Who Participates in Risk Transfer Markets? The Role of Transaction Costs and Counterparty Risk

Eric Stephens
University of Alberta

James Thompson
University of Waterloo

June 2012
Who Participates in Risk Transfer Markets? The Role of Transaction Costs and Counterparty Risk

Eric Stephens  
University of Alberta  
eric.stephens@ualberta.ca

James R. Thompson*  
University of Waterloo  
james@uwaterloo.ca

First Version: February 2012  
This Version: June 2012

Abstract

We analyze the role of transaction costs in risk transfer markets. For example, when these markets are in their infancy, they are characterized by few contracts and high transaction costs. In this case, we show that only highly risk-averse buyers (e.g., hedgers) exist in the market alongside high quality counterparties, and no asymmetric information can be present on either the quality of the risk being transferred or the quality of the counterparty to which the risk is ceded. With lower transaction costs, we show that less risk-averse buyers (e.g., speculators) will enter the market thereby increasing risk transfer; however, these buyers will choose to contract with less stable counterparties. When transaction costs are low, we show that asymmetric information on the quality of the risk being transferred and of the quality of the counterparties can exist in equilibrium. Finally, we analyze the effect of a transaction tax, which is viewed simply as an increase in transaction costs. Such a tax is shown to push relatively less risk-averse buyers out of the market, which tends to reduce the relative number of unstable counterparties. In addition, we show that it reduces the rents that can be extracted due to asymmetric information.

Keywords: Risk Transfer, Transaction Costs, Counterparty risk, Transaction Taxes.

*We would like to thank Alex Edmans, Hector Perez Saiz, as well as seminar participants at the Federal Reserve Board of Governors, the University of Pennsylvania (Wharton), Georgia State University (RMI), 2012 CEA meetings and the University of Waterloo for helpful comments. Note that part of this paper was written while Thompson was visiting the Wharton School for whom he would like to thank for their hospitality.
1 Introduction

The rate of development of risk transfer markets in the past 40 years has been extraordinary. Seemingly overnight, new instruments to disperse many forms of risk such as interest rate, credit and foreign exchange have arisen. A characterizing feature of the evolution of these markets is that they begin small, with high transaction costs. As more players join the market, transaction costs decrease as contracts are standardized and more efficient means to find counterparties are developed.\footnote{For example, it was recently estimated that in Europe, the average transaction cost in the OTC market has declined to as little as 55 Euros per 1 million Euro transaction, and this amount is expected to decrease further as electronic platforms become more prevalent (Deutsche Börse Group, “The Global Derivatives Market: An Introduction”, White Paper (2008)).} For example, consider the formation of the International Swaps and Derivative Association (ISDA) in 1985. Prior to this, each deal required a customized contract and great care needed to be taken to mitigate legal risk, a cost which ISDA standardization decreased considerably.\footnote{For a discussion of this and related issues see Ludwig (1993).} Over-the-counter (OTC) markets, which are the motivating example of this paper due to their bilateral nature and potential exposure to counterparty risk had a notional size of approximately 614 trillion dollars as of 2007, accounting for almost 84 percent of the total derivative market. The massive growth in OTC contracts can be seen by looking back as recently as 2001 when the estimated notional size was just under 100 trillion dollars.\footnote{Deutsche Börse Group, “The Global Derivatives Market: An Introduction”, White Paper (2008), BIS “Triennial Central Bank Survey: Foreign Exchange and Derivative Market Activity in 2001”, Basel: Bank for International Settlements (2002))} Further, it was OTC contracts such as credit default swaps which were in their infancy only a decade before, that played an important role in the credit crisis of 2007-2009. As such, understanding the relationship between transaction costs and counterparty risk is an important piece of the financial stability puzzle. In addition, the resurgence of interest in financial transaction taxes within the European Union and elsewhere makes the study of transaction costs of further relevance.

In this paper we model the effect of transaction costs in a simple market for risk transfer. The extant literature focuses largely on the affect of transaction costs on security demand, prices and volatility. In contrast, we abstract from these issues and study transaction costs from a different point of view. Namely, that decreased costs can bring new players into the market, and those new players may contract with very different counterparties. We find that when transaction costs are sufficiently high, only buyers that are very averse to risk, which we refer to as hedgers, can exist in the market and contract with relatively safe sellers (the party to which the risk is ceded). Furthermore, we show that with high transaction costs, the market cannot sustain asymmetric information regarding either the quality of the seller or the underlying risk. In other words, perfect information is a prerequisite for contracting to take place when transaction costs are high (e.g., when markets are in their infancy). When transaction costs are low, in addition to hedgers the market will also consist of buyers that are less risk-averse, which we refer to as speculators. We show that speculators contract with less stable counterparties, so that decreasing transaction costs tends to result in an increase in the relative number of unstable sellers. With the influx of speculators,
the size of the market will increase and asymmetric information on the quality of the risk being transferred can exist in equilibrium, unlike when transactions costs are high. Similarly, we show that asymmetric information on the quality of the seller (counterparty risk) can exist in equilibrium when transaction costs are sufficiently low and has two noteworthy effects on the relative number of unstable sellers in the market: a reduction due to the elimination of risk-averse buyers in the market, and an increase due to buyers who otherwise would have contracted with stable sellers under perfect information contract with unstable sellers with certainty instead of a pool of stable and unstable sellers when there is asymmetric information. Which effect dominates depends on competition amongst unstable sellers. Finally, we analyze a transaction tax which is effectively an increase in transaction costs. We show that such a tax decreases risk transfer by driving buyers with relatively low risk-aversion (e.g., speculators) out of the market, which consequently reduces the relative number of unstable counterparties (this requires some assumptions on competition when there is asymmetric information regarding seller type). Furthermore, we show that a tax will decrease the information rents that can be extracted when there is asymmetric information.

The intuition behind our results is as follows. A contingent contract is written on an underlying risk in which the buyer insures itself from a potential loss, much like an insurance contract. Sellers are risk neutral and possess a risky portfolio with the possibility of default. Building on the model of Stephens and Thompson (2011), we introduce good and bad sellers with different probabilities of default (counterparty risk). The bad sellers offer lower quality protection for a lower price, while the good sellers must charge more for superior protection. On the demand side of the market we first model a population of buyers who differ solely in their aversion to risk. Buyers that are highly risk-averse are willing to pay higher prices for better protection at the good sellers, while buyers that are less risk-averse tolerate more counterparty risk at the bad sellers in exchange for lower prices. When transaction costs are low, the market is partitioned between those who contract with bad sellers and those who contract with good ones. The most responsive buyers to transaction costs are those who are the least risk-averse. Consequently, when transaction costs increase, buyers that would have contracted with bad sellers are first to drop out of the market. Thus, the relative number of bad sellers in the market is decreasing in transaction costs.

The impact of asymmetric information is more subtle. To analyze the case of asymmetric information over seller type, we assume that a proportion of bad sellers have private information over their type and thus can pool with the good sellers while other bad sellers are revealed before contracts are written. Although the bad sellers that pool can extract rents since they can charge higher prices than they otherwise could have, it is also possible that the revealed bad sellers also earn rents. This occurs because when transaction costs are low, revealed bad sellers contract with buyers that strictly prefer them over contracting with the pool (i.e., those buyers that have little aversion to risk). This eases the competition among bad sellers since the pooling bad sellers compete with good sellers in the pool, instead of the revealed bad sellers. Thus revealed bad sellers can charge higher prices than if there was perfect information. We find that when transaction costs are low, there are two noteworthy effects on the relative number of good and bad sellers in the
market. First, since the revealed bad sellers charge higher prices, this drives the least risk-averse buyers (speculators) out of the market. Since these parties would have otherwise contracted with bad sellers under perfect information, this decreases the relative number of bad sellers. Conversely, contracting with the pool of good and bad sellers is less attractive than contracting with the good seller with certainty (which can be done in the perfect information case). As such, we show that some buyers that would contract with a good seller with perfect information opt for a bad seller over the pool with asymmetric information, thus increasing the relative number of bad sellers. Which effect dominates depends on the degree of competition among the revealed bad sellers. When there are sufficiently many, competition drives the price down to that which would have resulted under perfection information and the latter effect dominates. When transaction costs are sufficiently high, the market will cease to exist with asymmetric information of this type where it otherwise would have in its absence. This is the case in which with perfect information, only highly risk-averse buyers are in the market contracting with good sellers. With asymmetric information, the value of the contract goes down since the pool of sellers offers an inferior contract to that of the good seller (with certainty). With sufficiently high transaction costs, even the most risk averse buyers leave the market because of the inferior contract with the pool.

We also consider the more standard problem of asymmetric information regarding the underlying risk on which the contract is written. We assume that there are two types of buyers, one with lower risk, the other with higher risk. With high transaction costs, we demonstrate a lemons result in which low risk buyers are driven out of the market, so that asymmetric information cannot exist in equilibrium. This occurs because the low-risk buyers would be in the market if they could be identified as such, however when forced to pool with high-risk types, they drop out. Thus, asymmetric information cannot be sustained in equilibrium when transaction costs are high. When transaction costs decrease, low-risk buyers are willing to pool with high-risk ones since the lower transaction costs make the pooled contract more attractive. Thus, asymmetric information can be a feature of the equilibrium when transaction costs are sufficiently low.

Finally, we consider the implications of a transaction tax, which is interpreted as an increase in transaction costs. Such an increase causes the least risk-averse buyers to drop out of the market, thereby reducing the relative number of bad sellers. With both types of asymmetric information, a tax reduces the information rents. With asymmetric information over seller quality, the revealed bad sellers must lower their price since the transaction tax means that some of the buyers who would have otherwise contracted with them over the pool drop out of the market. This causes increased competition among the revealed sellers and so they decrease the price to attract more buyers, thereby lowering their information rents. By lowering the price, some buyers from the pool switch to bad sellers making it less likely that a pooling bad seller will obtain a contract and thus decreasing the rents of the pooling bad sellers as well. With asymmetric information over buyer risk type, an increase in transaction costs causes proportionally more low-risk buyers to leave the

\[\text{4} \text{When there is asymmetric information over seller quality this requires sufficient competition and is discussed in greater detail below.}\]
market than high-risk. To understand this result, consider a low risk and high risk buyer with the same level of risk aversion. Since they pay the same amount in the pool, the low risk buyers benefit less from the contract since they are less likely to receive a payment from the seller and so are more likely to leave the market. Therefore, the equilibrium beliefs of the sellers will put more weight on a buyer being a high-risk type when transaction costs increase, and so set a higher price. Since the price increases as transaction costs increase, the information rents of the high-risk types are decreasing in transaction costs.

**Literature Review**

This paper contributes to the literatures on transaction costs and counterparty risk. The transaction cost literature has largely focused on the effects that these costs have on demand, prices, and market volatility. Lo et al. (2005), Vayanos and Vila (1999), Vayanos (1998) and Allen and Gale (1994) develop GE models to determine the affect of transaction costs on these variables. Lo et al. (2005) demonstrate the intuitive illiquidity discount in asset markets and find that even small transaction costs can have significant effects. They consider a base model in which agents trade frequently (continuously) and show that with a fixed transaction cost, agents may considerably reduce their trade with some periods of no trade at all in equilibrium. Vayanos (1998) finds that asset prices can increase or decrease, depending on demand. If an increase in transaction costs causes buyers to hold their positions for longer, prices may actually increase. Vayanos and Vila (1999) show how one can get similar counterintuitive results based on the interplay between an illiquid asset that is affected by transaction costs and a liquid asset that is not. Similar to our work, Allen and Gale (1994) study limited market participation, however the focus in that work is on how limited participation interacts with market volatility. In contrast to these papers, we determine the type of players that enter and exit the market and study the effects on counterparty risk in multiple informational environments. As such, we do not consider the GE effects of security demand on each individual player and instead focus on the incentive aspect of transaction costs. This is done at the cost of being unable to make broad welfare statements, but it greatly benefits the analysis by elucidating the mechanisms behind our results as simply as possible.

In the literature on counterparty risk and incentives, Thompson (2010), Acharya and Bisin (2010) and Biais, Heider and Hoerova (2012) demonstrate moral hazards that may be present on the sell side of the market wherein the seller (or insurer) may take positions which increase counterparty risk and are not in the best interest of the buyer (or insured party). In our paper, we model counterparty risk exogenously, focusing on differences between sellers when there are heterogenous buyers and transaction costs, which these papers do not study. Stephens and Thompson (2011) analyze counterparty risk with multiple sellers and, similar to the current paper, show that a risk-averse buyer will tend to contract with a good seller. They also show that asymmetric information on the seller can endogenously increase the counterparty risk of good sellers as they are forced to compete with bad ones. In contrast, we model many buyers and sellers so that we can show how
changes in transaction costs affect the composition of buyers and sellers in the market. In addition, we consider asymmetric information on both sides of the market.

The rest of the paper is organized as follows. Section 2 outlines the model, Section 3 analyzes the equilibrium with full information, Section 4 introduces asymmetric information on seller quality, and Section 5 considers asymmetric information on buyer risk type. In Section 6 we analyze the effect of a transaction tax, and Section 7 concludes. The Appendix contains non-trivial proofs.

2 Model

The model builds on that of Thompson (2010) and Stephens and Thompson (2011). An individual (or institution) possesses a risky income stream that can be contracted on by some derivative product. For modeling convenience, we assume that the buyer makes an upfront payment which entitles them to a payout in the event that an observable loss occurs. An upfront payment is not crucial and the analysis can be viewed as any state-contingent contract between two parties.

2.1 Sellers

There is a measure $N$ of both two types of seller, which are referred to as "good" and "bad", denoted $j = \{G, B\}$. Sellers have initial assets (or wealth) $W_j$ and receive returns of $1 + r_j$ on all investments. The seller collects $P_j < 1$ from the buyer in return for a contingent payment of size 1, and we assume for simplicity that a seller only contracts with one buyer.\footnote{Our focus in this paper is not on the size of contract, but rather on determining which types of agents will purchase contracts and with which types of counterparties. Adding endogenous contract size and/or non-exclusive contracts would needlessly complicate our analysis and, as discussed above, these results have already been well addressed in the literature. For a generalization of the model that allows a contract size choice and the ability of an individual to contract with many sellers, see Stephens and Thompson (2012).} We introduce counterparty risk in the simplest way possible by assuming that sellers default with exogenous probability $1 - q_j$, in which case they receive zero return and default on the claim. The probability of the buyer making a claim is $1 - p$ and thus we characterize the sellers expected return (with a contract) as follows

$$E(\pi_j) = q_j[p(1 + r_j)(W_j + P_j) + (1 - p)((1 + r_j)(W_j + P_j) - 1)], \quad j \in \{B, G\}. \quad (1)$$

The following expression characterizes the zero-profit price $P_j^0$, which equates the expected return of a seller with and without a contract:

$$P_j^0 = (1 - p)/(1 + r_j). \quad (2)$$

An important restriction here is that the contract itself has no bearing on the probability of seller default, which substantially simplifies the analysis and permits closed-form solutions of equilibrium prices. Endogenous default does not affect the qualitative results and we refer the interested reader to Stephens and Thompson (2011) which allows for this feature. Less restrictive is the assumption
that there is no recovery value and no collateral when sellers cannot pay and simply note that allowing for partial recovery and collateral has no interesting qualitative effects as this simply reduces counterparty risk.\footnote{Importantly, if collateral is inserted into the model, it cannot be perfect thus mitigating all counterparty risk. Realistically, collateral tends to decrease in value in times of stress so assuming that collateral can always perfectly eliminate counterparty risk is not realistic.} Finally, we assume throughout the paper that $r_B > r_G$ and that $q_G > q_B$. Thus a bad seller can be interpreted as having a higher return portfolio, which in turn allows them to charge a lower zero-profit price, but at the cost of a higher risk of default. This is the interesting case in which neither seller dominates the other as would be true if a particular seller could charge lower prices as well as provide better quality protection.

### 2.2 Buyers and Transaction Costs

There are many buyers who all share the same underlying asset risk but who’s preferences differ over the desire to shed that risk. The asset can take one of two state contingent values: with probability $p$ it returns $R_B > 1$ (the high state) and with probability $1 - p$ it returns nothing (the low state). As described above, the buyer can contract with a seller to transfer the risk of the low state. We assume for simplicity that the risk transfer contract is of size 1.\footnote{The fixed contract size assumption is discussed in more detail in Thompson (2010) and Stephens and Thompson (2011).} If a buyer $k$ incurs the loss from the low state and has not transferred this risk it suffers the cost $Z_k \geq 0$. We assume that there is a measure $\mathcal{Z}$ of buyers such that $Z_k \in [0, \mathcal{Z}]$. This cost could represent an endogenous reaction to a shock to the buyer’s portfolio; however, we will not model this here. It is this cost that makes the buyer averse to holding risk.\footnote{For a more formal discussion of $Z$ and its close relationship with risk aversion, see Stephens and Thompson (2011).} With only one asset type, whether $Z$ is known to sellers is unimportant. This is not the case in Section 5, where we allow for two asset types and make the natural assumption that $Z$ is not known to sellers. When the buyer experiences a loss and is under contract with seller $j$, the buyer receives 1 if the seller is solvent and suffers the cost $Z_k$ if the seller is insolvent.\footnote{The cost of counterparty risk and the cost of not entering the market are set equal for simplicity and can be allowed to differ without changing the qualitative results.} We assume that entering into a contract comes with a cost $\Delta$. As discussed above, this cost can be interpreted in a number of ways including a search cost, legal fees or as will be discussed below, a transaction tax. Given competition among sellers, the cost must be absorbed by the buyers, however this is not crucial. Note that, as in Stephens and Thompson (2011), for what follows we will refer to buyers with high $Z_k$ as “hedgers” and those with low $Z_k$ as “speculators”. This approach is in line with Keynes (1930) and Hicks (1946), who present early work that differentiates hedgers from speculators according to their risk aversion, as in our interpretation of $Z$. Subsequent literature such as Hirshleifer (1975, 1977) has also emphasized informational asymmetries and beliefs in addition to risk aversion when modeling speculation. To analyze beliefs would require a full market microstructure model which is beyond the scope of this
We note that the results below will obtain when there is divergent beliefs, provided that
the hedger has a higher willingness to pay due to risk aversion. In other words, the hedger must
treat the contract as more than just a bet and thus be willing to pay a premium to be relieved of
the risk. The following expression characterizes the expected payoff of buyer type \(k\) that contracts
with seller type \(j\):

\[
E(\pi_{kj}) = pR_B + (1 - p)q_j - (1 - p)(1 - q_j)Z_k - P_j - \Delta \quad \forall Z_k \in [0, Z].
\] (3)

3 Equilibrium with Full Information

Each buyer chooses a seller type with whom to contract. Given that the underlying asset risk is
the same for each buyer, their choice depends solely on their preference parameter \(Z_k\). Since those
with high \(Z_k\) suffer a larger cost when they do not transfer risk, and when they do transfer risk
but the seller is unable to fulfill the contract, they are the most willing to enter the market and
the most willing to contract with a good seller. We assume that the measure of each type of seller
is sufficiently large so that competition under perfect information drives prices down to that which
earn zero profit. Define \(\hat{Z}\) as the buyer that is indifferent between contracting with a good and a
bad seller as follows:

\[
\hat{Z} = \frac{P^0_G - P^0_B}{(1 - p)(q_G - q_B)} - 1.
\] (4)

We assume for simplicity that when a buyer is indifferent between a good and bad seller, the good
is chosen. To focus on the interesting case in which both sellers types are potentially active in
equilibrium, we let \(Z > \hat{Z}\). This assumption does not imply that both types of sellers must be
active, since (4) does not consider the effect of transaction costs on market participation. Define \(Z_j\)
as the buyer that is indifferent between contracting with seller \(j\) and not contracting at all. These
two conditions are given as follows

\[
Z_G = \frac{P_G + \Delta}{q_G(1 - p)} - 1
\] (5)

\[
Z_B = \frac{P_B + \Delta}{q_B(1 - p)} - 1.
\] (6)

The following proposition analyzes the effect of transaction costs on the composition of buyers and
sellers in the market.

**Proposition 1** The effect of transaction costs on market composition is characterized as follows:

i. When \(\Delta > (1 + \overline{Z})(1 - p)q_G - P^0_G\), there is no market for risk transfer.

---

\(^{10}\) One could model beliefs of the buyer through the \(Z\) parameter in addition to capturing risk aversion. Those
buyers that have a more favorable view of the contract would have a higher \(Z\). We do not pursue this interpretation
further in this paper since it would complicate the analysis by requiring a richer model of both buyers and sellers.
ii. When \( \frac{(P_0^G - P_0^B)q_G}{q_G - q_B} - P_0^G \leq \Delta \leq (1 + \bar{Z})(1 - p)q_G - P_0^G \), only good sellers exist in the market.

iii. When \( \Delta < \frac{(P_0^G - P_0^B)q_G}{q_G - q_B} - P_0^G \), both good and bad sellers exist in the market.

When \( \Delta > (1 + \bar{Z})(1 - p)q_G - P_0^G \), transaction costs are prohibitively high and no buyer wishes to contract with any seller. Thus, the risk transfer market does not exist. In the intermediate case, only those buyers for which \( Z_k \geq \hat{Z} \) contract in the market. In other words, those buyers that have stronger hedging motives participate in the market, and in this case these are precisely the buyers that contract with good sellers. When transaction costs are relatively low, buyers for which \( Z_k < \hat{Z} \) participate in the market and contract with bad sellers. These are what we call speculators who enter only when transaction costs are sufficiently low. For what follows, we define total risk transfer as the total number of buyers that contract in the market. The following corollary to Proposition 1 summarizes the effect of transaction costs on risk transfer and counterparty risk.

**Corollary 1**

i. When \( \Delta \leq (1 + \bar{Z})(1 - p)q_G - P_0^G \) (i.e., when a market exists), risk transfer increases (decreases) as transaction costs decrease (increase).

ii. When \( \Delta < \frac{(P_0^G - P_0^B)q_G}{q_G - q_B} - P_0^G \) (i.e., when both types of seller are active), the relative number of bad sellers increases (decreases) as transaction costs decrease (increase).

As transaction costs decrease, this relaxes the participation constraint of all buyers’ and permits some who otherwise would not find it optimal to contract to enter the market. When transaction costs are sufficiently low, further decreases in such costs will bring in buyers who prefer to contract with bad sellers, and thus the relative number of bad sellers increases as transaction costs decrease.

4 Asymmetric Information on Seller Quality

An important feature of many markets for financial risk transfer is that the buyer may not have perfect information as to the counterparty risk of the seller, whereas the seller is better informed about its own risk.\(^{11}\) To analyze asymmetric information on seller quality, we restrict ourself to the most interesting case in which there exists a market under full information. This requires that transaction costs are not so high that even the most risk averse buyer does not participate. This is ensured by the following assumption (see Proposition 1).

**Assumption 1** \( \Delta < (1 + \bar{Z})(1 - p)q_G - P_0^G \).

\(^{11}\)An example of the opacity in these types of markets comes from the credit crisis of 2007-2009, in which we saw the rapid and repeated downgrading by rating agencies of large sellers of credit default swaps such as Ambac, MBIA and AIG. See Acharya and Bisin (2010) for a more in-depth discussion.
To introduce asymmetric information, we consider the situation in which buyers are only able to identify a fraction of the bad sellers as being bad. In particular, let the exogenous variable $\phi \in [0, 1]$ represent the fraction of bad sellers whose type is initially unknown to buyers (and thus they have the opportunity to pool with good sellers or reveal their type), while $1 - \phi$ represents the fraction who’s type is revealed. We assume without loss of generality that good sellers type information is never revealed. Thus, $\phi = 0$ is the full information environment explored above, while $\phi = 1$ is the case in which the market is completely opaque. Recall that the measure of sellers of each type is $N$. In this section we assume that $N \geq Z + 1$, which implies perfect competition when there is no asymmetric information.\footnote{This is a simplifying assumption that provides a sufficient condition for a unique solution in Lemma 4 discussed below, and can be relaxed while still obtaining our results.}

The analysis is broken down into two cases, the first in which no revealed bad seller can exist in the market, and the second in which they can. Lemma 1 describes conditions on $\Delta$ which yield these two cases.

**Lemma 1** For sufficient transaction cost $\Delta \geq \frac{(P^0_G - P^0_B)(\phi q_B + q_G)}{q_G - q_B} - P^0_G$, no bad seller that has been revealed can exist in the market. When $\Delta < \frac{(P^0_G - P^0_B)(\phi q_B + q_G)}{q_G - q_B} - P^0_G$, revealed bad sellers can be active.

**Proof.** See Appendix.

### 4.1 Asymmetric Information on Seller Quality: High Transaction Costs

We first consider the case in which no buyer would knowingly contract with a bad seller, as detailed in Lemma 1 this will occur for sufficiently high transaction costs. Given competition among good sellers, each charges $P^G_0$ in equilibrium. Since charging a price too low would reveal themselves, the bad sellers who are not revealed simply charge the price $P^G_0$ and pool with the good sellers. In this case, any equilibrium involves only the pool of sellers and any bad seller that is revealed is not in the market. To ensure that the market exists we must put an upper bound on transaction costs. The following condition on $\Delta$ ensures that there are buyers in the market who contract with the pool of sellers.

$$\frac{(P^0_G - P^0_B)(\phi q_B + q_G)}{q_G - q_B} - P^0_G \leq \Delta < (1 + Z)(1 - p)\frac{\phi q_B + q_G}{1 + \phi} - P^0_G,$$

The expression on the right hand side represents the value of $\Delta$ at which the most risk averse individual will choose not to participate. This value is lower than the full information case and leads to the following proposition.

**Proposition 2** When $$(1 + Z)(1 - p)\frac{\phi q_B + q_G}{1 + \phi} - P^0_G < \Delta < (1 + Z)(1 - p)q_G - P^0_G$$ the market collapses due to the presence of asymmetric information over seller quality.
Thus, when transaction costs are sufficiently high, the market collapses (the left hand side of the expression in the proposition) where it otherwise would have existed with perfect information (the right hand side of the expression in the proposition). The intuition behind this result is that with perfect information, buyers can contract with a good seller with certainty. With asymmetric information, the expected quality of the pool is less than that of a good seller while price remains unchanged.

4.2 Asymmetric Information on Seller Quality: Low Transaction Costs

As detailed in Lemma 1, for sufficiently low transaction costs bad sellers can exist in equilibrium. When revealed bad sellers can be active, bad sellers that are not revealed can either charge the price \( P_G^0 \) and pool with the good sellers (as above), or reveal themselves alongside those that nature has already revealed. Bad sellers, whether revealed by nature or by choice, set a price \( P_B^* \) given competition from other revealed bad sellers. The following Lemma characterizes properties of the price of a bad seller that must hold in equilibrium.

**Lemma 2** The equilibrium price at the bad sellers \( P_B^* \) must satisfy:

i. \( N(1-\phi) = \hat{Z}_{pl}(P_B^*) - Z_B(P_B^*) \) if \( N(1-\phi) < \hat{Z}_{pl}(P_B^0) - Z_B(P_B^0) \)

ii. \( P_B^* = P_B^0 \) if \( N(1-\phi) \geq \hat{Z}_{pl}(P_B^0) - Z_B(P_B^0) \).

Where, analogous to \( \hat{Z} \) described in (4), we define

\[
\hat{Z}_{pl}(P_B) = \frac{(P_G^0 - P_B)(1 + \phi)}{(1 - p)(q_G - q_B)} - 1, \tag{8}
\]

which represents the buyer that is indifferent between contracting with the pool of sellers or with a revealed bad seller and

\[
Z_B(P_B) = \frac{P_B + \Delta}{q_B(1 - p)} - 1 \tag{9}
\]

represents the buyer that is indifferent between contracting with a bad seller and dropping out of the market. Lemma 2 determines the equilibrium price that a revealed bad seller will charge. To obtain this price, there cannot be an excess of buyers wishing to contract with the revealed sellers. To understand this, consider the problem of the sellers. If each seller charges \( P_B^0 \) and there are more than \( N(1-\phi) \) buyers that wish to contract with them, they could raise the price until only \( N(1-\phi) \) buyers wish to do so. Since the payoff to the sellers from taking the contract is strictly increasing in price, the sellers would always raise their price until \( N(1-\phi) = \hat{Z}_{pl}(P_B^*) - Z_B(P_B^*) \) (item i of Lemma 2). When there are too many revealed sellers, competition drives prices down to that which earn zero profit (item ii of Lemma 2). Given Lemma 2, we define \( \phi \) such that \( \phi \leq \hat{\phi} \).
implies $P_B^* = P_B^0$ and $\phi > \hat{\phi}$ implies $P_B^* > P_B^0$. We rewrite item i of Lemma 2 as follows:

$$
(1 - \phi)N = \frac{(P_G^0 - P_B^*) (1 + \phi)}{(1 - p)(q_G - q_B)} - \frac{P_B^* + \Delta}{q_B(1 - p)}.
$$

Rearranging for $P_B^*$ yields:

$$
P_B^* = \frac{P_G^0 (1 + \phi)}{(1 - p)(q_G - q_B)} - \frac{(1 - \phi)N - \Delta}{q_B(1 - p)} \frac{1}{q_B(1 - p) + \frac{(1 + \phi)}{(1 - p)(q_G - q_B)}}.
$$

The following lemma describes some useful properties of $P_B^*$.

**Lemma 3**

i. $P_B^* < P_G^0$

ii. $\frac{dP_B^*}{d\phi} \geq 0$, where the inequality is strict for $P_B^* > P_B^0$.

**Proof.** See Appendix.

The first item is relatively straightforward. The price that a revealed bad seller can charge must be below that of which the pool is charging, otherwise no buyer would ever contract with a revealed bad seller. The second item says that the more bad sellers that pool with the good sellers, the higher the price that the revealed bad sellers can charge. This occurs because the competition among revealed bad sellers decreases when there are less of them. The price charged by the revealed bad sellers is limited however; it can not rise too high or bad sellers that were not revealed would wish to reveal themselves alongside those that were revealed by nature. Therefore, in equilibrium the upper bound on $P_B^*$ is determined by the optimization problem of the bad sellers who are not revealed by nature. In particular, these sellers have the choice to reveal themselves by charging a price below $P_G^0$. An unrevealed bad seller will do so if the expected profit from revealing exceeds the expected profit from remaining in the pool. Thus, the following condition must hold in equilibrium:

$$
(1 + r_B)P_B^* - (1 - p) \leq \frac{Z - Z_{pl}(P_B^*)}{N(1 + \phi)} [(1 + r_B)P_G^0 - (1 - p)].
$$

Where $(Z - Z_{pl}(P_B^*))/N(1 + \phi)$ is the probability that a pooling bad seller obtains a contract (recall that there are $N$ good sellers and $\phi N$ potential pooling bad sellers), so that the right hand side of (12) is the expected profit of a pooling bad seller. When (12) is not satisfied, a pooling bad seller has the incentive to reveal itself. The following result determines the maximum $\phi$ for which the pooling sellers would not wish to reveal themselves, which we denote $\overline{\phi}$. Furthermore, this result shows that whenever nature draws $\phi > \overline{\phi}$, pooling bad sellers switch so that the equilibrium proportion of pooling bad sellers, which we denote $\phi^*$ satisfies $\phi^* = \overline{\phi}$.
Lemma 4  There exists a unique $\phi \in (\hat{\phi}, 1]$ that solves:

$$(1 + r_B)P_B^*(\phi) - (1 - p) = \frac{Z - \hat{Z}_{pl}(P_B^*)}{N(1 + \phi)} \left[ (1 + r_B)P_G^0 - (1 - p) \right].$$  \hspace{1cm} (13)

When $\phi > \hat{\phi}$, pooling bad sellers reveal such that in equilibrium $\phi^* = \hat{\phi}$. Alternatively, when $\phi < \hat{\phi}$, $\phi^* = \phi$, so that the only revealed bad sellers in equilibrium are those revealed exogenously.

Proof.  See Appendix.

It is straightforward to show that $\hat{\phi} < \phi$, and thus it follows that there exists a $\phi \in (\hat{\phi}, \phi)$ such that no pooling bad seller would wish to reveal itself ($\phi^* = \phi$). As such, there exists an equilibrium in which the revealed bad sellers can earn positive profit since $P_B^* > P_G^0$ whenever $\phi^* > \hat{\phi}$. For $N$ sufficiently large, $\phi^* = \hat{\phi}$ and the profit of revealed sellers will necessarily equal zero. As $N$ approaches infinity, all of the information rents of the unrevealed bad sellers vanish since the probability of an unrevealed bad seller obtaining a pooled contract goes to zero in such a case. Lemma 4 implies that whenever $\phi > \hat{\phi}$, pooling bad sellers will have the incentive to reveal themselves until $\phi^* = \hat{\phi}$. Thus, in equilibrium there can never be more than $\hat{\phi}$ pooling sellers. In other words, although ex-ante $\phi$ can be greater than $\hat{\phi}$, this can never hold in equilibrium.

When considering the effect of asymmetric information on counterparty risk, we wish to look beyond the obvious effect, namely that buyers can no longer contract with good sellers with certainty. Instead, the following proposition highlights two less obvious effects on the relative number of good and bad sellers.

Proposition 3

i. There are $\hat{Z}_{pl} - Z$ buyers that contract with good sellers with perfect information but choose bad sellers over the pool with asymmetric information.

ii. There are $Z_B(P_B^*) - Z_B(P_B^0)$ buyers that contract with bad sellers with perfect information but drop out of the market with asymmetric information.

For $N$ sufficiently large, $P_B^* = P_B^0$ as described in item (ii) of Lemma 2. Thus, $Z_B(P_B^*) - Z_B(P_B^0) = 0$ and the first effect (i) dominates the second (ii), so that the relative number of unstable sellers increases due to asymmetric information over seller type.

Given the information friction, buyers who would contract with good sellers under perfect information may contract with bad sellers instead of the pool of good and bad under asymmetric information. These buyers switch to bad sellers to avoid the information rents implicit in the pooled price (i.e., the lower quality coverage for the same price as would have been paid at good sellers with perfect information) thereby increasing the relative number of bad sellers in the market. Given that competition between revealed bad sellers is now less than it is under perfect information, those that are revealed can also extract rents by charging higher prices. In this case risk transfer decreases...
as those with low $Z$ drop out of the market. Thus, some contracts which would have been at bad sellers under perfect information no longer exist with asymmetric information, thereby decreasing the relative number of bad sellers. In the case in which revealed sellers cannot extract any rents (i.e., when there are many revealed sellers so that competition drives prices down to that which earns zero profit), the first effect is the only effect that remains.

5 Asymmetric Information on the Underlying Asset

We now turn to the more traditional problem of asymmetric information regarding the quality of the underlying asset. To model this situation, we return to the case in which seller quality is known and add one new feature to the model of Section 2. Assume now that a buyer possess one of two types of assets with equal probability; (R)isky and (S)afe, indexed by $i \in \{R, S\}$. Each asset yields return $R_B$ with probability $p_i$, otherwise it defaults with probability $1 - p_i$ and returns nothing. It is assumed that the risky asset is less likely to return $R_B$, so that $p_R \leq p_S$. There are a measure $Z$ of each buyer type that differ by their preference parameter $Z$, again distributed over $[0, Z]$. We assume that $Z$ is not known to the sellers.\(^\text{13}\) Whereas in Section 2.1, each seller knew the probability that the asset would return zero, they now form a belief $b_j$ as to the probability of the low state. We continue to assume competition such that a seller charges their zero profit price given beliefs, which is denoted $P^0_j(b_j)$.

When buyer type is unknown, a seller can only charge one pooling price.\(^\text{14}\) To characterize beliefs, let $\theta_G (\theta_B)$ represent the probability that the buyer of the contract at a good (bad) seller is the safe type. We modify the zero profit prices as defined in (2) to explicitly account for beliefs,

$$P^0_j(b_j) = \frac{b_j}{1 + r_j} = \frac{(1 - p_S)\theta_j + \theta_j - (1 - p_R)(1 - \theta_j)}{1 + r_j}.$$ \hspace{1cm} (14)

In what follows, we denote prices $P^0_j(\theta_j)$ where convenient. Analogous to (4) under perfect information, we define the $Z$ for which a safe and risky type buyer is indifferent between contracting

\(^{13}\)For the sellers to observe $Z$, they would have to know the motivation of the buyers to trade, which would be an extreme assumption. We could obtain our qualitative results in such a case, however the analysis would be more tedious. For a formal market microstructure model that allows for multiple unknown attributes of traders, see Gervais (1997).

\(^{14}\)Note that we rule out non-linear pricing and thus the possibility of a separating equilibrium. This is reasonable given that financial risk transfer contracts are non-exclusive, i.e., the seller cannot preclude the buyer from purchasing the same contract from other sellers. Stephens and Thompson (2012) show that if the buyer can split the contract over many sellers, separation cannot be achieved through a menu of contracts. However, separation may be possible if there is aggregate risk to which a bad seller is subjected. In the current paper, the fixed contract size rules out separation through menus of contracts, and since there is no aggregate risk, extending the model in the spirit of Stephens and Thompson (2012) would justify focusing on only pooling in equilibrium.
with a good and bad seller

$$\hat{Z}^S = \frac{P^0_G(\theta_G) - P^0_B(\theta_B)}{(1 - pS)(qG - qB)} - 1$$  \hspace{1cm} (15)$$

$$\hat{Z}^R = \frac{P^0_G(\theta_G) - P^0_B(\theta_B)}{(1 - pR)(qG - qB)} - 1.$$  \hspace{1cm} (16)$$

Under full information, sellers offer prices $P^0_j(1 - pS)$ and $P^0_j(1 - pR)$ if they contract with a safe or risky type. As in Section 3, we analyze the most interesting case in which some members of both buyer types would potentially contract with both seller types under perfect information (actual participation in the market with either seller type will depend on transaction costs). This requires $Z > \hat{Z}^S > \hat{Z}^R$ for any set of beliefs, where the first inequality is true for sufficient $Z$ and the second is true by definition, as can be seen by comparing (15) and (16). Note that because $Z > \hat{Z}^S > \hat{Z}^R$, the most risk-averse buyer $Z$ of either underlying risk type will always choose the good seller when participating in the market. The following assumption, which is analogous to Assumption 1, ensures that at least some positive measure of both buyer types wish to participate under full information.

**Assumption 2** \hspace{1cm} $\Delta < (1 + Z)(1 - p_i)qG - P^0_G(1 - p_i), \ i \in \{R, S\}.$

We now turn to market participation and define $Z^S_j$ ($Z^R_j$) as the $Z$ for which a safe (risky) type buyer is indifferent between contracting at seller $j$ and not entering in the market:

$$Z^S_j = \min \left( Z, \frac{P^0_j(\theta_j) + \Delta}{(1 - pS)q_j} - 1 \right)$$  \hspace{1cm} (17)$$

$$Z^R_j = \frac{P^0_j(\theta_j) + \Delta}{(1 - pR)q_j} - 1.$$  \hspace{1cm} (18)$$

Given $P^0_j((1 - pS)\theta_j + (1 - pR)(1 - \theta_j)) \leq P^0_j(1 - pR)$, Assumption 2 implies that there will be risky types in the market, or equivalently $Z^R_j < Z$. The safe types however may or may not be in the market when they are forced to pool with the risky types. Using the preceding definitions, the following expression provides implicit expressions for $\theta_G$ and $\theta_B$ which are the proportions (probabilities) of safe buyers at the good and bad sellers respectively. Note that these expressions are only defined when there are buyers at the respective sellers.

$$\theta_G = \frac{Z - \hat{Z}^S}{2Z - (Z^S + Z^R)} \in [0, 1/2]$$  \hspace{1cm} (19)$$

$$\theta_B = \frac{Z^S - Z^S_B}{Z^S - Z^S_B + Z^R - Z^R_B} \in [1/2, 1]$$  \hspace{1cm} (20)$$

That the proportion of safe buyers fall into these ranges can be shown simply by substituting expressions (15)-(18) into the definitions of $\theta_G$ and $\theta_B$.\footnote{Note that $\theta_G = \frac{1}{2}$ and $\theta_B = \frac{1}{2}$ only in the trivial case when $p_S = p_R.$} The following proposition represents the
main result of this section.

**Proposition 4** When \((1 + Z)(1 - p_S)q_G - P^0_G(1 - p_S) \geq \Delta \geq (1 + Z)(1 - p_S)q_G - P^0_G(1 - p_R)\), the market contains both underlying risk types under full information, but only risky types under asymmetric information. When \(\Delta < (1 + Z)(1 - p_S)q_G - P^0_G(1 - p_R)\), risk transfer can occur with asymmetric information in which both types of buyers participate in the market.

**Proof.** See Appendix.

Without asymmetric information, Assumption 2 ensures that both safe and risky buyers will be in the market. However, with asymmetric information, if the transaction costs are too high \((1 + Z)(1 - p_S)q_G - P^0_G(1 - p_S) \geq \Delta \geq (1 + Z)(1 - p_S)q_G - P(1 - p_R)\), the safe buyers do not wish to pool with the risky buyers and so drop out of the market. Therefore, in equilibrium only risky buyers are left and thus the sellers know with certainty with whom they are contracting so that asymmetric information ceases to exist. In this case, the amount of risk transfer decreases relative to the full information case. The mechanism behind this result is that of a lemons problem first explored in Akerlof (1970).

6 A Transaction Tax

Transaction taxes have received much attention since the onset of the credit crisis in 2007. The consideration of such a tax in our model is relatively straightforward, and allows for insights not currently found in the literature. A transaction tax can be viewed as an increase in transaction costs, which as we have seen has important implications for market outcomes. It is important to emphasize that the intention of this section is to add new elements into the debate over transaction taxes. Since we do not pursue a welfare analysis here, we cannot make unconditional statements as to whether such a tax is desirable. The following proposition summarizes the effect of a transaction tax (i.e., an increase in transaction costs) on both the composition of buyers and sellers in the market, as well as the rents that arise under asymmetric information.

**Proposition 5** The introduction of a transaction tax (i.e., an increase in transaction costs):

i. Causes the lowest \(Z_k\) buyers to drop out of the market, thereby reducing risk transfer in all informational environments considered.

ii. Reduces the relative number of bad sellers in the market under perfect information and asymmetric information on the underlying asset (assuming bad sellers are initially present). With asymmetric information of seller quality, this holds when \(\phi \leq \hat{\phi}\) (or equivalently when \(N\) is sufficiently large).

---

16 For an in-depth discussion of transaction taxes see Matheson (2011).
iii. Reduces (weakly) the information rent that can be extracted when there is asymmetric information on either seller quality or underlying asset type.

Proof. See Appendix.

In all cases, a tax drives the lowest \( Z \) buyers out of the market because the effective cost of protection increases due to the tax and those buyers that are less risk-averse find it too expensive to contract. When transaction costs are sufficiently small to begin with, those that drop out can be interpreted as “speculators”, i.e., those buyers who have a \( Z_k \) close to zero. Importantly, low \( Z \) buyers are precisely the ones that choose to contract with relatively unstable sellers and thus the proportion of bad to good sellers falls with the tax.\(^{17}\)

The effect of a tax on information rents varies depending on which type of asymmetric information is considered. When the underlying asset quality is unknown to sellers, transaction costs have a heterogeneous affect on high and low risk buyers. More of the low risk buyers will drop out of the market as they find protection less necessary/beneficial than the high risk buyers. As such, the quality of the pool decreases, and so the equilibrium price gets closer to that which the high risk buyers would pay under perfect information, and thus their information rents decrease.

When there is asymmetric information on seller quality, there are more cases to consider. When transaction costs are so high that revealed sellers do not participate, a tax will drive buyers out of the market and thus lower the probability of a bad seller obtaining a contract, which in turn reduces expected rents. With revealed sellers present and earning zero profit, the tax has no effect on expected rents since revealed sellers earn zero profit and pooling sellers still obtain a contract with the same probability. When revealed bad sellers earn positive profits, the tax leads to a decrease in the number of buyers with which they would otherwise contract. This in turn increases competition and leads to a reduction in revealed sellers price (and thus rents), to attract buyers back. Lower prices at revealed sellers lead some buyers that were initially at the pool, to opt for revealed sellers. This lowers the probability that pooling sellers will obtain a contract and thus reduces their expected rents as well.

7 Conclusion

In this paper we analyze transaction costs in a simple market for risk transfer. We show that when transaction costs are high (which would be typical for a risk transfer market in its infancy), the market is used for hedging, as opposed to speculating, and only high quality counterparties can be

\(^{17}\)In the unknown seller case, the result is ambiguous when \( \phi > \hat{\phi} \), as an increase in transaction costs can have the opposite effect. Such a change precipitates the exit of the lowest \( Z \) buyers as described above. Competition between revealed sellers then forces the price down until the number of buyers equals the number of revealed bad sellers once again. In doing this, they attract both some buyers that dropped out of the market, and some of the buyers that were contracting with the pool. Thus, the number of revealed sellers remains constant, while the number of contracts sold in the pool decreases causing the (expected) relative number of bad sellers to increase. Although potentially interesting, such a case depends crucially on the form of competition, namely that revealed bad sellers can earn sufficient positive profit. Since this is fairly limited in scope, we do not pursue the details here.
present. We show that a prerequisite for a market to exist with high transaction costs is that there be no asymmetric information on either the quality of the counterparty or the quality of the risk being transferred. As transaction costs decrease we show that less risk averse buyers (e.g., speculators) will join the market leading to an increase in the relative number of lower quality counterparties that are active. A transaction tax is then analyzed as simply an increase in transaction costs. We show that such an increase can reduce speculation, the relative number of unstable sellers and the information rents that can be extracted when there is asymmetric information.

8 Appendix

Proof of Lemma 1

The condition on $\Delta$ is determined by considering the case of a buyer who is indifferent between contracting with a bad seller or the pool and when they are just indifferent between doing this or leaving the market (any higher value for $\Delta$ will ensure that all remaining buyers choose the pool). The former is characterized by $\hat{Z}_{pl} = (P_G^0 - P_B^0)/(q_{pl} - q_B) - 1$, while the latter is characterized by $Z_{pl} = (P_G^0 + \Delta)/[q_{pl}(1 - p)] - 1$. Note that $q_{pl} = (\phi q_B + q_G)/(1 + \phi)$ is the probability that a pooled seller is solvent.

\[
\Rightarrow \frac{P_G^0 + \Delta}{(\phi q_B + q_G)(1 - p)} \geq \frac{(P_G^0 - P_B^0)(1 + \phi)}{(1 - p)(q_G - q_B)} \geq \Delta \geq \frac{(P_G^0 - P_B^0)(\phi q_B + q_G)}{q_G - q_B} - P_G^0
\]

(21)

Which is exactly as described in the Lemma. Note that the price hypothetically offered by risky sellers is taken to be $P_B^0$, which is true if there is enough competition between revealed risky sellers. Thus the condition is merely sufficient, $\Delta$ satisfies the expression for $P_B^0$, then it is also satisfied for all other potential $P_B \in [P_B^0, P_G^0]$.

Proof of Lemma 3

For the first part of the Lemma, we re-write our expression for $P^*_B$ (11) as follows

\[
P^*_B = P_G^0 \left[ \frac{(1 + \phi)}{1 - (q_G - q_B)(1 - p)} + \frac{(1 + \phi)}{q_B(1 - p)} \right] - \left[ \frac{(1 - \phi)N + \Delta}{q_B(1 - p)} \right] < P_G^0
\]

18
since
\[
\frac{(1+\varphi)}{(1-p)(q_G-q_B)} \left( \frac{1}{q_B(1-p)} + \frac{(1+\varphi)}{(1-p)(q_G-q_B)} \right) < 1.
\] (22)

In proving the second part of the Lemma we focus on \( P_B^* > P_B^0 \), since the latter is constant. Define the following two variables:

\[
\eta_1 = \frac{P_B^0 (1+\varphi)}{(1-p)(q_G-q_B)} - (1-\varphi)N - \frac{\Delta}{q_B(1-p)}
\]
(23)

\[
\eta_2 = \frac{1}{q_B(1-p)} + \frac{(1+\varphi)}{(1-p)(q_G-q_B)}
\]
(24)

Thus,
\[
\frac{dP_B^*}{d\phi} = \frac{P_B^0}{(1-p)(q_G-q_B)} + \frac{N \eta_1}{\eta_2} - \frac{\eta_1}{(1-p)(q_G-q_B)} \eta_2^2 \]
(25)
\[
= \left( \frac{1}{\eta_2(1-p)(q_G-q_B)} \right) \left( P_B^0 + N(1-p)(q_G-q_B) - P_B^* \right)
\]
(26)
\[
> 0,
\]
(27)

using \( P_B^* = \eta_1/\eta_2 \) and \( P_B^0 > P_B^* \) which was shown above.

\[ \blacksquare \]

**Proof of Lemma 4**

Define the return for revealed and unrevealed sellers as follows:

\[
F(\varphi) = (1+r_B)P_B^* - (1-p), \quad G(\varphi) = \frac{Z - \hat{Z}_{pl}}{N(1+\varphi)} \left( (1+r_B)P_G^0 - (1-p) \right),
\]
(28)

and define \( H(\varphi) = G(\varphi) - F(\varphi) \). We now show that \( H'(\cdot) \leq 0 \), which ensures that \( \bar{\varphi} \in (\hat{\varphi},1] \) is unique.

\[
H'(\varphi) = \frac{dP_B^*}{d\phi} \left( \frac{(1+r_B)P_G^0 - (1-p)}{N(1-p)(q_G-q_B)} - (1+r_B) \right) - \frac{(1+\overline{Z})(1+r_B)P_G^0 - (1-p)}{N(1+\varphi)^2}.
\]
(29)

It was shown in Lemma 3 that \( dP_B^*/d\phi > 0 \), thus a sufficient condition for \( H'(\cdot) < 0 \) is

\[
\frac{(1+r_B)P_G^0 - (1-p)}{N(1-p)(q_G-q_B)} - (1+r_B) \leq 0.
\]
(30)
Inserting the zero profit prices defined in (2) and simplifying we have
\[
\frac{r_B - r_G}{(1 + r_B)(1 + r_G)(q_G - q_B)} = \hat{Z} + 1 < Z + 1 \geq N. \tag{31}
\]

To see why this ensures the result, consider the values taken by \( H(\cdot) \) at the endpoints. At \( H(\hat{\phi}) \), from Lemma 2 and equation (2), we have \( P_B^* = P_B^0 = (1 - p)/(1 + r_B) \), so that \( F(\hat{\phi}) = 0 \). Thus, \( H(\hat{\phi}) > 0 \) whenever the probability of obtaining a contract in the pool is non-zero. If \( H(1) \) is positive, \( \tilde{\phi} = 1 \) since \( H(\cdot) \) is non-increasing. If \( H(1) \) is negative, then there is a unique interior value \( \tilde{\phi} < 1 \) which satisfies \( H(\tilde{\phi}) = 0 \).

**Proof of Proposition 4**

Consider when \((1 + Z)(1 - p_S)q_G - P_G^0(1 - p_S) \geq \Delta \geq (1 + Z)(1 - p_S)q_G - P_G^0(1 - p_R)\). The left side ensures that \(\Delta\) is low enough to ensure market participation of safe types under full information (this also implies participation of risky types). The right hand side is the threshold value for \(\Delta\) such that a higher transaction cost results in a complete exodus of safe buyers. This can be seen from (17) as follows:
\[
Z \leq \frac{P_G^0(\theta_G) + \Delta}{(1 - p_S)q_G} - 1 \Rightarrow \Delta \geq (1 + Z)(1 - p_S)q_G - P_G^0(\theta_G) \tag{32}
\]

where \(\theta_G = 0\) and so \(b_G = 1 - p_R\) as indicated in Proposition 4. When \(\Delta < (1 + Z)(1 - p_S)q_G - P_G^0(\theta_G)\), safe types can exist in equilibrium. To show this, we rewrite the expression for the fraction of safe types at the good and bad sellers, initially given in (19) and (20) as follows:

\[
\theta_G = \mathcal{G}(\theta_G, \theta_B) = \frac{1 + Z - \left( \frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_S)(q_G - q_B)} \right)}{2(1 + Z) - \left( \frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_S)(q_G - q_B)} \right) + \left( \frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_R)(q_G - q_B)} \right)} \tag{33}
\]

\[
\theta_B = \mathcal{B}(\theta_G, \theta_B) = \frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_S)(q_G - q_B)} - \frac{P_B^0(\theta_B) + \Delta}{qb(1 - p_S)} + \frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_R)(q_G - q_B)} - \frac{P_B^0(\theta_B) + \Delta}{qb(1 - p_R)}. \tag{34}
\]

Define \(\mathcal{F}(\theta_G, \theta_B) = \mathcal{G}(\theta_G, \mathcal{B}(\theta_G, \theta_G)) - \theta_G = 0\). It follows that since prices \(P_j(\theta)\) are linear in \(\theta_j\), that \(\mathcal{F}(\cdot)\) is a continuous function that maps \([0, 1/2] \times [1/2, 1]\) into itself. Thus an equilibrium exists by Brouwer’s fixed point theorem. Note that \(\theta_G\) and \(\theta_B\) are only defined when there are buyers at both types of sellers. For sufficiently high \(Z\), \(\theta_G^* > 0\) and for sufficiently low \(\Delta\), \(\theta_B^* > 0\). The case in which there are no buyers at the bad seller and so \(\theta_B\) is undefined is possible. To characterize the equilibrium in this case, we must define off equilibrium beliefs for bad sellers to pin down \(P_B^0\) and thus \(\theta_G(\theta_B)\).
Proof of Proposition 5

i. With perfect information, it is straightforward to show that the threshold value for \( Z \) in which a particular buyer participates in the market, given by (5) or (6), are both increasing in \( \Delta \). Analogous conditions hold when there is asymmetric information over seller quality or asset risk (conditional on asset type in the latter case). We omit these for brevity.

ii. With perfect information, this follows from Corollary 1. With asymmetric information on the underlying asset, we define \( \theta_{B,G} \) as the proportion of bad sellers in the market given by:

\[
\theta_{B,G} = \frac{(\hat{Z}^S - Z_j^S) + (\hat{Z}^R - Z_j^R)}{(Z - \hat{Z}^S + \hat{Z} - \hat{Z}^R) + (\hat{Z}^S - Z_j^S) + (\hat{Z}^R - Z_j^R)} \tag{35}
\]

\[
= 1 - \left[ \frac{2Z - (\hat{Z}^S + \hat{Z}^R)}{2Z - (Z_j^S + Z_j^R)} \right] \tag{36}
\]

From (15) and (16) it follows that \( \hat{Z}^S \) and \( \hat{Z}^R \) are independent of \( \Delta \). From (17) and (18), it can be seen that \( Z_j^S \) and \( Z_j^R \) are increasing in \( \Delta \). It follows that \( \frac{d\theta_{B,G}}{d\Delta} < 0 \) so that the proportion of good (bad) sellers is increasing (decreasing) in transaction costs. With asymmetric information on seller quality, there are two cases to be analyzed. When no revealed bad seller can exist in equilibrium (as per Lemma 1), bad sellers exist only probabilistically in the pool. An increase in transaction costs then has no affect on the (expected) relative number of bad sellers. With revealed bad sellers in the market, \( \phi \leq \hat{\phi} \) implies that these sellers charge their zero profit price \( P_0^B \). Define \( \gamma \) as the fraction of revealed bad sellers in the market, given by:

\[
\gamma = \frac{\hat{Z}_{pl}(P_0^B) - Z_B(P_0^B)}{Z - Z_B(P_0^B)} = 1 - \left[ \frac{Z - \hat{Z}_{pl}(P_0^B)}{Z - Z_B(P_0^B)} \right] \tag{37}
\]

From (8), \( \hat{Z}_{pl}(P_0^B) \) is independent of \( \Delta \) while from (9) \( Z_B(P_0^B) \) is increasing in \( \Delta \). Thus, \( \frac{d\gamma}{d\Delta} < 0 \) and so the fraction of bad sellers decreases as transaction costs increase.

iii. Consider first the case of asymmetric information on seller quality. If nature draws \( \phi \leq \hat{\phi} \), there is no effect. This is because the price of revealed bad sellers is \( P_0^B = P_0^B \) and there are no information rents, while the payoff of pooling bad sellers remains unchanged since their payoff is independent of \( \Delta \) when \( P_0^B \) is constant. When nature draws \( \bar{\phi} > \phi > \hat{\phi} \) it follows from (11) that \( \frac{dP_0^B}{d\Delta} < 0 \) since \( \frac{d\phi^*}{d\Delta} = 0 \), thus information rents decrease for revealed bad sellers. The expected payoff of a pooling bad seller, denoted \( E(\pi_{pl}^B) \) is

\[
E(\pi_{pl}^B) = \frac{Z - \hat{Z}_{pl}(P_0^*)}{N(1 + \phi)} \left[ (1 + r_B)P_0^G - (1 - p) \right] \tag{38}
\]

\[
= \frac{Z - \left( \frac{(P_0^B - P_0^*)^2}{(1-p)(q_G - q_B)} - 1 \right)}{N(1 + \phi)} \left[ (1 + r_B)P_0^G - (1 - p) \right], \tag{39}
\]
and it follows that \( \frac{dE(\phi_B)}{d\Delta} < 0 \) since \( \frac{dP^*_B}{d\Delta} < 0 \) so that rents of the pooling bad sellers decrease as well. Finally, consider the case in which nature draws \( \phi > \bar{\phi} \). Information rents for the revealing bad sellers must decrease since (we claim that) \( P^*_B \) must decrease. Assume to the contrary that \( P^*_B \) remains the same. Then, Lemma 2 part i. implies that \( \phi \) must increase. However, if \( \phi \) increases and \( P^*_B \) remains unchanged, condition (13) from Lemma 4 cannot be satisfied. Thus the revealing price and consequently, information rents of revealed bad sellers must decrease. Given that \( \phi \) must increase and \( P^*_B \) decreases, it follows from (39) that rents of the pooling bad sellers decrease as well.

From (17) and (18), it is clear that with asymmetric information on the underlying asset, an increase in \( \Delta \) causes a greater reduction of safe types in the market than risky, which in turn reduces \( \theta_G \) and \( \theta_B \). It is straightforward to show that the pooling price is strictly decreasing in \( \theta_G \) and \( \theta_B \) and so the information rents of risky buyers are decreasing in \( \Delta \) (ignoring the trivial case where \( \theta_G = 0 \) and \( \theta_B = 0 \) in which there are no information rents).

9 References


### Working Paper Series

#### Department of Economics, University of Alberta


---

<table>
<thead>
<tr>
<th>Year</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012-11</td>
<td>The Long Run Impact of Biofuels on Food Prices</td>
<td>Chakravorty, U., Hubert, M., Nestbakken, L.</td>
</tr>
<tr>
<td>2012-10</td>
<td>Exclusive Dealing and Market Foreclosure: Further Experimental Results</td>
<td>Landeo, C., Spier, K.</td>
</tr>
<tr>
<td>2012-09</td>
<td>Playing against an Apparent Opponent: Incentives for Care, Litigation, and Damage Caps under Self-Serving Bias</td>
<td>Landeo, C., Nikitin, M., Izmalkov, S.</td>
</tr>
<tr>
<td>2012-08</td>
<td>Separation Without Mutual Exclusion in Financial Insurance</td>
<td>Stephens, E., Thompson, J.</td>
</tr>
<tr>
<td>2012-07</td>
<td>Outcome Uncertainty, Reference-Dependent Preferences and Live Game Attendance</td>
<td>Coates, D., Humphreys, B., Zhou, L.</td>
</tr>
<tr>
<td>2012-06</td>
<td>Patent Protection with a Cooperative R&amp;D Option</td>
<td>Che, X.</td>
</tr>
<tr>
<td>2012-04</td>
<td>Commercial Revitalization in Low-Income Urban Communities: General Tax Incentives vs Direct Incentives to Developers</td>
<td>Zhou, L.</td>
</tr>
<tr>
<td>2012-03</td>
<td>Native Students and the Gains from Exporting Higher Education: Evidence from Australia</td>
<td>Zhou</td>
</tr>
<tr>
<td>2012-02</td>
<td>The Overpricing Problem: Moral Hazard and Franchises</td>
<td>Eckert, H, Hannweber, van Egteren</td>
</tr>
<tr>
<td>2012-01</td>
<td>Institutional Factors, Sport Policy, and Individual Sport Participation: An International Comparison</td>
<td>Humphreys, Maresova, Ruseski</td>
</tr>
<tr>
<td>2011-23</td>
<td>The Supply and Demand Factors Behind the Relative Earnings Increases in Urban China at the Turn of the 21st Century</td>
<td>Gao, Marchand, Song</td>
</tr>
<tr>
<td>2011-22</td>
<td>Tariff Pass-Through and the Distributional Effects of Trade Liberalization</td>
<td>Ural Marchand</td>
</tr>
<tr>
<td>2011-21</td>
<td>The Effect of Parental Labor Supply on Child Schooling: Evidence from Trade Liberalization in India</td>
<td>Humphreys, Maresova, Ruseski</td>
</tr>
<tr>
<td>2011-20</td>
<td>Estimating the Value of Medal Success at the 2010 Winter Olympic Games</td>
<td>Humphreys, Johnson, Mason, Whitehead</td>
</tr>
<tr>
<td>2011-19</td>
<td>Riding the Yield Curve: A Spanning Analysis</td>
<td>Galvani, Landon</td>
</tr>
<tr>
<td>2011-18</td>
<td>The Effect of Gambling on Health: Evidence from Canada</td>
<td>Humphreys, Nyman, Ruseski</td>
</tr>
<tr>
<td>2011-17</td>
<td>Lottery Participants and Revenues: An International Survey of Economic Research on Lotteries</td>
<td>Perez, Humphreys</td>
</tr>
<tr>
<td>2011-16</td>
<td>The Belief in the “Hot Hand” in the NFL: Evidence from Betting Volume Data</td>
<td>Paul, Weinbach, Humphreys</td>
</tr>
<tr>
<td>2011-15</td>
<td>From Housing Bust to Credit Crunch: Evidence from Small Business Loans</td>
<td>Huang, Stephens</td>
</tr>
<tr>
<td>2011-14</td>
<td>CEO Turnover: More Evidence on the Role of Performance Expectations</td>
<td>Humphreys, Paul, Weinbach</td>
</tr>
<tr>
<td>2011-13</td>
<td>External Balance Adjustment: An Intra-National and International Comparison</td>
<td>Smith</td>
</tr>
<tr>
<td>2011-12</td>
<td>Prize Structure and Performance: Evidence from NASCAR</td>
<td>Frick, Humphreys</td>
</tr>
<tr>
<td>2011-11</td>
<td>Spatial Efficiency of Genetically Modified and Organic Crops</td>
<td>Ambec, Langinier, Marcoult</td>
</tr>
</tbody>
</table>

**Please see above working papers link for earlier papers**

[www.economics.ualberta.ca](http://www.economics.ualberta.ca)