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# Nonlinear Pricing on Private Roads with Congestion and Toll Collection Costs

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## Abstract

Nonlinear pricing (a form of second-degree price discrimination) is widely used in transportation and other industries but it has been largely overlooked in the road-pricing literature. This paper explores the incentives for a profit-maximizing toll-road operator to adopt some simple nonlinear pricing schemes when there is congestion and collecting tolls is costly. Users are assumed to differ in their demands to use the road. Regardless of the severity of congestion, an access fee is always profitable to implement either as part of a two-part tariff or as an alternative to paying a toll. Use of access fees for profit maximization can increase or decrease welfare relative to usage-only pricing. Hence a ban on access fees could reduce welfare.

*Key words*: Congestion pricing; Two-part pricing; Private roads; Toll collection costs

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# 1 INTRODUCTION

There is a vast and growing literature on congestion pricing of roads and other public facilities. Efficient usage calls for a Pigouvian tax per trip or per unit of usage. There is also a substantial body of work on pricing of private facilities as well as public facilities with a budget constraint for which revenue generation is a goal in addition to efficient usage. This second stream of work has focused on linear pricing schemes and third-degree price discrimination whereby higher tolls are charged to market segments with less price-elastic demand.

A third body of research has recently developed on nonlinear pricing of telecommunications and other services. Nonlinear pricing has, however, been largely overlooked in the road pricing literature. As De Borger (2001) remarks, this is surprising given the prevalence of both fixed charges and usage fees for road transport. Indeed, nonlinear pricing schemes are often used on toll roads. Some examples are presented in Table 1. All existing area-based road pricing schemes (in Singapore, Norway, London, Stockholm, and Milan) either feature some form of nonlinear pricing or did so at some point in their history. So do a number of toll roads in the US and Canada. State Route 91 in Orange County, California, offers optional plans that comprise two-part or three-part tariffs. During Phase I of the Value Pricing project on Interstate 15, monthly permits were sold to allow drivers of Single Occupancy Vehicles (SOVs) to use the High Occupancy Vehicle (HOV) lanes.[1] The High Occupancy Toll (HOT) lane experiment on Interstate 15 in Utah is following a similar plan. A number of facilities offer quantity discounts in the form of reduced prices for advance purchase of multiple trips, or *ex post* discounts based on cumulative usage over an accounting period. Nonlinear pricing schemes are easier to implement using Electronic Tolling Collection (ETC) technology than with conventional toll booths, and are likely to grow in popularity as ETC technology spreads.

The goal of this paper is to help fill the gap in the literature by studying nonlinear pricing on toll roads. Although nonlinear pricing can be used by public toll-road operators to boost revenues, or pursue equity objectives, the analysis is focused on private roads. Private-sector involvement in the construction and operation of roads is growing worldwide. Although tolls on most private roads are regulated in some way, it will be assumed that the private operator can

[1] In Phase II permits were replaced by tolls.

pursue unconstrained profit maximization. This polar case serves as an analytically tractable and insightful counterpart to the bulk of the literature on road pricing which considers public operation with welfare maximization as the objective.

There are several streams of literature on nonlinear pricing. Oi (1971) is the classic study on the use of two-part tariffs by a monopolist. Oi shows that, with identical users, the profit-maximizing solution is to charge a price equal to marginal cost and to extract all consumers' surplus with an access or membership fee. If individuals have different demands the monopolist typically sets price above marginal cost and faces a tradeoff between profits generated from marginal and inframarginal consumers. Using a club theory model with identical individuals Scotchmer (1985a,b) shows how a two-part tariff supports an efficient competitive equilibrium when facilities are congestible.

The more recent literature on access pricing generally deals with industries such as telecommunications, gas, and electricity where a vertically-integrated monopoly controls supply of a key input to its competitors (e.g., Laffont and Tirole, 1994; Armstrong et al., 1996; Domon and Ota, 2001). Essegaier et al. (2002) investigates the effects of service capacity and consumer demand heterogeneity on a firm's choice between usage pricing, access pricing (i.e., a flat fee) and a two-part tariff. They show that the choice depends on the relative importance of light-demand users and heavy-demand users. Sundararajan (2004) studies a seller's choice between usage pricing, access pricing, and letting users choose between the two schemes. His model includes a transactions cost for usage pricing, but abstracts from congestion. Sundararajan shows that offering an access pricing plan is always profitable — either alone or in combination with usage pricing.

There is also a stream of research on internet pricing. Much of this concentrates on marginal-cost pricing by packet, cell, or byte transmitted and/or received (e.g., Borella et al., 1999; Altmann and Chu, 2001). Nevertheless, three-part tariffs are commonly used for internet pricing in Europe. These tariffs are defined by an access fee, an allowance of free minutes, and a price per minute for connection time beyond the allowance. The potential advantages of three-part tariffs over two-part tariffs are derived theoretically by Masuda and Whang (2006) and Bagh and Bhargava (2008), and demonstrated empirically by Lambrecht et al. (2007).

Attention in this paper is restricted to road pricing and a set of five tolling schemes: (1) usage-only pricing, (2) access-only pricing, (3) a choice between a usage charge and an access fee, (4) a two-part tariff, and (5) a choice between a two-part tariff and a usage charge. Three-part tariffs, block pricing, and other nonlinear pricing schemes are left for future research. There are several reasons for focusing on these five schemes. First, they are all used on toll roads (cf. Table 1). Second, simple multi-part tariffs can be nearly as profitable as optimal (continuous) nonlinear pricing schemes (Wilson, 1993). And third, consumers have shown a preference for simple pricing schemes in diverse industries, and firms have responded by adopting simple structures even though they lose some control over the pattern of demand (Bonsall et al., 2007).

The five schemes considered in the paper are depicted in Figure 1 by plotting outlay (i.e., total expenditure) against individual consumption measured by number of trips, $q$. With usage-only pricing ($U$) the outlay is $E_U = \tau_U q$, where $\tau_U$ is the price per trip. The outlay curve is a ray through the origin with slope $\tau_U$. With access-only pricing ($A$) the outlay curve is a horizontal line, $E_A = A_A$, where $A_A$ is the access fee. If users are given a choice between the access fee and the usage charge (scheme *AorU*), those who choose to take less than $q_{AU}$ trips are better off with the usage charge and those who travel more than $q_{AU}$ prefer the access fee. The relevant outlay curve is the lower envelope of the usage-only and access-only curves, $Min\left(\tau_U q, A_A\right)$, shown in Figure 1 by the long-dash line with a kink at point $B$.[2] For the fourth scheme, a two-part tariff ($T$), the outlay curve is $E_T = A_T + \tau_T q$. For the final scheme, users are given a choice between the two-part tariff and a usage charge (*TorU*). The relevant outlay curve is $Min\left(A_T + \tau_T q, \tau_U q\right)$ shown by the short-dash line with a kink at point $C$. Users who choose less than $q_{TU}$ trips pay less with the usage charge, and those who travel more than $q_{TU}$ pay less with the two-part tariff.[3]

The analysis in the paper is organized around three sets of questions. First and foremost: which of the five pricing schemes is most profitable? How does the choice depend on congestion? Does usage-only pricing necessarily dominate if congestion is severe?

A second question is: how do toll collection costs act as a counterforce to congestion in favour of access pricing? Transactions costs are incurred in segmenting consumers, identifying

[2] Note that the optimal access fee for scheme *AorU* typically differs from the optimal access fee for access-only pricing, and the optimal usage charge for scheme *AorU* typically differs from the optimal usage charge for usage-only pricing. A similar observation applies to the two-part tariff and to the choice between two-part tariff and usage charge — considered immediately below.

[3] If a choice were offered between $T$ and $A$, the breakeven point would be at Point D in Figure 1.

price elasticities and preventing resale, and they are significant in many industries (Leeson and Sobel, 2008; Levinson and Odlyzko, 2008). Transactions costs are also recognized to be a key determinant of whether fixed fees are more profitable than usage pricing (Nahata et al., 1999; Sundararajan, 2004). Despite advances in ETC technology, tolling is still costly. Indeed, operating costs are a significant proportion of revenues for some schemes.[4] Electronic toll collection involves a variety of tasks: vehicle detection, classification, and identification; charge determination; posting toll transactions and customer payments to accounts; violation enforcement; maintaining customer accounts, and correcting billing errors; handling customer inquiries; and operating retail outlets and other payment channels. It is difficult to infer from publicly available information what proportion of costs are allocable to usage pricing, let alone to determine the differential cost per trip between usage pricing and access pricing.[5] Still, at least some tasks such as customer payments, maintaining accounts, correcting billing errors, and handling inquiries are likely to be lower for access-only accounts. [6] Indeed, transponders may not be required.[7]

The final question addressed in the paper is: How does the use of access fees by a private operator affect social welfare? The literature provides mixed evidence on this question. It is well known that third-degree price discrimination can be welfare-enhancing or welfare-reducing (Varian, 1989; Winter, 1997). As far as nonlinear pricing, perfect (first-degree) price discrimination is welfare-enhancing relative to no price discrimination (i.e., linear pricing) because marginal price and marginal revenue coincide and a monopolist is induced to supply the

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[4] For area-based tolling schemes operating costs as a fraction of revenues are 21 percent for Singapore, 10 percent for the Oslo toll ring, 5 percent for the Bergen toll ring, about 50 percent for London, and 22 percent for the Stockholm Trial (Lindsey, 2008, p.19). On State Route 91 the figure is 57 percent (91 Express Lanes, 2009, p.15).

[5] Some toll roads such as Interstate 15 in San Diego and Highway 407 in Toronto charge monthly transponder lease fees and account maintenance fees. It is unknown how these fees compare with the respective costs.

[6] According to State Route 91's 2007 annual report (91 Express Lanes, 2007, p.21) every month its Customer Service Center "handles an average of 34,000 customer phone calls, processes an average of 4,000 e-mails, and issues an average of 1,000 new 91 Express Lanes transponders." Customers who pay access-only fees presumably have less reason to communicate with the Center than those who pay tolls. State Route 91's 2009 annual report (91 Express Lanes, 2009) does not provide comparable information about the Customer Service Center.

[7] In July, 2008, express lanes were established on I-95 in Florida. Registered hybrid vehicles and carpool cars are allowed to use the lanes free and need only display a decal (http://www.95express.com/home/registration.shtm [January 2010])

first-best output while expropriating all surplus from consumers. However, the welfare ranking of two-part pricing and linear pricing is ambiguous. Income effects aside, the fixed charge is an efficient tool for extracting consumer's surplus. But the fixed charge creates a deadweight loss if it induces low-demand users to drop out of the market.[8] Fixed charges are also inferior to tolls for controlling congestion. They do not deter usage at the margin. And if they induce low-demand consumers to drop out, these consumers are likely to forego some trips for which the marginal benefit exceeds the marginal social cost.[9]

Toll collection and other administration costs create another wedge between private and public incentives for using access fees rather than tolls. A firm is willing to expend resources up to the amount of its gain from tolling: the sum of consumers' surplus captured and the elimination of any deadweight loss due to overusage in the absence of tolls. But the social gain from tolling is only the reduction in deadweight loss (Leeson and Sobel, 2008). A firm is therefore willing to incur higher toll collection costs than is a public operator.

All this suggests that the private sector is biased towards excessive usage of both access fees and tolls. Whether it is more biased towards one instrument than the other is unclear *a priori*.

We conclude the introduction by identifying how our analysis differs from previous work. De Borger (2001) studies the role of two-part tariffs for pricing road transport when external costs are important. His analysis differs from ours in considering public rather than private operation, in including welfare-distributional concerns in the public operator's objective function, and in dealing with the choice between ownership taxes and variable charges rather than between access fees and tolls on a single facility. Salas et al. (2009) study the use of two-part tariffs on congested road networks where the tariff consists of an access charge for each trip plus a charge based on the distance traveled. Nonlinear pricing is thus applied to individual trips rather than an accounting period during which many trips can be taken. Their model also differs in considering the social optimum rather than profit maximization, in using stochastic user equilibrium as the solution concept, and in abstracting from toll collection costs.

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[8] Oi (1971) overlooked the dropout problem when he claimed that the two-part tariff is socially preferable to a single monopoly price because it creates a smaller distortion between price and marginal cost.

[9] The efficiency loss depends, *inter alia*, on how demand varies over time. Monthly permits were used during Phase I of the Value Pricing program on Interstate 15 (see Table 1). As Gómez-Ibáñez and Small (1998, p.231) remark "Although a flat monthly fee may not appear to be congestion pricing, there is little incentive for anyone to purchase a permit except for use during peak hours. Thus the price of the monthly permit serves as a somewhat crudely targeted congestion toll."

Essegaier et al. (2002) compare three of the pricing schemes considered here: access-only pricing, usage-only pricing, and two-part tariffs. Their model differs fundamentally in featuring inelastic individual demand and a facility with a (strict) capacity constraint rather than congestion in the form of gradual deterioration in the quality of service with aggregate usage. Unlike Essegaier et al. (2002), Mason (2000) does consider elastic individual demands but he deals with a competitive market structure rather than monopoly. His model allows for congestion, but he assumes that this negative externality is outweighed by positive network externalities from usage so that individual utility is an increasing function of total usage. Like Essegaier et al. (2002), Sundararajan (2004) considers three pricing schemes: access-only pricing, usage-only pricing, and a choice between the two. He also allows for an administration cost per unit sold under the usage-based contract. But he excludes congestion and does not consider two-part tariffs. His usage-based contract also differs from the usage fee considered here in that the marginal price can depend on quantity purchased.

The next section of the paper sets out the general model and derives several general properties of the pricing schemes. Section 3 adopts a linear version of the general model in which users differ in their willingness to pay for trips. Profit-maximizing solutions for the five pricing schemes are derived and compared numerically. Section 4 briefly analyzes a variant of the model in Section 3 in which users differ in their trip frequencies rather than their willingness to pay for trips. Section 5 summarizes the main results and provides ideas for further research.

## 2 THE GENERAL MODEL

### 2.1 Model specification

To be concrete the tolled facility will be called a highway and demand will be measured by numbers of trips. Conditional on using the highway, individual demand is given by a function $q\left(p,T,x\right)$ where $p$ is the monetary cost of a trip, $T$ is travel time, and $x$ is an index of individuals. (A notational glossary is provided at the end of the text.) Income effects on demand are ignored so that consumer's surplus is an exact measure of welfare. This assumption can be justified on the grounds that, for most users, annual expenditure on toll roads is a very small fraction of income. Individuals with a larger *x* are assumed to have a lower demand[10]:

$$q(p,T,x_2) < q(p,T,x_1) \text{ for all } p \geq 0,\ T \geq 0 \text{ and } x_2 > x_1 \text{ such that } q(p,T,x_1) > 0. \quad (1)$$

An individual of type *x* will sometimes be called "type *x*". Assumption (1) allows for several dimensions of heterogeneity including reservation prices, trip frequencies, and values of travel time. Heterogeneity in reservation prices will be analyzed in Section 3, and heterogeneity in trip frequencies more briefly in Section 4.[11] Individual demand is assumed to be a differentiable and strictly decreasing function of *T* and *p*: $\partial q(p,T,x)/\partial T < 0$ and $\partial q(p,T,x)/\partial p < 0$ whenever $q(p,T,x) > 0$. To guarantee concavity of the profit functions with respect to tolls it is also assumed that the absolute price elasticity of demand is increasing in *p*.

The distribution of *x* in the population of potential users is assumed to have a support $[0,\overline{x}]$, $\overline{x} > 0$, and a continuously differentiable cumulative distribution $F(x)$ with a density $f(x) = \partial F(x)/\partial x$. $F(x)$ is assumed to be such that for all five pricing schemes first-order conditions identify a unique global profit maximum. Without loss of generality vehicle operating costs are normalized to zero so that the monetary cost of a trip equals the toll, $\tau$. Let $\overline{p}(T,x)$ denote the choke price on demand for type *x* given travel time *T* (i.e., $q(\overline{p}(T,x),T,x) = 0$).[12] Given an access fee of *A*, type *x* receives a consumer's surplus of

$$S(x) = Max\left(\int_{p=\tau}^{\overline{p}(T,x)} q(p,T,x)dp - A, 0\right). \quad (2)$$

[10] Assumption (1) rules out demand curves that cross. This is a standard assumption in the mechanism design literature. As Oi (1971) shows for uncongested facilities, if demand curves cross a profit-maximizing two-part tariff can have a price less than marginal cost.

[11] Heterogeneity in values of time (VOT) is not examined. For a given value of *T*, individuals with a higher VOT will have a lower willingness to pay and demands will differ in a similar way to a specification with differences in reservation prices when there is no congestion. In practice, VOT tends to be positively correlated with reservation price since higher-income individuals have both a higher willingness to pay and a higher VOT than individuals with lower incomes. If individuals differ in both characteristics, assumption (1) can be violated since a high-income type 1 can have a higher demand than a low-income type 2 when *T* is low, but a lower demand when *T* is high.

[12] The choke price is assumed to be finite. This assumption does not affect results of interest.

Eq. (2) incorporates the obvious condition that type $x$ travels only if $S(x) \geq 0$.[13]

Travel time is assumed to be an increasing and differentiable function of total usage (measured by the number of trips), $T(Q)$: $\partial T / \partial Q > 0$.[14] Vehicles are thus assumed to be identical in the congestion delay they impose. Third-degree price discrimination — in which users are charged different tolls on the basis of their (observable) vehicle or personal characteristics (see Varian, 1989) — is ruled out. Following Sundararajan (2004) it is assumed that the highway operator incurs a cost $k \geq 0$ per trip for usage pricing. Any fixed toll collection costs are assumed to be independent of the type of tolling scheme, and the common cost is normalized to zero. Any fixed costs of operating the toll road itself are ignored, as are any costs of usage incurred by the operator for operations and maintenance.[15]

### 2.2 Choice between usage and access pricing

As a first step in assessing the relative merits for profit maximization of usage charges and access fees it is useful to compare the two simple pricing schemes, usage-only pricing (*U*) and access-only pricing (*A*), that each feature only one pricing instrument. Three factors within the scope of the model affect the relative profitability of the two schemes: congestion, toll collection costs, and user heterogeneity. Clearly, congestion favors usage pricing while collection costs favor access pricing. User heterogeneity favors usage pricing because — as noted in the introduction — access fees induce low-demand users to drop out of the market. A general analysis of these three factors turns out to be tedious and not very illuminating. Some useful insights can be obtained from a simple case with homogeneous users and zero collection costs in which only congestion and the shape of the individual travel demand curve comes into play.

In this simple specification profits from usage-only pricing are

$$\pi_U = \tau Q_U . \tag{3}$$

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[13] The access fee is a fixed charge that is independent of usage and can be interpreted as an admission or membership fee that has to be paid for the right to use the road. The magnitude of the fee will depend on the accounting period; it will be larger for a year than for a month, and for a month than for a day. Unlike tolls, an admission fee cannot be varied by time of day to track changes in congestion. This difference in flexibility of the two pricing instruments does not come into play here since the model is static.

[14] Unless necessary for clarity the dependence of *T* on *Q* will be suppressed.

[15] Operations and maintenance costs per trip can be deducted from willingness to pay. These costs do not affect results of primary interest since they do not affect the choice of tolling scheme.

Let $\bar{N}$ denote the number of (identical) individuals who are potential users. Under usage-only pricing all individuals will use the facility and total usage, $Q_U$, is given implicitly by the equation[16]

$$Q_U = \bar{N} q\left(\tau, T\left(Q_U\right)\right). \tag{4}$$

The first-order condition for maximum profits is

$$\frac{\partial \pi_U}{\partial \tau} = Q_U + \tau \frac{\partial Q_U}{\partial \tau} = 0. \tag{5}$$

From Eq. (4),

$$\frac{\partial Q_U}{\partial \tau} = \frac{\bar{N}\left(\partial q / \partial \tau\right)}{1 - \bar{N}\left(\partial q / \partial T\right)\left(dT/dQ\right)}. \tag{6}$$

Substituting Eq. (6) into Eq. (5), and rearranging terms, yields a formula for the toll:

$$\tau = \underbrace{\frac{\bar{N} q\left(\partial q/\partial T\right)\left(dT/dQ\right)}{\partial q/\partial \tau}}_{Congestion\ charge} - \underbrace{\frac{q}{\partial q/\partial \tau}}_{Markup}. \tag{7}$$

The profit-maximizing toll has two components.[17] The first component is a congestion charge, equal to the marginal external cost of congestion, with the same functional form as the Pigouvian toll that supports the social optimum. The second component is a markup that is inversely proportional to the price elasticity of demand. Total profits are obtained by substituting Eq. (7) into Eq. (3):

$$\pi_U = \left( \frac{\bar{N} q\left(\partial q/\partial T\right)\left(dT/dQ\right)}{\partial q/\partial \tau} - \frac{q}{\partial q/\partial \tau} \right) Q_U. \tag{8}$$

With access-only pricing the access fee is set to extract all consumer's surplus. There are two cases to consider. In one, the operator chooses to serve all $\bar{N}$ individuals. Total usage, $Q_A$, is given implicitly by the equation

$$Q_A = \bar{N} q\left(0, T\left(Q_A\right)\right). \tag{9}$$

From Eq. (2) the access fee is

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[16] In this subsection the type index is omitted as an argument of the demand function since users are identical.

[17] This decomposition is analyzed in Small and Verhoef (2007, Section 6.1).

$$A = \int_{p=0}^{\bar{p}(T(Q_A))} q\left(0, T\left(Q_A\right)\right) dp, \tag{10}$$

and profits are therefore

$$\pi_A = \bar{N}A = \bar{N} \int_{p=0}^{\bar{p}(T(Q_A))} q\left(0, T\left(Q_A\right)\right) dp. \tag{11}$$

In the second case the operator sets the access fee above the level given by Eq. (10) when all $\bar{N}$ individuals are users. Doing so reduces the number of users below $\bar{N}$, but because congestion is reduced the customer base stabilizes at a positive level as long as *A* is not set too high.

A common assumption in the literature is that individual trip demand depends on $\tau$ and $T$ additively through the generalized cost of travel, $\tau + vT$, where *v* is the value of time. With this specification the conventional diagram of congestion pricing[18] can be used to compare usage pricing with the two cases of access pricing as well as to compare all three schemes with the social optimum. This is done in Figure 2 with the normalization $v = 1$. Aggregate demand is plotted against total trips. The average time cost of a trip is $T\left(Q\right)$, and the marginal social cost is $T\left(Q\right) + Q\left(dT/dQ\right)$. With free access, equilibrium occurs at point *n*: the intersection of the aggregate demand curve, $\bar{N}q\left(p, T\left(0\right)\right)$, and $T\left(Q\right)$. In the social optimum all individuals use the facility. Optimal usage, $Q_O$, occurs at the usage level where the marginal social cost matches willingness to pay (point *k*). The optimum can be supported by imposing a toll $\tau_0$ equal to distance $\overline{kl}$.

With usage-only pricing the toll, $\tau_U$, is set by the private operator to maximize the area of the rectangle contained between the demand curve, the average travel time cost curve, and the vertical axis. As shown, the toll equals distance $\overline{gj}$, and profits, $\pi_U$, equal the lightly-shaded area *bgjf*. The congestion charge component of the toll in Eq. (7) equals distance $\overline{hj}$, and the markup is $\overline{gh}$. Total usage, $Q_U$, is less than optimal usage, $Q_O$.

If access pricing is implemented without excluding users then equilibrium usage, $Q_A\left(\bar{N}\right)$, is the same as with free access and profits, $\pi_A$, equal area *anc*. If the access fee is set at a higher

[18] See Small and Verhoef (2007, Section 4.1).

level, the number of users drops to some number *N*, $N < \bar{N}$, the aggregate demand curve for active users shifts in from $\bar{N}q\left(p, T(0)\right)$ to $Nq\left(p, T(0)\right)$, and profits equal area *amd*. Area *amd* exceeds area *anc* if $T(Q)$ is sufficiently steep. Profits from the more profitable form of access-only pricing can be bigger or smaller than profits from usage-only pricing. Access-only pricing will be the more profitable if congestion is not too severe and demand is not too elastic in the range of high generalized costs so that area *agb* is significant. Clearly these insights remain relevant if user heterogeneity and toll transaction costs are introduced. This leads to:

> **Proposition 1**: The profits from usage-only pricing and access-only pricing cannot be ranked in general. Access-only pricing may be more profitable if congestion is not too severe and demand is not too elastic so that substantial consumer's surplus can be expropriated with the access fee. If access pricing is implemented, and congestion is moderately severe, it may be profitable to set the access fee high enough to exclude some individuals from using the highway in order to provide a better quality of service by reducing congestion.

The welfare ranking of usage-only pricing and access-only pricing (as implemented by the private operator) can differ from their profit ranking. Social surplus, *W*, from usage-only pricing is given by area *agjf* which exceeds profits by area *agb*. In contrast, social surplus matches profits for both cases of access-only pricing because the private operator expropriates all consumer's surplus with the access fee. Hence there is a possibility that $\pi_A > \pi_U$ whereas $W_A < W_U$ so that the private operator is biased in favor of access pricing. However, if user heterogeneity and toll transaction costs are added back in to the model, the bias can go the other way. This will be demonstrated in Section 3.6.

### 2.3 Combining usage and access pricing

Section 2.2 uses a simple variant of the general model to focus on the exclusive choice between usage charges and access fees. For the remainder of Section 2 the general model will be used to address the first question posed in the introduction: under what conditions is it profitable to implement both usage pricing and access pricing? The three composite pricing schemes implement usage pricing and access pricing in different ways. The two-part tariff (scheme *T*) combines an access fee and a usage charge in one payment option. Scheme *AorU* offers them as alternative forms of payment between which consumers can choose. And scheme *TorU* offers consumers a choice between paying a two-part tariff that combines both types of prices, and paying just a usage charge. Subsection 2.3.1 assesses the profitability of the two-part tariff compared to access-only pricing and usage-only pricing. Subsection 2.3.2 does likewise for scheme *AorU*. Finally, Subsection 2.3.3 compares the profitability of scheme *AorU* and scheme *TorU*.

### 2.3.1 The two-part tariff (Scheme *T*)

Profits with a two-part tariff, $\pi_T$, are[19]

$$\pi_T = \int_{x=0}^{\hat{x}} \left( A + (\tau - k) q(\tau, T, x) \right) f(x) dx, \tag{12}$$

where $\hat{x}$ is the highest consumer type that is served or the "market reach". In general $\hat{x} \leq \overline{x}$. A general property of monopolistic screening problems is that the monopolist extracts all surplus from the consumer type with the lowest demand who chooses to participate in the market. Type $\hat{x}$ therefore receives zero surplus. Given Eq. (2), type $\hat{x}$ is defined implicitly by the condition:

$$S(\hat{x}) = \int_{p=\tau}^{\overline{p}(T,\hat{x})} q(p, T, \hat{x}) dp - A = 0. \tag{13}$$

It is assumed here and in the rest of Section 2 that $\hat{x} < \overline{x}$ so that some types are excluded from consumption. The total number of trips taken by all users is

$$Q = \int_{x=0}^{\hat{x}} q(\tau, T(Q), x) f(x) dx. \tag{14}$$

As in Section 2.2., Eq. (14) is an implicit equation for *Q*. Provided toll collection costs are not too high the operator will charge a toll. From Eq. (12) the first-order condition for maximizing profits with respect to the toll is

$$\frac{\partial \pi_T}{\partial \tau} = \int_{x=0}^{\hat{x}} \left( \underbrace{q(\tau, T, x)}_{(a)} + \underbrace{(\tau - k)\left( \frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial T}\frac{dT}{dQ}\frac{dQ}{d\tau} \right)}_{(b)} \right) f(x) dx$$

[19] To ease notation subscript *T* is omitted from *A* and $\tau$.

$$+\underbrace{\left(A+\left(\tau-k\right)q\left(\tau,T,\hat{x}\right)\right)f\left(\hat{x}\right)\frac{d\hat{x}}{d\tau}}_{(c)}=0\,. \qquad (15)$$

The first line in Eq. (15) corresponds to the gain in profit (i.e., toll revenue net of toll collection costs) from inframarginal users. Term (a) is the gain in toll revenue from the initial volume of trips, and term (b) is the loss of profit due to a reduction in volume. The loss of profit is moderated by a reduction in travel time as usage falls since $dT/dQ>0$ by assumption, and $dQ/d\tau<0$ as shown in Appendix A. Term (c) is the loss in combined access fee and toll profit from individuals who stop using the highway because their consumer's surplus becomes negative. Term (c) has the same sign as the derivative $d\hat{x}/d\tau$. Now

$$\frac{d\hat{x}}{d\tau}\overset{s}{=}-q\left(\tau,T,\hat{x}\right)+\frac{dT}{dQ}\left(\underbrace{\int_{x=0}^{\hat{x}}\frac{\partial q\left(\tau,T,x\right)}{\partial\tau}f\left(x\right)dx}_{(d)}\bullet\underbrace{\int_{p=\tau}^{\bar{p}(T,\hat{x})}\frac{\partial q\left(p,T,\hat{x}\right)}{\partial T}dp}_{(e)}\right.$$

$$\left.-\underbrace{q\left(\tau,T,\hat{x}\right)}_{(f)}\bullet\underbrace{\int_{x=0}^{\hat{x}}\frac{\partial q\left(\tau,T,x\right)}{\partial T}f\left(x\right)dx}_{(g)}\right), \qquad (16)$$

where $\overset{s}{=}$ means "has the same sign as". Expression (16) is clearly negative if $dT/dQ=0$ since it reduces to $-q\left(\tau,T,\hat{x}\right)$. Term (d) is the reduction in total demand induced by a marginal increase in toll. Term (e) is the increase in demand by the marginal type, $\hat{x}$, induced by the resulting reduction in travel time. Term (f) is the reduction in consumer's surplus for the marginal type due to the toll increase, and term (g) is the rebound effect of reduced congestion on total travel. In the specific model used in Sections 3 and 4, (d)•(e) = (f)•(g) and $d\hat{x}/d\tau<0$. But in the general model here the expression in brackets can be negative if the value of time (VOT) of the marginal type exceeds the average VOT of inframarginal types, so that term (e) is relatively large in absolute value and term (g) is relatively small. If so, an increase in toll attracts marginal types because they value the travel time reduction more than the higher cost.[20] It is therefore conceivable that $d\hat{x}/d\tau>0$.

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[20] This possibility also arises in models with pure usage pricing; e.g. Edelson (1971), Foster (1974), and Glazer and Niskanen (2000).

Now consider the choice of access fee. From Eq. (12) the first-order condition for maximizing profits with respect to *A* is

$$\frac{\partial \pi_T}{\partial A} = \int_{x=0}^{\hat{x}} \left( \underbrace{1}_{(a)} + \underbrace{(\tau - k)\frac{\partial q}{\partial T}\frac{dT}{dQ}\frac{dQ}{dA}}_{(b)} \right) f(x)\,dx + \underbrace{\left(A + (\tau - k)\, q(\tau, T, \hat{x})\right) f(\hat{x})\frac{d\hat{x}}{dA}}_{(c)} = 0\,. \qquad (17)$$

As shown in Appendix A, $dQ/dA < 0$ and $d\hat{x}/dA < 0$. Unlike for a toll increase, an increase in the access fee unambiguously reduces the market reach. Term (a) in Eq. (17) is the marginal gain in access fee revenue derived from inframarginal users. Term (b) is the gain in toll profit from inframarginal users due to reduced congestion. Term (c) is the reduction in combined access fee and toll profit from the marginal type who drops out.

The optimal two-part tariff defined by the first-order conditions Eq. (15) and Eq. (17) is characterized by:

**Proposition 2**: If toll collection costs are not too high, then the profit-maximizing two-part tariff features a strictly positive access fee and a usage charge above marginal cost: $A > 0$ and $\tau - k > 0$. If toll collection costs are sufficiently high, then the two-part tariff is degenerate. It has no usage charge and an access fee that is strictly positive.

Proof: See Appendix A. Proposition 2 is consistent with Oi's (1971) result for uncongested facilities and no money collection costs that the profit-maximizing access fee is positive and the profit-maximizing price exceeds marginal cost.[21] Proposition 2 shows that this result extends to congestible facilities as long as toll collection costs are not too high. Oi's intuitive explanation still applies. With homogeneous users it is efficient to impose a usage charge equal to the marginal external cost of congestion created by each trip, and expropriate remaining consumer's surplus with the access fee. When users have heterogeneous demands the operator faces a dropout problem for marginal users. A marginal increase in the toll above the marginal external cost of congestion generates more than enough profit to compensate for the reduction in the access fee required to retain business of the marginal user. But a positive access fee remains

[21] Ng and Weisser (1974) derive a general version of this result using a model with income effects.

profitable as a way to extract consumer's surplus from inframarginal users because it does not reduce their trip demand (further) below efficient levels.

### 2.3.2 Choice between access fee and usage charge (Scheme *AorU*)

With the *Access or Usage* (*AorU*) pricing scheme the operator allows users to choose between paying an access fee for unlimited usage and paying a toll per trip. Profits are[22]

$$\pi_{AU} = \int_{x=0}^{y} Af(x)\,dx + \int_{x=y}^{\hat{x}} (\tau - k)\, q(\tau, T, x)\, f(x)\,dx, \tag{18}$$

where $y$ is the type indifferent between paying the access fee and paying the toll. Types $x \in [0, y)$ choose the access fee, and types $x \in (y, \hat{x}]$ choose the toll.[23] Type *y* is defined implicitly by the condition:

$$\int_{p=0}^{\bar{p}(T,y)} q(p, T, y)dp - A = \int_{p=\tau}^{\bar{p}(T,y)} q(p, T, y)dp,$$

or

$$A = \int_{p=0}^{\tau} q(p, T, y)dp. \tag{19}$$

Total usage is defined by the implicit equation

$$Q = \int_{x=0}^{y} q(0, T(Q), x)\, f(x)\,dx + \int_{x=y}^{\hat{x}} q(\tau, T(Q), x)\, f(x)\,dx. \tag{20}$$

From Eq. (18) the first-order condition for $\tau$ is

$$\frac{\partial \pi_{AU}}{\partial \tau} = \underbrace{\int_{x=y}^{\hat{x}} q(\tau, T, x)\, f(x)\,dx}_{(a)} + \underbrace{(\tau - k)\int_{x=y}^{\hat{x}} \left( \frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial T}\frac{dT}{dQ}\frac{dQ}{d\tau} \right) f(x)\,dx}_{(b)}$$

$$+ \underbrace{\left( A - (\tau - k)\, q(\tau, T, y) \right) f(y) \frac{dy}{d\tau}}_{(c)} = 0. \tag{21}$$

Note that $q(\tau, T, \hat{x}) = 0$ since type $\hat{x}$ receives zero consumer's surplus and does not pay the access fee.

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[22] To simplify notation no subscript to denote the *AorU* scheme is added to *A* or $\tau$.

[23] Users pool into two groups because there is a continuum of types and only two pricing schemes to choose between.

Term (a) in Eq. (21) is the marginal gain in toll revenue from trips taken by usage-pricing customers. Term (b) is the marginal loss of profit from the toll due to a reduction in the number of trips they take. Similar to the two-part tariff, this loss is moderated by the drop in congestion. Term (c) is the marginal increase in profit derived from users who switch from usage pricing to access pricing. As shown in Appendix A, $dQ/d\tau < 0$ and $dy/d\tau > 0$. Since terms (a) and (c) are positive, and term (b) is nonnegative if $\tau \le k$, it follows that $\tau > k$ if usage pricing is used.

From Eq. (18) the first-order condition for $A$ is

$$\frac{\partial \pi_{AU}}{\partial A} = \underbrace{F(y)}_{(a)} + \underbrace{(\tau - k)\int_{x=y}^{\hat{x}} \frac{\partial q}{\partial T}\frac{dT}{dQ}\frac{dQ}{dA} f(x)dx}_{(b)} + \underbrace{\left(A - (\tau - k)q(\tau, T, y)\right) f(y)\frac{dy}{dA}}_{(c)} = 0. \qquad (22)$$

Term (a) is the marginal gain in access-fee revenue from access-pricing customers. Term (b) is the marginal gain in toll profit from usage-pricing customers due to reduced congestion. Term (c) is the marginal profit loss from customers who switch from access pricing to usage pricing. As shown in Appendix A, $dQ/dA < 0$ and $dy/dA < 0$.

The optimal access fee and toll for the *Access or Usage* scheme, defined by Eq. (21) and Eq. (22), is characterized by:

> **Proposition 3**: If the profit-maximizing *Access or Usage* scheme offers users a non-degenerate choice between a usage charge and an access fee then $A > 0$ and $\tau - k > 0$. If congestion is severe, and toll collection is not too costly, only usage pricing is used with $\tau - k > 0$. If toll collection is sufficiently costly, then only access pricing is used with $A > 0$.

Proof: See Appendix A. According to Proposition 3 it is not necessarily profitable to offer access pricing in the *Access or Usage* scheme. This contrasts with Proposition 2 which established that the optimal two-part tariff always features a strictly positive access fee. The reason for the difference is similar to that underlying Proposition 1: all consumers pay a toll in the two-part pricing scheme, whereas access-pricing customers do not pay a toll and thus take excessive numbers of trips. If congestion is severe, access pricing is not profitable because the loss from not pricing usage outweighs the gain from being able to extract consumer's surplus from inframarginal users.[24]

### *2.3.3* Choice between Scheme *AorU* and Scheme *TorU*

Schemes *AorU* and *TorU* both offer a choice of payment. Scheme *TorU* has an additional degree of freedom because the two part tariff option features a usage charge as well as an access fee. This suggests that — if toll collection costs are sufficiently small — *TorU* is more profitable than *AorU*. This is indeed the case, and the ranking is formalized as:

> **Proposition 4**: Suppose toll collection costs are zero. Then, if either users are heterogeneous or the highway is congestible (or both), scheme *TorU* is strictly more profitable than scheme *AorU*.

Proof: See Appendix A. The proof entails showing that any *AorU* scheme (whether or not it is profit-maximizing) can be replaced by a *TorU* scheme that yields higher profits both from the set of individuals who choose the usage-pricing option in the *AorU* scheme and from the set of individuals who choose the access-pricing option. Scheme *TorU* has two advantages over *AorU*. First, it better controls congestion since all users face a usage charge. Second, the combination of usage charge and access fee in the two-part pricing option is more effective at expropriating consumer's surplus without dissuading low-demand consumers from participation. Naturally, these advantages of the *TorU* scheme can be outweighed by toll collection costs. This will be demonstrated in Sections 3 and 4.

Further analytical results with the general model are difficult to derive. In part, this is because both profit-maximizing and socially-optimal pricing schemes depend on the distribution of types in the population.[25] To enable a fuller comparison of the five pricing schemes in terms of profits and welfare, a specific version of the general model is now adopted.

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[24] Proposition 3 also differs from Sundararajan's (2004) Proposition 2 that it is always profitable to offer a fixed-fee contract. This is because there is no congestion in his model.

[25] By contrast, to solve the optimal single-part tariff it is only necessary to know the price elasticity of aggregate demand (Oi, 1971, p.87).

## 3 SPECIFIC MODEL WITH HETEROGENEOUS RESERVATION PRICES

### 3.1 Model specification

To enhance analytical tractability a linear version of the model is now adopted. Travel time is assumed to be a linear function of total usage:[26]

$$T(Q) = eQ\,,\ e \geq 0\,. \tag{23}$$

Free-flow travel time is normalized without loss of generality to zero. Conditional on receiving nonnegative consumer's surplus the demand function of a generic individual is assumed to have the linear form

$$q = Max\left(1 - \tau - T - x, 0\right), \tag{24}$$

where $x \geq 0$ and the value of time is the same for all individuals and normalized to unity. Type index *x* determines reservation price to use the highway. An individual's type depends on distance traveled before or after using the highway, availability of alternative routes, public transit accessibility, personal preferences for public transit, and other factors. To emphasize the spatial interpretation of this specification, type will sometimes be called "location". An individual located at *x* uses the highway only if $x < 1 - \tau - T$ . For sake of tractability *x* is assumed to have a uniform distribution $x \sim U\left[0, \bar{x}\right]$ where $\bar{x} \leq 1$.[27]

The first two pricing schemes considered are free access and the social optimum. These schemes serve as benchmarks against which the five profit-maximizing schemes can be compared.

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[26] Linear delay functions are sometimes assumed for computer networks (Roughgarden and Tardos, 2002). By contrast, highway travel time functions are usually assumed to be strictly convex functions of traffic flows. However, in the model here demand is measured by numbers of trips and in dynamic models equilibrium costs can be approximately linear functions of the number of trips. This is true of the bottleneck model (Arnott *et al*., 1998). Eq. (23) can be viewed as a reduced-form, static representation of such a model.
[27] Uniform distributions are commonly assumed in models of price discrimination (Armstrong, 2006). Even with this simplification, solutions for some pricing schemes are characterized by third- and higher-degree polynomials and have to be solved numerically.

### 3.2 Free access (Scheme *E*)

Under free access the highway can be used without paying either an access fee or a toll. Given Eq. (23), and $\tau = 0$ in Eq. (24), trip demand by type *x* is $q = Max\left(1 - eQ_E - x, 0\right)$ where $Q_E$ is total usage in the free-access equilibrium. Both demand and consumer's surplus are zero for type $\tilde{x}_E = 1 - eQ_E$. If $\overline{x} \le \tilde{x}_E$ then all types use the highway in equilibrium. If $\overline{x} > \tilde{x}_E$, then types $\left[0, \tilde{x}_E\right)$ use it, and types $\left[\tilde{x}_E, \overline{x}\right]$ do not. Hence the market reach is $\hat{x}_E = Min\left(\overline{x}, \tilde{x}_E\right)$. Total usage is given by the implicit equation

$$Q_E = \int_{x=0}^{\hat{x}_E} \left(1 - eQ_E - x\right) dx = \left(1 - eQ_E\right)\hat{x}_E - \frac{\hat{x}_E^2}{2}$$

which simplifies to

$$Q_E = \frac{\hat{x}_E\left(1 - \hat{x}_E / 2\right)}{1 + e\hat{x}_E}. \tag{25}$$

Social surplus is the sum of individual consumer's surpluses:

$$W_E = \int_{x=0}^{\hat{x}_E} \frac{\left(1 - eQ_E - x\right)^2}{2} dx = \frac{1}{6}\left[\left(1 - eQ_E\right)^3 - \left(1 - eQ_E - \hat{x}_E\right)^3\right]. \tag{26}$$

Substituting Eq. (25) into Eq. (26) yields

$$W_E = \frac{1}{6\left(1 + e\hat{x}_E\right)^3}\hat{x}_E\left(1 + e\hat{x}_E\right)\left[3\left(1 + \frac{e\hat{x}_E^2}{2}\right)\left(1 - \hat{x} - \frac{e\hat{x}_E^2}{2}\right) + \hat{x}_E^2\left(1 + e\hat{x}_E\right)^2\right]. \tag{27}$$

Using the formula $\tilde{x}_E = 1 - eQ_E$ and Eq. (25) one obtains $\tilde{x}_E = \left(\sqrt{1 + 2e} - 1\right)/e$. Hence

$$\hat{x}_E = Min\left(\overline{x}, \frac{\sqrt{1 + 2e} - 1}{e}\right). \tag{28}$$

If $\overline{x} \le \left(\sqrt{1 + 2e} - 1\right)/e$, then social surplus is given by eqn. (27) with $\hat{x} = \overline{x}$. If $\overline{x} \ge \left(\sqrt{1 + 2e} - 1\right)/e$, then eqn. (27) simplifies to

$$W_E = \frac{\sqrt{1 + 2e} - 1 + e\left(\sqrt{1 + 2e} - 3\right)}{e^3}. \tag{29}$$

With no congestion ($e = 0$), Eq. (27) applies with $\hat{x} = \overline{x}$. If $e = 0$ and $\overline{x} = 1$, Eq. (27) simplifies to $W_E = 1/6$.

### 3.3 The social optimum (Scheme *O*)

Social surplus with a toll, $\tau_O$, is given by:

$$W_O = \int_{x=0}^{\hat{x}} \left\{ \underbrace{\frac{\left(1-eQ_O-\tau_O-x\right)^2}{2}}_{\text{Consumers' surplus}} + \underbrace{\left(\tau_O-k\right)\left(1-eQ_O-\tau_O-x\right)}_{\text{Toll revenue}} \right\} dx$$

$$= \frac{1}{6}\left[\left(1-eQ_O-\tau_O\right)^3 - \left(1-eQ_O-\tau_O-\hat{x}_O\right)^3\right] + \left(\tau_O-k\right)Q_O . \tag{30}$$

Here, $\hat{x}_O$ is the market reach in the social optimum; it is defined by the equations $\hat{x}_O = Min\left(\overline{x}, \tilde{x}_O\right)$ and $\tilde{x}_O = 1-eQ_O-\tau_O$. Total usage is given by the implicit equation

$$Q_O = \int_{x=0}^{\hat{x}} \left(1-eQ_O-\tau_O-x\right)dx = \left(1-eQ_O-\tau_O\right)\hat{x}_O - \frac{\hat{x}_O^2}{2} = \frac{\left(1-\tau_O\right)\hat{x}_O - \hat{x}_O^2/2}{1+e\hat{x}_O} . \tag{31}$$

Given Eq. (23), total travel cost is $eQ^2$, the marginal social cost of a trip is $2eQ$, and the marginal external congestion cost is $eQ$. The optimal Pigouvian toll is therefore $\tau_O = k + eQ_O$. Substituting this equation into Eq. (31) gives total usage as a function of the market reach:

$$Q_O = \frac{1-k-\hat{x}_O/2}{1+2e\hat{x}_O}\hat{x}_O . \tag{32}$$

Substituting Eq. (32) into $\tau_O = k + eQ_O$ gives

$$\tau_O^* = \frac{1+e\hat{x}_O}{1+2e\hat{x}_O}k + \frac{e\hat{x}_O}{1+2e\hat{x}_O}\left(1-\frac{\hat{x}_O}{2}\right). \tag{33}$$

The Pigouvian toll is a weighted average of the toll collection cost, *k*, and the average reservation price of individuals who travel, $1-\hat{x}_O/2$. The weight on the average reservation price increases towards one half as the congestion coefficient, *e*, increases. Substituting Eq. (32) and Eq. (33) into $\tilde{x}_O = 1-eQ_O-\tau_O$, and using the formula $\hat{x}_O = Min\left(\overline{x}, \tilde{x}_O\right)$, yields

$$\hat{x}_O = Min\left(\overline{x}, \frac{\sqrt{1+4e\left(1-k\right)}-1}{2e}\right). \tag{34}$$

Underlying Eqs. (30)-(34) is the assumption that tolling is socially beneficial (i.e., $W_O > W_E$) which is true only if toll collection costs are not too high relative to the costs of congestion. The threshold value of the collection cost, *k*, is plotted against *e* in the lower curve of Figure 3.[28] The relationship is concave over most of the range of *e* and reaches $k \cong 0.32$ at $e = 12$. Of more practical interest is the ratio of the toll collection cost to the toll, $k/\tau_O^*$, shown in the upper curve of Figure 3. The ratio rises from zero to about 0.57 at $e = 12$ which is higher than collection costs in most existing toll systems.[29] To get a sense of the magnitude of congestion with $e = 12$, note that with $\bar{x} = 1$ and $e = 12$ the market reach with free access is $\hat{x}_E = 1/3$ compared to $\hat{x}_E = 1$ without congestion, and total usage is just one ninth of its value without congestion. A value of $e = 12$ thus serves as a reasonable upper bound on the severity of congestion that could be encountered on a highway.[30]

### 3.4 Usage-only pricing (Scheme *U*)

Usage-only pricing was considered in Section 2 for a case with homogeneous users and no toll collection costs. Under the model assumptions in this section profits are

$$\pi_U = \int_{x=0}^{\hat{x}_U} (\tau_U - k)(1 - eQ_U - \tau_U - x)dx = (\tau_U - k)(1 - eQ_U - \tau_U - \hat{x}_U/2)\hat{x}_U, \qquad (35)$$

where $\hat{x}_U = Min(\bar{x}, \tilde{x}_U)$ and $\tilde{x}_U = 1 - eQ_U - \tau_U$. Total usage is given by the analogue to Eq. (31):

$$Q_U = \frac{(1-\tau_U)\hat{x}_U - \hat{x}_U^2/2}{1 + e\hat{x}_U}. \qquad (36)$$

Substituting Eq. (36) into Eq. (35), and solving the first-order condition $\partial \pi_U / \partial \tau_U = 0$, yields

$$\tau_U^* = \frac{1}{2}k + \frac{1}{2}\left(1 - \frac{\hat{x}_U}{2}\right). \qquad (37)$$

The profit-maximizing toll is an unweighted average of the toll collection cost and the average reservation price on demand, and it is independent of the congestion coefficient, *e*. It is

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[28] The equation for the threshold value is given by a fourth-order polynomial and has to be solved numerically. Toll collection costs could be reduced by charging only a fraction of users. This possibility is ruled out. It is also possible that an access fee — either standalone or in combination with a toll — would be more efficient than pure usage pricing because of lower collection costs. This possibility too is not examined. It is readily shown that, if users are identical, a pure access fee is beneficial (albeit less efficient than usage pricing when *k* is small) if $e > 1$.

[29] See footnote 4.

[30] Since the model excludes free-flow travel time it is not possible to judge the level of congestion by comparing congested and uncongested travel times.

independent because greater congestion discourages usage by just enough to offset the larger marginal external congestion cost of a trip at any given level of total usage.[31] The operator passes on only half the collection cost (absorbing the rest) because demand becomes more elastic as the price of a trip rises.

For a given market reach, $\tau_U^*$ is higher than the Pigouvian toll in Eq. (33) which assigns a weight over one half to the collection cost. As in Section 2.2, $\tau_U^*$ can be decomposed into a congestion charge and a markup. Substituting Eq. (37) into Eq. (36) gives

$$Q_U = \frac{1-k-\hat{x}_U/2}{2\left(1+e\hat{x}_U\right)}\hat{x}_U . \tag{38}$$

At this level of usage the Pigouvian toll would be $k+eQ_U$. Subtracting the Pigouvian toll from $\tau_U^*$ in Eq. (37) gives the markup as

$$Markup = \frac{1-k-\hat{x}_U/2}{2\left(1+e\hat{x}_U\right)} .$$

Given Eq. (38) the markup is positive whenever total usage is positive.

Substituting Eq. (37) into Eq. (35) gives profits as a function of the market reach:

$$\pi_U^* = \frac{1}{4}\left(1-k-\frac{\hat{x}_U}{2}\right)^2 \frac{\hat{x}_U}{1+e\hat{x}_U} . \tag{39}$$

Differentiating $\pi_U^*$ with respect to $\hat{x}_U$ one obtains

$$\hat{x}_U^* = Min\left(\overline{x}, \frac{Z_U - 3}{4e}\right), \tag{40}$$

where $Z_U \equiv \sqrt{9+16\left(1-k\right)e}$. If $\overline{x} \le \frac{Z_U-3}{4e}$ then Eq. (37) and Eq. (39) apply with $\hat{x}_U = \overline{x}$. If $\overline{x} > \frac{Z_U-3}{4e}$ then

$$\tau_U^* = \frac{1}{2}\left(1+k-\frac{Z_U-3}{8e}\right), \tag{41}$$

and

[31] The independence of $\tau_U^*$ from $e$ is specific to the linearity of the demand function.

$$\pi_U^* = \frac{1}{256e^3}\left(8(1-k)e+3-Z_U\right)^2 \frac{Z_U-3}{1+Z_U}. \tag{42}$$

If $e=0$ and $\bar{x}=1$, then $\tau_U^* = (2/3)k+1/3$, $\hat{x}_U^* = (2/3)(1-k)$ and $\pi_U^* = (2/27)(1-k)^3$. The operator passes on 2/3 of the cost of toll collection, and the markup is $(1-k)/3$. A final point to note is that usage pricing is profitable as long as $k<1$. This is because the operator can charge a toll of almost unity and still attract an individual located at $x=0$. By contrast, Figure 3 shows that tolling is socially unwarranted even with severe congestion if toll collection costs are very high. This accords with the general observation in the introduction that a private operator is willing to spend more resources on tolling than is a public operator.

### 3.5 Other profit-maximizing schemes

To economize on space, derivations of the remaining four profit-maximizing schemes are relegated to Appendix B.

### 3.6 Numerical comparison of pricing schemes

As a first step in the comparison it is instructive to consider the limiting case of no congestion $(e=0)$ and zero toll collection costs ($k=0$), and to vary the range of types, $\bar{x}$, parametrically.[32] Although this comparison does not address the role of pricing for congestion relief, it does illustrate the efficiency loss from suppressed trips due to tolls and access fees when they are used for profit maximization.

Profits for the five profit-maximizing schemes are plotted against $\bar{x}$ in Figure 4. For small $\bar{x}$ individuals are fairly homogeneous and, as shown by Oi (1971), the access fee is a relatively efficient tool for expropriating consumers' surplus without discouraging usage. The two-part tariff yields the highest profit although it has a minimal advantage over access pricing for $\bar{x} \le 0.2$. Beyond $\bar{x}=0.4$, scheme *TorU* is the most profitable. Several other features of Figure 4 are worth noting:

[32] This comparison extends Coyte and Lindsey (1988) who compare schemes *U*, *T*, and *TorU* in a model of spatial competition without congestion, where $\bar{x}$ is the distance between firms in a one-dimensional space.

- Neither usage-only pricing nor access-only pricing is ever the most profitable. This illustrates Proposition 2 in Section 2 and the general principle that a monopolist always benefits from using additional pricing instruments (Armstrong, 2006) — at least if transactions costs are unimportant.
- As indicated by the left-most vertical line in Figure 4, access-only pricing profits reach a maximum at $\bar{x} = 1/3$. Consequently, if $\bar{x} > 1/3$ the profit-maximizing market reach with access-only pricing remains at 1/3 and the operator excludes types $x > 1/3$ from usage.
- For the other four pricing schemes profits reach a maximum at: $\bar{x} = 2/5$ (scheme *T*), $\bar{x} = 4/7$ (scheme *TorU*), $\bar{x} = 3/5$ (scheme *AorU*) and $\bar{x} = 2/3$ (scheme *U*). By comparison, the socially optimal market reach is 1 and no one is excluded from consumption.
- Maximum profits from usage-only pricing and access-only pricing are equal at $\pi \cong 0.0741$.[33] Similarly, maximum profits from the two-part tariff and scheme AorU are equal at $\pi$ =0.08.

Figure 5 compares the schemes in terms of welfare. Except for access-only pricing with $\bar{x} \le 1/3$, welfare is reduced by pricing because free access is optimal if there is no congestion. For $\bar{x} > 1/3$, access-only pricing is welfare-reducing as well because it dissuades individuals located beyond $x = 1/3$ from usage. Indeed, access-only pricing is the least efficient scheme for $\bar{x} \ge 0.57$. For $\bar{x} < 0.57$, usage-only pricing is the least efficient.[34] Thus, one of the two pure pricing schemes always ranks last in terms of profits, and one of them also always ranks last in terms of welfare.

Suppose now that the highway is congestible: $e > 0$. To economize on space it is henceforth assumed that the range of types, $\bar{x}$, is sufficiently great that all pricing schemes result in exclusion of some individuals who would travel with free access.[35]

To begin, assume that toll collection costs are zero. Figure 6 displays profits with pricing schemes *A*, *AorU*, *T*, and *TorU* relative to profits from usage-only pricing as a function of the

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[33] It can be shown that profits from schemes *A* and *U* are also equal if the individual demand curve has the nonlinear functional form $q = (1-\tau-T-x)^{\gamma}$, $\gamma > 0$. Schemes *A* and *U* are also equally profitable if the density function of types has the triangular form $f(x) = 1+(2x-1)\lambda$, $\lambda \in [-1,1]$. But the equality of profits breaks down if the two generalizations are combined with $\gamma \ne 1$ and $\lambda \ne 0$. Thus, as pointed out in Section 2.2, usage-only pricing and access-only pricing are not equally profitable in general.

[34] The welfare loss from usage-only pricing is one half of profits for all values of $\bar{x}$.

[35] Since the market reach is a decreasing function of parameters *e* and *k* it suffices to assume $\bar{x} \ge 2/3$.

congestion-cost coefficient, $e$. As expected, access-only pricing is the least profitable scheme although the gap narrows for high values of $e$ because the highway becomes relatively unprofitable regardless of how it is priced. Also in line with expectations, scheme *AorU* is less profitable than the two-part tariff because individuals who choose access pricing in scheme *AorU* do not pay a toll and take too many trips. Scheme *AorU* also decreases in profitability relative to usage-only pricing as $e$ increases, and above $e = 2.5$ it is unprofitable to offer access pricing as an option. Consistent with Proposition 4, given $k = 0$ scheme *TorU* is more profitable than scheme *AorU* for all values of $e$. Indeed, *TorU* is the most profitable scheme although its advantage over schemes *AorU*, *T*, and *U* becomes minimal at high congestion levels.

Figures 7-9 reveal large differences between schemes in tolls, access fees and market reach. Scheme *TorU* has a large spread between the toll charged for the two-part tariff, $\tau_{TU}^{T}$, and the standalone toll, $\tau_{TU}^{U}$ (Figure 7). The toll for the standalone two-part tariff is intermediate in value. The access fee is highest in scheme *AorU* and much lower for the two-part tariffs in schemes *T* and *TorU* (Figure 8). Amongst the profit-maximizing schemes, usage-only pricing supports the largest market reach, and access pricing the smallest (Figure 9). Of the combined pricing schemes, the two-part tariff — which does not offer consumers a choice of payment — has a smaller market reach than *AorU* or *TorU*.

Figure 10 compares the efficiency of the schemes using the metric $w_i \equiv \left(W_i - W_E\right)/\left(W_O - W_E\right)$ where $i$ indexes schemes. Index $w_i$ takes a value between 0 and 1 if scheme $i$ increases social surplus relative to the free-access equilibrium, and a negative value if the scheme reduces social surplus. With $e = 0$, all schemes have a relative efficiency of $-\infty$ because they impose a welfare loss and $W_O = W_E$ . Access-only pricing performs very poorly relative to the other four schemes which are closely bunched. Usage-only pricing ranks first, and scheme *TorU* second. The welfare impact of the four schemes becomes positive with $e \cong 1$. With $e = 1$ the market range with free access is reduced by about 27 percent from its level of 1 with $e = 0$, and the number of trips is reduced by about 46 percent. This suggests that profit-maximization using any of the pricing schemes is welfare-improving only when congestion is fairly severe. Nevertheless, all schemes except access-only pricing asymptotically approach full efficiency as $e$ increases.

Figure 11 presents results parallel to Figure 6 with the toll collection cost set at *k*=0.05. Access-only pricing performs somewhat better than in Figure 6, and it is more profitable than usage-only pricing for $e \leq 1.5$. Scheme *AorU* also improves in relative performance because some users choose the access fee, and *AorU* is more profitable than scheme *TorU* for $e \leq 1.8$. If *k* is increased further to *k*=0.10, scheme *AorU* becomes the most profitable scheme over the whole range of *e*.[36]

Panel (a) of Figure 12 displays the profit-maximizing pricing scheme over a rectangular range of *k* and *e*. Consistent with Proposition 4, scheme *TorU* is the most profitable when toll collection costs are zero or relatively low. Scheme *AorU* is the most profitable in the rest of the space. Thus, the two schemes that offer a choice of payment dominate the three schemes that do not offer a choice.

Panel (b) of Figure 12 identifies the profit-maximizing pricing scheme that yields the highest welfare.[37] The map is similar to that in panel (a) except that with high toll collection costs access-only pricing is welfare superior to *AorU*. This is consistent with the demonstration in Figure 3 that usage-only pricing is not socially beneficial if toll collection costs are very high. The dark curve toward the bottom of panel (b) of Figure 12 divides the parameter space into two regions: a region below which the profit-maximizing strategy is welfare-reducing (lightly shaded for ease of reference), and a region above which it is welfare-improving. As expected, the region of welfare improvement encompasses high congestion levels and low toll collection costs.

Figure 12 demonstrates that the most profitable pricing scheme is not always the most socially beneficial. This is true not only of the five schemes collectively, but also of usage-only pricing and access-only pricing. Section 2.2 presented an example in which the private operator is biased in favor of access pricing over usage pricing because the access fee can be used to expropriate consumers' surplus. In the specific model used here the bias can go in the opposite direction. For example, with $k = 0.05$ usage-pricing is more profitable than access pricing for $e > 1.6$ whereas usage-pricing yields higher social surplus only for $e > 2$. The private operator is therefore biased in favor of usage pricing for $e \in (1.6, 2.0)$.

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[36] To economize on space the corresponding figure is not shown.

[37] This is not the pricing scheme that a welfare-maximizing public toll-road operator would choose, but rather the privately implemented tolling scheme that yields the highest welfare gain, or lowest welfare loss.

## 4 SPECIFIC MODEL WITH HETEROGENEOUS TRIP FREQUENCIES

Section 3 analyzed a specific version of the model in which users differ in their reservation prices for trips. This section briefly considers the case in which users have the same reservation prices but differ in their trip frequencies.[38] In place of Eq. (24) individual demand is assumed to have the form

$$q = (1-x)\,Max(1-\tau-T,0),$$

where $x \sim U\left[0,\bar{x}\right]$ and $\bar{x} \le 1$. Variable $x$ is now a type index for trip frequency with larger values of $x$ corresponding to lower frequency. An individual's $x$ depends on such characteristics as working full-time or part-time, frequency of travel for business, shopping and other purposes, etc.

*A priori*, one might expect the model specification with heterogeneous trip frequencies to behave quite differently from the specification with heterogeneous reservation prices. With heterogeneous reservation prices individuals located far away from the highway are deterred by congestion from using it even in the free-access equilibrium. As the generalized cost of usage rises, individuals drop out and the market reach decreases. With heterogeneous trip frequencies, by contrast, everyone uses the highway if there is no access fee. Furthermore, everyone has the same toll elasticity of demand because individual demand curves differ only in scale.

Despite these differences, the two specifications of heterogeneity turn out to yield rather similar results. Derivations for the no-toll equilibrium, the social optimum, and the five profit-maximizing schemes are described in Appendix C. Discussion is limited here to the main similarities and differences that arise with the two dimensions of heterogeneity.

Figures 13 and 14 present counterparts to Figures 4 and 5 for profits and welfare as a function of the range of types. The main difference is that market reaches are larger in Figure 13 than in Figure 4. With access-only pricing the market reach is 1/2 compared to 1/3, and with the

[38] This specification is analyzed using numerical examples in Wang et al. (2005) for schemes *U*, *A*, and *AorU*. In the case of commuting trips, heterogeneity in trip frequencies is arguably less important than heterogeneity in reservation prices since most workers commute five days a week and use the same mode each day.

two-part tariff it is 2/3 rather than 2/5. For the other three schemes the market reach is unity: no one is excluded.

Relative profits of the pricing schemes in Figure 13 are very similar to those in Figure 4, and the welfare losses in Figure 14 are very similar to those in Figure 5. Indeed, usage-only pricing and access-only pricing again yield the same maximum profits ($\pi = 0.125$), and similarly for the two-part tariff and scheme *AorU* ($\pi \cong 0.1481$).

As a final comparison, Figure 15 presents a counterpart to Figure 12.[39] As shown in panel (a), profits are again maximized with either scheme *TorU* or *AorU*. Consistent with Proposition 4, scheme *TorU* dominates scheme *AorU* when toll collection costs are low. The range in which *TorU* dominates *AorU* is larger than in Figure 12 because — for a given value of parameter $e$ — congestion is more severe with heterogeneous trip frequencies. The welfare-preferred choices shown in panel (b) of Figure 15 differ in two respects from Figure 12. First, access-only pricing is preferred only above a higher threshold of toll collection costs. Second, usage-only pricing is preferred to *TorU* when congestion is severe and toll collection costs are not too high.[40] Thus, both pure pricing schemes are welfare-preferred for certain parameter values whereas profits are always maximized with one of the composite schemes. The two-part tariff is neither profit-maximizing nor welfare-preferred for any parameter values.

## 5 CONCLUDING REMARKS

Nonlinear pricing is routinely used by private-sector firms to boost profits. It also allows public enterprises to meet budget constraints with less allocative efficiency loss than with uniform pricing schemes. Yet nonlinear pricing has been largely overlooked in the literatures on road pricing and private toll roads. The goal of this paper is to help fill the gap by studying the use of simple nonlinear pricing schemes on privately-operated, congestion-prone roads. Three questions are addressed: What combination of access fees and usage charges will a private operator

[39] As noted in Section 3, with heterogeneous reservation prices and $\bar{x} = 1$, market reach with free access is reduced to 1/3 with the congestion-cost coefficient set at $e = 12$. With heterogeneous trip frequencies the corresponding market reach of 1/3 obtains with $e = 4$. To obtain a more complete picture in Figure 15 the range is extended to $e = 5$ and the range of toll collection costs is extended to $k = 0.3$.

[40] The boundary between scheme *U* and scheme *TorU* is independent of parameter *k* because all users pay a toll in each scheme.

choose? How important are toll collection costs as a consideration in favour of access pricing? And how does the use of access fees for profit maximization affect welfare?

Four general results are derived regarding the first question. (1) Profits from usage-only pricing (scheme $U$) and access-only pricing (scheme $A$) cannot be ranked in general (Proposition 1). Access-only pricing may be more profitable if congestion is not too severe, and demand is not too elastic, so that substantial consumer's surplus can be expropriated with the access fee. (2) Regardless of the severity of congestion, an access fee is always profitable to implement as part of a two-part tariff (Proposition 2). (3) If access fees and tolls are offered as separate payment choices (scheme $AorU$), the combination can be less profitable than using either instrument in isolation (Proposition 3). (4) If toll collection costs are zero it is always more profitable to offer a choice between a two-part tariff and usage pricing (scheme $TorU$) than a choice between a pure access fee and usage pricing (scheme $AorU$) (Proposition 4). Numerical comparisons using a linear version of the model suggest that regardless of toll collection costs the most profitable strategy is either scheme $TorU$ or scheme $AorU$.

Regarding the second question, toll collection costs strongly favour access pricing. If toll collection costs amount to 10 percent or more of maximum willingness to pay for a trip, scheme $AorU$ is more profitable than scheme $TorU$ because $AorU$ avoids collection costs for consumers who choose the access fee.

With respect to the third question it turns out that use of access fees for profit maximization can increase or decrease welfare relative to usage-only pricing. Usage-only pricing for profit maximization can perform poorly against alternative schemes — particularly if the highway is not heavily congested or if individuals are relatively homogeneous. This suggests that bans on access fees could be socially harmful.

This paper provides only a first pass at studying nonlinear pricing on private toll roads. Toll collection costs are considered only insofar as usage pricing is more costly per trip than access pricing. Fixed toll collection costs are disregarded, as are costs that depend on the number of parameters that define a scheme (e.g., two tolls and one access charge for scheme $TorU$). Alternative specifications of individual demand curves and the distribution of consumer types warrant investigation. Additional pricing schemes such as block tariffs and three-part tariffs also deserve attention.

Alternative assumptions about how users choose between pricing schemes are another avenue for further research. Following the standard rationality paradigm it has been assumed that users make utility-maximizing choices. Yet it is not uncommon for consumers to choose dominated alternatives (e.g., Amromin et al., 2007) — whether out of ignorance, due to inertia, or because of difficulties in making comparisons. According to economic theory, a two-part tariff is equivalent to a particular type of quantity discount. But people may view the fixed fee of a two-part tariff as a loss, and the subsequent saving in tolls as a gain, and weigh the loss more heavily than the gain. Ho and Zhang (2008) provide experimental evidence of this framing effect in the context of a manufacture-retailer channel. As far as toll roads this suggests that drivers may be more favorably disposed to a two-part tariff if it can be presented as a quantity discount.

Another potentially important influence on consumer choice is demand uncertainty. Individuals may be uncertain about future gasoline prices, employer attitudes towards telecommuting, and other factors that influence how much they are likely to use a toll road. This may dissuade them from paying a non-refundable access fee out of fear that they will not recoup the cost in toll savings.

## 6 NOTATIONAL GLOSSARY

| | |
|---|---|
| $A$ | access fee |
| $\mathcal{A}$ | access-pricing scheme |
| $AorU$ | pricing scheme with choice between access fee and usage charge |
| $e$ | congestion cost parameter |
| $E$ | free-access scheme |
| $F(x)$ | cumulative distribution of types |
| $f(x)$ | density function of types |
| $k$ | toll collection cost per trip |
| $\bar{N}$ | potential number of users |
| $O$ | system-optimum scheme |
| $p$ | monetary cost of a trip |
| $\bar{p}$ | choke price on demand |
| $q$ | individual number of trips |
| $Q$ | total number of trips |
| $S$ | consumer's surplus |
| $T$ | travel time |
| $\mathcal{T}$ | two-part tariff pricing scheme |
| $TorU$ | pricing scheme with choice between two-part tariff and usage charge |
| $U$ | usage-only-pricing scheme |
| $w_i$ | relative efficiency of scheme *i* in terms of welfare |
| $W$ | social surplus |
| $x$ | index or "type" of individual |
| $\tilde{x}$ | type that receives zero consumer's surplus |
| $\hat{x}$ | market reach |
| $\bar{x}$ | highest type in population |
| $y$ | type indifferent between paying access fee and usage charge |

*Greek characters*

| | |
|---|---|
| $\pi$ | profits |
| $\tau$ | toll per use |

# 7 APPENDIXES

## 7.1 Appendix A: Profit-maximization in the general model

### 7.1.1 Two-part tariff: Proof of Proposition 2

Differentiating Eq. (13) and Eq. (14) in the text with respect to $\tau$ one obtains

$$-q\left(\tau,T,\hat{x}\right)+\int_{p=\tau}^{\bar{p}(T,\hat{x})}\left(\frac{\partial q}{\partial T}\frac{dT}{dQ}\frac{dQ}{d\tau}+\frac{\partial q}{\partial x}\frac{d\hat{x}}{d\tau}\right)dp=0\,, \tag{A1}$$

$$\frac{dQ}{d\tau}=\int_{x=0}^{\hat{x}}\left(\frac{\partial q}{\partial \tau}+\frac{\partial q}{\partial T}\frac{dT}{dQ}\frac{dQ}{d\tau}\right)f\left(x\right)dx+q\left(\tau,T,\hat{x}\right)f\left(\hat{x}\right)\frac{d\hat{x}}{d\tau}\,. \tag{A2}$$

To economize on writing define $I_T\equiv\frac{dT}{dQ}\int_{x=0}^{\hat{x}}\frac{\partial q\left(\tau,T,x\right)}{\partial T}f\left(x\right)dx$, $I_\tau\equiv\int_{x=0}^{\hat{x}}\frac{\partial q\left(\tau,T,x\right)}{\partial \tau}f\left(x\right)dx$,

$J_T\equiv\frac{dT}{dQ}\int_{p=\tau}^{\bar{p}(T,\hat{x})}\frac{\partial q\left(\tau,T,\hat{x}\right)}{\partial T}dp$ and $J_x\equiv\int_{p=\tau}^{\bar{p}(T,\hat{x})}\frac{\partial q\left(\tau,T,\hat{x}\right)}{\partial x}dp$. (All four expressions are negative.)

Eq. (A1) and Eq. (A2) can then be written compactly in matrix form:

$$\begin{bmatrix} J_T & J_x \\ 1-I_T & -q\left(\tau,T,\hat{x}\right)f\left(\hat{x}\right)\end{bmatrix}\begin{bmatrix}\dfrac{dQ}{d\tau}\\ \dfrac{d\hat{x}}{d\tau}\end{bmatrix}=\begin{bmatrix}q\left(\tau,T,\hat{x}\right)\\ I_\tau\end{bmatrix}.$$

Hence

$$\frac{dQ}{d\tau}=\frac{1}{\Delta}\left(-q^2\left(\tau,T,\hat{x}\right)f\left(\hat{x}\right)-I_\tau J_x\right)<0\,, \tag{A3}$$

$$\frac{d\hat{x}}{d\tau}=\frac{1}{\Delta}\left(I_\tau J_T-q\left(\tau,T,\hat{x}\right)\left(1-I_T\right)\right), \tag{A4}$$

where $\Delta\equiv-q\left(\tau,T,\hat{x}\right)f\left(\hat{x}\right)J_T-J_x\left(1-I_T\right)>0$. Substituting Eq. (A3) and Eq. (A4) into Eq. (6) yields

$$\frac{\partial \pi_T}{\partial \tau}=\int_{x=0}^{\hat{x}}\left(q\left(\tau,T,x\right)+\left(\tau-k\right)\frac{\partial q}{\partial \tau}\right)f\left(x\right)dx+A\frac{f\left(\hat{x}\right)}{\Delta}Y$$

$$+\frac{\tau-k}{\Delta}\left[I_T\left(-q^2\left(\tau,T,\hat{x}\right)f\left(\hat{x}\right)-I_\tau J_x\right)+q\left(\tau,T,\hat{x}\right)f\left(\hat{x}\right)Y\right], \tag{A5}$$

where $Y \equiv -q(\tau, T, \hat{x})(1 - I_T) + I_\tau J_T$. Differentiating Eq. (13) and Eq. (14) in the text with respect to $A$:

$$\frac{dT}{dQ}\int_{p=\tau}^{\bar{p}(T,\hat{x})}\frac{\partial q}{\partial T}dp \bullet \frac{dQ}{dA} + \int_{p=\tau}^{\bar{p}(T,\hat{x})}\frac{\partial q}{\partial x}dp \bullet \frac{d\hat{x}}{dA} = 1,$$

$$\frac{dQ}{dA} = \int_{x=0}^{\hat{x}}\frac{\partial q}{\partial T}f(x)dx\frac{dT}{dQ}\frac{dQ}{dA} + q(\tau, T, \hat{x})f(\hat{x})\frac{d\hat{x}}{dA},$$

or in matrix form

$$\begin{bmatrix} J_T & J_x \\ 1 - I_T & -q(\tau, T, \hat{x})f(\hat{x}) \end{bmatrix}\begin{bmatrix} \dfrac{dQ}{dA} \\ \dfrac{d\hat{x}}{dA} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Hence

$$\frac{dQ}{dA} = -\frac{1}{\Delta}q(\tau, T, \hat{x})f(\hat{x}) < 0, \tag{A6}$$

$$\frac{d\hat{x}}{dA} = -\frac{1}{\Delta}(1 - I_T) < 0. \tag{A7}$$

Substituting Eq. (A6) and Eq. (A7) into Eq. (17) in the text yields

$$\frac{\partial \pi_T}{\partial A} = F(\hat{x}) + \frac{f(\hat{x})}{\Delta}\left(-A(1 - I_T) - (\tau - k)q(\tau, T, \hat{x})\right). \tag{A8}$$

If $A = 0$, Eq. (13) in the text implies $q(\tau, T, \hat{x}) = 0$, and from Eq. (A8) $\partial\pi_T/\partial A = F(\hat{x}) > 0$ which is inconsistent with profit maximization. Hence $A_T^* > 0$ as asserted in Proposition 2. Substituting Eq. (A8) with $\partial\pi_T/\partial A = 0$ into Eq. (A5), and rearranging terms, one obtains an equation for $\tau - k$:

$$\frac{\partial \pi_T}{\partial \tau} = \underbrace{\int_{x=0}^{\hat{x}} q(\tau, T, x)f(x)dx - q(\tau, T, \hat{x})F(\hat{x})}_{(1)} + \underbrace{\frac{I_\tau J_T}{1 - I_T}F(\hat{x})}_{(2)} +$$

$$\left(I_\tau - \frac{f(\hat{x})}{(1 - I_T)\Delta}q(\tau, T, \hat{x})Y + \frac{I_T\left(q^2(\tau, T, \hat{x})f(\hat{x})I_\tau J_x\right) + q(\tau, T, \hat{x})f(\hat{x})Y}{\Delta}\right)(\tau - k) = 0. \tag{A9}$$

Eq. (A9) is of the form $Z_1 + Z_2(\tau - k) = 0$. Since both numbered terms in $Z_1$ are positive, $Z_1 > 0$. Multiplying $Z_2$ through by $\Delta > 0$ one has

$$Z_2 \overset{s}{=} \Delta I_\tau + \left(1 - \frac{1}{1-I_T}\right) q(\tau, T, \hat{x})\, f(\hat{x}) Y + I_T \left(q^2(\tau, T, \hat{x})\, f(\hat{x}) + I_\tau J_x\right)$$

$$= -\left(q(\tau, T, \hat{x})\, f(\hat{x}) J_T + J_x (1 - I_T)\right) I_\tau - \frac{I_T}{1-I_T} q(\tau, T, \hat{x})\, f(\hat{x}) \left(I_\tau J_T - q(\tau, T, \hat{x})(1 - I_T)\right)$$

$$= \underbrace{-q(\tau, T, \hat{x})\, f(\hat{x}) I_\tau J_T \left(1 + \frac{I_T}{1-I_T}\right)}_{(3)} \underbrace{- J_x (1 - I_T) I_\tau}_{(4)} \underbrace{+ q^2(\tau, T, \hat{x})\, f(\hat{x}) I_T}_{(5)} . \qquad \text{(A10)}$$

Since all three numbered terms in Eq. (A10) are negative, $Z_2 < 0$. Hence $\tau - k = -Z_1 / Z_2 > 0$ as per Proposition 2.

### 7.1.2 Access or usage pricing: Proof of Proposition 3

Differentiate Eq. (19) and Eq. (20) in the text with respect to $\tau$ to obtain

$$\int_{p=0}^{\tau} \frac{\partial q}{\partial y} dp \bullet \frac{dy}{d\tau} + \int_{p=0}^{\tau} \frac{\partial q}{\partial T} dp \bullet \frac{dT}{dQ} \frac{dQ}{d\tau} + q(\tau, T, y) = 0 , \qquad \text{(A11)}$$

$$\frac{dQ}{d\tau} = \int_{x=0}^{\hat{x}} \frac{\partial q}{\partial T} f(x) dx \bullet \frac{dT}{dQ} \frac{dQ}{d\tau} + \int_{x=y}^{\hat{x}} \frac{\partial q}{\partial \tau} f(x) dx + \left(q(0, T, y) - q(\tau, T, y)\right) f(y) \frac{dy}{d\tau} . \qquad \text{(A12)}$$

Define $I_T \equiv \frac{dT}{dQ} \int_{x=0}^{\hat{x}} \frac{\partial q(\tau, T, x)}{\partial T} f(x) dx$, $I_\tau \equiv \int_{x=y}^{\hat{x}} \frac{\partial q(\tau, T, x)}{\partial \tau} f(x) dx$,

$J_T \equiv \frac{dT}{dQ} \int_{p=0}^{\tau} \frac{\partial q(\tau, T, y)}{\partial T} dp$ and $J_y \equiv \int_{p=0}^{\tau} \frac{\partial q(\tau, T, y)}{\partial y} dp$. (All four expressions are negative.)

Eq. (A11) and Eq. (A12) can then be written:

$$\begin{bmatrix} J_T & J_y \\ 1 - I_T & -\left(q(0, T, y) - q(\tau, T, y)\right) f(y) \end{bmatrix} \begin{bmatrix} \frac{dQ}{d\tau} \\ \frac{dy}{d\tau} \end{bmatrix} = \begin{bmatrix} -q(\tau, T, y) \\ I_\tau \end{bmatrix} .$$

Hence

$$\frac{dQ}{d\tau} = \frac{1}{\Delta} \left(-I_\tau J_y + q(\tau, T, y)\left[q(0, T, y) - q(\tau, T, y)\right] f(y)\right) < 0 ,$$

$$\frac{dy}{d\tau} = \frac{1}{\Delta} \left((1 - I_T) q(\tau, T, y) + I_\tau J_T\right) > 0 ,$$

where $\Delta \equiv -\left(q(0, T, y) - q(\tau, T, y)\right) f(y) J_T - J_y (1 - I_T) > 0$.

Differentiating Eq. (19) and Eq. (20) in the text with respect to *A*:

$$\int_{p=0}^{\tau} \frac{\partial q}{\partial y} dp \bullet \frac{dy}{dA} + \int_{p=0}^{\tau} \frac{\partial q}{\partial T} dp \bullet \frac{dT}{dQ}\frac{dQ}{dA} = 1,$$

$$\frac{dQ}{dA} = \int_{x=0}^{y} \frac{\partial q}{\partial T} f(x) dx \bullet \frac{dT}{dQ}\frac{dQ}{dA} + \int_{x=y}^{\hat{x}} \frac{\partial q}{\partial T} f(x) dx \bullet \frac{dT}{dQ}\frac{dQ}{dA} + \left(q(0,T,y) - q(\tau,T,y)\right) f(y)\frac{dy}{dA},$$

or in matrix form

$$\begin{bmatrix} J_T & J_y \\ 1 - I_T & -\left(q(0,T,y) - q(\tau,T,y)\right) f(y) \end{bmatrix} \begin{bmatrix} \dfrac{dQ}{dA} \\ \dfrac{dy}{dA} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Hence

$$\frac{dQ}{dA} = -\frac{1}{\Delta}\left(q(0,T,y) - q(\tau,T,y)\right) f(y) < 0, \tag{A13}$$

$$\frac{dy}{dA} = -\frac{1}{\Delta}\left(1 - I_T\right) < 0. \tag{A14}$$

Substituting Eq. (A13) and Eq. (A14) into Eq. (22) in the text yields

$$\frac{\partial \pi_{AU}}{\partial A} = F(y) + (\tau - k)\frac{f(y)\left(I_T\left(q(0,T,y) - q(\tau,T,y)\right) + \left(1 - I_T\right)\left(A - (\tau - k)q(\tau,T,y)\right)\right)}{\Delta}. \tag{A15}$$

Suppose the access fee is set at the level $A = \int_{p=0}^{\tau} q(p,T,0)dp$ so that all customers choose usage pricing and the consumer at $y = 0$ is indifferent between paying *A* and paying the toll. Multiplying Eq. (A15) through by $\Delta > 0$ one obtains

$$-\frac{\partial \pi_{AU}}{\partial A} \overset{s}{=} \underbrace{\left(\int_{p=0}^{\tau} q(p,T,0)dp - (\tau - k)q(\tau,T,0)\right)\left(1 - I_T\right) f(0)}_{(a)}$$

$$+ \underbrace{(\tau - k) I_T\left(q(0,T,y) - q(\tau,T,y)\right) f(0)}_{(b)}. \tag{A16}$$

Term (a) in (A16) is the profit that would be gained by marginally reducing A and inducing consumers at $y = 0$ to switch from usage pricing to access pricing. Term (b) is the reduction in toll revenue from all other customers due to the increase in congestion caused by increased usage of customers at $y = 0$. (Note that $I_T$ is proportional to $dT/dQ$.) If congestion is severe enough it is unprofitable to attract any users to access pricing.

### 7.1.3 Choice between Scheme *AorU* and Scheme *TorU*: Proof of Proposition 4

For brevity let AU denote scheme *AorU* and TU denote scheme *TorU*. The proof proceeds as follows. Let $(A_{AU}, \tau_{AU})$ be any *AorU* scheme, and let *y* be the user who is indifferent between paying access fee $A_{AU}$ and paying usage charge $\tau_{AU}$. Consider an alternative *TorU* scheme with two-part tariff $(A_{TU}, \tau_{TU}^{T})$, and usage charge $\tau_{TU}^{U} > 0$. Set $\tau_{TU}^{U} = \tau_{AU}$ and assume $(A_{TU}, \tau_{TU}^{T})$ is chosen so that user *y* is indifferent between paying the two part tariff and paying user charge $\tau_{TU}^{U}$. The same set of high-demand types who choose access-pricing with scheme *AorU* will choose the two-part tariff with scheme *TorU*. Since $\tau_{TU}^{U} > 0$ the high-demand types will take fewer trips with *TorU* and impose less congestion. Therefore, since $\tau_{TU}^{U} = \tau_{AU}$ the low-demand types will take more trips with *TorU* and generate more toll revenue. Some types with lower-yet demands that did not travel with *AorU* may also start to travel and generate additional revenue. Nevertheless, total trips by all types must be lower with *TorU* than with *AorU*. The proof is completed by showing that the two-part tariff option of *TorU* can be chosen not only to make type *y* indifferent between the two-part tariff and the usage charge, but also to generate more revenue from high-demand types than the access fee of *AorU*.

To prove this last step let $T_{AU}$ denote $T(Q_{AU})$ and $T_{TU}$ denote $T(Q_{TU})$. With scheme *AorU* user type *y* is indifferent between access fee $A_{AU}$ and usage charge $\tau_{AU}$ so that

$$\int_{p=0}^{\bar{p}(T_{AU},y)} q(p, T_{AU}, y)dp - A_{AU} = \int_{p=\tau_{AU}}^{\bar{p}(T_{AU},y)} q(p, T_{AU}, y)dp . \tag{A17}$$

With scheme *TorU* user type *y* is indifferent between two-part tariff $(A_{TU}, \tau_{TU}^{T})$ and usage charge $\tau_{TU}^{U} = \tau_{AU}$ so that

$$\int_{p=\tau_{TU}^{T}}^{\bar{p}(T_{TU},y)} q(p, T_{TU}, y)dp - A_{TU} = \int_{p=\tau_{AU}}^{\bar{p}(T_{TU},y)} q(p, T_{TU}, y)dp . \tag{A18}$$

Given (A17) and (A18)

$$A_{AU} = \int_{p=0}^{\tau_{AU}} q(p, T_{AU}, y)dp ,$$

$$A_{TU} = \int_{p=\tau_{TU}^{T}}^{\tau_{AU}} q(p, T_{TU}, y)dp ,$$

and hence

$$A_{TU} - A_{AU} = \int_{p=\tau_{TU}^{T}}^{\tau_{AU}} q\left(p, T_{TU}, y\right)dp - \int_{p=0}^{\tau_{AU}} q\left(p, T_{AU}, y\right)dp .$$

Individual demand is strictly decreasing with travel time, $T_{TU} < T_{AU}$ , and $\tau_{TU}^{U} = \tau_{AU}$ . Therefore

$$A_{TU} - A_{AU} > -\int_{p=0}^{\tau_{TU}^{T}} q\left(p, T_{TU}, y\right)dp . \tag{A19}$$

Now profits for the two schemes are

$$\pi_{AU} = A_{AU} F\left(y\right) + \tau_{AU} \int_{x=y}^{\hat{x}_{AU}} q\left(\tau_{AU}, T_{AU}, x\right) f\left(x\right)dx ,$$

$$\pi_{TU} = A_{TU} F\left(y\right) + \tau_{TU}^{T} \int_{x=0}^{y} q\left(\tau_{TU}^{T}, T_{TU}, x\right) f\left(x\right)dx + \tau_{AU} \int_{x=y}^{\hat{x}_{TU}} q\left(\tau_{AU}, T_{TU}, x\right) f\left(x\right)dx$$

As argue above, usage by low-demand users is greater with *TorU* than *AorU*:

$$\int_{x=y}^{\hat{x}_{TU}} q\left(\tau_{AU}, T_{TU}, x\right) f\left(x\right)dx > \int_{x=y}^{\hat{x}_{AU}} q\left(\tau_{AU}, T_{AU}, x\right) f\left(x\right)dx .$$

Therefore

$$\pi_{TU} - \pi_{AU} > \left(A_{TU} - A_{AU}\right) F\left(y\right) + \tau_{TU}^{T} \int_{x=0}^{y} q\left(\tau_{TU}^{T}, T_{TU}, x\right) f\left(x\right)dx .$$

Given inequality (A19),

$$\pi_{TU} - \pi_{AU} > -\int_{p=0}^{\tau_{TU}^{T}} q\left(p, T_{TU}, y\right)dp \bullet F\left(y\right) + \tau_{TU}^{T} \int_{x=0}^{y} q\left(\tau_{TU}^{T}, T_{TU}, x\right) f\left(x\right)dx$$

$$\stackrel{s}{=} \underbrace{\frac{1}{F\left(y\right)} \int_{x=0}^{y} q\left(\tau_{TU}^{T}, T_{TU}, x\right) f\left(x\right)dx}_{(a)} - \underbrace{\frac{1}{\tau_{TU}^{T}} \int_{p=0}^{\tau_{TU}^{T}} q\left(p, T_{TU}, y\right)dp}_{(b)} . \tag{A20}$$

Term (a) in Eq. (A20) is mean usage for high-demand individuals given toll $\tau_{TU}^{T}$ and travel time $T_{TU}$ . Term (b) is mean usage for type *y* when the toll ranges between zero and $\tau_{TU}^{T}$ . If $\tau_{TU}^{T}$ is sufficiently small, term (a) exceeds term (b). This completes the proof.

The intuition for the last step of the proof is the same as in Oi (1971): with heterogeneous users it is always more profitable to levy a positive usage charge in combination with an access fee than to charge an access fee alone.

### *7.2* Appendix B: Equilibria in the specific model with heterogeneous reservation prices

Equilibria for the free access, social optimum and usage-only pricing schemes are discussed in the text. The remaining four tolling schemes are described here. These schemes differ in the extent to which analytical solutions can be derived and the complexity of the formulas. The summaries differ accordingly.

*Access-only pricing (Scheme A)*

The access fee is chosen to extract all consumer's surplus from a user located at $\hat{x}$:

$$A_A = \frac{(1-eQ-\hat{x})^2}{2}.$$

Profits are

$$\pi_A = A_A\hat{x} = \frac{(1-eQ-\hat{x})^2}{2}\hat{x}. \tag{B1}$$

Aggregate usage is given by Eq. (36) in the text with $\tau_U = 0$:

$$Q = \frac{\hat{x}-\hat{x}^2/2}{1+e\hat{x}}. \tag{B2}$$

Substitution of Eq. (B2) into Eq. (B1) gives

$$\pi_A = \frac{1}{2}\left(\frac{1-\hat{x}-e\hat{x}^2/2}{1+e\hat{x}}\right)^2\hat{x}.$$

If $\hat{x} \le \overline{x}$, $\hat{x}$ is given by the cubic equation $1-3\hat{x}-e\hat{x}\left(9+(7/2)\hat{x}+(3/2)\hat{x}^2\right)=0$. If $e>0$, then $\hat{x}_A^* < 1/3$. If $e=0$, $\hat{x}_A^* = 1/3$, $A_A^* = 2/9$ and $\pi_A^* = 2/27$.

*Choice between access fee and usage charge (Scheme AorU)*

Let *y* be the location of a user who is indifferent between the access fee and the usage charge. *y* is defined by the condition

$$\frac{(1-eQ-y)^2}{2} - A_{AU} = \frac{(1-eQ-\tau_{AU}-y)^2}{2}. \tag{B3}$$

Users $x \in [0,y)$ prefer the access fee, and users $x \in (y,\hat{x}]$ prefer the usage charge. Profits are therefore

$$\begin{aligned}\pi_{AU} &= \int_{x=0}^{y} A_{AU}\,dx + \int_{x=y}^{\hat{x}} (\tau_{AU}-k)(1-eQ-\tau_{AU}-x)\,dx \\ &= A_{AU}y + (\tau_{AU}-k)\left(1-eQ-\tau_{AU}-\frac{y+\hat{x}}{2}\right)(\hat{x}-y).\end{aligned} \tag{B4}$$

Solving for *Q* and substituting into Eq. (B4) gives

$$\pi_{AU} = A_{AU} y + \frac{\tau_{AU} - k}{1 + e\hat{x}}\left(1 - \tau_{AU} - \frac{y + \hat{x}}{2} - e\left(\tau_{AU} + \frac{\hat{x}}{2}\right) y\right)(\hat{x} - y).$$

The optimal access fee is derived from the first-order condition

$$\frac{\partial \pi_{AU}}{\partial A_{AU}} + \frac{\partial \pi_{AU}}{\partial y}\frac{\partial y}{\partial A_{AU}} = 0,$$

where the derivative $\partial y / \partial A_{AU}$ is obtained from Eq. (B3). The resulting formula can be rearranged to obtain a formula for *y*:

$$y^*_{AU} = \frac{\tau^2_{AU} + 2k(1 - \tau_{AU}) + e\left(-\tau^2_{AU} + k(2\tau_{AU} + \hat{x})\right)\hat{x}}{2(\tau_{AU} + k) + 2e\left(\tau_{AU}(2k + \hat{x}) + k\hat{x}\right)}.$$

The first-order conditions for $\tau_{AU}$ and $\hat{x}^*_{AU}$ are complicated and require numerical solution. Define $Z_{AU} \equiv \sqrt{1 + 16k + 24k^2}$. If $e = 0$ and $\hat{x}^*_{AU} < \overline{x}$, the solution is $\tau^*_{AU} = \frac{1 + Z_{AU} - 2k}{5}$,

$A^*_{AU} = \frac{(1 - 2k + Z_{AU})\left(6 - 3k + 66k^2 + (6 + 13k) Z_{AU}\right)}{50(1 + 3k + Z_{AU})}$, $y^*_{AU} = \frac{26k + 24k^2 + (1 - 7k) Z_{AU}}{5(1 + 3k + Z_{AU})}$,

$\hat{x}^*_{AU} = \frac{4 + 2k - Z_{AU}}{5}$.

*Two-part tariff (Scheme T)*

Profits in this scheme are

$$\pi_T = \int_{x=0}^{\hat{x}} \left(A_T + (\tau_T - k)(1 - eQ - \tau_T - x)\right) dx = A_T \hat{x} + (\tau_T - k)\left(1 - eQ - \tau_T - \frac{\hat{x}}{2}\right)\hat{x}. \qquad \text{(B5)}$$

Aggregate usage is

$$Q = \frac{(1 - \tau_T - \hat{x}/2)\hat{x}}{1 + e\hat{x}}. \qquad \text{(B6)}$$

The user located at $\hat{x}$ receives zero consumer's surplus so that

$$A_T = \frac{(1 - eQ - \tau_T - \hat{x})^2}{2} = \frac{1}{2}\left(\frac{1 - \tau_T - (1 + e\hat{x}/2)\hat{x}}{1 + e\hat{x}}\right)^2. \qquad \text{(B7)}$$

Substituting Eq. (B6) and Eq. (B7) into Eq. (B5) and solving the first-order condition $\partial \pi_T / \partial \tau_T = 0$ yields

$$\tau^*_T = k + \frac{1 + 2e(1 - k)}{1 + 2e\hat{x}}\frac{\hat{x}}{2}. \qquad \text{(B8)}$$

Substituting Eq. (B8) into Eq. (B7) gives $A_T^* = \frac{1}{2}\left(\frac{1-k-(3/2+e\hat{x})\hat{x}}{1+2e\hat{x}}\right)^2$. The market reach works out to $x_T^* = Min\left(\bar{x}, \frac{\sqrt{25+48(1-k)e}-5}{12e}\right)$. If $e=0$ and $\bar{x} > \frac{\sqrt{25+48(1-k)e}-5}{12e}$, the solution simplifies to $\tau_T^* = \frac{1+4k}{5}$, $A_T^* = \frac{2}{25}(1-k)^2$, $x_T^* = \frac{2}{5}(1-k)$.

*Choice between two-part tariff and usage charge (Scheme TorU)*

Let *y* be the location of a user who is indifferent between the two-part tariff fee and the usage charge. *y* is defined by the condition

$$\frac{\left(1-eQ-\tau_{TU}^T-y\right)^2}{2} - A_{TU} = \frac{\left(1-eQ-\tau_{TU}^U-y\right)^2}{2}.$$

Users $x \in [0, y)$ prefer the two-part tariff, and users $x \in (y, \hat{x}]$ prefer the usage charge. Profits are therefore

$$\begin{aligned}\pi_{TU} &= \int_{x=0}^{y}\left(A_{TU} + \left(\tau_{TU}^T - k\right)\left(1-eQ-\tau_{TU}^T - x\right)\right)dx + \int_{x=y}^{\hat{x}}\left(\tau_{TU}^U - k\right)\left(1-eQ-\tau_{TU}^U - x\right)dx \\ &= A_{TU}y + \left(\tau_{TU}^T - k\right)\left(1-eQ-\tau_{TU}^T - \frac{y}{2}\right)y + \left(\tau_{TU}^U - k\right)\left(1-eQ-\tau_{TU}^U - \frac{y+\hat{x}}{2}\right)(\hat{x}-y). \qquad \text{(B9)}\end{aligned}$$

Solving for *Q*, substituting the formula into Eq. (B9), deriving the first-order condition $\partial\pi_{TU}/\partial A_{TU} = 0$, and rearranging the resulting expression gives a formula for *y*:

$$y_{TU}^* = \frac{\tau_{TU}^T(1+e\hat{x}) + \tau_{TU}^U(1-e\hat{x})}{2(1+e\hat{x})}.$$

The first-order conditions for $\tau_{TU}^T$, $\tau_{TU}^U$ and $\hat{x}_{TU}$ are complicated. But if $x_{TU}^* < \bar{x}$, they can be solved to yield $\tau_{TU}^{T*} = \frac{(43+96e(1-k)-Z_{TU})(Z_{TU}-7)}{192e(2+Z_{TU})}$, $\tau_{TU}^{U*} = \frac{(35+96e(1-k)-5Z_{TU})(5+Z_{TU})}{192e(2+Z_{TU})}$, $y_{TU}^* = \frac{\hat{x}_{TU}^*}{2}$ and $\hat{x}_{TU}^* = \frac{Z_{TU}-7}{12e}$ where $Z_{TU} \equiv \sqrt{49+96e(1-k)}$. (The formula for $A_{TU}^*$ and $\pi_{TU}^*$ are long.) If $e=0$ and $\hat{x}_{TU}^* < \bar{x}$, the solution is $\tau_{TU}^{T*} = \frac{1+6k}{7}$, $\tau_{TU}^{U*} = \frac{3+4k}{7}$, $A_{TU}^* = \frac{6}{49}(1-k)^2$, $y_{TU}^* = \frac{2}{7}(1-k)$, $\hat{x}_{TU}^* = \frac{4}{7}(1-k)$ and $\pi_{TU}^* = \frac{4}{49}(1-k)^3$.

### 7.3 Appendix C: Equilibria in the specific model with heterogeneoues trip frequencies

*Free access (Scheme E)*

In the absence of tolls all individuals with $x<1$ use the highway. Social surplus equals total consumers' surplus:

$$W_E = \int_{x=0}^{\bar{x}} \frac{(1-eQ)^2}{2}(1-x)\,dx = \frac{1}{2}(1-eQ)^2\,\bar{x}(1-\bar{x}/2) = \frac{1}{2}(1-eQ)^2\,G(\bar{x}), \tag{C1}$$

where $G(x) \equiv x(1-x/2)$. Total usage is

$$Q = \int_{x=0}^{\bar{x}} (1-eQ)(1-x)\,dx = (1-eQ)G(\bar{x}) = \frac{G(\bar{x})}{1+eG(\bar{x})}. \tag{C2}$$

Substituting (C2) into (C1) yields $W_E = \dfrac{G(\bar{x})}{2\left(1+eG(\bar{x})\right)^2}$. If $\bar{x}=1$, then $W_E = 1/(2+e)^2$. If $e=0$ as well, then $W_E = 1/4$.

*The social optimum (Scheme O)*

If usage is rationed only by a toll, then as with free access all individuals with $x<1$ use the highway. Social surplus is

$$W_O = \int_{x=0}^{\bar{x}} \left( \frac{(1-eQ-\tau_O)^2}{2} + (\tau_O - k)(1-eQ-\tau_O) \right)(1-x)dx .$$

$$= \frac{1-eQ-\tau_O}{2}(1-eQ+\tau_O)G(\bar{x}). \tag{C3}$$

Total usage is

$$Q = \int_{x=0}^{\bar{x}} (1-eQ-\tau_O)(1-x)dx = \frac{(1-\tau_O)G(\bar{x})}{1+eG(\bar{x})}. \tag{C4}$$

Substituting Eq. (C4) into Eq. (C3) gives

$$W_O = \frac{(1-\tau_O)\left(1-k+\tau_O\left(1+2eG(\bar{x})\right)\right)}{2\left(1+eG(\bar{x})\right)^2}. \tag{C5}$$

The first-order condition $\partial W_O / \partial \tau_O = 0$ yields

$$\tau_O^* = \frac{1+eG(\bar{x})}{1+2eG(\bar{x})}k + \frac{eG(\bar{x})}{1+2eG(\bar{x})}. \tag{C6}$$

Similar to the case with heterogeneous reservation prices, the optimal toll is a weighted average of the toll collection cost and the average reservation price on demand (which is 1). Using Eq. (C6), Eq. (C5) simplifies to $W_O = \frac{G(\bar{x})}{2(1+eG(\bar{x}))}(1-k)^2$. If $\bar{x}=1$, then $\tau_O^* = \frac{2+e}{2(1+e)}k + \frac{e}{2(1+e)}$ and $W_O = \frac{(1-k)^2}{4(1+e)}$.

*Usage-only pricing (Scheme U)*

Profits are

$$\pi_U = \int_{x=0}^{\bar{x}} (\tau_U - k)(1-eQ-\tau_U)(1-x)dx = (\tau_U - k)(1-eQ-\tau_U)G(\bar{x}), \tag{C7}$$

and total usage is

$$Q = \int_{x=0}^{\bar{x}} (1-eQ-\tau_U)(1-x)\,dx = \frac{(1-\tau_U)G(\bar{x})}{1+eG(\bar{x})}. \tag{C8}$$

Substituting Eq. (C8) into Eq. (C7), and solving the first-order condition $\partial\pi_U / \partial\tau_U = 0$, yields

$$\tau_U^* = \frac{1}{2}k + \frac{1}{2}. \tag{C9}$$

Again as with heterogeneous reservation prices the profit-maximizing toll is an unweighted average of the toll collection cost and the reservation price on demand. Substituting Eq. (C8) and Eq. (C9) into Eq. (C7) gives $\pi_U^* = \frac{(1-k)^2}{4}\frac{G(\bar{x})}{1+eG(\bar{x})}$. If $\bar{x}=1$, then $\pi_U^* = \frac{(1-k)^2}{4(2+e)}$. If $e=0$ as well, then $\pi_U^* = \frac{(1-k)^2}{8}$.

*Access-only pricing (Scheme A)*

The access fee extracts all consumer's surplus from the user at $\hat{x}$:

$$A_A = \frac{(1-eQ)^2}{2}(1-\hat{x}).$$

Profits are

$$\pi_A = A_A\hat{x} = \frac{(1-eQ)^2}{2}\hat{x}(1-\hat{x}). \tag{C10}$$

Total usage is

$$Q = (1 - eQ) G(\hat{x}) = \frac{G(\hat{x})}{1 + eG(\hat{x})}. \tag{C11}$$

Substitution of Eq. (C11) into Eq. (C10) gives $\pi_A = A_A \hat{x} = \frac{1}{2}\left(\frac{1}{1+eG(\hat{x})}\right)^2 \hat{x}(1-\hat{x})$. $\hat{x}_A^*$ is defined implicitly by the cubic equation $1 - 2\hat{x} - e\hat{x}\left(1 - (3/2)\hat{x} + \hat{x}^2\right) = 0$. If $e > 0$, then $A_A^* = 1/4$, $\hat{x}_A^* = 1/2$ and $\pi_A^* = 1/8$.

*Choice between access fee and usage charge (Scheme AorU)*

Profits are

$$\begin{aligned} \pi_{AU} &= \int_{x=0}^{y} A_{AU}\, dx + \int_{x=y}^{\hat{x}} (\tau_{AU} - k)(1 - eQ - \tau_{AU})\, dx \\ &= A_{AU} y + (\tau_{AU} - k)(1 - eQ - \tau_{AU})\left(G(\hat{x}) - G(y)\right). \end{aligned} \tag{C12}$$

Solving for $Q$ and substituting into Eq. (C12), gives

$$\pi_{AU} = A_{AU} y + \frac{\tau_{AU} - k}{1 + eG(\hat{x})}\left(1 - \tau_{AU}\left(1 + eG(y)\right)\right)\left(G(\hat{x}) - G(y)\right).$$

The user at $y$ is indifferent between the access fee and the usage charge:

$$\frac{(1-eQ)^2}{2}(1-y) - A_{AU} = \frac{(1 - eQ - \tau_{AU})^2}{2}(1-y). \tag{C13}$$

Using the first-order condition $\frac{\partial \pi_{AU}}{\partial A_{AU}} + \frac{\partial \pi_{AU}}{\partial y}\frac{\partial y}{\partial A_{AU}} = 0$, and Eq. (C13), yields a cubic equation for $y_A^*$. The first-order condition for $\tau_{AU}$ requires numerical solution. Define $Z_1 \equiv \sqrt{1 + 6k - 3k^2}$, $Z_2 \equiv 3 - 6k + 3k^2$ and $Z_3 \equiv 2 + 12k - 6k^2$. If $e = 0$ and $\bar{x} = 1$, the solution is

$$\tau_{AU}^* = \frac{1+k}{2} + (1-k)\frac{Z_1}{Z_2},$$

$$A_{AU}^* = \frac{1}{2}\left(1 - \frac{4Z_1 - Z_3}{2Z_2}\right)\left(\frac{1+k}{2} + (1-k)\frac{4Z_1 - Z_3}{4Z_2}\right)\left(\frac{3-k}{2} - (1-k)\frac{4Z_1 - Z_3}{2Z_2}\right),$$

$$y_{AU}^* = \frac{4Z_1 - Z_3}{2Z_2}.$$

The formula for $\pi_{AU}^*$ is cumbersome.

*Two-part tariff (Scheme T)*

Profits are

$$\pi_T = \int_{x=0}^{\hat{x}} \left(A_T + \left(\tau_T - k\right)\left(1 - eQ - \tau_T\right)\right)\left(1 - x\right) dx = A_T \hat{x} + \left(\tau_T - k\right)\left(1 - eQ - \tau_T\right) G\left(\hat{x}\right). \qquad \text{(C14)}$$

Total usage is

$$Q = \frac{\left(1 - \tau_T\right) G\left(\hat{x}\right)}{1 + eG\left(\hat{x}\right)}. \qquad \text{(C15)}$$

The user at $\hat{x}$ has zero consumer's surplus so that

$$A_T = \frac{\left(1 - eQ - \tau_T\right)^2}{2}\left(1 - \hat{x}\right) = \frac{1 - \hat{x}}{2}\left(\frac{1 - \tau_T}{1 + eG\left(\hat{x}\right)}\right)^2. \qquad \text{(C16)}$$

Substituting Eq. (C15) and Eq. (C16) into Eq. (C14) gives

$$\pi_T = \frac{1 - \tau_T}{1 + eG\left(\hat{x}\right)} \frac{\hat{x}}{2}\left(\left(1 - \hat{x}\right)\frac{1 - \tau_T}{1 + eG\left(\hat{x}\right)} + \left(\tau_T - k\right)\left(2 - \hat{x}\right)\right).$$

Solving the first-order condition $\partial \pi_T / \partial \tau_T = 0$ yields

$$\tau_T^* = k + \frac{\left(1 - k\right)\left(\hat{x} + e\left(2 - \hat{x}\right) G\left(\hat{x}\right)\right)}{2\left(1 + e\left(2 - \hat{x}\right) G\left(\hat{x}\right)\right)}. \qquad \text{(C17)}$$

Substituting Eq. (C17) into Eq. (C16) gives $A_T^* = \frac{1 - \hat{x}}{8}\left(\frac{\left(1 - k\right)\left(2 - \hat{x}\right)}{1 + e\left(2 - \hat{x}\right) G\left(\hat{x}\right)}\right)^2$. The profit-maximizing market reach is $\hat{x}_T^* = Min\left(\overline{x}, 2/3\right)$. If $\overline{x} \geq 2/3$ then $\hat{x}_T^* = 2/3$,

$\tau_T^* = k + \frac{1 - k}{2}\frac{18 + 16e}{27 + 16e}$, $A_T^* = \frac{2}{27}\frac{\left(1 - k\right)^2}{\left(1 + \left(16/27\right)e\right)^2}$ and $\pi_T^* = \frac{4}{27}\frac{\left(1 - k\right)^2}{1 + \left(16/27\right)e}$. If $e = 0$, the solution simplifies to $\tau_T^* = \left(1 + 2k\right)/3$, $A_T^* = \left(2/27\right)\left(1 - k\right)^2$ and $\pi_T^* = \left(4/27\right)\left(1 - k\right)^2$.

*Choice between two-part tariff and usage charge (Scheme TorU)*

Profits are

$$\begin{aligned}\pi_{TU} &= \int_{x=0}^{y} \left(A_{TU} + \left(\tau_{TU}^T - k\right)\left(1 - eQ - \tau_{TU}^T\right)\right)\left(1 - x\right) dx + \int_{x=y}^{\hat{x}} \left(\tau_{TU}^U - k\right)\left(1 - eQ - \tau_{TU}^U\right)\left(1 - x\right) dx \\ &= A_{TU} y + \left(\tau_{TU}^T - k\right)\left(1 - eQ - \tau_{TU}^T\right) G\left(y\right) + \left(\tau_{TU}^U - k\right)\left(1 - eQ - \tau_{TU}^U\right)\left(G\left(\overline{z}\right) - G\left(y\right)\right).\end{aligned}$$

User $y$ is indifferent between the two-part tariff and the pure usage charge:

$$\frac{\left(1-eQ-\tau_{TU}^{T}\right)^2}{2}\left(1-y\right)-A_{TU}=\frac{\left(1-eQ-\tau_{TU}^{U}\right)^2}{2}\left(1-y\right).$$

If $\overline{x}<1$, the solution must be solved numerically. If $\overline{x}=1$, then $\tau_{TU}^{T*}=k+\frac{4+5e}{16+10e}\left(1-k\right)$, $\tau_{TU}^{U*}=k+\frac{12+5e}{16+10e}\left(1-k\right)$, $A_{TU}^{*}=\frac{8}{\left(8+5e\right)^2}\left(1-k\right)^2$, $y_{TU}^{*}=1/2$, $\pi_{TU}^{*}=\frac{5}{32+20e}\left(1-k\right)^2$. If $e=0$, the solution simplifies to $\tau_{TU}^{T*}=\left(1+3k\right)/4$, $\tau_{TU}^{U*}=\left(3+k\right)/4$, $A_{TU}^{*}=\left(1-k\right)^2/8$, $y_{TU}^{*}=1/2$, $\pi_{TU}^{*}=\left(5/32\right)\left(1-k\right)^2$.

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## 10 REFERENCES

91 Express Lanes. 2007. Annual Report (http://www.91expresslanes.com/Generalinfo/91annualreport.Pdf [August 2008]).

91 Express Lanes. 2009. Annual Report (http://www.91expresslanes.com/Generalinfo/91annualreport.Pdf [January 2010]).

Altmann, J., Chu, K., 2001. How to charge for network services - flat-rate or usage-based? *Computer Networks-The International Journal of Computer and Telecommunications Networking* 36 (5-6), 519-531.

Amromin, G., Jankowski, C., Porter, R.D., 2007. Transforming payment choices by doubling fees on the Illinois Tollway. *Economic Perspectives* 2Q, 22-47.

Armstrong, M., Doyle, C., Vickers J., 1996. The access pricing problem: A Synthesis. *The Journal of Industrial Economics* 44, 131-150.

Armstrong, M., 2006. Recent developments in the economics of price discrimination. In R. Blundell, W.K. Newey and T. Persson (eds), *Advances in Economics and Econometrics: Theory and Applications*, New York: Cambridge University Press, 96-141.

Arnott, R., de Palma, A., Lindsey, R., 1998. Recent developments in the bottleneck model. In K.J. Button and E.T. Verhoef (eds.), *Road Pricing, Traffic Congestion and the Environment: Issues of Efficiency and Social Feasibility*, Aldershot: Edward Elgar, 79-110.

Bagh, A., Bhargava, H., 2008. Doing more with less: Price discrimination with three-part vs two-part tariffs. ( SSRN-id1134387.pdf [August 2008]).

Bonsall, P, Shires, J., Maule, J., Matthews, B., Beale, J. (2007), “Responses to complex pricing signals: Theory, evidence and implications for road pricing”, *Transportation Research Part A* 41**(7),** 672-683.

Borella, M.S., Upadhyay, V., Sidhu, I., 1999. Pricing framework for a differential services Internet. *European Transactions on Telecommunications* 10 (3), 275-288.

Colle, M., 2006. Review of customer service: Highway 407 ETR. (www.mto.gov.on.ca/english/pubs/colle407/colle407.pdf [September 2007]).

Coyte, P.C., Lindsey, R., 1988. Spatial monopoly and spatial monopolistic competition with two-part pricing. *Economica* 55, 461-477.

De Borger, B., 2001. Discrete choice models and optimal two-part tariffs in the presence of externalities: Optimal taxation of cars. *Regional Science and Urban Economics* 31, 471-504.

Domon, K., Ota, K., 2001. Access pricing and market structure. *Information Economics & Policy* 13(1), 77-93.

Edelson, N. M., 1971. Congestion tolls under monopoly. *The American Economic Review* 61(5), 873-882.

Essegaier, S., Gupta, S., Zhang, Z. J., 2002. Pricing access services. *Marketing Science* 21(2), 139-159.

Foster, C., 1974. The regressiveness of road pricing. *International Journal of Transport Economics* 1, 133-141.

Glazer, A., Niskanen, E., 2000. Which consumers benefit from congestion tolls? *Journal of Transport Economics and Policy* 34(1), 43-54.

Gómez-Ibáñez, J., Small, K.A., 1998. Road pricing for congestion management: The transition from theory to policy. In K.J. Button and E.T. Verhoef (eds.), *Road Pricing, Traffic Congestion and the Environment: Issues of Efficiency and Social Feasibility*, Aldershot: Edward Elgar, 213-246.

Ho, T-H., Zhang, J., 2008. Designing pricing contracts for boundedly rational customers: Does the framing of the fixed fee matter? *Management Science* 54(4), 686-700.

Laffont, J.J., Tirole, J., 1994. Access pricing and competition. *European Economic Review* 38(9), 1673-1710.

Lambrecht, A., Seim, K., Skiera, B., 2007. Does uncertainty matter? Consumer behaviour under three-part tariffs. *Marketing Science* 26(5), 698-710.

Leeson, P.T., Sobel, R., 2008. Costly price discrimination. *Economics Letters* 99(1), 206-208.

Levinson, D., Odlyzko, A., 2008. Too expensive to meter: The influence of transaction costs in transportation and communication. ", *Philosophical Transactions of the Royal Society A* 366(1872), 2033-2046.

Lindsey, R., 2008. Prospects for urban road pricing in Canada. In G. Burtless and J. Rothenberg Pack (eds.), *Brookings Wharton Papers on Urban Affairs: 2008*, 235-293.

Mason, R., 2000. Simple competitive Internet pricing. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 358 (1773), 2309-2318.

Masuda, Y., Whang, S., 2006. On the optimality of fixed-up-to tariff for telecommunication service. *Information Systems Research* 17(3), 247-253.

Nahata, B., Ostaszewski, K., Sahoo, P., 1999. Buffet pricing. *Journal of Business* 72(2), 215-228.

Ng, Y-K., Weisser, M., 1974. Optimal pricing with a budget constraint - The case of the two-part tariff. *Review of Economic Studies* 41(3), 337-345.

Oi, W., 1971. A Disney dilemma: Two-part tariffs for a Mickey Mouse monopoly. *Quarterly Journal of Economics* 85, 77-90.

Rotarisa, L., Danielisa, R., Marcucci, E., Massianic, J., 2009. The urban road pricing scheme to curb pollution in Milan: A preliminary assessment. Working Paper 122, Università degli Studi di Trieste, Dipartimento di Scienze Economiche e Statistiche (http://www2.units.it/~nirdses/sito_inglese/working%20papers/files%20for%20wp/wp122.pdf, [December 2009]).

Roughgarden, T., Tardos, E., 2002. How bad is selfish routing? *Journal of the ACM* 49(2), 236-259.

Salas, M., Robuste, F., Sauri, S. 2009. Impact of two-part pricing scheme on social welfare for congested metropolitan expressway networks. 88th Annual Meeting of the Transportation Research Board, Washington, D.C. Conference CD Paper No. 09-2158.

Scotchmer, S., 1985a. Profit maximizing clubs. *Journal of Public Economics* 27, 25-45.

Scotchmer, S., 1985b. Two-tier pricing of shared facilities in a free-entry equilibrium. *Rand Journal of Economics* 16(4), 456-472.

Small, K.A., Verhoef, E.T. 2007. *The Economics of Urban Transportation*. London: Routledge.

Sundararajan, A., 2004. Non-linear pricing of information goods. *Management Science* 50(12), 1660-1673.

Tretvik, T., 2003. Urban road pricing in Norway: Public acceptability and travel behaviour. In J. Schade and B. Schlag (eds.), *Acceptability of transport pricing strategies*, Amsterdam: Elsevier, 77-92.

Varian, H.R., 1989. Price discrimination. In R. Schmalensee and R.D. Willig (eds.), *Handbook of Industrial Organization* I, Elsevier, 597-654.

Wang, J.Y.T., Yang, H., Lindsey, R., 2005. Two-part congestion pricing on public and private highways", working paper.

Wilson, R., 1993. *Nonlinear pricing*, New York: Oxford University Press.

Winter, R.A., 1997. Colluding on relative prices. *Rand Journal of Economics* 28(2), 359-371.

**Table 1: Examples of nonlinear pricing of roads**

| | |
|---|---|
| **Access fees** | |
| [a] Singapore Area License (1975-1998) | $1.50-$2.50/day to enter restricted area. |
| [b] Oslo and Bergen toll rings | 1-month, 6-month and annual passes;<br>also prepaid fixed numbers of crossings. |
| [c] European vignette system | Passes of varying duration to use expressways. |
| [d] Milan EcoPass Scheme | Annual passes for residents. Cost depends on engine type. |
| [a] Interstate 15, San Diego, Express Lanes. Phase I (Dec 1996 - Mar 1998) | Limited number of monthly permits sold for $50 allowing SOVs to use HOV lanes. |
| [e] Interstate 15, Utah, HOV lanes | $50/month for SOVs to use HOV lanes. |
| [f] Maine Turnpike Commuter Plan | Quarterly commuter passes for unlimited toll-free travel. Ratio of pass fee to single-trip toll varies by entry/exit pair and direction. |
| **Two-part tariffs** | |
| [g] State Route 91, Convenience Plan | One-time $75 fee per transponder with no monthly minimum toll outlay. |
| [g] State Route 91, Express Club | $20/transponder/month for $1.00 discount per trip. |
| **Three-part tariffs** | |
| [g] State Route 91, Standard Plan | Minimum $7 toll outlay/month. |
| [g] State Route 91, Discount Programs | One week of driving toll-free. |
| **Toll caps** | |
| [b] Trondheim toll ring (1991-2005) | Caps of one payment/hr and 75 payments/month. |
| [h] Stockholm charge | Cap of 60 SEK/day. |
| **Quantity discounts** | |
| [i] London congestion charge | 20 days (monthly) for price of 17 days (15% discount).<br>252 days (annual) for price of 212 days (16% discount). |
| [c] M5, Hungary | 40% reductions for regular users. |
| [d] Milan EcoPass Scheme | 50% discount for first 50 entries, 40% discount for next 50 entries. |
| [j] Dulles Greenway, VIP Miles Frequent Rider Program | Rebate cheques issued based on annual mileage: 5% (2,800-6,999), 7.5% (7,000-10 499), 10% (10 500-13 999), 12.5% (14 000-17 499), 15% (17 500+). |
| [k] Delaware SR-1 Frequent User Plan | 50% discount on cash toll for 30+ trips using same transponder during rolling 30-day period. |
| [l] Ambassador Bridge (US-Canada border). Commuter cards.<br>Single crossing: $3.25 | $90 for 30 crossings: 8% discount. 3 month expiration.<br>Eligibility: trips for school, work, medical care. |
| [m] Ogdensburg-Prescott Bridge (US-Canada border). Commuter cards.<br>Single crossing: $2.75 | $37 for 20 crossings: 33% discount. (6 mon. expir.).<br>$100 for 60 crossings: 39% discount (6 mon. expir.).<br>$90 for 75 crossings: 56% discount (2 mon. expir.) |
| [m] Ogdensburg-Prescott Bridge (US-Canada border). Truck charge rebates | $20 000-$29 999: 5% discount<br>$30 000-$39 999: 8% discount |

| | |
|---|---|
| based on annual toll expenditures | $40 000+: 10% discount |
| [n] Highway 407 ETR , Toronto | Frequent-user discounts. Outcome of settlement agreement with Ontario government in 2006. |

All figures in US dollars unless otherwise indicated.

Sources:
[a] Gómez-Ibáñez and Small (1998, p.215)
[b] Tretvik (2003)
[c] http://www.worldbank.org/transport/roads/toll_rds.htm
[d] Rotarisa et al. (2009).
[e] http://www.udot.utah.gov/expresslanes/index.php
[f] https://ezpassmaineturnpike.com/EZPass
[g] http://www.91expresslanes.com
[h] http://en.wikipedia.org/wiki/Stockholm_congestion_tax
[i] http://www.tfl.gov.uk/assets/downloads/pay-congestion-charge-form.pdf
[j] http://dullesgreenway.com/overview-of-the-vip-program.html
[k] http://www.deldot.gov/public.ejs?command=PublicTollRateGrid
[l] http://www.ambassadorbridge.com/an_overview_express.html#COMMUTER%20CARDS
[m] http://www.ogdensport.com/bridge.html
[n] Colle (2006)

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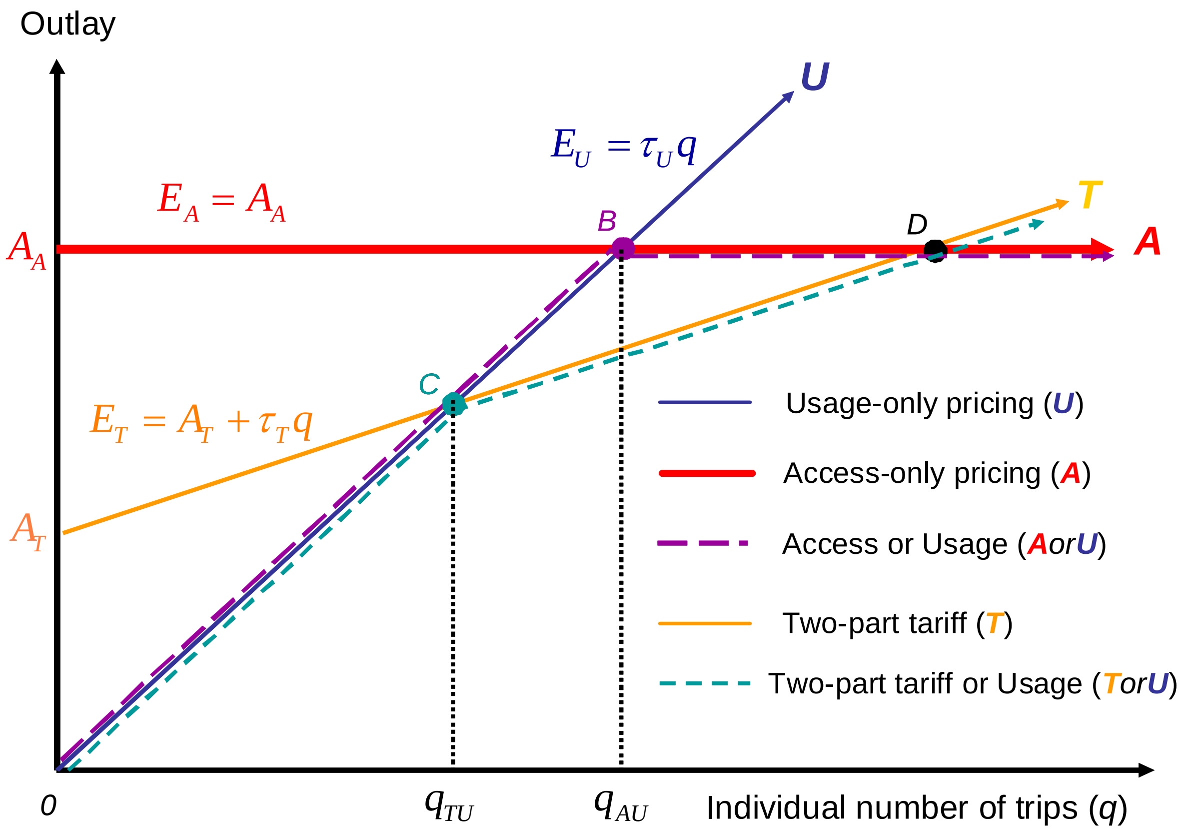



**Figure 1: Outlay curves for five pricing schemes**

Source: Authors' construction

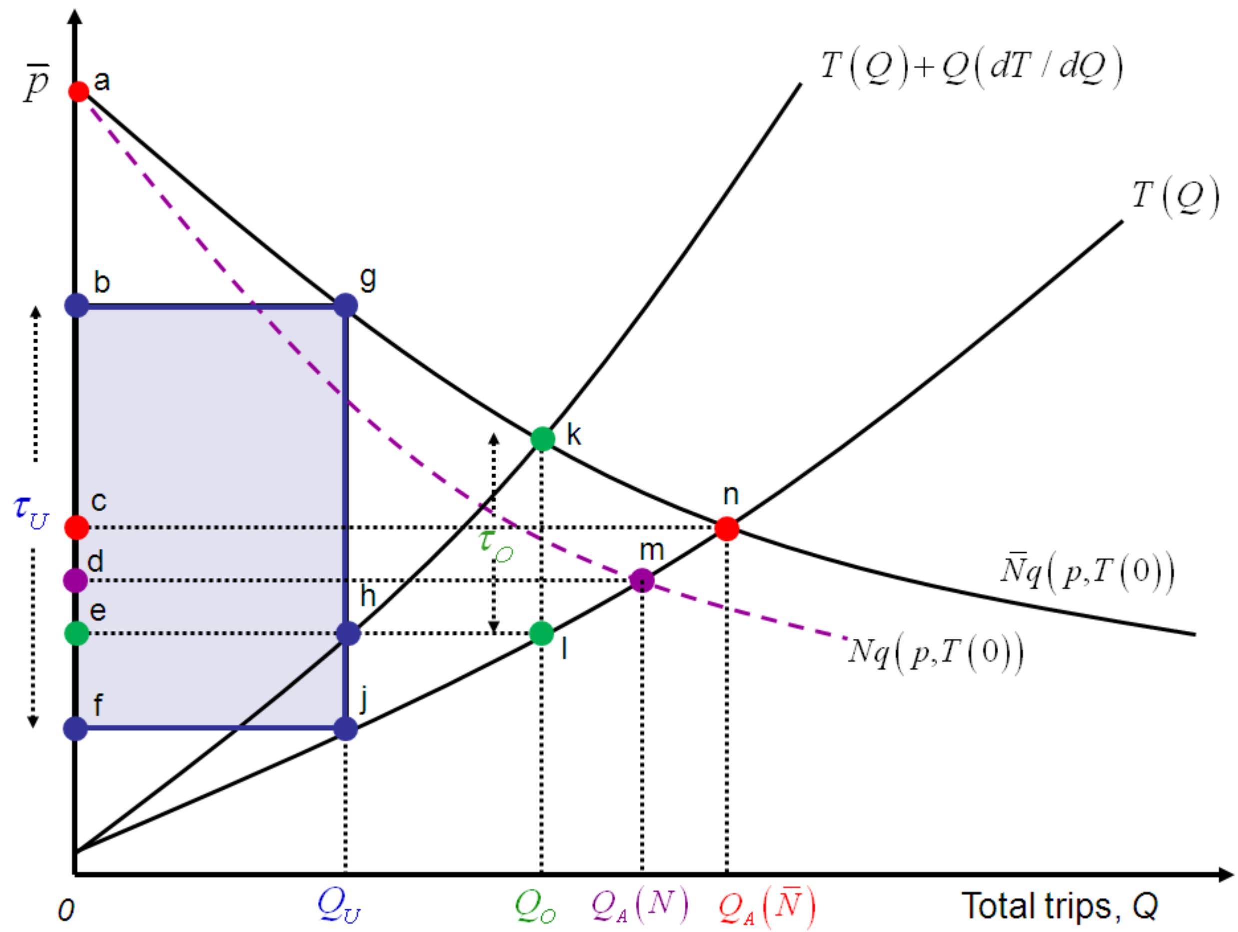



**Figure 2: Comparison of profits and welfare for usage-only pricing and access-only pricing**

Source: Authors' construction

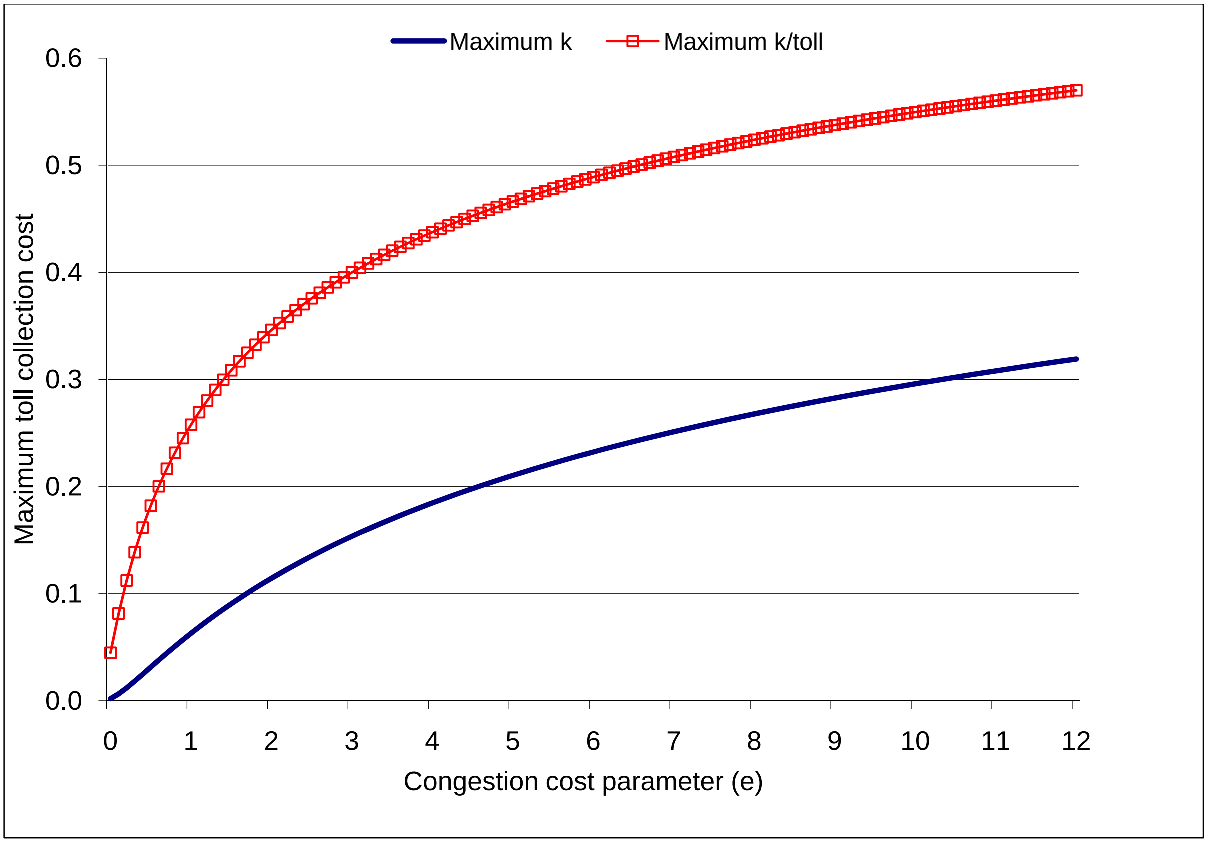



**Figure 3: Maximum toll collection cost for which Pigouvian toll is welfare-improving (heterogeneous reservation prices)**

Source: Authors' calculation

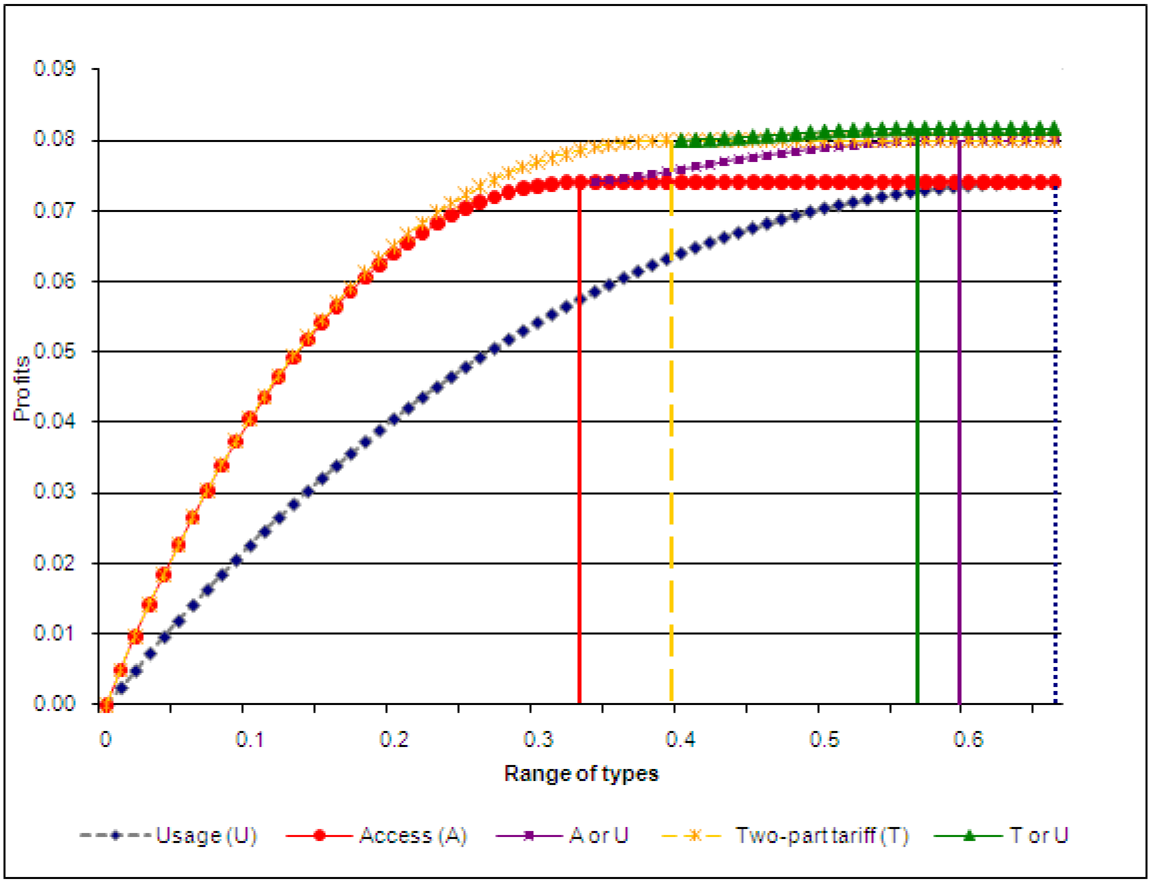



**Figure 4: Profits with heterogeneous reservation prices (*e*=0, *k*=0)**

Source: Authors' calculation

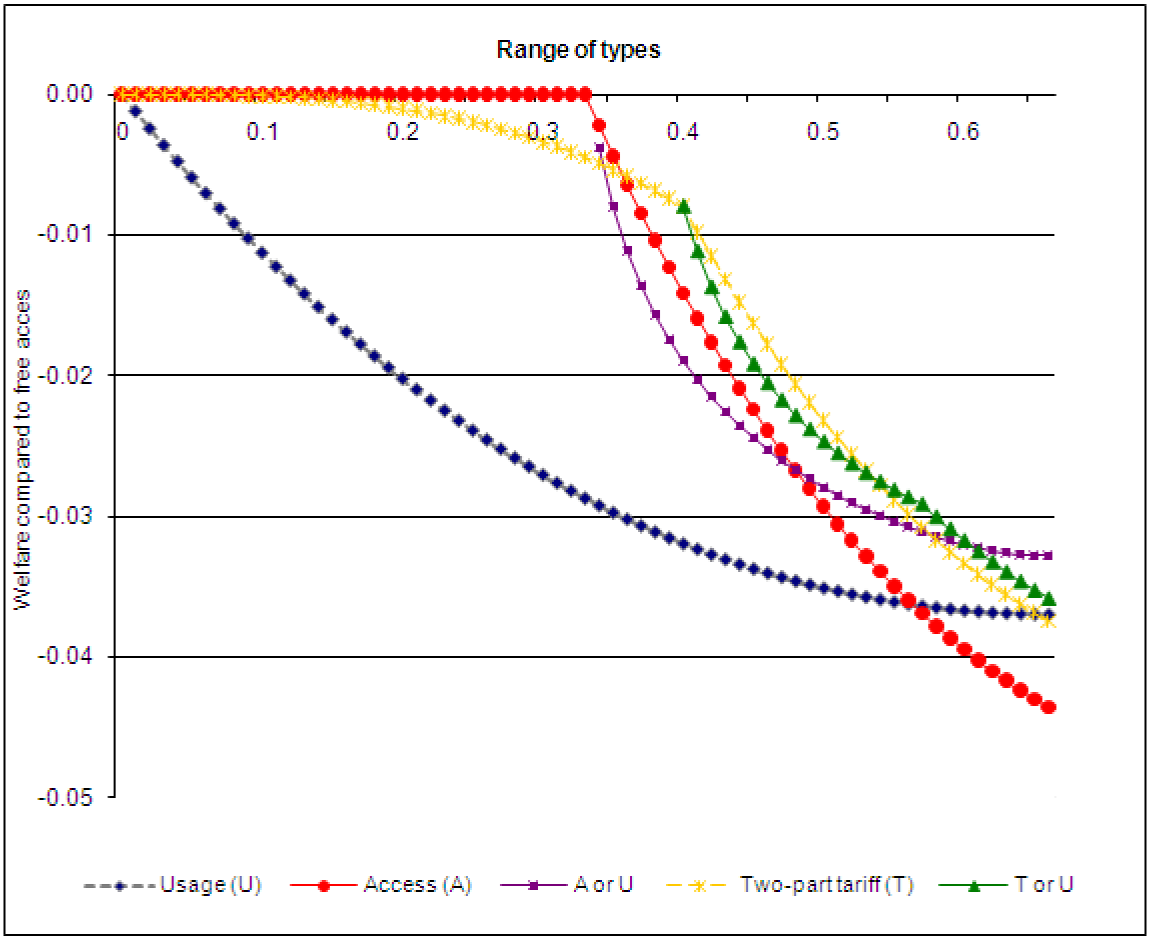



**Figure 5: Welfare compared to free access with heterogeneous reservation prices (*e*=0, *k*=0)**

Source: Authors' calculation

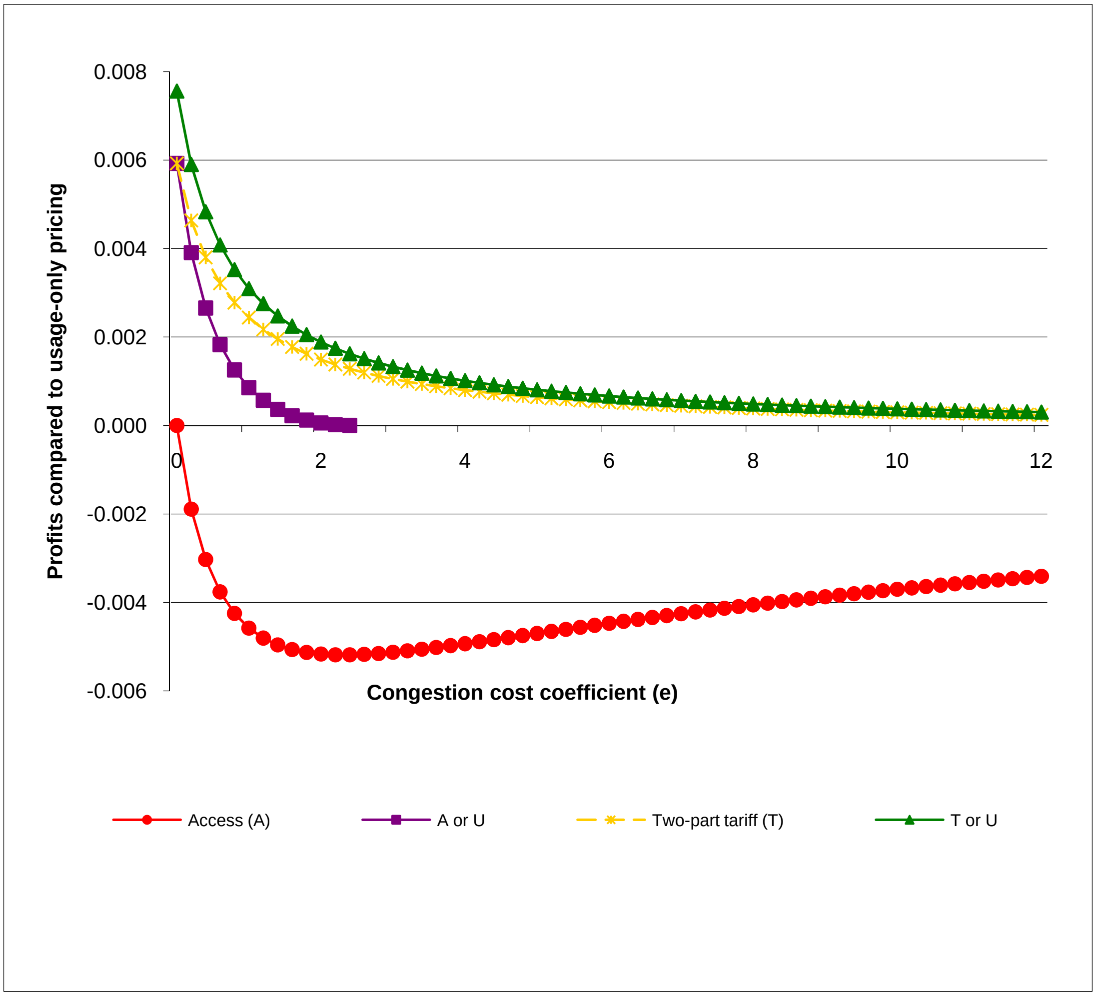
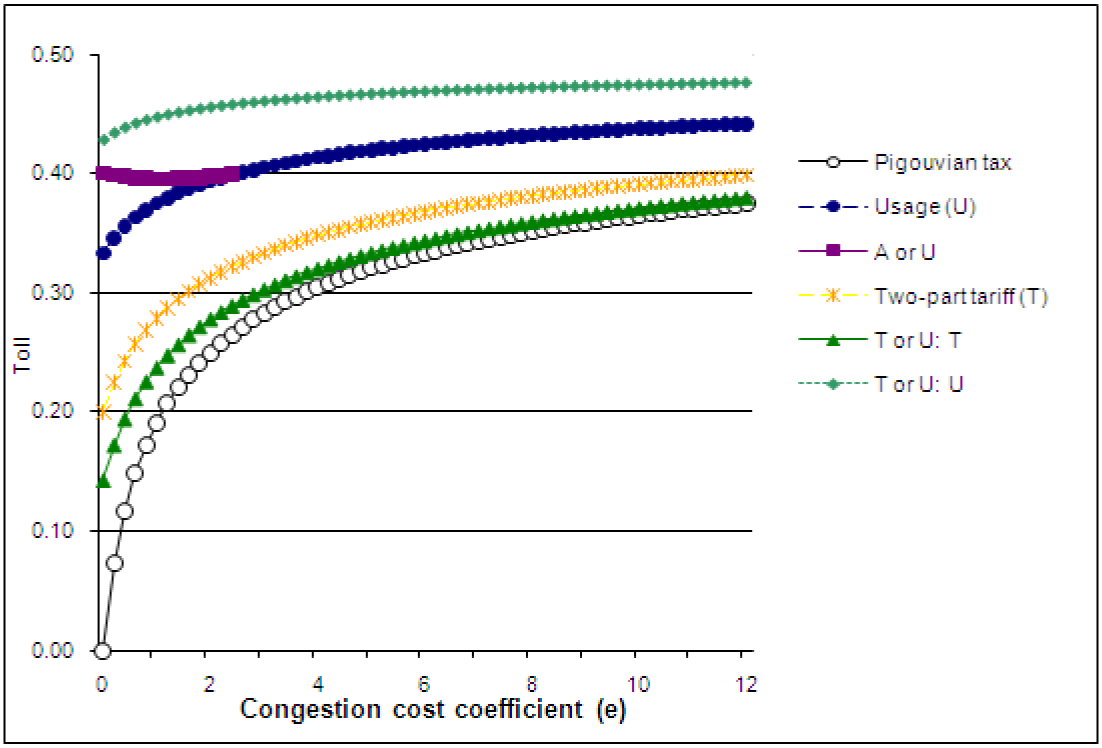



**Figure 6: Profits compared to usage-only pricing (heterogeneous reservation prices, *k*=0)**

Source: Authors' calculation

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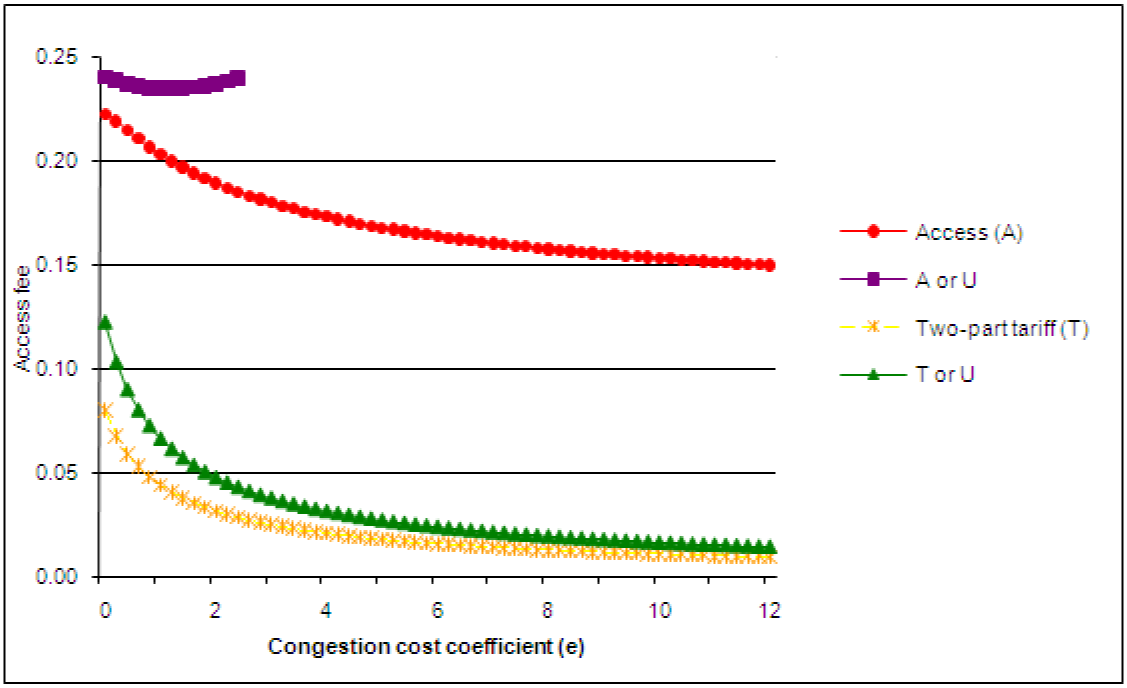
Source: Authors' calculation

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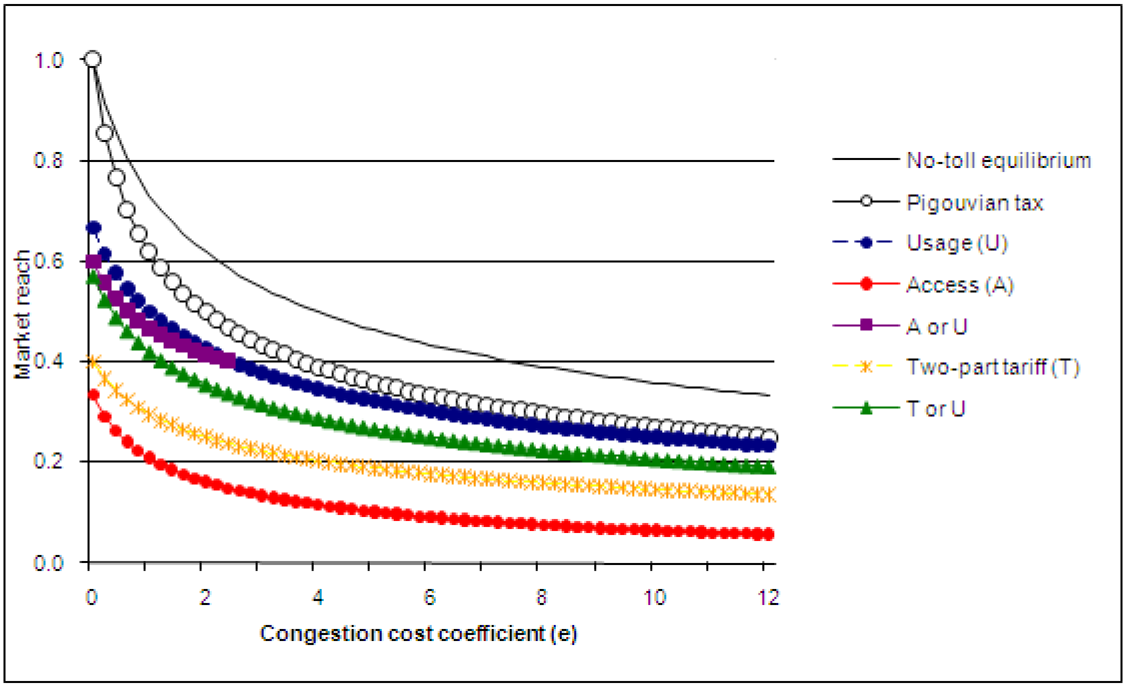
Source: Authors' calculation

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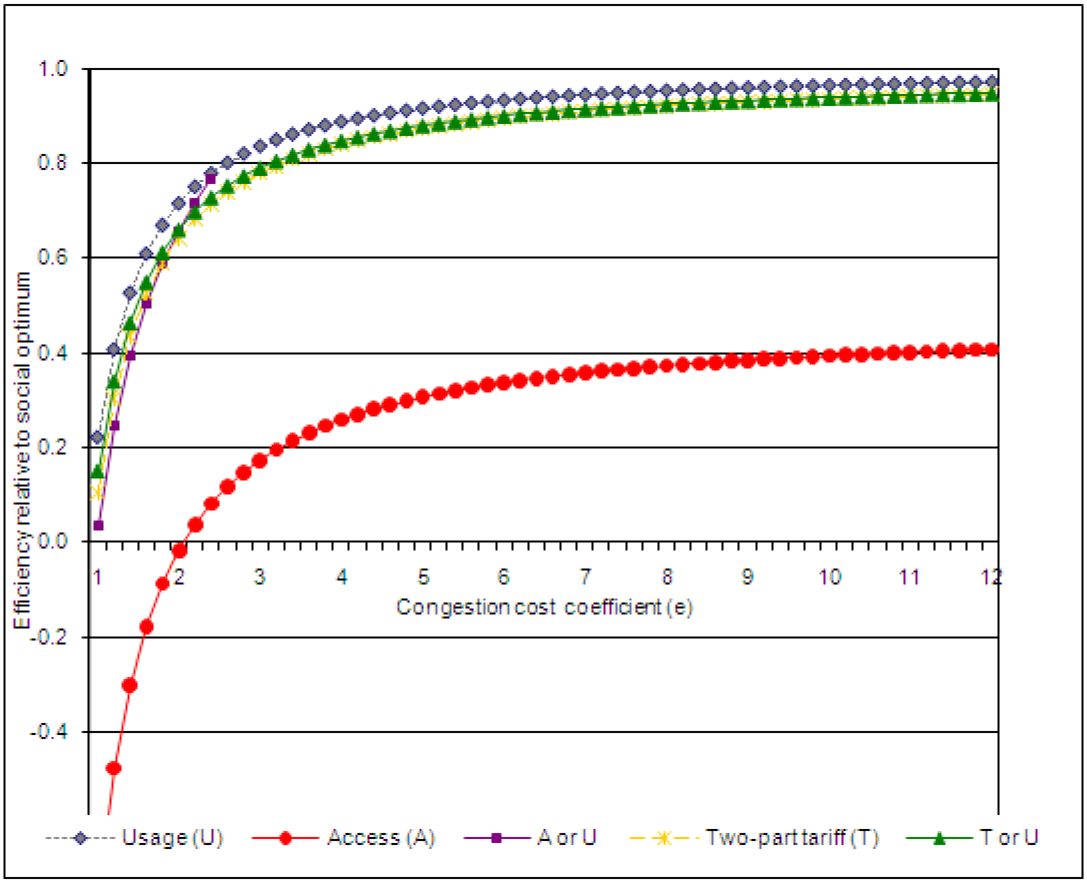
Source: Authors' calculation

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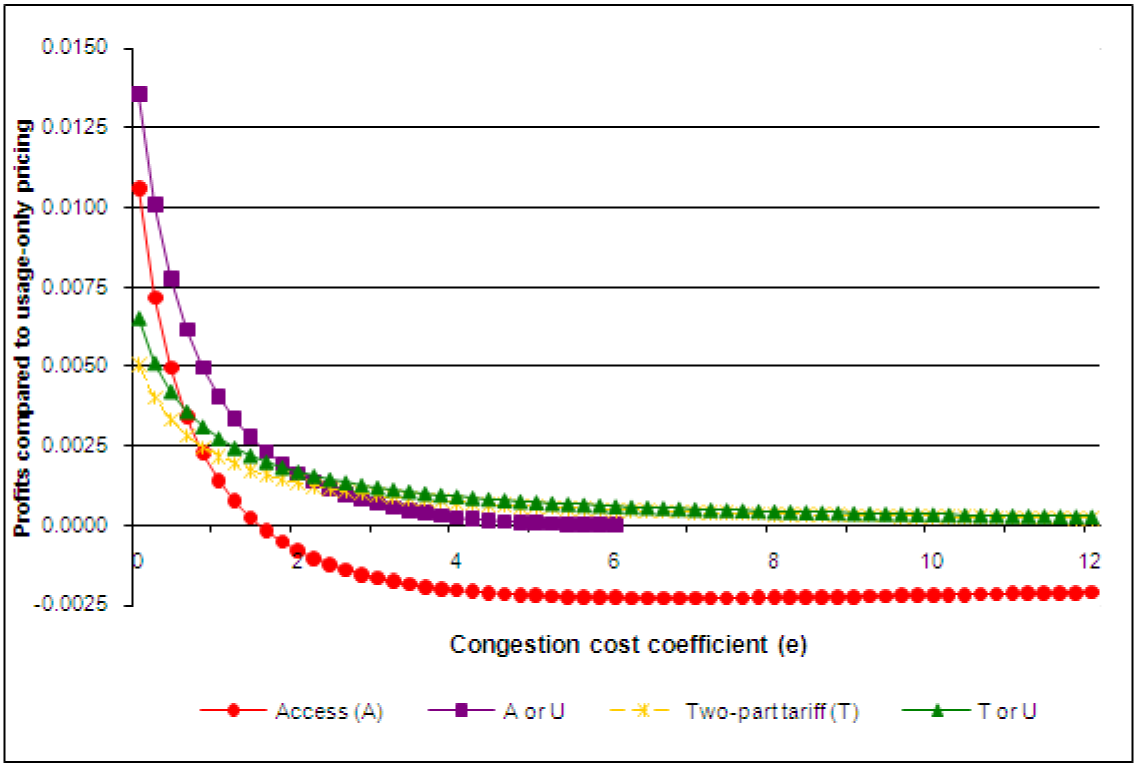
Source: Authors' calculation

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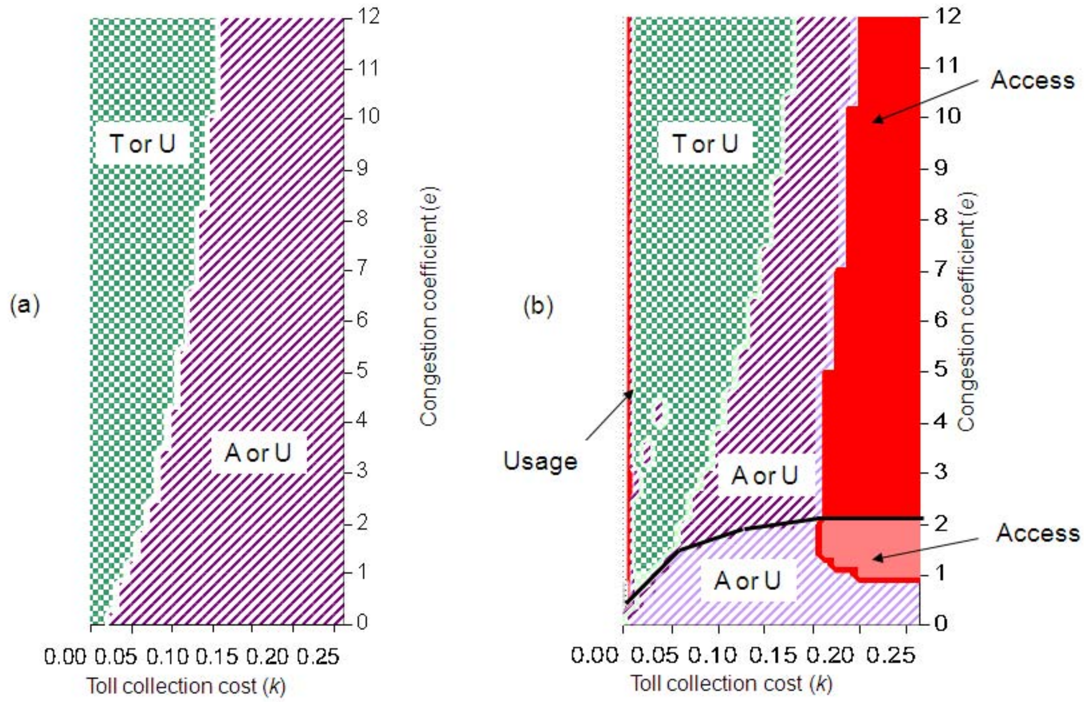
Source: Authors' calculation

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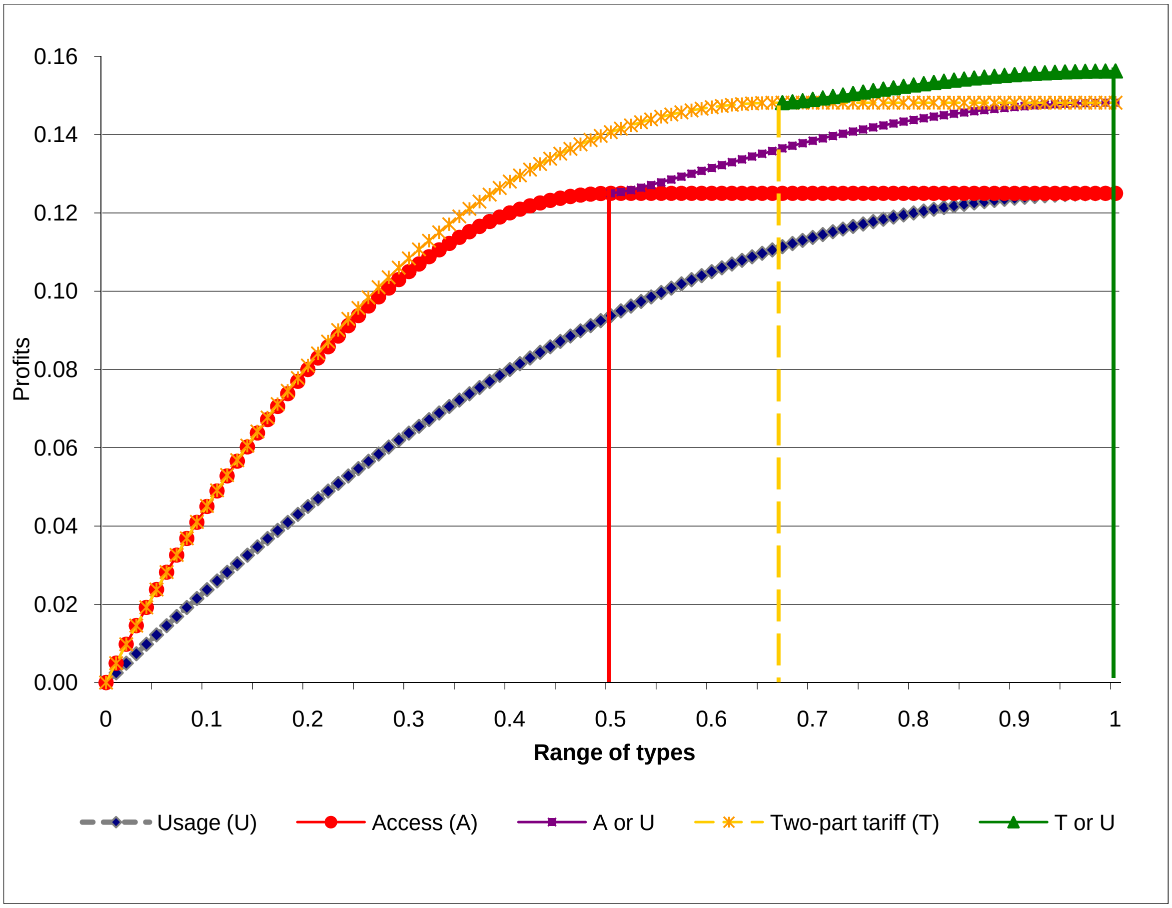
Source: Authors' calculation

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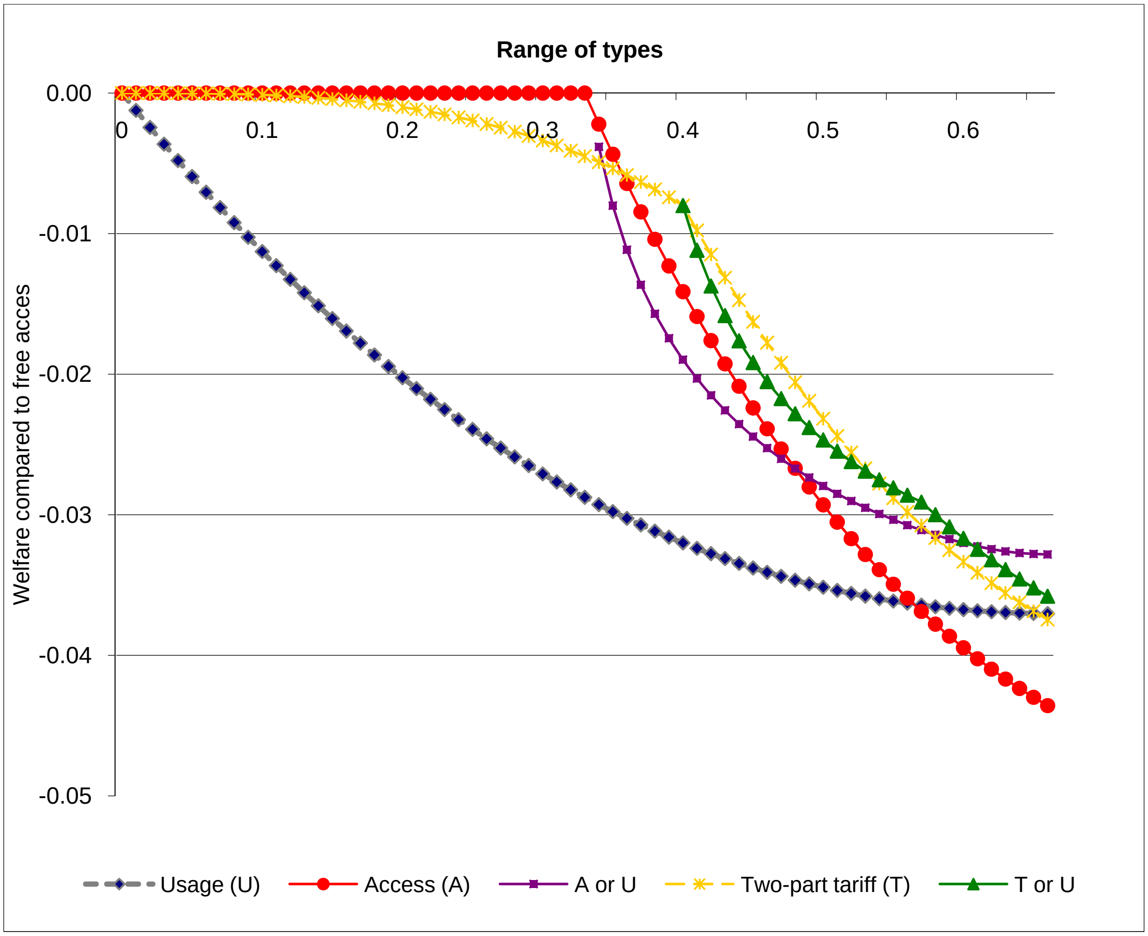
Source: Authors' calculation

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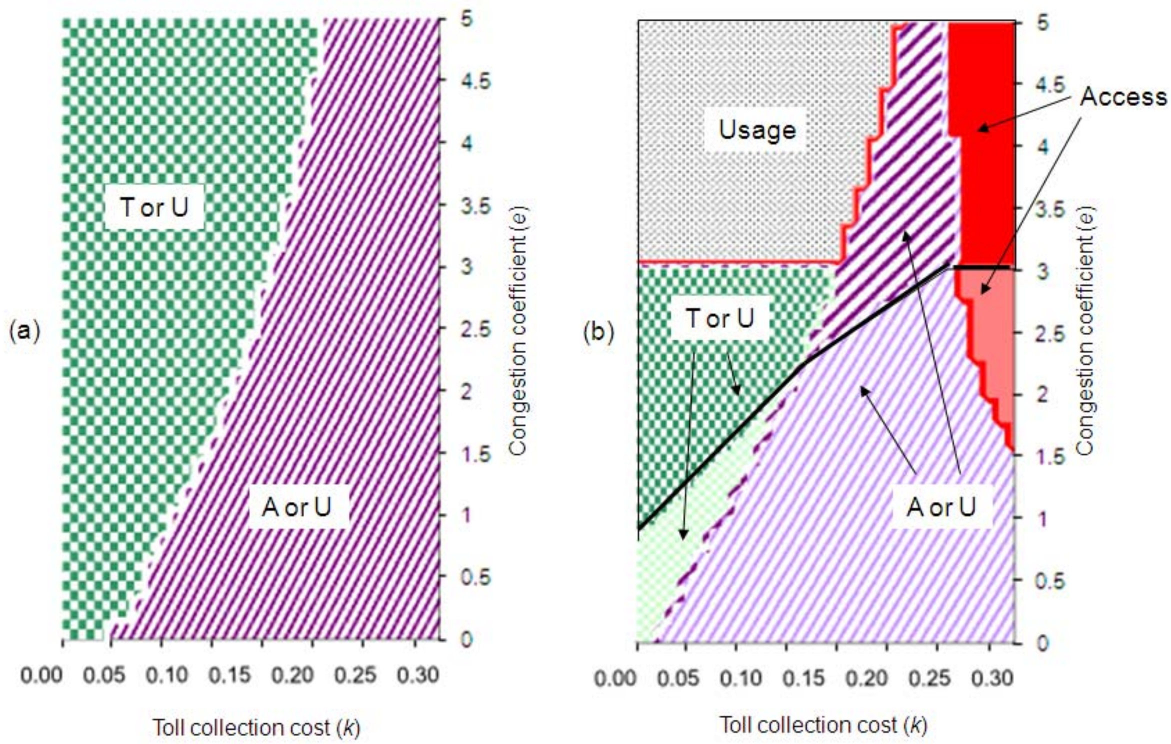
Source: Authors' calculation

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Source: Authors' calculation

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