_j\left(s^{t-1}\right) \leq b\left(s^{t-1}\right)$. Here, $\tilde{b}_j\left(s^{t-1}\right)$ can be interpreted as sales of bonds or, alternatively, as repurchasing agreements (repos).[15] We abstract from haircuts on collateral.[16]

[14] In a case study of aerospace plants, Ramey and Shapiro (2001) show that the decrease in the value of installed capital at plants that discontinued operations is higher than the actual depreciation rate. In addition, Eisfeldt and Rampini (2006) provide evidence that costs of capital reallocation are strongly countercyclical.

[15] While we model $\tilde{b}_j\left(s^{t-1}\right)$ as bond sales, incorporating explicitly the repurchase of bonds—which is typical in a repo agreement—would yield identical results. Specifically, if no bankruptcy occurs, then intermediaries have the resources necessary to repurchase the bonds from the counterparty. This simply amounts to a reshuffling of profits among intermediaries, before these profits are paid as returns to equityholders. When some intermediaries go bankrupt, they are unable to repurchase the bonds and the counterparty keeps them, as is true in the data. Equityholders receive no returns from bankrupt intermediaries. In either case, payments to equityholders are identical regardless of whether we model the repurchase of bonds or not.

[16] A repo transaction may require the borrower to pledge collateral in excess of the loan received. For example, Krishnamurthy, Nagel, and Orlov (2014) document that average haircuts vary between 2 and 7 percent by type of collateral. Currently, our model abstracts from haircuts in the repo market. Introducing a fixed haircut in the model would not change our results, since the equilibrium repo price, $\tilde{p}\left(s^{t-1}\right)$, adjusts with the size of the haircut so that resources obtained through the repo market remain unchanged.

The assumption that interbank (repo) borrowing is collateralized, $\tilde{b}_j\left(s^{t-1}\right) \leq b\left(s^{t-1}\right)$, is motivated by a debt enforcement problem à la Kiyotaki and Moore (1997). Namely, lenders in the interbank market cannot force borrowers to repay debts, unless these debts are secured by collateral.

Our model is consistent with evidence that repos are an important margin of balance sheet adjustment by intermediaries (Adrian and Shin (2010)) and that repo lending allows participants to "hedge against market risk exposures arising from other activities" (Financial Stability Board (2012)). In our model, the redistribution of resources using the repo market is socially beneficial as it allows financial intermediaries to change their risk exposure in response to new information on the productivity of their investments. Resources are reallocated towards intermediaries who are expected to be more productive, and who lower their holdings of bonds to invest additional resources in their risky projects. Resources flow towards the high-risk intermediaries in an expansion and towards the low-risk intermediaries in a contraction.

While repo borrowing is beneficial, it also enables intermediaries to take advantage of their limited liability and overinvest in risky projects. Intermediaries' ability to increase risky investments is limited by their bond holdings. Higher purchases of bonds make balance sheets seem safer initially, but may lead to increased risk taking through the repo market.

Although intermediaries start out as identical each period, the funds they receive from households vary with the aggregate state, allowing the model to capture interesting dynamics over time such as sustained high levels of investment into high-risk projects.

Labor Demand and Production

Once the aggregate shock, $s_t \in \{\overline{s}, \underline{s}\}$, is realized, financial intermediaries choose labor demand, $l_j\left(s^t\right)$, to equate the wage rate, $W_j\left(s^t\right)$, with the marginal product of labor, $(1-\theta-\alpha)\, q_j\left(s_t\right)\left[k_j\left(s^{t-1}\right)\right]^{\theta}\left[l_j\left(s^t\right)\right]^{-\theta-\alpha}$. Production takes places using capital, $k_j\left(s^{t-1}\right)$, chosen at the second stage portfolio decision and labor, $l_j\left(s^t\right)$. Finally, returns to assets are paid and bankruptcy may occur.

We note that labor is an essential input into production. If we abstract from labor, then expected returns to financial sector equity in our model are larger than expected returns to deposits, pushing households to choose zero deposits, which is counterfactual. We assume the labor input is chosen after the intermediaries know j and s_t, for computational simplicity.

2.2 Nonfinancial sector

There is a measure π_m of identical nonfinancial firms funded entirely through household equity. Each nonfinancial firm enters period t with equity $M\left(s^{t-1}\right)/\pi_m$ from households which is invested into capital. Hence, $k_m\left(s^{t-1}\right) = M\left(s^{t-1}\right)/\pi_m$. Equity returns depend on the productivity of the production technology in the nonfinancial sector, $q_m\left(s_t\right)$ which satisfies: $q_h\left(\overline{s}\right) \geq q_m\left(\overline{s}\right) > q_l\left(\overline{s}\right) \geq q_l\left(\underline{s}\right) > q_m\left(\underline{s}\right) > q_h\left(\underline{s}\right)$.

The problem of a nonfinancial firm is to choose capital and labour to solve:

$$\begin{aligned}
&\max\left\{y_m\left(s^t\right) + q_m\left(s_t\right)\left(1-\delta\right)k_m\left(s^{t-1}\right) - R^m\left(s^t\right)k_m\left(s^{t-1}\right) - W_m\left(s^t\right)l_m\left(s^t\right)\right\} \\
&\text{subject to: } y_m\left(s^t\right) = q_m\left(s_t\right)\left[k_m\left(s^{t-1}\right)\right]^{\theta}\left[l_m\left(s^t\right)\right]^{1-\theta}
\end{aligned}$$

where $R^m\left(s^t\right)$ is the return to capital (equity) invested in the nonfinancial sector, $l_m\left(s^t\right)$ is the labor employed in the nonfinancial sector and $W_m\left(s^t\right)$ is the wage rate.

The nonfinancial sector is introduced to allow our model to be consistent with U.S. data showing a high equity to deposit ratio for households, a low equity to deposit ratio in the financial sector and to match the relative importance of the two sectors in U.S. production.

2.3 Households

There is a measure one of identical households, who maximize expected utility subject to a budget constraint which equates current wealth, $w\left(s^t\right)$, to expenditures on consumption,

$C\left(s^{t}\right)$, and investments that will pay returns next period.

$$
\begin{aligned}
& \max \sum_{t=0}^{\infty} \sum_{s^{t}} \beta^{t} \varphi\left(s^{t}\right) \log C\left(s^{t}\right) \\
\text{subject to: } w\left(s^{t}\right) &= R^{d}\left(s^{t-1}\right) D_{h}\left(s^{t-1}\right)+R^{z}\left(s^{t}\right) Z\left(s^{t-1}\right)+R^{m}\left(s^{t}\right) M\left(s^{t-1}\right) \\
& +\pi_{m} W_{m}\left(s^{t}\right)+\left(1-\pi_{m}\right) \pi_{l} W_{l}\left(s^{t}\right)+\left(1-\pi_{m}\right) \pi_{h} W_{h}\left(s^{t}\right)+T\left(s^{t}\right) \\
w\left(s^{t}\right) &= C\left(s^{t}\right)+M\left(s^{t}\right)+D_{h}\left(s^{t}\right)+Z\left(s^{t}\right)
\end{aligned}
$$

Here, β is the discount factor and $\varphi\left(s^{t}\right)$ is the probability of history s^{t}.

At the beginning of period t, the aggregate state s_{t} is revealed and household wealth—comprised of returns on previous period investments, wage income and lump-sum taxes $\left(T\left(s^{t}\right)<0\right)$ or transfers $\left(T\left(s^{t}\right) \geq 0\right)$ from the government—is realized.

Investments take the form of deposits, financial sector equity, and nonfinancial sector equity. Deposits, $D_{h}\left(s^{t-1}\right)$, earn a fixed return, $R^{d}\left(s^{t-1}\right)$, which is guaranteed by deposit insurance. Equity invested in the financial sector, $Z\left(s^{t-1}\right)$, is a risky investment which gives households a state-contingent claim to the profits of the intermediaries. The return per unit of equity is $R^{z}\left(s^{t}\right)=\frac{1}{z\left(s^{t-1}\right)} \sum_{j \in\{h, l\}} \pi_{j} V_{j}\left(s^{t}\right)$. Similarly, the equity invested in the nonfinancial sector, $M\left(s^{t-1}\right)$, receives a state-contingent return, $R^{m}\left(s^{t}\right)$. An interior solution in which households invest in all three assets requires that expected returns to deposits and equity are equalized. Formally, $\sum_{s^{t+1} \mid s^{t}} \frac{\beta^{t+1} \varphi\left(s^{t+1}\right)}{C\left(s^{t+1}\right)}\left[R^{z}\left(s^{t+1}\right)-R^{d}\left(s^{t}\right)\right]= \sum_{s^{t+1} \mid s^{t}} \frac{\beta^{t+1} \varphi\left(s^{t+1}\right)}{C\left(s^{t+1}\right)}\left[R^{z}\left(s^{t+1}\right)-R^{m}\left(s^{t+1}\right)\right]=0$.

Each household supplies one unit of labour inelastically. We assume that labour markets are segmented. Fraction π_{m} of a household's time is spent working in the nonfinancial sector, and fraction $1-\pi_{m}$ is spent in the financial sector. Within the financial sector, a household's time is split between high-risk and low-risk intermediaries according to shares π_{j}, where $\pi_{h}+\pi_{l}=1$. Given that there are measure one of households and measure one of firms, labour supplied to each firm is one unit, for any realization of the aggregate state.

2.4 Government

The government issues bonds that financial intermediaries hold as an investment or use as a medium of exchange on the repo market.[17] At the end of period $t-1$, the government sells bonds, $B\left(s^{t-1}\right)$, at price, $p\left(s^{t-1}\right)$ and deposits the proceeds with financial intermediaries.[18] Each financial intermediary purchases risk-free assets $b\left(s^{t-1}\right) = B\left(s^{t-1}\right) / \left(1-\pi_m\right)$ and receives $D_g\left(s^{t-1}\right) / \left(1-\pi_m\right)$ of government deposits, where $D_g\left(s^{t-1}\right) = p\left(s^{t-1}\right) B\left(s^{t-1}\right)$.

To guarantee the fixed return on deposits the government provides deposit insurance at zero price which is financed through household taxation.[19] The government balances its budget after the production takes place at the beginning of period t.[20]

$$T\left(s^t\right) + B\left(s^{t-1}\right) + \Delta\left(s^t\right) = R^d\left(s^{t-1}\right) D_g\left(s^{t-1}\right)$$

Here, $\Delta\left(s^t\right)$ is the amount of deposit insurance necessary to guarantee the fixed return on deposits, $R^d\left(s^{t-1}\right)$. Given the limited liability of intermediaries, if they are unable to pay $R^d\left(s^{t-1}\right)$ on deposits, they pay a smaller return on deposits which ensures they break-even. The rest is covered by deposit insurance.

2.5 Market clearing

The *labour market clearing conditions* state that labour demanded by financial intermediaries and nonfinancial firms equals labour supplied by households: $\pi_m l_m\left(s^t\right) = \pi_m$ and $\left(1-\pi_m\right)\pi_j l_j\left(s^t\right) = \left(1-\pi_m\right)\pi_j$ for each $j \in \{h, l\}$. This implies $l_m\left(s^t\right) = l_h\left(s^t\right) = l_l\left(s^t\right) = 1$.

[17] This model focuses on the role that government bonds provide as collateral in the repo market. We abstract from the tax smoothing role of government debt.

[18] Alternatively, the proceeds from the bond sales could be transferred to households.

[19] Pennacchi (2006, pg. 14) documents that, since 1996 and prior to the crisis, deposit insurance has been essentially free for U.S. banks. In our model, the assumption of a zero price of deposit insurance is not crucial. What matters is that the insurance is not priced in a way to eliminate moral hazard. This means, for example, that deposit insurance can not be contingent on the portfolio decisions of the intermediaries.

[20] We concentrate on new issuance of (one period) bonds and abstract from outstanding bonds for computational reasons. Considering the valuation effects of interest rate policy in the presence of outstanding bonds may be an interesting extension of the model.

The *goods market clearing condition* equates total output produced with aggregate consumption and investment. Output produced by nonfinancial firms is $\pi_m q_m(s^t)(k_m(s^{t-1}))^\theta$, while output produced by financial firms is $(1-\pi_m)\sum_{j\in\{l,h\}}\pi_j q_j(s^t)[k_j(s^{t-1})]^\theta$, where $k_j(s^{t-1})$ are resources allocated to the risky projects after repo market trading.

$$\begin{aligned} C\left(s^t\right)+M\left(s^t\right)+D_h\left(s^t\right)+Z\left(s^t\right) &= (1-\pi_m)\sum_{j\in\{l,h\}}\pi_j q_j(s_t)\left\{\left[k_j\left(s^{t-1}\right)\right]^\theta+(1-\delta)k_j\left(s^{t-1}\right)\right\} \\ &\quad +\pi_m q_m(s_t)\left[\left[k_m\left(s^{t-1}\right)\right]^\theta+(1-\delta)k_m\left(s^{t-1}\right)\right] \end{aligned}$$

There are four *financial market clearing conditions.* Deposits demanded by intermediaries equal deposits from the households and the government: $D_h(s^{t-1})+D_g(s^{t-1})=D(s^{t-1})=(1-\pi_m)d(s^{t-1})$. In the bond market, total bond sales by the government equal the bond purchases by financial intermediaries: $B(s^{t-1})=(1-\pi_m)b(s^{t-1})$. In the interbank repo market, trades between the different types of intermediaries must balance: $\sum_{j\in\{l,h\}}\pi_j\tilde{b}_j(s^{t-1})=0$. Lastly, total equity invested by households in the financial and nonfinancial sectors are distributed over the firms: $M(s^{t-1})=\pi_m k_m(s^{t-1})$ and $Z(s^{t-1})=(1-\pi_m)z(s^{t-1})$.

2.6 Government Optimal Policy

The main policy instrument is the price of government bonds. The government chooses the bond price, $p^*(s^{t-1})$, or alternatively the bond return, $1/p^*(s^{t-1})$, that maximizes the welfare of the households in the decentralized economy given in problem $(P3)$. The government satisfies any demand for bonds given this price.

$$p^*\left(s^{t-1}\right)=\arg\max_{p(s^{t-1})}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\varphi\left(s^t\right)\log C\left(s^t\right) \tag{P3}$$

subject to: $C\left(s^t\right)$ is part of a competitive equilibrium given policy $p\left(s^{t-1}\right)$

2.7 Social Planner Problem

We consider a social planner's problem as a reference point for our decentralized economy. To make the social planner's environment comparable to the decentralized one, we maintain the timing assumption. In a slight abuse of language, we refer to the technologies available to the social planner as belonging to financial and nonfinancial sectors.

At the beginning of period t, the aggregate state, s_t, is revealed and production takes place using capital that the social planner has allocated to the different technologies of production: $k_m(s^{t-1})$ for the nonfinancial sector, $k_h(s^{t-1})$ and $k_l(s^{t-1})$ for the high-risk and low-risk technologies of the financial sector. Output is then split between consumption and capital to be used in production at $t+1$. At the time of this decision, the social planner does not distinguish between the high-risk and low-risk technologies of the financial sector used in production next period, and simply allocates resources, $k_b(s^t)$, to both of them. Once their type is revealed, the social planner reallocates resources between the two technologies.

The social planner solves:

$$\max E\sum_{t=0}^{\infty}\beta^t \log C\left(s^t\right)$$

subject to:

$$\begin{aligned}
C\left(s^t\right)+\pi_m k_m\left(s^t\right)+(1-\pi_m)\,k_b\left(s^t\right) & \\
&= \pi_m q_m(s_t)\left[\left(k_m\left(s^{t-1}\right)\right)^{\theta}+(1-\delta)\,k_m\left(s^{t-1}\right)\right] \\
&\quad +(1-\pi_m)\,\pi_l q_l(s_t)\left[\left(k_l\left(s^{t-1}\right)\right)^{\theta}+(1-\delta)\left(k_l\left(s^{t-1}\right)\right)\right] \\
&\quad +(1-\pi_m)\,\pi_h q_h(s_t)\left[\left(k_h\left(s^{t-1}\right)\right)^{\theta}+(1-\delta)\,k_h\left(s^{t-1}\right)\right] \\
k_l\left(s^t\right) &= k_b\left(s^t\right)-\frac{\pi_h}{\pi_l}n\left(s^t\right) \\
k_h\left(s^t\right) &= k_b\left(s^t\right)+n\left(s^t\right)
\end{aligned}$$

where $n(s^t)$ is the amount of resources given to (or taken from) each high-risk production technology. To achieve this reallocation, $\frac{\pi_h}{\pi_l}n(s^t)$ resources need to be taken away from (or given to) each low-risk technology.

From a social planner's perspective, it is optimal for resources to flow to high-risk intermediaries during expansion periods and to low-risk intermediaries during contractions. To induce these reallocation flows in the decentralized economy, bond prices, $p(s^t)$, need to be appropriately chosen by the monetary authority.

2.8 Measurement of Risk Taking

We use our model to assess whether and how interest rate policy influences risk taking of intermediaries. To this end, we make our notion of risk taking precise. We define risk taking as the percentage deviation in resources invested in the high-risk projects in a competitive equilibrium relative to the social planner. Formally,

$$r\left(s^{t-1}\right) = \frac{k_h^{CE}\left(s^{t-1}\right) - k_h^{SP}\left(s^{t-1}\right)}{k_h^{SP}\left(s^{t-1}\right)} \cdot 100 \tag{3}$$

where $k_h^{CE}(s^{t-1})$ is the capital invested in high-risk projects in the competitive equilibrium for a given interest rate policy and $k_h^{SP}(s^t)$ is the capital the social planner invests in the high-risk technology.

If the social planner's allocation can be implemented with a competitive equilibrium, the value of $r(s^{t-1})$ in equation (3) is zero. Otherwise, a positive value of $r(s^{t-1})$ tells us that there is excessive risk taking in the competitive equilibrium, while a negative value indicates too little risk taking. We define an aggregate measure of risk taking, averaged over expansions and contractions, as $r \equiv E[r(s^{t-1})]$.

Alternatively, the risk taking measure in equation (3) can be defined as the percentage deviation in *the share* of resources invested in high-risk projects in a competitive equilibrium relative to the social planner. This entails replacing $k_h^i(s^{t-1})$ for $i \in \{CE, SP\}$ in equation (3) with $\frac{k_h^i\left(s^{t-1}\right)}{k^i(s^{t-1})}$, where $k^i(s^{t-1})$ represents the total capital in environment $i \in \{CE, SP\}$. The advantage of this alternative measure is that it takes into account any potential differences between the total capital stocks in the two environments. We find that the two measures yield similar quantitative predictions (see footnotes 29 and 31 in Section 4.2). For this reason, the results we report use the risk taking measure as defined in equation (3).

3 Competitive Equilibrium Properties

First, we present results which relate equilibrium bond prices and the return to deposits. In Section 3.2, we discuss additional results based on a simplified version of our model.

3.1 Bond Prices and the Return to Deposits

We introduce some useful language to help describe equilibrium properties of our model. We refer to the repo market as being unconstrained, if for a given history of shocks, s^t, and policy, $p\left(s^t\right)$, all financial intermediaries choose to pledge only a fraction of bonds as collateral in the repo market, i.e. $\tilde{b}_j\left(s^t\right) < b\left(s^t\right)$, while keeping the remainder on their balance sheet. A constrained repo market is one in which either high-risk or low-risk intermediaries have a binding collateral constraint, i.e. $\tilde{b}_j\left(s^t\right) = b\left(s^t\right)$ for some j and the Lagrange multipliers on these constraints are strictly positive. In this case, either high-risk or low-risk intermediaries have zero bonds on their balance sheet after the repo trades take place.

Proposition 1 relates equilibrium bond prices and the return to deposits, derived in the full model introduced in Section 2.

Proposition 1 *Equilibrium bond prices and the return to deposits satisfy:* $p\left(s^{t-1}\right) = \tilde{p}\left(s^{t-1}\right)$ *and* $R^d\left(s^{t-1}\right) \geq \frac{1}{p\left(s^{t-1}\right)}$. *The last inequality is strict in the case of a constrained repo market.*

Proof. These results follow from the first order conditions of the financial intermediaries' problems. Appendix A.1 outlines the proof. ■

Proposition 1 formalizes the intuitive result that bond prices and repo prices are equal, since there are no regulatory constraints in the model.[21] In addition, returns to deposits are weakly greater than returns to bonds, since otherwise there would be a profit opportunity. Namely, an intermediary would have incentives to pay a slightly higher deposit return to attract additional deposits and be able to invest more into bonds. The result $R^d\left(s^{t-1}\right) \geq \frac{1}{p(s^{t-1})}$ can also be interpreted in terms of the option value provided by bonds to intermediaries (beyond their asset return) because they can be retraded on the repo market. Whenever some intermediaries are constrained by the amount of collateral they hold, bonds trade at a discount: $R^d\left(s^{t-1}\right) > \frac{1}{p(s^{t-1})}$. However, if both high-risk and low-risk intermediaries have sufficient bonds, then $R^d\left(s^{t-1}\right) = \frac{1}{p(s^{t-1})}$, since the option value of bonds is zero.[22]

Proposition 1 is important for two reasons. First, it shows that interest rate policy has a direct effect on the repo market. The close relationship between the policy rate, $1/p\left(s^{t-1}\right)$, and the repo rate, $1/\tilde{p}\left(s^{t-1}\right)$, is supported by U.S. evidence, as shown in Bech, Klee, and Stebunovs (2012). Second, the return to depositors is bounded below by the interest rate on government bonds. Thus, the interest rate policy not only affects the choices financial intermediaries make, but also affects the investment choices of households.

3.2 Analytical Results from a Simplified Version of the Model

In this section, we consider a special case of our full model from Section 2 to gain intuition about the qualitative trade-offs implied by the portfolio and collateral channels in regard to the equilibrium behavior of risk taking. We make simplifying assumptions which allow us to derive analytical results, at the cost of losing some of the rich dynamics of the full model.

Assumptions $A1:$ (i) The aggregate productivity shock, s_t, is i.i.d. The probability of the

[21] Introducing a capital regulation constraint, for example, would generate a wedge between the equilibrium bond price and the repo price.

[22] The result $R^d\left(s^{t-1}\right) \geq \frac{1}{p(s^{t-1})}$ also has a liquidity interpretation à la Krishnamurthy and Vissing-Jorgensen (2012). In our model, bonds can be viewed as being more liquid compared to deposits, since bonds can be converted into risky assets in the interbank market, whereas deposits are only available at the first stage of the portfolio choice. If bonds are scarce (i.e. collateral constraint binds), intermediaries assign a high value to the liquidity attributes of bonds. As a result, the return to bonds is strictly lower than the return to deposits. If the supply of bonds is plentiful, the liquidity value of bonds is zero.

good aggregate state, $\overline{s}$, is ϕ and the probability of the bad aggregate state, $\underline{s}$, is $1-\phi$. (ii) Households are risk neutral. (iii) There is full depreciation, $\delta = 1$, and (iv) there is no nonfinancial sector, $\pi_m = 0$.

It is easy to show that, under assumptions $A1$, the optimal investments into the high-risk and low-risk technologies do not vary with the aggregate state. The social planner allocates $k_j^{SP} = \{\beta\theta \cdot [\phi q_j(\overline{s}) + (1-\phi) q_j(\underline{s})]\}^{\frac{1}{1-\theta}}$, for $j \in \{h, l\}$, for any time period. This result follows immediately from the equalization of the expected marginal products of capital across the different technologies of production.

We summarize the competitive equilibrium predictions for risk taking in our simplified model in two propositions. Proposition 2 derives conditions under which the social optimum can be implemented as a competitive equilibrium and provides intuition for why our full calibrated model is not efficient (as discussed further in Section 4.2). Proposition 3 characterizes the risk taking behavior of intermediaries when the competitive equilibrium is not efficient.

Proposition 2 *Under assumptions $A1$, the interest rate policy $1/p = 1/\beta$ implements the social planner's allocation as a competitive equilibrium. The competitive equilibrium features either (i) a repo market in which either high-risk or low-risk intermediaries pledge all their bond holdings as collateral, no bankruptcy and zero household deposits into financial intermediaries (only equity investments) or (ii) an unconstrained repo market, no bankruptcy and equity from households which satisfies:* $z \geq k\left(1 - \frac{\theta+\alpha}{\theta} \cdot \frac{1}{\phi\frac{q_h(\overline{s})}{q_h(\underline{s})} + 1 - \phi}\right)$.

Proof. Available in Appendix A.1. ■

Proposition 2 shows that whenever equity is sufficiently high to guarantee that no bankruptcy occurs in equilibrium, the competitive equilibrium allocation is efficient. The intuition behind this result is that, with enough equity, the moral hazard problem of financial intermediaries is reduced and intermediaries do not go bankrupt, as most of their liabilities are state-contingent. We note that when we calibrate our full model from Section 2 to the U.S.

economy, the equilibrium household equity under the optimal interest rate policy is not high enough to implement the social optimum (see Section 4.2).

Proposition 3 establishes results on the intermediaries' risk taking behavior when the competitive equilibrium is not efficient and features bankruptcy. We focus on the case when the collateral constraint of intermediaries binds, since this is the relevant case for the numerical simulations of our full model (see Section 4.2).

Proposition 3 *Under assumptions A1, in an equilibrium with a constrained repo market,* (i) *during a good aggregate state* $(s_t = \overline{s})$*, lower policy interest rates lead to a reduction in risk taking, as defined in equation* (3) , *while* (ii) *during a bad aggregate state* $(s_t = \underline{s})$*, lower policy interest rates lead to an increase in risk taking, as defined in equation* (3).

Proof. Available in Appendix A.1. ■

The results of Proposition 3 can be interpreted in terms of the portfolio and the collateral risk taking channels of monetary policy. Purchases of bonds are positively related to bond returns, which means that, at low interest rates, all intermediaries invest more capital into risky projects during the first stage of the portfolio decision (*portfolio channel*). However, the amount of risk taking assumed by financial intermediaries also depends on the volume of interbank market transactions (*collateral channel*). The effect of lower bond returns on repo market activity differs depending on the aggregate state of the economy.

When the repo market is constrained, the portfolio reallocation between intermediaries is restricted due to scarce collateral (i.e. fewer bonds purchased in the bond market at low interest rates). During an expansion, high-risk intermediaries would like to invest more in high-risk projects, but they are constrained from borrowing more. Lower policy rates lead to lower investments in risky capital in the competitive equilibrium, i.e. $\frac{\partial k_h^{CE}}{\partial(1/p)} > 0$, and a reduction in risk taking as defined in equation (3), since k_h^{SP} is fixed (i.e. $k_h^{SP} = \{\beta\theta \cdot [\phi q_h(\overline{s}) + (1-\phi) q_h(\underline{s})]\}^{\frac{1}{1-\theta}}$). By the same token, during a contraction, fewer resources are reallocated from the high-risk to the low-risk intermediaries, and there is an increase in risk taking.

One main insight from the analytical results is that if equity of financial intermediaries is sufficiently high, the competitive equilibrium is efficient. This result no longer holds when equity is relatively low, consistent with U.S. data. In this case, the equilibrium features either excessive or insufficient risk taking, depending on the level of interest rates.

When we calibrate our full model from Section 2 to the U.S. economy, we find results similar to those of Proposition 3. However, the mechanism through which lower interest rates reduce (increase) risk taking during periods of good (bad) aggregate shocks differs slightly. When the aggregate shock is persistent, conditional on boom periods, there is a negative correlation between the interest rate ($1/p$) and the investments in high-risk capital in the competitive equilibrium (k_h^{CE}). This result is consistent with the empirical literature showing that when interest rates are low intermediaries take on more risk. As discussed, the simple analytical model is not able to capture this result, illustrating why it is necessary to analyze the full model. In our full model, as the interest rate declines during persistent periods of good times, both k_h^{CE} and k_h^{SP} rise. Numerical results of our full model are necessary to shed light on whether the risk taking behavior in this environment is excessive.

4 Quantitative Analysis

4.1 Calibration

We calibrate the model to the U.S. economy and solve it numerically to evaluate its quantitative predictions for risk taking. We calibrate the following parameters: β, θ, the aggregate shock transition matrix, Φ, and π_h. The remaining parameters, π_m, α, δ, $q_h(\overline{s})$, $q_h(\underline{s})$, $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$, are determined jointly using a minimum distance estimator. Tables 1 and 2 summarize the parameter values.

We identify our model's total output with the U.S. business sector value added and our model's nonfinancial sector with the U.S. corporate nonfinancial sector.[23] Unless otherwise

[23] We treat the remainder of the U.S. business sector (the corporate financial and the noncorporate busi-

noted, we use U.S. quarterly data from the BEA and the Flow of Funds for the period $1987Q1$ to $2015Q2$ when inflation was low and stable.

We choose the discount factor, β, to ensure an annual real interest rate of 4 percent in our quarterly model. We set θ to match an average capital income share for the U.S. business sector of 0.29 from 1948 to 2014. To calibrate the transition matrix for the aggregate state, we use the Harding and Pagan (2002) approach of identifying peaks and troughs in real value added of the U.S. business sector from $1947Q1$ to $2015Q2$.[24] Using information on the number of contractions and expansions and their average duration, we calculate: (i) the probability of switching from a contraction to an expansion, $\Phi\left(s_t = \overline{s} | s_{t-1} = \underline{s}\right) = 0.256$, and (ii) the probability of switching from an expansion to a contraction, $\Phi\left(s_t = \underline{s} | s_{t-1} = \overline{s}\right) = 0.05$.

The share of financial intermediaries with high-risk projects, $\pi_{h,}$ is more difficult to determine. The model's idiosyncratic shock—which determines the type of risky projects intermediaries invest in—is assumed to be i.i.d. to retain tractability of the model. The motivation behind the i.i.d. assumption is that the subset of U.S. financial intermediaries who are considered the most risky changes considerably over time. In the context of the recent financial crisis, one can think of brokers and dealers as a proxy for high-risk intermediaries, although the widespread use of off-balance sheet activities among other institutions suggests that this definition may be too narrow.

To determine π_h, we use U.S. Flow of Funds data which shows that financial assets of brokers and dealers were, on average, 5 percent of the financial assets of all financial institutions and 21 percent of the financial assets of depository institutions.[25] We set π_h between these two estimates and perform sensitivity analysis with respect to this value.

We determine the remaining parameters—the importance of the nonfinancial sector, π_m,

nesses) as the model's financial intermediation sector. In U.S. data, noncorporate businesses are strongly dependent on the financial sector for funding. In the past three decades, bank loans and mortgages were 60 to 80 percent of noncorporate businesses' liabilities. For simplicity, we do not model these loans, and assume that the financial intermediary is endowed with the technology of production of noncorporate businesses.

[24] The business cycles we identify closely mimic those determined by the NBER.

[25] The 21 percent average masks large variations, ranging from 8.6 percent in $1987Q1$ to 35 percent in $2007Q2$ and down to 15.6 percent in $2015Q2$.

the fixed factor in the production function of the financial sector, α, the depreciation rate, δ, and the productivity parameters, $q_h(\underline{s})$, $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$—jointly using a minimum distance estimator to match eight U.S. data moments described below. We normalize the productivity of the high-risk intermediary in the good aggregate state, $q_h(\overline{s}) = 1$, since the absolute level of productivity is not important in our model.

Let Ω_i be a model moment and let $\tilde{\Omega}_i$ be the corresponding data moment. We choose the set of parameters Q^* to solve problem $(P4)$ below, where the optimal bond price, p^*, is the solution to problem $(P3)$ shown in Section 2.6 and where the productivity parameters are ordered across the different technology types as discussed in the model section.[26]

$$\begin{aligned} Q^* &= \arg \min_{\substack{Q=\{q_m(\overline{s}),q_m(\underline{s}),q_l(\overline{s}),\\ q_l(\underline{s}),q_h(\underline{s}),\delta,\alpha,\pi_m\}}} \sum_{i=1}^{8} \left(\frac{\Omega_i - \tilde{\Omega}_i}{\tilde{\Omega}_i} \right)^2 \qquad \text{(P4)} \\ \text{s.t.} &: \quad q_h(\underline{s}) < q_m(\underline{s}) < q_l(\underline{s}) \leq q_l(\overline{s}) < q_m(\overline{s}) \leq q_h(\overline{s}) \ \text{ and} \\ &\quad \Omega_i \text{ is implied in a competitive equilibrium given policy } p^* \end{aligned}$$

The eight data moments that we target are: (i) the average value added share for the corporate nonfinancial sector, (ii) the average equity to total asset ratio for corporate financial businesses, (iii) the average capital depreciation rate, (iv) the average peak-to-trough decline in business sector real value added, (v) the coefficient of variation of business sector value added, (vi) the coefficient of variation of household net worth, (vii) the average ratio of household deposits to total financial assets and $(viii)$ the recovery rate during bankruptcy. We note that the average peak-to-trough decline in business sector real value added is taken across all contraction periods since 1947Q1. Moreover, the recovery rate during bankruptcy is from Acharya, Bharath, and Srinivasan (2003), who show the average recovery rate on corporate bonds in the United States during 1982 to 1999 was 42 cents on the dollar.

[26] Given an initial set of parameter values, call it Q_1^*, and an initial guess for our competitive equilibrium allocation, we find the optimal bond price, p_1^*, using problem $(P3)$. Then, given p_1^*, we find parameters Q_2^* which solve problem $(P4)$. We continue this iterative process until convergence is achieved. We choose this two-step procedure because our model is highly nonlinear and the initial guess is very important in finding a competitive equilibrium solution. The initial guess we start with is the social planner's allocation.

Table 2 shows that the model matches the data moments well. The first three data moments help pin down π_m, α and δ, respectively. We chose moment (ii) since the parameter α influences the returns to equity in our model's financial sector, which, in turn, depend on the equity to total assets ratio of intermediaries. We also note that δ is chosen so that our model's stochastic depreciation rate, $\frac{\pi_m q_{m,t}\delta k_{m,t}+(1-\pi_m)\left(\pi_h q_{h,t}\delta k_{h,t}+\pi_l q_{l,t}\delta k_{l,t}\right)}{\pi_m k_{m,t}+(1-\pi_m)\left(\pi_h k_{h,t}+\pi_l k_{l,t}\right)}$, matches the data. The remaining moments help pin down productivity parameters.

The productivity parameters estimated (see Table 2) show that low-risk projects are virtually riskless, while high-risk projects have a large variance of returns. This suggests the moral hazard problem is important for high-risk intermediaries.

4.2 Simulation Results

In this section, we present simulation results from our competitive equilibrium and contrast them with the social optimum. We discuss why the social optimum is not implementable in our calibrated model, and then solve for the optimal interest rate policy. Moreover, we consider upward and downward shifts in the interest rate schedule, to evaluate risk taking behavior when interest rates are either lower or higher than optimal.

The simplified version of our model in Section 3.2 illustrated that when equity is sufficiently high, the moral hazard of financial intermediaries is reduced, there is no bankruptcy, and the competitive equilibrium is efficient (see Proposition 2). However, the social planner allocation can no longer be implemented as a competitive equilibrium when our full model is calibrated to match the average U.S. equity to asset ratio of financial intermediaries of about 23%. Indeed, implementing the social planner allocation as a competitive equilibrium in our calibrated model would require that, in a bad aggregate state, the returns to deposits and bonds satisfy: $R^d < 1/p$, which violates the result of Proposition 1.[27] The interpretation

[27] Implementing the social optimum has two implications for competitive equilibrium returns. First, in a bad aggregate state, it is optimal to shift resources from high-risk to low-risk intermediaries, who are expected to be relatively more productive. To provide incentives for high-risk intermediaries to buy a large value of bonds in the interbank market, bond returns need to be sufficiently high (or bond prices need to be sufficiently low) in a bad aggregate state. Second, returns to deposits need to be relatively low so that

of our finding is that interest rate policy alone cannot eliminate the moral hazard problem of the high-risk financial intermediaries.

Given that the social planner allocation is not implementable, we solve for the optimal bond price, $p^*(s^{t-1})$, that maximizes the welfare of the representative consumer, i.e. the price which solves problem $(P3)$. Moreover, we consider uniform upward or downward shifts in the interest rate schedule relative to the optimal policy to assess how the competitive equilibrium changes in environments with higher or lower than optimal interest rates. In all these experiments, we use two metrics to compare the competitive equilibrium results to the social planner allocation. First, we use the risk taking measure defined in Section 2.8 to determine whether a particular interest rate policy implies too much or too little risk taking relative to the social planner. Second, we consider a standard welfare measure, the lifetime consumption equivalent (LTCE), defined as the percentage decrease in the optimal consumption from the planner's allocation required to give the consumer the same welfare as the consumption from the competitive equilibrium with a given interest rate policy.

We employ a collocation method with occasionally binding non-linear constraints to solve our model, due to the limited liability of financial intermediaries and the possibility of a constrained repo market (for details, see Appendix $A.2$).

Experiment 1: Optimal interest rate policy, $[p^*(s^{t-1})]^{-1}$. We optimize over the bond price policy function numerically, as shown in problem $(P3)$. We find that when the interest rate policy is chosen optimally, the competitive equilibrium has a constrained repo market and features bankruptcy of high-risk intermediaries. The collateral constraint binds for high-risk intermediaries during periods of good aggregate shocks ($s_t = \overline{s}$), and for low-risk intermediaries during periods of bad aggregate shocks ($s_t = \underline{s}$). The intuition is that optimal policy aims to restrict risk taking of financial intermediaries, who otherwise may take advantage of their limited liability and overinvest in risky projects. An effective way to restrict risk taking and potential bankruptcy losses is to limit the amount of bonds, so that

intermediaries can pay back depositors. In combination, returns would have to satisfy $R^d < 1/p$.

collateral for future trading in the repo market is scarce.

Figure 2 presents simulation results from the competitive equilibrium and the social planner's problem, for a sequence of one hundred draws of the aggregate shock. We find that the optimal interest rate policy is procyclical. The intuition is as follows. Returns to bonds are linked to returns to deposits (recall Proposition 1), which in turn are linked to expected returns to equity through non-arbitrage conditions. Low returns to bonds in contractions allow returns to deposits to be low and ensure that potential bankruptcy costs are minimized.[28] In addition, whenever returns to bonds are low, the supply of government bonds is also low (see second row of subplots in Figure 2). As a result, the equilibrium value of government bonds, pB, falls, which reduces the value of collateral that can be used to borrow in contractions.

The third row of subplots in Figure 2 shows there is excessive risk taking in the competitive equilibrium, as more resources are invested in high-risk projects compared to the amount allocated by the social planner. During periods with good realizations of the aggregate state, the value of collateral is high and resources in the repo market are reallocated from the low-risk to the high-risk projects, which are expected to be more productive. Risk taking in contractions is lower than in expansions, but is still in excess of the social planner optimum. In contractions, a lower collateral value limits the reallocation of resources from high-risk to low-risk projects, leaving high-risk intermediaries with higher than optimal investment in risky projects.

Figure 2 also shows that output produced in the competitive equilibrium is higher relative to the social optimum, because more resources are invested in productive high-risk projects. However, the consumption paths in the two environments track each other closely.

Lastly, as the optimal interest rate policy, $1/p$, declines during extended periods of good

[28] The average difference between the return to deposits, R^d, and the return to bonds, $1/p$, in our experiment with optimal interest rate policy is 200 basis points per year. As argued in footnote 22, some of this difference can be assigned to the liquidity attributes of bonds in our model, in line with Krishnamurthy and Vissing-Jorgensen (2012). These authors find that the U.S. Treasury convenience yield (i.e. the value assigned to the liquidity and safety attributes of Treasuries) is 73 basis points per year for the period $1926 - 2008$.

aggregate shocks, the resources allocated to high-risk investments in the competitive equilibrium, k_h^{CE}, increase (see Figure 2). In all our simulations, conditional on boom periods, the correlation between the interest rate policy and high-risk investments is negative, consistent with findings of the empirical literature (for paper references see footnote 1). However, during persistent periods of good aggregate shocks the optimal amount of resources that are allocated to high-risk projects $\left(k_h^{SP}\right)$ also increases. This suggests that not all of the increase in k_h^{CE} is suboptimal.

To measure welfare losses and risk taking in our competitive equilibrium relative to the social planner, we average over the results of 500 simulations of 750 periods each. Table 3 shows that, at the optimal interest rate policy, investments in high-risk projects are about 5 percent higher, on average, in the competitive equilibrium relative to the social optimum.[29] The average is taken over expansion and contraction periods. The excessive risk taking leads to a small welfare loss of 0.0188% in LTCE.[30]

Our model has other interesting implications. First, repo market reallocation is beneficial, as it brings the economy closer to the social optimum. Shutting down the interbank repo market in the competitive equilibrium reduces welfare relative to the social planner by an amount equivalent to lowering consumption throughout the lifetime by about 1 percent. Second, government seigniorage from bond issuance, i.e. $R^d\left(s^{t-1}\right) D_g\left(s^{t-1}\right) - B\left(s^{t-1}\right)$, is always positive. Specifically, using $D_g\left(s^{t-1}\right) = p\left(s^{t-1}\right) B\left(s^{t-1}\right)$, seigniorage becomes $\left[R^d\left(s^{t-1}\right) p\left(s^{t-1}\right) - 1\right] \cdot B\left(s^{t-1}\right)$, which is always positive in a model with a constrained repo market because $R^d\left(s^{t-1}\right) > 1/p\left(s^{t-1}\right)$, as shown in Proposition 1. In good times, when there is no bankruptcy and deposit insurance in zero, all the seigniorage revenue is trans-

[29] As mentioned in section 2.8, instead of comparing the level of investments in high-risk projects $\left(\text{i.e. } k_h^{CE} \text{ and } k_h^{SP}\right)$ to gauge the amount of risk taking—see equation (3)—one could compare investments in high-risk projects as a share of the total capital stock $\left(\text{i.e. } k_h^{CE}/k^{CE} \text{ and } k_h^{SP}/k^{SP}\right)$. Under this alternative measure of risk taking, we obtain the same qualitative result that the competitive equilibrium features excessive risk taking at the optimal interest rate policy. Quantitatively, the amount of excessive risk taking is slightly lower than under our benchmark measure of risk taking defined in equation (3). Namely, at the optimal interest rate policy, the share of investments in high-risk projects is about 3.8 percent higher, on average, in the competitive equilibrium relative to the social optimum.

[30] Augmenting the model with a capital regulation constraint (as in Basel II) delivers similar results.

ferred to households. In bad times, seigniorage revenue covers a part of deposit insurance, while the rest is covered via lump-sum household taxation. Numerically, at the optimal interest rate, seigniorage is, on average, 0.27 percent of average household consumption, while household transfers are, on average, -0.36 percent of average household consumption.

Experiment 2: Level shifts in the optimal interest rate policy. We consider uniform shifts in the bond returns schedule: $\left[p^*\left(s^{t-1}\right)\right]^{-1} \pm \psi$, where $p^*\left(\cdot\right)$ is the optimal bond price and ψ is a constant, say 0.5 percentage points. In all of these experiments, the equilibrium also features a constrained repo market. Results from these alternate policies are presented in Figures 3 and 4.

Figure 3 compares risk taking and welfare results from a wide range of experiments with different values of ψ with results from Experiment 1. Similar to the results displayed in Table 3, the welfare and risk taking in Figure 3 are averages over 500 simulations of 750 periods each. In both subplots, the x-axis shows deviations from the optimal equilibrium policy, $\left[p^*\left(s^{t-1}\right)\right]^{-1}$ ranging from -2 to $+2$ percentage points at annual rates. The zero mark on the x-axis shows results for Experiment 1, where policy is optimally chosen. We find that small deviations from the optimal policy, say 50 basis points, entail relatively small welfare losses, but sizable changes in risk taking. Higher than optimal bond returns lead to more risk taking relative to the optimum, while lower than optimal bond returns lead to reductions in risk taking (also see Table 3).[31]

Whenever interest rates are sufficiently below the optimum, there is too little risk taking in the competitive equilibrium relative to the optimum, $r \equiv E\left[r\left(s^{t-1}\right)\right] < 0$ (see Figure 3). Here is the intuition for this result. At low interest rates, intermediaries purchase fewer government bonds (*the portfolio channel*). When aggregate productivity is expected to be high, high-risk intermediaries would like to invest more in their risky projects. However, a low

[31] The qualitative risk taking results are identical if the risk taking measure is alternatively defined in terms of the share of high-risk investments in total capital (i.e. by comparing k_h^{CE}/k^{CE} and k_h^{SP}/k^{SP}) as discussed in Section 2.8. The quantitative results are only slightly different and, for this reason, are not reported. Also, see footnote 29.

quantity and a low value of collateral constrain their portfolio adjustment in the interbank market (*the collateral channel*). Quantitatively, the collateral channel dominates. Thus, during expansion periods, whenever interest rates are sufficiently low, investments in high-risk projects in the competitive equilibrium are lower than the social planner optimum (see Figure 4 for simulation of optimal policy minus 50 basis points at annual rate). Conversely, during contractions, risk taking is in excess of the social optimum. Our model is calibrated to be consistent with the fact that U.S. expansion periods are longer than contractions. As a result, aggregate risk taking, defined as an average over expansions and contractions in our simulations, is lower than the social optimum, whenever policy rates are sufficiently low.

Higher than optimal interest rates entail larger welfare losses in our environment (Figure 3). At higher interest rates, intermediaries purchase more safe bonds (*portfolio channel*), which leads to a relaxation of their collateral constraint and allows for more borrowing in the interbank market (*collateral channel*). Our paper makes an important contribution by highlighting that relaxing collateral constraints increases risk taking with adverse effects for real activity and welfare. This insight is in contrast to Kiyotaki and Moore (1997) where shocks to credit-constrained firms are amplified and transmitted to output through changes in collateral values. In their framework, relaxing collateral constraints is beneficial.

The final observation from Figure 3 is that large reductions in bond returns result in a shutdown of the repo market in good times. This result is similar to Boissay, Collard, and Smets (2016) who show that sufficiently low interest rates lead to interbank market freezes. In our numerical experiments, deviations of at least 160 basis points below the optimal policy lead intermediaries to demand no bonds in good times. The portfolio channel is quantitatively important here, as the collateral channel is eliminated. Even though high-risk intermediaries invest all resources in risky assets in good times, they are still underinvesting relative to the social planner. As the bond market shuts down in good times, the households give slightly more resources to financial intermediaries. This result generates the kink in the subplots of Figure 3. To the left of the kink, risk taking is still lower compared to the social planner, but less so.

Sensitivity analysis: Share of High-Risk Intermediaries In the numerical results presented so far, high-risk financial intermediaries represented 15 percent of all intermediaries (or 5.2 percent of all firms in the economy). We examine how our results on welfare and risk taking change when high-risk intermediaries are a smaller or bigger fraction of all intermediaries, i.e. π_h is 13% or 17%. In both cases, we re-optimize the policy rate.

The results from the sensitivity analysis are reported in Table 4. The quantitative results change with π_h. Higher π_h leads to slightly higher risk taking and slightly larger welfare losses at the optimal interest rate policy. However, the qualitative result remains the same: lower than optimal interest rates lead to reductions in risk taking relative to the social planner.

To summarize, we examined a model in which a moral hazard problem enables intermediaries to take on excessive risks at the optimal interest rate policy. To shed light on recent debates in the literature, we use our framework to examine how the intermediaries' incentives to take on risks are altered by changes in the interest rate policy. A key insight is that collateralized interbank borrowing acts as a safeguard against increases in risk taking, especially at low interest rates. Moreover, higher than optimal interest rates lead to a relaxation of collateral constraints and induce higher risk taking and larger welfare losses compared to lower than optimal interest rates.

5 Conclusion

The recent financial crisis has stirred interest in the relationship between lower than optimal interest rates and the risk taking behavior of financial institutions. We examine this relationship in a dynamic general equilibrium model that features deposit insurance, limited liability of financial intermediaries, and heterogeneity in the riskiness of intermediaries' portfolios.

There are two channels through which interest rate policy influences risk taking in our model. The portfolio channel illustrates the idea that lower than optimal policy rates reduce the returns to safe assets and lead intermediaries to shift investments towards riskier assets. Given fewer bond purchases in the bond market, intermediaries have less collateral available for repo market transactions. Hence, the collateral channel constrains the ability of intermediaries to take on more risk through the repo market, after they receive further information regarding the riskiness of their projects. In order to determine the quantitative importance of the two channels, we calibrate our model to U.S. data and show that, our decentralized economy with optimal interest rate policy features excessive risk taking and has welfare that is close to, though below, the social optimum. While both risk taking channels lead to important changes in the intermediaries' portfolios, for reasonably large variations around the optimal policy, the collateral channel dominates quantitatively. Thus, lower than optimal interest rates lead to reductions in risk taking relative to the social optimum.

Our results are consistent with the empirical finding that intermediaries take on more risks when interest rates are low. Expansions in our decentralized economy are characterized by optimally declining interest rates and feature higher investments in risky projects. However, the socially optimal amount of investments in risky assets also rises in expansions. This suggests that, when interest rates are low, some increase in risky investments is optimal and government policy should not aim to eliminate it.

A response to the financial crisis has been the global adoption of stricter capital and leverage regulations aimed to promote financial sector stability (Basel Committee on Banking Supervision (2011)). Concurrently, there have been discussions about integrating financial stability objectives in monetary policy decision making (Stein (2014), Woodford (2012), Kocherlakota (2014)). A natural extension of our analysis is to introduce financial regulation constraints in the intermediaries' problem. In our current framework, the amount of equity to risky capital investments and the intermediaries' leverage varies with the cycle. It is therefore straightforward to introduce capital and leverage constraints and to examine the interaction between monetary policy and financial regulations in shaping intermediaries' decisions. We have shown that optimal monetary policy alone cannot eliminate the excessive risk taking of financial intermediaries relative to the social optimum. In Cociuba, Shukayev, and Ueberfeldt (2015), we show that state contingent capital regulation or state contingent leverage regulation can be used in conjunction with monetary policy to eliminate excessive risk taking and achieve the social optimum.

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A Appendix

A.1 Proofs

To simplify notation in our derivations, we use subscript $t-1$ as short hand notation for the history, s^{t-1}. For example, $\tilde{b}_{j,t-1} \equiv \tilde{b}_j\left(s^{t-1}\right)$ and $b_{t-1} \equiv b\left(s^{t-1}\right)$.

Proof of Proposition 1.

Deriving the relationship between bond prices and the return to deposits in our model involves analyzing three possible outcomes on the repo market. Transactions of bonds either satisfy: (i) $\tilde{b}_{j,t-1} < b_{t-1}$ for both $j \in \{h, l\}$ or (ii) $\tilde{b}_{h,t-1} = b_{t-1}$ and $\tilde{b}_{l,t-1} < b_{t-1}$ or (iii) $\tilde{b}_{l,t-1} = b_{t-1}$ and $\tilde{b}_{h,t-1} < b_{t-1}$. We sketch the proof of Proposition 1 for case (ii). The proof is obtained in an analogous fashion for cases (i) and (iii) and is omitted here for brevity.[32]

In case (ii), the high-risk intermediary increases the amount of resources allocated to risky investments by selling all bond holdings in the repo market.

Step 1: Some Key Relationships

In characterizing the equilibrium, it is useful to define x_{t-1} as the share of resources a financial intermediary retains for risky investment at the first stage of the portfolio decision.

$$
\begin{aligned}
k_{t-1} &= x_{t-1}\left(z_{t-1} + d_{t-1}\right) && (4)\\
b_{t-1} &= \frac{1 - x_{t-1}}{p_{t-1}}\left(z_{t-1} + d_{t-1}\right) && (5)
\end{aligned}
$$

where the second equation was obtained using (4) and equation (1) in the main text.

For case (ii), high-risk intermediaries use all their bonds as collateral in the repo market, while low-risk intermediaries give resources against this collateral.

$$
\begin{aligned}
\tilde{b}_{h,t-1} &= b_{t-1} = \frac{1 - x_{t-1}}{p_{t-1}}\left(z_{t-1} + d_{t-1}\right) && (6)\\
\tilde{b}_{l,t-1} &= -\frac{\pi_h}{\pi_l} b_{t-1} = -\frac{\pi_h}{\pi_l}\frac{1 - x_{t-1}}{p_{t-1}}\left(z_{t-1} + d_{t-1}\right) && (7)
\end{aligned}
$$

[32] The full derivation is available upon request from the authors.

Using equations $(4)-(7)$, the resources allocated to risky investments by high-risk and low-risk intermediaries after the repo market trades are given by (8) and (9).

$$k_{h,t-1} \equiv k_{t-1} + \tilde{p}_{t-1}\tilde{b}_{h,t-1} = \left[x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right](z_{t-1}+d_{t-1}) \tag{8}$$

$$k_{l,t-1} \equiv k_{t-1} + \tilde{p}_{t-1}\tilde{b}_{l,t-1} = \left[x_{t-1} - \frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right](z_{t-1}+d_{t-1}) \tag{9}$$

Step 2: Equilibrium Conditions for the Financial Sector

In what follows, we make use of the equilibrium result $l_{h,t} = l_{l,t} = 1$.

We rewrite the repo market problem given in $(P2)$ as below:

$$\max_{\tilde{b}_{j,t-1}} \textstyle\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \left\{ \begin{array}{l} q_{j,t}\left[\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta} + (1-\delta)\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)\right] \\ +\left(b_{t-1}-\tilde{b}_{j,t-1}\right) - R^d_{t-1}d_{t-1} - W_{j,t} \end{array} \right\}$$

where $\tilde{b}_{j,t-1} \in \left[-\frac{k_{t-1}}{\tilde{p}_{t-1}}, b_{t-1}\right]$ and $1_{j,t}$ is an indicator function which equals 1 if the profits of intermediaries (the terms in the curly brackets above) are strictly positive, or 0 otherwise.

The first order conditions with respect to bond trades, $\tilde{b}_{h,t-1}$ and $\tilde{b}_{l,t-1}$, are given by:

$$\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \left\{ q_{j,t}\tilde{p}_{t-1}\left[\theta\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta-1} + 1-\delta\right] - 1\right\} - \mu_{j,t-1} = 0 \tag{10}$$

where $\mu_{j,t-1}$ for $j \in \{h,l\}$ are the Lagrange multipliers on the constraints $\tilde{b}_{j,t-1} \le b_{t-1}$ and they satisfy the complimentary slackness conditions: $\mu_{j,t-1} \ge 0$, $\mu_{j,t-1}\left(b_{t-1}-\tilde{b}_{j,t-1}\right) = 0$.[33]

For case (ii), $\mu_{l,t-1} = 0$ and $\mu_{h,t-1} \ge 0$. Using this, along with the expressions in (8) and (9), we can rewrite equation (10) for $j \in \{h,l\}$ as (11) and (12) below:

$$\theta\left[\left(x_{t-1} - \frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1} + 1-\delta = \frac{\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t}{\tilde{p}_{t-1}\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t}} \tag{11}$$

$$\theta\left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1} + 1-\delta \ge \frac{\sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t}{\tilde{p}_{t-1}\sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t}} \tag{12}$$

[33] In equilibrium, the constraint $-\frac{k_{t-1}}{\tilde{p}_{t-1}} \le \tilde{b}_{j,t-1}$ does not bind as returns to capital invested in risky projects would become infinite.

Notice that equation (11) can be equivalently written as:

$$\left[x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right]\left(z_{t-1}+d_{t-1}\right)=\left[\frac{1}{\theta}\left(\frac{\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t}{\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t}\tilde{p}_{t-1}}-1+\delta\right)\right]^{\frac{1}{\theta-1}} \tag{13}$$

Using equations $(4)-(9)$ we rewrite the bond market problem $(P1)$ as below:

$$\max_{\substack{x_{t-1}\in[0,1]\\ d_{t-1}\geq 0}} \sum_{j\in\{h,l\}}\pi_j\sum_{s^t|s^{t-1}}\lambda_t V_{j,t}$$

$$\text{subject to: } V_{l,t}=\max\left\{\left[\begin{array}{l} q_{l,t}\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta}\\ +q_{l,t}\left(1-\delta\right)\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\\ +\frac{1}{\pi_l}\frac{\left(1-x_{t-1}\right)}{p_{t-1}}\left(z_{t-1}+d_{t-1}\right)-R^d_{t-1}d_{t-1}-W_{l,t}\end{array}\right],\ 0\right\}$$

$$V_{h,t}=\max\left\{\left[\begin{array}{l} q_{h,t}\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta}\\ +q_{h,t}\left(1-\delta\right)\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)-R^d_{t-1}d_{t-1}-W_{h,t}\end{array}\right],\ 0\right\}$$

The first order conditions with respect to x_{t-1} and d_{t-1} are given by (14) and (15), respectively.[34]

$$\begin{aligned}&\frac{1}{p_{t-1}}\sum_{s^t|s^{t-1}}\lambda_t 1_{l,t}= \\ &\left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\left(1+\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_l\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t} \\ &+\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_h\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t}\end{aligned} \tag{14}$$

$$\begin{aligned}&R^d_{t-1}\sum_{j\in\{h,l\}}\pi_j\sum_{s^t|s^{t-1}}1_{j,t}\lambda_t= \\ &\left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\pi_l\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t} \\ &+\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\pi_h\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t}\end{aligned} \tag{15}$$

[34] In order to obtain equation (15), we derive the first order condition with respect to deposits and simplify it by using equation (14).

Step 3: Bond Prices

Using (13) , the equilibrium condition for the choice of x_{t-1}, equation (14), becomes:

$$\begin{aligned}
&\left(\frac{1}{p_{t-1}}-\frac{\pi_l}{\tilde{p}_{t-1}}\left(1+\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\right)\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t \\
=\;&\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_h\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t}
\end{aligned}$$

Using $\pi_l+\pi_h=1$, the equation above is simplified to:

$$\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\cdot\Xi=0 \tag{16}$$

where $\Xi\equiv\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\pi_h\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t}+$ $+\frac{\pi_l\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t}{\tilde{p}_{t-1}}$. Notice that $\Xi>0$, unless all financial intermediaries go broke. Then, equation (16) implies that the bond price and the repo price are equated, $\tilde{p}_{t-1}=p_{t-1}$.

Step 4: Bond Price and Return to Deposits

Next, we subtract equation (15) from equation (14) and find equation (17) .

$$\begin{aligned}
&\frac{1}{p_{t-1}}\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t-R^d_{t-1}\sum_{j\in\{h,l\}}\pi_j\sum_{s^t|s^{t-1}}1_{j,t}\lambda_t \\
=\;&\left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\pi_h\frac{\tilde{p}_{t-1}}{p_{t-1}}\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t} \\
&-\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\frac{\tilde{p}_{t-1}}{p_{t-1}}\pi_h\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t}
\end{aligned} \tag{17}$$

Using (11) and (12) , equation (17) becomes $R^d_{t-1}\geq\frac{1}{p_{t-1}}$. This completes the proof of Proposition 1 for the case in which the high-risk intermediary sells all bonds in the repo market. The other cases are derived analogously, but are omitted here for brevity.

Proof of Proposition 2.

In the case of an *unconstrained repo market* $\tilde{b}_{j,t-1} < b_{t-1}$, and the Lagrange multipliers on these constraints are $\mu_{j,t-1} = 0$ for both $j \in \{h, l\}$. Using equation (10), we obtain:

$$k_{j,t-1} \equiv k_{t-1} + \tilde{p}_{t-1}\tilde{b}_{j,t-1} = \left[\frac{1}{\theta}\left(\frac{\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t}{\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t q_{j,t}\tilde{p}_{t-1}} - 1 + \delta\right)\right]^{\frac{1}{\theta-1}} \tag{18}$$

where, as before, $1_{j,t}$ is an indicator which equals 0 when intermediaries are bankrupt and 1 otherwise. Under assumptions $A1$, we know that $\lambda_t = \begin{cases} \phi \text{ if } s_t = \overline{s} \\ 1-\phi \text{ if } s_t = \underline{s} \end{cases}$ and $\delta = 1$. Assuming no bankruptcy of financial intermediaries, i.e. $1_{j,t} = 1$, we see from equation (18) that $k_{j,t-1} = \left[\theta\tilde{p}_{t-1}\left(\phi q_j\left(\overline{s}\right) + (1-\phi)\, q_j\left(\underline{s}\right)\right)\right]^{\frac{1}{1-\theta}}$. Using Proposition 1, in an unconstrained repo market $p_{t-1} = \tilde{p}_{t-1}$ and $R^d_{t-1} = 1/p_{t-1}$. Moreover, under assumptions $A1$, the Euler equation in our simplified model is $R^d_t = 1/\beta$. Hence, $\tilde{p}_{t-1} = \beta$ and resources allocated to high-risk and low-risk technologies coincide with the social planner's allocations: $k_{j,t-1} = \left[\theta\beta\left(\phi q_j\left(\overline{s}\right) + (1-\phi)\, q_j\left(\underline{s}\right)\right)\right]^{\frac{1}{1-\theta}}$ for $j \in \{h, l\}$.

No intermediary becomes bankrupt if and only if equation (19) holds for all j and all aggregate states.

$$q_{j,t}\left(k_{t-1} + \tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta} + \left(b_{t-1} - \tilde{b}_{j,t-1}\right) - R^d_{t-1}d_{t-1} - W_{j,t} > 0 \tag{19}$$

Equation (19) simplifies to $(\theta+\alpha)\, q_{j,t}\left(k_{j,t-1}\right)^{\theta} - \frac{k_{j,t-1}}{\beta} + \frac{z_{t-1}}{\beta} > 0$ for all j and all t, where we used the fact that $W_{j,t}$ is the marginal product of labor, the expression for b_{t-1} given in equation (5), the expression for $\tilde{b}_{j,t-1} = \frac{k_{j,t-1}-k_{t-1}}{\tilde{p}_{t-1}}$ from equations (8) and (9), the expression for k_{t-1} from equation (4) and the results that $p_{t-1} = \tilde{p}_{t-1} = \beta$ and $R^d_{t-1} = 1/p_{t-1}$.

To guarantee no intermediary becomes bankrupt, it suffices to check that high-risk intermediaries do not go bankrupt in the bad aggregate state. This is equivalent to
$z_{t-1} \geq k_{t-1}\left(1 - \frac{\theta+\alpha}{\theta} \cdot \frac{1}{\phi\frac{q_h(\overline{s})}{q_h(\underline{s})} + 1 - \phi}\right)$.

In the case of a constrained repo market, we focus on the situation in which $\tilde{b}_{h,t-1} = b_{t-1}$ and $\tilde{b}_{l,t-1} < b_{t-1}$. The proof for the case $\tilde{b}_{h,t-1} < b_{t-1}$ and $\tilde{b}_{l,t-1} = b_{t-1}$ is derived analogously.

Interiority of $\tilde{b}_{l,t-1}$ yields, as before, the expression for $k_{l,t-1}$ given in equation (18). To find $k_{h,t-1}$ we use equation (15) along with equations (8) and (9), the expression for $k_{l,t-1}$ in equation (18) and the assumption $\delta = 1$.

$$k_{h,t-1} = \left(\frac{\pi_h \theta \sum_{s^t|s^{t-1}} 1_{h,t} \lambda_t q_{h,t}}{R^d_{t-1} \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t - \frac{\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t}{\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t q_{j,t}\tilde{p}_{t-1}} \pi_l \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t}} \right)^{\frac{1}{1-\theta}} \tag{20}$$

Using assumptions $A1$, the results that $p_{t-1} = \tilde{p}_{t-1}$ and $R^d_t = 1/\beta$, and given different bankruptcy scenarios for intermediaries, we can simplify equations (18) and (20) as below.

$$k_{l,t-1} = \begin{cases} \left[p_{t-1}\theta q_l(\bar{s})\right]^{\frac{1}{1-\theta}} & \text{if both types go broke when } s_t = \underline{s} \\ \left\{p_{t-1}\theta\left[\phi q_l(\bar{s}) + (1-\phi)\, q_l(\underline{s})\right]\right\}^{\frac{1}{1-\theta}} & \text{if only high-risk type goes broke if } s_t = \underline{s} \\ \left\{p_{t-1}\theta\left[\phi q_l(\bar{s}) + (1-\phi)\, q_l(\underline{s})\right]\right\}^{\frac{1}{1-\theta}} & \text{if neither type goes broke when } s_t = \underline{s} \end{cases} \tag{21}$$

$$k_{h,t-1} = \begin{cases} \left(\frac{\pi_h\theta q_h(\bar{s})}{\frac{1}{\beta} - \pi_l \frac{1}{p_{t-1}}}\right)^{\frac{1}{1-\theta}} & \text{if both types go broke when } s_t = \underline{s} \\ \left(\frac{\pi_h\theta\phi q_h(\bar{s})}{\frac{1}{\beta}(\pi_l + \pi_h\phi) - \pi_l \frac{1}{p_{t-1}}}\right)^{\frac{1}{1-\theta}} & \text{if high-risk type goes broke when } s_t = \underline{s} \\ \left(\frac{\pi_h\theta\left[\phi q_h(\bar{s}) + (1-\phi) q_h(\underline{s})\right]}{\frac{1}{\beta} - \pi_l \frac{1}{p_{t-1}}}\right)^{\frac{1}{1-\theta}} & \text{if neither type goes broke when } s_t = \underline{s} \end{cases} \tag{22}$$

If neither type of intermediary is bankrupt in a bad aggregate state ($s_t = \underline{s}$), the interest rate policy $\frac{1}{p_{t-1}} = \frac{1}{\beta}$ implements the social planner's allocation (see equations (21) and (22)). It can be shown that no financial intermediary is bankrupt if and only if households invest only equity into intermediaries (household deposits are zero).

Proof of Proposition 3.

First, we prove result (i). In an equilibrium with a constrained repo market, during a good aggregate state ($s_t = \bar{s}$), high-risk financial intermediaries are selling all their bonds in the interbank market, i.e. $\tilde{b}_{h,t-1} = b_{t-1}$. The low-risk financial intermediaries purchase bonds, i.e. $\tilde{b}_{l,t-1} < 0 < b_{t-1}$. Using the expression for $k_{h,t-1}$ in equation (22), it is easy to show that lower policy interest rates lead to less high-risk investments in the competitive equilibrium during good aggregate states, i.e. $\frac{\partial k_{h,t-1}^{CE}}{\partial\left(\frac{1}{p_{t-1}}\right)} > 0$. Since the social planner's allocation, $k_{h,t-1}^{SP}$ does not change (i.e. $k_h^{SP} = \{\beta\theta \cdot [\phi q_h(\bar{s}) + (1-\phi) q_h(\underline{s})]\}^{\frac{1}{1-\theta}}$), lower k_h^{CE} also means lower risk taking during good aggregate states.

The proof for result (ii) is derived analogously. In an equilibrium with a constrained repo market, during a bad aggregate state ($s_t = \underline{s}$), low-risk financial intermediaries are selling all their bonds in the interbank market, i.e. $\tilde{b}_{l,t-1} = b_{t-1}$. The high-risk financial intermediaries purchase bonds, i.e. $\tilde{b}_{h,t-1} < 0 < b_{t-1}$. An expression for $k_{h,t-1}$ can be derived in this case (the analog for equation (22) when $\tilde{b}_{l,t-1} = b_{t-1}$ and $\tilde{b}_{h,t-1} < b_{t-1}$). It is then easy to show that lower interest rates increase risk taking during bad aggregate states, $\frac{\partial k_{h,t-1}^{CE}}{\partial\left(\frac{1}{p_{t-1}}\right)} < 0$.

A.2 Computation of Equilibrium

We compute a recursive formulation of the model, where the state variables are the aggregate state, s_t, and the household wealth, w_t. We solve for consumption as a function of the state variables using a collocation method with linear spline functions. To improve the accuracy and the speed of the computation, we use an endogenous grid method à la Carroll (2006).

We separate the household problem into two parts: an intertemporal consumption choice and a portfolio choice. The household's portfolio decision involves allocation of resources to the nonfinancial and financial sectors so that expected returns across sectors are equalized. Given the overall resources allocated to the financial sector, the split between equity and deposits is determined so that expected returns from these two types of investments are equalized (for details, see Carroll (2012), Section 7 on multiple control variables).

There are two main challenges when solving the financial sector problem: (i) some financial intermediaries may be constrained in their repo market trades and (ii) financial intermediaries may go bankrupt when the aggregate state is realized. We consider all the possible combinations in sequence and verify which is an equilibrium. For example, we assume that when the aggregate state switches from good to bad, high risk intermediaries are constrained in their repo market trade and go bankrupt, while the low risk intermediaries are unconstrained and do not go bankrupt. After solving the financial intermediaries' problems, we check whether the outcome is consistent with the assumed behavior.

B Tables

Table 1: Calibrated Parameters

Parameter/Value	Moment[1]
$\beta = \left(\frac{1}{1.04}\right)^{1/4}$	Annual real interest rate of 4 percent
$\theta = 0.29$	Capital income share
$\Phi = \begin{bmatrix} 0.95 & 0.05 \\ 0.256 & 0.744 \end{bmatrix}$	Average length and number of expansions and contractions of U.S. business sector
$\pi_l = 0.85$, $\pi_h = 1 - \pi_l = 0.15$	Financial assets of brokers and dealers; Sensitivity analysis

[1]Sources of data: U.S. National Income and Product Accounts and U.S. Flow of Funds accounts. Further details are provided in Section 4.1.

Table 2: Jointly Calibrated Parameters

Panel A

Parameter	Value
Share of nonfinancial firms	$\pi_m = 0.65358$
Depreciation rate	$\delta = 0.02578$
Fixed factor income share	$\alpha = 0.00342$
Productivity parameters	
nonfinancial firms	$q_m(\overline{s}) = 0.98241$
	$q_m(\underline{s}) = 0.94173$
low-risk financial firms	$q_l(\overline{s}) = 0.94177$
	$q_l(\underline{s}) = 0.94175$
high-risk financial firms	$q_h(\overline{s}) = 1$
	$q_h(\underline{s}) = 0.35547$

Panel B

Moments Targeted[1]	Data in %	Model in %
Average value added share of corporate nonfinancial sector	67.6	70.0
Average equity to asset ratio of the financial sector	23.0	21.4
Average capital depreciation rate in economy	2.5	2.5
Average peak-to-trough decline in output during contractions[2]	6.4	8.5
Coefficient of variation of output[3]	4.6	4.2
Coefficient of variation of household net worth[3]	8.5	9.3
Average deposits over total household financial assets	16.4	17.6
Recovery rate in bankruptcy	42.0	40.4

[1]Sources of data: U.S. National Income and Product Accounts and U.S. Flow of Funds accounts. The recovery rate in bankruptcy is from Acharya, Bharath, and Srinivasan (2003). [2]Total output is measured as the real value added for the U.S. business sector. We detrend output by the average growth rate over the period $1947Q1 - 2015Q2$. [3]We calculate statistic after detrending the variable by the average growth rate over the period $1987Q1 - 2015Q2$.

Table 3: Model Welfare and Risk Taking Relative to the Social Planner[1]

Experiment	LTCE[2] in %	Risk taking[3] in %
Optimal policy −0.5 percentage points at annual rate	−0.0301	−5.67
Optimal policy −0.2 percentage points at annual rate	−0.0207	0.57
Optimal interest rate policy	−0.0188	5.11
Optimal policy +0.2 percentage points at annual rate	−0.0208	9.99
Optimal policy +0.5 percentage points at annual rate	−0.0321	17.98

[1]The statistics are averages over 500 simulations of 750 periods each of the model economy and the social planner's problem. [2]Lifetime Consumption Equivalents (LTCE) is the percentage decrease in the optimal consumption from the social planner problem needed to generate the same welfare as the competitive equilibrium with a given interest rate policy. [3]Risk taking is the percentage deviation in the amount of resources invested in the high-risk projects in the competitive equilibrium relative to the social planner's choice. The numbers reported here are averages over expansions and contractions in our calibrated model. A positive number indicates too much risk taking, on average, relative to the social planner, while a negative number indicates too little risk taking.

Table 4: Sensitivity Analysis for Fraction of High Risk Intermediaries
Welfare and Risk Taking Results Relative to the Social Planner[1]

	LTCE[2] in %		
Experiment / π_h value	0.13	**0.15**	0.17
Optimal policy −0.5 percentage points	−0.0267	**−0.0301**	−0.0349
Optimal interest rate policy	−0.0137	**−0.0188**	−0.0248
Optimal policy +0.5 percentage points	−0.0295	**−0.0321**	−0.0364
	Risk taking[3] in %		
Experiment / π_h value	0.13	**0.15**	0.17
Optimal policy −0.5 percentage points	−7.92	**−5.67**	−3.70
Optimal interest rate policy	4.45	**5.11**	5.83
Optimal policy +0.5 percentage points	19.69	**17.98**	16.95

[1]The statistics are averages over 500 simulations of 750 periods each of the model economy and the social planner's problem. [2,3]See definitions given in notes to Table 3.

C Figures

Figure 1: Timing of Model Events

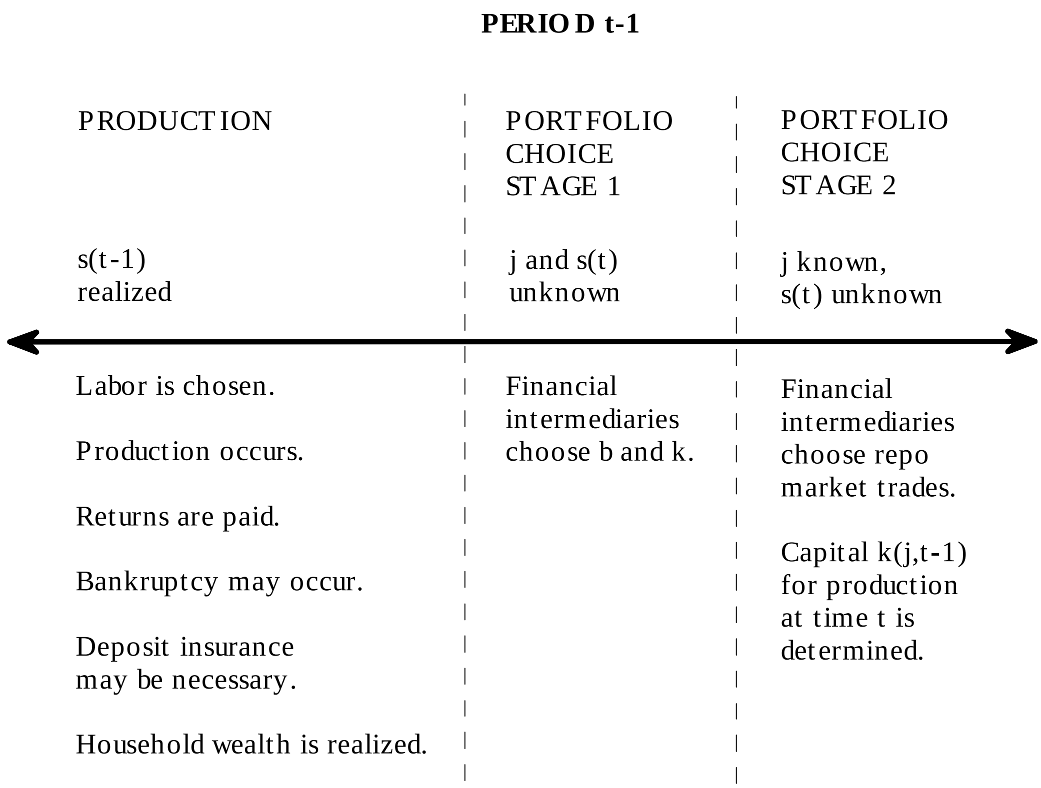


Figure 2: Simulation Results: Model with Optimal Interest Rate Policy and Social Planner Allocations

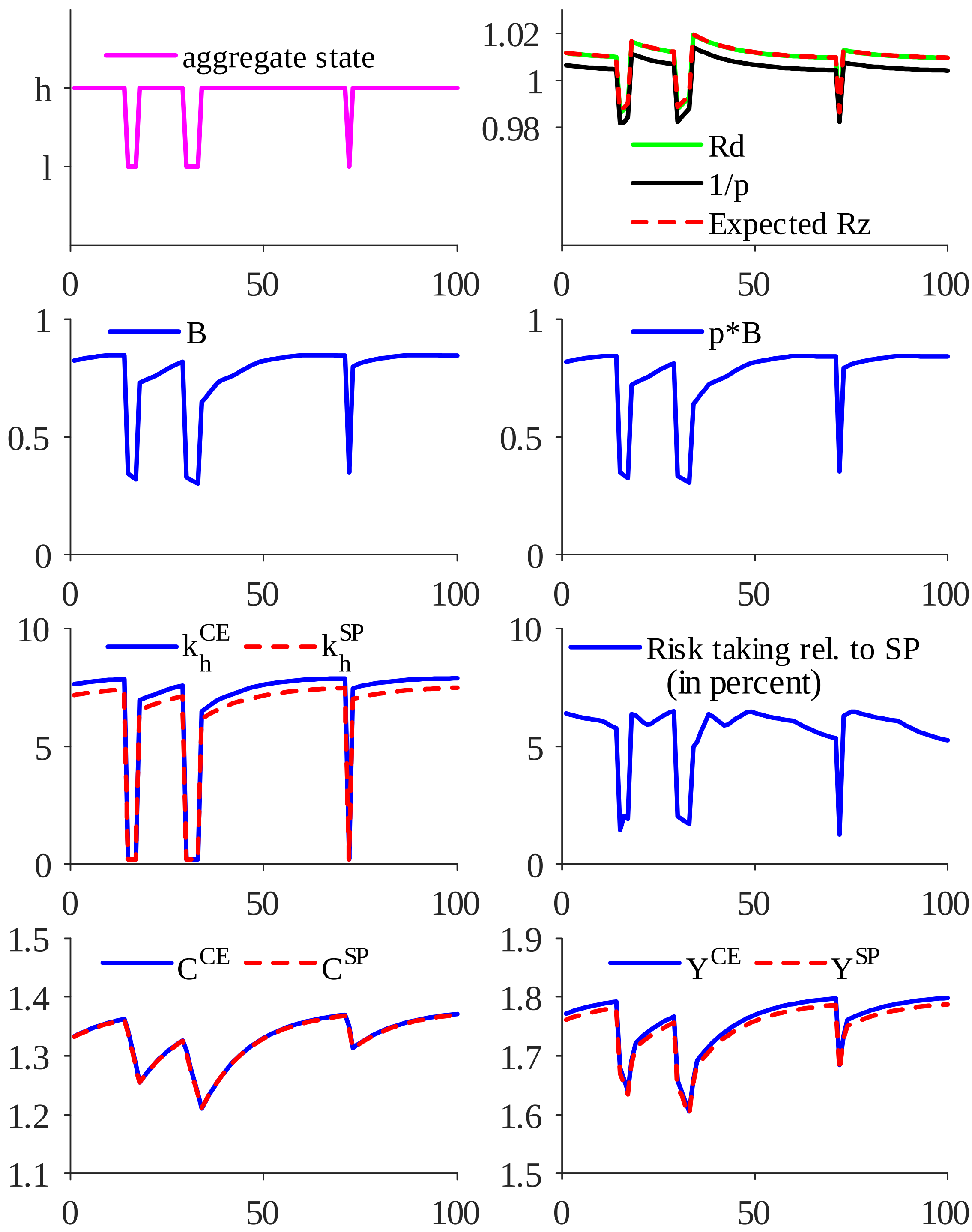


Figure 3: Model Welfare and Risk Taking Relative to the Social Planner

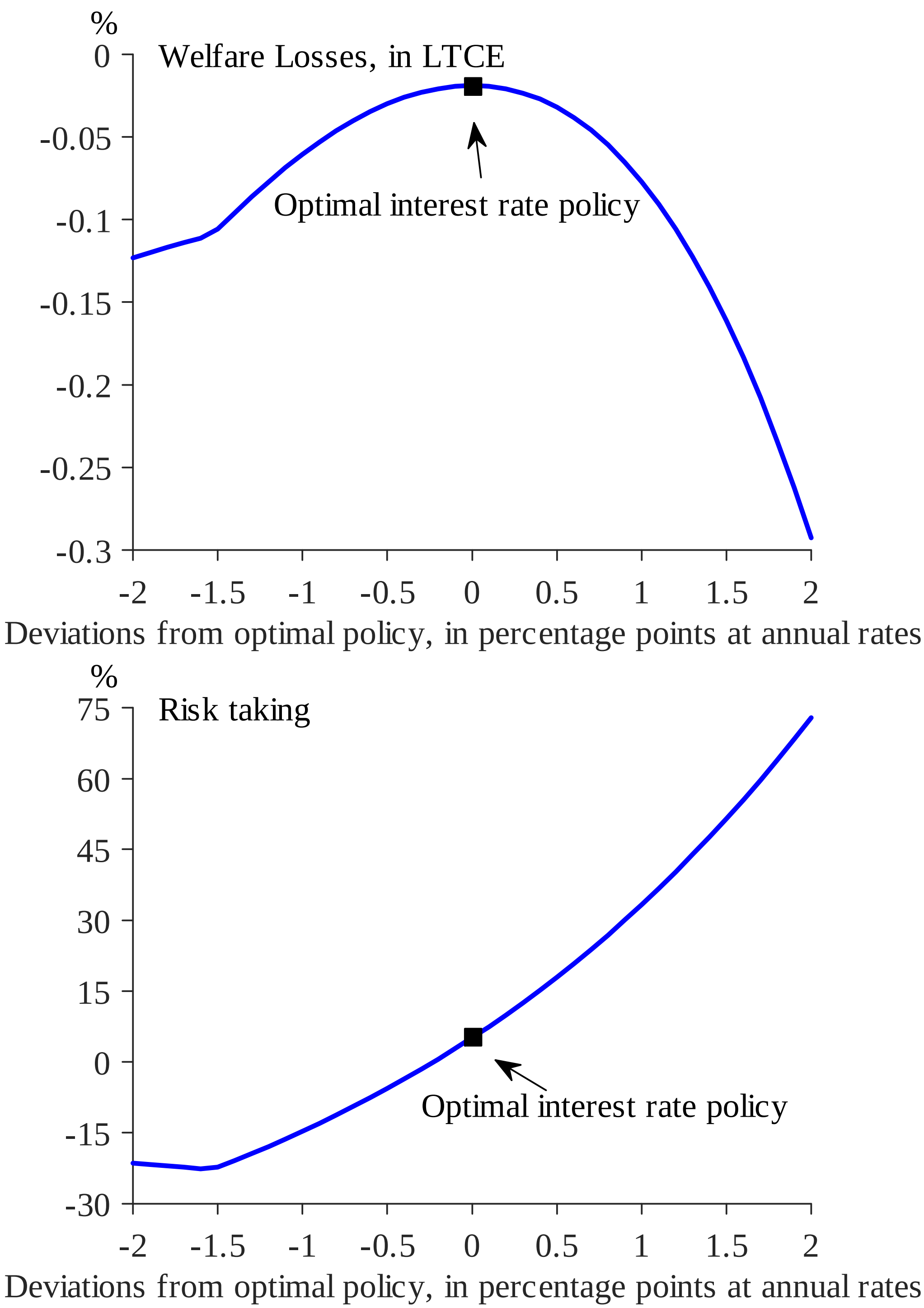
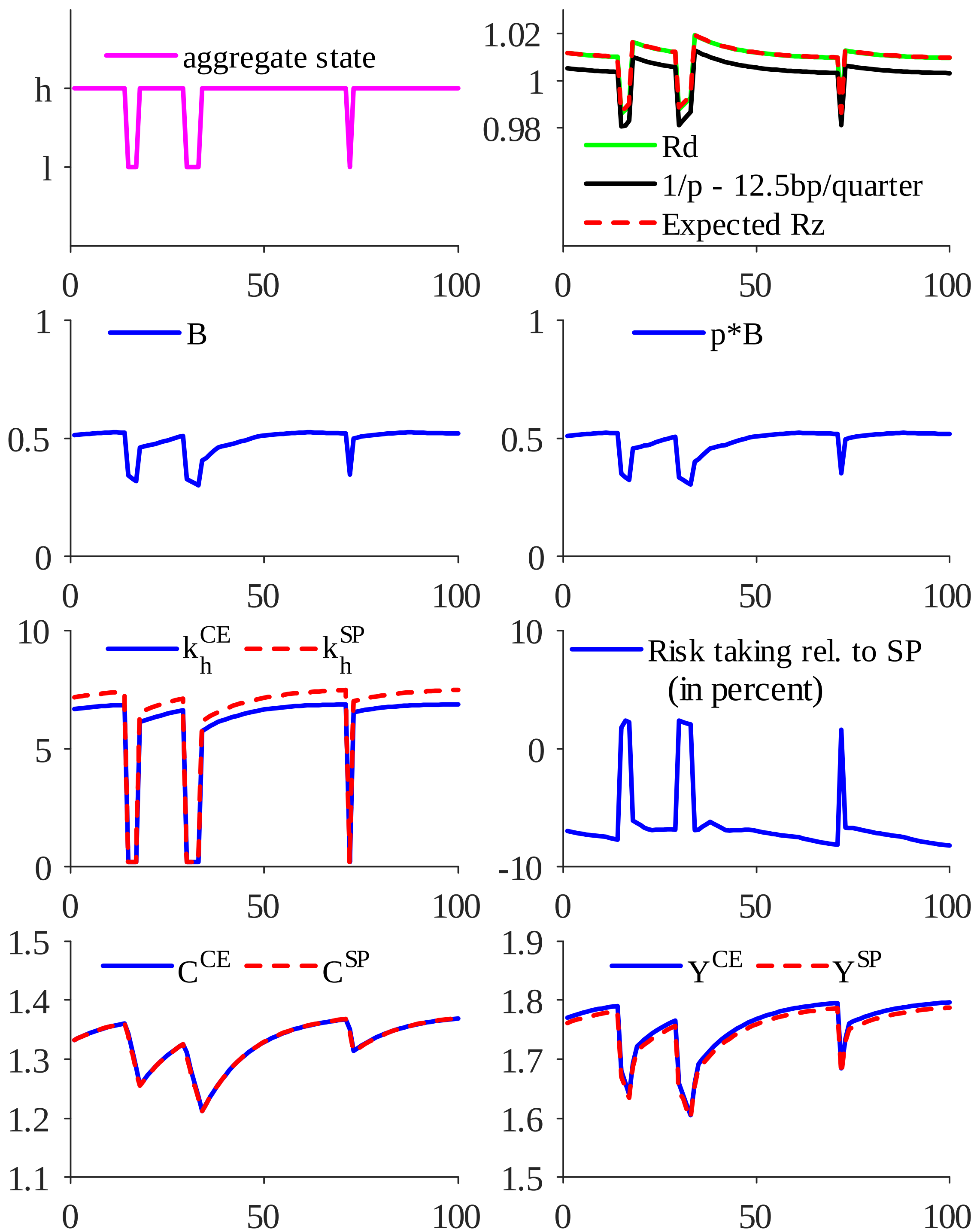


Figure 4: Simulation Results: Model with Lower than Optimal Interest Rates and Social Planner Allocations



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