UNIVERSITY OF ALBERTA
FACULTY OF ARTS
Department of Economics

Working Paper No. 2011-19

Riding the Yield Curve: A Spanning Analysis

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November 2011

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Abstract The average return on long-term bonds exceeds the return on short-term bills by a large amount over short investment horizons. A riding-the-yield-curve investment strategy takes advantage of the higher returns on longer term bonds. This strategy involves the purchase of bonds with maturities longer than the investment horizon and the sale of these bonds, before they mature, at the end of the investment horizon. Most of the literature that evaluates this strategy compares only *ex post* average returns or Sharpe ratios. In this paper, we use spanning tests to provide formal statistical evidence on the benefits of investing in long bonds when the investment horizon is short. The results for both the US and Canada indicate that an investor with a short horizon is better off investing in short-term debt instruments than long-term bonds.

Keywords North American Bond Market; Portfolio Diversification; Mean-Variance Spanning; Yield Curve

JEL Classifications G11 G12 G15

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1 Introduction

This paper presents tests of whether it is beneficial for an investor with a short investment horizon to hold long-term bonds. The principal motivation for this analysis is the large average discrepancy between the returns on long bonds and short-term bills. As shown in Table 1, Part A, from June 1982 through September 2008, the 1-month excess holding period return averaged 5.88 percentage points for 10-year US Treasury bonds, but was only .19 percentage points for 3-month Treasury bills. The large average relative excess returns associated with long maturity bonds are not unique to the US market. For a somewhat shorter period, the average 1-month excess return on 10-year Government of Canada bonds was 3.97 percentage points, while the excess return on 3-month bills was only .42 (Table 1, Part A).[1] Similar differences between the excess returns on short-term bills and long-term bonds are also evident for a 3-month investment horizon (Table 1, Part B).

The higher average excess returns of longer maturity bonds have induced some practitioners to adopt a riding-the-yield-curve investment strategy (Pelaez 1997).[2] This strategy involves purchasing assets with maturities longer than the investment horizon and selling these assets before they mature.[3] A riding-the-yield-curve strategy takes advantage of the higher average yields on longer term assets, but is not without risk.[4] An unexpected upward shift in the yield curve during

[1] Bolder et al. (2004) produce a similar table for an investment horizon of 180 days.
[2] The analysis of technical trading strategies, of which riding-the-yield curve is one example, is more common in the equity trading literature. See, for example, Li and Wang (2007), Marshall et al. (2008) and Wang et al. (2009).
[3] An alternative strategy which also exploits the upward slope in the yield curve is a bond ladder. This involves holding a balanced portfolio of bond maturities at all times. A portion of investments are redeemed each period and the proceeds re-invested to maintain the maturity structure of the ladder. This strategy takes advantage of the upward sloping yield curve, while minimizing liquidity risk (Siegel 2008). The key difference between a bond ladder and riding-the-yield-curve is that, with a bond ladder, bonds are held to maturity, while the riding strategy involves selling bonds before maturity. A key shortcoming of a laddering strategy, relative to riding-the-yield-curve, is that it assumes that only a fraction of resources are likely to be needed at short notice. While a bond ladder may be a useful strategy for cash managers who plan to rollover funds, it is not a useful strategy for investors who have a fixed amount of cash to invest for a short period.
[4] A riding-the-yield-curve investment strategy can only yield excess returns if the simple version of the expectations hypothesis does not hold. Bieri and Chincarnini (2005) note that, while still controversial, numerous studies have

the holding period will cause a capital loss when the longer maturity asset is sold at the end of the investment horizon. These losses are larger the longer the maturity of the asset held relative to the investment horizon (Roll 1971). The greater risk of holding long-term bonds over a one-month investment horizon, despite the higher average return, is reflected in the typically lower value of the Sharpe ratio for long bonds. For example, the Sharpe ratio for the 6-month US Treasury bill is .41, while that for the 10-year Treasury bond is only .15 (Table 1, Part A).

Beginning with Dyl and Joehnk (1981), several studies have examined the benefits of a riding-the-yield-curve investment strategy.[5] The principal methodology used in this literature is the comparison of *ex post* returns and Sharpe ratios.[6] While some studies find that riding-the-yield-curve is profitable, in the sense that it boosts average returns, other authors emphasize that these increased returns may be outweighed by increased risk.[7] Moreover, the conclusions reached by the different studies appear to be sensitive to the length of the investment horizon, the maturity of the asset chosen for the ride, and the sample period. Several studies examine whether the slope of the yield curve can be used as a signal of when it is best to pursue a riding-the-yield-curve strategy, and in this way to improve the risk-return trade-off, but the results of these analyses are not conclusive.[8]

The current study contributes to the riding-the-yield-curve literature in several ways. First, rather than comparing *ex post* returns or Sharpe ratios, as is done in much of the existing literature, the results presented in this paper are obtained by employing formal tests that evaluate the merits of

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rejected the expectations hypothesis, so there may be an opportunity for riding-the-yield-curve to increase returns. See In and Batten (2005) for a recent study in which the expectations hypothesis cannot be rejected.

[5] In addition to Dyl and Joehnk (1981), see Grieves and Marcus (1992), Pelaez (1997), Ang et al. (1998), Grieves et al. (1999), Bieri and Chincarini (2005), Chua et al. (2005), and Pilotte and Sterbenz (2006). Bieri and Chincarini (2005) provide a comprehensive review of the literature.

[6] Other types of comparisons are also made. Grieves and Marcus (1992) examine whether a riding strategy can stochastically dominate a buy-and-hold strategy. Grieves et al. (1999) calculate the coefficient of relative risk aversion for which an investor would not benefit from riding. Ang et al. (1998) extend Grieves and Marcus (1992) by calculating coefficients of variation of returns as well as measures of first- and second-order stochastic dominance.

[7] Dyl and Joehnk (1981), Ang et al. (1998) and Chua et al. (2005) provide evidence both in support of and against riding. Pelaez (1997) and Bieri and Chincarini (2005) conclude that the increase in risk from riding outweighs the benefit, while the results in Grieves and Marcus (1992), Grieves et al. (1999) and Mercer et al. (2009) tend to support a riding strategy.

[8] Dyl and Joehnk (1981) and Grieves et al. (1999) find that conditioning on the slope of the yield curve improves the success of a riding strategy, while Grieves and Marcus (1992) and Ang et al. (1998) find it does not.

portfolio diversification. These tests allow us to ascertain whether the expansion of a portfolio of short-term bonds to include long maturity debt instruments entails statistically significant gains. As noted by DeRoon and Nijman (2001), while a comparison of *ex post* returns and Sharpe ratios, as in the riding-the-yield-curve literature, may seem to indicate differences in the mean-variance frontier for different portfolios, these differences may be the result of random factors. Formal tests allow a researcher to determine if a shift in the efficient frontier is statistically significant.

The methodology employed here involves the use of spanning tests, an econometric technique that evaluates the benefits of portfolio diversification within a mean-variance (MV) framework.[9] Spanning tests gauge the gains from portfolio diversification by testing for statistically significant shifts in the MV efficient frontier. If the efficient frontier generated by an initial set of assets (the "benchmark" investments) is statistically indistinguishable from that associated with an expanded set of assets, an MV investor does not benefit from including the new assets in the benchmark portfolio. In this case, the benchmark investments are said to span the additional assets. In contrast, whenever the expansion of the investment opportunities set leads to a significant shift in the MV efficient frontier, so that the initial assets do not span the added assets, investors with MV preferences are better off expanding their benchmark portfolios to include the new assets.

To examine the profitability of a riding-the-yield-curve strategy, we ask whether short-term bills span longer maturity bonds. If this is not the case, the addition of longer maturity bonds to a portfolio of short-term bills leads to a significant expansion of the mean-variance frontier, so a riding-the-yield-curve strategy would be attractive to MV investors. On the other hand, if the short-maturity bills span the long-term bonds, MV investors do not benefit from the inclusion of the long-term bonds in their portfolio and it is optimal for the long-term assets to be assigned zero weight.

We also investigate the benefits of broadening a portfolio of long-term bonds to include short-maturity bills when the investment horizon is short. If the long-term bonds span the short-

[9] See Huberman and Kandel (1987), Jobson and Korkie (1989), DeRoon and Nijman (2001) and Kan and Zhou (2008).

term bills, an investor pursuing a riding-the-yield curve investment strategy and holding long-term bonds only would be no worse off, from a MV perspective, than if they held both short and long-term assets. If the long bonds do not span the short bills, a riding-the-yield-curve strategy, which involves holding only long bonds, would leave an MV investor worse off.

The second major contribution of this study is that we augment the portfolio of short-term assets with assets that are of longer maturity than those considered in most of the riding-the-yield-curve literature. Existing studies have concentrated on comparisons of short-term assets and have rarely considered assets with a maturity of more than two years.[10] The advantage of considering longer term assets is that, as can be seen from Table 1, the average excess returns of longer maturity bonds are much higher than the excess returns of shorter maturity bonds, so there may be a greater benefit to adding longer term assets to a portfolio. Further, the correlations between the returns of long-term and short-term assets are lower (see Table 2), which suggests that, *ceteris paribus*, diversification across longer maturities may have greater benefits. Given the observed differences in average returns and correlations, riding longer term bonds may be beneficial to investors even if riding shorter maturity bonds is not.

Another novelty of the current analysis is that we identify whether the benefits from expanding a portfolio of short maturity bills to include long bonds, or the benefits from expanding a portfolio of long-term bonds to include short-term bills, are due to a pure risk reduction, as measured by a shift in the Global Minimum Variance (GMV) portfolio of the efficient frontier, or to a change in the return to risk bearing, as measured by the Sharpe ratio. That is, we distinguish

[10] For example, Grieves and Marcus (1992) and Grieves, Mann, Marcus and Ramanlal (1999) consider only investments in 6-month, 9-month and 12-month bills. Ang et al. (1998) use a 3-month horizon and consider investments in a six-month bill or a one-year horizon and a two-year bond. Dyl and Joehnk (1981) analyze investments only in bills of maturities of less than 26 weeks, while Mercer et al. (2009) consider only 91 and 182-day Treasury bills. Pelaez (1997) considers only investments in a two-year bond, Pilotte and Sterbenz (2006) and Chua et al. (2005) analyze investments in a five-year bond. Bieri and Chincarini (2005) considers the broadest set of maturities – 6 months, 9 months, 1 year, 2 years, 5 years, 7 years and 10 years.

whether adding long (short) maturity bonds to a portfolio of short (long) term bills is beneficial in terms of an overall decrease in risk exposure or an improvement in the risk-return tradeoff.

Fourth, this study evaluates the profitability of adding long-term bonds to a portfolio of short-term assets using data for two countries, the US and Canada. Most previous studies have examined one country in isolation.[11] The use of two different samples makes it possible to determine whether the results are unique to one market and one debt issuer or whether the conclusions can be applied more broadly.

The literature typically examines riding strategies that involve riding only one long-term asset at a time, rather than a portfolio of assets, although investments in a portfolio of assets of different maturities could potentially provide diversification benefits and reduce the risk of riding.[12] The fifth contribution of this study is to test whether it is beneficial to add a *portfolio* of long-term bonds to a portfolio of short-term assets.

Finally, we undertake spanning tests for the case in which it is possible for investors to take short positions in assets as well as for the case in which short sales are not allowed. This distinction is important as it may be difficult or costly for investors to enter into short sales contracts. The role of short sales and the impact of restrictions on these sales are rarely addressed in either the riding-the-yield-curve or spanning literatures. To investigate the role of short sales restrictions, we use a modified spanning test developed by DeRoon et al. (2001).

For both the US and Canada, our empirical analysis indicates that there is generally no advantage to augmenting a portfolio of short-term bills with long-term bonds when the investment horizon is short (one or three months). The results are particularly strong with respect to the risk-return tradeoff and when short-selling is not allowed. These findings cast doubt on the benefits of a

[11] Exceptions are Ang et al. (1998) and Bieri and Chincarini (2005).
[12] Petrella (2005) looks at small, medium and large cap stocks and finds some benefit to size diversification even when returns are highly correlated.

riding-the-yield-curve investment strategy and may be important to cash managers with funds to invest over short horizons.

The tests used in this paper are unconditional in the sense that they compare the efficient frontier generated by the benchmark assets with the frontier of an expanded set of assets. Some of the recent literature on portfolio diversification focuses on tests for conditional spanning in which pricing-relevant information is embedded in the efficient frontier.[13] Since unconditional spanning implies conditional spanning, our findings indicate that the use of "smart" information to form portfolios of long-term bonds is unlikely to deliver significant benefits over a portfolio of short-term bills as the short-term assets unconditionally span the long-term bonds.

Although adding long-term bonds to a portfolio of short-term bills is not found to significantly expand the efficient frontier, the opposite is not the case – augmenting a portfolio of long-term bonds with short-term bills leads to a significant expansion of the frontier. Thus, an investor with a short investment horizon is worse off, from a mean-variance perspective, if they pursue a riding-the-yield-curve investment strategy and hold long-term bonds only.

The paper is organized as follows. In Section 2, the spanning test methodology is reviewed. The data and details of the empirical approach are described in Section 3. Test results are presented in Section 4 using US data and in Section 5 for Canadian data. A brief summary and conclusions follow in Section 6.

2 Spanning Test Methodology

In a seminal paper, Huberman and Kandel (1987) develop a test for mean-variance spanning in a multivariate regression analysis framework. Versions of this test have been employed to

[13] A well-understood approach to conditional spanning is to augment the investment opportunities set in the unconditional case with scaled returns as proposed in Bekaert and Urias (1996). A scaled return is the payoff of a portfolio that is managed on the basis of the realizations of selected indicators. Ferson and Siegel (2001, 2009) generalized the use of scaled returns by identifying an efficient use of conditioning information. As tests for spanning are linked to the estimation of the volatility bound on the stochastic discount factor, the literature that analyzes the role of conditioning information is large (see, for example, DeRoon and Nijman (2001) and Bekaert and Liu (2004)).

evaluate the merits of expanding the set of assets in a portfolio to include international as well as domestic equities (Bekaert and Urias 1996; Errunza et al. 1999; De Roon et al. 2001; Kai et al. 2003). Other studies use spanning tests to investigate the gains from portfolio diversification across different classes or types of assets (Petrella 2005; Eun et al. 2009; Galvani and Plourde 2010).

Spanning tests statistically evaluate whether the addition of N risky "test" assets to a portfolio of K "benchmark" assets significantly expands the mean-variance efficient frontier. The K benchmark assets are said to span the N test assets if the addition of the test assets to the portfolio does not lead to a significant shift in the frontier.

Huberman and Kandel (1987) show that spanning can be evaluated using tests of linear restrictions on the parameters from a regression of the excess returns (the gross return minus the risk-free return) of the test assets on the excess returns of the benchmark assets:

$$\mathbf{r}_{Nt} = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{r}_{Kt} + \varepsilon_t, \qquad t = 1, 2, \ldots, T, \tag{1}$$

where $\mathbf{r}_{Nt}$ is an N×1 vector of excess returns on the N test assets, $\mathbf{r}_{Kt}$ is a K×1 vector of the excess returns on the K benchmark assets, $\boldsymbol{\varepsilon}_t$ is an N×1 vector with $\mathrm{E}[\boldsymbol{\varepsilon}_t] = \mathbf{0}_N$, $\mathbf{0}_N$ is an N-dimensional vector of zeros, $\boldsymbol{\alpha}$ is an N×1 vector, and $\boldsymbol{\beta}$ is a N×K matrix. The vector $\boldsymbol{\alpha}$ is equal to the difference – $\boldsymbol{\mu}_N$ - $\boldsymbol{\beta\mu}_K$, where $\boldsymbol{\mu}_N$ and $\boldsymbol{\mu}_K$ are the vectors of the expected excess returns on the test assets and the benchmark assets, respectively. A test that the K benchmark assets span the N test assets – a test that the mean-variance frontier generated by the benchmark securities does not shift significantly with the inclusion of the N test assets in the investment opportunities set – involves testing the 2N linear restrictions:

$$\boldsymbol{\alpha} = \mathbf{0}_N, \tag{2a}$$

$$\boldsymbol{\beta\iota}_K = \boldsymbol{\iota}_N, \tag{2b}$$

where $\boldsymbol{\iota}_K$ and $\boldsymbol{\iota}_N$ are K×1 and N×1 vectors of ones, respectively. If these restrictions hold exactly, the benchmark assets can perfectly replicate the returns on the test assets and there is no advantage to expanding the benchmark portfolio. In fact, investors would be worse off holding both the benchmark and test assets as this would increase the variance of the returns without raising the mean return of the portfolio (Kan and Zhou 2008).

Jobson and Korkie (1982, 1984, 1989) show that the mean-variance frontier associated with the augmented portfolio of the $N+K$ test and benchmark assets has a Sharpe ratio that is statistically indistinguishable from the Sharpe ratio of the benchmark frontier whenever the linear restrictions given in (2a) cannot be rejected. In other words, a test of restriction (2a) indicates whether there is a significant difference in the return per unit of risk for the benchmark and augmented portfolios. Kan and Zhou (2008) note that, when borrowing and lending at the risk free rate are unconstrained, investors are concerned only with the risk-return trade-off (the tangency portfolio of risky assets). In this case, investors would focus only on a test of (2a). If borrowing and lending at the risk free rate is constrained, investors are interested in the whole efficient frontier and, thus, in joint tests of restrictions (2a) and (2b).

Whenever the restrictions given in (2b) cannot be rejected, Kan and Zhou (2008) show that the Global Minimum Variance (GMV) portfolio of the benchmark efficient frontier does not differ significantly from that of the frontier associated with the expanded set of investments. The GMV portfolio of a collection of risky assets is the efficient portfolio that yields the lowest standard deviation of excess returns. Van Zijl (1987) calls the risk associated with this portfolio the unavoidable level of risk because no other portfolio of the risky assets has a smaller variance. As such, movements in the GMV portfolio are relevant from the perspective of market participants with an interest in hedging against risk or with high risk aversion (Kempf and Memmel 2005).

From the above discussion, it is clear that the null hypothesis of spanning can be decomposed into assessments of whether there are significant changes in the risk-return trade-off or the level of unavoidable risk. There is spanning whenever the test assets fail to improve both the return to risk and the level of unavoidable risk offered by the benchmark portfolio. By testing for both spanning and the equality of the Sharpe ratios (equality of the risk-return trade-off), it is possible to ascertain whether, when spanning is rejected, this is due to a change in the minimum level of risk or the return to risk taking.

3 Data and the Empirical Application

Tests of spanning (a joint test of (2a) and (2b)) and the equality of the risk-return trade-off (a test of restriction (2a)) are employed to evaluate the benefit of adding long-term bonds to a portfolio of short-term bills when the investment horizon is short. These tests are undertaken for investment horizons of one month and three months using monthly US data for the period June 1982 to September 2008 and Canadian data for June 1986 through June 2008. The short-term benchmark portfolio consists of 3-month, 6-month and 1-year Treasury bills when the investment horizon is one month in duration, but is limited to the 6-month and 1-year bills when the horizon is three months.[14] So that the test results for the one and three month investment horizons can be more easily compared, we also consider a short term portfolio that includes only the 6-month and 1-year bills for the case of the one month investment horizon.

The long-term test assets used to augment the short-term benchmark portfolio are 5-year, 7-year and 10-year bonds. We test whether the short-term portfolio can span an equally weighted portfolio of these long bonds as well as each long bond individually. The use of equally weighted portfolios is common due to the multicollinearity of returns (see, for example, Huberman and Kandel (1987)).

The spanning tests proposed by Huberman and Kandel (1987) are formulated in terms of excess returns – the gross return less the risk free return. The gross returns used here are holding period yields calculated from the last-business-day-of-the-month zero-coupon bond yield estimates

[14] It is important to use Treasury bills and bonds only as Reilly et al. (2010) find significant differences across bonds with different ratings. Use of bonds with variable ratings could confuse a rating effect with the riding-the-yield curve effect.

of the US Federal Reserve and the Bank of Canada.[15] The risk free return is represented by the one month and three month Treasury-bill rates when the investment horizon is one month and three months, respectively.

Over short horizons, long-term bonds can exhibit extremely large holding period returns. As a consequence, it is possible for the results of the regression-based statistical inference to be driven by a few large outliers. To ensure that this is not the case, we have removed a small number of outliers from the sample.[16] In only a few cases are test results affected by these outlier observations and *none* of the general conclusions change if the outliers are included in the sample.

This study employs data for non-overlapping investment horizons only, so the returns are calculated for buy-and-hold strategies with a duration equal to the length of the investment horizon (either one month or three months).[17] As the data are monthly, it is possible to calculate three different 3-month holding period return series – for example, one starting at the end of January, one starting at the end of February and one starting at the end of March. We report test results for each of these three series.

[15] More detailed information on the data and the method of calculating returns is provided in the *Appendix*. Bieri and Chincarini (2005) and Korn and Koziol (2006) also use holding period returns based on zero-coupon yield estimates.
[16] An outlier is defined as an observation that is more than 2.5 standard deviations away from the mean. The standard deviations of US and Canadian excess bond returns are large. For example, for the US, the average one month holding period excess return is a mere 19.4 percent of the corresponding standard deviation. For Canada, it is only 13.3 percent. As the standard deviations of US and Canadian bond returns are large, the outlier definition used here eliminates few observations. The exact percentage of outliers deleted from the sample in each case is reported in the tables of results and is generally below 5 percent of the available observations.
[17] With overlapping investment horizons, since neighboring observations rely, to some extent, on common information, the regression errors in the spanning test equation may be correlated across time, which violates the standard independence assumption underlying the test. This does not occur with non-overlapping observations. Bremer et al. (2011) also note that overlapping horizons can bias test results. The disadvantage of utilizing non-overlapping observations is that the parameters and test statistics are estimated with a smaller number of observations than if the observations are allowed to overlap. None of the conclusions change if we use overlapping observations rather than non-overlapping observations. (The results with overlapping observations are available by request from the authors.) Some riding-the-yield-curve studies, such as Grieves and Marcus (1992), also do not use overlapping observations, but many studies, particularly those with longer investment horizons, employ samples with overlapping observations (for example, Grieves et al. (1999), Ang et al. (1998), Bolder et al. (2004), and Bieri and Chincarini (2005)).

The hypotheses are evaluated using multivariate Wald tests. The Wald statistics are corrected for heteroskedasticity and autocorrelation as in Newey and West (1987). [18] Following the recommendation of Kan and Zhou (2001), we use bootstrapped p-values rather than asymptotic p-values.[19] In almost all cases, the bootstrapped and asymptotic p-values yield identical results.

4 Empirical results – US data

This section reports test results using US data while the following section provides corresponding results for Canadian data. In all cases, the test statistics are derived from regressions of the excess returns of the test assets – the assets being added to the portfolio – on the excess returns of the benchmark assets (as in equation (1)).

4.1 Is it advantageous to add longer maturity bonds to a portfolio of short-term bills?

An investor has a 1-month investment horizon and a benchmark portfolio of short-term US Treasury bills (the short-term portfolio). Given the much higher average excess returns on long-term bonds, the investor considers adding longer maturity bonds to the short-term portfolio. Table 3, Part A, gives the results for tests of spanning (a joint test of (2a) and (2b)) and the equality of the Sharpe ratios (a test of (2a)) when the short-term "benchmark" portfolio consists of Treasury bills with maturities of 6-months and 1- year. The results in Part B of Table 3 are for the case in which the short-term benchmark portfolio also includes the 3-month bill. The "test" assets – the 5-year, 7-year and 10-year bonds – are evaluated individually and as part of an equally-weighted portfolio.

The results in Table 3, Parts A and B, indicate that the hypothesis that the benchmark portfolio of short-term bonds spans the long-term bonds cannot be rejected. This is the case whether the long-term bonds are examined individually or combined in an equally-weighted

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[18] Estimation of the multivariate regression is undertaken using an exactly identified two-step Generalized Method of Moments methodology that incorporates the Newey-West spectral matrix with $T^{1/4}$ lags.
[19] We use semi-parametric bootstrapping techniques as described in Davidson and MacKinnon (2004, Section 4.6).

portfolio, and whether or not there are restrictions on short sales. Tests for the equality of the Sharpe ratios indicate that none of the portfolios augmented with the long-term bonds yields a significantly different Sharpe ratio than the benchmark portfolio of short-term assets. Hence, the results in Table 3 clearly indicate that broadening the set of investment possibilities to include long-term bonds when the investment horizon is one month in length neither improves the risk-return trade-off (the Sharpe ratio) nor decreases the overall level of risk (the GMV).

The results for a 3-month investment horizon are given in Table 4. In this case, the benchmark portfolio consists of the 6-month and 1-year Treasury bills only. Further, as noted above, there exist three different 3-month holding period return series depending on whether the first observation in the series is for an investment that begins at the end of January, the end of February or the end of March. The test results for each of these three series are presented in Parts A, B and C of Table 4, respectively.

The results in Parts B and C of Table 4 are consistent with the results in Table 3. Neither spanning nor the equality of the Sharpe ratios can be rejected with or without short selling (except for the case of the 7-year bond with short selling in Part C). These results imply that, in general, three month investments initiated at the end of February or the end of March gain no benefit, both in terms of the risk-return tradeoff or the minimum level of risk, from the addition of long-term bonds to a portfolio of short-term bills. In contrast, the time series of returns for three month investments that begin at the end of January point to significant diversification gains from the addition of long-term bonds (Table 4, Part A). These diversification gains vanish when short sales restrictions are imposed, which suggests that the diversification gains observed for trades starting in January can only be realized by short selling some assets.

The results reported in Tables 3 and 4 indicate that there is little benefit to including long-term bonds in a portfolio of short-term bills when the investment horizon is short. The evidence of insignificant diversification gains is particularly compelling when we consider restrictions on short selling and when the investment horizon is one month in duration. These findings cast doubt on the profitability of riding-the-yield-curve strategies that attempt to exploit the return differential between longer and shorter maturity bonds. Nevertheless, given the results in Tables 3 and 4, it may still be possible for a portfolio of longer maturity assets to provide the same risk-return trade-off and minimum level of risk exposure as a portfolio of short-term assets. In this case, the long-term assets would span the short maturity portfolio and a strategy of riding-the-yield-curve, investing in long-term bonds when the investment horizon is short, would be no worse, from a mean-variance perspective, than investing in both long and short-term assets.

4.2 Is it advantageous to add shorter maturity bills to a portfolio of long-term bonds?

To determine whether holding long-term bonds only is no worse than holding both short-term and long-term assets over a short horizon, we reverse the testing procedure and evaluate whether a portfolio of long-term bonds can span the short-term bills (those with maturities of 3-months, 6-months and 1-year). To undertake this test, the baseline regression, equation (1), is estimated with the short-term assets as the test assets and the long-term bonds (the 5, 7, and 10 year bonds) as the benchmark assets. Each of the test assets (the 3-month, 6-month and 1-year Treasury bills) is considered individually as well as aggregated in an equally-weighted portfolio.[20]

The results of these tests are given in Table 5 for a one-month investment horizon and in Table 6, Parts A, B and C, for a three-month investment horizon. In all cases, the results indicate that adding short-term bills to a portfolio of long-term bonds has a statistically significant impact on the efficient frontier. The long-term bonds do not span any of the short-term bills and the addition of the short-term bills to the portfolio improves the risk-return tradeoff (as measured by the Sharpe

[20] For consistency with the previous section, we present two equally-weighted (EW) portfolios of short-term debt instruments for the one-month investment horizon. The first EW portfolio includes the returns of the 3-month, 6-month and 1-year Treasury bills, while the second EW portfolio excludes the 3-month bill.

ratio). These findings hold for the 1-month and 3-month investment horizons, across all three series of three-month holding period returns, and whether or not restrictions on short selling are imposed.

[Figure 1a]

The test results given in Tables 5 and 6 suggest that there is a significant shift in the efficient frontier following the addition of an equally-weighted portfolio of short-term assets to a portfolio of long-term bonds. This shift in the efficient frontier is illustrated in Figure 1a. For comparison purposes, in Figure 1b we plot the corresponding shift in the efficient frontier when an equally-weighted portfolio of long-term bonds is added to a portfolio of short-term bills. In this case, there is almost no movement in the efficient frontier, which is consistent with the test results given in Tables 3 and 4.[21] Figure 1 and the test results in Tables 5 and 6 suggest that an investor with either a 1-month or 3-month investment horizon would be worse off choosing a portfolio that contained long-term bonds only despite the higher average returns associated with these bonds.

[Figure 1b]

The results of this section imply that a portfolio of short-term US bills spans augmented portfolios that include long-term US bonds when the investment horizon is one month or three months in duration. Thus, the inclusion of long-term bonds in a portfolio of short-term assets is not beneficial when the investment horizon is short. In contrast, a portfolio of long-term bonds cannot span the short-term bills, so the addition of short-term bills to the long-term portfolio improves the risk return trade-off of the augmented portfolio relative to a portfolio that includes only long-term bonds. This suggests that an investor would be worse off pursuing a riding-the-yield-curve

[21] The expanded frontiers (the dotted lines) in Figures 1a and 1b are associated with the same set of long *and* short-term maturities. Nevertheless, these lines differ to a small extent because the alternative portfolio in each case is an equally-weighted portfolio of the alternative assets, rather than an unconstrained portfolio. The imposition of the equal weights constraint for the two alternative portfolios, which is necessitated by the multicollinearity of returns, causes the small difference in the dotted lines. The dotted lines look more different in these figures than they actually are because the scales on the vertical and horizontal axes of the two figures are different.

investment strategy that involved holding long-term bonds only. Further, as the short-term assets span the long-term bonds, an investor with a short investment horizon is better off holding short-term assets only since the addition of long-term assets to the portfolio would increase the variance of the portfolio without increasing the mean return.

5 Empirical results – Canadian data

The tests for spanning using Canadian data reinforce the conclusions obtained for the US. The results reported in Tables 7 and 8 indicate that long-term bonds fail to improve significantly the investment opportunities offered by short-term assets. Table 7 shows that augmenting the short-term portfolio with longer maturity assets does not improve the risk-return tradeoff or the level of minimum risk for portfolio managers with a one-month investment horizon. The same result holds, broadly, for investors with a three-month horizon, although, in a few cases, long-term bonds seem to entail some diversification gains over portfolios of short-term assets through a decrease in the minimum level of risk (Table 8). These gains vanish when short selling restrictions are taken into account.[22]

As with the results for the US, the spanning tests that employ Canadian data imply a significant shift in the efficient frontier with the addition of short-term bills to a portfolio of long-term bonds (Tables 9 and 10). For investors with an investment horizon of one-month, the gain from the addition of the short-term instruments consists of both an improved tradeoff between risk and return and a decreased minimum level of risk. Investors with a three-month horizon benefit only from a significantly lower level of minimum risk. Thus, there exist significant diversification gains when short-term bills are added to a portfolio of long term assets.

[Figure 2a]

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[22] To investigate whether the lack of diversification gains offered by a portfolio of longer maturity bonds is robust for different sample sub-periods, the tests for spanning and equality of the Sharpe ratios were re-evaluated for all possible rolling windows of 10 years in length, spaced one month apart, for both the US and Canada. The results of these tests are almost always consistent with the results for the whole sample reported in the tables.

The findings described in this section can be graphically summarized by two figures. Figure 2a illustrates the significant shift in the efficient frontier when an equally-weighted index of short-term bills is added to a portfolio of long maturity bonds. In contrast, Figure 2b shows that the addition of an equally-weighted portfolio of long-term bonds to a portfolio of short maturity assets yields almost no change in the efficient frontier. Given the similarities between the test results for the US and Canada, it is not surprising that Figures 2a and 2b strongly resemble Figures 1a and 1b.

[Figure 2b]

6 Summary and Conclusion

This study addresses the question of whether the addition of long maturity bonds to a portfolio of short-term bills can benefit investors with one or three month investment horizons. Several descriptive statistics suggest that this could be the case – the average 1-month and 3-month excess returns on long-term bonds are relatively high and have low rates of correlation with the returns of shorter maturity bills.

Test results using US data suggest that the risk-return tradeoff offered by short-term debt instruments cannot be improved by broadening the portfolio to include long-term bonds. Moreover, long-term bonds fail to decrease the overall level of risk exposure – that is, the level of unavoidable risk (Van Zijl 1987). This is the case for both the one and three month investment horizons, although there is a January anomaly in the three month case that disappears when short selling restrictions are taken into account.

The analysis of Canadian data is generally consistent with our findings for the US. The addition of long-term bonds to a portfolio of shorter maturity Canadian debt instruments fails to deliver an improved risk-return trade-off. Further, for the one-month investment horizon, the addition of long bonds does not significantly lower the level of unavoidable risk associated with portfolios of short-term instruments. For the three month investment horizon, there is some evidence that augmenting the portfolio with long bonds can decrease the portfolio's minimum level of risk, but only if short selling is allowed.

While this evidence suggests that broadening a portfolio of short-term bills to include longer maturities fails to yield significant diversification benefits, the opposite is not the case. For both the US and Canada, and for both the one and three month investment horizons, the inclusion of short-term debt instruments in a portfolio of long-term bonds yields a significant shift in the efficient frontier. That is, extending portfolios of long-term bonds to include relatively low-return low-risk short-term instruments appears to deliver substantial gains to investors with mean-variance preferences. Thus, a riding-the-yield-curve investment strategy that involves holding long-term bonds only would make investors worse off from a mean-variance perspective.

The overall message of this analysis is that an investor should hold short-term debt instruments rather than long-term bonds when the investment horizon is short. It is not optimal to hold a portfolio that includes long-term bonds as well as short term bills since this portfolio would have the same mean return as a portfolio that includes only short-term assets, but a larger variance. The insignificance of the gains from including long-term bonds in a portfolio of short-term debt instruments appears to hold both for investors interested in maximizing the return to risk (i.e., the Sharpe ratio) and investors concerned with the overall risk exposure of their portfolio holdings (i.e. the Global Minimum Variance). These results are generally robust to the use of data for the US or Canada, to whether the investment horizon is one or three months in duration, and to whether or not short selling is allowed. The results presented here provide no support for investment strategies that involve holding longer maturity bonds when the investment horizon is short, such as a riding-the-yield-curve investment strategy.

Acknowledgments We thank Constance Smith and two anonymous referees for helpful comments on an earlier draft.

Appendix 1: Description of the Data and Data Sources

One month holding period yields (*HPY*) are calculated as in Bolder, Johnson and Metzler (2004, 25-6), except that we use the exact number of days between monthly observations rather than an approximation:

$$HPY(t,N,T) = \frac{1}{d}(T \cdot z(t,t+T) - (T-d) \cdot z(t+d,t+T))$$

where *HPY(t,N,T)* = the holding period yield on a bond maturing in *T* years that is purchased at time *t* and held for *N* days,

d = *N/(*number of days in the year*)*,

z(t,t+T) = the continuously compounded yield at time *t* on a zero-coupon bond that matures in *T* years.

The holding period yield reported for a particular date (observation) is the *ex post* yield realized from an investment made one month (three months) prior to that date, where one month (three months) is the length of the investment horizon. All yields are continuously compounded at annual rates.

The US yields used to calculate the holding period yields are constant maturity nominal US Treasury yields as of the last business day of the month. These were downloaded from the website of the Board of Governors of the US Federal Reserve. The methodology used to estimate the zero-coupon Treasury yield curves is described on the website of the Office of Debt Management, Department of the Treasury.

The Canadian zero-coupon yields are for the last business day of each month and were downloaded from the website of the Bank of Canada. The methodology used to calculate the Government of Canada zero-coupon bond yields is described in Bolder, Johnson and Metzler (2004).

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Table 1: Mean excess holding period returns and Sharpe ratios by maturity for government bills and bonds (returns are percent per annum)

Part A: One month excess returns

	US		Canada	
Maturity	Excess holding-period return	Sharpe ratio	Excess holding-period return	Sharpe ratio
3-months	0.19	.24	0.42	.47
6-months	0.79	.41	0.36	.16
1-year	1.22	.30	0.74	.15
2-year	2.45	.28	1.31	.14
3-year	2.91	.22	1.79	.13
5-year	4.11	.19	2.70	.13
7-year	5.10	.17	3.34	.12
10-year	5.88	.15	3.97	.11

Part B: Three month excess returns

	US		Canada	
Maturity	Excess holding-period return	Sharpe ratio	Excess holding-period return	Sharpe ratio
6-months	0.55	.63	0.14	.17
1-year	1.11	.45	0.46	.18
2-year	2.32	.41	1.03	.18
3-year	2.83	.32	1.51	.19
5-year	4.07	.29	2.27	.19
7-year	5.09	.27	2.89	.18
10-year	5.82	.23	3.74	.19

Sample period: US, June 1982 - September 2008; Canada, June 1986 - June 2008.

Holding period returns are calculated as of the last business day of the month for non-overlapping one-month or three month periods. Excess holding period returns are calculated by subtracting the one or three month Treasury bill yield from the gross holding period return. For the US and Canada, respectively, the 1-month Treasury bill yield is from Fama and French (downloaded from the CRSP database) and Statistics Canada (Cansim identifier V122529). The 3-month yields are the 3-month zero-coupon Treasury bill yields calculated by the Federal Reserve and the Bank of Canada, respectively (see the *Appendix* for more details).

The Sharpe ratio is the average excess holding period return divided by the standard deviation of the excess holding period returns for the sample period.

Table 2: Correlations between excess US holding period returns for bills and bonds of different maturities

Part A: 1-month excess holding period returns

	3-month	6-month	1-year	2-year	3-year	5-year	7-year	10-year
3-month	1							
6-month	0.583335	1						
1-year	0.411712	0.907375	1					
2-year	0.286129	0.776586	0.943241	1				
3-year	0.258201	0.729261	0.905374	0.984788	1			
5-year	0.230323	0.678293	0.858546	0.953731	0.983941	1		
7-year	0.2422	0.65907	0.828692	0.924004	0.958556	0.986822	1	
10-year	0.235526	0.626035	0.792273	0.88859	0.93002	0.968167	0.986869	1

Part B: 3-month (non-overlapping) excess holding period returns

	6-month	1-year	2-year	3-year	5-year	7-year	10-year
6-month	1						
1-year	0.925564	1					
2-year	0.821016	0.956418	1				
3-year	0.773361	0.922981	0.991322	1			
5-year	0.727938	0.883172	0.969427	0.989869	1		
7-year	0.707129	0.855376	0.94243	0.969251	0.990663	1	
10-year	0.684479	0.83127	0.917521	0.947246	0.977158	0.994129	1

Sample period: June 1982 – September 2008

Table 3: Tests for spanning and equality of the Sharpe ratio when longer maturity US Treasury bonds are added to a portfolio of short term US Treasury bills – 1 month investment horizon

Part A: The short term (benchmark) portfolio consists of 6-month and 1-year US Treasury bills

Maturity of added longer maturity bond (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
5-year	0.22	0.00	1.26	3.4
	(0.89)	(0.95)	(0.16)	
7-year	0.41	0.01	0.68	3.7
	(0.85)	(0.87)	(0.24)	
10-year	0.87	0.57	0.01	4.7
	(0.67)	(0.45)	(0.42)	
Equally Weighted	0.47	0.17	0.25	4.3
Portfolio	(0.84)	(0.66)	(0.26)	

Part B: The short term (benchmark) portfolio consists of 3-month, 6-month and 1-year US Treasury bills

Maturity of added longer maturity bond (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
5-year	0.93	0.07	2.13	5.9
	(0.55)	(0.84)	(0.20)	
7-year	1.92	0.05	3.49	6.2
	(0.36)	(0.79)	(0.18)	
10-year	3.16	0.22	0.17	7.1
	(0.21)	(0.64)	(0.50)	
Equally Weighted	1.77	0.01	1.94	6.8
Portfolio	(0.36)	(0.90)	(0.19)	

Spanning hypothesis: The assets in the benchmark portfolio span the test assets. This is a joint test of (2a) and (2b).

Sharpe ratio hypothesis: The Sharpe ratio of the benchmark portfolio is equal to the Sharpe ratio of the augmented portfolio. This is a test of (2a).

* Rejects the hypothesis at 5 percent.

The tests employ excess returns over the 1-month US Treasury bill rate.

The values reported are Wald test statistics. The values in brackets are p-values which are based on Newey-West standard errors calculated using $T^{1/3}$ lags where T is the number of observations (see Davidson and MacKinnon, 2004, Section 9.3).

Number of observations: 321 before outliers are removed.

Table 4: Tests for spanning and equality of the Sharpe ratio when longer maturity US Treasury bonds are added to a portfolio of short term US Treasury bills – 3 month investment horizon

Maturity of added longer maturity bond (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
Part A: The first 3-month investment horizon begins at the end of January				
5-year	13.56*	4.19*	1.16	3.7
	(0.00)	(0.01)	(0.13)	
7-year	11.35*	4.13*	0.69	4.7
	(0.00)	(0.04)	(0.20)	
10-year	16.95*	2.79	0.19	4.7
	(0.00)	(0.11)	(0.33)	
Equally Weighted	11.59*	3.19	0.52	3.7
Portfolio	(0.02)	(0.08)	(0.22)	
Part B: The first 3-month investment horizon begins at the end of February				
5-year	7.61	2.53	0.46	5.6
	(0.06)	(0.11)	(0.33)	
7-year	6.53	2.48	0.23	5.6
	(0.06)	(0.15)	(0.26)	
10-year	7.68	1.81	0.01	5.6
	(0.05)	(0.14)	(0.45)	
Equally Weighted	7.44	2.22	0.12	5.6
Portfolio	(0.08)	(0.22)	(0.32)	
Part C: The first 3-month investment horizon begins at the end of March				
5-year	5.62	0.29	0.47	2.8
	(0.09)	(0.60)	(0.29)	
7-year	7.32*	0.49	0.28	2.8
	(0.04)	(0.43)	(0.35)	
10-year	7.12	0.10	0.0001	2.8
	(0.06)	(0.73)	(0.52)	
Equally Weighted	7.13	0.24	0.86	2.8
Portfolio	(0.09)	(0.64)	(0.34)	

See notes to Table 3.

The short term (benchmark) portfolio consists of 6-month and 1-year US Treasury bills.

All tests employ excess returns calculated relative to the 3-month US Treasury bill rate.

Number of observations: 106 before outliers are removed.

Table 5: Tests for spanning and equality of the Sharpe ratio when shorter maturity US Treasury bills are added to a portfolio of long term US Treasury bonds – 1 month investment horizon

Maturity of added short maturity bill (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
3-month	>500* (0.00)	8.47* (0.00)	341.55* (0.00)	5.9
6-month	>500* (0.00)	59.29* (0.00)	375.87* (0.00)	4.6
1-year	>500* (0.00)	25.11* (0.00)	351.54* (0.00)	4.3
Equally Weighted Portfolio (EW)	>500* (0.00)	35.10* (0.00)	359.23* (0.00)	4.3
EW excluding 3-month	>500* (0.00)	40.00* (0.00)	367.94* (0.00)	4.3

See the notes to Table 3.

The long term (benchmark) portfolio consists of 5-year, 7-year and 10-year US Treasury bonds.

Table 6: Tests for spanning and equality of the Sharpe ratio when shorter maturity US Treasury bills are added to a portfolio of long term US Treasury bonds – 3 month investment horizon

Maturity of added short maturity bill (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
Part A: The first 3-month investment horizon begins at the end of January				
6-month	>500* (0.00)	66.25* (0.00)	93.43* (0.00)	5.6
1-year	>500* (0.00)	14.65* (0.00)	85.08* (0.00)	3.7
Equally Weighted Portfolio	>500* (0.00)	21.97* (0.00)	90.40* (0.00)	4.7
Part B: The first 3-month investment horizon begins at the end of February				
6-month	>500* (0.00)	18.15* (0.04)	107.77* (0.00)	4.7
1-year	>500* (0.00)	68.86* (0.00)	95.85* (0.00)	5.6
Equally Weighted Portfolio	>500* (0.00)	32.55* (0.00)	102.84* (0.00)	5.6
Part C: The first 3-month investment horizon begins at the end of March				
6-month	>500* (0.00)	85.97* (0.00)	99.87* (0.00)	1.8
1-year	>500* (0.00)	23.12* (0.00)	89.76* (0.00)	2.8
Equally Weighted Portfolio	>500* (0.00)	45.67* (0.00)	95.16* (0.00)	1.8

See notes to Table 3.

The long term (benchmark) portfolio consists of 5-year, 7-year and 10-year US Treasury bonds.

All tests employ excess returns calculated relative to the 3-month US Treasury bill rate.

Number of observations: 106 before outliers are removed.

Table 7: Tests for spanning and equality of the Sharpe ratio when longer maturity Canadian Treasury bonds are added to a portfolio of short term Canadian Treasury bills – 1 month investment horizon

Part A: The short term (benchmark) portfolio consists of 6-month and 1-year Canadian Treasury bills

Maturity of added longer maturity bond (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
5-year	4.33	0.35	1.33	4.4
	(0.13)	(0.52)	(0.60)	
7-year	5.67	0.17	4.00	4.4
	(0.08)	(0.68)	(0.59)	
10-year	7.34	0.34	3.55	4.1
	(0.06)	(0.50)	(0.49)	
Equally Weighted	7.14*	0.26	4.07	4.4
Portfolio	(0.04)	(0.71)	(0.51)	

Part B: The short term (benchmark) portfolio consists of 3-month, 6-month and 1-year Canadian Treasury bills

Maturity of added longer maturity bond (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
5-year	4.91	4.81*	0.15	4.8
	(0.13)	(0.03)	(0.32)	
7-year	3.37	3.37	0.20	6.2
	(0.24)	(0.05)	(0.38)	
10-year	1.75	1.72	0.42	7.1
	(0.39)	(0.18)	(0.23)	
Equally Weighted	2.67	2.66	0.28	5.1
Portfolio	(0.21)	(0.08)	(0.31)	

See the notes to Table 3.

The tests employ excess returns over the 1-month Canadian Treasury bill rate.

Number of observations: 270 before outliers are removed.

Table 8: Tests for spanning and equality of the Sharpe ratio when longer maturity Canadian Treasury bonds are added to a portfolio of short term Canadian Treasury bills – 3 month investment horizon

Maturity of added longer maturity bond (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
Part A: The first 3-month investment horizon begins at the end of January				
5-year	24.41*	0.02	0.40	3.4
	(0.00)	(0.85)	(0.29)	
7-year	12.07*	0.04	0.76	3.4
	(0.02)	(0.85)	(0.19)	
10-year	6.47	0.08	1.22	3.4
	(0.08)	(0.72)	(0.19)	
Equally Weighted	10.78*	0.03	0.89	3.4
Portfolio	(0.03)	(0.88)	(0.16)	
Part B: The first 3-month investment horizon begins at the end of February				
5-year	19.21*	0.07	0.00	4.5
	(0.00)	(0.78)	(0.66)	
7-year	7.00*	0.00	0.00	4.5
	(0.02)	(0.98)	(0.75)	
10-year	5.22	0.02	0.0025	4.5
	(0.20)	(0.84)	(0.53)	
Equally Weighted	7.32	0.00	0.001	3.4
Portfolio	(0.06)	(0.96)	(0.53)	
Part C: The first 3-month investment horizon begins at the end of March				
5-year	11.95*	1.25	0.23	3.4
	(0.01)	(0.17)	(0.33)	
7-year	8.89*	2.81	0.34	4.5
	(0.03)	(0.11)	(0.27)	
10-year	9.73*	4.95*	1.89	5.7
	(0.03)	(0.03)	(0.08)	
Equally Weighted	10.38*	3.46	0.69	4.5
Portfolio	(0.00)	(0.05)	(0.27)	

See notes to Table 3.

The short term (benchmark) portfolio consists of 6-month and 1-year Canadian Treasury bills.

All tests employ excess returns calculated relative to the 3-month Canadian Treasury bill rate.

Number of observations: 88 before outliers are removed.

Table 9: Tests for spanning and equality of the Sharpe ratio when shorter maturity Canadian Treasury bills are added to a portfolio of long term Canadian Treasury bonds – 1 month investment horizon

Maturity of added short maturity bill (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
3-month	>500* (0.00)	79.39* (0.00)	281.23* (0.00)	4.4
6-month	>500* (0.00)	8.48* (0.00)	275.01* (0.00)	4.1
1-year	>500* (0.00)	6.69* (0.01)	240.15* (0.00)	4.4
Equally Weighted Portfolio (EW)	>500* (0.00)	14.20* (0.00)	272.88* (0.00)	4.3
EW excluding 3-month	>500* (0.00)	8.12* (0.00)	260.90* (0.00)	4.4

See the notes to Table 3.

The long term (benchmark) portfolio consists of 5-year, 7-year and 10-year Canadian Treasury bonds.

Number of observations: 270 before outliers are removed.

Table 10: Tests for spanning and equality of the Sharpe ratio when shorter maturity Canadian Treasury bills are added to a portfolio of long term Canadian Treasury bonds – 3 month investment horizon

Maturity of added short maturity bill (the "test" asset)	*Spanning hypothesis*	*Sharpe ratio hypothesis*	*Spanning hypothesis with no short selling*	*Outliers removed (percent of sample)*
Part A: The first 3-month investment horizon begins at the end of January				
6-month	>500* (0.00)	0.06 (0.74)	136.9* (0.00)	3.4
1-year	>500* (0.00)	0.63 (0.38)	190.5* (0.00)	3.4
Equally Weighted Portfolio	>500* (0.00)	0.39 (0.53)	193.1* (0.00)	3.4
Part B: The first 3-month investment horizon begins at the end of February				
6-month	>500* (0.00)	0.39 (0.58)	121.69* (0.00)	4.5
1-year	>500* (0.00)	1.14 (0.29)	129.79* (0.00)	4.5
Equally Weighted Portfolio	>500* (0.00)	0.84 (0.29)	122.40* (0.00)	4.5
Part C: The first 3-month investment horizon begins at the end of March				
6-month	>500* (0.00)	0.13 (0.70)	70.10* (0.00)	4.5
1-year	>500* (0.00)	0.46 (0.49)	71.62* (0.00)	5.6
Equally Weighted Portfolio	>500* (0.00)	0.45 (0.52)	71.63* (0.00)	4.5

See notes to Table 3.

The long term (benchmark) portfolio consists of 5-year, 7-year and 10-year Canadian Treasury bonds.

All tests employ excess returns calculated relative to the 3-month Canadian Treasury bill rate.

Number of observations: 88 before outliers are removed.

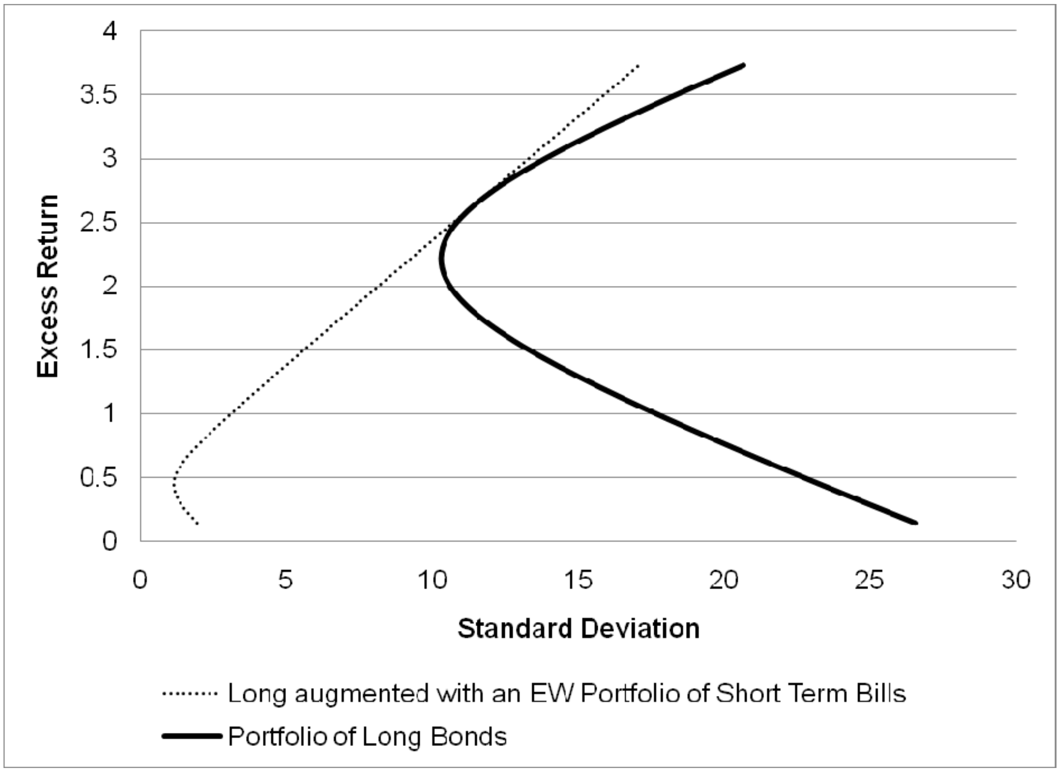

Fig 1a Efficient frontiers when long maturity bonds are the benchmark – US case. The solid line represent the efficient frontier for a portfolio of long bonds. The dotted line is the efficient frontier when this portfolio of long bonds is augmented with an equally-weighted portfolio of short term bills

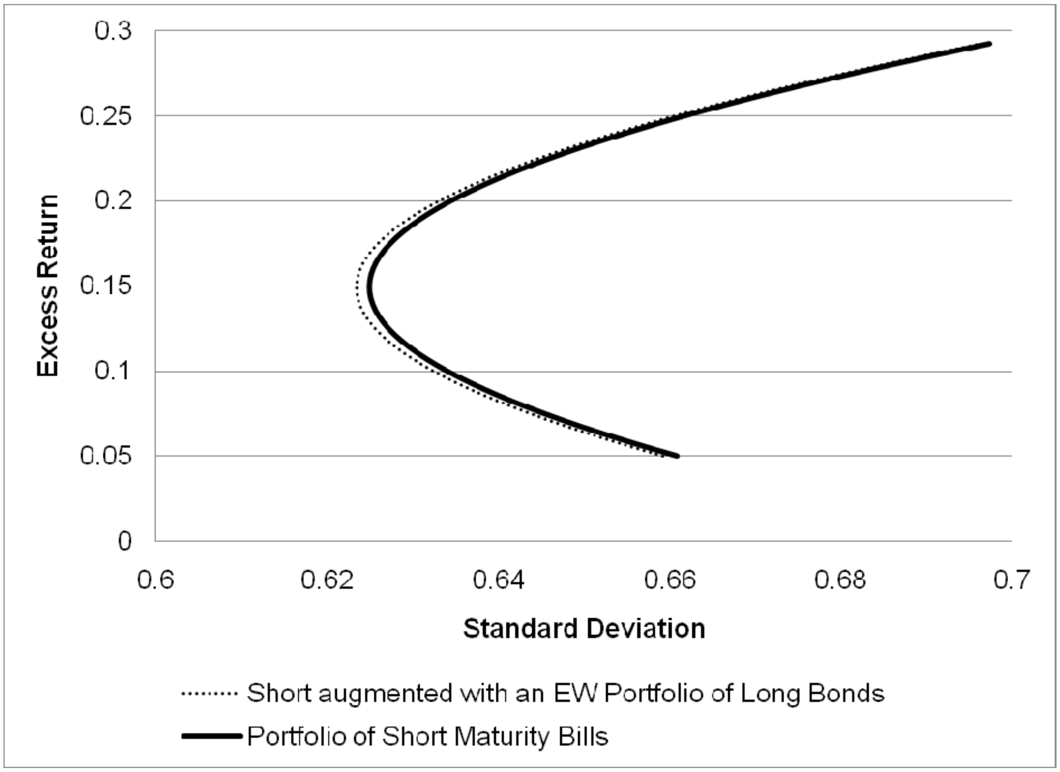

Fig 1b Efficient frontiers when short maturity bills are the benchmark – US case. The solid line represent the efficient frontier for a portfolio of short maturity bills. The dotted line is the efficient frontier when this portfolio of short term bills is augmented with an equally-weighted portfolio of long term bonds

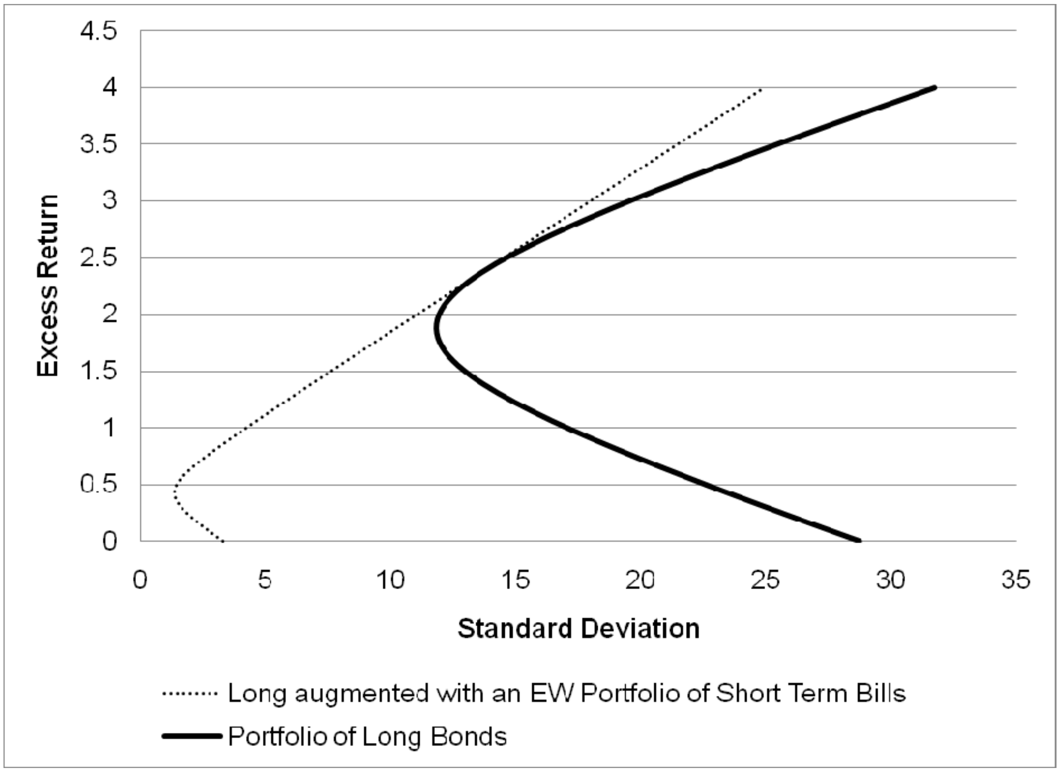

Fig 2a Efficient frontiers when long maturity bonds are the benchmark – Canada case. The solid line represent the efficient frontier for a portfolio of long bonds. The dotted line is the efficient frontier when this portfolio of long bonds is augmented with an equally-weighted portfolio of short term bills

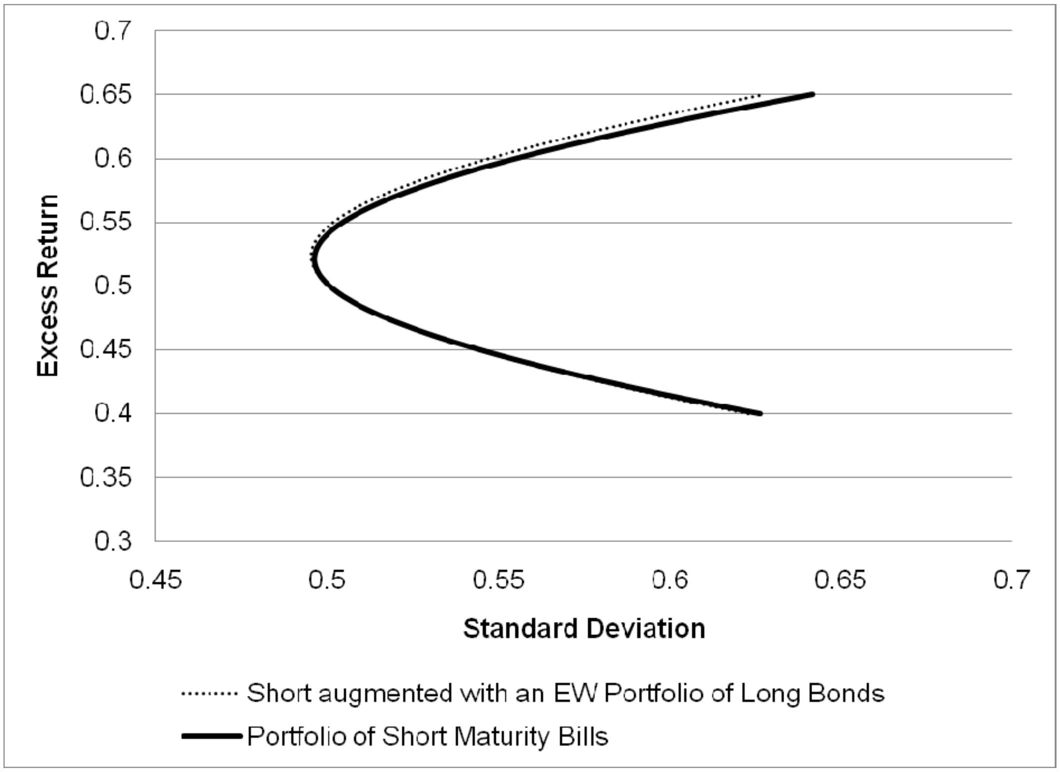

Fig 2b Efficient frontiers when short maturity bills are the benchmark – Canada case. The solid line represent the efficient frontier for a portfolio of short maturity bills. The dotted line is the efficient frontier when this portfolio of short term bills is augmented with an equally-weighted portfolio of long term bonds

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