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On the Optimal Design of Demand Response Policies

by

David P. Brown* and David E. M. Sappington**

Abstract

We characterize the optimal regulatory policy to promote demand response in the electricity sector. Demand response arises when consumers reduce their purchases of electricity in times of peak demand, when the utility's marginal cost of supplying electricity is relatively high. The optimal policy differs systematically from the policy in the U. S. Federal Energy Regulatory Commission's (FERC's) Order 745. Under plausible conditions, implementation of the FERC's policy can reduce welfare substantially below the level secured by the optimal demand response policy.

Keywords: electricity pricing, demand response

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1 Introduction

The cost of supplying electricity can vary substantially from day to day and even from hour to hour. This is the case because generating units with relatively high operating costs often must be called upon to produce electricity during times of peak demand. In contrast to the ever-changing cost of supplying electricity, the retail price of electricity typically varies little, if at all, for long periods of time. Such time-invariant pricing reflects historic difficulty in measuring the precise time at which electricity is consumed and ongoing consumer resistance to time-sensitive pricing now that smart meters render such pricing feasible.

To help overcome the inefficiencies that arise when the retail price of electricity diverges substantially from the marginal cost of supplying electricity (Borenstein and Holland, 2005), U.S. regulators have, at the urging of Congress, implemented demand response policies.[1,2] In essence, demand response policies compensate electricity customers for reducing their purchases of electricity below historic norms during periods of peak electricity demand. Of central concern in the design of demand response policies is the compensation that is provided to consumers who reduce their electricity consumption.

The Federal Energy Regulatory Commission (FERC) has concluded that compensation for reduced electricity consumption should reflect the utility's marginal cost of supplying electricity.[3] Although such marginal-cost compensation may seem natural, many industry experts argue that it is overly generous and will induce excessive demand response. In particular, experts suggest that the unit compensation for demand response should be set equal to the difference between the utility's marginal cost of supplying electricity and the

[1] § 1252(f) of the Energy Policy Act of 2005 (Pub. L. No. 109-58, 119 STAT. 966 (2005)) states that "It is the policy of the United States that time-based pricing and other forms of demand response, whereby electricity customers are provided with electricity price signals and the ability to benefit by responding to them, shall be encouraged."

[2] The U.S. Department of Energy (2006) defines demand response to encompass "Changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized."

[3] In particular, the FERC has concluded that a "demand response resource must be compensated for the service it provides to the energy market at the market price for energy, referred to as the locational marginal price (LMP)" (FERC, 2011, ¶2).

prevailing unit retail price of electricity.[4] The reduced compensation for demand response effectively imposes on consumers the cost of purchasing electricity from the utility before allowing them to re-sell the electricity to the utility at a price equal to the utility's marginal cost of supply.[5]

Although the arguments presented by the industry experts are compelling, the arguments have not been accompanied by a fully-specified, formal model of the relevant economic environment. We provide such a model and employ it to characterize the optimal regulatory policy.

Our model provides substantial support for the experts' conclusions. We identify plausible conditions under which the optimal unit compensation for demand response is precisely the level recommended by the experts. In general, however, the optimal rate of compensation can differ from this level. A different rate of compensation is optimal, for example, when electricity production entails social losses from externalities, as it typically does in practice. An alternative rate of compensation also is optimal when society values asymmetrically the welfare of different consumer groups, e.g., consumers who can readily replace demand response with on-site generation of electricity and consumers who lack this capability.

Industry experts also have suggested that a demand response policy will play no useful role when retail prices can adjust rapidly to reflect the prevailing marginal cost of supplying electricity.[6] Our formal analysis of this issue again provides considerable support for the

[4] For example, Chao (2011, p. 78) concludes that "the optimal level of demand-response payment ... equals the difference between the wholesale price and the [retail rate] RR." Bushnell et al. (2009), Hogan (2009, 2010), Borlick (2010), Chao (2010), and Borlick et al. (2012), among others, offer corresponding conclusions.

[5] Borlick et al. (2012, p. 2) advise that "Retail customers that reduce their consumption should not be paid as if they generated the electricity they merely declined to buy. Instead, retail customers should be compensated as if they had entered into a long-term contract to purchase electricity at their retail rate but instead, during a peak demand period, resold the electricity to others at the market rate ... Simply put, the customer must be treated as if it had first purchased the power it wishes to resell to the market."

[6] To illustrate, Chao (2011, p. 79) observes that in "the special case where the [retail price of electricity] equals the wholesale price, the optimal demand response payment would be zero. Therefore, for consumers on dynamic retail pricing, there is no longer any reason to pay them for demand reduction." Chen et al. (2010) and Li et al. (2011) demonstrate the optimality of setting the price of electricity equal to its instantaneous marginal cost of production and propose an iterative algortithm to achieve the optimal outcome in the presence of limited information.

experts' conclusion, but identifies conditions under which an optimally-designed demand response policy can enhance welfare even in the absence of any restrictions on retail prices. The incremental value of a demand response policy in this setting arises because the prevailing retail price affects consumption by all consumers whereas the prevailing compensation for demand response only affects the actions of consumers who provide demand response. The ability to differentially affect the behavior of a subset of consumers can be valuable when consumers employ different technologies for on-site electricity production.

In addition to formally characterizing the optimal demand response policy, we investigate the welfare gains that an optimally designed policy can secure. We also examine the welfare losses that can arise when the FERC's marginal-cost compensation policy is implemented in place of the optimal policy. We find that the welfare gains from an optimal policy can be substantial under plausible conditions, as can the losses from the FERC's policy.

We develop and explain these findings as follows. Section 2 reviews the key elements of our model. Section 3 characterizes the optimal regulatory policy in the setting of primary interest where the retail price of electricity does not vary with the realized state of demand for electricity, where there are no distributional concerns, and where consumers cannot influence the baseline level of electricity consumption above which they are compensated for providing demand response. Section 4 identifies the changes to the optimal policy that arise when each of these restrictions is relaxed. Section 5 illustrates the welfare gains that an optimally designed demand response policy can secure and the welfare losses that arise when the FERC's marginal-cost compensation policy is implemented in place of the optimal policy. Section 6 concludes and discusses directions for further research. The proofs of all formal conclusions are presented in Appendix A. Appendix B provides both the details of the analysis that underlies the numerical solutions presented in section 5 and additional numerical solutions.

2 Model

A regulated utility produces and delivers electricity to consumers. The utility's cost of producing and delivering X units of electricity is $C(X)$, which is an increasing, convex function (so $C'(X) > 0$ and $C''(X) \geq 0$ for all $X > 0$). This cost structure reflects the utility's need to employ progressively less efficient generating units as the demand for electricity increases above the utility's baseload capacity.[7]

Consumer $i \in \{1, ..., N\}$ derives value $V_i(x_i, \theta)$ from consuming x_i units of electricity in state θ. $V_i(\cdot)$ is a strictly increasing, strictly concave function of x_i in each state. Furthermore, each consumer's total and marginal valuation of electricity increases with the state (so $\frac{\partial V_i(\cdot)}{\partial \theta} > 0$ and $\frac{\partial^2 V_i(\cdot)}{\partial \theta \partial x_i} > 0$ for all $x_i > 0$). The state might reflect the extent of temperature extremes, for example. Particularly high (low) temperatures typically increase the marginal value of electricity that is employed to power air conditioning (heating) units. The state θ is the realization of a random variable that has strictly positive support on the interval $[\,\underline{\theta}, \,\overline{\theta}\,]$, with density function $g(\theta)$ and distribution function $G(\theta)$.

Every consumer can purchase electricity from the regulated supplier. Some consumers also can produce their own electricity. They might do so, for example, by installing solar panels, or perhaps a small-scale wind or natural gas turbine, to generate electricity on-site. x_i^u will denote the amount of electricity consumer i purchases from the regulated utility. x_i^o will denote the amount of electricity consumer i generates (and consumes) himself at cost $C_i(x_i^o)$.[8] $C_i(\cdot)$ is a strictly increasing, strictly convex function.

Each consumer pays a fixed charge (R) for the right to purchase electricity from the utility. The amounts of electricity a consumer purchases and produces are assumed to be unaffected by R. In contrast, consumer i's choices of x_i^u and x_i^o are affected by the regulated unit price (r) of electricity purchased from the utility and by the prevailing compensation for demand response. Consumer i's demand response, x_i^d, is the extent to which the consumer reduces the amount of electricity he purchases from the utility below a specified baseline level, $\underline{x}_i$. Formally, $x_i^d \equiv \max\{0, \underline{x}_i - x_i^u\}$. $\underline{x}_i$ might reflect the average amount of electricity

[7] In practice, a utility's production costs may increase discontinuously at output levels where less efficient auxiliary generating units are brought on line. We assume $C(\cdot)$ is continuously differentiable for analytic tractability. This assumption does not alter our primary qualitative conclusions.

[8] We abstract from the possibility that a consumer might supply electricity to other consumers or sell electricity to the regulated utility.

consumer i has purchased from the utility historically, for example. To focus on the pricing issues of central interest, we assume initially that consumer i perceives $\underline{x}_i$ to be an exogenous parameter.[9]

$m(\theta)$ denotes the payment a consumer receives from the utility for each unit of demand response he provides in state θ. Because this compensation for demand response can vary with the state, it can be set at a relatively high level when θ is high, for example, to encourage consumers to reduce the amount of electricity they purchase from the utility when the utility's marginal cost of producing electricity is relatively high.

Electricity production can generate social losses from externalities. e_i will denote the social loss associated with each unit of electricity that consumer i produces himself.[10] The unit loss can vary across consumers because different consumers may employ different technologies to generate electricity. $e(X)$ will denote the total social loss from externalities that arises when the utility produces X units of electricity.[11]

The regulator chooses her policy instruments $\{r, R, m(\theta)\}$ to maximize expected social welfare while ensuring non-negative expected profit for the utility. Social welfare is the difference between aggregate consumer welfare and the social loss from externalities. Aggregate consumer welfare is the difference between: (i) the sum of the value that all consumers derive from their electricity consumption and the compensation they receive for the demand response they provide; and (ii) the sum of consumers' payments to the utility and the costs consumers incur in producing electricity themselves. Formally, when consumer i produces $x_i^o(\cdot, \theta)$ units of electricity and purchases $x_i^u(\cdot, \theta)$ units of electricity from the utility in state

[9] We thereby abstract initially from the possibility that, as in Chao (2009, 2011) and Chao and DePillis (2012), a consumer's choice of x_i^u in one period might affect the value of $\underline{x}_i$ that is established in future periods. Section 4 considers the possibility that consumers might be able to influence their baseline consumption levels.

[10] This linear structure for the losses from externalities due to electricity production by consumers is adopted for analytic and expositional simplicity. The key qualitative conclusions drawn below persist under non-linear structures.

[11] $e(X)$ is an increasing function. For simplicity, we abstract from the possibility that the social loss from externalities due to production by the utility might vary with the amount of electricity that consumers produce.

θ, aggregate expected consumer welfare is:

$$E\,\{\,U(\cdot)\,\} \;=\; \sum_{i=1}^{N} \int_{\underline{\theta}}^{\overline{\theta}} [\, V_i(\, x_i^u(\cdot,\theta) + x_i^o(\cdot,\theta), \theta\,) \,-\, r\; x_i^u(\cdot,\theta) \\ +\; m(\theta)\, x_i^d(\cdot,\theta) \,-\, C_i(x_i^o(\cdot,\theta))\,]\, dG(\theta) - N\,R\,. \tag{1}$$

The corresponding expected social loss from externalities is:

$$E\,\{\,L(\cdot)\,\} \;=\; \int_{\underline{\theta}}^{\overline{\theta}} \left[\, \sum_{i=1}^{N} e_i\, x_i^o(\cdot) \,+ e\,(X^u) \right] dG(\theta)\,, \tag{2}$$

where $X^u \;\equiv\; \sum_{i=1}^{N} x_i^u(\cdot)\,.$

The utility's expected profit is the difference between its expected revenues and its expected costs (which include payments to consumers for the demand response they provide). Formally:

$$E\,\{\,\pi\,\} \;=\; N\,R + \sum_{i=1}^{N} \int_{\underline{\theta}}^{\overline{\theta}} [\, r\; x_i^u(\cdot) -\; m(\theta)\, x_i^d(\cdot)\,]\, dG(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} C\,(X^u)\; dG(\theta)\,. \tag{3}$$

The regulator's formal problem, denoted [RP], is to choose r, R, and $m(\theta)$ to:

$$\text{Maximize} \quad E\,\{\,U(\cdot)\,\} - E\,\{\,L(\cdot)\,\} \quad \text{subject to} \;\; E\,\{\,\pi\,\} \;\geq\; 0\,, \tag{4}$$

where, given r, R, and $m(\theta)$, consumer i chooses $x_i^u(\cdot,\theta)$ and $x_i^o(\cdot,\theta)$ to:

$$\text{Maximize} \quad V_i(x_i^u(\cdot,\theta) + x_i^o(\cdot,\theta), \theta\,) - R - r\; x_i^u(\cdot,\theta) + m(\theta)\, x_i^d(\cdot,\theta) \,-\, C_i(x_i^o(\cdot,\theta))\,. \tag{5}$$

Ω_i^D (Ω_i^{-D}) will denote the set of $\theta \in [\,\underline{\theta},\, \overline{\theta}\,]$ realizations for which consumer i provides (does not provide) demand response at the solution to [RP].[12] To focus on the settings of primary interest, much of the ensuing analysis considers settings where the optimal regulatory policy induces some demand response.[13]

The timing in the model is the following. First, each consumer's baseline level of electricity purchase from the utility ($\underline{x}_i$) is specified. Second, the regulator sets r, R, and $m(\theta)$. Third, the state (θ) is realized. Fourth, each consumer determines how much electricity to

[12] Formally, Ω_i^D (Ω_i^{-D}) is the set of $\theta \in [\,\underline{\theta},\, \overline{\theta}\,]$ for which $\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^u}\big|_{x_i^u = \underline{x}_i} < (\geq)\; r + m(\theta)$ at the solution to [RP].

[13] Formally, unless otherwise noted, we assume $\Omega_i^D \neq \{\varnothing\}$ for some $i \in \{1, ..., N\}$.

produce himself on-site and how much to purchase from the utility. Fifth, the utility supplies all of the electricity that consumers demand, receives the associated revenue, and delivers the required payments to consumers for the demand response they provide.

3 The Optimal Demand Response Policy

Before characterizing the optimal regulatory policy, we examine how the unit compensation for demand response, $m(\theta)$, affects a consumer's actions. Lemma 1 reports that when a consumer is initially purchasing some electricity from the utility, producing some electricity himself, and providing some demand response, the consumer will reduce his purchase from the utility and increase his own production of electricity as $m(\theta)$ increases. Furthermore, due to the increasing marginal cost of self-generation, the consumer will increase his production of electricity by less than he curtails his purchases from the utility. Consequently, an increase in $m(\theta)$ induces a reduction in the sum of the consumer's purchase and production of electricity.

Lemma 1. *Suppose* $x_i^u(\cdot,\theta) > 0$, $x_i^o(\cdot,\theta) > 0$, *and* $x_i^d(\cdot,\theta) > 0$. *Then* $\frac{dx_i^u(\cdot,\theta)}{dm(\theta)} < 0$, $\frac{dx_i^o(\cdot,\theta)}{dm(\theta)} > 0$, *and* $\frac{d\left(x_i^u(\cdot,\theta)+x_i^o(\cdot,\theta)\right)}{dm(\theta)} < 0$.

Proposition 1 and Corollaries 1 and 2 now identify the key features of the optimal regulatory policy.

Proposition 1. *At the solution to [RP]:*

$$m(\theta) \;=\; C'\left(X^u\right) - r + e'\left(X^u\right) - \frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^{N} \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|}\,; \tag{6}$$

$$r \;=\; \frac{\sum\limits_{i=1}^{N} \int_{\Omega_i^{-D}} \left\{ \left[C'\left(X^u\right) + e'\left(X^u\right) \right] \frac{\partial x_i^u(\cdot)}{\partial r} + e_i \frac{\partial x_i^o(\cdot)}{\partial r} \right\} dG(\theta)}{\sum\limits_{i=1}^{N} \int_{\Omega_i^{-D}} \frac{\partial x_i^u(\cdot)}{\partial r}\, dG(\theta)}\,; \quad \textit{and} \tag{7}$$

$$R \;=\; \frac{1}{N} \left[\int_{\underline{\theta}}^{\overline{\theta}} C\left(X^u\right) dG(\theta) + \sum_{i=1}^{N} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ m(\theta)\, x_i^d(\cdot) \;-\; r\; x_i^u(\cdot) \right\} dG(\theta) \right]. \tag{8}$$

Corollary 1. *Suppose there are no losses from externalities (so $e_i = 0$ for $i = 1, ..., N$ and $e(X) = 0$ for all $X \geq 0$). Then $m(\theta) = C'(X^u) - r$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ at the solution to [RP].*

Corollary 2. *If on-site electricity production is prohibitively costly for consumers, then $m(\theta) = C'(X^u) - r + e'(X^u)$ at the solution to [RP].*

Equation (8) in Proposition 1 indicates that the optimal regulatory policy will grant the utility only the minimum expected profit required to ensure the utility's operation (i.e., $E\{\pi(\theta)\} = 0$). This conclusion reflects the regulator's concern with maximizing consumer welfare. Equation (6) in Proposition 1 characterizes the optimal unit compensation for demand response in terms of the optimal unit retail price of electricity, r^* (which is specified in equation (7)). The optimal specification of $m(\theta)$ is best understood by first abstracting from the effects of externalities.

Corollary 1 reports that in the absence of social losses from externalities, $m(\theta)$ is optimally set equal to the difference between the utility's marginal cost of production and r^*. This conclusion reflects the fact that to induce each consumer who provides demand response to act so as to maximize welfare, the effective marginal cost he incurs from purchasing electricity from the utility should be equal to the social marginal cost of electricity production by the utility in each state. A consumer's effective marginal cost of purchasing electricity from the utility is the sum of the retail price of electricity (r) and the unit compensation for demand response (m) the consumer foregoes when he decides to purchase the marginal unit of electricity from the utility rather than increase his demand response. In the absence of social losses from externalities, the social marginal cost of producing electricity is simply the utility's marginal production cost ($C'(\cdot)$). Therefore, the optimal policy equates $r + m(\theta)$ and $C'(\cdot)$ by setting $m(\theta) = C'(X^u) - r$.

Corollary 1 supports the critics of the FERC's marginal-cost compensation policy. As the critics note, the FERC's policy effectively awards to consumers the full value of a commodity (i.e., reduced electricity consumption) without first requiring them to pay anything for the commodity (since they are not required to purchase electricity at the prevailing retail price before effectively selling it to the utility). Therefore, the FERC's policy induces more than the welfare-maximizing level of demand response, *ceteris paribus*.

Proposition 1 and Corollary 2 report that when externalities are present, the optimal unit compensation for demand response is increased above $C'(X^u) - r$ by the extent to which reduced production by the utility reduces social losses from externalities. In the case where consumers do not produce electricity on-site or where such production does not generate externalities, $m(\theta)$ is optimally increased by $e'(X^u)$, the rate at which social losses from externalities decline as the utility's production of electricity declines. Equation (6) in Proposition 1 reports that, more generally, this increase in $m(\theta)$ is reduced by the extent to which reduced production by the utility increases social losses from externalities due to increased electricity production by consumers. This adjustment becomes more pronounced as e_i increases and as consumers become more likely to replace the electricity they do not purchase from the utility with electricity they produce themselves (i.e., as $\frac{\partial x_i^o(\cdot)}{\partial m(\theta)}$ increases relative to $\left|\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}\right|$).[14]

Having characterized the optimal regulatory policy, we now consider its efficacy in inducing consumers to undertake efficient purchase (x^u) and production (x^o) decisions. Given the actions of other consumers, the efficient actions (x_i^u and x_i^o) for consumer i in state θ equate relevant marginal private benefits and social costs, and so are determined by:

$$\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^u} - C'(X^u) - e'(X^u) \ \leq \ 0, \qquad x_i^u[\cdot] \ = \ 0; \quad \text{and}$$

$$\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^o} - C_i'(x_i^o) - e_i \ \leq \ 0, \qquad x_i^o[\cdot] \ = \ 0. \tag{9}$$

[14]Recall from Lemma 1 that $\frac{\partial x_i^o(\cdot)}{\partial m(\theta)} < \left|\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}\right|$ for all $i = 1, ..., N$. Therefore, $e'(X^u) - \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\left|\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}\right|} > 0$ when $e'(X) = e_i = \underline{e}$, a constant, for all $i = 1, ..., N$. Therefore, equation (6) implies that $m(\cdot)$ is optimally increased above $C'(\cdot) - r$ when the marginal social loss from externalities is constant and identical for all sources of electricity production. The increase in $m(\theta)$ serves to reduce social losses from externalities because the increase in the amount of electricity consumers produce on-site as their demand response increases is less than the amount of their demand response.

Corollary 3 first considers the relatively simple setting where the utility is the only cost-effective source of electricity.

Corollary 3. *Suppose electricity production by consumers is prohibitively costly. Then, given the actions of other consumers, the actions of each consumer who provides demand response are efficient at the solution to [RP]. In contrast, the actions of consumers who do not provide demand response generally are not efficient.*

Corollary 3 reflects the fact that when consumers do not produce electricity themselves (or when such production generates no externalities), the regulator chooses $m(\theta)$ to ensure that each consumer who provides demand response delivers the efficient level of demand response in each state. However, because the retail price structure does not vary with the state, the regulator typically cannot induce consumers who do not provide demand response to purchase the efficient level of electricity from the utility in each state.

An additional consideration arises when consumers produce electricity and this production generates losses from externalities. Consumers do not consider these losses when deciding how much electricity to produce. Consequently, because the regulator is not endowed with the ability to impose consumer-specific taxes on consumers for the electricity (and externalities) they produce, the regulator cannot induce consumers to produce the efficient levels of electricity, as Corollary 4 reports.

Corollary 4. *Suppose $x_i^o > 0$ for some consumer $i \in \{1, ..., N\}$ at the solution to [RP] identified in Proposition 1. Then, given the actions of other consumers, the actions of consumer i at the identified solution are not efficient if $e_i > 0$.*

4 Extensions

We now examine how the optimal regulatory policy changes when distributional concerns arise, when consumers can influence their baseline consumption levels, and when retail prices can vary with the realized state.

4.1 Distributional Concerns

First consider the possibility that the regulator might value differently the welfare of consumers who can provide demand response and those who cannot. For example, implementation costs may limit participation in a demand response program to large commercial and industrial consumers, and the regulator may be particularly concerned with the welfare of small residential consumers.[15] Let $\widetilde{\alpha}$ denote the weight the regulator assigns to the welfare of the $\widetilde{N}$ consumers who can provide demand response, and let $\widetilde{x}^d(\cdot)$ and $\widetilde{x}^o(\cdot)$, respectively, denote demand response and electricity production by these consumers. In addition, let $\widehat{\alpha}$ denote the weight the regulator assigns to the welfare of the $\widehat{N}$ consumers who cannot provide demand response (where $\widetilde{N} + \widehat{N} = N$). [RP-D] will denote the regulator's problem in this setting with distributional concerns.[16] Proposition 2 characterizes the optimal unit compensation for demand response in this setting.

Proposition 2. *At the solution to [RP-D], given the optimal unit retail price r:*[17]

$$\begin{aligned} m(\theta) \;&=\; C'(\cdot) - r + \left[\frac{\widetilde{N} + \widehat{N}}{\widetilde{\alpha}\, N + \widehat{\alpha}\, \widehat{N}} \right] e'(\cdot) \\ &\quad - \frac{\left[\widetilde{N} + \widehat{N} \right] \sum_{i=1}^{N} e_i \, \frac{\partial x_i^o}{\partial m(\theta)} + \widehat{N} \left[\widehat{\alpha} - \alpha \right] \sum_{i=1}^{N} x_i^d(\cdot)}{\left[\widetilde{\alpha}\, \widetilde{N} + \widehat{\alpha}\, \widehat{N} \right] \sum_{i=1}^{N} \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|} \, . \end{aligned} \tag{10}$$

Therefore, if there are no losses from externalities (so $e_i = 0$ for $i = 1, ..., N$ and $e(X) = 0$ for all $X \geq 0$):

$$m(\theta) \;=\; C'(\cdot) - r - \frac{\widehat{N} \left[\widehat{\alpha} - \widetilde{\alpha} \right] \sum_{i=1}^{N} x_i^d(\cdot)}{\left[\widetilde{\alpha}\, \widetilde{N} + \widehat{\alpha}\, \widehat{N} \right] \sum_{i=1}^{N} \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|} \, . \tag{11}$$

[15] Borlick (2011) notes that the marginal-cost compensation for demand response advised by the FERC requires consumers who do not provide demand response to subsidize those who do.

[16] The regulator seeks to maximize the relevant weighted average of the expected welfare of the two types of consumers while ensuring non-negative profit for the regulated utility. The proof of Proposition 2 includes a formal statement of [RP-D].

[17] The argument of $C'(\cdot)$ and $e'(\cdot)$ in the statement of Proposition 2 is the total amount of electricity purchased from the utility.

Proposition 2 indicates that, *ceteris paribus*, the regulator will reduce the compensation for demand response when she values relatively highly the welfare of consumers who cannot provide demand response (i.e., when $\widehat{\alpha} > \widetilde{\alpha}$). Although the reduced compensation induces less than the (unweighted) surplus-maximizing level of demand response, it permits reductions in the charges (r and R) imposed on consumers who do not provide demand response. Equation (11) indicates that, *ceteris paribus*, the reduction in $m(\theta)$ tends to be more pronounced as: (i) $\widehat{\alpha}$ increases, so the regulator values more highly the welfare of consumers who cannot provide demand response; (ii) $\widehat{N}$ increases, so there are more consumers who cannot provide demand response; (iii) $\sum_{i=1}^{N} x_i^d(\cdot)$ increases, so the magnitude of the equilibrium demand response increases; and (iv) $\sum_{i=1}^{N} \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|$ declines, so a reduction in $m(\theta)$ causes a smaller increase in the demand for electricity from the utility (and an associated smaller increase in the utility's marginal cost of production).

4.2 Endogenous Baseline Consumption Levels

Now consider the possibility that consumer i might undertake action a_i at personal cost $D_i(a_i)$ to increase his baseline consumption level, $\underline{x}_i$. For example, as Chao (2011) and Chao and DePillis (2012) posit, a consumer might purchase more than the level of electricity that maximizes his contemporary welfare in early periods, recognizing that doing so will increase his baseline consumption level in later periods.[18] We assume $\underline{x}_i$ is an increasing, concave function of a_i and $D_i(\cdot)$ is a strictly increasing, strictly convex function for all $i = 1, ..., N$.[19]

The regulator first specifies $\{R, r, m(\theta)\}$ and the rule that will be employed to establish baseline consumption levels. Consumers then choose their actions to influence their baseline consumption levels. Finally, consumers determine how much electricity they will purchase from the utility and how much electricity they will produce themselves. The regulator seeks

[18] Chao (2011, p. 75) observes that "a customer can artificially increase its consumption during "normal" consumption periods to create a higher baseline in order to collect demand reduction payments without actually reducing load."

[19] We further assume that, for all $i = 1, ..., N$, consumer i's expected welfare is a strictly concave function of a_i and consumer i chooses $a_i > 0$.

to maximize aggregate expected consumer welfare while ensuring non-negative expected profit for the utility.[20]

Let [RP-a] denote the regulator's formal problem in this setting.[21] Also let $\delta_{i\theta} = 1$ if $\theta \in \Omega_i^{Da}$ and $\delta_{i\theta} = 0$ otherwise, where Ω_i^{Da} is the set of $[\underline{\theta}, \overline{\theta}]$ for which consumer i provides demand response at the solution to [RP-a]. For expositional ease, Proposition 3 characterizes the optimal compensation for demand response in this setting for the case where $\Omega_i^{Da} \neq \{\varnothing\}$ for each $i = 1, ..., N$.

Proposition 3. *At the solution to [RP-a]:*

$$m(\theta) = \frac{C'(X^u) - r + e'(X^u) - \left[\sum_{i=1}^{N} e_i \frac{dx_i^o(\cdot)}{dm(\theta)} \right] / \sum_{i=1}^{N} \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|}{\left[\sum_{i=1}^{N} \left\{ \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + \delta_{i\theta} \frac{\partial \underline{x}_i}{\partial a_i} \frac{\partial a_i}{\partial m(\theta)} \right\} \right] / \sum_{i=1}^{N} \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|} . \tag{12}$$

It is readily shown that an increase in $m(\theta)$ induces consumers who provide demand response to devote more effort to increasing their baseline consumption levels (so $\frac{\partial a_i}{\partial m(\theta)} > 0$ for all $i = 1, ..., N$) at the solution to [RP-a].[22] Therefore, the denominator of the fraction in equation (12) exceeds 1. Consequently, Propositions 1 and 3 indicate that, *ceteris paribus*, the optimal compensation for demand response is scaled down systematically when consumers can influence their baseline consumption levels. The reduction in $m(\theta)$ limits incentives to artificially inflate baseline consumption, but leads to distortions where they otherwise would not arise, as Corollary 5 reports.

Corollary 5. *Given the actions of other consumers, the actions of a consumer who provides demand response generally are not efficient at the solution to [RP-a], even if electricity production by consumers generates no social losses from externalities.*

[20] Consumer i's welfare now includes both the personal cost of action a_i and the impact of this action on $\underline{x}_i$.

[21] The proof of Proposition 3 includes a formal statement of [RP-a].

[22] See the proof of Proposition 3.

4.3 State-Specific Pricing

Now suppose the regulator can set a state-specific unit retail price, $r(\theta)$, in addition to R and $m(\theta)$. Let [RP-s] denote the regulator's formal problem in this setting, where she seeks to maximize aggregate expected consumer welfare while ensuring non-negative expected profit for the utility. Proposition 4 identifies conditions under which a demand response policy admits no strict welfare gains in this setting.

Proposition 4. *At the solution to [RP-s], $r(\theta) = C'(X^u) + e'(X^u)$ and $m(\theta) = 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ if: (i) no consumer produces electricity (so $x_i^o = 0$ for all $i = 1, ..., N$); (ii) consumer production of electricity entails no externalities (so $e_i = 0$ for $i = 1, ..., N$); or (iii) all consumers provide demand response in all states (so $x_i^d(\cdot) > 0$ for all $i = 1, ...N$ and for all $\theta \in [\underline{\theta}, \overline{\theta}]$).*

Proposition 4 indicates that when the regulator sets the optimal state-specific retail prices for electricity, a demand response policy will not enhance welfare if consumers do not produce electricity on-site or if such production entails no externalities. Under these conditions, the regulator can maximize surplus by setting the retail price of electricity equal to its social marginal cost of production in each state. Consequently, non-zero compensation for demand response would only reduce expected welfare in this event.[23]

The same is true when all consumers provide demand response in every state. In this case, an increase in $r(\theta)$ has the same impact as an increase in $m(\theta)$ on each consumer's electricity purchase and production decisions. Consequently, a demand response policy offers no strict welfare gains when the regulator sets the optimal state-specific retail prices for electricity.

In contrast, identical changes in $r(\theta)$ and $m(\theta)$ do not affect symmetrically the actions of all consumers who produce electricity on-site when only some of them provide demand response. Therefore, as Corollary 6 indicates, the regulator optimally increases $m(\theta)$ above

[23] Chao (2011, p. 79) observes that "In the special case where the [retail price of electricity] equals the wholesale price, the optimal demand response payment would be zero. Therefore, for consumers on dynamic retail pricing, there is no longer any reason to pay then for demand reduction."

0 in states where, relative to corresponding effects on the demand for electricity from the utility, an increase in $r(\theta)$ increases losses from externalities due to increased electricity production by consumers more rapidly than does an increase in $m(\theta)$. The increase in $m(\theta)$ permits a less pronounced increase in electricity (and externality) production by consumers than would an increase in $r(\theta)$.

Corollary 6. *Suppose $x_i^o(\cdot) > 0$ for some consumers and $x_i^d(\cdot) > 0$ for some, but not all, consumers at the solution to [RP]. Then:*

$$m(\theta) \gtreqless 0 \quad as \quad \frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N} \left| \frac{\partial x_i^u(\cdot)}{\partial r(\theta)} \right|} \gtreqless \frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^{N} \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|} \quad \text{at the solution to [RP].} \tag{13}$$

As is the case in other settings, the regulator's inability to impose consumer-specific taxes on on-site electricity (and externality) production in the present setting often precludes her from inducing efficient actions, as Corollary 7 reports.

Corollary 7. *Suppose $x_i^o > 0$ for some consumer $i \in \{1, ..., N\}$ at the solution to [RP-s] identified in Proposition 4. Then, given the actions of other consumers, the actions of consumer i at the identified solution: (i) are efficient if electricity production by consumers entails no externalities; and (iii) are not efficient if $e_i > 0$.*

Corollary 7 implies that when consumers produce electricity and generate social losses from externalities in doing so, the optimal regulatory policy generally does not induce efficient actions even when the regulator can set state-specific retail prices.

5 Welfare Gains and Losses

We now illustrate the welfare gains that can arise when an optimally designed demand response policy is implemented. We also illustrate the welfare losses that can arise when compensation for demand response is instead set equal to the utility's marginal cost of producing electricity. To do so, we consider the following *benchmark setting* in which the utility is the only producer of electricity and production entails no losses from externalities.

The utility's cost of producing X units of electricity is $C(X) = F + a\,X + b\,X^2$, where a, b, and F are nonnegative constants.

There are N_H identical "H consumers" and N_L identical "L consumers." The former (e.g., non-residential consumers) value electricity more highly than do the latter (e.g., residential consumers). Each i ($\in \{L, H\}$) consumer derives value $V_i(x_i, \theta) = v_i \left[\frac{\theta (x_i)^{1+\alpha_i} - \overline{V}_i}{1+\alpha_i} \right]$ from x_i units of electricity in state θ, where $\overline{V}_i \geq 0$ is a constant. v_L is normalized to 1 and v_H is set equal to 1.88, reflecting the estimated relative values of lost load for residential and non-residential electricity consumers (London Economics, 2013). We set $\frac{1}{\alpha_L} = -0.15$ and $\frac{1}{\alpha_H} = -0.20$, reflecting common estimates of the short-run price elasticity demand for electricity for residential and non-residential customers, respectively.[24]

The demand parameter θ reflects the extent to which the daily high temperature ($\overline{T}$) exceeds an upper threshold ($78^o\,F$) and the daily low temperature ($\underline{T}$) falls below a lower threshold ($65^o\,F$) in our sample. Thus, higher values of θ typically will be associated with increased demand for electricity for cooling and heating. Formally, $\theta = 1 + \max\{\,0,\, \overline{T} - 78\,\} + \max\{\,0,\, 65 - \underline{T}\,\}$. Our sample consists of the daily temperature realizations in 2013 in all states in the PJM Interconnection region.[25,26] (Appendix B presents the results of corresponding analyses that reflect conditions in the California and ISO New England regions.)[27] $\theta \in [\,0, 70\,]$ in this sample, and maximum likelihood estimation reveals that the distribution of θ is well-approximated by a gamma distribution with scale parameter 3.064 and shape

[24] See, for example, King and Chatterjee (2003), Espey and Espey (2004), Narayan and Smyth (2005), Taylor et al. (2005), Wade (2005), Bernstein and Griffin (2006), and Paul et al. (2009). It is readily verified that consumer i's price elasticity of demand for electricity in the setting is $\frac{1}{\alpha_i}$.

[25] PJM Interconnection is the "regional transmission organization (RTO) that coordinates the movement of wholesale electricity in all or parts of Delaware, Illinois, Indiana, Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia and the District of Columbia" (www.pjm.com/about-pjm/who-we-are.aspx).

[26] The temperature data are drawn from the National Oceanic and Atmospheric Administration (2014).

[27] ISO New England is "the independent, not-for-profit corporation responsible for keeping electricity flowing across the six New England states and ensuring that the region has reliable, competitively priced wholesale electricity" (www.iso-ne.com/about). We investigate potential outcomes in the California, ISO New England, and PJM Interconnection regions because Busnell (2007) provides estimates of the cost parameters a and b in these three regions. We focus on outcomes in the PJM Interconection region in the text for brevity and because this region is the largest and the most populous of the three regions.

parameter 8.021.[28]

$\underline{x}_i$ is the amount of electricity an i consumer would purchase in this benchmark setting under the optimal regulatory policy in the absence of any demand response (DR) program.[29] $N_L + N_H$ is set to ensure that expected demand is equal to the average hourly load in the PJM Interconnection region in 2013.[30] $\frac{N_L}{N_L+N_H}$ is set equal to 0.879, the fraction of U.S. electricity customers classified as residential customers in the PJM Interconnection region in 2012 (Energy Information Administration, 2014a).

The utility's fixed cost of production (F) is taken to be \$39,252,470. This number reflects the 46% of revenue collected annually from ratepayers in the PJM Interconnection region that is estimated to be employed to cover the fixed costs of installing generation capacity and maintaining and upgrading the region's transmission and distribution network.[31] The remaining cost parameters are set at $a = 0.0$ and $b = 0.00045$, the parameter values that Bushnell (2007) estimates for this region.

Table 1 reports outcomes in this benchmark setting: (i) in the absence of any DR policy (so $m(\theta) = 0$ for all θ); (ii) under the optimal marginal-cost compensation ("FERC") policy (where $m(\theta) = C'(\cdot)$ for all θ); and (iii) under the optimal DR policy (i.e., at the solution to [RP]). The first two rows of data in Table 1 report the unit price of electricity (r) and the associated fixed charge (R).[32] The third row presents the expected DR com-

[28] The data reveal that the distribution of θ is also approximated reasonably well by a generalized extreme value (GEV) distribution with parameters $(\mu,\ \sigma,\ \xi) = (18.460,\ 10.928,\ -0.029)$. The key qualitative conclusions reported below are unchanged when this GEV distribution is employed instead of the identified gamma distribution.

[29] The optimal regulatory policy in the absence of a demand response policy is characterized in Lemma B1 in Appendix B.

[30] This average hourly load, 90,314 megawatts, is total annual consumption (791,152,262 megawatt hours) in the PJM Interconnection region in 2013 divided by 8,760, the number of hours in a year (PJM, 2014).

[31] ISO-NE (2006) and Thomas et al. (2014) estimate that variable energy production costs constitute between 48% and 60% (an average of 54%) of ratepayer revenue. Revenue is calculated as the product of the average retail rate for electricity and the total load in the PJM Interconnection region in 2013 (PJM, 2014).

[32] r is reported in dollars per megawatt hour. Thus, $r = 83.19$ denotes a price of approximately \$0.083 per kilowatt hour. R is reported in dollars per year. Thus, $R = 299.96$ represents a monthly fixed charge of approximately \$25.

pensation payment ($E\{m(\theta)\}$).[33] The fourth row reports expected peak-load production costs ($E\{C^P(\cdot)\}$), which are the utility's expected costs (in millions of dollars) in states in which strictly positive demand response arises.[34] The last row presents the level of aggregate expected consumer welfare ($E\{W\}$) in millions of dollars.[35]

	No DR Policy	Optimal DR Policy	FERC DR Policy
r	83.19	75.20	78.10
R	299.96	323.36	307.57
$E\{m(\theta)\}$	0	21.29	86.28
$E\{C^P(\cdot)\}$	8.49	6.99	5.13
$E\{W\}$	29.16	34.23	30.27

Table 1. Outcomes in the Benchmark Setting.

Table 1 reveals that the optimal DR policy in the benchmark setting increases welfare by 17.4% above the corresponding level achieved in the absence of any DR policy.[36] The welfare gain reflects in part the 17.6% reduction in expected peak-load production costs the optimal DR policy secures.[37] The cost reductions, in turn, permit a lower unit price for electricity. Consumers also benefit from the compensation they receive for their demand response, which nearly offsets the increase in the fixed charge.

The optimal DR policy increases expected welfare by 13.1% above the level secured under the optimal FERC policy. This welfare increase arises even though the optimal FERC

[33] $E\{m(\theta)\} = \int_{\theta_m}^{\overline{\theta}} m(\theta)\, dG(\theta)$, where $\theta_m = 42.5$ is the smallest realization of θ for which demand response is provided both at the solution to [RP] and under the optimal FERC policy in the benchmark setting. The qualitative conclusions drawn below are robust to alternative plausible definitions of peak-load production costs.

[34] Formally, $E\{C^P(\theta)\} = \int_{\theta_m}^{\overline{\theta}} C(\cdot)\, dG(\theta)$.

[35] $E\{W\} = \sum_{i=1}^{N} \int_{\underline{\theta}}^{\overline{\theta}} [\, V_i(\, x_i^u(\cdot,\theta), \theta\,) - r\, x_i^u(\cdot,\theta) + m(\theta)\, x_i^d(\cdot,\theta)\,]\, dG(\theta) - N R$, reflecting equation (1).

[36] Larger percentage increases in expected welfare arise in the settings analyzed in Appendix B.

[37] Reported percentage changes may not reflect the entries in Table 1 exactly because these entries are rounded.

policy reduces expected peak-load production costs by 26.6% below the corresponding costs under the optimal DR policy. The optimal FERC policy reduces electricity consumption excessively, causing the value that consumers derive from consuming electricity to decline by more than the corresponding reduction in production costs.

The welfare gains secured under an optimal DR policy typically increase as the convexity of the utility's cost function increases. The enhanced gains arise because the expected cost savings from curtailing peak-load consumption become more pronounced as the utility's marginal cost increases more rapidly with output. To illustrate this more general conclusion, Table 2 reports the levels of expected welfare that arise as b increases and decreases by 10%, 20%, and 30% above and below its value (0.00045) in the benchmark setting.[38] The table reveals, for example, that when b increases by 20% (from 0.00045 to 0.00054), the increase in expected welfare secured under the optimal DR policy (relative to the welfare secured in the absence of any DR policy) increases from 17.4% to 23.9%.[39] In contrast, a 20% reduction in b (from 0.00045 to 0.00036) reduces this gain in expected welfare from 17.4% to 15.4%.

[38] All other parameter values are held constant at their levels in the benchmark setting.

[39] Systematic increases in the marginal cost of production (i.e., increases in a) also enhance the welfare gains generated by an optimal DR policy. To illustrate, suppose a increases from 0 to 20, while all other parameters are held constant at their levels in the benchmark setting. (The average value of a in the settings considered in Appendix B is approximately 23.). The increase in expected welfare that the optimal DR policy generates in this case (relative to no DR policy) rises to 33.6% (from the 17.4% generated in the benchmark setting). Bushnell's (2007) estimate of $a = 0$ in the PJM region reflects in part substantial supply by nuclear generators. Some of these generators are scheduled for retirement in the near future, which will tend to increase a. However, increased supply of energy from renewable sources may reduce a.

b	No DR Policy	Optimal DR Policy	FERC DR Policy
0.000585	3.10	3.89	3.50
0.000540	14.29	17.71	15.78
0.000495	14.88	18.22	16.20
0.000450	29.16	34.23	30.27
0.000405	29.90	34.78	30.42
0.000360	31.64	36.52	31.16
0.000315	40.25	45.84	39.35

Table 2. Expected Welfare as b Changes.

When the utility's marginal cost of production increases sufficiently slowly with output, even an optimally designed FERC policy can reduce welfare below the level achieved in the absence of any DR policy.[40] This conclusion is illustrated in the last two rows of data in Table 2. These data indicate that when b declines by 20% or 30% below its level in the benchmark setting, the excessive demand reduction the FERC policy induces reduces the value that consumers derive from consuming electricity by more than it reduces peak-load production costs.

6 Conclusions

We have characterized the optimal demand response policy in several settings, including settings with a fixed retail price for electricity and settings where the retail price can fully reflect the utility's marginal cost of production. Our findings generally support industry experts who advocate compensation for demand response that reflects the difference between the utility's marginal cost of supplying electricity and the prevailing retail price of electric-

[40] A value of b substantially below Bushnell's (2007) estimate might arise, for example, from pronounced reductions in the price of natural gas, which often is employed to power peak-load production units. The U.S. experienced sharp reductions in the price of natural gas between 2007 and 2009 (www.infomine.com/investment/metal-prices/natural-gas/all/). The ongoing replacement of (low cost) coal generation by natural gas generation in the PJM region can introduce a countervailing effect on b.

ity. However, the optimal policy typically entails further adjustments to account for the externalities associated with electricity production. The optimal policy generally is not the marginal-cost compensation policy advocated by the FERC.

We have shown that the optimal demand response policy can secure substantial increases in expected welfare under plausible conditions. The FERC's demand response policy often generates a significantly smaller increase in welfare, and can even reduce welfare below the level that arises in the absence of any demand response policy. Therefore, the expressed concerns about the FERC's policy would seem to merit serious consideration.

Our illustrations of the performance of the optimal demand response policy and the FERC's policy did not account explicitly for losses from externalities associated with electricity production.[41] A full accounting for these losses could alter the relative performance of the FERC's demand response policy. Observe from Proposition 1 that, *ceteris paribus*, the difference between the marginal compensation under the FERC's policy and the corresponding optimal compensation declines as the marginal social loss from externalities associated with electricity production by the utility increases, after adjusting for relevant social losses from externalities associated with increased electricity production by consumers. Accurate estimation of social losses from externalities requires detailed knowledge of the particular technologies being employed to generate electricity at all relevant output levels. Such estimation and development of the associated implications for the relative performance of different demand response policies await further research.

In closing, we note two additional extensions of our analysis that merit further research. First, the optimal demand response policy should be characterized in settings where the retail price of electricity partially reflects the utility's marginal cost of production, e.g., in the presence of time-of-day pricing. Our findings in the settings with a fixed retail price

[41] A utility's marginal cost of supplying electricity can reflect relevant social losses from externalities. This will be the case if the utility (or the competitive suppliers from which the utility procures electricity) faces costs that reflect relevant social losses from externalities. These costs might take the form of emissions taxes or the costs or emission permits, for example. Fabra and Reguant (2014) find that a large fraction of emissions costs are passed on to consumers in the form of higher retail prices for electricity.

and fully state-specific retail pricing (recall Propositions 1 and 4) suggest that the optimal compensation for demand response will continue to reflect differences between the utility's marginal cost of production and the prevailing retail price of electricity. We conjecture that our key qualitative conclusions also will persist in settings with restructured electricity markets, where the utility secures electricity from non-affiliated generators.

Second, additional policy instruments warrant consideration. The optimal design of a demand response policy is best viewed as an element of a broader exercise that includes, for example, the optimal design of distributed generation, energy conservation, and renewable energy portfolio policies. The key qualitative conclusions drawn above seem likely to persist in the context of this more general analysis, but the details of the analysis remain to be determined.

Appendix A. Proofs of the Formal Conclusions

Proof of Lemma 1.

Given θ, consumer i chooses x_i^u and x_i^o to maximize:

$$M \;\equiv\; V_i(x_i^u + x_i^o, \theta) - R - r\; x_i^u + m(\theta)\; x_i^d \;-\; C_i(x_i^o) \;\;\text{where}\;\; x_i^d \;\equiv\; \max\{\, 0,\, \underline{x}_i - x_i^u \,\}\,. \tag{14}$$

Therefore, at an interior optimum:

$$\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^u} \;=\; \begin{cases} r + m(\theta) & \text{if } \left.\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^u}\right|_{x_i^u = \underline{x}_i} \;<\; r + m(\theta) \\ r & \text{otherwise}\,; \end{cases}$$

$$\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^o} \;\leq\; C_i^{'}(x_i^o) \quad\text{and}\quad x_i^o\left[\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^o} - C_i^{'}(x_i^o)\right] \;=\; 0\,. \tag{15}$$

Define $M_j \;\equiv\; \frac{\partial M}{\partial j}$ and $M_{jl} \;\equiv\; \frac{\partial^2 M}{\partial j \partial l}$. Then (15) implies that when consumer i chooses $x_i^u \;>\; 0$ and $x_i^o \;>\; 0$, these choices are characterized by:

$$M_{x_i^u} \;=\; \frac{\partial V_i(\cdot)}{\partial x_i^u} - r + m(\theta)\,\frac{\partial x_i^d}{\partial x_i^u} \;=\; 0 \;\;\text{and}\;\; M_{x_i^o} \;=\; \frac{\partial V_i(\cdot)}{\partial x_i^o} - C_i'(x_i^o) \;=\; 0$$

$$\Rightarrow\quad \left[\widehat{M}\right]\begin{bmatrix} dx_i^u \\ dx_i^o \end{bmatrix} \;=\; \begin{bmatrix} -\frac{\partial x_i^d}{\partial x_i^u} \\ 0 \end{bmatrix} dm(\theta)\,,\quad\text{where}\;\; \left[\widehat{M}\right] \;\equiv\; \begin{bmatrix} M_{x_i^u x_i^u} & M_{x_i^u x_i^o} \\ M_{x_i^o x_i^u} & M_{x_i^o x_i^o} \end{bmatrix},$$

$$M_{x_i^u x_i^u} \;=\; \frac{\partial^2 V_i(\cdot)}{\partial\,(x_i^u)^2} \;=\; \frac{\partial^2 V_i(\cdot)}{\partial x_i^u \partial x_i^o} \;=\; M_{x_i^u x_i^o} \;=\; M_{x_i^o x_i^u},\;\text{and}\; M_{x_i^o x_i^o} \;=\; \frac{\partial^2 V_i(\cdot)}{\partial\,(x_i^o)^2} - C_i''(x_i^o)\,. \tag{16}$$

(16) implies:

$$\left|\widehat{M}\right| \;=\; \frac{\partial^2 V_i(\cdot)}{\partial\,(x_i^u)^2}\left[\frac{\partial^2 V_i(\cdot)}{\partial\,(x_i^o)^2} - C_i''(x_i^o)\right] - \left[\frac{\partial^2 V_i(\cdot)}{\partial\,(x_i^u)^2}\right]^2 \;=\; -\,C_i''(x_i^o)\,\frac{\partial^2 V_i(\cdot)}{\partial\,(x_i^u)^2} \;>\; 0\,. \tag{17}$$

Since $\frac{\partial x_i^d}{\partial x_i^u} \;=\; -1$ when $x_i^d \;>\; 0$, (16) and (17) imply that when $x_i^d \;>\; 0$:

$$\frac{dx_i^u}{dm(\theta)} \;=\; \frac{1}{\left|\widehat{M}\right|}\begin{vmatrix} 1 & M_{x_i^u x_i^o} \\ 0 & M_{x_i^o x_i^o} \end{vmatrix} \;=\; \frac{\frac{\partial^2 V_i(\cdot)}{\partial (x_i^o)^2} - C_i''(x_i^o)}{-\,C_i''(x_i^o)\,\frac{\partial^2 V_i(\cdot)}{\partial (x_i^u)^2}} \;<\; 0,\;\;\text{and} \tag{18}$$

$$\frac{dx_i^o}{dm(\theta)} \;=\; \frac{1}{\left|\widehat{M}\right|}\begin{vmatrix} M_{x_i^u x_i^u} & 1 \\ M_{x_i^o x_i^u} & 0 \end{vmatrix} \;=\; \frac{-\,\frac{\partial^2 V_i(\cdot)}{\partial (x_i^u)^2}}{-\,C_i''(x_i^o)\,\frac{\partial^2 V_i(\cdot)}{\partial (x_i^u)^2}} \;=\; \frac{1}{C_i''(x_i^o)} \;>\; 0\,. \tag{19}$$

(18) and (19) imply:

$$\frac{d\,(x_i^u + x_i^o)}{dm(\theta)} \;=\; \frac{\frac{\partial^2 V_i(\cdot)}{\partial (x_i^o)^2} - C_i''(x_i^o) - \frac{\partial^2 V_i(\cdot)}{\partial (x_i^u)^2}}{-\,C_i''(x_i^o)\,\frac{\partial^2 V_i(\cdot)}{\partial (x_i^u)^2}} \;=\; \frac{1}{\frac{\partial^2 V_i(\cdot)}{\partial (x_i^u)^2}} \;<\; 0\,. \quad \blacksquare \tag{20}$$

<u>Proof of Proposition 1 and Corollaries 1 and 2</u>.

Let $\lambda \geq 0$ denote the Lagrange multiplier associated with the utility's participation constraint $(E\{\pi\} \geq 0)$. Then the Lagrangian function associated with [RP] is:

$$\pounds \;=\; E\{\,U(\cdot)\,\} - E\,\{\,L(\cdot)\,\} + \lambda\; E\{\,\pi\,\}\;. \tag{21}$$

Pointwise optimization of (21) with respect to $m(\theta)$, using (1), (2), (3), and the envelope theorem provides:

$$\begin{aligned} &[\,1-\lambda\,] \sum_{i=1}^{N} x_i^d(r, m(\theta), \theta)\; g(\theta) - e'\,(X^u) \sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}\, g(\theta) - \sum_{i=1}^{N} e_i\, \frac{\partial x_i^o}{\partial m(\theta)}\; g(\theta) \\ &-\;\; \lambda\, C'\,(X^u) \sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}\; g(\theta) + \lambda \sum_{i=1}^{N} \left[\, r\, \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} - m(\theta)\, \frac{\partial x_i^d(\cdot)}{\partial m(\theta)} \right] g(\theta) \;=\; 0\,. \end{aligned} \tag{22}$$

Since the value of R does not affect consumption decisions, differentiating (21) with respect to R, using (1), (2), and (3), provides $-N + \lambda\,N \;=\; 0 \;\Rightarrow\; \lambda \;=\; 1$. Also, $\frac{\partial x_i^d(\cdot)}{\partial m(\theta)} = -\,\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}$ since $\frac{\partial x_i^u(\cdot)}{\partial m(\theta)} = 0$ if $x_i^u(\cdot) > \underline{x}_i$. Therefore, (22) can be written as:

$$[\,r + m(\theta) - e'\,(X^u) - C'\,(X^u)\,] \sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \;-\; \sum_{i=1}^{N} e_i\, \frac{\partial x_i^o}{\partial m(\theta)} \;=\; 0\,. \tag{23}$$

$\sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \;<\; 0$ because $\frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \;<\; 0$ when $x_i^d(\cdot) \;>\; 0$ and $\frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \;\leq\; 0$ when $x_i^d(\cdot) \;=\; 0$. Therefore, (6) follows from (23). The Corollaries follow immediately from (6).

Let $\Omega_i^=$ denote the set of $\theta \in [\,\underline{\theta},\, \overline{\theta}\,]$ for which $\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^u}|_{x_i^u = \underline{x}_i} \;=\; r + m(\theta)$ at the solution to [RP]. Observe that:

$$\begin{aligned} &V_i(\,x_i^u(r, m(\theta), \theta) + x_i^o(\cdot), \theta) \,-\, r\; x_i^u(r, m(\theta), \theta) + \;m(\theta)\,[\,\underline{x}_i - x_i^u(r, m(\theta), \theta)\,] \\ &\qquad\qquad \;=\; V_i(\,x_i^u(r, \theta) + x_i^o(\cdot), \theta) \,-\, r\; x_i^u(r, \theta) \;\text{ for all }\; \theta \,\in\, \Omega_i^=\,. \end{aligned} \tag{24}$$

Further observe that (1) can be written as:

$$E\{U(\cdot)\} \;=\; \int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} w_i(\theta)\, d\,G(\theta) - N\,R \;\;\text{where}\;\; w_i(\theta) \;\equiv\; \begin{cases} w_i^D(\theta) & \text{if } \theta \in \Omega_i^D \\ w_i^{-D}(\theta) & \text{if } \theta \in \Omega_i^{-D}, \end{cases}$$

$$w_i^D(\theta) \;\equiv\; V_i(\,x_i^u(r, m(\theta), \theta) + x_i^o(\cdot), \theta) \,-\, r\, x_i^u(r, m(\theta), \theta)$$
$$+\;\; m(\theta)\,[\,\underline{x}_i - x_i^u(r, m(\theta), \theta)\,] - C_i(x_i^o(\cdot)), \;\;\text{and}$$

$$w_i^{-D}(\theta) \;\equiv\; V_i(\,x_i^u(r, \theta) + x_i^o(\cdot), \theta) \,-\, r\, x_i^u(r, \theta) \,-\, C_i(x_i^o(\cdot)). \tag{25}$$

(24) implies that for any $\widehat{\theta} \in \Omega_i^{=}$:

$$\lim_{\theta \to \widehat{\theta}^-} \sum_{i=1}^{N} w_i^D(\theta) \;=\; \lim_{\theta \to \widehat{\theta}^+} \sum_{i=1}^{N} w_i^{-D}(\theta) \quad \text{and} \quad \lim_{\theta \to \widehat{\theta}^-} \sum_{i=1}^{N} w_i^{-D}(\theta) \;=\; \lim_{\theta \to \widehat{\theta}^+} \sum_{i=1}^{N} w_i^D(\theta)\,.$$

Consequently, $\sum_{i=1}^{N} w_i(\theta)$ is continuous in θ for all θ.

Similarly, from (3):

$$E\,\{\,\pi\,\} \;=\; N\,R + \int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} \widetilde{\pi}_i(\theta)\, d\,G(\theta) -\, F$$

$$\text{where}\;\; \widetilde{\pi}_i(\theta) \;\equiv\; \begin{cases} \widetilde{\pi}_i^D(\theta) - C\left(\sum_{j=1}^{N} x_j^u(\cdot)\right) & \text{if } \theta \in \Omega_i^D \\ \widetilde{\pi}_i^{-D}(\theta) - C\left(\sum_{j=1}^{N} x_j^u(\cdot)\right) & \text{if } \theta \in \Omega_i^{-D}, \end{cases}$$

$$\widetilde{\pi}_i^D(\theta) \;\equiv\; r\, x_i^u(r, m(\theta), \theta) - m(\theta)\;[\,\underline{x}_i - x_i^u(r, m(\theta), \theta)\,]\,, \text{ and } \;\widetilde{\pi}_i^{-D}(\theta) \;\equiv\; r\, x_i^u(r, \theta)\,. \tag{26}$$

(24) implies that for any $\widehat{\theta} \in \Omega_i^{=}$:

$$\lim_{\theta \to \widehat{\theta}^-} \sum_{i=1}^{N} \widetilde{\pi}_i^D(\theta) \;=\; \lim_{\theta \to \widehat{\theta}^+} \sum_{i=1}^{N} \widetilde{\pi}_i^{-D}(\theta) \quad \text{and} \quad \lim_{\theta \to \widehat{\theta}^-} \sum_{i=1}^{N} \widetilde{\pi}_i^{-D}(\theta) \;=\; \lim_{\theta \to \widehat{\theta}^+} \sum_{i=1}^{N} \widetilde{\pi}_i^D(\theta)\,.$$

Consequently, $\sum_{i=1}^{N} \widetilde{\pi}_i(\theta)$ is continuous in θ for all θ.

The established continuity and Leibnitz' rule, along with (2), (25), and (26), ensure that differentiation of (21) with respect to r provides:

$$-\;\sum_{i=1}^{N}\;\int_{\Omega_i^D} x_i^u(r, m(\theta), \theta)\, d\,G(\theta) - \sum_{i=1}^{N}\;\int_{\Omega_i^{-D}} x_i^u(r, \theta)\, d\,G(\theta)$$

$$
- \int_{\underline{\theta}}^{\overline{\theta}} \left[\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial r} + e'(X^u) \sum_{j=1}^{N} \frac{\partial x_j^u(\cdot)}{\partial r} \right] dG(\theta)
$$

$$
+ \lambda \sum_{i=1}^{N} \int_{\Omega_i^D} \left[r \frac{\partial x_i^u(\cdot)}{\partial r} + x_i^u(r, m(\theta), \theta) + m(\theta) \frac{\partial x_i^u(\cdot)}{dr} \right] dG(\theta)
$$

$$
+ \lambda \sum_{i=1}^{N} \int_{\Omega_i^{-D}} \left[r \frac{\partial x_i^u(\cdot)}{dr} + x_i^u(r, \theta) \right] dG(\theta) - \lambda \int_{\underline{\theta}}^{\overline{\theta}} C'(X^u) \sum_{j=1}^{N} \frac{\partial x_j^u(\cdot)}{\partial r} dG(\theta) = 0 . \quad (27)
$$

Because $\lambda = 1$, (27) can be written as:

$$
\sum_{i=1}^{N} \int_{\Omega_i^D} \left\{ [r + m(\theta) - C'(X^u) - e'(X^u)] \frac{\partial x_i^u(\cdot)}{\partial r} - e_i \frac{\partial x_i^o(\cdot)}{\partial r} \right\} dG(\theta)
$$

$$
+ \sum_{i=1}^{N} \int_{\Omega_i^{-D}} \left\{ [r - C'(X^u) - e'(X^u)] \frac{\partial x_i^u(\cdot)}{\partial r} - e_i \frac{\partial x_i^o(\cdot)}{\partial r} \right\} dG(\theta) = 0 . \quad (28)
$$

From (15), for $i = 1, ..., N$, $\frac{\partial x_i^u(\cdot)}{\partial r} = \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}$ for all $\theta \in \Omega_i^D$. Therefore, from (23), for all $i = 1, ..., N$, for $\theta \in \Omega_i^D$:

$$
\sum_{i=1}^{N} \left\{ [r + m(\theta) - C'(X^u) - e'(X^u)] \frac{\partial x_i^u(\cdot)}{\partial r} - e_i \frac{\partial x_i^o(\cdot)}{\partial r} \right\} = 0 . \quad (29)
$$

(28) and (29) imply:

$$
\sum_{i=1}^{N} \int_{\Omega_i^{-D}} \left\{ [r - C'(X^u) - e'(X^u)] \frac{\partial x_i^u(\cdot)}{\partial r} - e_i \frac{\partial x_i^o(\cdot)}{\partial r} \right\} dG(\theta) = 0
$$

$$
\Rightarrow r \sum_{i=1}^{N} \int_{\Omega_i^{-D}} \frac{\partial x_i^u(\cdot)}{\partial r} dG(\theta)
$$

$$
= \sum_{i=1}^{N} \int_{\Omega_i^{-D}} \left\{ [C'(X^u) + e'(X^u)] \frac{\partial x_i^u(\cdot)}{\partial r} + e_i \frac{\partial x_i^o(\cdot)}{\partial r} \right\} dG(\theta) . \quad (30)
$$

(7) follows directly from (30). ■

Proof of Corollary 3.

First suppose $x_i^u < \underline{x}_i$ for some $i \in \{1, ..., N\}$. Then, since $x_i^o(\cdot) = 0$ for all $i = 1, ..., N$, (6) and (15) imply that at the solution to [RP] identified in Proposition 1, x_i^u is determined by:

$$\frac{\partial V_i(x_i^u, \theta)}{\partial x_i^u} \;=\; r + m(\theta) \;=\; C'\left(X^u\right) + e'\left(X^u\right) . \tag{31}$$

(9) and (31) imply that, given the actions of other consumers, the actions of consumer i are efficient.

Now suppose $x_i^u(\cdot) > \underline{x}_i$. Then (7) and (15) imply that at the solution to [RP] identified in Proposition 1, x_i^u is determined by:

$$\frac{\partial V_i(x_i^u, \theta)}{\partial x_i^u} \;=\; r \;=\; \frac{\sum_{i=1}^{N} \int_{\Omega_i^{-D}} \left\{ \left[C'\left(X^u\right) + e'\left(X^u\right)\right] \frac{dx_i^u(\cdot)}{dr} \right\} d\,G(\theta)}{\sum_{i=1}^{N} \int_{\Omega_i^{-D}} \frac{dx_i^u(\cdot)}{dr}\, d\,G(\theta)} . \tag{32}$$

(9) and (32) imply that, given the actions of other consumers, the actions of consumer i are efficient if and only if, for all $\theta \in [\underline{\theta}, \overline{\theta}\,]$:

$$\begin{aligned} &\sum_{i=1}^{N} \int_{\Omega_i^{-D}} \left[C'\left(X^u\right) + e'\left(X^u\right)\right] \frac{dx_i^u(\cdot)}{dr}\; d\,G(\theta) \\ &\qquad = \left[\sum_{i=1}^{N} \int_{\Omega_i^{-D}} \frac{dx_i^u(\cdot)}{dr}\, d\,G(\theta) \right] \left[C'\left(X^u\right) + e'\left(X^u\right)\right] . \end{aligned} \tag{33}$$

The equality in (33) typically will not hold because $x_i^u(\cdot)$, and thus X^u, vary with θ. ■

<u>Proof of Corollary 4</u>.

(15) implies that at the solution to [RP]:

$$\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^o} \;=\; C_i'(x_i^o) \;<\; C_i'(x_i^o) + e_i \quad \text{when} \;\; e_i \;>\; 0 . \tag{34}$$

The corollary follows from (9) and (34). ■

<u>Proof of Proposition 2</u>.

Letting "$\widetilde{\cdot}$" ("$\widehat{\cdot}$") denote variables for consumers who can (cannot) provide demand response, expected weighted consumer welfare in this setting is:

$$\begin{aligned} E\left\{ U^{\alpha}(\cdot) \right\} \;=\; \widetilde{\alpha} \Bigg\{ \sum_{i=1}^{\widetilde{N}} \int_{\underline{\theta}}^{\overline{\theta}} &\left[V_i(\, \widetilde{x}_i^u(r, m(\theta), \theta) + \widetilde{x}_i^o(\cdot), \theta) \;-\; r\, \widetilde{x}_i^u(\cdot) \right. \\ &\left. +\;\; m(\theta)\, \widetilde{x}_i^d(\cdot) \;-\; C_i(\widetilde{x}_i^o(\cdot)) \right] d\,G(\theta) - \widetilde{N} R \Bigg\} \end{aligned}$$

$$+\,\widehat{\alpha}\left\{\,\sum_{i=1}^{\widehat{N}}\,\int_{\underline{\theta}}^{\overline{\theta}}[\,V_i(\,\widehat{x}_i^u(r,\theta)-r\,\widehat{x}_i^u(\cdot)\,]\,dG(\theta)-\widehat{N}R\right\}. \tag{35}$$

The utility's expected profit is:

$$\begin{aligned} E\,\{\pi^\alpha\} \;&=\; R\left[\widetilde{N}+\widehat{N}\right]+\sum_{i=1}^{\widetilde{N}}\int_{\underline{\theta}}^{\overline{\theta}}[\,r\,\widetilde{x}_i^u(\,r,m(\theta),\theta)\;-\;m(\theta)\,\widetilde{x}_i^d(\cdot)\,]\,dG(\theta) \\ &+\;\sum_{i=1}^{\widehat{N}}\int_{\underline{\theta}}^{\overline{\theta}} r\,\widehat{x}_i^u(\,r,\theta)\,dG(\theta)-\;\int_{\underline{\theta}}^{\overline{\theta}} C\left(\sum_{i=1}^{\widetilde{N}}\widetilde{x}_i^u(\cdot)+\sum_{i=1}^{\widehat{N}}\widehat{x}_i^u(\cdot)\right)dG(\theta)\,. \end{aligned} \tag{36}$$

Expected social losses from externalities are:

$$E\,\{\,L^\alpha(\cdot)\,\}\;=\;\int_{\underline{\theta}}^{\overline{\theta}}\left[\sum_{i=1}^{\widetilde{N}}e_i\,\widetilde{x}_i^o(\cdot)+\sum_{i=1}^{\widehat{N}}e_i\,\widehat{x}_i^u(\cdot)\;+e\left(\sum_{i=1}^{\widetilde{N}}\widetilde{x}_i^u(\cdot)+\sum_{i=1}^{\widehat{N}}\widehat{x}_i^u(\cdot)\right)\right]dG(\theta)\,. \tag{37}$$

The regulator's problem, [RP-D], is to choose $\{\,R,\,r,\,m(\theta)\,\}$ to maximize $E\,\{\,U^\alpha(\cdot)\,\}-E\,\{\,L^\alpha(\cdot)\,\}$ while securing non-negative expected profit for the utility. Let $\lambda_\alpha\geq 0$ denote the Lagrange multiplier associated with the utility's participation constraint ($E\,\{\,\pi^\alpha\,\}\,\geq\,0$). Then the Lagrangian function associated with [RP-D] is:

$$\pounds_\alpha \;=\; E\,\{\,U^\alpha(\cdot)\,\}-E\,\{\,L^\alpha(\cdot)\,\}+\lambda_\alpha\,E\,\{\,\pi^\alpha\,\}\;. \tag{38}$$

Since the value of R does not affect consumption decisions, differentiating (38) with respect to R, using (35), (36), (37), provides:

$$-\left[\widetilde{\alpha}\,\widetilde{N}+\widehat{\alpha}\,\widehat{N}\right]+\lambda_\alpha\left[\widetilde{N}+\,\widehat{N}\right]\;=\;0\quad\Rightarrow\quad\lambda_\alpha\;=\;\frac{\widetilde{\alpha}\,\widetilde{N}+\widehat{\alpha}\,\widehat{N}}{\widetilde{N}+\,\widehat{N}}\,. \tag{39}$$

Since $\frac{\partial\widehat{x}_i^u(\cdot)}{\partial m(\theta)}=0$ for all $i=1,...,\widehat{N}$, pointwise optimization of (38) with respect to $m(\theta)$, using (35), (36), (37), Leibnitz' rule, and the established continuity of consumer welfare and profit (recall the proof of Proposition 1) provides:

$$\begin{aligned} &[\,\widetilde{\alpha}-\lambda_\alpha\,]\sum_{i=1}^{\widetilde{N}}\widetilde{x}_i^d(r,m(\theta),\theta)\;g(\theta)-e'\,(\cdot)\,\sum_{i=1}^{\widetilde{N}}\frac{\partial\widetilde{x}_i^u(\cdot)}{\partial m(\theta)}\,g(\theta)-\;\sum_{i=1}^{\widetilde{N}}e_i\,\frac{\partial\widetilde{x}_i^o}{\partial m(\theta)}\,g(\theta) \\ &-\;\;\lambda_\alpha\,C'\,(\cdot)\,\sum_{i=1}^{\widetilde{N}}\frac{\partial\widetilde{x}_i^u(\cdot)}{\partial m(\theta)}\,g(\theta)+\lambda_\alpha\,\sum_{i=1}^{\widetilde{N}}\left[r\,\frac{\partial\widetilde{x}_i^u(\cdot)}{\partial m(\theta)}-m(\theta)\,\frac{\partial\widetilde{x}_i^d(\cdot)}{\partial m(\theta)}\right]g(\theta)\;=\;0\,. \end{aligned} \tag{40}$$

From (39):

$$\widetilde{\alpha} - \lambda_\alpha \;=\; \frac{\widetilde{\alpha}\,\widetilde{N} + \widetilde{\alpha}\,\widehat{N} - \widetilde{\alpha}\,\widetilde{N} - \widehat{\alpha}\,\widehat{N}}{\widetilde{N} + \widehat{N}} \;=\; \frac{\widehat{N}\,[\,\widetilde{\alpha} - \widehat{\alpha}\,]}{\widetilde{N} + \widehat{N}}\,.$$

Therefore, (40) can be written as:

$$\frac{\widehat{N}\,[\,\widetilde{\alpha} - \widehat{\alpha}\,]}{\widetilde{N} + \widehat{N}} \sum_{i=1}^{\widetilde{N}} \widetilde{x}_i^d(\cdot) + \left[\frac{\widetilde{\alpha}\,\widetilde{N} + \widehat{\alpha}\,\widehat{N}}{\widetilde{N} + \widehat{N}}\right] \left[r - C'(\cdot) - \left(\frac{\widetilde{N} + \widehat{N}}{\widetilde{\alpha}\,\widetilde{N} + \widehat{\alpha}\,\widehat{N}}\right) e'(\cdot) \right] \sum_{i=1}^{\widetilde{N}} \frac{\partial \widetilde{x}_i^u(\cdot)}{\partial m(\theta)}$$

$$- \left[\frac{\widetilde{\alpha}\,\widetilde{N} + \widehat{\alpha}\,\widehat{N}}{\widetilde{N} + \widehat{N}}\right] \sum_{i=1}^{\widetilde{N}} m(\theta)\, \frac{\partial \widetilde{x}_i^d(\cdot)}{\partial m(\theta)} - \sum_{i=1}^{\widetilde{N}} e_i\, \frac{\partial \widetilde{x}_i^o}{\partial m(\theta)} \;=\; 0\,. \tag{41}$$

Since $\frac{\partial \widetilde{x}_i^d(\cdot)}{\partial m(\theta)} \;=\; -\frac{\partial \widetilde{x}_i^u(\cdot)}{\partial m(\theta)}$, (41) can be written as:

$$\left[\widetilde{\alpha}\,\widetilde{N} + \widehat{\alpha}\,\widehat{N}\right] \left[r + m(\theta) - C'(\cdot) - \left(\frac{\widetilde{N} + \widehat{N}}{\widetilde{\alpha}\,\widetilde{N} + \widehat{\alpha}\,\widehat{N}}\right) e'(\cdot) \right] \sum_{i=1}^{\widetilde{N}} \frac{\partial \widetilde{x}_i^u(\cdot)}{\partial m(\theta)}$$

$$- \left[\widetilde{N} + \widehat{N}\right] \sum_{i=1}^{\widetilde{N}} e_i\, \frac{\partial \widetilde{x}_i^o}{\partial m(\theta)} + \widehat{N}\,[\,\widetilde{\alpha} - \widehat{\alpha}\,] \sum_{i=1}^{\widetilde{N}} \widetilde{x}_i^d(\cdot) \;=\; 0$$

$$\Rightarrow \quad r + m(\theta) - C'(\cdot) - \left[\frac{\widetilde{N} + \widehat{N}}{\widetilde{\alpha}\,\widetilde{N} + \widehat{\alpha}\,\widehat{N}}\right] e'(\cdot)$$

$$= \frac{\left[\widetilde{N} + \widehat{N}\right] \sum_{i=1}^{\widetilde{N}} e_i\, \frac{\partial \widetilde{x}_i^o}{\partial m(\theta)} - \widehat{N}\,[\,\widetilde{\alpha} - \widehat{\alpha}\,] \sum_{i=1}^{\widetilde{N}} \widetilde{x}_i^d(\cdot)}{\left[\widetilde{\alpha}\,\widetilde{N} + \widehat{\alpha}\,\widehat{N}\right] \sum_{i=1}^{\widetilde{N}} \frac{\partial \widetilde{x}_i^u(\cdot)}{\partial m(\theta)}}\,. \tag{42}$$

(10) follows immediately from (42) because $\frac{\partial \widetilde{x}_i^u(\cdot)}{\partial m(\theta)} < 0$ when $\widetilde{x}_i^d(\cdot) > 0$ and $\frac{\partial \widetilde{x}_i^u(\cdot)}{\partial m(\theta)} \leq 0$ when $\widetilde{x}_i^d(\cdot) = 0$. ■

Proof of Proposition 3.

(25) implies that aggregate consumer welfare in this setting is:

$$E\{\,U^a(\cdot)\,\} \;=\; \int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} w_i(\theta)\, d\,G(\theta) - N\,R - D(a_i)\,. \tag{43}$$

Since $\sum\limits_{i=1}^{N} w_i(\theta)$ is continuous in θ for all θ (recall the proof of Proposition 1), (25), (43), and Leibnitz' rule imply that a_i is determined by:

$$H_i(a_i, r, m(\theta), \theta)\,) \;\equiv\; \int_{\underline{\theta}}^{\widetilde{\theta}_i} m(\theta)\, \frac{\partial \underline{x}_i}{\partial a_i}\, dG(\theta) - D_i'(a_i) \;=\; 0\,. \tag{44}$$

By assumption:

$$\frac{\partial H_i(\cdot)}{\partial a_i} \;=\; \frac{d\widetilde{\theta}_i(\cdot)}{da_i}\, m(\widetilde{\theta}_i)\, \frac{\partial \underline{x}_i}{\partial a_i}\, g(\widetilde{\theta}_i) + \int_{\underline{\theta}}^{\widetilde{\theta}_i} m(\theta)\, \frac{\partial^2 \underline{x}_i}{\partial\,(a_i)^2}\, dG(\theta) - D_i''(a_i) \;<\; 0\,. \tag{45}$$

(44) implies:

$$\frac{\partial H_i(\cdot)}{\partial m(\theta)} \;=\; \begin{cases} \frac{\partial \underline{x}_i}{\partial a_i}\, g(\theta) \;>\; 0 & \text{if } \theta \in \Omega_i^D \\ 0 & \text{otherwise.} \end{cases} \tag{46}$$

(44), (45), and (46) imply:

$$\frac{\partial a_i}{\partial m(\theta)} \;=\; -\,\frac{\partial H_i / \partial m(\theta)}{\partial H_i / \partial a_i} \;\geq\; 0\,. \tag{47}$$

The regulator's problem, [RP-a], is to choose $\{\,R,\, r,\, m(\theta)\,\}$ to maximize $E\{\,U^a(\cdot)\,\} - E\,\{\,L(\cdot)\,\}$ while securing non-negative expected profit for the utility. Let $\lambda_a \geq 0$ denote the Lagrange multiplier associated with the utility's participation constraint ($E\{\,\pi^a\,\} \geq 0\,$). Then the Lagrangian function associated with [RP] is:

$$\pounds_a \;=\; E\{\,U^a(\cdot)\,\} - E\,\{\,L(\cdot)\,\} + \lambda_a\, E\{\,\pi^a\,\}\,. \tag{48}$$

Let $\frac{dx_i^j(\cdot)}{dm(\theta)} \;=\; \frac{\partial x_i^j(\cdot)}{\partial m(\theta)} + \frac{\partial x_i^j(\cdot)}{\partial a_i}\, \frac{\partial a_i}{\partial m(\theta)}$ for $j \in \{u, d, o\}$. For the reasons identified in the proof of Proposition 1, expected consumer welfare and the firm's expected profit are both continuous functions of θ. Consequently, Leibnitz' rule, (2), (26), and (43) imply that pointwise optimization of (48) with respect to $m(\theta)$ provides:

$$\begin{aligned} &[\,1 - \lambda_a\,] \sum_{i=1}^{N} x_i^d(\,r, m(\theta), \theta)\; g(\theta) - e'\,(X^u) \sum_{i=1}^{N} \frac{dx_i^u(\cdot)}{dm(\theta)}\, g(\theta) - \sum_{i=1}^{N} e_i\, \frac{dx_i^o}{dm(\theta)}\, g(\theta) \\ &-\; \lambda_a\, C'\,(X^u) \sum_{i=1}^{N} \frac{dx_i^u(\cdot)}{dm(\theta)}\, g(\theta) + \lambda_a \sum_{i=1}^{N} \left[\, r\, \frac{dx_i^u(\cdot)}{dm(\theta)} - m(\theta)\, \frac{dx_i^d(\cdot)}{dm(\theta)} \right] g(\theta) \;=\; 0\,. \end{aligned} \tag{49}$$

Since the value of R does not affect consumption decisions, differentiating (48) with respect to R, using (2), (26), and (43), provides $-\,N + \lambda_a\, N \;=\; 0 \;\Rightarrow\; \lambda_a \;=\; 1$. Therefore, (49) can be written as:

$$[\, r - e'\,(X^u) - C'\,(X^u)\,] \sum_{i=1}^{N} \frac{dx_i^u(\cdot)}{dm(\theta)} \;-\; \sum_{i=1}^{N} e_i \, \frac{dx_i^o}{dm(\theta)} \;=\; m(\theta) \sum_{i=1}^{N} \frac{dx_i^d(\cdot)}{dm(\theta)} \,. \tag{50}$$

(18) implies $\frac{\partial x_i^d(\cdot)}{\partial m(\theta)} = - \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} > 0$. Also, (15) implies that $x_i^u(\cdot)$ does not vary with $\underline{x}_i$, given r and $m(\theta)$. Therefore:

$$\frac{dx_i^u(\cdot)}{dm(\theta)} \;=\; \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \quad \text{and} \quad \frac{\partial x_i^d(\cdot)}{\partial a_i} \;=\; \begin{cases} \frac{\partial \underline{x}_i}{\partial a_i} & \text{if } x_i^u(\cdot) \,\le\, \underline{x}_i \\ 0 & \text{if } x_i^u(\cdot) \,>\, \underline{x}_i \,. \end{cases} \tag{51}$$

$$\Rightarrow \quad \frac{dx_i^d(\cdot)}{dm(\theta)} \;=\; \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)} \;>\; 0 \,. \tag{52}$$

(12) follows from (50), (51), and (52). ■

Proof of Corollary 5.

(15) implies that when $e_i = 0$ for all $i = 1, ..., N$, $x_i^u \,<\, \underline{x}_i$ at the solution to [RP-a] identified in Proposition 3 is determined by:

$$\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^u} \;=\; r + m(\theta) \;=\; r + \frac{[\, C'\,(X^u) - r + e'\,(X^u)\,] \sum_{i=1}^N \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|}{\sum_{i=1}^N \left\{ \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)} \right\}}$$

$$= \frac{[\, C'\,(X^u) + e'\,(X^u)\,] \sum_{i=1}^N \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + r \, \sum_{i=1}^N \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)}}{\sum_{i=1}^N \left\{ \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)} \right\}} \,. \tag{53}$$

(9) and (53) imply that, given the actions of other consumers, consumer i's actions are efficient only if:

$$\frac{[\, C'\,(X^u) + e'\,(X^u)\,] \sum_{i=1}^N \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + r \, \sum_{i=1}^N \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)}}{\sum_{i=1}^N \left\{ \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)} \right\}} \;=\; C'\,(X^u) + e'\,(X^u)$$

$$\Leftrightarrow \quad [\, C'\,(X^u) + e'\,(X^u)\,] \left[\frac{\sum_{i=1}^N \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right|}{\sum_{i=1}^N \left\{ \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)} \right\}} - 1 \right]$$

$$+ \; \frac{r \, \sum_{i=1}^N \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)}}{\sum_{i=1}^N \left\{ \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)} \right\}} \;=\; 0$$

$$\Leftrightarrow \quad [\, r - C'(X^u) - e'(X^u)\,] \left[\frac{\sum_{i=1}^{N} \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)}}{\sum_{i=1}^{N} \left\{ \left| \frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \right| + \delta_{i\theta} \, \frac{\partial \underline{x}_i}{\partial a_i} \, \frac{\partial a_i}{\partial m(\theta)} \right\}} \right] = 0 \, . \tag{54}$$

(52) implies that (54) holds if and only if $r = C'(X^u) - e'(X^u)$ for each $\theta \in [\underline{\theta}, \overline{\theta}\,]$. These inequalities typically will not all hold because $x_i^u(\cdot)$, and thus X^u, vary with θ. ■

Proof of Proposition 4.

Expected consumer welfare in this setting is:

$$E\{U^s(\cdot)\} = \sum_{i=1}^{N} \int_{\underline{\theta}}^{\overline{\theta}} [\, V_i(\, x_i^u(r(\theta), m(\theta), \theta) + x_i^o(\cdot), \theta) - r(\theta)\, x_i^u(\cdot)$$
$$+ \; m(\theta)\, x_i^d(\cdot) - C_i(x_i^o(\cdot))\,]\, dG(\theta) - NR \, . \tag{55}$$

The utility's expected profit is:

$$E\{\pi^s\} = NR + \sum_{i=1}^{N} \int_{\underline{\theta}}^{\overline{\theta}} [\, r(\theta)\, x_i^u(\, r, m(\theta), \theta) - m(\theta)\, x_i^d(\cdot)\,]\, dG(\theta)$$
$$- \int_{\underline{\theta}}^{\overline{\theta}} C\left(\sum_{i=1}^{N} x_i^u(\cdot) \right) dG(\theta) \, . \tag{56}$$

Expected social losses from externalities are:

$$E\{L^s(\cdot)\} = \int_{\underline{\theta}}^{\overline{\theta}} \left[\sum_{i=1}^{N} e_i\, x_i^o(\cdot) + e\left(\sum_{i=1}^{N} x_i^u(\cdot) \right) \right] dG(\theta) \, . \tag{57}$$

The regulator's problem, [RP-s], is to choose $\{R, r(\theta), m(\theta)\}$ to maximize $E\{U^s(\cdot)\} - E\{L^s(\cdot)\}$ while securing non-negative expected profit for the utility. Let $\lambda_s \geq 0$ denote the Lagrange multiplier associated the utility's participation constraint ($E\{\pi^s\} \geq 0$). Then the Lagrangian function associated with [RP-s] is:

$$\pounds_s = E\{U^s(\cdot)\} - E\{L^s(\cdot)\} + \lambda_s\, E\{\pi^s\} \, . \tag{58}$$

Since the value of R does not affect consumption decisions, differentiating (58) with respect to R, using (55), (56), and (57) provides $-N + \lambda_s N = 0 \Rightarrow \lambda_s = 1$.

Pointwise optimization of (58) with respect to $m(\theta)$, using (55), (56), (57) and the envelope theorem provides:

$$[\,1-\lambda_s\,]\sum_{i=1}^{N} x_i^d(r(\theta),m(\theta),\theta)\; g(\theta) - e'\,(X^u)\;\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}\; g(\theta) - \;\sum_{i=1}^{N} e_i\;\frac{\partial x_i^o}{\partial m(\theta)}\; g(\theta)$$

$$-\;\;\lambda_s\, C'\,(X^u)\;\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}\; g(\theta)+\lambda_s\;\sum_{i=1}^{N}\left[r(\theta)\,\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}-m(\theta)\,\frac{\partial x_i^d(\cdot)}{\partial m(\theta)}\right]g(\theta)\;=\;0\,. \quad (59)$$

Since $\lambda_s\;=\;1,$ (59) can be written as:

$$r(\theta)\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}-m(\theta)\sum_{i=1}^{N}\frac{\partial x_i^d(\cdot)}{\partial m(\theta)}-[\,e'\,(\cdot)+C'\,(\cdot)\,]\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}\;-\;\sum_{i=1}^{N} e_i\;\frac{\partial x_i^o}{\partial m(\theta)}\;=\;0$$

$$\Rightarrow\quad m(\theta)\;=\;\frac{[\,r(\theta)-e'\,(\cdot)-C'\,(\cdot)\,]\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}-\sum_{i=1}^{N} e_i\,\frac{\partial x_i^o}{\partial m(\theta)}}{\sum_{i=1}^{N}\frac{\partial x_i^d(\cdot)}{\partial m(\theta)}}$$

$$=\;C'\,(\cdot)-r(\theta)+e'\,(\cdot)+\frac{\sum_{i=1}^{N} e_i\,\frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}}\,. \quad (60)$$

The last equality in (60) holds because $\frac{\partial x_i^d(\cdot)}{\partial m(\theta)}\;=\;-\,\frac{\partial x_i^u(\cdot)}{\partial m(\theta)}$, since $x_i^d(\cdot)\,=\,\max\{\,0,\,\underline{x}_i-x_i^u(\cdot)\,\}$.

Pointwise optimization of (58) with respect to $r(\theta)$, using (55), (56), (57) and the envelope theorem provides:

$$[\,\lambda_s-1\,]\;\sum_{i=1}^{N} x_i^u(\cdot)\;g(\theta)-\sum_{i=1}^{N} e_i\;\frac{\partial x_i^o(\cdot)}{\partial r(\theta)}\,g(\theta)-e'(\cdot)\;\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}\,g(\theta)$$

$$-\;\lambda_s\;C'(\cdot)\;\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}\;g(\theta)+\lambda_s\,r(\theta)\;\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}\;g(\theta)-\lambda_s\,m(\theta)\sum_{i=1}^{N}\frac{\partial x^d(\cdot)}{\partial r(\theta)}\;g(\theta)\;\;=\;\;0\,. \quad (61)$$

Since $\lambda_s\;=\;1,$ (61) can be written as:

$$r(\theta)\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}\;\;=\;\;m(\theta)\sum_{i=1}^{N}\frac{\partial x^d(\cdot)}{\partial r(\theta)}+[\,C'(\cdot)+e'\,(\cdot)\,]\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}+\sum_{i=1}^{N} e_i\;\frac{\partial x_i^o(\cdot)}{\partial r(\theta)}$$

$$\Rightarrow\quad r(\theta)\;\;=\;\;C'\,(\cdot)+e'\,(\cdot)+m(\theta)\;\left[\frac{\sum_{i=1}^{N}\frac{\partial x_i^d(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}\right]+\frac{\sum_{i=1}^{N} e_i\,\frac{\partial x_i^o}{\partial r(\theta)}}{\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}$$

$$=\;\;C'\,(\cdot)+e'\,(\cdot)+m(\theta)\;\left[\frac{\sum_{i=1}^{N}\frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)}\,\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}\right]+\frac{\sum_{i=1}^{N} e_i\,\frac{\partial x_i^o}{\partial r(\theta)}}{\sum_{i=1}^{N}\frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}\,. \quad (62)$$

The last equality in (62) holds because, since $x_i^d(\cdot) = \max\{0, \underline{x}_i - x_i^u(\cdot)\}$: (i) $\frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} = -1$ when $x_i^d(\cdot) > 0$; and (ii) $\frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} = 0$ and $\frac{\partial x_i^d(\cdot)}{\partial r(\theta)} = 0$ when $x_i^d(\cdot) \leq 0$.

Using (62), (60) can be written as:

$$m(\theta) = -\frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} - m(\theta) \frac{\sum_{i=1}^N \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} + \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}}$$

$$\Leftrightarrow \quad m(\theta) \left[\frac{\sum_{i=1}^N \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\theta)} \right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right] = \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}} - \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}$$

$$\Leftrightarrow \quad m(\theta) = \left[\frac{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\theta)} \right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right] \left[\frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}} - \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right]. \tag{63}$$

Using (63), (62) can be written as:

$$\begin{aligned}
r(\theta) &= C'(X^u) + e'(X^u) + \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \\
&\quad + \left[\frac{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\theta)} \right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right] \left[\frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}} - \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right] \frac{\sum_{i=1}^N \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \\
&= C'(\cdot) + e'(\cdot) + \left[\frac{\sum_{i=1}^N \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \left(1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\theta)} \right) \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right] \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}} \\
&\quad + \left[1 - \frac{\sum_{i=1}^N \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\theta)} \right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right] \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \\
&= C'(\cdot) + e'(\cdot) + \left[\frac{\sum_{i=1}^N \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\theta)} \right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right] \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}} \\
&\quad + \left[\frac{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\theta)} \right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} \right] \frac{\sum_{i=1}^N e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^N \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}
\end{aligned}$$

$$= C'(X^u) + e'(X^u) + \frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N} \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\theta)}\right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}$$

$$+ \left[\frac{\sum_{i=1}^{N} \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N} \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)}\right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}\right] \frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}} . \tag{64}$$

Conclusions (i) and (ii) of the proposition follow directly from (63) and (64) because $e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)} = e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)} = 0$ when consumers do not produce electricity or when their production entails no externalities. Conclusion (iii) of the proposition follows from (63) and (64) because $\frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} = -1$, $\frac{\partial x_i^u(\cdot)}{\partial m(\theta)} = \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}$, and $\frac{\partial x_i^o(\cdot)}{\partial m(\theta)} = \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}$ when $x_i^d(\cdot) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and for all $i = 1, ...N$. ■

Proof of Corollary 6.

(13) follows immediately from (63) because $\frac{\partial x_i^u(\cdot)}{\partial r(\theta)} < 0$, $\frac{\partial x_i^u(\cdot)}{\partial m(\theta)} \leq 0$, and $\frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} \in \{0, -1\}$. ■

Proof of Corollary 7.

Since $x_i^u(\cdot) > 0$, (15), (63), and (64) imply that at the solution to [RP] identified in Proposition 4, x_i^u and x_i^o are determined by:

$$\frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^u} = r(\theta) + m(\theta) = C'(X^u) + e'(X^u)$$

$$+ \frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N} \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)}\right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}} + \left[\frac{\sum_{i=1}^{N} \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N} \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)}\right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}\right] \left[\frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}}\right]$$

$$+ \left[\frac{\sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N} \left[1 + \frac{\partial x_i^d(\cdot)}{\partial x_i^u(\cdot)}\right] \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}\right] \left[\frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial m(\theta)}}{\sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial m(\theta)}} - \frac{\sum_{i=1}^{N} e_i \frac{\partial x_i^o(\cdot)}{\partial r(\theta)}}{\sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial r(\theta)}}\right]$$

$$\text{and} \qquad \frac{\partial V_i(x_i^u + x_i^o, \theta)}{\partial x_i^o} = C_i'(x_i^o) . \tag{65}$$

(9) and (65) imply that, given the actions of other consumers, the actions of consumer i at the identified solution: (i) are efficient if $e_j = 0$ for all $j \in \{1, ..., N\}$; and (ii) are not efficient if $e_i > 0$. ■

Appendix B. Elements of the Numerical Solutions

This appendix has three sections. The first section presents the analytic conclusions that underlie the analysis of the benchmark setting considered in Section 5. The second section describes the techniques employed to derive explicit solutions to the relevant problems in the benchmark setting. The third section presents estimates of the welfare gains that demand response policies might secure in settings that reflect conditions in the California and the ISO New England regions.

1. Analytic Conclusions for the Benchmark Setting.

In the benchmark setting analyzed in Section 5, for any given θ, consumer i chooses x_i^u to maximize:

$$\theta\, v_i \left[\frac{(x_i^u)^{\alpha_i+1} - \overline{V}_i}{\alpha_i + 1} \right] - R - r\, x_i^u + m(\theta)\, x_i^d(\cdot). \tag{66}$$

Therefore, at an interior optimum:

$$v_i\, \theta\, [\, x_i^u\,]^{\alpha_i} \;=\; \begin{cases} r + m(\theta) & \text{if } \; v_i\, \theta\, [\, \underline{x}_i\,]^{\alpha_i} \;<\; r + m(\theta) \\ r & \text{otherwise,} \end{cases} \tag{67}$$

so the amount of electricity consumer i purchases from the utility is:

$$x_i^u(\cdot) \;=\; \begin{cases} \left[\frac{r + m(\theta)}{v_i\, \theta} \right]^{\frac{1}{\alpha_i}} & \text{if } \; v_i\, \theta\, [\, \underline{x}_i\,]^{\alpha_i} \;<\; r + m(\theta) \\ \left[\frac{r}{v_i\, \theta} \right]^{\frac{1}{\alpha_i}} & \text{otherwise.} \end{cases} \tag{68}$$

(67) implies that consumer i will be indifferent between providing demand response and consuming his baseline level of consumption, $\underline{x}_i$, when $\theta = \widetilde{\theta}_i$, where:

$$v_i\, \widetilde{\theta}_i\, [\, \underline{x}_i\,]^{\alpha_i} \;=\; r + m(\theta) \quad \Rightarrow \quad \widetilde{\theta}_i \;=\; \frac{r + m(\theta)}{v_i\, [\, \underline{x}_i\,]^{\alpha_i}}\,. \tag{69}$$

Brown and Sappington (2015) demonstrate that $\widetilde{\theta}_i \in [\, \underline{\theta},\, \overline{\theta}\,]$ will be unique and $\Omega_i = (\widetilde{\theta}_i,\, \overline{\theta}\,]$ (so individual i provides DR only for $\theta \in (\widetilde{\theta}_i,\, \overline{\theta}\,])$ if b is sufficiently large (so the utility's marginal cost of production increases sufficiently rapidly with output) and $\overline{V}_i$ is sufficiently large for all i (which ensures that each consumer's valuation of electricity is strictly positive).

First consider the setting where there is no demand response program. The regulator's problem in this setting, [RP-n], is to choose $\{R, r\}$ to maximize expected consumer welfare while ensuring the utility secures non-negative expected profit.

Lemma B1. *At the solution to [RP-n]:*

$$r = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} \left[a + 2b \sum_{j=1}^{N} \left(\left[\frac{r}{v_j \theta} \right]^{\frac{1}{\alpha_j}} \right) \right] [v_i \theta]^{-\frac{1}{\alpha_i}} \frac{1}{\alpha_i} r^{\frac{1-\alpha_i}{\alpha_i}} \, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} [v_i \theta]^{-\frac{1}{\alpha_i}} \frac{1}{\alpha_i} r^{\frac{1-\alpha_i}{\alpha_i}} \, dG(\theta)}; \quad \text{and} \tag{70}$$

$$\begin{aligned} R = \frac{1}{N} \Bigg[F + \int_{\underline{\theta}}^{\overline{\theta}} \left\{ a \sum_{i=1}^{N} \left[\frac{r}{v_i \theta} \right]^{\frac{1}{\alpha_i}} + b \left(\sum_{i=1}^{N} \left[\frac{r}{v_i \theta} \right]^{\frac{1}{\alpha_i}} \right)^2 \right\} dG(\theta) \\ - \frac{\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} \left[a + 2b \sum_{j=1}^{N} \left(\left[\frac{r}{v_j \theta} \right]^{\frac{1}{\alpha_j}} \right) \right] [v_i \theta]^{-\frac{1}{\alpha_i}} \frac{1}{\alpha_i} r^{\frac{1-\alpha_i}{\alpha_i}} \, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} [v_i \theta]^{-\frac{1}{\alpha_i}} \frac{1}{\alpha_i} r^{\frac{1-\alpha_i}{\alpha_i}} \, dG(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} \left(\frac{r}{v_i \theta} \right)^{\frac{1}{\alpha_i}} dG(\theta) \Bigg]. \end{aligned} \tag{71}$$

<u>Proof</u>. Let $\lambda_n \geq 0$ denote the Lagrange multiplier associated with the utility's participation constraint. Then the Lagrangian function associated with [RP-n] is:

$$\pounds = E\{U(\cdot)\} + \lambda_n E\{\pi\}. \tag{72}$$

Since the value of R does not affect consumption decisions, differentiating (72) with respect to R, using (1) and (3) (with $m(\theta) = 0$ for all θ), provides $-N + \lambda_n N = 0 \Rightarrow \lambda_n = 1$.

From (68):

$$\frac{\partial x_i^u(r, \theta)}{\partial r} = [v_i \theta]^{-\frac{1}{\alpha_i}} \frac{1}{\alpha_i} \left[r^{\frac{1-\alpha_i}{\alpha_i}} \right]. \tag{73}$$

Since $C(X) = aX + bX^2$, (73) implies:

$$\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} C'(\cdot) \frac{\partial x_i^u(\cdot)}{\partial r} \, dG(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} \left[a + 2b \sum_{i=1}^{N} x_i^u(\cdot) \right] [v_i \theta]^{-\frac{1}{\alpha_i}} \frac{1}{\alpha_i} r^{\frac{1-\alpha_i}{\alpha_i}} \, dG(\theta), \tag{74}$$

and

$$\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial r} \, dG(\theta) \;=\; \int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} [\, v_i \, \theta \,]^{-\frac{1}{\alpha_i}} \, \frac{1}{\alpha_i} \, r^{\frac{1-\alpha_i}{\alpha_i}} \, dG(\theta) \, . \tag{75}$$

Differentiating (72) with respect to r, using (1) and (3) (with $m(\theta) = 0$ for all θ) and the envelope theorem, provides:

$$\sum_{i=1}^{N} \int_{\underline{\theta}}^{\overline{\theta}} - x_i^u(r, \theta) \, dG(\theta) + \lambda_n \sum_{i=1}^{N} \int_{\underline{\theta}}^{\overline{\theta}} x_i^u(r, \theta) \, dG(\theta)$$

$$+ \lambda_n \int_{\underline{\theta}}^{\overline{\theta}} r \sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial r} \, dG(\theta) - \lambda_n \int_{\underline{\theta}}^{\overline{\theta}} C'(\cdot) \sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial r} \, dG(\theta) \;=\; 0 \, . \tag{76}$$

Because $\lambda_n = 1$, (76) can be written as:

$$\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} [\, r - C'(\cdot) \,] \, \frac{\partial x_i^u(\cdot)}{\partial r} \, dG(\theta) \;=\; 0 \quad \Rightarrow \quad r \;=\; \frac{\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} C'(\cdot) \, \frac{\partial x_i^u(\cdot)}{\partial r} \, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \sum_{i=1}^{N} \frac{\partial x_i^u(\cdot)}{\partial r} \, dG(\theta)} \, . \tag{77}$$

(70) follows from (68), (73), (74), (75), and (77).

Since $\lambda_n > 0$, (3) implies:

$$N \, R + \int_{\underline{\theta}}^{\overline{\theta}} r \sum_{i=1}^{N} x_i^u(\, r, \theta) \, dG(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} C\,(\cdot) \; dG(\theta) - F \;=\; 0 \, . \tag{78}$$

(71) follows directly from (68), (73), and (78). ■

Now consider the setting of primary interest where the regulator chooses $\{ \, R, \, r, \, m(\theta) \, \}$ to maximize expected consumer welfare while ensuring the utility secures non-negative expected profit. Let [RP-E] denote the regulator's problem in this setting for the example under consideration.

Lemma B2. *At the solution to [RP-E]:*

$$m(\theta) \;=\; \begin{cases} a + 2\, b \sum_{i=1}^{N} \left[\frac{r + m(\theta)}{v_i \, \theta} \right]^{\frac{1}{\alpha_i}} - r & \text{if } \; v_i \, \theta \, [\, \underline{x}_i \,]^{\alpha_i} \; < \; r + m(\theta) \\ a + 2\, b \sum_{i=1}^{N} \left[\frac{r}{v_i \, \theta} \right]^{\frac{1}{\alpha_i}} - r & \text{otherwise;} \end{cases} \tag{79}$$

$$r \;=\; \frac{\sum\limits_{i=1}^{N} \int\limits_{\widetilde{\theta}_i(\cdot)}^{\overline{\theta}} \left[a + 2\,b \sum_{j=1}^{N} \left(\frac{r}{v_j\,\theta} \right)^{\frac{1}{\alpha_j}} \right] [\, v_i\,\theta \,]^{-\frac{1}{\alpha_i}} \; \frac{1}{\alpha_i}\, r^{\frac{1-\alpha_i}{\alpha_i}} \; dG(\theta)}{\sum\limits_{i=1}^{N} \int\limits_{\widetilde{\theta}_i(\cdot)}^{\overline{\theta}} [\, v_i\,\theta \,]^{-\frac{1}{\alpha_i}} \; \frac{1}{\alpha_i}\, r^{\frac{1-\alpha_i}{\alpha_i}} \; dG(\theta)}\,; \quad \text{and} \tag{80}$$

$$\begin{aligned} R \;=\; & \frac{1}{N} \Bigg[F + \int_{\underline{\theta}}^{\overline{\theta}} \left[\, a\, X^u + b\,(X^u)^2 \,\right] dG(\theta) - \sum_{i=1}^{N} \int_{\widetilde{\theta}_i}^{\overline{\theta}} r \left(\frac{r}{v_i\,\theta} \right)^{\frac{1}{\alpha_i}} dG(\theta) \\ & - \sum_{i=1}^{N} \int_{\underline{\theta}}^{\widetilde{\theta}_i} \left\{ r \left(\frac{r \,+\, m(\theta)}{v_i\,\theta} \right)^{\frac{1}{\alpha_i}} - \; m(\theta)\; \underline{x}_i \,+\, m(\theta) \left(\frac{r \,+\, m(\theta)}{v_i\,\theta} \right)^{\frac{1}{\alpha_i}} \right\} dG(\theta) \Bigg]. \end{aligned} \tag{81}$$

Proof. The proof follows directly from (68), (69), (73), (74), (75), and Proposition 1. ■

Now consider the regulator's problem of choosing $\{\, R,\, r \,\}$ to maximize expected consumer welfare in the benchmark setting while ensuring the utility secures non-negative expected profit when the unit compensation for demand response is set equal to the utility's prevailing marginal cost of supplying electricity, i.e., $m(\theta) \;=\; C'(\sum_{i=1}^{N} x_i^u(\cdot))$ for all $\theta \in [\, \underline{\theta},\, \overline{\theta}\,]$. Call this problem [RP-F].

Lemma B3. *At the solution to [RP-F]:*

$$m(\theta) \;=\; \begin{cases} a + 2\,b\, \sum_{i=1}^{N} \left[\frac{r \,+\, m(\theta)}{v_i\,\theta} \right]^{\frac{1}{\alpha_i}} & \text{if } \; v_i\,\theta\,[\, \underline{x}_i \,]^{\alpha_i} \;<\; r + m(\theta) \\ a + 2\,b\, \sum_{i=1}^{N} \left[\frac{r}{v_i\,\theta} \right]^{\frac{1}{\alpha_i}} & \text{otherwise;} \end{cases} \tag{82}$$

$$r \;=\; \frac{\sum_{i=1}^{N} \int_{\widetilde{\theta}_i}^{\overline{\theta}} \left[a + 2\,b\, \sum_{j=1}^{N} \left(\frac{r}{v_j\,\theta} \right)^{\frac{1}{\alpha_j}} \right] [\, v_i\,\theta \,]^{-\frac{1}{\alpha_i}} \; \frac{1}{\alpha_i}\, r^{\frac{1-\alpha_i}{\alpha_i}} \; dG(\theta)}{\sum_{i=1}^{N} \int_{\underline{\theta}}^{\widetilde{\theta}_i} [\, v_i\,\theta \,]^{-\frac{1}{\alpha_i}} \; \frac{1}{\alpha_i} \left[\frac{r + m(\theta)}{r} \right]^{\frac{1-\alpha_i}{\alpha_i}} dG(\theta) + \sum_{i=1}^{N} \int_{\widetilde{\theta}_i}^{\overline{\theta}} [\, v_i\,\theta \,]^{-\frac{1}{\alpha_i}} \; \frac{1}{\alpha_i}\, r^{\frac{1-\alpha_i}{\alpha_i}} \; dG(\theta)}\,; \tag{83}$$

and

$$\begin{aligned} R \;=\; & \frac{1}{N} \Bigg[F + a \sum_{i=1}^{N} \int_{\underline{\theta}}^{\widetilde{\theta}_i} \left(\frac{r + m(\theta)}{v_i\,\theta} \right)^{\frac{1}{\alpha_i}} dG(\theta) + a \sum_{i=1}^{N} \int_{\widetilde{\theta}_i}^{\overline{\theta}} \left(\frac{r}{v_i\,\theta} \right)^{\frac{1}{\alpha_i}} dG(\theta) \\ & + b \int_{\underline{\theta}}^{\overline{\theta}} \left(\sum_{i=1}^{N} x_i^u(\cdot) \right)^2 dG(\theta) - \sum_{i=1}^{N} \int_{\widetilde{\theta}_i}^{\overline{\theta}} [\, v_i\,\theta \,]^{-\frac{1}{\alpha_i}} \; r^{\frac{1+\alpha_i}{\alpha_i}} \, dG(\theta) \end{aligned}$$

$$- \sum_{i=1}^{N} \int_{\underline{\theta}}^{\widetilde{\theta}_i} \left\{ [\, r + m(\theta) \,] \left(\frac{r + m(\theta)}{v_i \, \theta} \right)^{\frac{1}{\alpha_i}} - m(\theta) \, \underline{x}_i \right\} dG(\theta) \Bigg]. \tag{84}$$

Proof. (82) follows immediately from the assumptions that $C(X) = a\,X + b\,X^2$ and $m(\theta) = C'(X^u)$ for all $\theta \in [\,\underline{\theta}, \overline{\theta}\,]$. The proof of (83) parallels the proof of (7).

As in the proof of Proposition 1, it is readily verified that the utility's participation constraint binds at the solution to [RP-F]. Therefore, from (68):

$$\begin{aligned} R \; = \; & \frac{1}{N} \Bigg[F + \int_{\underline{\theta}}^{\overline{\theta}} C\,(X^u)\; dG(\theta) - \sum_{i=1}^{N} \int_{\underline{\theta}}^{\widetilde{\theta}_i} \{ r\; x_i^u(\cdot) \; - \; m(\theta) \, [\, \underline{x}_i - x_i^u(\cdot) \,] \, \} \, dG(\theta) \\ & \qquad - \sum_{i=1}^{N} \int_{\widetilde{\theta}_i}^{\overline{\theta}} r\; x_i^u(\cdot)\; dG(\theta) \Bigg] \\ = \; & \frac{1}{N} \Bigg[F + \int_{\underline{\theta}}^{\overline{\theta}} \{\, a\,X^u + b\;(X^u)^2 \,\} \; dG(\theta) \\ & - \sum_{i=1}^{N} \int_{\underline{\theta}}^{\widetilde{\theta}_i} \left\{ r \; \left(\frac{r + m(\theta)}{v_i\,\theta} \right)^{\frac{1}{\alpha_i}} - \; m(\theta)\; \underline{x}_i + m(\theta) \left(\frac{r + m(\theta)}{v_i\,\theta} \right)^{\frac{1}{\alpha_i}} \right\} dG(\theta) \\ & \qquad - \sum_{i=1}^{N} \int_{\widetilde{\theta}_i}^{\overline{\theta}} r \left(\frac{r}{v_i\,\theta} \right)^{\frac{1}{\alpha_i}} \; dG(\theta) \Bigg]. \end{aligned} \tag{85}$$

(84) follows immediately from (85). ■

2. Numerical Solution Techniques.

We now briefly describe the techniques employed to derive explicit solutions to problems [RP-n], [RP-E], and [RP-F]. (Additional details are available upon request from the authors.)

First, the Newton-Raphson's Iteration Method ("Newton's Method") was employed to solve (70) and (71) for the values of r and R that the regulator will implement in the absence of any DR policy. These values, in turn, were employed to identify $\underline{x}_L$ and $\underline{x}_H$, the baseline consumption levels for the two consumer types.

Second, employing the identified values for $\underline{x}_L$ and $\underline{x}_H$, Newton's method was employed to solve (79) and (80) for the values of r and $m(\theta)$ (and thus $\widetilde{\theta}_L$ and $\widetilde{\theta}_H$) at the solution to [RP-E]. Gaussian Quadrature Numerical Integration was employed to perform integration over the density function $g(\theta)$, which was estimated using maximum likelihood estimation.

Third, (81) was solved to identify the value of R at the solution to [RP-E]. Quasi-Monte Carlo Integration (Owen, 2009) was employed to perform integration over the $m(\theta)$ function that is defined implicitly in (79).

Fourth, corresponding techniques were employed to solve (82), (83), and (84) for the values of $m(\theta)$, r, and R that constitute the solution to [RP-F]. In particular, Quasi-Monte Carlo Integration was first employed to solve (82) for $m(\theta)$ as a function of r. Newton's method and Gaussian Quadrature Numerical Integration were then employed to solve for the optimal values of r, $\widetilde{\theta}_L$, and $\widetilde{\theta}_H$. Given these values, R defined in (84) was then computed, employing Quasi-Monte Carlo Integration over the implicitly defined function $m(\theta)$ in (82).

Finally, numerical integration was employed to calculate the relevant expected values of welfare, DR compensation, and peak-load production costs.

3. Estimated Outcomes in Additional Settings.

We now present estimates of the welfare gains that demand response (DR) policies can produce in two additional settings: the *CA benchmark setting* and the *NE benchmark setting*. These settings parallel the benchmark setting analyzed in Section 5, except that relevant parameter values are modified to reflect conditions in the California and the ISO New England regions. Five primary modifications are implemented.

First, the cost parameters a and b reflect Bushnell's (2007) estimates for the the two regions. Second, the minimum ($\underline{\theta}$), maximum ($\overline{\theta}$), shape (h), and scale (ζ) parameters of the (gamma) distribution reflect 2013 temperature data specific to each of the two regions.[42] Third, $N_L + N_H$ is set to ensure that expected demand is equal to the average hourly load in the relevant region in 2013.[43] Fourth, $\frac{N_L}{N_L+N_H}$ reflects the fraction of residential customers in the relevant regions.[44] Fifth, F is set at 46% of the revenue collected from ratepayers in

[42] The region-specific temperature data are drawn from the National Oceanic and Atmospheric Administration (2014).

[43] Region-specific load data are derived from EIA (2014c).

[44] These region-specific data are derived from EIA (2014a).

the relevant regions in 2013.[45] Table B1 presents these parameter values.

Parameter	CA Benchmark Setting	NE Benchmark Setting
a	28.53	18.13
b	0.0015	0.0021
$\underline{\theta}$	0	0
$\overline{\theta}$	61	66
h	10.629	10.241
ζ	2.187	3.494
$N_L + N_H$	$29,334$	$13,741$
$\frac{N_L}{N_L+N_H}$	0.8775	0.8734
F	$19,589,413$	$7,856,599$

Table B1. Parameters in the CA and NE Benchmark Settings.

Tables B2 and B3 present the outcomes corresponding to the outcomes reported in Table 1 for the CA benchmark setting and the NE benchmark setting, respectively.[46]

[45] Revenue in a region is calculated as the product of the average retail rate for electricity and the total load in the region (EIA, 2014b,c).

[46] Recall that $E\{C^P(\cdot)\}$ is the utility's expected production cost in states $\theta \in [\theta_m, \overline{\theta}]$, where θ_m is the smallest realization of θ for which strictly positive demand response arises both at the solution to [RP] and under the optimal FERC policy. $\theta_m = 31.2$ in the California benchmark setting. $\theta_m = 34.2$ in the NE benchmark setting.

	No DR Policy	Optimal DR Policy	FERC DR Policy
r	120.95	108.62	114.40
R	417.83	443.53	437.39
$E\{m(\theta)\}$	0	27.04	105.43
$E\{C^P(\cdot)\}$	7.31	5.34	4.53
$E\{W\}$	11.02	13.72	11.86

Table B2. Outcomes in the CA Benchmark Setting.

	No DR Policy	Optimal DR Policy	FERC DR Policy
r	81.17	72.35	74.59
R	413.85	437.03	433.03
$E\{m(\theta)\}$	0	15.58	76.50
$E\{C^P(\cdot)\}$	2.71	1.56	1.18
$E\{W\}$	4.90	6.37	5.84

Table B3. Outcomes in the NE Benchmark Setting.

Tables B2 and B3 indicate that the optimal DR policy increases expected welfare above the level achieved in the absence of any DR policy by 24.5% in the California benchmark setting and by 30.0% in the NE benchmark setting. These percentage gains exceed the corresponding gain in the (PJM) benchmark setting analyzed in Section 5. The more pronounced gains reflect in part the utility's more steeply sloped marginal cost function (i.e., the larger values of a and b) in the California and the NE benchmark settings.

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