DEPARTMENT OF
ECONOMICS
UNIVERSITY OF ALBERTA

Working Paper No. 2010-04

State-dependent congestion pricing with reference-dependent preferences

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January 2010

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December 9, 2009

Demand and capacity fluctuations are common for roads and other congestible facilities. With ongoing advances in pricing technology and ways of communicating information to prospective users, state-dependent congestion pricing is becoming increasingly practical. But it is still rare or nonexistent in many potential applications. One explanation is that people dislike uncertainty about how much they will pay. To explore this idea a model of reference-dependent preferences is developed based on Kőszegi and Rabin (2006). Using a facility yields an "intrinsic" utility and a "gain-loss" utility measured relative to the probability distribution over states of utility outcomes. Two types of preferences are analyzed: *bundled* preferences in which gains and losses are perceived for overall utility, and *unbundled* preferences in which gains and losses are perceived separately for the toll and other determinants of utility.

Tolls are chosen to maximize total expected utility plus revenues. With bundled preferences the toll is set above the Pigouvian level when usage conditions are good, and below it when conditions are bad, to reduce gains and losses from fluctuations in utility. With unbundled preferences the direction of toll adjustment is less clear and depends on whether supply or demand is variable. For both types of preferences tolls are sensitive to the strength of gain-loss utility. If gain-loss utility is moderately strong, a state-independent toll can be optimal.

Key words: Congestion pricing; State-dependent pricing; Reference-dependent preferences

1 INTRODUCTION

Demand and capacity fluctuations are common in road and air transportation, at recreational areas, in communications networks, and at other congestible facilities. According to the principles of social marginal cost pricing congestion tolls or admission fees at public facilities should vary with usage conditions. Private-sector operators also have an incentive to vary prices in order to ration demand profitably and maintain service quality. Reliable service and predictable waiting or service times are often highly valued by consumers. For example, drivers are strongly averse to travel time uncertainty (De Palma and Picard, 2005) and consider unpredictable travel time more onerous than predictable time (Small et al., 2005). State-dependent pricing can help to alleviate uncertainty by reducing both the average level and the variability of demand at peak times.

State-dependent pricing is widespread in the air travel, hotel, and some other industries. Yet despite advances in pricing technology and the growing ease of communicating information to prospective users, state-dependent pricing is still rare or nonexistent in many potential applications. In road transportation it has only been implemented on a few High Occupancy Toll (HOT) lanes in the U.S. On other roads tolls are either set according to a predictable schedule or are constant over time.[1]

There are several possible explanations for the dearth of state-dependent pricing. One is that the infrastructure and operating costs are (still) too high for it to be cost-effective (Levinson and Odlyzko, 2008). Another is that individuals prefer simple and predictable pricing schemes and would find it difficult to deal with changing and possibly complex charges (Bonsall et al., 2007). A third and related obstacle is that state-dependent pricing can ration access efficiently only if prices are set far enough in advance of usage that individuals can adapt their usage plans. Legal barriers may also prevent charges being varied — perhaps on the grounds that doing so would be discriminatory.

This paper examines another explanation for the rarity of state-dependent pricing: that individuals dislike price uncertainty per se. According to standard consumer theory individuals are approximately risk-neutral with respect to small monetary outlays. Yet there is a large body

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[1] See http://www.tollroadsnews.com/node/3975 [February 11, 2009].

of evidence that even for stakes of just a few dollars individuals weigh monetary losses more than monetary gains. Individuals can also object strongly to price hikes that are not accompanied by an increase in supply or that do not seem to be justified by higher supplier costs (Frey and Pommerehne, 1993; Xia et al., 2004). Raising prices to ration demand for a fixed supply, particularly in response to an infrequent and severe shortage such as in the aftermath of a hurricane, may even be considered repugnant (Roth, 2007).

There is also a large body of evidence that attitudes towards variations in either service quality or prices are strongly influenced by expectations. The importance of expectations in determining attitudes towards congestion has long been recognized in the literature on outdoor recreation (Clawson and Knetsch, 1966). Expectations can influence perceptions of crowding and willingness to pay for a visit more strongly than do actual encounters.[2] Expectations also influence assessments of prices. Prices previously paid or prices charged at comparable facilities or under similar circumstances establish a benchmark or reference point against which new prices are judged.[3] Prices that exceed the benchmark may be considered excessive and may induce opposition or steep drops in demand.

When individuals are averse to fluctuations in service quality and fluctuations in prices, there is a tradeoff between varying prices according to Pigouvian principles to smooth congestion and keeping prices relatively constant to reduce price uncertainty. As noted above dynamic tolling is applied on a few HOT lanes and drivers are willing to forego certainty about payment in return for predictable (and shorter) travel times. However, HOT lanes offer drivers a readily understood and attractive option. Since tolls are adjusted to maintain free-flow conditions on the toll lanes the link between service quality and the level of the toll is clear. Furthermore, tolls are posted near the entrance to HOT lanes and drivers can choose at the last minute whether to pay the toll or take the adjacent toll-free lanes. State-dependent pricing may be less readily accepted for

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[2] See Stankey (1972), Shelby et al. (1983), and Michael and Reiling (1997). It has long been alleged that employees at Disneyland parks overstate expected waiting times for rides. One explanation is that this practice encourages people to spend time at restaurants or other revenue-generating activities rather than joining queues. Another explanation is that visitors feel elated when waiting times turn out to be shorter than predicted. I am grateful to Terry Daniel for this information.

[3] See Kahneman et al. (1986), Reiling et al. (1988), McCarville and Crompton (1987), and McCarville (1996) inter alios.

other types of road-pricing schemes or at other congestible facilities if it is difficult to avoid paying charges when they are high (Golob, 2001).

To explore the implications for state-dependent pricing of aversion to variations in service quality and prices this paper develops a model based on Kőszegi and Rabin's (2006) general model of reference-dependent preferences. To emphasize the generality of the model, users or prospective users of the facility will be called "agents". Using a facility yields an "intrinsic" utility and a "gain-loss" utility measured relative to the probability distribution over states of outcomes that result from the usage decisions an agent makes. Two types of preferences are analyzed: *bundled* preferences in which agents perceive gains and losses with respect to overall utility, and *unbundled* preferences in which they perceive gains and losses separately for the toll and other determinants of utility including time costs. For both types of preferences tolls are chosen to maximize the sum of expected utility and revenues. Tolls thus reflect public rather than private-sector objectives.

Studies of loss aversion typically find that reference-dependent effects decay as agents become familiar with gains and losses and can become negligible with sufficient experience (List, 2003). This may explain why established practices of dynamically pricing airline tickets and access to HOT lanes have become widely accepted. To capture the effects of experience in the model it is assumed that a fraction of agents are "experienced" and do not perceive gains and losses. The model can be interpreted as one of overlapping generations in which a fraction of the population retires each period and is replaced by new, inexperienced agents.

Two studies that analyze some aspects of state-dependent pricing deserve mention. Emmerink et al. (1998) examine road pricing in a model where drivers are risk averse with respect to travel time but not the toll they pay. If drivers are informed about travel time before they start a trip they incur no uncertainty cost and the congestion toll is the standard, deterministic Pigouvian toll for the travel conditions that prevail that day. If no drivers are informed the toll is independent of the state and it is set above the Pigouvian toll to reflect the additional external cost that drivers impose by increasing travel time uncertainty. Finally, if a fraction of drivers are informed the toll is set above the Pigouvian charge when travel conditions are bad and below it when travel conditions are good. Doing so is beneficial because it reduces uncertainty in travel time for drivers who are not informed. As discussed later these results contrast with results generated in the current paper.

Rotemberg (2008) develops a model of private-sector pricing that is similar in several respects to the model here. His model is based on the assumption that consumers give a firm the benefit of the doubt if they believe that the firm attaches some altruistic value for their wellbeing. Consumers' beliefs are upset if the firm behaves in a contradictory way such as by raising the price of a good during a shortage. Consumers experience a psychological cost from price shocks that depends on the difference between the price they actually pay and the price they expected to pay. Rotemberg's model does not feature congestion but rather a fixed quantity of a good that has to be rationed somehow.

In the model used here the facility operator has full information about supply and demand and uses the information to set the toll. State-dependent pricing of this sort differs from responsive pricing as envisaged by Vickrey (1971) whereby the price is set according to a formula based on occupancy or some other measure of congestion. Responsive pricing operates in real time or near real time, but uses only limited, contemporaneous information about usage conditions that may be less than what agents themselves know.[4] For this reason, the term "state-dependent pricing" is used here rather than "responsive pricing" or "dynamic pricing".

Section 2 develops the model, characterizes optimal usage decisions for inexperienced agents, and derives two general results on existence and uniqueness of individual or "personal" equilibrium decisions. Section 3 provides a more detailed analysis of usage decisions when there are two states that differ in terms of supply or demand conditions. Section 4 derives optimal tolls for two states and works through a numerical example. Section 5 summarizes the main results and identifies ways in which the model of reference-dependent pricing developed here could be generalized or modified.[5]

[4] Courty and Pagliero (2008) develop a model of responsive pricing in which agents decide how long to use a facility. They show that responsive pricing is generally not fully efficient because it fails to convey information about future prices that agents would like to know when they decide whether to initiate consumption.

[5] A preliminary and partial version of this paper is developed in Lindsey (2009). The current paper goes further by treating demand as well as capacity fluctuations, allowing some agents to be experienced, and deriving general results on existence and uniqueness of equilibrium.

2 THE MODEL

2.1 States and usage decisions

There is a single congestible facility. Both the state of the facility and demand can vary from day to day. Let Ω denote the (discrete) set of possible states on a given day and h_w the probability of state $w \in \Omega$. The cost of usage in state *w* is $c_w = C_w(N)$ where *N* is the number of users. ($C_w(\ )$ varies with capacity shocks but not demand shocks.) The full price or generalized cost of usage is $p_w = c_w + \tau_w$ where τ_w is the user charge or toll in state *w*. Gross utility is v_w and intrinsic utility is $u_w = v_w - p_w = v_w - c_w - \tau_w = s_w - \tau_w$ where $s_w = v_w - c_w$ is surplus gross of the toll. Utility from no usage is normalized to zero.

Agents make one decision: whether to use the facility. They learn the state before they have to decide. For state *w* the decision, Yes (*Y*) or No (*N*), is recorded by an indicator variable $d_w \in \{Y, N\}$. The decision vector for all states is $\boldsymbol{d} = (d_w, w \in \Omega)$ and the set of possible $\boldsymbol{d}$ is denoted by $\boldsymbol{D}$. Gross utility in state *w* is therefore $v_w^d = v_w$ if $d_w = Y$ and $v_w^d = 0$ if $d_w = N$. Variables c_w^d, τ_w^d, u_w^d, and s_w^d are defined analogously. The "outcome" of state *w* is denoted $z_w^d = \left(v_w^d, c_w^d, \tau_w^d\right)$.

Usage decisions for experienced agents depend solely on intrinsic utility. An agent uses the facility in state *w* if and only if $u_w \geq 0$. Usage decisions for inexperienced agents depend not only on intrinsic utility but also on reference-dependent preferences. Following Kőszegi and Rabin (2006) these preferences are described by "gain-loss utility" where gains and losses are measured relative to a reference point decision vector $\boldsymbol{r} \in \boldsymbol{D}$. Given a gain-loss utility function $\theta(\ )$, gain-loss utility in state *w* can be written in terms of the outcome of decision $\boldsymbol{d}$ in state *w* and the outcomes of $\boldsymbol{r}$ in all states: $\sum_{\sigma \in \Omega} h_\sigma \theta\left(z_\sigma^{\boldsymbol{r}}, z_w^{\boldsymbol{d}}\right)$. Overall utility (henceforth simply "utility") is the sum of intrinsic utility, u_w^d, and gain-loss utility.

2.2 Assumptions on gain-loss utility

Several assumptions about the gain-loss utility function will be imposed at various stages of the analysis. Written in terms of generic outcomes the first two assumptions are:

Assumption G1: $$\theta(z,z)=0\,. \tag{1}$$

Assumption G2: $$\theta(z,z')+\theta(z',z'')\leq\theta(z,z'')\,. \tag{2}$$

Assumption G1 implies that gain-loss utility depends only on outcomes and not the decision rule that was used. Assumption G2 reflects aversion to change. A direct shift from outcome z to outcome z'' is preferred to a shift that passes through an intermediate outcome, z'. Setting $z''=z$, and imposing Assumption G1, one obtains from Assumption G2:

$$\theta(z,z')+\theta(z',z)\leq 0\,. \tag{3}$$

A shift from outcome z to z' and back again leaves an agent indifferent or worse off. This implies that there is no "utility pump".

A third assumption is that gain-loss utility depends only on differences in outcomes:

Assumption G3: $$\theta(z,z')=\theta(z'-z)\,. \tag{4}$$

A focus of the paper is whether gains and losses are perceived separately with respect to all three components of an outcome: v_w, c_w, and τ_w. Empirical evidence is mixed on whether gains and losses for goods and money are perceived separately or together and the matter remains unsettled (Bateman *et al*., 2005). This question has received little attention in the transportation literature[6] and it seems reasonable to entertain various possibilities. Two will be explored here. In the first case, called *bundled* preferences, all three components of an outcome are integrated so that gains and losses depend only on intrinsic utility. In the second case, called *unbundled* preferences, gains and losses with respect to the toll are perceived separately from the other two components.[7]

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[6] See De Borger and Fosgerau (2008) for a recent study in the context of valuing travel time. Econometric studies of HOT lane usage (e.g., Brownstone et al., 2003; Small et al., 2005) have assumed that utility is a linear function of the toll and therefore do not provide evidence on attitudes towards toll variation.

[7] In both cases it is assumed that gains and losses depend on v_w and c_w only through surplus, $s_w=v_w-c_w$. One justification for this assumption is that, regardless of how access to the facility is priced, agents incur costs to use the facility and will have acclimated to this.

The structure of the gain-loss utility function depends on whether preferences are bundled or unbundled. In both cases Assumptions G1-G3 are assumed to hold. For unbundled preferences the gain-loss utility function is assumed to take the form:

Assumption G4: $$\theta(z,z') = \mu(s'-s) + \mu(\tau-\tau'), \quad (5)$$

where $\mu(0)=0$ and $\mu(\;)$ is a strictly increasing function.[8]

For bundled preferences the gain-loss utility function takes the form:

Assumption G5: $$\theta(z,z') = \mu(u'-u), \quad (6)$$

where again $\mu(0)=0$ and $\mu(\;)$ is a strictly increasing function.

For most of the analysis in later sections $\mu(\;)$ in eqn. (5) or eqn. (6) is assumed to have the linear form adopted by Kőszegi and Rabin (2006)[9]:

Assumption G6: $$\mu(x) = \eta x \;\; for \;\; x \ge 0, \; \mu(x) = \eta\lambda x \;\; for \;\; x \le 0 \; \text{ where } \lambda \ge 1. \quad (7)$$

Parameter η reflects the strength of gain-loss utility and $\lambda - 1$ reflects the relative importance of loss aversion.

2.3 Personal equilibrium

The last element of the model is Kőszegi and Rabin's solution concept of personal equilibrium. As noted above, utility is the sum of intrinsic utility and gain-loss utility. Given decision rule ***d*** and a reference point decision vector ***r***, utility in state *w* is

$$U_w^{\boldsymbol{d}}\big|\boldsymbol{r} = u_w^{\boldsymbol{d}} + \sum\nolimits_{\sigma\in\Omega} h_\sigma \theta\left(z_\sigma^{\boldsymbol{r}}, z_w^{\boldsymbol{d}}\right). \quad (8)$$

Expected utility over all states is

$$E \cdot U^{\boldsymbol{d}}\big|\boldsymbol{r} = \sum\nolimits_{w\in\Omega} h_w U_w^{\boldsymbol{d}}\big|\boldsymbol{r} = \sum\nolimits_{w\in\Omega} h_w \left(u_w^{\boldsymbol{d}} + \sum\nolimits_{\sigma\in\Omega} h_\sigma \theta\left(z_\sigma^{\boldsymbol{r}}, z_w^{\boldsymbol{d}}\right)\right). \quad (9)$$

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[8] The same gain-loss utility function is assumed to apply for surplus and for the toll. In a stated-preference study of value of travel time De Borger and Fosgerau (2008) estimate a reference-dependent gain-loss utility function and find more loss aversion with respect to time than with respect to money. Assumption G4 is adopted for tractability and for ease of comparing bundled and unbundled preferences.

[9] Assumption G6 is their Assumption A3. Whereas Assumptions G1-G5 do not impose continuity on functions $\theta(\;)$ or $\mu(\;)$, Assumption G6 does embody continuity. This implies that agents do not experience a discrete gain or loss when outcomes differ by an arbitrarily small amount.

Kőszegi and Rabin (2006) define a decision rule $\boldsymbol{d}$ to be an "unacclimating personal equilibrium" (UPE)[10] if $\boldsymbol{d}$ is optimal conditional on the reference point decision rule that stems from $\boldsymbol{d}$; i.e. with $\boldsymbol{r} = \boldsymbol{d}$. A UPE is therefore defined by the condition

$$E \cdot U^{\boldsymbol{d}} \Big| \boldsymbol{d} \geq E \cdot U^{\boldsymbol{d}'} \Big| \boldsymbol{d} \text{ for all } \boldsymbol{d}' \in \boldsymbol{D}. \tag{10}$$

2.4 Existence and uniqueness of personal equilibrium

As explained above, usage decisions for experienced agents are governed by intrinsic utility alone. An experienced agent uses the facility in state *w* if and only if $u_w \geq 0$: a condition that depends only on state *w*. A UPE therefore always exists for experienced agents. Usage decisions for inexperienced agents are complicated by gain-loss utility which depends on decisions made in all states *as per* eqn. (8). However, as noted earlier, Assumption G2 on the gain-loss utility function embodies aversion to change or inertia in the sense of preferring the status quo. This turns out to guarantee that at least one UPE exists:

Proposition 1: If the gain-loss utility function satisfies Assumptions G1 and G2, then a UPE exists.

Proof: See Appendix A.

Assumptions G1 and G2 guarantee that a UPE exists, but not that it is unique, and multiple UPE can in fact exist.[11] This will be demonstrated in Section 3. Assumptions G1 and G2 also fail to guarantee that a UPE exists in which the facility is used if and only if $u_w \geq 0$ (i.e., for the decision rule used by experienced agents). As shown in Appendix A, such an equilibrium may not exist even if u_w is positive in every state or negative in every state.

However, if preferences are bundled as per Assumption G5, then there does exist a unique UPE in which the facility is used if and only if $u_w \geq 0$. This result is formalized in

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[10] The modifier "unacclimating" is added by Kőszegi and Rabin (2007).

[11] As Kőszegi and Rabin (2006) note, multiple equilibria are a generic characteristic of their model. This is also true of Courty and Pagliero's (2008) model of responsive pricing in which users' expectations about future usage conditions influence their decisions whether to continue using a facility if it is currently severely congested.

Proposition 2: If the gain-loss utility function satisfies Assumptions G1, G2, and G5, then a unique UPE exists in which the facility is used in each state with positive intrinsic utility and not used in each state with negative intrinsic utility.

Proof: See Appendix B.

To facilitate further analysis, and the derivation of optimal tolls, it is assumed for the balance of the paper that there are two states. Personal equilibria for bundled and unbundled preferences are derived in Section 3. In Section 4, individual decision rules are aggregated and used to derive optimal tolls.

3 PERSONAL EQUILIBRIA WITH TWO STATES

Assume there are two states: *Good* days (w=G) and *Bad* days (w=B) with respective probabilities $h_G = 1-\pi$ and $h_B = \pi$. *Good* days and *Bad* days may differ with respect to conditions that affect gross utility from usage such as weather and special events and/or in terms of factors that affect the cost of usage such as fuel prices and accidents. All agents are assumed to get a higher intrinsic utility from usage on *Good* days than on *Bad* days in equilibrium (i.e., $u_G > u_B$) although even on *Good* days they may prefer no usage. Consequently, there are three candidate UPE for any agent: $\boldsymbol{d} = (d_G, d_B) = (Y, Y)$, $\boldsymbol{d} = (Y, N)$, and $\boldsymbol{d} = (N, N)$. For brevity, these candidates will be denoted *YY*, *YN*, and *NN*.

3.1 Candidate personal equilibria

In this subsection necessary and sufficient conditions for each candidate UPE are derived under Assumption G3 which encompasses both bundled and unbundled preferences. Section 3.2 carries the analysis further for bundled preferences, and Section 3.3 does so for unbundled preferences.

Candidate personal equilibrium YY

If usage on both *Good* and *Bad* days is the reference point, utility from usage on *Good* and *Bad* days is (cf eqn. (8)):

$$U_G^Y\big|YY = u_G + (1-\pi)\theta(z_G, z_G) + \pi\theta(z_B, z_G) = u_G + \pi\theta(z_B, z_G), \tag{11}$$

$$U_B^Y\big|YY = u_B + (1-\pi)\theta(z_G, z_B) + \pi\theta(z_B, z_B) = u_B + (1-\pi)\theta(z_G, z_B). \tag{12}$$

Gain-loss utility on *Good* days in eqn. (11), $\theta(z_B, z_G)$, is weighted by the fraction, π, of days that are *Bad*. If $\theta(z_B, z_G) > 0$, "elation" is experienced from usage on a *Good* day rather than a *Bad* one. However, even with $u_G > u_B$ gain-loss utility is not necessarily positive unless preferences are bundled because the toll can be higher on *Good* days than *Bad* days and the loss from paying a higher toll may exceed the gain from enjoying a higher surplus on *Good* days. Gain-loss utility on *Bad* days in eqn. (12), $\theta(z_G, z_B)$, is weighted by the fraction, $1-\pi$, of days that are *Good*. Given $u_B < u_G$, $\theta(z_G, z_B) < 0$, and "disappointment" is experienced from usage on a *Bad* day rather than a *Good* day. [12]

An agent who deviates from candidate *YY*, and foregoes usage on either type of day, gets a utility

$$U_G^N\big|YY = U_B^N\big|YY = 0 + (1-\pi)\theta(z_G, 0) + \pi\theta(z_B, 0).$$

Since $U_G^Y\big|YY > U_B^Y\big|YY$, *YY* is a UPE if and only if

$$U_B^Y\big|YY \geq U_B^N\big|YY:\ u_B - (1-\pi)\big(\theta(z_G, 0) - \theta(z_G, z_B)\big) - \pi\theta(z_B, 0) \geq 0. \tag{13}$$

Candidate personal equilibrium YN

For candidate *YN*, the facility is only used on *Good* days. Utility on *Good* and *Bad* days is

$$U_G^Y\big|YN = u_G + \pi\theta(0, z_G), \tag{14}$$

$$U_B^N\big|YN = (1-\pi)\theta(z_G, 0). \tag{15}$$

Deviating from *YN* by forgoing usage on a *Good* day yields a utility

$$U_G^N\big|YN = (1-\pi)\theta(z_G, 0). \tag{16}$$

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[12] This is obvious for bundled preferences. For unbundled preferences (Assumption G4) $\theta(z_G, z_B) = \mu(s_B - s_G) + \mu(\tau_G - \tau_B)$. Given $u_B < u_G$, $s_B - s_G < \tau_B - \tau_G \Rightarrow \mu(s_B - s_G) < \mu(\tau_B - \tau_G)$. Given inequality (3), $\mu(\tau_G - \tau_B) + \mu(\tau_B - \tau_G) \leq 0 \ \Rightarrow \mu(\tau_G - \tau_B) \leq -\mu(\tau_B - \tau_G)$. Hence $\mu(s_B - s_G) + \mu(\tau_G - \tau_B) < 0$.

Deviating from *YN* by undertaking usage on a *Bad* day yields a utility

$$U_B^Y \big| YN = u_B + (1-\pi)\theta(z_G, z_B) + \pi\theta(0, z_B). \tag{17}$$

Candidate *YN* is a UPE if and only if (using eqns. (14) and (16))

$$U_G^Y \big| YN \ge U_G^N \big| YN: \; u_G + \pi\theta(0, z_G) - (1-\pi)\theta(z_G, 0) \ge 0, \tag{18}$$

and (using eqns. (15) and (17))

$$U_B^N \big| YN \ge U_B^Y \big| YN: \; u_B + (1-\pi)\big(\theta(z_G, z_B) - \theta(z_G, 0)\big) + \pi\theta(0, z_B) \le 0. \tag{19}$$

Candidate personal equilibrium NN

For this candidate

$$U_G^N \big| NN = U_B^N \big| NN = 0. \tag{20}$$

Since $U_G^Y \big| NN > U_B^Y \big| NN$, *NN* is a UPE if and only if

$$U_G^Y \big| NN \le U_G^N \big| NN: \; u_G + (1-\pi)\theta(0, z_G) + \pi\theta(0, z_G) = u_G + \theta(0, z_G) \le 0. \tag{21}$$

In summary, the conditions for UPE are (from eqns. (13), (18), (19), and (21)):

$$YY: \; u_B - (1-\pi)\big(\theta(z_G, 0) - \theta(z_G, z_B)\big) - \pi\theta(z_B, 0) \ge 0, \tag{22}$$

$$YN: \; \begin{array}{l} u_G + \pi\theta(0, z_G) - (1-\pi)\theta(z_G, 0) \ge 0 \\ u_B + (1-\pi)\big(\theta(z_G, z_B) - \theta(z_G, 0)\big) + \pi\theta(0, z_B) \le 0 \end{array}, \tag{23a,b}$$

$$NN: \; u_G + \theta(0, z_G) \le 0. \tag{24}$$

3.2 Bundled preferences

If gain-loss utility is bundled, then by Proposition 2 there exists a unique UPE in which the facility is used if and only if intrinsic utility is positive in the realized state. Consistent with Proposition 2, given Assumption G5, conditions (22)-(24) simplify to

$$YY: \; u_B \ge 0, \tag{25}$$

$$YN: \; \begin{array}{l} u_G \ge 0 \\ u_B \le 0 \end{array}, \tag{26a,b}$$

$$NN: \; u_G \le 0. \tag{27}$$

Conditions (25)-(27) are mutually exclusive (except when they hold as equalities) and collectively exhaustive, and they coincide with the usage conditions for experienced agents who do not perceive gain-loss utility. This is true regardless of how *Good* days and *Bad* days differ in terms of supply and demand conditions.

3.3 Unbundled preferences

If gain-loss utility is not bundled, conditions (22)-(24) are not mutually exclusive and it is possible for two, or even all three, of *YY*, *YN*, and *NN* to be UPE. The signs of the gain-loss utility components in conditions (22)-(24) depend on whether *Good* and *Bad* days differ with respect to supply or demand, and the two cases need to be analyzed separately.

3.3.1 Variable supply

If usage conditions vary only respecting supply, the user cost functions $C_G(\ )$ and $C_B(\ )$ differ from each other but an agent's gross utility, *v*, is the same on *Good* and *Bad* days. To facilitate analysis the gain-loss utility function is assumed to satisfy Assumption G6. Conditions (22)-(24) for UPE become

$$YY: \quad v \geq c_B + \frac{1+\eta\left(\pi+(1-\pi)\lambda\right)}{1+\eta\lambda}\tau_B - \frac{(1-\pi)\eta(\lambda-1)}{1+\eta\lambda}\tau_G \equiv \bar{V}_B^{YY}, \tag{28}$$

$$YN: \quad \begin{array}{l} v \geq c_G + \dfrac{1+\eta\left(1-\pi+\pi\lambda\right)}{1+\eta\left(\pi+(1-\pi)\lambda\right)}\tau_G \equiv \bar{V}_G^{YN} \\ v \leq c_B - \dfrac{(1-\pi)\eta(\lambda-1)}{1+\eta\left(\pi+(1-\pi)\lambda\right)}\tau_G + \dfrac{1+\eta\lambda}{1+\eta\left(\pi+(1-\pi)\lambda\right)}\tau_B \equiv \bar{V}_B^{YN} \end{array}, \tag{29a,b}$$

$$NN: \quad v \leq c_G + \frac{1+\eta\lambda}{1+\eta}\tau_G \equiv \bar{V}_G^{NN}. \tag{30}$$

Since $\lambda > 1$, condition (28) is looser than condition (25); i.e. $\bar{V}_B^{YY} < c_B + \tau_B$. This is because an agent who is habituated to usage on both days would perceive a loss from staying home on *Bad* days. $\bar{V}_B^{YY}$ is a decreasing function of τ_G because a higher τ_G results in less disappointment from usage on a *Bad* day relative to a *Good* day. The size of this effect is proportional to the probability of *Good* days ($1-\pi$), the strength of gain-loss utility (η), and the degree of loss aversion ($\lambda - 1$). Similarly, condition (30) is looser than condition (27) ; i.e. $\bar{V}_G^{NN} > c_G + \tau_G$.[13] YY and NN are therefore more prevalent as UPE for unbundled preferences than for bundled preferences. Moreover, since $\bar{V}_B^{YY} < \bar{V}_B^{YN}$ and $\bar{V}_G^{YN} < \bar{V}_G^{NN}$, there exists a range of v for which *YY* and *YN* are both UPE, and another range of v for which *YN* and *NN* are both UPE. Depending on parameter values the two ranges may overlap so that *YY*, *YN,* and *NN* are all UPE. Multiple UPE exist with unbundled preferences because if agents are loss averse separately with respect to user costs and the toll they are averse to changes in each component and candidate UPE are more robust to deviations.[14]

To keep the analysis tractable it is assumed that a UPE of type *NN* prevails over any UPE of types *YN* and *YY*, and a UPE of type *YN* prevails over a UPE of type *YY*.[15] The boundary gross utility for which *YY* and *YN* are both UPE is therefore $\bar{V}_B^{YN}$ in condition (29b), and the corresponding boundary gross utility for *YN* and *NN* is $\bar{V}_G^{NN}$ in condition (30).

3.3.2 Variable demand

If demand varies, but not supply, the user cost functions $C_G(\ )$ and $C_B(\ )$ are the same on *Good* and *Bad* days but $v_G > v_B$ for any agent. Aggregate usage is therefore greater on *Good* days so that $c_G > c_B$ and the Pigouvian toll is larger on *Good* days as well. It is assumed that gain-loss utility is not strong enough to reverse this ranking so that $\tau_G > \tau_B$. Given Assumption G6, conditions (22)-(24) become:

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13 The coefficient on τ_G in $\bar{V}_G^{NN}$ exceeds one because a higher τ_G increases the perceived loss from paying the toll on a *Good* day if the agent is habituated to not using the facility.

14 One possible way to select between UPE is to use Kőszegi and Rabin's (2006) concept of a preferred personal equilibrium (PPE). As elaborated in the conclusions, decision $\boldsymbol{d}$ is a PPE if $U^{\boldsymbol{d}}\big|\boldsymbol{d} \geq U^{\boldsymbol{d}'}\big|\boldsymbol{d}'$ for all $\boldsymbol{d}' \in \boldsymbol{D}$; i.e. $\boldsymbol{d}$ is the preferred decision when each feasible decision vector is evaluated relative to itself. PPE is typically unique. Unfortunately, in the model here the PPE depends on parameter values and is therefore not a very tractable equilibrium selection approach.

15 Attention is thus restricted to unique UPE that result in lower aggregate usage than alternative UPE. This is reasonable insofar as no usage is the status quo before an agent has used a facility for the first time.

$$YY: \quad v_B \geq c_B + \frac{1+\eta}{1+\eta\lambda}\tau_B \equiv \tilde{V}_B^{YY}, \tag{31}$$

$$YN: \quad \begin{array}{l} v_G \geq c_G + \dfrac{1+\eta\left(1-\pi+\pi\lambda\right)}{1+\eta\left(\pi+\left(1-\pi\right)\lambda\right)}\tau_G \equiv \tilde{V}_G^{YN} \\ v_B \leq c_B + \dfrac{1+\eta\left(1-\pi+\pi\lambda\right)}{1+\eta\left(\pi+\left(1-\pi\right)\lambda\right)}\tau_B \equiv \tilde{V}_B^{YN} \end{array}, \tag{32a,b}$$

$$NN: \quad v_G \leq c_G + \frac{1+\eta\lambda}{1+\eta}\tau_G \equiv \tilde{V}_G^{NN}. \tag{33}$$

Conditions (32a) and (33) are the same as conditions (29a) and (30) for variable supply, but conditions (31) and (32b) differ from (28) and (29b). Since $\tilde{V}_B^{YY} < \tilde{V}_B^{YN}$ and $\tilde{V}_G^{YN} < \tilde{V}_G^{NN}$, multiple UPE exist for certain ranges of v_B and v_G. Once again, it is assumed that a UPE of type *NN* prevails over any UPE of types *YN* and *YY*, and a UPE of type *YN* prevails over a UPE of type *YY*. The boundary gross utility for which *YY* and *YN* are both UPE is therefore $\tilde{V}_B^{YN}$ in condition (32b), and the corresponding boundary gross utility for *YN* and *NN* is $\tilde{V}_G^{NN}$ in condition (33).

4 OPTIMAL TOLLS WITH TWO STATES

To derive optimal tolls it is necessary to specify the distribution of preferences of potential users. The fraction of agents who are inexperienced will be denoted by f. For both experienced and inexperienced agents the distribution of gross utility differs between *Good* days and *Bad* days when demand is variable. For experienced agents the cumulative distribution on *Good* days will be denoted $H_G^E\left(v\right)$ with a corresponding density $h_G^E\left(v\right)$. The number of experienced agents is therefore $\bar{N}_G^E \equiv H_G^E\left(\infty\right)$ and the number of experienced users on *Good* days is $N_G^E = \bar{N}_G^E - H_G^E\left(p_G\right)$. Using analogous notation, the number of experienced users on *Bad* days is $N_B^E = \bar{N}_B^E - H_B^E\left(p_B\right)$. Inexperienced agents are assumed to have the same normalized distribution of gross utility as experienced agents;[16] hence $H_G^I(v) = f/(1-f)H_G^E(v)$ and $H_B^I(v) = f/(1-f)H_B^E(v)$. The numbers of inexperienced agents on *Good* days is $N_G^I = \bar{N}_G^I - H_G^I(\bar{V}_G)$ where $\bar{V}_G = p_G$ with bundled preferences, $\bar{V}_G = \bar{V}_G^{NN}$ with unbundled preferences and variable supply, and $\bar{V}_G = \tilde{V}_G^{NN}$ with unbundled preferences and variable demand. Analogously, the number of inexperienced agents on *Bad* days is $N_B^I = \bar{N}_B^I - H_B^I(\bar{V}_B)$ where $\bar{V}_B = p_B$ with bundled preferences, $\bar{V}_B = \bar{V}_B^{NN}$ with unbundled preferences and variable supply, and $\bar{V}_B = \tilde{V}_B^{NN}$ with unbundled preferences and variable demand.

Welfare will be measured by the sum of aggregate expected utility and expected toll revenue.[17] As in Section 3, bundled preferences and unbundled preferences are analyzed separately and the gain-loss utility function is assumed to satisfy Assumption G6.

4.1 Bundled preferences

With bundled preferences, variable capacity and variable demand can be treated simultaneously. Expected utility for an experienced *YY* agent is

$$E \cdot U^E \big| YY = (1-\pi)(v_G - p_G) + \pi(v_B - p_B), \tag{34}$$

expected utility for an experienced *YN* agent is

$$E \cdot U^E \big| YN = (1-\pi)(v_G - p_G), \tag{35}$$

and expected utility for an experienced *NN* agent is $E \cdot U^E \big| NN = 0$.

For inexperienced *YY* agents, expected utility is $E \cdot U^I \big| YY = (1-\pi)U_G^Y \big| YY + \pi U_B^Y \big| YY$. Given eqns. (11) and (12),

$$E \cdot U^I \big| YY = (1-\pi)(v_G - p_G) + \pi(v_B - p_B) - \pi(1-\pi)\rho(v_G - v_B + p_B - p_G), \tag{36}$$

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[16] As noted in the introduction, this is consistent with an overlapping generations interpretation of the model in which a fraction *f* of the user population retires from the market each period and is replaced by new, inexperienced agents.

[17] Expected utility for inexperienced agents is defined by their overall utility. Their gain-loss utility is therefore included in the welfare measure — thus respecting consumer sovereignty.

where $\rho \equiv \eta(\lambda - 1)$. For inexperienced *YN* agents expected utility is $E \cdot U^I \Big| YN =$ $(1-\pi) U_G^Y \Big| YN + \pi U_B^N \Big| YN$. Given eqns. (14) and (15)

$$E \cdot U^I \Big| YN = (1-\pi)(1-\pi\rho)(v_G - p_G). \tag{37}$$

Finally, for inexperienced *NN* agents expected utility is $E \cdot U^I \Big| NN = 0$.

With bundled preferences the boundary gross utilities for inexperienced agents, p_B and p_G, are the same as for experienced agents. Consequently, expected utilities for experienced and inexperienced agents can be combined in the same integral. Indexing experienced and inexperienced agents together in order of decreasing gross utility, the expected utility for all agents can be written

$$\begin{aligned} E \cdot U^{Tot} = \int_{i=0}^{N_B} &\Big((1-\pi)(v_{Gi} - p_G) + \pi(v_{Bi} - p_B) - f\pi(1-\pi)\rho(v_{Gi} - v_{Bi} + p_B - p_G) \Big) di \\ &+ \int_{i=N_B}^{N_G} (1-\pi)(1 - f\pi\rho)(v_{Gi} - p_G) \, di \end{aligned} \tag{38}$$

where $N_B \equiv N_B^E + N_B^I$ and $N_G \equiv N_G^E + N_G^I$. Expected toll revenue is

$$E \cdot TR = (1-\pi)\tau_G N_G + \pi\tau_B N_B, \tag{39}$$

and welfare is

$$W = E \cdot U^{Tot} + E \cdot TR. \tag{40}$$

Optimal tolls are derived by maximizing *W* in (40) with respect to τ_G and τ_B. The solution (see Appendix C) is

$$\tau_G = \underbrace{C_G' N_G}_{\tau_G^{Pigou}} + \underbrace{\frac{\pi\eta(\lambda-1) fN_G}{h_G(p_G)}}_{(a)}, \tag{41}$$

$$\tau_B = \underbrace{C_B' N_B}_{\tau_B^{Pigou}} - \underbrace{\frac{(1-\pi)\eta(\lambda-1) fN_B}{h_B(p_B)}}_{(b)}, \tag{42}$$

where $h_G(p_G) \equiv h_G^E(p_G) + h_G^I(p_G)$ and $h_B(p_B) \equiv h_B^E(p_B) + h_B^I(p_B)$. In eqns. (41) and (42), τ_G^{Pigou} and τ_B^{Pigou} are standard Pigouvian tolls. Terms (a) and (b) make adjustments for reference-dependent preferences. The toll on *Good* days is raised above τ_G^{Pigou}, and the toll on *Bad* days is set below τ_B^{Pigou}. Adjusting tolls in this way brings generalized costs on *Good* days and *Bad* days closer together and therefore reduces the degree of disappointment experienced on *Bad* days.[18] The adjustments are proportional to the absolute strength of loss aversion, $\rho = \eta(\lambda - 1)$, and the number of inexperienced agents: fN_G on *Good* days and fN_B on *Bad* days. The adjustments are inversely proportional to the densities of agents at the margin of usage, $h_G(p_G)$ and $h_B(p_B)$, because the marginal welfare gains from pricing congestion are proportional to these densities.[19]

Equations (41) and (42) reveal that tolls play a dual role when agents have reference-dependent preferences. One role is as a Pigouvian tax to control congestion, and the other is to smooth utility.

4.2 Unbundled preferences

With unbundled preferences, variable supply and variable demand have to be analyzed separately.

4.2.1 Variable supply

As described in Section 3.3.1, with unbundled preferences and variable supply the boundary gross utilities determining usage decisions for inexperienced agents are $\overline{V}_B^{YN}$ and $\overline{V}_G^{NN}$. These boundary values differ from the boundary values for experienced agents, p_B and p_G. Consequently, expected utilities for experienced and inexperienced agents need to be tallied separately. Indexing experienced and inexperienced agents separately in order of decreasing gross utility, aggregate expected utility can be written

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[18] As noted in the introduction, Emmerink et al. (1998) find that tolls are adjusted in the opposite direction. Their model features two states and travellers who are risk averse with respect to travel time but not the toll they pay. Travel time is lower in their Good state than their Bad state. To reduce uncertainty in travel time the toll is set below the Pigouvian toll in the Good state in order to increase usage and travel time, and raised above the Pigouvian toll in the Bad state to suppress usage and reduce travel time.

[19] Eqns. (41) and (42) apply only for parameter values such that $u_G > u_B$ since otherwise the direction of gains and losses would be reversed.

$$E \cdot U^{Tot} = \int_{i=0}^{N_B^E} \left[(1-\pi)(v_{Gi} - c_G - \tau_G) + \pi(v_{Bi} - c_B - \tau_B) \right] di + \int_{i=N_B^E}^{N_G^E} \left[(1-\pi)(v_{Gi} - c_G - \tau_G) \right] di$$
$$\int_{i=0}^{N_B^I} \left[(1-\pi)(v_{Gi} - c_G - \tau_G) + \pi(v_{Bi} - c_B - \tau_B) - \pi(1-\pi)\rho(c_B + \tau_B - c_G - p_G) \right] di \qquad . \quad (43)$$
$$+\int_{i=N_B^I}^{N_G^I} \left[(1-\pi)(v_{Gi} - c_G - \tau_G) - \pi(1-\pi)\rho(v_{Gi} - c_G + \tau_G) \right] di$$

Expected toll revenue and welfare are again given by eqns. (39) and (40). First-order conditions for the tolls are written out in Appendix D; they do not simplify enough to be readily interpreted. It is possible to show that if all agents are inexperienced ($f = 1$) the toll on *Good* days may be below τ_G^{Pigou}. This contrasts with the toll for bundled preferences in eqn. (41) which is unambiguously above τ_G^{Pigou}. However, at least for small values of ρ, $\tau_B < \tau_B^{Pigou}$ which is consistent with the result for bundled preferences in eqn. (42).

4.2.2 Variable demand

Similar to the case with variable supply, with variable demand expected utilities for experienced and inexperienced agents need to be tallied separately. Aggregate expected utility is

$$E \cdot U^{Tot} = \int_{i=0}^{N_B^E} \left[(1-\pi)(v_{Gi} - c_G - \tau_G) + \pi(v_{Bi} - c_B - \tau_B) \right] di + \int_{i=N_B^E}^{N_G^E} \left[(1-\pi)(v_{Gi} - c_G - \tau_G) \right] di$$
$$\int_{i=0}^{N_B^I} \left[\begin{array}{l} (1-\pi)(1-\pi\rho)(v_{Gi} - c_G) + \pi\left(1+(1-\pi)\rho\right)(v_{Bi} - c_B) \\ -(1-\pi)(1+\pi\rho)\tau_G - \pi\left(1-(1-\pi)\rho\right)\tau_B \end{array} \right] di \qquad . \ (44)$$
$$+\int_{i=N_B^I}^{N_G^I} \left[(1-\pi)(1-\pi\rho)(v_{Gi} - c_G) - (1-\pi)(1+\pi\rho)\tau_G \right] di$$

The first line of eqn. (44) for experienced agents is the same as in eqn. (43), but the terms for inexperienced agents differ. Expected toll revenue and welfare are again given by eqns. (39) and (40). First-order conditions for the tolls are given in Appendix D where it is shown that for small values of ρ, τ_G is likely to be below τ_G^{Pigou}, and τ_B is likely to be above τ_B^{Pigou}.

4.3 Summary

The analytical results derived in Sections 4.1 and 4.2 are summarized in Table 1. Of note is the contrast between bundled preferences, and unbundled preferences with variable demand. With bundled preferences, $\tau_G > \tau_G^{Pigou}$ and $\tau_B < \tau_B^{Pigou}$. Tolls are adjusted this way even if $\tau_G^{Pigou} > \tau_B^{Pigou}$ because agents are better off on *Good* days and reducing the discrepancy between utility on *Good* and *Bad* days decreases net gain-loss disutility. In contrast, with unbundled preferences and variable demand, tolls tend to be adjusted in the opposite direction. The divergence in results highlights the importance of understanding not only how agents perceive gains and losses but also of identifying the source of variations in usage conditions.

4.4 Numerical examples

The analysis so far provides insights into whether tolls are increased or decreased when agents have reference-dependent preferences, but it does not indicate how much tolls are adjusted. To assess this, a simple numerical example is developed with one specification for variable supply and another for variable demand. The example is broadly descriptive of an urban automobile commute with utility measured in dollars.

4.4.1 Calibration

Variable supply

Gross utility from usage is assumed to be uniformly distributed over the interval $[0,36]$ on both *Good* days and *Bad* days and for both experienced and inexperienced agents. User cost is a linear function of aggregate usage. On *Good* days it is $c_G = 8 + N_G / 3000$, and on *Bad* days it is $c_B = 8 + N_B / 1500$.[20] The probability of *Bad* days is set to $\pi = 0.2$. The combined density of experienced and inexperienced agents is $h_G = h_B = 500$ so that $\bar{N}_G^E + \bar{N}_G^I = \bar{N}_B^E + \bar{N}_B^I = 18{,}000$.

There is little evidence on which to draw in selecting plausible values for η and λ. The importance of gain-loss utility will depend on the nature of demand and supply shocks, agents' experience with state-dependent pricing, and other context-specific considerations.[21] In a theoretical paper, Laciana and Weber (2008) make the "somewhat arbitrary assumption" (p.10) that gain-loss utility does not exceed 10% of intrinsic utility. This is equivalent to assuming

[20] The doubling of the slope on *Bad* days might happen if half road capacity becomes unavailable.

[21] There is evidence that price adjustment is less acceptable for demand shocks than for supply shocks that are accompanied by an increase in supplier costs.

$\eta \leq 0.1$ here.[22] By contrast, Rotemberg (2008) uses an example with parameters equivalent to $\eta = 1$. As far as the loss aversion parameter, values for λ of 2 or more have been estimated, or assumed, in various studies. To be conservative, λ is set to 2. In each scenario optimal tolls were calculated for values of η ranging from zero up to the largest value consistent with the utility rankings assumed in the scenario.[23]

Variable demand

For variable demand, the slope of the user cost function is assumed to be $c_G = c_B = 8 + N/2000$. The slope is intermediate between the slopes on *Good* and *Bad* days in the variable supply scenario. On *Bad* days the aggregate distribution of gross utility is the same as in the variable supply scenario, whereas on *Good* days the gross utility of every agent doubles so that it is uniformly distributed over the interval $[0, 72]$. *Good* and *Bad* days are assumed to be equally likely so that $\pi = 0.5$. In other respects the variable demand scenario is the same as the variable supply scenario.

4.4.2 Results

Figures 1 to 4 present tolls for scenarios in which all agents are inexperienced. Figure 1 shows the results for bundled preferences and variable supply. With $\eta = 0$, the tolls are τ_G=\$3.50 and τ_B=\$5.60. As η rises, τ_G increases and τ_B decreases (consistent with eqns. (41) and (42)), and at $\eta \cong 0.14$ the tolls converge at about \$4.00. Thus, when the weight on gains and losses reaches just one seventh the (implicit) weight of unity on intrinsic utility it is optimal to set the same tolls on *Good* and *Bad* days and the tolls are no longer state-dependent. A similar pattern occurs with unbundled preferences and variable supply in Figure 2 except that τ_G

[22] Laciana and Weber's example is developed in the context of regret theory, whereas Kőszegi and Rabin's (2006) model features disappointment rather than regret. It is not obvious in general whether utility is more or less strongly influenced by regret than by disappointment.

[23] For Figure 3 calculations were performed only up to $\eta = 0.25$.

remains relatively constant[24] while τ_B declines more steeply, and the common toll ends up at $3.55 rather than $4.00. This is due to the greater loss aversion with respect to τ_G that agents perceive when preferences are unbundled.

Tolls for bundled and unbundled preferences with variable demand are shown in Figures 3 and 4. With $\eta = 0$, tolls are τ_G =$6.40 and τ_B =$4.67. The toll is higher on *Good* days than on *Bad* days because demand and congestion are greater while supply conditions are the same. With bundled preferences the gap between the tolls rises as η increases. In contrast, with unbundled preferences the gap quickly shrinks and tolls converge with $\eta \cong 0.05$. The contrast between bundled and unbundled preferences with variable demand is much more pronounced than with variable supply.

Figures 1-4 display tolls for scenarios in which all agents are inexperienced ($f = 1$). Given the observed tendency of reference-dependent preferences to decay with experience, it is of interest to determine how optimal tolls behave when there is a mix of experienced and inexperienced agents. With bundled preferences, the threshold utilities that determine usage decisions are p_G and p_B for both types of agents. Moreover, the fraction of agents who are inexperienced, *f*, enters eqns. (41) and (42) multiplicatively with parameter η. Optimal tolls for any value of *f* can therefore be determined from Figures 1 and 3. For example, if $\eta = 0.1$ and $f = 0.5$, optimal tolls are the same as with $\eta = 0.05$ and $f = 1$. The influence of inexperienced agents in determining the toll is therefore proportional to their relative numbers.

With unbundled preferences, the effects of f have to be determined numerically. This was done for variable supply and variable demand by fixing parameter η at the value shown in Figures 2 and 4 at which the tolls converge, and computing equilibria over the range $f \in [0,1]$. As shown in Figures 5 and 6, tolls on both *Good* and *Bad* days vary approximately linearly with *f*. Thus, similar to the case with bundled preferences, the influence of inexperienced agents in determining tolls is roughly proportional to their relative numbers.

The finding that inexperienced agents "matter" as much as experienced agents in determining optimal tolls is surprising in one respect. Haltiwanger and Waldman (1985) and Fehr and Tyran

[24] τ_B drops very slightly from $3.50 to $3.494 before turning up.

(2005) have shown that experienced agents have a disproportionately large influence in settings where choices are strategic substitutes such as the congestible facility considered here. However, "experience" in their context refers to knowledge about usage conditions. In the context here, all agents are perfectly informed and "experience" determines preferences instead. Similar to the market trading setting considered by List (2003), inexperience has a first-order effect on usage decisions and optimal congestion pricing policy.

5 CONCLUSIONS

Supply and demand shocks are common on roads and at recreational areas and other congestible facilities. According to social marginal cost pricing principles, congestion tolls or admission fees should vary with usage conditions. Yet, despite advances in pricing technology and growing ease of communicating information, state-dependent pricing is still rare. One explanation is that individuals dislike varying prices. This paper analyzes aversion to price variation by developing a model of reference-dependent preferences based on Kőszegi and Rabin (2006). Using a facility yields an "intrinsic" utility and a "gain-loss" utility measured relative to the probability distribution over states of outcomes that result from the usage decisions an agent makes. Two types of reference-dependent preferences are considered: *bundled* preferences in which agents perceive gains and losses with respect to overall utility, and *unbundled* preferences in which they perceive gains and losses separately for the toll and other determinants of utility.

Usage decisions are assumed to be individually optimal if an agent is at an unacclimating personal equilibrium (UPE). Under weak assumptions on the gain-loss utility function, a UPE always exists. With bundled preferences UPE is unique, but with unbundled preferences multiple UPE can exist. Individually optimal usage decisions depend on the strength of gain-loss utility and whether usage conditions vary with supply or demand.

Optimal tolls are derived for a special case of the model with two states, a given fraction of agents who are inexperienced and perceive gains and losses, and a linear gain-loss utility function. With bundled preferences the toll pattern is clear-cut: on *Good* days the toll is set above the benchmark Pigouvian toll, and on *Bad* days the toll is set below it. Adjusting tolls in this way reduces variability in utility and the magnitude of net gain-loss disutility. The size of the adjustments is proportional to the fraction of agents who are inexperienced. With unbundled preferences, optimal toll policy depends on whether supply or demand is variable. Under plausible conditions the toll on *Bad* days is set below the Pigouvian toll if supply is variable, but above the Pigouvian toll if demand is variable. A numerical example shows that for both types of preferences tolls are sensitive to the strength of gain-loss utility and vary roughly in proportion to the fraction of agents who are inexperienced. Except with unbundled preferences and variable demand, *Good*-day and *Bad*-day tolls converge when all agents are inexperienced and the weight on "gain-loss" utility reaches a modest fraction of the weight on intrinsic utility. State-independent tolling is then optimal.

There are various directions in which the modeling of reference-dependent pricing could be generalized or modified. A few are identified here.

Perfect information

One restrictive assumption of the model is that prospective users are perfectly informed about the state when they make their usage decisions. This assumption has the advantage that the model is applicable not only to stochastic fluctuations in usage conditions but also to predictable, recurring fluctuations such as weekly peaks in demand or seasonal variations in access to recreational areas. The model applies to deterministic peak-load pricing if the probability distribution of states, h_w, is interpreted to be the fraction of time that each condition applies.[25]

Another reason for assuming perfect information is that state-dependent pricing would be pointless if agents know nothing about the state.[26] Nevertheless, it would be more realistic to suppose that agents have access only to imperfect information such as regional weather reports or news bulletins about accidents. Studies of outdoor recreation, for example, find that infrequent

[25] Another consequence of the perfect-information assumption is that the model excludes regret. When gain-loss utility $\theta(z, z')$ is negative, an agent experiences "disappointment" that outcome z' occurred rather than outcome z. Conversely, if $\theta(z, z')$ is positive, the agent feels "elation". But agents never experience regret from taking one decision when another decision would have yielded a higher utility in the state that actually occurs.

[26] This should be qualified by the observation that if agents learn the price before making a usage decision, they may infer something about the state from whether the price is high or low.

visitors often fail to predict congestion levels accurately and may not know what to expect (Prince and Ahmed, 1988; Shelby et al., 1983).[27]

Nature of the reference point

In Kőszegi and Rabin's (2006) model, the reference point is the complete lottery of outcomes that result from an agent's usage decisions. Alternative specifications of the reference point have been proposed in the literature. Gul (1991) uses the certainty equivalent of a lottery, Grant and Kajii (1998) use the best outcome, and Sugden (2003) assumes that outcomes are compared with the reference lottery state by state rather than with the whole lottery.[28]

The assumption that agents evaluate outcomes relative to the full set of possibilities is conceivable if there are few states, but it is unlikely that people have the cognitive capacity or motivation to assess satisfaction relative to many reference points. Evaluation using multiple outcomes is more plausible if the stakes are high, and if reference points are easily compared (Ordóñez et al., 2000); e.g. if they are defined just by travel time and a toll. Recent experience is probably more salient than experience in the distant past. For example, McCarville (1996) found that price last paid strongly influences current price expectations for visitors to recreational facilities.

Use of unacclimating personal equilibrium as the solution concept

In a UPE as defined in eqn. (10), alternative decision rules ($\boldsymbol{d}'$) are evaluated using the candidate UPE decision rule ($\boldsymbol{d}$) as the reference point. Kőszegi and Rabin (2006) develop another solution concept that they call *preferred personal equilibrium* (PPE). Decision rule $\boldsymbol{d}$ is a PPE if it is a UPE and if it is preferred to every alternative decision rule when each rule is evaluated relative to itself:

$$U^{\boldsymbol{d}}\Big|\boldsymbol{d} \geq U^{\boldsymbol{d}'}\Big|\boldsymbol{d}' \text{ for all } \boldsymbol{d}' \in \boldsymbol{D}. \tag{45}$$

A PPE is typically unique even if there are multiple UPE. Kőszegi and Rabin (2007) develop a third solution concept called *choice-acclimating personal equilibrium* (CPE). CPE is defined by

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[27] Agents may also have biased perceptions. For example, attendance at New England ski areas during the winter of 1989 was low because people mistakenly thought that snow conditions were poor (Shelby et al., 1989).

[28] The counterpart to eqn. (9) in Sugden's (2003) model is $E \cdot U^{\boldsymbol{d}}\Big|\boldsymbol{r} = \sum_{w\in\Omega} h_w \left(u_w^{\boldsymbol{d}} + \theta\left(z_w^{\boldsymbol{r}}, z_w^{\boldsymbol{d}}\right)\right)$.

the same condition (45) but without the requirement that the CPE also be a UPE. UPE and PPE are solution concepts that take expectations and the reference point as given. By contrast, in a CPE the agent recognizes that selecting a decision rule determines not only the vector of possible outcomes but also the reference point. Which of the three solution concepts (UPE, PPE, CPE) is appropriate in a given setting depends on various considerations including how rapidly agents adapt their expectations after a choice has been made.

Including gain-loss utility in the welfare function

The welfare function used in Section 4 to derive optimal tolls includes the gain-loss utility perceived by inexperienced agents, and thus respects the principle of consumer sovereignty. No consensus has yet emerged in the literature on whether the principle should be followed for policy assessment when agents have non-standard preferences.[29] One could argue that tolls should be based only on intrinsic preferences since once agents have gained experience with state-dependent pricing, Pigouvian tolls will be optimal given their actual preferences. The need to periodically adjust tolls while agents are learning would be avoided. However, several opposing arguments can be made. First, individual agents may never completely adapt. Second, there is likely to be ongoing turnover in the population of potential users. (The model allows for this by including experienced and inexperienced agents.) Third, state-dependent pricing may not be implemented at all unless agents accept it. Adjusting tolls to reflect current, albeit transitory preferences may help to clear the acceptability hurdle.

6 ACKNOWLEDGMENTS

For helpful comments I am grateful to participants at the 44th Annual Conference of the Canadian Transportation Research Forum, participants at the fourth Kuhmo-Nectar Conference and Summer School and seminar participants at the University of British Columbia (Sauder School of Business). Thanks are also due to Gillian Schafer for able research assistance.

[29] See Kahneman and Sugden (2005), Bernheim and Rangel (2007), and Sugden (2008).

7 ROLE OF THE FUNDING SOURCE

Financial support from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. The SSHRC was not involved in any specific aspects of this review or in the decision to submit the paper for publication.

8 LIST OF TABLES AND FIGURES

9 NOTATIONAL GLOSSARY

B	bad day
c_w	cost of usage in state *w*
$C_w(\;)$	cost function for usage in state *w*
d_w	indicator of decision for state *w*
$\boldsymbol{d}$	decision vector for all states
$\boldsymbol{D}$	set of possible $\boldsymbol{d}$
f	fraction of agents who are inexperienced
G	good day
h_w	probability of state w
$H_B^E(v)$	cumulative distribution of gross utility for experienced agents on *Bad* days
$H_G^E(v)$	cumulative distribution of gross utility for experienced agents on *Good* days
$H_B^I(v)$	cumulative distribution of gross utility for inexperienced agents on *Bad* days
$H_G^I(v)$	cumulative distribution of gross utility for inexperienced agents on *Good* days
N	number of users
N_B^E	number of experienced users on *Bad* days
N_G^E	number of experienced users on *Good* days
N_B^I	number of inexperienced users on *Bad* days
N_G^I	number of inexperienced users on *Good* days
N	decision not to use facility
p_w	full price or generalized cost of usage in state *w*
$\boldsymbol{r}$	reference point decision vector
s_w	surplus gross of the toll
TR	expected toll revenue
u_w	intrinsic utility
$U_w^{\boldsymbol{d}}$	utility in state *w* given d_w

v_w gross utility in state *w*

v_w^d gross utility in state *w* given d_w

$\bar{V}_w^{\boldsymbol{d}}$ critical level of gross utility given decision ***d***, state *w* and variable supply

$\tilde{V}_w^{\boldsymbol{d}}$ critical level of gross utility given decision ***d***, state *w* and variable demand

w state

W welfare

Y decision to use facility

z_w^d outcome of state *w* given d_w

η parameter measuring strength of gain-loss utility

$\theta(\ )$ general gain-loss utility function

λ parameter measuring relative importance of loss aversion

$\mu(\ )$ specific gain-loss utility function

$\rho \equiv \eta(\lambda - 1)$

τ_w user charge or toll in state *w*.

Ω set of possible states

10 APPENDIXES

10.1 Appendix A: Proof of Proposition 1

Consider the generic reference point defined by Ω^N:

$$\boldsymbol{d}: \; d_w = \begin{cases} N, & w \in \Omega^N \\ Y, & w \in \Omega^Y \equiv \Omega - \Omega^N \end{cases}.$$

Let $\boldsymbol{d}^* \big| \boldsymbol{d}$ denote the optimal decision rule given reference point $\boldsymbol{d}$. The following lemma shows that if an agent prefers no usage in state *w* given reference point $\boldsymbol{d}$, then the agent also prefers no usage in state *w* given a reference point that entails usage in fewer states than does $\boldsymbol{d}$.

Lemma:

Suppose $d_w^* \big| \boldsymbol{d} = N$ for $w \in \Omega^N$. Consider $\hat{\Omega}^N \supset \Omega^N$ and the decision rule

$$\hat{\boldsymbol{d}}: \; d_w = \begin{cases} N, & w \in \hat{\Omega}^N \\ Y, & w \in \Omega - \hat{\Omega}^N \end{cases}.$$

Then $d_w^* \big| \hat{\boldsymbol{d}} = N$ for $w \in \Omega^N$.

Proof of Lemma: To economize on writing let $\theta_{zz'}$ stand for $\theta(z, z')$. Given $d_\sigma^* \big| \boldsymbol{d} = N$ for $\sigma \in \Omega^N$, the agent prefers no usage in state σ:

$$u_\sigma + \sum\nolimits_{w \in \Omega^Y} h_w \theta_{w\sigma} + \sum\nolimits_{w \in \Omega^N} h_w \theta_{0\sigma} \le \sum\nolimits_{w \in \Omega^Y} h_w \theta_{w0}.$$

This implies

$$\begin{aligned} u_\sigma + \sum\nolimits_{w \in \Omega - \hat{\Omega}^N} h_w \theta_{w\sigma} + \sum\nolimits_{w \in \hat{\Omega}^N} h_w \theta_{0\sigma} + \left(\sum\nolimits_{w \in \hat{\Omega}^N - \Omega^N} h_w \theta_{w\sigma} - \sum\nolimits_{w \in \hat{\Omega}^N - \Omega^N} h_w \theta_{0\sigma} \right) \\ \le \sum\nolimits_{w \in \Omega - \hat{\Omega}^N} h_w \theta_{w0} + \sum\nolimits_{w \in \hat{\Omega}^N - \Omega^N} h_w \theta_{w0} \end{aligned}.$$

Hence

$$u_\sigma + \sum\nolimits_{w \in \Omega - \hat{\Omega}^N} h_w \theta_{w\sigma} + \sum\nolimits_{w \in \hat{\Omega}^N} h_w \theta_{0\sigma}$$

$$\le \sum\nolimits_{w \in \Omega - \hat{\Omega}^N} h_w \theta_{w0} + \sum\nolimits_{w \in \hat{\Omega}^N - \Omega^N} h_w \left(\theta_{w0} + \theta_{0\sigma} - \theta_{w\sigma} \right) \le \sum\nolimits_{w \in \Omega - \hat{\Omega}^N} h_w \theta_{w0}$$

where the last inequality follows from Assumption G2. ■

The remainder of the proof shows how a UPE can be found by iteration. Set $\Omega^N = \varnothing$. If $d_w^* \big| \boldsymbol{d} = Y$ for $w \in \Omega$ then $\boldsymbol{d}$ is a UPE. If not, then $d_w^* \big| \boldsymbol{d} = N$ for $w \in \Omega_N^Y$ for some $\Omega_N^Y \subseteq \Omega^Y$.

Update Ω^N by setting $\Omega^N = \Omega^N \cup \Omega_N^Y$. By the Lemma, $d_w^* | \boldsymbol{d} = N$ for $w \in \Omega^N$. If $d_w^* | \boldsymbol{d} = Y$ for $w \in \Omega^Y$ then $\boldsymbol{d}$ is a UPE. If not, update Ω^N again following the same procedure. The process ends at a UPE with either $\Omega^N = \Omega$ or $\Omega^N \subset \Omega$. ■

Personal equilibrium when intrinsic utility has same sign in every state

As noted in the text, although multiple UPE can exist a UPE in which the facility is used if and only if $u_w \geq 0$ may not exist even if u_w is positive in every state, or negative in every state. To show this let $\Omega^+ \equiv \{w \in \Omega | u_w \geq 0\}$ denote the set of states with positive intrinsic utility and $\Omega^- \equiv \Omega - \Omega^+$. And let $\Omega^Y \equiv \{w \in \Omega | d_w = Y\}$ be the set of states in which usage occurs, and $\Omega^N \equiv \Omega - \Omega^Y$.

Claim 1: Suppose $\Omega^+ = \Omega$. Given Assumptions G1 and G2, $\Omega^Y = \Omega$ is not necessarily a UPE.

Proof: Set $d_w = Y$, $\forall w \in \Omega$. The difference in utility between usage and no usage in state *w* is

$$U_w^Y | \boldsymbol{d} - U_w^N | \boldsymbol{d} = u_w + \sum\nolimits_{\sigma \in \Omega} h_\sigma \theta_{\sigma w} - \sum\nolimits_{\sigma \in \Omega} h_\sigma \theta_{\sigma 0} = u_w + \sum\nolimits_{\sigma \in \Omega} h_\sigma \left(\theta_{\sigma w} - \theta_{\sigma 0}\right). \qquad \text{(A1)}$$

Given only Assumptions G1 and G2 it is not possible to sign $\theta_{\sigma w} - \theta_{\sigma 0}$ and the sign of (A1) is therefore also indeterminate.

Claim 2: Suppose $\Omega^- = \Omega$. Given Assumptions G1 and G2, $\Omega^N = \Omega$ is not necessarily a UPE.

Proof: Set $d_w = N$, $\forall w \in \Omega$. The difference in utility between usage and no usage in state *w* is

$$U_w^Y | \boldsymbol{d} - U_w^N | \boldsymbol{d} = u_w + \sum\nolimits_{\sigma \in \Omega} h_\sigma \theta_{0w} = u_w + \theta_{0w}. \qquad \text{(A2)}$$

Given only Assumptions G1 and G2 it is not possible to sign θ_{0w} and the sign of (A2) is therefore also indeterminate.

10.2 Appendix B: Proof of Proposition 2

Define Ω^+, Ω^-, Ω^Y and Ω^N as in Appendix A. In addition, let $\Omega^+(w) \equiv \{\sigma \in \Omega | u_\sigma \geq u_w\}$ denote the set of states with positive and higher intrinsic utility than state *w*, and $\Omega^-(w) \equiv \Omega - \Omega^+(w)$.

The first step of the proof is to show that $\Omega^{Y} = \Omega^{+}$ is a UPE. Let $\boldsymbol{d}$: $d_w = Y$ for $w \in \Omega^{Y}$ be the reference point. First consider $w \in \Omega^{+}$. Utility from usage in state *w* is

$$U_w^Y \left| \boldsymbol{d} = u_w + \sum_{\sigma \in \Omega^-} h_\sigma \mu^+ \left(u_w - 0\right) + \sum_{\sigma \in \Omega^+ \cap \Omega^-(w)} h_\sigma \mu^+ \left(u_w - u_\sigma\right) + \sum_{\sigma \in \Omega^+(w)} h_\sigma \mu^- \left(u_w - u_\sigma\right) \right. \quad \text{(B1)}$$

where the superscript on $\mu(\ )$ identifies the sign of the argument if known. Utility from no usage in state *w* is

$$U_w^N \left| \boldsymbol{d} = 0 + \sum_{\sigma \in \Omega^-} h_\sigma \underbrace{\mu\left(0 - 0\right)}_{=0} + \sum_{\sigma \in \Omega^+} h_\sigma \mu^- \left(0 - u_\sigma\right). \right. \quad \text{(B2)}$$

Subtracting (B2) from (B1):

$$\begin{aligned} U_w^Y \left| \boldsymbol{d} - U_w^N \right| \boldsymbol{d} = u_w &+ \sum_{\sigma \in \Omega^-} h_\sigma \mu^+ \left(u_w\right) + \sum_{\sigma \in \Omega^+ \cap \Omega^-(w)} h_\sigma \left(\mu^+ \left(u_w - u_\sigma\right) - \mu^- \left(-u_\sigma\right)\right) \\ &+ \sum_{\sigma \in \Omega^+(w)} h_\sigma \left(\mu^- \left(u_w - u_\sigma\right) - \mu^- \left(-u_\sigma\right)\right) \end{aligned} . \quad \text{(B3)}$$

All four terms in (B3) are nonnegative since $u_w \geq 0$. Hence usage is preferable to no usage for $w \in \Omega^{+}$.

Now consider $w \in \Omega^{-}$. Utility from usage in state *w* is

$$U_w^Y \left| \boldsymbol{d} = u_w + \sum_{\sigma \in \Omega^-(w)} h_\sigma \mu^- \left(u_w - 0\right) + \sum_{\sigma \in \Omega^- \cap \Omega^+(w)} h_\sigma \mu^- \left(u_w - 0\right) + \sum_{\sigma \in \Omega^+} h_\sigma \mu^- \left(u_w - u_\sigma\right). \right. \quad \text{(B4)}$$

Utility from no usage is given by (B2). Subtracting (B2) from (B4):

$$\begin{aligned} U_w^Y \left| \boldsymbol{d} - U_w^N \right| \boldsymbol{d} = u_w &+ \sum_{\sigma \in \Omega^-(w)} h_\sigma \mu^- \left(u_w\right) + \\ \sum_{\sigma \in \Omega^- \cap \Omega^+(w)} h_\sigma \mu^- \left(u_w\right) &+ \sum_{\sigma \in \Omega^+} h_\sigma \left(\mu^- \left(u_w - u_\sigma\right) - \mu^- \left(-u_\sigma\right)\right) \end{aligned} . \quad \text{(B5)}$$

All four terms in (B5) are negative since $u_w < 0$. Hence no usage is preferable to usage for $w \in \Omega^{-}$.

The second step of the proof is to show that $\Omega^{Y} = \Omega^{+}$ is the only UPE. Consider the following four sets of states: $\Omega^{N-} \equiv \Omega^{N} \cap \Omega^{-}$, $\Omega^{Y-} \equiv \Omega^{Y} \cap \Omega^{-}$, $\Omega^{Y+} \equiv \Omega^{Y} \cap \Omega^{+}$ and $\Omega^{N+} \equiv \Omega^{N} \cap \Omega^{+}$. In any alternative UPE at least one of sets Ω^{Y-} and Ω^{N+} is non-empty. Consider $w \in \Omega^{N+}$. To see that usage is warranted in this state note that

$$U_w^Y = u_w + \sum_{\sigma\in\Omega^{N-}} h_\sigma \mu^+(u_w) + \sum_{\sigma\in\Omega^{Y-}} h_\sigma \mu^+(u_w - u_\sigma) + \sum_{\sigma\in\Omega^{Y+}\cap\Omega^-(w)} h_\sigma \mu^+(u_w - u_\sigma)$$
$$+ \sum_{\sigma\in\Omega^{Y+}\cap\Omega^+(w)} h_\sigma \mu^-(u_w - u_\sigma) + \sum_{\sigma\in\Omega^{N+}} h_\sigma \mu^+(u_w),$$

$$U_w^N = 0 + \sum_{\sigma\in\Omega^{N-}} h_\sigma \underbrace{\mu(0)}_{=0} + \sum_{\sigma\in\Omega^{Y-}} h_\sigma \mu^+(-u_\sigma) + \sum_{\sigma\in\Omega^{Y+}} h_\sigma \mu^-(-u_\sigma) + \sum_{\sigma\in\Omega^{N+}} h_\sigma \underbrace{\mu(0)}_{=0},$$

and

$$U_w^Y - U_w^N = u_w + \sum_{\sigma\in\Omega^{N-}} h_\sigma \mu^+(u_w)$$
$$+ \sum_{\sigma\in\Omega^{Y-}} h_\sigma \left(\mu^+(u_w - u_\sigma) - \mu^+(-u_\sigma)\right) + \sum_{\sigma\in\Omega^{Y+}\cap\Omega^-(w)} h_\sigma \left(\mu^+(u_w - u_\sigma) - \mu^-(-u_\sigma)\right) \quad \text{(B6)}$$
$$+ \sum_{\sigma\in\Omega^{Y+}\cap\Omega^+(w)} h_\sigma \left(\mu^-(u_w - u_\sigma) - \mu^-(-u_\sigma)\right) + \sum_{\sigma\in\Omega^{N+}} h_\sigma \mu^+(u_w).$$

All six terms in (B6) are positive since $u_w \geq 0$. The proof that usage is not warranted for $w \in \Omega^{Y-}$ is similar. ■

10.3 Appendix C: Optimal tolls with bundled preferences

Welfare is given by eqns. (38)-(40) in the text

$$W = \int_{i=0}^{N_B} \Big((1-\pi)(v_{Gi} - p_G) + \pi(v_{Bi} - p_B) - f\pi(1-\pi)\rho\left(v_{G_i} - v_{Bi} + p_B - p_G\right) \Big) di$$
$$+ \int_{i=N_B}^{N_G} (1-\pi)(1-f\pi\rho)(v_{Gi} - p_G)\, di + (1-\pi)\tau_G N_G + \pi\tau_B N_B.$$

Using the relations $v_{GN_G} - p_G = 0$, and $\partial N_B / \partial \tau_G = 0$, the first-order condition for τ_G is

$$\frac{\partial W}{\partial \tau_G} = \int_{i=0}^{N_B} \left[-(1-\pi) + f\pi(1-\pi)\rho\right] \frac{\partial p_G}{\partial \tau_G} di$$
$$+ \int_{i=N_B}^{N_G} \left[-(1-\pi)(1-f\pi\rho)\frac{\partial p_G}{\partial \tau_G}\right] di + (1-\pi)\left(N_G + \tau_G \frac{\partial N_G}{\partial \tau_G}\right) = 0$$

which simplifies to

$$N_G - \left[(1-f\pi\rho)N_G + \tau_G h_G(p_G)\right]\frac{\partial p_G}{\partial \tau_G} = 0. \quad \text{(C1)}$$

Now $\frac{\partial N_G}{\partial \tau_G} = -h_G(p_G)\frac{\partial p_G}{\partial \tau_G}$. Hence

$$\frac{\partial p_G}{\partial \tau_G} = 1 + C_G' \frac{\partial N_G}{\partial \tau_G} = 1 - C_G' h_G(p_G)\frac{\partial p_G}{\partial \tau_G} = \left(1 + C_G' h_G(p_G)\right)^{-1}, \quad \text{(C2)}$$

and

$$\frac{\partial N_G}{\partial \tau_G} = -h_G\left(p_G\right)\left(1 + C_G' h_G\left(p_G\right)\right)^{-1}. \tag{C3}$$

Substituting (C2) and (C3) into (C1) yields eqn. (41) for τ_G.

Using the relations $v_{GN_G} - p_G = 0$, $v_{BN_B} - p_B = 0$, and $\partial N_G / \partial \tau_B = 0$, the first-order condition for τ_B is

$$\frac{\partial W}{\partial \tau_B} = \int_{i=0}^{N_B} \left[-\pi \frac{\partial p_B}{\partial \tau_B} - f\pi\left(1-\pi\right)\rho \frac{\partial p_B}{\partial \tau_B} \right] di$$

$$+\left[\left(1-\pi\right)\left(v_{GN_B} - p_G\right) - f\pi\left(1-\pi\right)\rho\left(v_{GN_B} - p_G\right)\right]\frac{\partial N_B}{\partial \tau_B}$$

$$-\left(1-\pi\right)\left(1-f\pi\rho\right)\left(v_{GN_B} - p_G\right)\frac{\partial N_B}{\partial \tau_B} + \pi\left(N_B + \tau_B \frac{\partial N_B}{\partial \tau_B}\right) = 0$$

which simplifies to

$$N_B - \left[\left(1 + f\pi\rho\right)N_B + \tau_B h_B\left(p_B\right)\right]\frac{\partial p_B}{\partial \tau_B} = 0. \tag{C4}$$

Given $\frac{\partial N_B}{\partial \tau_B} = -h_B\left(p_B\right)\frac{\partial p_B}{\partial \tau_B}$,

$$\frac{\partial p_B}{\partial \tau_B} = 1 + C_B' \frac{\partial N_B}{\partial \tau_B} = 1 - C_B' h_B\left(p_B\right)\frac{\partial p_B}{\partial \tau_B} = \left(1 + C_B' h_B\left(p_B\right)\right)^{-1}, \tag{C5}$$

and

$$\frac{\partial N_B}{\partial \tau_B} = -h_B\left(p_B\right)\left(1 + C_B' h_B\left(p_B\right)\right)^{-1}. \tag{C6}$$

Substituting (C5) and (C6) into (C4) yields eqn. (42) for τ_B.

10.4 Appendix D: Optimal tolls with unbundled preferences

10.4.1 Variable supply

Welfare is given by eqns. (39), (40) and (43):

$$W=\int_{i=0}^{N_B^E}\left[(1-\pi)(v_{Gi}-c_G-\tau_G)+\pi(v_{Bi}-c_B-\tau_B)\right]di+\int_{i=N_B^E}^{N_G^E}\left[(1-\pi)(v_{Gi}-c_G-\tau_G)\right]di$$
$$\int_{i=0}^{N_B^I}\left[(1-\pi)(v_{Gi}-c_G-\tau_G)+\pi(v_{Bi}-c_B-\tau_B)-\pi(1-\pi)\rho(c_B+\tau_B-c_G-p_G)\right]di$$
$$+\int_{i=N_B^I}^{N_G^I}\left[(1-\pi)(v_{Gi}-c_G-\tau_G)-\pi(1-\pi)\rho(v_{Gi}-c_G+\tau_G)\right]di+(1-\pi)\tau_G N_G+\pi\tau_B N_B$$

where $N_B^I=\bar{N}_B^I-H_B^I\left(\bar{V}_B^{YN}\right)$ and $N_G^I=\bar{N}_G^I-H_G^I\left(\bar{V}_G^{NN}\right)$. By eqn. (29b),

$\bar{V}_B^{YN}=c_B-\underbrace{\frac{(1-\pi)\eta(\lambda-1)}{1+\eta(\pi+(1-\pi)\lambda)}}_{\equiv\alpha_{BG}}\tau_G+\underbrace{\frac{1+\eta\lambda}{1+\eta(\pi+(1-\pi)\lambda)}}_{\equiv\alpha_{BB}}\tau_B$, and by eqn. (30), $\bar{V}_G^{NN}=c_G+\underbrace{\frac{1+\eta\lambda}{1+\eta}}_{\equiv\alpha_{GG}}\tau_G$.

The first-order condition for τ_G is

$$\begin{aligned}
\frac{\partial W}{\partial\tau_G}&=\int_{i=0}^{N_B^E}\left[-(1-\pi)\left(1+C_G'\frac{\partial N_G}{\partial\tau_G}\right)-\pi C_B'\frac{\partial N_B}{\partial\tau_G}\right]di-(1-\pi)\int_{i=N_B^E}^{N_G^E}\left(1+C_G'\frac{\partial N_G}{\partial\tau_G}\right)di\\
&+\int_{i=0}^{N_B^I}\left[-(1-\pi)\left(1+C_G'\frac{\partial N_G}{\partial\tau_G}\right)-\pi C_B'\frac{\partial N_B}{\partial\tau_G}-\pi(1-\pi)\rho\left(C_B'\frac{\partial N_B}{\partial\tau_G}-1-C_G'\frac{\partial N_G}{\partial\tau_G}\right)\right]di\\
&+\left[(1-\pi)\left(v_{N_B^I}-c_G-\tau_G\right)+\pi\left(v_{N_B^I}-c_B-\tau_B\right)-\pi(1-\pi)\rho(c_B+\tau_B-c_G-\tau_G)\right]\frac{\partial N_B^I}{\partial\tau_G}\\
&-\left[(1-\pi)\left(v_{N_B^I}-c_G-\tau_G\right)-\pi(1-\pi)\rho\left(v_{N_B^I}-c_G+\tau_G\right)\right]\frac{\partial N_B^I}{\partial\tau_G}\\
&+\left[(1-\pi)\left(v_{N_G^I}-c_G-\tau_G\right)-\pi(1-\pi)\rho\left(v_{N_G^I}-c_G+\tau_G\right)\right]\frac{\partial N_G^I}{\partial\tau_G}\\
&+(1-\pi)\int_{i=N_B^I}^{N_G^I}\left[-\left(1+C_G'\frac{\partial N_G}{\partial\tau_G}\right)-\pi\rho\left(1-C_G'\frac{\partial N_G}{\partial\tau_G}\right)\right]di\\
&+(1-\pi)\left(N_G+\tau_G\frac{\partial N_G}{\partial\tau_G}\right)+\pi\tau_B\frac{\partial N_B}{\partial\tau_G}=0
\end{aligned}\quad. \qquad \text{(D1)}$$

Using the relations $v_{GN_G^E}-p_G=0$, $v_{BN_B^E}-p_B=0$, $\partial N_G^I/\partial\tau_B=0$ and $\partial N_G/\partial\tau_B=0$ the first-order condition for τ_B is

$$
\begin{aligned}
\frac{\partial W}{\partial \tau_B} &= \int_{i=0}^{N_B^E} -\pi\left(1+C_B'\frac{\partial N_B}{\partial \tau_B}\right)di \\
&+\int_{i=0}^{N_B^I}\left[-\pi\left(1+C_B'\frac{\partial N_B}{\partial \tau_B}\right)-\pi(1-\pi)\rho\left(1+C_B'\frac{\partial N_B}{\partial \tau_B}\right)\right]di \\
&+\left[(1-\pi)\left(v_{N_B^I}-c_G-\tau_G\right)+\pi\left(v_{N_B^I}-c_B-\tau_B\right)-\pi(1-\pi)\rho(c_B+\tau_B-c_G-\tau_G)\right]\frac{\partial N_B^I}{\partial \tau_B} \\
&-\left[(1-\pi)\left(v_{N_B^I}-c_G-\tau_G\right)-\pi(1-\pi)\rho\left(v_{N_B^I}-c_G+\tau_G\right)\right]\frac{\partial N_B^I}{\partial \tau_B}+\pi\left(N_B+\tau_B\frac{\partial N_B}{\partial \tau_G}\right)=0
\end{aligned}
\quad . \qquad \text{(D2)}
$$

Equations (D1) and (D2) do not simplify enough to be amenable to interpretation. If all users are inexperienced ($f=1$) the tolls work out to

$$
\tau_G=\frac{1-\pi\rho}{\alpha_{GG}-\pi\rho(1+\alpha_{GG})}C_G'N_G+\frac{\pi\rho(1+C_G'h_G)\big((1+\eta\lambda)(2N_B-N_G)-(1-\pi)\rho N_B\big)}{(1+\eta\lambda)\big(\alpha_{GG}-\pi\rho(1+\alpha_{GG})\big)h_G\alpha_{GG}}, \qquad \text{(D3)}
$$

$$
\tau_B=\frac{\big(\alpha_{BG}-(1-\pi)\rho(2-\alpha_{BG})\big)\tau_G+C_B'N_B\big(1+(1-\pi)\rho\big)-\dfrac{(1-\pi)\rho N_B(1+C_B'h_B)}{h_B\alpha_{BB}}}{\alpha_{BB}+(1-\pi)\rho(\alpha_{BB}-1)}. \qquad \text{(D4)}
$$

It is straightforward but tedious to show that

$$
\tau_G-C_G'N_G \overset{s}{=} -(1-\pi)h_GC_G'N_G+\pi(1+C_G'h_G)(2N_B-N_G) \qquad \text{(D5)}
$$

where $\overset{s}{=}$ means identical in sign. The right-hand side of (D5) can be positive or negative. Similarly, straightforward but tedious algebra reveals that in the limit as $\eta \to 0$

$$
\tau_B-C_B'N_B \overset{s}{=} -(1-\pi+\pi h_BC_B')N_B-(1-\pi)h_B\tau_G<0\,. \qquad \text{(D6)}
$$

10.4.2 Variable demand

Welfare is given by eqns. (39), (40) and (44):

$$
\begin{aligned}
W &= \int_{i=0}^{N_B^E}\left[(1-\pi)(v_{Gi}-c_G-\tau_G)+\pi(v_{Bi}-c_B-\tau_B)\right]di+\int_{i=N_B^E}^{N_G^E}\left[(1-\pi)(v_{Gi}-c_G-\tau_G)\right]di \\
&+\int_{i=0}^{N_B^I}\begin{bmatrix}(1-\pi)(1-\pi\rho)(v_{Gi}-c_G)+\pi\big(1+(1-\pi)\rho\big)(v_{Bi}-c_B) \\ -(1-\pi)(1+\pi\rho)\tau_G-\pi\big(1-(1-\pi)\rho\big)\tau_B\end{bmatrix}di \\
&+\int_{i=N_B^I}^{N_G^I}\left[(1-\pi)(1-\pi\rho)(v_{Gi}-c_G)-(1-\pi)(1+\pi\rho)\tau_G\right]di+(1-\pi)\tau_GN_G+\pi\tau_BN_B
\end{aligned}
$$

where $N_B^I = \bar{N}_B^I - H_B^I\left(\tilde{V}_B^{YN}\right)$ and $N_G^I = \bar{N}_G^I - H_G^I\left(\tilde{V}_G^{NN}\right)$. By eqn. (32b), $\tilde{V}_B^{YN} = c_B + \underbrace{\frac{1+\eta\left(1-\pi+\pi\lambda\right)}{1+\eta\left(\pi+\left(1-\pi\right)\lambda\right)}}_{\equiv\beta_{BB}}\tau_B$, and by eqn. (33), $\tilde{V}_G^{NN} = c_G + \underbrace{\frac{1+\eta\lambda}{1+\eta}}_{\equiv\beta_{GG}}\tau_G$.

Optimal toll on Good days

The first-order condition for τ_G is

$$\begin{aligned}\frac{\partial W}{\partial \tau_G} &= \int_{i=0}^{N_G^E}\left[-\left(1-\pi\right)\left(1+C_G'\frac{\partial N_G}{\partial \tau_G}\right)\right]di + \int_{i=0}^{N_G^I}\left[-\left(1-\pi\right)\left(1-\pi\rho\right)C_G'\frac{\partial N_G}{\partial \tau_G} - \left(1-\pi\right)\left(1+\pi\rho\right)\right]di \\ &\quad + \left[\left(1-\pi\right)\left(1-\pi\rho\right)\left(v_{GN_G^I} - c_G\right) - \left(1-\pi\right)\left(1+\pi\rho\right)\tau_G\right]\frac{\partial N_G^I}{\partial \tau_G} + \left(1-\pi\right)\left(N_G + \tau_G\frac{\partial N_G}{\partial \tau_G}\right) = 0\end{aligned} \tag{D7}$$

which simplifies to

$$\left(\tau_G - C_G' N_G\right)\frac{\partial N_G}{\partial \tau_G} + \left(\beta_{GG} - 1 - \pi\rho\left(1+\beta_{GG}\right)\right)\tau_G\frac{\partial N_G^I}{\partial \tau_G} - \pi\rho N_G^I\left(1 - C_G'\frac{\partial N_G}{\partial \tau_G}\right) = 0\,. \tag{D8}$$

If the left-hand side (LHS) of (D8) is evaluated at $\tau_G = C_G' N_G$ one obtains the expression

$$F_G \equiv \left(\beta_{GG} - 1 - \pi\rho\left(1+\beta_{GG}\right)\right)\tau_G\frac{\partial N_G^I}{\partial \tau_G} - \pi\rho N_G^I\left(1 - C_G'\frac{\partial N_G}{\partial \tau_G}\right). \tag{D9}$$

On the plausible assumption that the LHS of (D8) is a decreasing function of τ_G the sign of $\tau_G - C_G' N_G$ matches the sign of F_G in (D9). Now $\partial N_G^I / \partial \tau_G = \Delta_{GG}^{-1} h_G^I\left(h_G^E C_G'\left(1-\beta_{GG}\right) - \beta_{GG}\right)$ and $\partial N_G / \partial \tau_G = -\Delta_{GG}^{-1}\left(h_G^E + h_G^I \beta_{GG}\right)$ where $\Delta_{GG} \equiv 1 + \left(h_G^E + h_G^I\right)C_G'$. Substituting these formulas into (D9) and evaluating the expression at $\eta = 0$ gives

$$F_G \stackrel{s}{=} -\left(1-\pi\right)C_G' h_G^I N_G - \pi N_G^I - \frac{\pi C_G'}{N_G N_G^I}\left(2\frac{h_G^E + h_G^I}{N_G} - \frac{h_G^I}{N_G^I}\right). \tag{D10}$$

The sign of F_G is ambiguous in general. But $F_G < 0$ in three identifiable cases: (i) $C_G' = 0$, (ii) $h_G^E = N_G^E = 0$, and (iii) $\pi = 0$. Under any of these conditions τ_G is set below the Pigouvian toll for *Good* days. In case (i) this is obvious because with $C_G' = 0$ the marginal congestion externality is zero, the toll has no role to play as a Pigouvian tax, and reducing the toll reduces gain-loss disutility.

Optimal toll on Bad days

Using the relations $v_{GN_G^E} - p_G = 0$, $v_{BN_B^E} - p_B = 0$, $\partial N_G^I / \partial \tau_B = 0$, $\partial N_B^I / \partial \tau_G = 0$, $\partial N_G / \partial \tau_B = 0$ and $\partial N_G^I / \partial \tau_B = 0$ the first-order condition for τ_B is

$$\begin{aligned}\frac{\partial W}{\partial \tau_B} = \int_{i=0}^{N_B^E} \left[-\pi \left(1 + C_B' \frac{\partial N_B}{\partial \tau_B} \right) \right] di + \int_{i=0}^{N_B^I} \left[-\pi \left(1 + (1-\pi)\rho\right) C_B' \frac{\partial N_B}{\partial \tau_B} - \pi \left(1 - (1-\pi)\rho\right) \right] di \\ + \pi \left[\left(1 + (1-\pi)\rho\right)\left(v_{BN_B^I} - c_B \right) - \left(1 - (1-\pi)\rho\right) \tau_B \right] \frac{\partial N_B^I}{\partial \tau_B} + \pi \left(N_B + \tau_B \frac{\partial N_B}{\partial \tau_G} \right) = 0\end{aligned} \quad . \text{(D11)}$$

which simplifies to

$$\left(\tau_B - C_B' N_B\right) \frac{\partial N_B}{\partial \tau_B} + \left(\beta_{BB} - 1 + (1-\pi)\rho\left(1 + \beta_{BB}\right)\right) \tau_B \frac{\partial N_B^I}{\partial \tau_B} + (1-\pi)\rho N_B^I \left(1 - C_B' \frac{\partial N_B}{\partial \tau_B} \right) = 0 \, . \quad \text{(D12)}$$

If the LHS of (D12) is evaluated at $\tau_B = C_B' N_B$ one obtains the expression

$$F_B \equiv \left(\beta_{BB} - 1 + (1-\pi)\rho\left(1 + \beta_{BB}\right)\right) \tau_B \frac{\partial N_B^I}{\partial \tau_B} + (1-\pi)\rho N_B^I \left(1 - C_B' \frac{\partial N_B}{\partial \tau_B} \right). \quad \text{(D13)}$$

Since the LHS of (D12) is a decreasing function of τ_B, the sign of $\tau_B - C_B' N_B$ matches the sign of F_B in (D13). Now $\partial N_B^I / \partial \tau_B = -\Delta_{BB}^{-1} h_B^I \left(\left(\beta_{BB} - 1\right) h_B^E C_B' + \beta_{BB} \right)$ and $\partial N_B / \partial \tau_B = -\Delta_{BB}^{-1} \left(h_B^E + h_B^I \beta_{BB} \right)$ where $\Delta_{BB} \equiv 1 + \left(h_B^E + h_B^I \right) C_B'$. Substituting these formulas into (D13), and evaluating the expression at $\eta = 0$, gives

$$F_B \overset{s}{=} -C_B' N_B h_B^I + (1-\pi) N_B^I \left(1 + 2 C_B' \left(h_B^E + h_B^I \right) \right). \quad \text{(D14)}$$

The sign of F_B is ambiguous in general. But it is either definitely, or likely, positive in the three parallel cases considered for τ_G:

(i) If $C_B' = 0$ then $F_B > 0$ since the toll has no role to play as a Pigouvian tax and increasing τ_B reduces gain-loss disutility from the disparity between τ_B and τ_G.

(ii) If $h_B^E = N_B^E = 0$, then $F_B \overset{s}{=} 1 - \pi + (1 - 2\pi) C_B' h_B^I$ which is positive if $\pi \le 1/2$.

(iii) If $\pi = 0$, $F_B \overset{s}{=} \left(N_B\right)^{-1} + C_B' \left(2 \frac{h_B^E + h_B^I}{N_B} - \frac{h_B^I}{N_B^I} \right)$ which is likely to be positive.

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Table 1: Analytical results for optimal tolls

Preferences	$\tau_G - \tau_G^{Pigou}$	$\tau_B - \tau_B^{Pigou}$
Bundled	> 0	< 0
Unbundled, variable supply	$\gtreqless 0$	< 0 for weak gain-loss preferences
Unbundled, variable demand	Plausibly < 0 for weak gain-loss preferences	Plausibly > 0 for weak gain-loss preferences

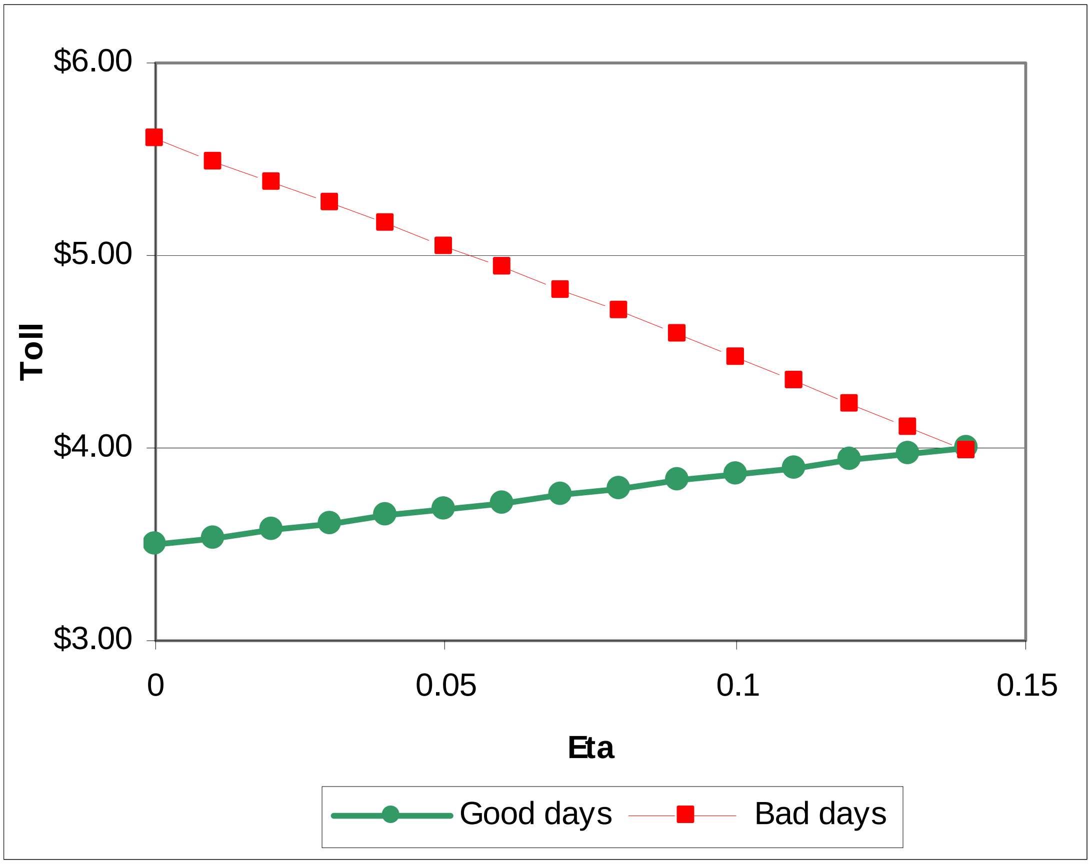

Figure 1: Optimal tolls as a function of η with all agents inexperienced; bundled preferences and variable supply with $\pi = 0.2$

Source: Author's calculation

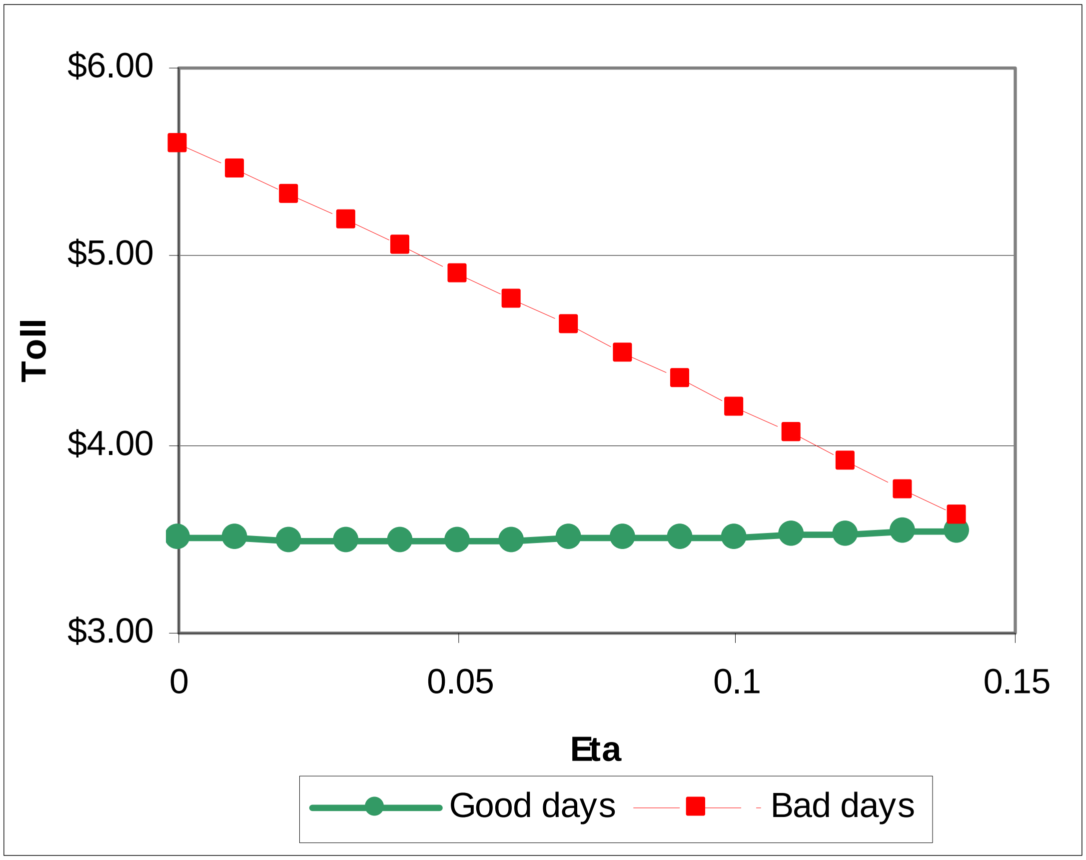

Figure 2: Optimal tolls as a function of η with all agents inexperienced; unbundled preferences and variable supply with $\pi = 0.2$

Source: Author's calculation

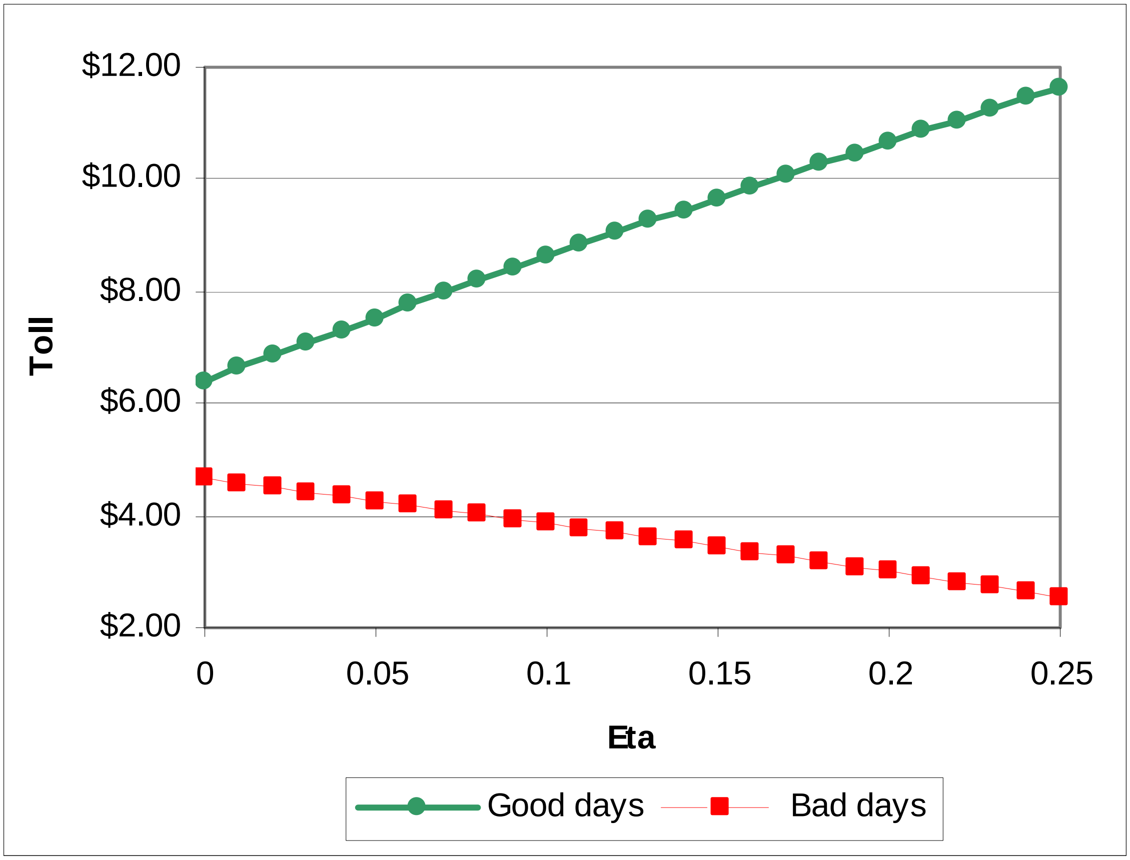

Figure 3: Optimal tolls as a function of η **with all agents inexperienced; bundled preferences and variable demand with** $\pi = 0.5$

Source: Author's calculation

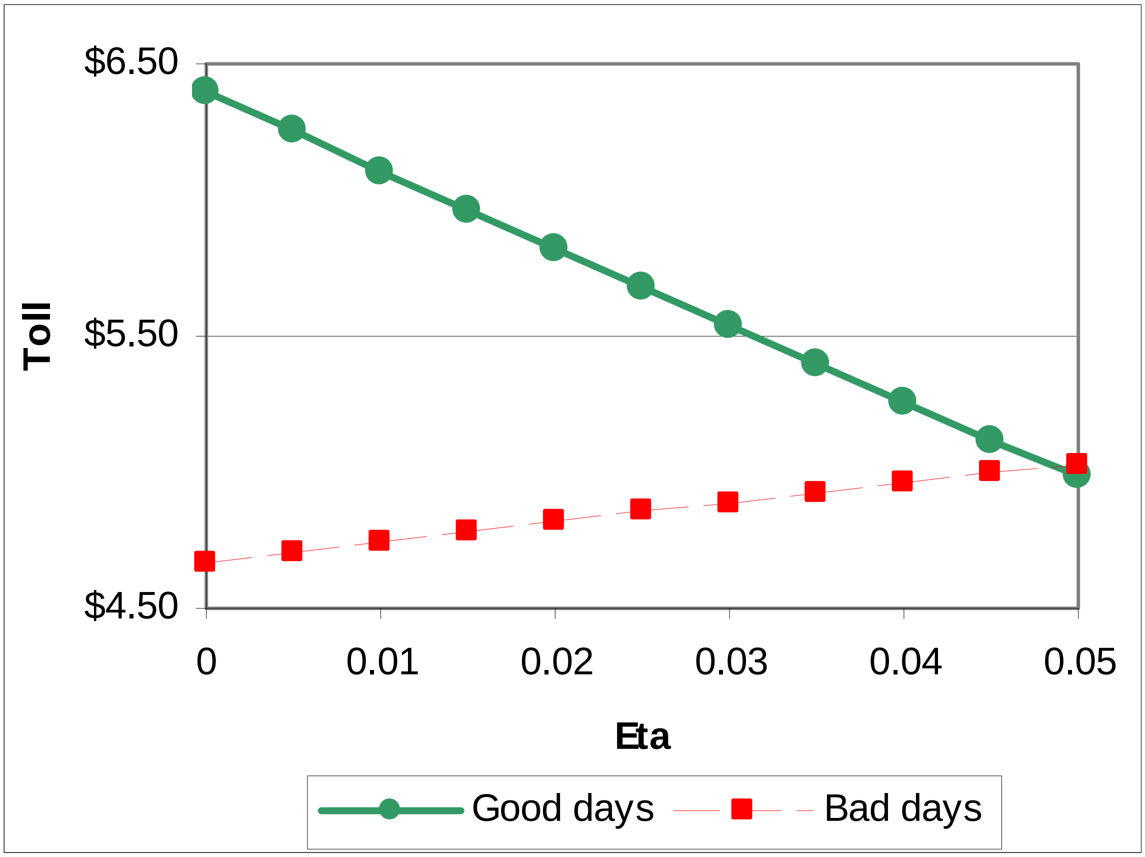

Figure 4: Optimal tolls as a function of η with all agents inexperienced; unbundled preferences and variable demand with $\pi = 0.5$

Source: Author's calculation

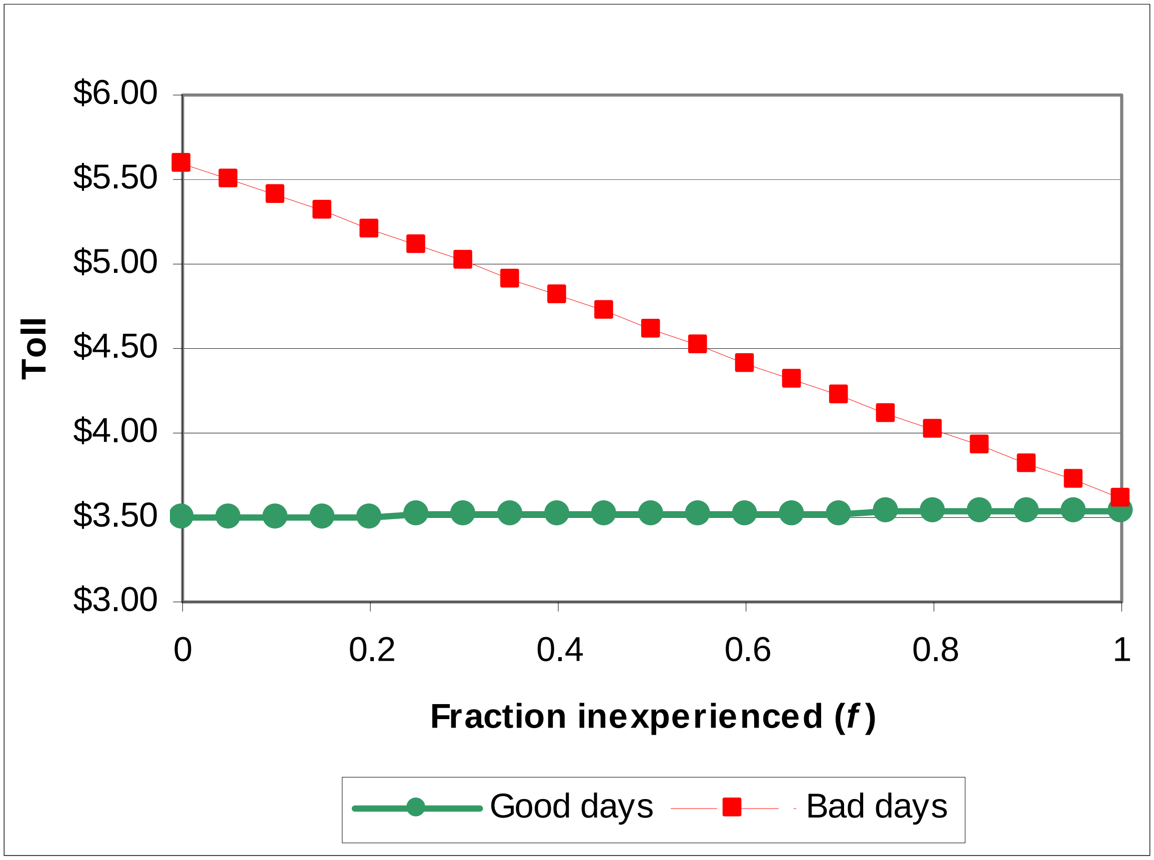

Figure 5: Optimal tolls as a function of the fraction of agents who are inexperienced; unbundled preferences and variable supply with $\pi = 0.2$, $\eta = 0.14$

Source: Author's calculation

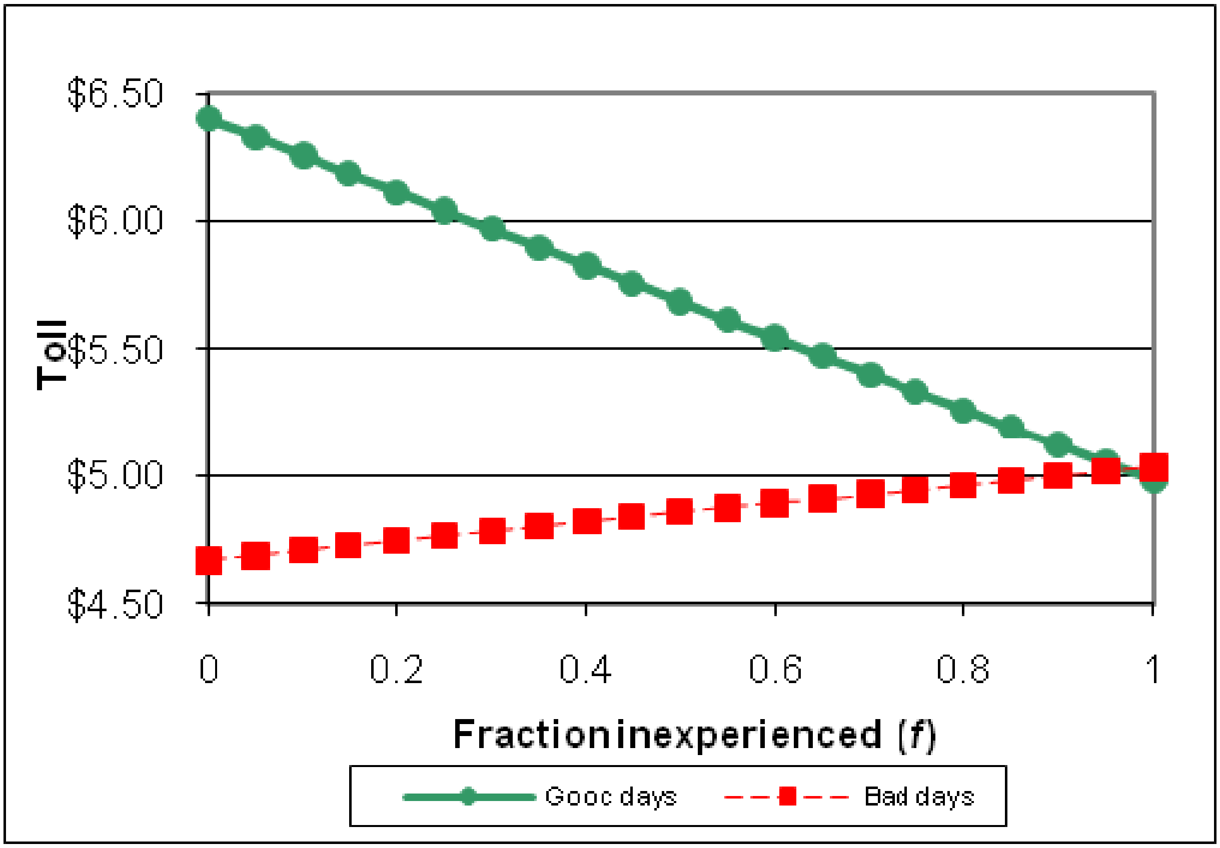

Figure 6: Optimal tolls as a function of the fraction of agents who are inexperienced; unbundled preferences and variable demand with $\pi = 0.5$, $\eta = 0.05$

Source: Author's calculation

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