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Department of Economics

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Separation Without Mutual Exclusion in Financial Insurance

Eric Stephens
University of Alberta

James Thompson
University of Waterloo

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Separation Without Mutual Exclusion in Financial Insurance

Eric Stephens
University of Alberta
eric.stephens@ualberta.ca

James R. Thompson*
University of Waterloo
University of Pennsylvania
james@uwaterloo.ca

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Abstract

In traditional economic models of insurance, sellers typically employ a non-linear pricing scheme to elicit type information from buyers. In financial insurance contracts, such a policy is not possible since contracts are non-exclusive. In addition, counterparty risk in financial contracts can be particularly problematic relative to traditional insurance. Accordingly, we relax the standard assumption of contract exclusivity and allow the insured to contract with many sellers, some of which may be unstable. In contrast to the traditional insurance model, we show that separation of risk types among insured parties can be achieved with linear pricing when there is aggregate counterparty risk. This result is shown to collapse when contracts are cleared through a central counterparty, suggesting that such an arrangement can create opacity.

Keywords: Insurance, Separation, Mutual Exclusion, Counterparty Risk.

JEL Classification Numbers: G22, D82, G21.

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1 Introduction

In their seminal papers, Rothschild and Stiglitz (1976) and Wilson (1977) develop a framework in which insurers can screen individuals through a menu of contracts. Screening provides the means to mitigate an asymmetric information problem facing insurance providers, as unobservable insured party risk types will reveal themselves through their choice of contract. The existence of a separating equilibrium depends crucially on the assumption that insured parties cannot purchase insurance from more than one provider. Without this assumption, which is commonly referred to as mutual exclusivity, separation of risk types through a market mechanism cannot generally be achieved. In markets for financial insurance (e.g., credit default swaps), similar to that for life insurance, it is not possible for a seller to restrict a buyer from purchasing insurance elsewhere. In this case the exclusion assumption is implausible and sellers are restricted to linear pricing.¹

In this paper we model a market for risk transfer in which an insured party may purchase protection on the same risk from multiple insurers. With insured parties that differ according to their privately known risk type, we show that they may choose to separate on insurer choice, even when they can divide their contract among as many insurers as they wish and linear pricing is employed. We show that separation cannot exist when contracts must flow through a central clearing counterparty (CCP), which is of particular relevance given the recent passage of the Dodd-Frank bill in the U.S. that mandates a CCP for many financial insurance contracts.²

The existence of a separating equilibrium in our model hinges on a departure from the standard insurance literature; namely the possibility of insurer default (counterparty risk), with a subset of the insurers having a greater exposure to aggregate risk of default. This is a particularly important feature of financial risk transfer markets, as became all too clear in the wake of the recent credit crisis in which numerous sellers of protection experienced extreme instability at the same time.³ The insured party (buyer) in the model can be of one of two types, risky or safe, and this type is not known to the insurers. The buyer chooses a contract size and can split it over a large number of stable ('good') and unstable ('bad') insurers, where contracts are limited to linear pricing. Consider the credit default swap (CDS) market as an example. Arora, Gandhi and Longstaff (2009) report that counterparty risk is priced in CDS contracts, meaning that less stable insurers charge lower prices (premiums) to compete in the market. This is very intuitive because if insured parties know that an insurer is unstable, the only way it can exist in equilibrium is if it can offer the contract for a cheaper price than a more stable insurer. To capture this intuition in our model, we endow bad insurers with a higher risk of default but also an investment opportunity that yields more

¹For a further discussion of non-exclusivity and linear pricing in the context of life insurance, see Hoy and Polborn (2000) and Villeneuve (2003).

²Europe is moving in a similar direction with the European Market Infrastructure Regulation (EMIR).

³For example, AIG and many monoline insurers experienced nothing short of a crisis when too many contracts that they had sold required payment. Note that it is not crucial that the insurer actually default, although we will refer to counterparty risk in our paper as such. For example, even though the monolines never formally defaulted, their downgrade caused many parties that had purchased protection with them to have to replace those contracts with a higher quality insurer. Given that the number of high quality insurers became scarce in this market at the same time as demand for them increased, many buyers no doubt suffered a cost when replacing the contract.

than that to which the good insurer invests. As such, our bad insurers can charge lower premia to (potentially) attain a contract.

Initially, we assume that the risk of each insurer defaulting is idiosyncratic. Given a large number of each type of insurer, we show that an insured party will diversify its counterparty risk by insuring with the full set of bad insurers or the full set of good insurers. By doing this, the insured party perfectly diversifies the counterparty risk and so learns exactly what percentage of its contract will not perform. In the case when the bad insurers can offer a sufficient price (premium) advantage over the good insurers, the insured party can replicate the *true* (i.e., expected) coverage that they would have received had they contracted with the good insurers simply by increasing the contract size that they split over the bad insurers. Alternatively, in the case in which the bad insurers do not offer a sufficient price advantage, all coverage will occur with good insurers. In either case, the insurers do not learn the type of insured party with which they are contracting, and so a pooling price prevails.

In addition to the pooling equilibria described above, the presence of aggregate risk creates the possibility of a separating equilibrium. Consider the case in which the bad insurers' default risk is correlated with each other. We show that there exists an equilibrium in which the safe insured party divides its contract over the bad insurers, whereas the risky insured party divides over the good insurers. This occurs because counterparty risk cannot be fully diversified at the bad insurers. As such, an insured party cannot replicate good insurance by increasing the contract size and insuring with the bad insurers. This allows the safe insured party to be identified as it is less costly for them to bear the aggregate counterparty risk, in exchange for a lower premium. The risky insured party is willing to pay a higher premium since counterparty risk is more costly to them. Thus, the safe and risky insured party are revealed and the contracts at the good and bad insurers are priced accordingly. When contracts must be cleared through a CCP, all insurers pool their counterparty risk. In this case, provided that there are lots of contracts in the market, the counterparty risk to which an insured party is exposed is the same regardless of the type of insurer to which they contract. Thus, buyers become solely price driven and the separating equilibrium described above cannot exist.

Literature Review

In the insurance economics literature it is typically assumed that insurers always pay claims and that contracts are mutually exclusive. With respect to contract nonperformance, Doherty and Schlesinger (1990) and Agarwal and Ligon (1998) extend the standard insurance model to consider the implications of insurer default risk (but maintain the assumption of exclusivity). The former finds that partial insurance is optimal at a “fair price” when default is total (which is what we assume in this paper), while results are ambiguous in the partial default case.⁴ The latter

⁴Cummins and Mahul (2003) consider a related model in which they analyze the role of divergent beliefs regarding the probability of insurer insolvency. The authors find that the optimal marginal indemnity above the deductible is smaller (greater) than one if the buyer has more pessimistic (optimistic) beliefs than the insurer regarding the

provides conditions under which coverage and premium fall and characterizes the effect on the type of equilibrium. In a recent paper, Biffis and Millosovich (2011) consider endogenous insurer default and analyze the joint effect of counterparty risk on insurance premiums and indemnity schedules. As in Doherty and Schlesinger (1990), the model consists of one insurer, one buyer and symmetric information so that the asymmetric information problem we consider is not an issue. Stephens and Thompson (2011) develop a model of risk transfer to analyze the effects of competition, asymmetric information regarding insurer default, risk aversion and the imposition of a central clearing arrangement on (endogenous) insurer default risk. However, there is no asymmetric information regarding the probability of the underlying risk, which is the main focus of this paper. Finally, Phillips, Cummins and Allen (1998) study capital allocation and pricing across multiple lines of insurance when there is default risk. None of the above mentioned papers consider the mutual exclusion assumption which is the focus of this paper.

There are a few papers in the existing literature that analyze the role of contract nonperformance on the separation of insured risk types. Thompson (2010) presents a model with one insurer and one insured party in which the latter may reveal their type information in equilibrium when this influences the portfolio decision of the insurer (and therefore counterparty risk). Closer to the arguments presented here, Smith and Stutzer (1990) consider the role of aggregate uncertainty and adverse selection in a model in which mutual and stock insurers coexist. The key difference between the two sellers is that a mutual insurer shares some aggregate risk with the insured parties whereas the stock insurers do not. The authors provide a framework in which both types of contracts are purchased in equilibrium and where low-risk individuals reveal type information by sharing aggregate risk. Ligon and Thistle (2005) consider a similar environment but focus on the size of sharing arrangements in which smaller mutuals offer less risk-sharing than stock insurers. The results in Smith and Stutzer (1990) and Ligon and Thistle (2005) parallel ours, wherein mutuals may provide a mechanism in which unobservable type information can be revealed in equilibrium. The characterization of separating equilibria in the above contributions all make the assumption of mutual exclusivity, whereas the main contribution of this paper is to relax this assumption so that we can consider the market for financial insurance where buyers cannot be restricted from purchasing protection from multiple insurers.⁵

The paper proceeds as follows. Section 2 introduces the model framework. Section 3 analyzes the equilibrium in the market for risk transfer when there is no mutual exclusion; both with and without aggregate risk and provides a brief discussion on the introduction of a central counterparty arrangement in the former case. Concluding remarks are made in Section 4 and non-trivial proofs

default risk (the traditional case in which the marginal indemnity is equal to one is recovered when both buyer and insurer have the same perception of risk). Note that in that paper, it is assumed that the buyer does not have superior information regarding the loss distribution, so that unlike the analysis presented here there is no asymmetric information problem.

⁵Rothschild (2009) uses insurance on multiple contingencies (for example, insurance on both fire and flood) to achieve separation with linear pricing. In contrast, we achieve separation with only one contingency (insurance on the bad outcome of the risky asset), which is a common assumption in insurance economics and is the relevant assumption for the types of markets we consider.

for the results in the main paper can be found in Appendix A. Appendix B provides a formal model for the analysis of a CCP which serves to augment the discussion in Section 3.

2 The Model

The basic model is a generalization of that used in Stephens and Thompson (2011). There are two types of agents; a buyer (insured party) and a seller (insurer). This section describes the buyer and sellers' problem as well as the equilibrium concept to be employed.

2.1 Buyer

There is a single buyer who is endowed with one of two types of a risky asset with equal probability, (S)afe and (R)isky denoted $i \in \{R, S\}$. Both assets provide return $R_B > 1$ with probability p_i and zero with probability $1 - p_i$, where $1 > p_S > p_R > 0$. Henceforth we will refer to a buyer with a safe type of asset as safe, and a buyer with a risky type of asset as risky. The buyer is characterized by the desire to shed risk, which is captured by an increasing concave utility function $u(\cdot)$. Thus, we define the expected return for a buyer in the absence of insurance as follows:

$$p_i u(R_B) + (1 - p_i) u(0) \quad i = R, S. \quad (1)$$

We assume that $u'(0)$ is finite to ensure an interesting solution in the environment with aggregate insurer risk which is discussed in Section 3.2.⁶

2.2 Insurers

There are two types of insurers, (G)ood and (B)ad indexed $j = G, B$. Within each type, we assume that there are N individual sellers indexed by $k = 1, \dots, N$. Each insurer of type j is endowed with initial assets W_j that are subject to a random iid rate of return $r \sim F_j(\cdot)$, where F_j is a distribution representing a portfolio draw. To create heterogeneity among insurer types, we assume that $E_B(r) > E_G(r)$ and $F_G(-1) < F_B(-1)$. It will be made clear below that these two assumptions allow us to consider the interesting case in which good insurers are safer than bad, but bad insurers can offer a lower price (as discussed in Section 1).⁷ When entering into a contract with a buyer, each insurer forms a belief as to the probability of a claim b_{jk} and charges $P_{jk} < 1$ per unit of protection sold in return for a contingent payment of size γ_{jk} . The sellers expected return (when a contract is sold) is given by

⁶This is not restrictive and assumed for ease of exposition. Functions for which $u'(\cdot) = \infty$ would be permissible if we included an initial wealth term.

⁷The title of *bad* insurer should be interpreted only in that such an insurer has higher counterparty risk. As we will show in this paper, the lower premium that bad insurers can charge may make them attractive to contract with.

$$(1 - b_{jk})E_j[(1 + r)(W_j + P_{jk}\gamma_{jk})] + b_{jk}[E_j(1 + r)(W_j + P_{jk}\gamma_{jk}) - \gamma_{jk}] \quad (2)$$

The first term represents the state in which no claim is called in and the insurer simply receives the premium $P_{jk}\gamma_{jk}$ in addition to their initial endowment. The second term captures the state in which a claim is made and the insurer must make the payment γ_{jk} . Note that this term can be negative and we do not assume limited liability, rather we assume that even when the insurer is insolvent and cannot pay the claim, it still suffers the cost γ_{jk} . This can be interpreted as a simple form of bankruptcy costs and serves to reduce complication in the model but does not affect the qualitative results.⁸ We assume that there are many insurers and only one buyer, and that the market for insurance is competitive so that the expected return to the contract will be zero from the sellers' perspective. The zero expected profit price is that which equalizes (2) and $E_j[(1 + r)W_j]$, which is the expected return with no contract. The resulting zero profit premium is given by the following simple function of beliefs and the investment return,

$$P_{jk}^0 = \frac{b_{jk}}{E_j(1 + r)}. \quad (3)$$

Since $E_B(r) > E_G(r)$, this implies that $P_{Bk}^0 < P_{Gk}^0$ when the beliefs of both insurer types are the same, so that the bad insurers can charge lower premia than the good. To define the counterparty risk imposed by the seller, we first assume that default occurs when a contract is called in *and* the following condition holds

$$(1 + r_j)(W_j + P_{jk}\gamma_{jk}) < \gamma_{jk}. \quad (4)$$

This implies a threshold value r_{jk}^* , such that any draw of $r_{jk} \leq r_{jk}^*$ results in insurer default.⁹ Henceforth, we define counterparty risk as

$$1 - q_{jk} = F(r_{jk}^*) = F\left(\frac{\gamma_{jk}}{W_j + P_{jk}\gamma_{jk}} - 1\right) \quad (5)$$

Finally, we shall make use of the following qualitative relationship between contract size and the risk of insurer default.

Lemma 1 *Insurer counterparty risk is increasing with the amount of coverage they provide.*

This result obtains since $dr_{jk}^*/d\gamma_{jk} = W_j/(W_j + P_{jk}\gamma_{jk})^2 > 0$. As we might expect, the more insurance that an individual insurer sells the more unstable it becomes.

⁸The issue of limited liability in the context of insurer counterparty risk is discussed in detail in Thompson (2010).

⁹We assume that there is no partial payment of claims. This is relatively straightforward to include in the model but yields no additional insight.

2.3 Equilibrium

The equilibrium concept employed throughout the paper is that of a Bayesian Nash Equilibrium. An equilibrium in our setting is defined as a set of contracts for buyer type i denoted $\gamma^i = [\gamma_{B1}^i, \dots, \gamma_{BN}^i, \gamma_{G1}^i, \dots, \gamma_{GN}^i]$, a set of prices $P = [P_{B1}, \dots, P_{BN}, P_{G1}, \dots, P_{GN}]$ and beliefs $b = [b_{B1}, \dots, b_{BN}, b_{G1}, \dots, b_{GN}]$, such that

- i. Buyer type i chooses γ^i to maximize expected utility.
- ii. Insurer beliefs b are consistent with Bayes' rule where possible.
- iii. Given competition, prices are those which earn the insurer zero expected profit, as defined in equation (3).

3 Separation Without Mutual Exclusion

Risk aversion provides the buyer motivation to purchase insurance since all things equal they prefer more certain outcomes. While this drives the insurance decision in the standard way, it also creates the desire for buyers to purchase protection from a number of sellers. The benefit of insuring with more insurers is that it makes the buyers return more predictable by making tail events (e.g., every insurer defaulting) less likely. This intuition is formalized in the following lemma.

Lemma 2 *The buyer will spread its contract evenly over every insurer possible at a given insurer type j .*

Lemma 1 states that the counterparty risk of each individual insurer decreases as the amount which they insure decreases. Furthermore, due to the concavity of $u(\cdot)$ the buyer is risk averse, which makes this result unsurprising as it amounts to saying that a risk averse agent prefers more predictable returns is then a standard one. For both of these reasons, it is clear that buyers wish to insure with as many insurers as possible (within a given type) to reduce the contract size at each individual insurer.¹⁰ Since individual insurers of any given type are identical, Lemma 2 allows us to focus solely on decisions across the two types rather than the entire set of insurers. Define γ_j^i as the total insurance purchased by type i from *all* insurers of type j . The contract at each individual insurer of a given type is $\bar{\gamma}_j^i = \gamma_{j1}^i = \gamma_{j2}^i = \dots = \gamma_{jN}^i$, where $\bar{\gamma}_j^i = \gamma_j^i / N$. Furthermore, since insurers of a given type are identical in the contracts they provide, prices and beliefs can be simplified to

$$P_j = P_{jk}, \quad b_j = b_{jk}. \quad (6)$$

To illuminate the results as simply as possible, we focus on the case in which the number of insurers becomes arbitrarily large. In this situation, a buyer would face no uncertainty if the default risk

¹⁰Note that to obtain the result in Lemma 2 it is important that there are no transaction costs so that a buyer would insure with as many insurers as possible, no matter how large the number of insurers. With positive transaction costs, the insured party will reduce the number of insurers to which it contracts. For sufficiently large costs, they will contract with only one insurer (or none), which is akin to assuming mutual exclusion. Provided that transaction costs do not lead to mutual exclusion, we could add this feature without changing the qualitative results of the model.

posed by insurers is idiosyncratic. Since individual contracts are infinitesimal at each insurer, the default risk of both types of sellers, described in expression (5), are constant and henceforth denoted $1 - q_j = F_j(-1)$, where $q_B < q_G$ by assumption. Our primary interest is in characterizing the equilibrium in this market when there is aggregate risk present, which is done in Section 3.2 below, but first we consider the benchmark case when all risk is idiosyncratic.

3.1 No Aggregate Risk

The buyer chooses γ_j^i to maximize expected return. Given Lemma 2, as N becomes arbitrarily large an appeal to the law of large numbers allows for a relatively simple characterization of the buyer's problem since the percentage of its contracts that will not perform is certain. Thus we write the buyer's problem as follows:

$$\max_{\gamma_j^i} p_i u(R_B - P_B \gamma_B^i - P_G \gamma_G^i) + (1 - p_i) u((q_B - P_B) \gamma_B^i + (q_G - P_G) \gamma_G^i) \text{ s.t. } \gamma_j^i \geq 0. \quad (7)$$

The first term represents the state of the world in which no claim is made so that the payoff is R_B less the insurance premium paid to both types of insurers, $P_B \gamma_B^i + P_G \gamma_G^i$. The second term captures the state in which a claim. In this case, the payoff consists of a certain amount $q_G \gamma_G^i$ ($q_B \gamma_B^i$) from good (bad) insurers and the premium paid is the same as when there was no claim.

In the absence of mutual exclusion, buyers can simply replicate good insurance with a larger contract from bad insurers, or alternatively, can replicate bad insurance with a smaller contract from good insurers. Thus, the equilibrium boils down to determining which insurer can provide cheaper *true* (i.e., expected) coverage. This is described in the following lemma.

Lemma 3 *When there is no mutual exclusion and all risk is idiosyncratic, there are two possible pooling equilibria in which all insurance is provided solely by either the good or the bad insurers. There is no separating equilibrium.*

Proof. See Appendix A.

With no exclusivity in contracts, differences in the risk of default cannot serve to separate risk types. The only question is which insurer type is offering a “better deal”, which depends solely on the distribution functions $F_j(r)$. To illustrate this, consider the case when both insurers have the same (pooling) belief that the likelihood of contracting with either type of buyer is equally likely. Both buyer types pool at the bad insurer when $E_B(1 + r)q_B > E_G(1 + r)q_G$, otherwise both pool at the good insurer. Intuitively, pooling occurs at the bad insurers when the premium in which they can charge is sufficiently low ($E_B(1 + r)$ is high), and occurs at the good insurers when their counterparty risk is low relative to that of the bad insurers ($\frac{q_B}{q_G}$ is low). The general conditions that ensure the existence of a pooling equilibrium at either insurer (for any off equilibrium path beliefs) are described in the proof of Lemma 3.

3.2 Aggregate Risk

We enrich the modeling of insurer default and allow aggregate risk that affects bad insurers, but does not affect good insurers. Specifically, with probability $1 - q_A$, all of our bad insurers default at once.¹¹ With aggregate risk, the buyers problem described in (7) becomes

$$\begin{aligned} \max_{\gamma_j^i} \quad & p_i u(R_B - P_B \gamma_B^i - P_G \gamma_G^i) + (1 - p_i)[q_A u((q_B - P_B) \gamma_B^i + (q_G - P_G) \gamma_G^i) + \\ & (1 - q_A) u((q_G - P_G) \gamma_G^i - P_B \gamma_B^i)] \quad \text{subject to: } \gamma_j^i \geq 0. \end{aligned} \quad (8)$$

The first term in the buyer's problem is the same as in (7). The second term captures the state in which a claim is made, but with aggregate risk this state will contain some uncertainty if there is any insurance purchased through bad insurers. Income when a claim is made depends on the aggregate state; in the event of an aggregate shock (probability $1 - q_A$) any claims made to bad insurers are not paid out.

We now turn to the main result of the section. The following proposition shows that differences in exposure to aggregate risk creates the potential for a separating equilibrium in this market.

Proposition 1 *With aggregate risk of insurer default, there exists a separating equilibrium in which the buyer reveals its type. In such an equilibrium, a risky buyer insures with only good insurers, and a safe buyer insures with only bad insurers.*

Proof. See Appendix A.

The key difference between the case with aggregate risk versus that with no aggregate risk is that in the former, good insurance cannot be replicated by simply increasing the contract size and splitting it over an arbitrarily large number of bad insurers. Instead, any buyer that insures solely with the bad insurers will bear aggregate risk which cannot be diversified away. In the separating equilibrium, the safe buyer will contract solely with bad insurers and bear this risk in exchange for a lower premium, while the risky buyer will contract solely with good insurers and pay a higher premium in exchange for more complete protection.

3.3 Central Counterparty

In this section we consider how the introduction of a CCP can shut down the separating equilibrium described above.¹² The intuition behind the following result is relatively straightforward, however the introduction of a CCP necessitates some added complication. Since the intuition can

¹¹This structure is assumed for simplicity. The good insurers could be exposed to aggregate risk, but to a lesser degree. Furthermore, we could have partially correlated default risk instead of perfectly correlated risk. Neither simplification affects the qualitative results.

¹²We will not describe the intricacies of CCPs here. For a discussion relevant to the model to be outlined in Appendix B, see Stephens and Thompson (2011), and for a more general overview of CCPs, see Bliss and Steigerwald (2006).

be given without a formal model, we discuss the results in general in this section for ease of exposition. The interested reader can refer to Appendix B where we outline a model in the spirit of Stephens and Thompson (2011) that provides a more formal discussion.

For our purposes, a central counterparty can be thought of simply as a means to pool counterparty risk. In the framework outlined above, an insured party was exposed to the counterparty risk of each insurer to which it contracted. Under a CCP, an insured party is exposed to the pooled counterparty risk of every active insurer. Thus, a central counterparty acts to diversify counterparty risk, thereby reducing the need for individual insured parties to do so themselves. To understand how a CCP affects our previous results, we consider a market in which many insured parties are contracting with many insurers. We focus on the situation in which the number of active insurers and insured parties becomes arbitrarily large. One can imagine that each insured party splits its contract up as in the case without a CCP, or contracts solely with one insurer, however it is not integral to the result. Mechanically, even though a contract is cleared through a CCP, it is still initially written between two parties. However, after it is signed it is then cleared through a CCP in a process called novation. With novation, the risk of the contract is shifted to the CCP who simultaneously acts as the insurer to the insured party and the insured party to the insurer. Thus, the premium that an insured party pays can be different between contracts, however the counterparty risk cannot (which effectively shuts down the mechanism by which the separating equilibrium described above is achieved). In the following proposition, we refer to a separating equilibrium as one in which type information is completely revealed. In our framework, this describes the case in which all risky insured parties insure with only good insurers and all safe with only bad insurers in the presence of aggregate risk.

Proposition 2 *When contracts are cleared through a CCP, there is no separating equilibrium.*

The intuition behind this result is as follows. Without a CCP, a safe buyer reveals itself by insuring with only bad insurers and bearing some uncertainty due to the aggregate counterparty risk. This uncertainty is more costly for a risky type who insures with good insurers to minimize counterparty risk. With a CCP, the counterparty risk to which a buyer is exposed is that of the CCP, and not the individual insurers. As the number of insured parties becomes arbitrarily large, switching between insurers does not change the counterparty risk of the CCP and thus buyers become completely price-driven and no separating equilibrium can exist.

4 Conclusion

We analyze a simple market for insurance when contracts can be split over multiple providers. Contrary to standard results in the insurance economics literature, we show that with counterparty risk (some of which must be aggregate risk), the insured parties can separate and thus reveal type information based on the type of insurers with whom they contract. With a CCP, the separating equilibrium breaks down since such an arrangement forces the pooling of counterparty risk causing

buyers to focus solely on the premium charged for coverage, which is analogous to the standard insurance model under linear pricing.

5 Appendix A

Proof of Lemma 3

An interior solution to (7), for which $\gamma_j^i > 0$ for both $j = B, G$ requires

$$\frac{P_B}{q_B - P_B} = \frac{u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i)(1 - p_i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \frac{1 - p_i}{p_i} = \frac{P_G}{q_G - P_G}. \quad (9)$$

Therefore γ_j^i cannot be positive for both $j = B, G$ at the optimum, unless $P_B/(q_B - P_B) = P_G/(q_G - P_G)$ in which case the solution is indeterminate.¹³ Therefore, both insurer types cannot be active simultaneously so that there can be no separating equilibrium through in insurer choice. There are two corner solutions which correspond to the two potential pooling equilibria described in the Lemma. We focus on the case in which buyers only purchase contracts from bad insurers (the alternative case is analogous). Both buyer types pool at bad insurers when:

$$\frac{P_G^0}{q_G - P_G^0} > \frac{P_B^0}{q_B - P_B^0} \rightarrow P_G^0 q_B > P_B^0 q_G \rightarrow \frac{b_G q_B}{E_G(1 + r)} > \frac{b_B q_G}{E_B(1 + r)}, \quad (10)$$

where we substitute in the expressions for the zero profit prices defined in equation (3) to obtain last expression. The pooling beliefs for the bad insurers are $b_B = 1/2 \times (2 - p_R - p_S)$, while the good insurers' beliefs are defined off the equilibrium path. We consider the beliefs which make the pooling equilibrium least likely to exist; namely $b_G = 1 - p_S$. Thus, expression (10) becomes

$$E_B(1 + r)q_B > \frac{1 - \frac{1}{2}(p_R + p_S)}{1 - p_S} E_G(1 + r)q_G. \quad (11)$$

Examining expression (11) we see that the bad insurer will dominate if it enjoys a large enough expected relative return on investments. ■

Proof of Proposition 1

The optimal conditions for the problem described in (8) can be written in a similar form to the case of no aggregate risk in that we have corner solutions for γ_B^i and γ_G^i when the following inequalities are strict:

$$\frac{P_B}{q_B - P_B} \geq \left[\frac{(1 - p_i)q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i)}{p_i u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i) + (1 - p_i)(1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)} \right] \quad (12)$$

¹³We ignore the uninteresting case in which there is no insurance, where $\gamma_j^i = 0$ for both $j = B, G$.

$$\frac{P_G}{q_G - P_G} \geq \left(\frac{1 - p_i}{p_i} \right) \left[\frac{q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i) + (1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right]. \quad (13)$$

At $q_A = 1$ this is the no aggregate risk case described above in expression (9). When $q_A \neq 1$, all else equal, the good insurer is now relatively more attractive. We posit a separating equilibrium as one in which buyers contract solely with one type of seller and in so doing reveal the quality of the underlying asset in equilibrium. To characterize the conditions for such an equilibrium, consider the safe buyer contracting solely with bad insurers. Using expressions (12) and (13), this implies

$$\frac{P_B}{q_B - P_B} = \underbrace{\left[\frac{(1 - p_S)q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i)}{p_S u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i) + (1 - p_S)(1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)} \right]}_a \quad (14)$$

while

$$\begin{aligned} \frac{P_G}{q_G - P_G} &> \underbrace{\left(\frac{1 - p_S}{p_S} \right) \left[\frac{q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i) + (1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right]}_d \\ &= \underbrace{\left(\frac{1 - p_S}{p_S} \right) \left[\frac{q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right]}_b + \underbrace{\left(\frac{1 - p_S}{p_S} \right) \left[\frac{(1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right]}_c. \end{aligned} \quad (15)$$

$$(16)$$

Using the labels above we can rewrite the condition:

$$\frac{P_B}{q_B - P_B} = a, \text{ and } \frac{P_G}{q_G - P_G} > b + c. \quad (17)$$

By inspection, it follows that $b > a$ so that $\frac{P_G}{q_G - P_G} > b + c > a + c = \frac{P_B}{q_B - P_B} + c$. Thus a sufficient condition is

$$\frac{P_G}{q_G - P_G} - \frac{P_B}{q_B - P_B} > c = \left(\frac{1 - p_S}{p_S} \right) \left[\frac{(1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right]. \quad (18)$$

Now consider the conditions in which the buyer with the risky asset will insure solely with the good insurer. Again using (12) and (13) we have

$$\frac{P_B}{q_B - P_B} > \underbrace{\left[\frac{(1 - p_R)q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i)}{p_R u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i) + (1 - p_R)(1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)} \right]}_\alpha \quad (19)$$

while

$$\frac{P_G}{q_G - P_G} = \underbrace{\left(\frac{1 - p_R}{p_R} \right) \left[\frac{q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i) + (1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right]}_{\delta} \quad (20)$$

$$= \underbrace{\left(\frac{1 - p_R}{p_R} \right) \left[\frac{q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right]}_{\beta} + \underbrace{\left(\frac{1 - p_R}{p_R} \right) \left[\frac{(1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right]}_{\gamma}. \quad (21)$$

Rewrite expression (19) and multiply by -1 so that $\frac{P_B}{q_B - P_B} > \alpha \rightarrow -\frac{P_B}{q_B - P_B} < -\alpha$. Adding δ to both sides yields $\delta - \frac{P_B}{q_B - P_B} < \delta - \alpha$. Substituting $\frac{P_G}{q_G - P_G} = \delta$ we have

$$\frac{P_G}{q_G - P_G} - \frac{P_B}{q_B - P_B} < \left(\frac{1 - p_R}{p_R} \right) \left[\frac{q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i) + (1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right] \quad (22)$$

$$- \left[\frac{(1 - p_R)q_A u'((q_B - P_B)\gamma_B^i + (q_G - P_G)\gamma_G^i)}{p_R u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i) + (1 - p_R)(1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)} \right]. \quad (23)$$

A separating equilibrium can only exist when the following set of conditions is satisfied:

$$c < \frac{P_G}{q_G - P_G} - \frac{P_B}{q_B - P_B} < \delta - \alpha. \quad (24)$$

To prove existence, we show there are parameters which satisfy this set of inequalities by considering the limiting case in which $p_S \rightarrow 1$ and $p_R \rightarrow 0$ (i.e., when there is a large difference in the underlying risk). In doing this, we shall rewrite the middle expression of (24) using the zero-profit prices, as defined in (3), under the separating beliefs.

$$\frac{P_G}{q_G - P_G} - \frac{P_B}{q_B - P_B} = \frac{1 - p_R}{q_G E_G(1 + r) - (1 - p_R)} - \frac{1 - p_S}{q_B E_B(1 + r) - (1 - p_S)}. \quad (25)$$

The limits of each term in (24) are

$$\lim_{p_S \rightarrow 1, p_R \rightarrow 0} c = \lim_{p_S \rightarrow 1, p_R \rightarrow 0} \left(\frac{1 - p_S}{p_S} \right) \left[\frac{(1 - q_A)u'((q_G - P_G)\gamma_G^i - P_B\gamma_B^i)}{u'(R_B - P_B\gamma_B^i - P_G\gamma_G^i)} \right] = 0, \quad (26)$$

$$\lim_{p_S \rightarrow 1, p_R \rightarrow 0} \frac{1 - p_R}{q_G E_G(1 + r) - (1 - p_R)} - \frac{1 - p_S}{q_B E_B(1 + r) - (1 - p_S)} = \frac{1}{q_G E_G(1 + r) - 1}, \quad (27)$$

and finally, $\lim_{p_S \rightarrow 1, p_R \rightarrow 0} (\delta - \alpha)$ is

$$\lim_{p_S \rightarrow 1, p_R \rightarrow 0} \left(\frac{1 - p_R}{p_R} \right) \left[\frac{q_A u'((q_B - P_B) \gamma_B^i + (q_G - P_G) \gamma_G^i) + (1 - q_A) u'((q_G - P_G) \gamma_G^i - P_B \gamma_B^i)}{u'(R_B - P_B \gamma_B^i - P_G \gamma_G^i)} \right] \quad (28)$$

$$- \left[\frac{(1 - p_R) q_A u'((q_B - P_B) \gamma_B^i + (q_G - P_G) \gamma_G^i)}{p_R u'(R_B - P_B \gamma_B^i - P_G \gamma_G^i) + (1 - p_R) (1 - q_A) u'((q_G - P_G) \gamma_G^i - P_B \gamma_B^i)} \right] = \infty. \quad (29)$$

Given this, the limiting case of the separation condition described in expression (24) is

$$0 < \frac{1}{q_G E_G (1 + r) - 1} < \infty, \quad (30)$$

which is satisfied under appropriate restrictions on $F_G(\cdot)$. ■

6 Appendix B

To introduce a central counterparty (CCP), we use a setup similar in spirit to that of Stephens and Thompson (2011). For simplicity, let there be $M = N$ insured parties of each type (with iid risk) where N is the number of both good and bad insurers as in the model without a CCP. As before, let N (and thus M) get arbitrarily large to simplify the analysis. We consider the separating equilibrium described in Section 3.3 in which all type information is revealed in equilibrium, focusing on the case in which all buyers with risky (safe) assets purchase coverage $\gamma_G^R > 0$ and $\gamma_B^R = 0$ ($\gamma_B^S > 0$ and $\gamma_G^S = 0$). The size of the contract γ_j^i is not of consequence to our analysis in this section so we simply take this as given. As described in Lemma 2, buyers spread insurance evenly among all insurers of a given type and thus each individual good (bad) insurer covers a total amount γ_G^R (γ_B^S) consisting of arbitrarily small amounts with each buyer.¹⁴

The CCP forms a default pool and charges each insurer for participation. Assume that each insurer pays $c \in (0, 1)$ per unit of coverage.¹⁵ To analyze the consequences of the CCP for the separating equilibrium described above we consider the environment described in Section 3.2 in which there is aggregate risk. The counterparty risk of the CCP is conditional on whether or not the aggregate event has occurred (i.e., all the bad insurers default). First, we characterize the default probability of the CCP when there is no aggregate shock. Since there are an arbitrarily large number of insured parties and insurers, an appeal to the law of large numbers allows us to treat the percentage of claims that are made and insured parties that default as certain. Thus,

¹⁴With an equal number of buyers and sellers it is irrelevant whether or not each insured party splits its contract, or contract with its own insurer. In both cases, the contract size at each insurer would be the same.

¹⁵It is reasonable to think that the CCP should levy different charges on different types of insurers and different types of assets insured, depending on the information available. Including this feature would only affect prices and would not change the results to be discussed.

the CCP defaults when the amount it collects ex-ante is insufficient to cover its liabilities from the defaulting insurers. To characterize this, we define the threshold value c^* as that at which the CCP is just able to meet its obligations. Note that q_B and q_G are a function of the size of contracts that each individual insurer insures which, due to symmetry are equal across insurer type.¹⁶

$$c^* = \frac{(1 - q_B)(1 - p_S)\gamma_B^S + (1 - q_G)(1 - p_R)\gamma_G^R}{\gamma_B^S + \gamma_G^R}$$

Thus, we can define the counterparty risk of the CCP when there is no aggregate shock ($1 - q_{ccp}^{NA}$) as follows.

$$1 - q_{ccp}^{NA} = \begin{cases} 0 & \text{if } c \geq c^* \\ 1 & \text{if } c < c^* \end{cases}$$

Given this, we can characterize counterparty risk with the aggregate shock ($1 - q_{ccp}^A$) as the case in which $q_B = 0$. Thus, default in the state with the aggregate shock is

$$1 - q_{ccp}^A = \begin{cases} 0 & \text{if } c \geq c^{**} \\ 1 & \text{if } c < c^{**}, \end{cases}$$

where

$$c^{**} = \frac{(1 - p_S)\gamma_B^S + (1 - q_G)(1 - p_R)\gamma_G^R}{\gamma_B^S + \gamma_G^R} > \frac{(1 - q_B)(1 - p_S)\gamma_B^S + (1 - q_G)(1 - p_R)\gamma_G^R}{\gamma_B^S + \gamma_G^R} = c^*.$$

Thus, aggregate risk implies a higher charge from the CCP to ensure solvency as we might expect. We now relate our discussion to Proposition 2 by noting that both $1 - q_{ccp}^{NA}$ and $1 - q_{ccp}^A$ are taken as given by an individual buyer. Consider whether a risky buyer, contracting with good insurers would wish to deviate to bad insurers. If it did, it would pay a premium $P_B < P_G$,¹⁷ however the counterparty risk to which it would be exposed would remain the same as defined by $1 - q_{ccp}^{NA}$ and $1 - q_{ccp}^A$ above. This is because each individual buyer is small so that its choice of insurer quality does not affect the quality of the CCP. In other words, γ_B^S and γ_G^R remain unchanged. Thus, an individual's choice of insurer type will be based solely on the premium that can be offered so that the separating equilibrium cannot exist.

¹⁶To maintain consistency with the previous results, we assume that $q_G > q_B$ which can be ensured with appropriate restrictions on $F_G(\cdot)$ and $F_B(\cdot)$.

¹⁷Given a zero profit condition on insurers, the cost c would be born by the insured parties and thus these prices would include this cost in equilibrium.

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