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Search of Prior Art and Revelation of Information by Patent Applicants

Corinne Langinier
University of Alberta

Philippe Marcoul
University of Alberta

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Corinne Langinier†
Department of Economics
University of Alberta

Philippe Marcoul
Department of Rural Economy
University of Alberta

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Abstract

We examine the strategic non-revelation of information by patent applicants. In a model of a bilateral search of information, we show that patent applicants may conceal information, and that examiners make their screening intensity contingent upon the received information. We then analyze the effects of a double review policy and a policy in which examiners *ex ante* commit to screening efforts. The implementation of the former policy reduces strategic non-revelation, but its overall implication remains unclear. The latter policy involves equal screening intensity across all applications, requires a limited commitment power and induces truthful revelation.

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†Corresponding author: 8-14 Tory Building, University of Alberta, Edmonton AB T6G 2H4, Canada. Corinne.Langinier@ualberta.ca,. Tel: 780 492 2989.

1 Introduction

In an economy that strongly relies on patents to encourage innovation and spur knowledge dissemination, the existing set of related inventions or *prior art* plays a crucial role to gauge and reward the creative effort of innovators. From the early decision of the innovator to file for a patent to its final granting by the patent examiner, most of the innovation patenting process arguably revolves around finding, reporting, and checking the prior art surrounding the innovation under scrutiny. Ultimately, the quality of patent applications compounded with the search ability of examiners explain to a large extend the outcome of the patenting process.

In this paper, we build a theoretical model to analyze the patenting process with two important features in mind. First, we allow both innovators and examiners to search for the innovation's prior art and, through it, to learn about its patentability. Second, we allow innovators to naturally influence the examiner's search process through the patent application they draft. Specifically, applicants can decide to withhold citations to relevant prior art that is known to them. We argue that this strategic concealment of relevant citations can make the examiner's inference process harder, thereby, allowing the issuance of "low quality" patents.

The question of low quality patents is a recurrent one in patent policy. Recent studies have questioned the performance of the U.S. Patent and Trademark Office (PTO) performance and, in particular, examiners are often accused of granting dubious patents. The patent quality problem is arguably related to the number of prior art citations included in the patent. There exists empirical evidence that supports the fact that examiners are less informed than applicants about relevant prior art, and may face particular challenges in searching for it (Jaffe and Lerner, 2005). On the other hand, other studies point to the scarcity of prior art citations by applicants. For instance, Alcacer and Gittelman (2006) show that, over the period 2001-2003, forty percent of patents had all citations (i.e., prior art information) listed by examiners. These empirical observations are not necessarily in contradiction. Rather, they suggest, in a non-exclusive way, that patent applicants may not have the proper incentives to search and reveal prior art, and that examiners may have a hard time finding relevant prior art in some fields.[1]

[1] This may be especially true for emerging fields. For instance, at the beginning of 2000, e-commerce was a new technological field. See Coppel (2000), Kesan (2002), Lemley (2001), and Merges (1999) for a discussion on the relevance of the patent protection system in the e-commerce world. Many "business-method software" patents have little prior art. According to Greg Aharonian, slightly fewer than half of all patents, over a period

In the absence of thorough knowledge of existing innovations, it is difficult for patent examiners to assess the novelty of innovations. In the U.S. patent system, to be patentable, an innovation must be useful, novel and non-obvious. A patent application must contain references to previous literature and patents upon which the innovation improves or from which it diverges. The nature of prior art is diverse and any relevant piece of evidence can constitute prior art; for instance, it can be a thesis in a French university or an article in a scientific journal. Applicants should provide information to demonstrate that the innovation has not yet been patented or published prior to the time the patent is filed. Legally, they have a duty of candor in disclosing prior art information,[2] but they have no duty to search for it: applicants must only disclose information of which they are aware. Moreover, according to the doctrine of inequitable conduct, they should disclose all relevant information they have, and should not disclose false information with the intent to deceive the PTO. However, the PTO has made it clear that applications will not be investigated and rejected based on violation of the duty to disclose prior art, as the PTO is not well-equipped to enforce such a rule (Kesan, 2002). Hence, applicants have no explicit (and possibly weak) incentives to search for relevant information, and perhaps even fewer to reveal it.[3]

We investigate the determinants of patent quality by focusing on the information gathering processes performed by applicants and patent examiners. Our modeling framework of the patenting process is a sequential bilateral information gathering game. We model a patent examiner as an imperfect "auditor," whose task is to determine patent validity. Arguably, establishing the patentability of an innovation depends both on the quality of the information cues (e.g., innovation attributes, prior art cited) provided by the applicant and on the quality of the information sources used by the examiner (e.g., prior art database). An examiner knows that the

of 20 years, cite no non-patent prior art, and the average patent cites about two non-patent prior art items (http://www.bustpatents.com/).

[2] In the European patent system, an innovation must be novel, mark an inventive step and be commercially applicable. Unlike in the US system, innovators do not have to provide a full list of prior art. The European patent office provides a search report to the innovator, who decides whether or not to request the patent examination (Graham, Hall, Harhoff and Mowery, 2002).

[3] This is again consistent with empirical findings by Sampat (2009); many applicants do not search for, or fail to disclose, prior art, as suggested by the fact that examiners tend to insert relevant patents that are owned by the applicants themselves. It is unlikely that applicants are unable to find information about their own patents. Lerner (2002) presents examples of financial patents where the prior art cited seems clearly incomplete.

prior art reported in the application depends on the field to which the innovation belongs but also can vary across firms. For instance, in mature fields – say pharmaceuticals – an examiner often has rich prior art databases and is able to perform a quick and exhaustive prior art search. Moreover, she expects to receive more prior art, as she believes it is more abundant and freely available. This is unlike emerging fields where prior art, which may or may not be abundant, is often unavailable to examiners as it is more likely to appear in scientific publications or other sources.[4] However, to the extent that non cited prior art will more likely result in a legal suit after the patent has been granted, applicants who have a strong litigation skills will be less hesitant to conceal prior art.

Overall, the examiner does not (fully) know whether or not the innovation has a lot of prior art surrounding it. With this uncertainty in mind, she must process the application with what the applicant has chosen to disclose. In this setup, releasing less information cues essentially increases the screening cost of the examiner, who, in turn, performs less scrutiny on applications with low informational content. Applicants have private information about the patentability of their innovations, which creates incentives for those with "bad" innovations to conceal some information so as to maximize their chances of getting through the patent process.

Our objective is to address several questions: What is the behavior of applicants when applying for a patent? What should be the optimal patent examiner response to a given patent application? How are the applicant's search effort and the examiner's scrutiny related?

In the absence of scrutiny commitment, we show that the screening intensity of an examiner is contingent upon the level of prior art transmitted. Applicants with "bad" innovations tend to conceal some prior art to decrease the examiner's level of scrutiny. However, always concealing prior art when the innovation is not patentable sharply raises the quality of the applications with abundant prior art. Thus, full concealment cannot be an equilibrium either. Although our main focus is to develop a theoretical framework to analyze the behavior of patent applicants, we wonder whether our equilibrium analysis reflects existing behaviors.

In a recent contribution, Allison and Hunter (2006) describe the change in the behavior of applicants that has occurred after the PTO strengthened the application review procedure

[4] This is because in these fields, when prior art exists, it is usually not in the form of patents. As pointed out by Jaffe and Lerner (2004), "Patent examiners are not very good at finding non-patented prior art." Allison and Hunter (2006) and Sampat (2009) make similar observations.

for innovations belonging to a specific emerging field. This program, called the Second Pair of Eyes Review (SPER), consists of a second examination of patents granted in the main class 705 (e.g., data processing, financial, business practice). Allison and Hunter (2006) find that many applicants seem to alter the nature of their patent applications in order to avoid the additional scrutiny of the SPER program.[5] We analyze the effects of the introduction of a second review in our theoretical framework and show it indeed results in a change of applicants' behavior. Lastly, we introduce another policy, in which we investigate whether the examiner's *ex ante* commitment to a certain level of scrutiny can induce applicants to search for prior art, and to reveal their findings. We find that an examiner should not have different scrutiny levels but rather, should commit to an equal screening intensity across all applications. This simple rule has two advantages: first, it requires a limited commitment and, second, it induces truthful information transmission from applicants.

Most of the patent literature has focused on the importance of patent litigation (Lanjouw and Schankerman, 2001) or settlement in case of patent infringements (Crampes and Langinier, 2002). Many contributions are concerned with the patent rules that affect the value of a patent in the context of sequential innovation (Chang, 1995; Scotchmer, 1996; O'Donoghue, 1998). Recently, attention has been brought to the problem of search of prior art, yet no formal framework has emerged to analyze the patent granting process (Farrell and Merges, 2004; Kesan, 2002; Lemley, 2001; Merges, 1999). Our paper is a contribution to this literature. Little attention has been devoted to issues related to the search and revelation of prior art information in the patent literature. Related to our model, Caillaud and Duchêne (2005) are concerned with the "overload" problem facing the patent office. In their model, a patent examiner undertakes a costly search and examination that depend on the volume of applications. We do not account for the overload problem in our setting. A recent study is concerned with the incentives of innovators to search for prior art before undertaking any R&D investments, and after, but with no strategic revelation of information (Atal and Bar, 2008).

Because of the lack of accessible data, only recently has some empirical attention been focused

[5] In practice, the PTO sorts out applications according to relevant prior art categories. Each of these categories has a team of specialized examiners in a particular prior art field. As noted by Allison and Hunter (2006), this sorting process often gives rise to strategic drafting, as "skilled patent attorneys can often draft applications so as to opt out of a predefined category." For a concise description of the patenting process, see also Lerner (1995).

on prior art problems (Alcacer and Gittelman, 2006; Alcacer, Gittelman and Sampat, 2009). The roles of patent examiners and applicants have been studied and the findings from these analyses are mostly consistent with issues we are tackling in this paper. In a recent empirical analysis, Lampe (2008) shows that innovators conceal information about prior art that is closely related to their innovation. Although Lampe (2008) does not take into account the role of the PTO, these results validate our main findings that applicants have an incentive not to reveal all of the prior art.

Other remedies have been suggested to improve the patent system. New ways of rewarding patent examiners can be proposed, or an opposition system similar to what exists in Europe can be implemented (Merges, 1999).[6] However, it is not just the patent examiners' responsibility; applicants are liable, too. Because it is well-established that (non-patent) prior art information is not easily available and that the PTO does not (and cannot) enforce the "doctrine of inequitable conduct,"[7] applicants have no incentive to search for prior art, although intuition suggests that applicants, being familiar with their innovation, know where to search. Hence, in many instances, the relevant information exists and can be accessed at a cost.[8]

Another strand of related literature is the literature on auditing and monitoring. In the context of a principal-agent model, it is showed that contracts must sometimes reward agents for announcing bad news (Levitt and Snyder, 1997). Similar to our model, the principal commits *ex ante* to an inefficient *ex post* outcome. Also related to our analysis, Khalil's (1997) model studies the optimal contract when there is no commitment to an audit policy from the principal. More recently, the strategic effect of monitoring versus auditing has been studied as well as the problem of audit in a sequential game without commitment (Strausz, 2005; Mitusch, 2006). In common with this literature, we show that in the absence of any credible mechanism for committing to an auditing procedure, there is no plausible equilibrium in which only good applicants apply for patents.

[6] For details on the opposition system, see Graham, Hall, Harhoff and Mowery (2002) and Friebel, Koch, Prady and Seabright (2006) for a detailed report on the European Patent Office.

[7] See Kesan (2002) for details about the law. Farrell and Merges (2004) are also in favor of providing better incentives for applicants to find and disclose prior art.

[8] On the website Bountyquest.com, one can post an announcement to find prior art concerning a patent for a reward of from $10,000 to $30,000. There exist other websites where prior art information can also be added to a patent (e.g., PatentFizz).

The paper is organized as follows. In section 2, we present the model and the technologies of patent examination and prior art search. In section 3, we show that, in the non-commitment case, there is no equilibrium in pure strategies. An equilibrium exists in mixed strategies in which the applicant does not always reveal information. In section 4, we analyze the effects of the introduction of the SPER initiative. Section 5 is devoted to the analysis of a situation in which the examiner *ex ante* commits to certain scrutiny levels. We conclude and derive policy implications in section 6.

2 The Model

We consider a sequential game with two players: a patent applicant endowed with an innovation and a representative examiner who must judge the granting of a patent.[9] At the outset, neither the applicant nor the examiner knows the value of the innovation. A "good" innovation should be granted a patent as it is novel, non-obvious and useful. A "bad" innovation should be refused a patent, as it is a non-patentable innovation that infringes upon existing patents or is not novel. Both the applicant and the examiner share common prior beliefs about the nature of the innovation: it is good with probability p.

The private (respectively, social) value of a good innovation that is granted a patent is $\overline{V}_G$ (respectively, $\overline{W}_G$); whereas it is $\overline{V}_R$ (respectively, $\overline{W}_R$), when it is refused a patent, with $\overline{V}_G > \overline{V}_R$ ($\overline{W}_G > \overline{W}_R$).[10] The upper (respectively, lower) bar indicates a good (respectively, bad) innovation, and the subscript G (respectively, R) indicates that the innovation is granted (respectively, refused) a patent. The applicant derives a positive benefit from exploiting his non-patented good innovation because it is new. On the other hand, the private (respectively, social) value of a bad innovation that is granted a patent is $\underline{V}_G$ (respectively, $\underline{W}_G$) and 0 (respectively, $\underline{W}_R$) if a patent is refused, with $\underline{V}_G > 0$ ($\underline{W}_R > \underline{W}_G$). The applicant prefers to be granted a

[9] We consider that the PTO and patent examiners have the same objective functions and, thus, the PTO and PTO examiners represent the same decision maker. However, it is unlikely that they always have the same objectives. Langinier and Marcoul (2009) analyze the problem of examiners' career concerns.

[10] Note that since a patent result in monopoly rights for the applicant, *ex post* the PTO examiner should never grant any patent to a valuable innovation. Such a behavior would dampen any incentives to patent. As such, our assumption that $\overline{W}_G > \overline{W}_R$ means that our representative examiner has *ex ante* as well as *ex post* benefits in mind.

patent on a bad innovation, as the probability of it being invalidated in court is relatively small, or a trial may invalidate only part of the claims. Overall, $\overline{V}_G > \underline{V}_G$, and $\Delta V \equiv \overline{V}_G - \underline{V}_G - \overline{V}_R > 0$, that is, the difference in private value between good and bad innovations is greater when the innovation is patented. Furthermore, the granting of a patent on a good innovation has a higher (or potentially equal) social value than refusing a patent on a bad innovation, which, in turn, has a higher value than refusing a patent on a good innovation, which is better than granting a patent on a bad innovation, i.e., $\overline{W}_G \geq \underline{W}_R > \overline{W}_R > \underline{W}_G$. This assumption implies that $\overline{W}_G - \underline{W}_G > \overline{W}_R - \underline{W}_R$. Finally, we define $\Delta W \equiv \overline{W}_G - \overline{W}_R - \underline{W}_G + \underline{W}_R > 0$, which represents the social gain from avoiding errors (an error being refusing a patent on a good innovation, or granting a patent on a bad innovation).

In the model, we assume that the innovator has a better information about the inventive degree of his innovation than the PTO examiner. However, by scrutinizing the application, the latter will eventually find out whether the innovation is patentable or not.

To determine whether an innovation is patentable or not, the examiner must search for the existing prior art. The existence (or the non existence) of prior art is arguably an important determinant of the patentability of the innovation. At the outset, the examiner does not know whether there exists substantial prior art information surrounding the innovation under scrutiny. To capture this uncertainty, we denote by γ (respectively, $1 - \gamma$) the unconditional probability that the prior art related to the innovation is abundant (respectively, scarce).

We now detail each of the components of the model.

2.1 Search of Prior Art Information

Before filing for a patent, the applicant decides the level of effort to devote to prior art search. If he exerts an effort e he will find the amount of prior art x. To simplify, we assume that the effort of the applicant generates a probability of finding prior art $e \in [0, 1]$ and has a disutility equal to $c(e) = e^2/2$.

The amount of prior art found by the applicant can take several values and depends both on the underlying scarcity of prior art surrounding this particular innovation and the search effort put forth by the applicant. We assume that the set of values is $\{\underline{x}, x_i, \overline{x}\}$ with $0 \leq \underline{x} < x_i < \overline{x} \leq 1$ and we normalize $\underline{x} = 0$ and $\overline{x} = 1$. The quantity of relevant prior art that is found depends on how abundant the prior art surrounding the innovation is. When the prior art is abundant (respectively, scarce), the applicant finds 1 (respectively, x_i) with probability e, and in the course of his search, he learns whether his innovation is patentable or not.[11] This aspect underlies the fact that a fair amount of learning about novelty takes place when applicants prepare their applications (Trajtenberg et al., 2000). The discovery of the patentability is soft information and, if necessary, its content might be omitted without altering the overall amount of prior art submitted,[12] even though this is against the law. Independent of whether the prior art is abundant or scarce, with probability $(1-e)$ the applicant discovers no information and learns nothing about the patentability of his innovation. After having searched for prior art, he decides to apply for a patent and to reveal $\widetilde{x}$ in the application. In our game, $\widetilde{x}$ is a message that can take values in the set $\overline{X} = \{\underline{x}, x_i, \overline{x}\}$ if the applicant found $\overline{x}$ and $X_i = \{\underline{x}, x_i\}$ if the applicant found x_i. In other words, the applicants can decide not to report some of the prior art he found but he can never forge an application that would content more relevant prior art than he actually found. Likewise, this aspect captures the possibility that applicants can withhold citations to relevant prior art.

The examiner does not observe the effort of the applicant nor does she observe what he actually found, she is just aware of the announced level of prior art $\widetilde{x}$ when she receives a patent application, and (eventually) updates her beliefs accordingly. She then makes a scrutiny effort $E \in [0, 1]$ to search for complementary information in order to be able to assess the patentability of the innovation. We assume that if the application contains no prior art, it does not provide enough information to be eligible for evaluation.[13]

The examination technology is simple. After exerting effort, the examiner receives a signal that can only take two values: "patentable" or "non patentable." The content of this signal is the *only* information available to her.[14]

[11] We assume perfect learning by innovators but clearly all the model could be recast to allow this learning to be interpreted as a simple "hint" that some innovations are better than others. Similarly, one would obtain the same qualitative results by relaxing this assumption and assuming instead that only a fraction of innovators learn their type.

[12] Typically, the innovator can find a citation that may invalidate his patent or significantly reduce the scope of his claims. He may simply remove this citation and pretend he never encountered it. This behavior is illegal and might be prosecuted under the doctrine of inequitable conduct.

[13] This assumption simplifies the analysis. As will become clear later (see footnote 18), it can be relaxed without affecting our results qualitatively.

[14] Literally, after exerting scrutiny effort, this search technology amounts to the examiner receiving a letter

We assume that the informativeness of the signal received depends on the effort that the examiner exerts. More precisely, the signal indicates the true nature of the innovation with probability E, while with probability $(1-E)$, it is distributed according to the beliefs she holds prior to receiving the signal. Formally, this technology implies that if the examiner believes *ex ante* that a given application is patentable with probability μ, then conditionally on the innovation being patentable (good), the probability of rightly granting a patent is

$$P\left(Granting \mid good\right) = E + \left(1-E\right)\mu,$$

whereas the probability of (incorrectly) refusing a patent to a good innovation is

$$P(\mathrm{Re\,fu\,sin\,g} \mid good) = \left(1-E\right)\left(1-\mu\right).$$

Note that for any strictly positive effort, the informativeness of the signal obtained will be higher than her prior belief and thus, it is a strictly dominant strategy for the examiner to recommend her signal whatever the belief held prior to search. As such, a higher scrutiny effort E results in a good innovation being more often granted a patent and can, in the extreme, i.e., when $E = 1$, result in a perfect sorting between patentable and non-patentable innovations.

The cost of the examiner's search is

$$C_{\widetilde{x}}\left(E\right) = \tfrac{K}{\widetilde{x}(1-E)}, \tag{1}$$

where $\widetilde{x} \in \{x_i, 1\}$ and $K > 0$. The rationale for this cost function goes as follows. If the PTO wanted any potential innovation to be scrutinized with perfect accuracy, the cost would be infinite. More importantly perhaps, the cost is affected by the relevant prior art, $\widetilde{x}$, transmitted by the applicant. Clearly, an applicant endowed with a bad innovation can withhold citations to relevant prior art as this lowers the number of information cues obtained by the examiner and raises the overall cost of examination. All else being equal, the level of scrutiny is decreased and dubious innovations are more likely to be patented. This "complementarity" between the quality of the information provided by the applicant and examiner efficiency is arguably an important aspect of this relationship.[15]

with a message reading "patentable" or "non patentable." We thank an earlier anonymous referee for forcing to precisely explain this point.

[15] In a survey of 4269 EPO examiners, Friebel et al. (2006) reports that 63% of them "say that high quality of patent applications save a lot of their working time (p.112)."

Our model allows for applications to differ with respect to their informational content. Thus, it seems important to wonder how the examiner should react given two different applications. One the one hand, every application should receive an equal scrutiny to guarantee fairness across applicants. On the other hand, it seems rational to treat two applications differently when both have the same social value but one is more costly to process than the other. We emphasize this issue by considering three different patenting processes:

i) The non commitment case – the examiner only responds to the information transmitted,

ii) the SPER – the examiner puts more emphasis on certain patents with scarce prior art, for instance in emerging fields,

iii) the commitment case – the examiner commits *ex ante* to certain levels of scrutiny efforts.

To summarize, the timing of our game is the following:

- In cases *i*) and *ii*),
  - on date 1, the applicant decides how much effort e to put into prior art search. He finds prior art information x and learns whether his innovation is patentable or not. He then files a patent application with announced prior art $\widetilde{x}$;
  - on date 2, the examiner observes the applicant's announcement $\widetilde{x}$, and decides to undertake search effort E;
  - on date 3, depending on the signal received during the examination, the examiner decides whether or not to grant a patent.
- In case *iii*),
  - on date 1, the examiner *ex ante* commits to certain levels of scrutiny effort;
  - on date 2, knowing the levels of effort of the examiner, the applicant chooses his search effort level e. He finds x and learns whether his innovation is patentable or not. He files a patent application with announced prior art $\widetilde{x}$;
  - on date 3, the examiner grants or not a patent based on the announced information, and the signal received during the examination process.

In the rest of this section and the next, we focus on the commitment case (case i) where the examiner responds to the information transmitted in the patent application. The structure of examination technology in case of the SPER (case ii) and in the commitment case (case iii) will be introduced in later sections.

2.2 Examination Technologies and Prior Art Search

We now present the structure of the patent examination technology, and the applicant's prior art search technology when the examiner does not commit to any scrutiny level of efforts *ex ante* (case i).

2.2.1 Patent Examination Technology

The examiner receives a patent application that contains $\widetilde{x}$. She first updates her beliefs and then decides how much effort to put into checking the application. Her updated beliefs depend crucially on her anticipation of the applicant's behavior. More precisely, the examiner draws some inference about validity by considering the amount of prior art reported in the application. Note that if the applicant has an incentive to withhold prior art information, then a (rational) examiner has no reason to believe that a given application has a probability p of being patentable. Formally, $\mu_{\widetilde{x}}$ denotes the probability that the innovation is good, given that $\widetilde{x}$ has been revealed in the patent application. The maximization program of the examiner is

$$\underset{E}{Max}\ [B(E;\mu_{\widetilde{x}}) - C_{\widetilde{x}}(E)], \tag{2}$$

where her gross benefit is

$$\begin{aligned} B(E;\mu_{\widetilde{x}}) \quad = \quad & \mu_{\widetilde{x}}\,[E + (1-E)\,\mu_{\widetilde{x}}]\,\overline{W}_G + \mu_{\widetilde{x}}\,(1-E)\,(1-\mu_{\widetilde{x}})\,\overline{W}_R \\ & + (1-\mu_{\widetilde{x}})\,[E + (1-E)\,(1-\mu_{\widetilde{x}})]\,\underline{W}_R + (1-\mu_{\widetilde{x}})\,(1-E)\,\mu_{\widetilde{x}}\underline{W}_G. \end{aligned} \tag{3}$$

The gross benefit depends on the examiner's effort E and on her updated beliefs $\mu_{\widetilde{x}}$. The first part of $B(E;\mu_{\widetilde{x}})$ represents the expected social value of a good innovation. With probability $\mu_{\widetilde{x}}$, the innovation is good. The examiner receives a signal indicating that the innovation is good with probability $E+(1-E)\mu_{\widetilde{x}}$ and grants a patent to a good innovation; this generates a social value $\overline{W}_G$. She refuses a patent to a good innovation with probability $(1-E)\,(1-\mu_{\widetilde{x}})$ and this generates a social value $\overline{W}_R$. The second part of the gross benefit represents the expected social value of a bad innovation. Indeed, with probability $(1-\mu_{\widetilde{x}})$, the innovation is bad and the examiner receives a corroborating signal with probability $E+(1-E)(1-\mu_{\widetilde{x}})$ and refuses a patent. This generates a social value $\underline{W}_R$. On the other hand, she receives a misleading signal and wrongly grants a patent to a bad innovation with probability $(1-E)\mu_{\widetilde{x}}$; this is worth $\underline{W}_G$ to society.

To summarize, two types of errors can be made in the patent granting process with probability $\mu_{\widetilde{x}}(1-\mu_{\widetilde{x}})(1-E)$: either a type I error of refusing a patent to a good innovation, or a type II error of wrongly granting a patent to a bad innovation.

The solution to the maximization program (2) with gross benefit (3) and cost (1), can be written as

$$\mu_{\widetilde{x}}(1-\mu_{\widetilde{x}})\widetilde{x}\Delta W = \tfrac{K}{(1-E)^2}. \tag{4}$$

Solving (4) with respect to E leads to the generic level of effort

$$E^*_{\widetilde{x}} = 1 - \left[\tfrac{K}{\mu_{\widetilde{x}}(1-\mu_{\widetilde{x}})\widetilde{x}\Delta W}\right]^{\frac{1}{2}} < 1, \tag{5}$$

where $\widetilde{x} \in \{x_i, 1\}$ and $K < \mu_{\widetilde{x}}(1-\mu_{\widetilde{x}})\widetilde{x}\Delta W$ to insure interior solutions.

The examiner will exert a high level of scrutiny effort on a fraction $E^*_{\widetilde{x}} > 0$ of the applications that contains $\widetilde{x}$, whereas a fraction $(1-E^*_{\widetilde{x}})$ will not be scrutinized as thoroughly. Moreover, a higher level of relevant prior art will increase her scrutiny effort.

2.2.2 Applicant Prior Art Search

The applicant makes two sequential decisions: he first decides the level of effort to put in prior art search and, second, the quantity of prior art to report to the examiner.

We first determine what the announcement of the applicant must be. After undertaking a search effort, if he finds x_i or 1, he learns whether his innovation is patentable. For expositional purposes, we call a good (respectively, bad) applicant an applicant who has learned that he has a patentable (respectively, non-patentable) innovation.

As the applicant cannot report more than he actually found, if he gets no information he reports nothing. If he finds x_i he can either report 0 or x_i. However, there is no gain from reporting 0, since in our setting, no patent is granted if the application does not contain prior art. Therefore, whatever the patentability of the innovation, he should always report truthfully, $\widetilde{x} = x_i$.

If the applicant finds $x = 1$, he can announce either 0, x_i or 1. Since announcing nothing is a strictly dominated strategy, we still need to determine under what circumstances he reports 1 or x_i. Although the benefit from withholding information appears to be clear, we have so far neglected to introduce explicitly the costs associated with concealing prior art information. In particular, recent studies show that the strength of a patent depends on whether prior art has been concealed or not to obtain this patent. The difference in value of two otherwise identical patents stems from the fact that it is much easier for competitors to invalidate a patent on "forgotten" prior art than on cited prior art (Allison and Lemley, 1998). To model *ex post* punishment for non-compliance, we assume that the private value of the innovation $V = \{\overline{V}_G, \underline{V}_G\}$ is negatively affected by prior art strategic concealment. More precisely, when prior art is concealed, the value of the patented innovation is αV, with $\alpha \in (0, 1)$. The parameter α represents such things as the ability and the willingness of the PTO to establish an inequitable conduct, it can also describe the applicant's ability to legally fight a patent litigation. In the latter case, it has no reason to be identical across applicants and α is interpreted as a firm specific attribute.

If the applicant learns that his innovation is patentable, his expected gain is

$$E_{\widetilde{x}}\overline{V}_G + (1 - E_{\widetilde{x}})[\mu_{\widetilde{x}}\overline{V}_G + (1 - \mu_{\widetilde{x}})\overline{V}_R],$$

where $\overline{V}_R$ is the private value of a non-patented good innovation, $E_{\widetilde{x}}$ is the effort performed by the examiner conditional upon receiving $\widetilde{x}$, and $\mu_{\widetilde{x}}$ is the corresponding updated belief. Thus, a good applicant who finds the maximum amount of prior art reveals all of it if[16]

$$(E_1 + (1 - E_1)\mu_1)\overline{V}_G + (1 - E_1)(1 - \mu_1)\overline{V}_R > (E_i + (1 - E_i)\mu_i)\alpha\overline{V}_G + (1 - E_i)(1 - \mu_i)\overline{V}_R. \quad (6)$$

On the other hand, a bad applicant who has an expected gain $(1 - E_{\widetilde{x}})\mu_{\widetilde{x}}\alpha\underline{V}_G$, reports all his findings if

$$(1 - E_1)\mu_1 > (1 - E_i)\mu_i\alpha. \quad (7)$$

Inequalities (6) and (7) represent the conditions under which both types fully report their findings.

Finally, the patent applicant must choose his effort level in searching for prior art, which is

[16] By a slight abuse of notation, we denote E_{x_i} by E_i and μ_{x_i} by μ_i.

the solution of

$$\underset{e}{Max}\ \{\Pi(e) - c(e)\},$$

where the gross benefit $\Pi(e)$ will be defined in section 3.2.

3 Non Commitment Case: Equilibrium Outcomes

In this section, we begin by characterizing the applicant's optimal transmission of information after finding prior art information and learning whether his innovation is good or bad, and we derive the Perfect Bayesian equilibrium. Then, we determine the optimal search strategy of the applicant.

3.1 Optimal Reporting Strategies

We consider first that no matter what he learned, the applicant reports what he found. The updated beliefs of the examiner, consistent with this behavior, are

$$\begin{aligned}
&\mu_1 = \Pr(good \mid \widetilde{x} = 1) = \tfrac{p\gamma}{p\gamma + (1-p)\gamma} = p, \\
&\mu_i = \Pr(good \mid \widetilde{x} = x_i) = \tfrac{p(1-\gamma)}{p(1-\gamma) + (1-p)(1-\gamma)} = p, \\
&\Pr(good \mid \widetilde{x} = 0) = p,
\end{aligned}$$

where posterior beliefs are equal to prior beliefs.

The patent examiner chooses the optimal levels of effort depending on the transmitted information $\widetilde{x}$. When she observes $\widetilde{x}$, the first order condition (4) gives the optimal level of effort

$$E^*_{\widetilde{x}t} = 1 - \left[\tfrac{K}{\widetilde{x}p(1-p)\Delta W}\right]^{\frac{1}{2}}, \tag{8}$$

where t stands for truthful and $\widetilde{x} = \{x_i, 1\}$. The effort levels E^*_{1t} and E^*_{it} are increasing (respectively, decreasing) with the prior belief p, if p is smaller (respectively, higher) than 1/2. Beliefs and effort levels are substitutes in the mind of the examiner. Therefore, starting from a prior belief $p < 1/2$, the effort levels E^*_{1t} and E^*_{it} increase when the prior belief p becomes more "diffuse" and approaches 1/2. When $p \geq 1/2$ and increases further, the examiner chooses to devote less effort to scrutiny and gives more weight to her priors when deciding to grant a patent. If there exists an equilibrium in which both good and bad applicants report truthfully, then $E^*_{1t} > E^*_{it}$ *for* $p \in [\underline{p}_i, \overline{p}_i]$ (all the proofs are relegated to the appendix). The intuition is straightforward.

The examiner intensifies her scrutiny effort when she receives more information, as it is less costly to do so.

We now define the equilibrium revelation strategy of the applicant, taking into account the examiner's level of scrutiny. A good applicant who finds 1 has no incentive to transmit less information. Indeed, knowing that his innovation is patentable, he is better-off fostering the chance that the examiner will discover the exact type of his innovation.[17] Formally, the inequality (6) is always satisfied, as $\overline{V}_G > \overline{V}_R$. Indeed, if the inequality (6) is satisfied for $\alpha = 1$, it is always satisfied for any $\alpha < 1$. On the other hand, a bad applicant who finds 1 reports truthfully if the inequality (7) is satisfied. Using (7), we define α_1 such that

$$\alpha \leq \alpha_1 \equiv \frac{1 - E^*_{1t}}{1 - E^*_{it}} = x_i^{\frac{1}{2}}.$$

A bad applicant reports truthfully if $\alpha \leq \alpha_1$. On the contrary, for any $\alpha > \alpha_1$, he conceals information. In the latter case, the inequality (7) is never satisfied, since $E^*_{1t} > E^*_{it}$ for $p \in [\underline{p}_i, \overline{p}_i]$, and $E^*_{1t} > E^*_{it} = 0$ for $p \in]\underline{p}_1, \underline{p}_i] \cup [\overline{p}_i, \overline{p}_1[$ (see appendix for $\underline{p}_1$ and $\overline{p}_1$). Thus, when $\alpha > \alpha_1$, inequality (7) is never satisfied for $p \in]\underline{p}_1, \overline{p}_1[$, and there is no equilibrium in which the applicant reports truthfully. When the cost of not revealing the truth (in term of reputation, or direct sanction in the case of a lawsuit) increases, applicants are more likely to report truthfully. In fact, in a system where it would be possible to perfectly enforce the duty of candor, applicants would report all of the information they have. However, as explained in the introduction, the PTO does not currently enforce this rule, and it is likely that in many cases the parameter α is large.

We posit our first set of findings, when the cost of concealing information is high.

Lemma 1 (candid applicants) *If $\alpha \leq \alpha_1$, it is optimal for the applicant to report truthfully. The level of scrutiny exerted by the examiner depends on the amount of prior art transmitted $\widetilde{x}$. She always exerts a higher level of scrutiny effort when she receives more prior art ($E^*_{1t} > E^*_{it}$) for $p \in [\underline{p}_i, \overline{p}_i]$. Moreover, irrespective of the amount of prior art received, her effort is maximized when $p = 1/2$.*

[17] Again uncited prior art is a more effective tool for invalidating patents in court than cited prior art (Allison and Lemley, 1998). In other words, when it is known that the innovation is novel, not including prior art could result in a weaker patent if it is later litigated.

The restriction on prior beliefs $p \in [\underline{p}_i, \overline{p}_i]$, with $0 < \underline{p}_i < 1/2 < \overline{p}_i < 1$, insures interior and strictly positive solutions E^*_{1t} and E^*_{it} to the maximization program (2). The intuition of this result goes as follows. When the nature of the innovation is perceived as highly uncertain, the examiner exerts a higher level of scrutiny irrespective of the amount of prior art received. When the uncertainty is very low (i.e., for values of p close to 0 or to 1), she believes that the innovation is either patentable or not, and it becomes too costly to exert a substantial scrutiny effort. She then issues patents according to her prior beliefs.

When the cost of punishment for non-compliance is low, i.e., $\alpha > \alpha_1$, there exists no equilibrium in pure strategies in which a bad applicant who finds the full amount of prior art reports truthfully when $p \in]\underline{p}_1, \overline{p}_1[$ (these restrictions insure interior or null solutions). For values of p close to 0 or to 1, as the examiner makes no effort whatever the information received ($E^*_{1t} = E^*_{it} = 0$), the applicant is indifferent between revealing the truth or not. If we assume that the applicant always reveals the truth when he is indifferent, we have an equilibrium in which the applicant reports truthfully and the examiner makes no complementary effort. This is only true when there is almost no uncertainty.

In our setting, the inexistence of an equilibrium in pure strategies is akin to an audit without commitment (Khalil, 1997). When the patent examiner assumes that applicants report honestly, she exerts a higher scrutiny effort when the prior art is more abundant, i.e., $E^*_{1t} > E^*_{it}$. Hence, bad applicants will not comply. Conversely, non-compliance is not an equilibrium either since abundant prior art means a patentable innovation for sure, and the examiner should not exert any scrutiny (see appendix for the proof on the non-existence of equilibrium).

We summarize these findings in the following lemma.

Lemma 2 *If $\alpha > \alpha_1$, there exists no equilibrium in pure strategies when $(p, \gamma) \in \Psi_l$.*

When it is not costly to conceal information, applicants have no incentives to always report truthfully or to always lie, but they can sometimes report truthfully. Therefore, we consider a Perfect Bayesian Equilibrium in which a bad applicant randomizes his report decision. We let θ denote the probability that a bad applicant who finds the full amount of prior art reports truthfully, $\widetilde{x} = 1$, and conceals information with the complementary probability, $(1 - \theta)$.

When the examiner receives a patent application containing $\widetilde{x} = 1$, it either originates from a good or a bad applicant. When she receives a patent application containing $\widetilde{x} = x_i$, it can come from a good or a bad applicant who has discovered the intermediate level, but also from a bad applicant who has found $x = 1$ but has concealed some information. However, it cannot come from a good applicant who has found $x = 1$.

The updated beliefs of the examiner consistent with randomization are

$$\begin{aligned} &\mu_1(\theta) = \Pr(good \mid \widetilde{x} = 1) = \tfrac{p\gamma}{p\gamma+\theta(1-p)\gamma} = \tfrac{p}{p+\theta(1-p)}, \\ &\mu_i(\theta) = \Pr(good \mid \widetilde{x} = x_i) = \tfrac{p(1-\gamma)}{1-\gamma+(1-\theta)(1-p)\gamma}, \\ &\Pr(good \mid \widetilde{x} = 0) = p. \end{aligned} \tag{9}$$

In this equilibrium, a bad applicant must be indifferent between revealing the full amount of prior art and getting $\mu_1(1-E_1)\underline{V}_G$, and revealing the intermediate level and getting $\mu_i(1-E_i)\alpha\underline{V}_G$. Thus, there exists a value of θ that must satisfy

$$\mu_1(\theta)(1-E_1) = \mu_i(\theta)(1-E_i)\alpha, \tag{10}$$

which is

$$\theta^* = \tfrac{(1-E_1)(1-p\gamma)-(1-\gamma)p(1-E_i)\alpha}{(1-p)[(1-\gamma)(1-E_i)\alpha+\gamma(1-E_1)]}, \tag{11}$$

where $\theta^* \in [0,1]$.[18]

We can posit the following result.

Proposition 1 (prior art concealment) *If $\alpha > \alpha_1$, whenever the equilibrium effort levels of the examiner are such that $E_1 > E_i$, there exists a unique probability θ^* defined by (11) with which a bad applicant discloses the full amount of prior art. This probability decreases with p, α, and E_1, and increases with γ, and E_i and $\underline{V}_G$.*

This finding emphasizes the main strategic tension faced by an applicant who has discovered that his innovation is non-patentable. The probability θ^* measures applicant's level of candor.

[18] It seems in order to discuss now what would happen if we allow the applicant to patent without reporting any prior art. The first consequence is that it would *de facto* enrich the possibilities for strategic concealment. An applicant with $\overline{x}$ can now choose to report x_i with probability $\overline{\theta}_i$ but *also* $\underline{x}$ with probability $\overline{\theta}_{\underline{x}}$. He is truthful with probability $1-\overline{\theta}_i-\overline{\theta}_{\underline{x}}$. Similarly, an applicant with x_i would be able to report $\underline{x}$ with probability $\underline{\theta}$. This creates no less than three randomization parameters that would *not* be determined independently (applicant with "$\overline{x}$" would choose $\left(\overline{\theta}_i, \overline{\theta}\right)$ taking into account $\underline{\theta}$ chosen by applicant "$\underline{x}$" and vice versa). However, the logic of strategic reporting (i.e., the use mixed strategies) would carry over to this more complicated framework; applicants would continue to conceal prior art.

The effects are straightforward. First, when the value of a patented innovation, $\underline{V}_G$, increases the applicant tends to transmit more prior art. Also, the lower the cost α, the higher θ^*. In fact, when it becomes too costly to report strategically, $\theta^* = 1$, which corresponds to an equilibrium with truthful revelation of information. As argued in the introduction, these results are consistent with empirical findings by Lampe (2008); applicants do withhold citations to relevant prior art. However, the expression of θ^* also reveals that applicants take examiners' behavior into account when deciding to withhold information. For instance, when the examiner increases her scrutiny effort conditional on obtaining abundant prior art, a bad applicant tends to report less prior art in his application. Conversely, when the examiner focuses more on applications with less prior art (E_i^* higher), the applicant will transmit more information. Together, these effects suggest that to avoid strategic reporting, the examiner might commit to identical screening intensity, irrespective of the information provided by the applicant (the commitment case is developed in section 5).

We now determine the optimal levels of examiner effort and show that it is an equilibrium. Before turning to the description of the objective functions of the examiner, it is worth describing her equilibrium beliefs. We use the probability that the applicant announces the full amount of prior art θ^* to define the updated beliefs of the examiner. By replacing θ^*, as defined by (11) in the updated probabilities (9), we obtain the equilibrium beliefs,

$$\mu_1^* = \frac{p[(1-\gamma)(1-E_i)\alpha+\gamma(1-E_1)]}{(1-E_1)} > p, \tag{12}$$

when the full amount of prior art has been revealed and

$$\mu_i^* = \frac{p[(1-\gamma)(1-E_i)\alpha+\gamma(1-E_1)]}{(1-E_i)\alpha} < p, \tag{13}$$

when the intermediate amount has been revealed by the applicant. Thus, when the examiner knows that there is some prior art concealment, she can no longer hold a uniform belief p across all applications in equilibrium.

The equilibrium efforts of the examiner account for the applicant's strategic behavior. The first order conditions of the examiner's program (2) when she receives $\widetilde{x} = 1$ is

$$\frac{\partial B(E_1;\mu_1^*)}{\partial E_1} = \frac{K}{(1-E_1)^2}, \tag{14}$$

and

$$\frac{\partial B(E_i;\mu_i^*)}{\partial E_i} = \frac{K}{x_i(1-E_i)^2} \tag{15}$$

when she receives $\widetilde{x} = x_i$.[19]

In this setting, the effort levels of the examiner when she receives the full or intermediate levels of prior art are no longer independent. The level of effort E_1 is found by solving equation (14) using the equilibrium prior μ_1^* and the randomization parameter θ^*:

$$E_1^*(E_i) = 1 - \left[\frac{K - p(1-E_i)\alpha(1-\gamma)\left[p(1-E_i)\alpha\Delta W(1-\gamma) + (\overline{W}_G - \underline{W}_R)\right]}{\gamma p(1-p\gamma)\Delta W}\right]^{\frac{1}{2}}. \quad (16)$$

A further consideration of expression (16) shows that an increase in E_i simply decreases E_1^*.

We also obtain the level of effort E_i as a function of E_1 using (15), μ_1^*, and the randomization parameter θ^*,

$$E_i^*(E_1) = 1 - \left[\frac{K\alpha^2 - (1-E_1)px_i\gamma\left[p\gamma(1-E_1)\Delta W + \alpha(\overline{W}_G - \underline{W}_R)\right]}{px_i(1-\gamma)(p\gamma+1-p)\Delta W\alpha^2}\right]^{\frac{1}{2}}. \quad (17)$$

The derivative of $E_i^*(.)$ with respect to E_1 is also negative.

Without putting more restrictions on the parameters, by using the reaction functions (16) and (17), it is not possible to obtain a general explicit expression for the equilibrium levels E_i^* and E_1^*. However, we can state the following proposition that holds for interior solutions.

Proposition 2 *If $\alpha > \alpha_1$, there exists a semi-separating equilibrium in which a bad applicant randomizes his revelation, whereas a good applicant always reports truthfully. Furthermore, the level of scrutiny effort exerted by the examiner after receiving a patent application with all the relevant prior art is higher than after receiving a patent application that contains the intermediate level, that is, $E_1^* > E_i^*$ for $(\gamma, p) \in \Omega$.*

There exists an equilibrium in which a bad applicant who finds $x = 1$ randomizes his reporting decision. In this case, the examiner intensifies her complementary search when she receives more prior art for $(\gamma, p) \in \Omega$, which insures interior solutions.

To simplify, and in order to get analytical solutions, we assume that it is socially as beneficial to grant a patent on a good innovation as to refuse one on a bad innovation (i.e., $\overline{W}_G = \underline{W}_R$). Although this assumption may not be empirically sound, it allows us to obtain analytical expressions for optimal efforts without altering our (qualitative) results. We find the following optimal efforts:

$$E_1^* = 1 - \left[\frac{K\left[x_i(1-p(1-\gamma)) - p(1-\gamma)\alpha^2\right]}{p\gamma x_i(1-p)\Delta W}\right]^{\frac{1}{2}}, \quad (18)$$

[19] See appendix for the second order condition.

and

$$E_i^* = 1 - \left[\frac{K\left(\alpha^2(1-p\gamma)-p\gamma x_i\right)}{px_i(1-\gamma)(1-p)\alpha^2 \Delta W} \right]^{\frac{1}{2}}. \tag{19}$$

The comparison of these efforts confirms that $E_1^* > E_i^*$ when the type and the field of innovation are highly uncertain (for $(p,\gamma) \in \Gamma$, where Γ is defined in the appendix).

As we obtain explicit solutions for the optimal levels of effort, we can make some comparative statics. Some of these findings are derived in the general case, as well.

Corollary 1 *If $\alpha > \alpha_1$, $\overline{W}_G = \underline{W}_R$ and holding everything else constant,*

- *E_1^* is strictly increasing with p, and E_i^* is increasing (respectively, decreasing) with p for $(p,\gamma) \in \Gamma$ (respectively, for $(p,\gamma) \notin \Gamma$),*
- *E_1^* is decreasing (respectively, increasing) with γ for $p \in [x_i/(1+x_i), 1]$ (respectively, for $p \in [0, x_i/(1+x_i)]$), and E_i^* is decreasing (respectively, increasing) with γ for $p \in [0, 1/(1+x_i)]$ (respectively, for $p \in [1/(1+x_i), 1]$).*

The logic goes as follows. Let us start at the highest level of uncertainty concerning both the value of the innovation and the field to which it belongs (i.e., p and γ belong to a closed set containing $1/2$). For a given γ, both levels of effort increase with the probability of a good innovation p. The examiner is willing to put in more effort to make her judgment more accurate when she believes the innovation is worthwhile. However, after p reaches a certain threshold as it becomes even more likely that the innovation is good, and because of the strategic report from some applicants, the examiner makes less effort when she receives less information. Indeed, she prefers to concentrate on applications that contain more information.

For a given p close to $1/2$, as γ increases, both levels of effort decrease. The higher the probability that the innovation belongs to a mature field, the greater the chances the applicant will find relevant prior art. Therefore, because of the complementarity between the applicant and examiner efforts, she will reduce the intensity of her scrutiny. The effect is identical on both efforts, as the uncertainty about the value of the innovation is at its highest. For a given p small enough, the examiner intensifies her search effort when she receives full information, but reduces it when she receives intermediate information. Because it is more likely that the innovation is bad, it is expected that there will be more strategic reports from bad applicants. Therefore, the examiner will intensify her scrutiny effort when she receives more information to make a more accurate scrutiny, since she receives fewer applications with the full amount of information. Lastly, for a given p high enough, the converse will arise: as γ increases, the examiner reduces her search effort when she receives the full amount, and intensifies it otherwise. In this case, it is more likely that the innovation is good and, therefore, fewer strategic reports will be made by bad applicants.

We can also state the following results.

Corollary 2 *If $\alpha > \alpha_1$, $\overline{W}_G = \underline{W}_R$ and holding everything else constant,*

- *E_1^* is increasing with ΔW* and α, *and decreasing with K and x_i.*
- *E_i^* is increasing with ΔW and x_i, and decreasing with K* and α.

The higher the social gain from avoiding mistakes (i.e., ΔW), the higher the scrutiny efforts of the examiner. She intensifies her scrutiny effort when the stakes are higher. Not surprisingly, the higher K, the lower the scrutiny efforts. When it is more costly to search, the examiner cannot put too much effort into each application. The higher the intermediate amount of information x_i, the lower (higher) the scrutiny effort when she receives more (less) information. If she receives more information in the emerging field case (i.e., the emerging field becomes more mature), she will intensify her scrutiny effort and E_i^* becomes closer to E_1^*.

As it becomes more costly to conceal information (i.e., α decreases), a bad applicant has less incentive to report strategically ($(1-\theta^*)$ decreases), and the updated belief of the examiner when she receives $\widetilde{x} = 1$ decreases. This triggers the examiner to reduce her scrutiny effort, as more bad applicants will now file a patent application with $\widetilde{x} = 1$. On the other hand, as α decreases, the updated belief of the examiner when she receives $\widetilde{x} = x_i$ increases, as less strategic reporting will occur. She then intensifies her scrutiny effort when she receives x_i.

As a last point, we comment on an argument made by Lemley (2001) that the PTO should not be too worried if undeserved patents are granted. If we consider that it is, in the end, not too costly to grant bad patents, we posit the following result:

Corollary 3 *Assume that there is a social loss in granting a patent to a non-patentable innovation, that is $\underline{W}_G < 0$. If this loss becomes smaller, the examiner should exert less scrutiny effort.*

When the social loss of granting a bad patent $\underline{W}_G$ becomes smaller, the examiner should optimally rely more on her prior beliefs and less on an informed decision (E small). This intuitive result of our model replicates the argument made by Lemley (2001), mentioned above. Nevertheless, this argument hinges on the assumption that $\underline{W}_G$ is small, which remains an open question (Farrell and Merges, 2004). We now turn to the applicant's decision to search for prior art.

3.2 Search of Prior Art by the Applicant

It is often claimed that patent applicants do not have incentives to search for prior art. It is true that the current system does not *seem* to give much incentive for prior art search. The observation of the (low) amount of prior art reported in applications actually tend to confirm that applicants do not search for prior art. But is this really true?

From the previous section, we can state that the prior art found is "not equal" to the prior art reported by applicants. Thus, the scarcity of prior art reported in applications should not be taken as *prima facie* evidence that applicants do not search for prior art. In particular, when the cost of not reporting prior art decreases (i.e., α increases), search tends to bring *only* benefits; the applicant gains a better knowledge regarding patentability but at the same time keeps his option not to reveal what he found. This suggests that when α is high, more search will be associated with higher chances to be granted a patent. The rest of the section analyzes the applicant's problem and makes these arguments more palatable.

The applicant's first decision is to search for prior art information. While making his search decision, the applicant can correctly infer the issue of the game and, therefore, his optimal report and the examiner's optimal efforts.

For $\alpha > \alpha_1$, the gross benefit function of the applicant is

$$\begin{aligned}\Pi(e) = \quad & e\{\gamma p\left[(E_1 + (1-E_1)\mu_1)\,\overline{V}_G + (1-E_1)\,(1-\mu_1)\,\overline{V}_R)\right] + \gamma(1-p)\theta(1-E_1)\mu_1\underline{V}_G \\ & +\gamma(1-p)(1-\theta)(1-E_i)\mu_i\alpha\underline{V}_G + (1-\gamma)p[(E_i + (1-E_i)\mu_i)\,\overline{V}_G \\ & +(1-E_i)\,(1-\mu_i)\,\overline{V}_R)] + (1-\gamma)(1-p)(1-E_i)\mu_i\underline{V}_G\} + (1-e)p\overline{V}_R.\end{aligned}$$

His effort generates a probability e of finding prior art. After making an effort e, with probability $(1-e)$ he does not find any prior art and does not file for a patent application. If his innovation is good, he nevertheless gets a positive benefit $\overline{V}_R$. Otherwise, with complementary probability e, he finds 1 with probability γ if prior art is abundant but only x_i if prior art is scarce (with probability $1-\gamma$). In the former case, with probability p, the innovation is good and it is granted a patent with probability $E_1 + (1-E_1)\mu_1$ but (wrongly) rejected with probability $(1-E_1)(1-\mu_1)$. A non patentable innovation with abundant prior art is submitted without concealment with probability θ and it is granted a patent with probability $(1-E_1)\mu_1$. The applicant can, with probability $1-\theta$, submit it fewer prior art than found ($\widetilde{x} = x_i$) and get a patent with probability $(1-E_i)\mu_i$. In the latter case, that is if x_i was found, a good innovation is awarded a patent with probability $E_i + (1-E_i)\mu_i$ and rejected with probability $(1-E_i)(1-\mu_i)$. Finally, a bad innovation with x_i goes through the screening process with probability $(1-E_i)\mu_i$.

From the maximization program of the applicant

$$\underset{e}{Max} \ \{\Pi(e) - c(e)\},$$

we derive the effort

$$e^*(\alpha) = Min \ \{1, e_1(\alpha) - \sigma(\alpha)\}, \tag{20}$$

where

$$e_1(\alpha) = p(1-p)\left[\gamma E_1^*(\alpha) + (1-\gamma)E_i^*(\alpha)\right]\Delta V + p^2\Delta V + p\underline{V}_G \tag{21}$$

and

$$\sigma(\alpha) = p\gamma(1-\gamma)(1-\alpha)\left[1 - E_i^*(\alpha) - \frac{1-E_1^*(\alpha)}{\alpha}\right](p\Delta V + \underline{V}_G). \tag{22}$$

As $\theta^* < 1$ and $\Delta V = \overline{V}_G - \underline{V}_G - \overline{V}_R$, it follows that $\sigma(\alpha) > 0$.

The optimal effort can be decomposed into the two expressions $e_1(\alpha)$ and $\sigma(\alpha)$. The first expression, $e_1(\alpha)$, which represents the baseline search incentives, is increasing with the overall effort expanded by the examiner. This arises because when examiners increase their effort, they more likely identify and grant patents to good innovations (i.e., those who have a strictly higher value patented than not). Therefore, if screening is expected to increase, it creates more incentives to search for applicants. The expression $\sigma(\alpha)$ represents the strategic component of the search effort, it affects the applicant's effort negatively. A decrease in α results in a loss (i.e., $(1-\alpha)$ increases) since prior art concealment is more costly. It is compounded with a lower probability that a (bad) innovation is granted a patent when it is misrepresented; the term $1 - E_i^* - (1-E_1^*)/\alpha$ represents the difference in errors across fields. When α decreases further this difference vanishes as strategic concealment is no longer attractive. This suggests that the curve of $\sigma(\alpha)$ has an inverted U-shape.

If $\alpha = 1$, the second term disappears as $\sigma(1) = 0$ and, therefore, $e^*(1) = e_1(1) = p(1-p)(\gamma E_1^*(1) + (1-\gamma)E_i^*(1))\Delta V + p^2\Delta V + p\underline{V}_G$.

The next result presents how search incentives are affected when α varies.

Lemma 3 *If $\alpha > \alpha_1$, the optimal effort level $e^*(.)$ is a convex function of α. Moreover, when $\gamma \leq \gamma_1 \equiv \frac{\alpha^3}{\alpha^3+x_i}$ and α is close to 1, $\frac{\partial\sigma(\alpha)}{\partial\alpha} > 0$.*

The convexity is essentially driven by the shape of the cost function of strategic reporting (22): it is low when α is close to 1 (i.e., the first term of (22) is small) or when there is no incentive to report strategically (i.e., when E_1^* is close enough to E_i^* and the second term of (22) is small). The search effort is thus maximized either when α is large (maximum strategic reporting) or when α is small (no strategic reporting). The second part of the lemma simply illustrates that a high level of impunity (α high) is in fact often associated with strong marginal incentives to search. Note that a necessary condition for this result to hold is that $\gamma \leq \gamma_1$. Indeed, when prior art is abundant overall (γ large), the examiner is biased *against* applications with low prior art content; the examiner does not ignore strategic concealment and applications with few prior art tend to be suspicious in the examiner's mind. As a result, strategic concealment is less attractive and this reduces innovator's incentives to search for prior art *ex ante.*

For $\alpha \leq \alpha_1$, the gross benefit function of the applicant does not account for any strategic report,

$$\begin{aligned}\Pi(e) = \quad & e[\gamma p((E_1 + (1-E_1)p)\overline{V}_G + (1-E_1)(1-p)\overline{V}_R)) + \gamma(1-p)(1-E_1)p\underline{V}_G \\ & +(1-\gamma)p((E_i + (1-E_i)p)\overline{V}_G + (1-E_i)(1-p)\overline{V}_R)) + (1-\gamma)(1-p)(1-E_i)p\underline{V}_G] \\ & +(1-e)p\overline{V}_R,\end{aligned}$$

and, therefore, the optimal level of effort for the applicant is

$$e_t^* = p(1-p)\left[\gamma E_{1t}^* + (1-\gamma)E_{it}^*\right]\Delta V + p^2\Delta V + p\underline{V}_G. \tag{23}$$

The expression (23) is identical to (21) but, not surprisingly perhaps, the strategic component (22) has disappeared. Similar to the case with strategic concealment, an increase in the relative value of a patented innovation ΔV tends to enhance incentives to search for prior art but clearly the applicant's effort no longer depends on α. Further, the effort of the applicant is sensitive to examiner efforts, as $\partial e^*/\partial E_j^* > 0$ for $j = 1, i$. The greater the efforts of the examiner, the more effort the applicant will put into his search for relevant prior art. This result again emphasizes the link between examiner and applicant search efforts.

The applicant's effort also varies with the amount of intermediate prior art that he can find, x_i, with the probability of having a good innovation, p, and the probability of abundant prior art, γ. To see how a change in these parameters affects the effort of the applicant, we need to differentiate equation (20) with respect to these variables, taking into account that examiner efforts will also vary with them. For instance, an increase in x_i induces the examiner to intensify her research effort E_i^*, whereas she devotes less effort to search for complementary prior art when she has received the full amount of prior art, $\partial E_1^*/\partial x_i < 0$. Thus, there are two effects that work in opposite directions. On the one hand, if the intermediate level of prior art increases due to an increase in the examiner's effort, the applicant intensifies his effort. But on the other hand, due to the decrease in the examiner's effort if 1 is reported, the applicant reduces his effort. It is not clear which effect is greater than the other, and we cannot conclude.

We summarize these findings in the following lemma.

Lemma 4 *If $\alpha \leq \alpha_1$ and holding everything else constant, the effort of the applicant e_t^**

- Increases with γ and ΔV.
- *increases with optimal levels of effort of the PTO examiner.*

Finally, we need to offer some comparison of the search effort exerted when the applicant is truthful and strategic.

Lemma 5 *If $p > \frac{1}{1+x_i}$, then $e^*(1) > e_t^*$ for any $\gamma \in (0, 1)$.*

This sufficient condition, which is (much) stronger than necessary, offers the simple illustration that when prior art concealment is not costly ($\alpha = 1$), search has *strategic value* and applicants can actually search more than if they were induced to be truthful. Thus, in certain circumstances applicants are likely to know well all the prior art related to their innovation. Patent regulators can use this to their advantage. For instance, a possible way to elicit this information is to favor open review procedures where competitors can challenge patents prior to their issuance by submitting prior art references.[20]

In the next two sections, we study two possible solutions for remedying prior art concealment.

4 Second Pair of Eyes Review

In March 2000, in reaction to numerous quality-related criticisms about the granting of business-method patents (main class 705), the PTO began a quality patent improvement initiative involving several measures, such as the hiring of additional, better-trained examiners, the obligation for them to consult non-patent prior art information sources and, perhaps most importantly, a second-level examination applied only to patents granted within the main 705 classification. The effects of this initiative, called the "Second Pair of Eyes Review," have been empirically studied in Allison and Hunter (2006). They first argue that an applicant endowed with a business-method innovation has substantial latitude to choose whether to submit it in the main class (705) or in other main classes related to business methods (i.e., with secondary 705 classification). Then, they empirically analyze the composition of patent applications before and after the SPER initiative.

We develop a simple variant of our model that incorporates the Second Pair of Eyes Review (SPER) to investigate its impact on the patenting process and the behavior of applicants. We model the SPER initiative by allowing for a second review of awarded patents when prior art is scarce. In our context, innovations can be thought to belong to two technologically related fields of innovation. In one of them, there is less prior art to be found and, at the cost of strategic drafting, applicants can "opt out" the field where there is abundant prior art (see footnote 5). To simplify the analysis, we assume that when a patent is granted to a good innovation the second review does not invalidate it, whereas it eliminates a fraction $\beta \in [0, 1]$ of wrongly patented innovations during the first review. In this setup, a bad applicant who anticipates the second

[20]Interestingly, the PTO has recently opened the patent examination process to third parties. Any party can submit related prior art online through a pilot program called "Peer-to-Patent" (http://dotank.nyls.edu/communitypatent accessed on 05/25/2209). In Europe, open review procedures are one building block of the patenting process (Friebel, et al., 2006).

review chooses θ such that

$$\mu_1(\theta)\,(1-E_1) = \mu_i(\theta)\,(1-E_i)\,(1-\beta)\,\alpha.$$

By solving this last equality we obtain

$$\theta^*_{SPER} = \frac{(1-E_1)(1-p\gamma)-(1-\gamma)p(1-E_i)\alpha(1-\beta)}{(1-p)[(1-\gamma)(1-\beta)(1-E_i)\alpha+\gamma(1-E_1)]}.$$

Note that although β can take any value between 0 and 1, any $\beta \geq \overline{\beta}$ where $\overline{\beta} = 1-(1-E_1)/[(1-E_i)\alpha]$ induces $\theta^*_{SPER} = 1$ (no strategic revelation), whereas for $\beta \in \left[0, \overline{\beta}\right]$ we have $\theta^*_{SPER} \in [\theta^*, 1]$. It is straightforward to show that θ^*_{SPER} increases with β, so that an increase in the precision in the second review reduces the strategic concealment of information.

If $\overline{W}_G = \underline{W}_R$ and $\alpha > \alpha_1$, the equilibrium efforts of the examiner under the SPER regime are

$$E_1^{SPER} = 1 - \left[\frac{K\left[x_i(1-p(1-\gamma))\Delta W^{SPER}-(1-\gamma)p\alpha^2\Delta W(1-\beta)^2\right]}{p\gamma x_i(1-p)\Delta W \Delta W^{SPER}}\right]^{\frac{1}{2}}, \tag{24}$$

$$E_i^{SPER} = 1 - \left[\frac{K[(1-\beta)^2(1-p\gamma)\alpha^2\Delta W - x_i p\gamma\Delta W^{SPER}]}{p x_i(1-\gamma)(1-p)\alpha^2\Delta W\Delta W^{SPER}(1-\beta)^2}\right]^{\frac{1}{2}}, \tag{25}$$

where $\Delta W^{SPER} = \overline{W}_G - \overline{W}_R + (1-\beta)(\underline{W}_R - \underline{W}_G)$.

A comparison of the effort levels leads to the following findings.

Lemma 6 *If $\alpha > \alpha_1$, as β increases, the examiner reduces her scrutiny effort when she receives the full amount of prior art, whereas she increases her effort when she receives the intermediate amount. Furthermore, she exerts more effort when she receives more information, $E_1^{SPER} > E_i^{SPER}$ for $\beta < \overline{\beta}$.*

The implementation of the SPER initiative reduces the strategic reporting of information by allowing the examiner to intensify her search effort when she receives x_i and to reduce it whenever she gets more information.

For the sake of simplicity, we do not take into account the cost of implementing the SPER policy. In fact, introducing a second examination should be more costly to the PTO. However, the introduction of such cost would make our model intractable and would not change the flavor of the results. Indeed, our findings would be more dramatic, as it would make the screening of patents with abundant prior art costly and, therefore, would reduce the examiner's efforts when prior art is abundant.

To analyze the effects of the SPER initiative on patents, we compare the number of patent applications and granted patents for each type of application, as well as their quality, before and after the implementation of the SPER initiative. Our findings are summarized in the following proposition:

Proposition 3 *If $\alpha > \alpha_1$, for any $\beta > 0$, the implementation of the SPER initiative has the following effects:*

1. *Fewer patent applications with low prior art content are scrutinized;*

2. *More patent applications with high prior art content are scrutinized;*

3. *Fewer patents with abundant prior art are granted;*

4. *The ratio of good patented innovations over all patented innovations decreases (respectively, increases) for applications with abundant prior art (respectively, scarce prior art).*

These findings lead us to propose several potential testable assumptions for a reform in the spirit of SPER. Some of these hypotheses have already been explored in Allison and Hunter (2006). For instance, they show that there is a sharp decrease in the proportion of main-class 705 patents relative to the total class 705 patents (i.e., main plus secondary) in the years following the implementation of the SPER initiative.[21] They also show that the number of patents granted in the main 705 classification increased substantially after the implementation of the SPER initiative. Finally, they analyze the content of several patents granted in secondary class 705 and show that an unusual proportion of them would potentially fall in the main 705 patent category.[22]

We also consider the effect of the SPER initiative on the effort of the applicant, as derived in the previous section. For the applicant, efforts in both type of application are important in determining his incentives to search for prior art. Thus, as evidenced by equations (16) and (17), any reform that increases the search effort of examiners in one field is likely to decrease

[21] This is also true in our model.

[22] Unfortunately, the authors do not have data on validity to test whether, as suggested by our model, these patents are of inferior quality.

the search effort in another field (provided applicants engage in strategic drafting). From a theoretical standpoint, the effect on applicant search efforts is unclear. At the least, we expect applicants' efforts to change when such a reform is introduced. This would be consistent with other findings of Allison and Hunter (2006), that after SPER, the amount of prior art disclosed in patent applications is significantly higher in the main 705 classification, and also in patents with secondary 705 classification. In our model, rather than the amount of prior art discovered per patent, an increase in examiner effort increases the fraction of innovations for which substantial prior art is discovered; that is, there are more patents with $x \in \{x_i, 1\}$ than patents with no prior art.[23]

5 *Ex ante* commitment of the PTO

In the previous sections, we have assumed that the examiner is passive in the sense that her scrutiny level is a (optimal) response to every application she receives. While this may be realistic in many cases, we need to look at the opposite case where the representative examiner commits to specific levels of scrutiny that are contingent on the prior art reported. In other words, we are analyzing a situation in which the PTO assumes "leadership" with an *ex ante* commitment policy where *applicants search effort and concealment strategy are now responses to this commitment.*

We denote by E_{1c} and E_{ic} the levels of effort of the examiner when, respectively, the full amount and the intermediate amount of prior art are provided and, to simplify, we assume that concealment is costless ($\alpha = 1$). Two qualitatively different cases must be considered: whether $E_{ic} \geq E_{1c}$ or $E_{ic} < E_{1c}$.

We first consider the case where $E_{ic} < E_{1c}$. By finding the full amount of prior art, the applicant learns the type of his innovation. Therefore, a good applicant will report all prior art, whereas a bad applicant may decide to transmit only the intermediate amount. The next result is the first step in finding the optimal commitment policy of the PTO.

Lemma 7 *It is always possible to find a policy that strictly dominates a policy in which the examiner ex ante commits to levels of scrutiny* (E_{ic}, E_{1c}) *such that* $E_{ic} < E_{1c}$.

[23] In our model, for instance, we could assume that an increase in e also increases the quantities x_i and $\overline{x}$.

A policy that involves $E_{ic} < E_{1c}$ has two major drawbacks. First, it is cost-dominated by another policy with a lower gap between E_{1c} and E_{ic} (see appendix). Intuitively, reducing E_{1c} does not change the optimal reporting strategy (all applicants optimally conceal prior art) but it reduces scrutiny costs. A more subtle problem with such a policy is that the commitment problem of the examiner is very intense when $E_{ic} < E_{1c}$ simply because the gains of reneging on her commitment are high. Indeed, in this context, the examiner would gain $C(E_{1c})$ monetary units by reneging on her commitment, without altering the quality of screening.

The commitment policy involving $E_{ic} \geq E_{1c}$ induces truthful revelation of information. However, the underlying commitment problem alluded above is much less intense. Indeed, in a truthtelling equilibrium, it is certain that (E_{ic}, E_{1c}) are *ex post* inefficient but the gains from reneging on her commitment (i.e., switching to E_{1t}^{*} and E_{it}^{*}) are much lower. Thus, compared to the case $E_{ic} < E_{1c}$, the examiner faces a low-intensity commitment problem when $E_{ic} \geq E_{1c}$. We now proceed to the analysis of this case.

When $E_{1c} = E_{ic}$, the applicant who learns the nature of his innovation is indifferent between reporting $\widetilde{x} = 1$ (truthful reporting) and reporting only part of the found prior art (strategic reporting). For definitiveness, we make the assumption that when the applicant is indifferent, he reports truthfully. Given that the applicant is induced to be truthful, and if the applicant reports some prior art information, the gross benefit function of the examiner is

$$B_c\left(E_{1c}, E_{ic}\right) = \gamma B(E_{1c}; p) + (1-\gamma) B(E_{ic}; p),$$

where c stands for commitment, and $B(E_{jc}; p)$ for $j = 1, i$ is defined by equation (3).

The *ex ante* commitment policy inducing information search and truthful revelation by the applicant is the solution of the following program:

$$\max_{E_{1c}, E_{ic}} \{e\left(E_{1c}, E_{ic}\right)\left[B_c\left(E_{1c}, E_{ic}\right) - \gamma \tfrac{K}{(1-E_{1c})} - (1-\gamma)\tfrac{K}{x_i(1-E_{ic})}\right]\},$$

subject to

$$E_{1c} \leq E_{ic}. \tag{26}$$

The next result simplifies the analysis of the examiner's program when she wants to induce truthful reports.

Proposition 4 *If the examiner wants to induce truthful reports, the unique optimal commitment policy is such that $E_{1c} = E_{ic}$.*

We know that, if they exist, the *ex post* efficient levels of effort are such that $E_1^* > E_i^*$. By committing to levels of effort *ex ante*, the PTO would like to reduce *ex post* inefficiencies as much as possible. More precisely, the examiner would like to raise E_{1c} as much as possible. Therefore, among all of the possible levels of effort that satisfy $E_{1c} \leq E_{ic}$, those that minimize *ex post* inefficiency are such that $E_{1c} = E_{ic} = E_c$.

Using this result, the program of the examiner can now be stated as

$$\max_{E_c}\{e\left(E_c\right)\left[B_c\left(E_c\right) - \tfrac{\gamma K}{(1-E_c)} - \tfrac{(1-\gamma)K}{x_i(1-E_c)}\right]\},$$

where

$$e\left(E_c\right) = p[\Delta V(1-p)E_c + p\Delta V + \underline{V}_G]$$

and $B(E_{1c};p) = B(E_{ic};p) = B(E_c;p)$; thus,

$$B_c\left(E_c\right) = -p(1-p)\Delta W(1-E_c) + p(\overline{W}_G - \underline{W}_R) + \underline{W}_R.$$

We cannot obtain an analytical solution due to the complexity of the *ex ante* program. However, we can state the following result.

Corollary 4 *The applicant increases his search effort in the commitment case, i.e.,* $e\left(E_c\right) > e_t^*$.

The levels of effort of the examiner when they are performed *ex post* generate too few incentives on the part of the applicants. By committing to effort *ex ante,* the patent examiner can threaten to lower the applicant's benefit. The applicant is thus led to increase his search of prior art. The commitment thus helps the examiner to shift, to some extent, the burden of the information search to applicants.

The comparison between the optimal efforts in the commitment case and in the non-commitment case leads to $E_i^* < E_c < E_1^*$ when the uncertainty about the value of the innovation and the field it belongs to is the highest, i.e., for some constellation of parameters (p, γ) (see appendix). The commitment case allows the patent examiner to intensify (respectively, reduce) her search effort when she receives less (respectively, more) prior art, because there is no longer strategic revelation of information.

When there is an *ex ante* commitment policy, the ratio of good patented innovations patents over all patented innovations is the same across all type of applications while, obviously, when there is no commitment, the quality is not homogenous across all type of applications.

6 Conclusion

Patent examiners must assess the patentability of an innovation by comparing the application to related information that is already in the public domain, called prior art information. When the initial application contains little prior art information, and when the prior art is not easily accessible to examiners, it may be difficult to judge the novel content of an innovation. Prior art information may be more difficult for examiners to gather when the innovation belongs to an emerging technological field and much of the prior art is non-patent information. Sampat (2009) shows that patent examiners' abilities to locate prior art embodied in U.S. patents exceed their ability to locate other types of prior art (non-patent). Therefore, if most of the prior art is not related to U.S. patents, examiners may be unable to assess the patentability of an innovation. This, in turn, may lead to a reduction in patent quality, as more questionable patents are issued.

In the current U.S. patent system, while applying for a patent, an applicant is supposed to disclose the prior art information of which he is aware, but is not required to search for more prior art. Thus, applicants have little incentive to reveal the information they have and to search for more.

We analyze the issues related to the gathering of prior art information. In our setting, applicants decide to apply for patent protection after gaining private information about the value of their innovation. Their decisions depend both on the private information they gather and the patent policy adopted by the PTO. After receiving an application, the patent examiner undertakes a costly search effort that depends on her beliefs about the value of the innovation. To be as realistic as possible, we allow the examiner to make two kinds of mistakes: to refuse a patent on a good innovation and to grant a patent to a bad innovation.

We consider different policies: one in which the PTO cannot commit to any search effort, one in which there is a second review for certain type of application, and one in which the examiner *ex ante* commits to a screening intensity. In the former case, an applicant who has learned he has a non-patentable innovation might conceal some information from the examiner to increase his probability of being granted a patent. The optimal policy of the examiner is to intensify her scrutiny effort when she receives more prior art. Therefore, she devotes more effort to those applications that are more likely to come from good applicants. In the second case, the introduction of a more thorough screening when prior art is scarce reduces the strategic non-revelation of information, but its overall success remains unclear. Indeed, the imposition of a stricter screening when prior art is scarce may improve the quality of patents in this field, but will negatively affect the quality of patents in another field. We then go further into our investigation of policy remedies, and, in the latter case, we show that by *ex ante* committing to the same screening effort across all types of application, the examiner induces truthful revelation of information.

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Appendix

Proof of Lemma 1

We first determine for which values of prior beliefs the examiner makes a complementary search effort, depending on the prior art transmitted. Second, we compare the different levels of effort.

The optimal efforts

$$E^*_{\widetilde{x}t}(p) = 1 - (\tfrac{K}{p(1-p)\widetilde{x}\Delta W})^{\frac{1}{2}},$$

for $\widetilde{x} = x_i, 1$ are first increasing and then decreasing with p. As the second derivative is negative, the benefit function is concave and reaches a maximum for $p = 1/2$ (the last part of Lemma 1 is proved). We now define for what values of p the efforts are such that $E^*_{\widetilde{x}t}(p) = 0$. These values belong to the interval $[0, 1]$ as the limits of $E^*_{\widetilde{x}t}(p)$ are $-\infty$ when p tends toward 0 or 1, and are

$$\begin{aligned} \underline{p}_{\widetilde{x}} &= \tfrac{\widetilde{x}\Delta W - [(\widetilde{x}\Delta W)^2 - 4\widetilde{x}K\Delta W]^{\frac{1}{2}}}{2\widetilde{x}\Delta W}, \\ \overline{p}_{\widetilde{x}} &= \tfrac{\Delta W + [(\widetilde{x}\Delta W)^2 - 4\widetilde{x}K\Delta W]^{\frac{1}{2}}}{2\widetilde{x}\Delta W}, \end{aligned}$$

if $K < \widetilde{x}\Delta W/4$. It is easy to verify that $\underline{p}_1 < \underline{p}_i < \frac{1}{2} < \overline{p}_i < \overline{p}_1$ where $\underline{p}_i \equiv \underline{p}_{x_i}$ and $\overline{p}_i \equiv \overline{p}_{x_i}$. Hence, for values of $p \in]0, \underline{p}_1[\cup]\overline{p}_1, 1[$, the examiner does not make any effort. For values of $p \in]\underline{p}_1, \underline{p}_i[\cup]\underline{p}_i, \overline{p}_1[$, she does not make any effort after receiving x_i, but makes the effort $E^*_{1t} > 0$ after receiving $\widetilde{x} = 1$. And finally, for values of $p \in]\underline{p}_i, \overline{p}_i[$, she makes efforts E^*_{1t} and E^*_{it}. It is straightforward to show that $E^*_{1t} > E^*_{it}$, as $x_i < 1$.

Proof of Lemma 2: No equilibrium in pure strategies for $\alpha > \alpha_1$

Another pure strategy to consider is the one in which the same applicant always conceals part of the prior art found and reveals only an intermediate amount. To see whether this can be an equilibrium, we need to compute the updated beliefs of the PTO.

Consider first that the PTO believes that a bad applicant who finds $x = 1$ always conceals some information. The set of beliefs consistent with this behavior is computed as

$$\begin{aligned} &\mu_1 = \Pr(good \mid \widetilde{x} = 1) = \tfrac{p\gamma}{p\gamma} = 1, \\ &\mu_i = \Pr(good \mid \widetilde{x} = x_i) = p\tfrac{1-\gamma}{1-\gamma p} < p, \\ &\Pr(good \mid \widetilde{x} = 0) = p. \end{aligned}$$

If she receives the maximum amount of prior art, the PTO prefers not to make any complementary search effort, as only an applicant with a good innovation will reveal the full amount. If she observes the intermediate level, the first order condition (4) gives

$$E_{il}^* = 1 - \left[\frac{K}{\mu_i(1-\mu_i)x_i\Delta W}\right]^{\frac{1}{2}} < 1,$$

as long as $(p, \gamma) \in \Psi_l \equiv]\underline{p}(\gamma), \overline{p}(\gamma)[$ where

$$\begin{aligned}\underline{p}(\gamma) &= \frac{(1-\gamma)x_i\Delta W + 2K\gamma - [((1-\gamma)x_i\Delta W + 2K\gamma)^2 - 4K((1-\gamma)x_i\Delta W + \gamma^2 K)]^{\frac{1}{2}}}{2[(1-\gamma)x_i\Delta W + \gamma^2 K]}, \\ \overline{p}(\gamma) &= \frac{(1-\gamma)x_i\Delta W + 2K\gamma + [((1-\gamma)x_i\Delta W + 2K\gamma)^2 - 4K((1-\gamma)x_i\Delta W + \gamma^2 K)]^{\frac{1}{2}}}{2[(1-\gamma)x_i\Delta W + \gamma^2 K]}.\end{aligned}$$

We then show that $\underline{p}(\gamma)$ and $\overline{p}(\gamma)$ are increasing with γ. Therefore, as long as $(p, \gamma) \in \Psi_l$, the examiner makes a strictly positive effort, and $E_{il}^* > E_{1l}^* = 0$. Thus, unlike the previous case, revealing an intermediate amount yields more scrutiny effort from the examiner. If $(p, \gamma) \in \Psi_l$, not only must uncertainty as to the value of the innovation be high enough, uncertainty about the prior art field must also be high. A maximum is reached for $p = 1/(2-\gamma) > 1/2$ that solves $dE_{il}^*/dp = 0$. Otherwise, if $(p, \gamma) \notin \Psi_l$, the efforts are such that $E_i^* = E_1^* = 0$.

Can this strategy be part of an equilibrium? A good applicant has no incentive to deviate from revealing the truth, as the examiner infers that only good applicants reveal the truth. Formally, inequality (6) is always satisfied, as $\overline{V}_G > \overline{V}_R$. Nevertheless, a bad applicant is expected to always conceal some prior art. Thus, we should have $(1 - E_i^*)\mu_i\alpha > (1 - E_1^*)\mu_1$. Because $E_1^* = 0$ and $\mu_1 = 1$, this inequality is equivalent to $(1 - E_i^*)\mu_i\alpha > 1$, which is never satisfied. Therefore, the applicant may decide to report truthfully in order to fool the examiner, who believes that only good applicants report the full amount of prior art. By reporting as expected, a bad applicant can get a patent by chance, when the examiner grants a patent based on her updated beliefs. If he now decides to report all of the prior art he found, he definitively obtains a patent, as the examiner believes he has a good innovation and reports truthfully. This confirms that this is not an equilibrium.

Using the same argument, we show that the examiner will not make any effort when she observes the full amount of prior art, but she will not give a patent, either. When she observes the intermediate level of prior art, she makes a positive effort. A good applicant will not deviate, whereas one who has a bad innovation will deviate, as he can get a patent by fooling the examiner. This is not an equilibrium. The examiner can also believe that no applicants will report truthfully. By the same token we show that this cannot be an equilibrium.

Concavity of the benefit functions of the PTO

In the case of mixed strategies, we show that the benefit functions are concave in their own argument (namely, E_1 and E_i). If the examiner receives $\widetilde{x} = 1$ or $\widetilde{x} = x_i$, her payoffs are respectively $B(E_1; \mu_1) - K/\,(1 - E_1)$ and $B(E_i; \mu_i) - K/x_i\,(1 - E_i)$, where the gross benefit functions $B(.)$ are defined by (3), and μ_1 and μ_i are defined by equations (12) and (13). After rewriting these benefit functions, we calculate the first and second derivatives. When the examiner receives $\widetilde{x} = 1$ (we find a similar equation for $\widetilde{x} = x_i$), the second order condition is

$$-\frac{K-p^2(1-E_i)^2(1-\gamma)^2\Delta W-p(1-E_i)(1-\gamma)(\overline{W}_G-\underline{W}_R)}{(1-E_1)^3},$$

which is always negative at the equilibrium values. Therefore, the benefit functions are locally concave. This is the first step of the proof of the existence of solutions.

Subgame Perfect Bayesian Equilibrium

There exists a semi-separating equilibrium in which a bad applicant randomizes his revelation decision. Equations (10), (16) and (17) must be satisfied. From the first equation, $\mu_1(\theta)(1 - E_1) = \mu_i(\theta)(1 - E_i)\alpha$, we derive a unique $\theta^* \in [0, 1]$,

$$\theta^* = \frac{(1-E_1)(1-p\gamma)-(1-\gamma)p(1-E_i)\alpha}{(1-p)[(1-\gamma)(1-E_i)\alpha+\gamma(1-E_1)]},$$

where $\theta^* < 1$ is always satisfied if $E_1^* > E_i^*$, and $\theta^* > 0$ if $p < \overline{p}_\theta(\gamma) = (1 - E_1)/((1 - \gamma)(1 - E_i) + \gamma\,(1 - E_1))$. This last condition is equivalent to checking that $\mu_1(\theta^*) < 1$. Assume for the moment that $E_1^* > E_i^*$ and that $p < \overline{p}_\theta(\gamma)$ (we will determine under what circumstances this condition holds in the last part of the proof).

To prove the last part of the Lemma 3 we calculate the following derivatives $\partial\theta^*/\partial p < 0$, $\partial\theta^*/\partial\gamma > 0$, $\partial\theta^*/\partial E_1 < 0$, and $\partial\theta^*/\partial E_i > 0$.

Proof of Proposition 1: Semi-separating equilibrium

We need to check that

i. there exists a solution (E_1^*, E_i^*), where $E_1^* \in [0, 1]$ and $E_i^* \in [0, 1]$,

ii. this solution is unique in the interval considered.

Before doing so, recall that the two functions $E_1(E_i)$ and $E_i(E_1)$ are decreasing, and thus we rewrite them as

$$\begin{aligned} E_1(E_i) &= 1 - \phi_1(E_i)^{\frac{1}{2}}, \\ E_i(E_1) &= 1 - \phi_i(E_1)^{\frac{1}{2}}. \end{aligned}$$

By analogy to the Cournot model, we call these expressions 'reaction functions'. Some assumptions are crucial in order to have solutions; in particular, to insure real solutions, we need to impose that $\phi_1(E_i) \geq 0$ and $\phi_i(E_1) \geq 0$. So the crucial assumptions are $E_{\widetilde{x}}(1) < 1$ (or, equivalently, $K > 0$) and $E_{\widetilde{x}}(0) < 1$ (to insure that $\phi_{\widetilde{x}}(0) \geq 0$). Let us now detail the analysis of the existence and unicity of positive solutions.

i. <u>Existence</u>

We know that the benefit functions of the PTO are concave. We then have to prove that the solutions are positive. Ideally, we need to have $E_1^{-1}(0) > E_i(0)$ and $E_i^{-1}(0) > E_1(0)$. However, because of the complexity of the reaction functions, we will not explicitly find the constellation of parameters (γ, p) such that these two conditions hold.

We first consider a weaker condition. According to the reaction functions, if $E_1(1) > 0$ and $E_i(1) > 0$, then $E_1(0) > 0$ and $E_i(0) > 0$ and, thus, there exist interior solutions between 0 and 1. We can easily derive a constellation of parameters such that these two conditions are satisfied. However, these conditions are very restrictive, and if one, or even both, of them are not satisfied, we can still have positive solutions (we will consider that problem later). It is easy to check that $E_1(1) > 0$ if $p \in [r_1'(\gamma), r_1''(\gamma)]$ where $r_1'(\gamma) = [\Delta W - \sqrt{(\Delta W)^2 - 4K\Delta W}]/2\gamma\Delta W$ and $r_1''(\gamma) = [\Delta W + \sqrt{(\Delta W)^2 - 4K\Delta W}]/2\gamma\Delta W$. Similarly, $E_i(1) > 0$ is satisfied if $p \in [r_i'(\gamma), r_i''(\gamma)]$, where $r_i'(\gamma) = [x_i\Delta W - \sqrt{(x_i\Delta W)^2 - 4x_iK\Delta W}]/2(1-\gamma)x_i\Delta W$ and $r_i''(\gamma) = [x_i\Delta W + \sqrt{(x_i\Delta W)^2 - 4x_iK\Delta W}]/2(1-\gamma)x_i\Delta W$. We represent these functions in a graph (γ, p) (figure 1). Thus, for a constellation of parameters $(\gamma, p) \in \{(\gamma, p)/p \geq r_i'(\gamma), p \geq r_1'(\gamma)\}$, there exists a positive set of efforts. However, on the left of $r_i'(\gamma)$, for instance, we can still have a candidate pair. So the constellation (γ, p) such that $p = r_i'(\gamma)$ corresponds to $E_i(1) = 0$. For a given p, the limit $E_i(1)$ decreases as γ increases (i.e., $\partial E_i(1)/\partial\gamma < 0$), for $p < 1/2(1-\gamma)$. Hence, there exists a function $R_i(\gamma)$ such that for $p \leq R_i(\gamma)$, the optimal solution is $E_i^* \leq 0$. If

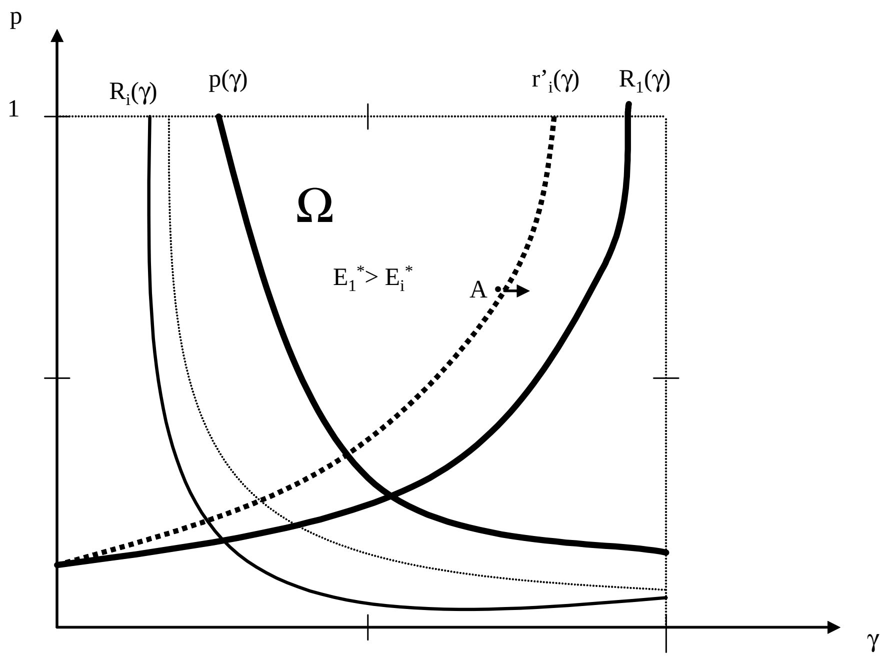

Figure 1:

we start from point A in figure 1 and increase γ, first the optimal solution can still be positive, but as the limit decreases there is one value of γ for which the optimal solution becomes negative. By the same token, the same reasoning applies to $E_1(1)$, and a function $R_1(\gamma)$ exists such that for $p < R_1(\gamma)$, the optimal solution is $E_1^* < 0$.

We have defined explicit functions, $R_i(\gamma)$ and $R_1(\gamma)$, such that there exists a pair of efforts $(E_1^*, E_i^*) \in [0,1] \times [0,1]$, that we represent in figure 1 for a constellation of parameters $(\gamma, p) \in \Upsilon = \{(\gamma, p) \mid p \geq R_i(\gamma), p \geq R_1(\gamma)\}$ (a set that is less restrictive than the previous set).

For some values of p, $E_1^* > 0$ is satisfied, whereas $E_i^* > 0$ is not. In other words, the examiner makes an effort E_1 after observing the maximum prior art, but no effort after observing the intermediate level. Equation (10) becomes $\mu_1(\theta)(1 - E_1) = \mu_i(\theta)$ and we derive

$$\theta_1^* = \frac{1-p-E_1(1-\gamma p)}{(1-p)(1-E_1\gamma)},$$

that belongs to $[0,1]$ if $p < \overline{p_1} = \frac{1-E_1}{1-E_1\gamma}$. We check that $\overline{p_1} > r_1''$, and thus $\theta_1^* \in [0,1]$.

For another constellation of parameters, none of the inequalities $E_1^* > 0$ and $E_i^* > 0$ are satisfied. Thus, the examiner will not make any effort and the applicant is indifferent if $\mu_1(\theta) =$

$\mu_i(\theta)$, which is only satisfied for $\theta = 1$. This is actually the equilibrium in pure strategies that we defined earlier.

ii. Uniqueness

To show that the solution is unique, we need to show that

$$\left|\frac{\partial^2(B_1(.)-C(.))}{\partial E_1^2}\right| > \left|\frac{\partial^2(B_1(.)-C(.))}{\partial E_1 \partial E_i}\right|.$$

After computation of these two second derivatives, a simple comparison shows that this is true as long as $E_1^* > E_i^*$. Thus, we have shown that there exists a unique pair of solutions $(E_1^*, E_i^*) \in [0,1] \times [0,1]$ for $(\gamma, p) \in \Upsilon$.

So far, we have assumed that $E_1^* > E_i^*$. We now have to show for which constellation of parameters (p, γ) the inequality $E_1^* > E_i^*$ holds, inside the constellation of parameters Υ. Let us first re-define the functions $E_1(E_i)$ and $E_i(E_1)$ as

$$\begin{aligned} E_1(E_i) &= 1 - \phi_1(E_i, p, \gamma)^{\frac{1}{2}}, \\ E_i(E_1) &= 1 - \phi_i(E_1, p, \gamma)^{\frac{1}{2}}. \end{aligned}$$

We then define the value of E_1 (respectively, E_i) when the reaction function $E_1(E_i)$ (respectively, $E_i(E_1)$) cuts the function $E_i = E_1$. In other words, we determine E such that $E_1(E) = E$, which is $(1-E)^2 = \phi_1(E, p, \gamma)$, and $E_i(E) = E$, which is $(1-E)^2 = \phi_i(E, p, \gamma)$. As $E_1 = E_i$, there exists a value of E such that $\phi_1(E, p, \gamma) - \phi_i(E, p, \gamma) = 0$. The solution of this equation is $E(p, \gamma)$. If we plug this solution back into the previous equation we have $\phi_1(E(p, \gamma), p, \gamma) - \phi_i(E(p, \gamma), p, \gamma) = 0 \equiv G(p, \gamma)$ that only depends on p and γ. By totally differentiating $G(.)$, we determine the sign of the function $p(\gamma)$ that represents all the values of (p, γ) $(p = p(\gamma))$ such that $E_1^* = E_i^*$. Indeed,

$$\frac{dp}{d\gamma} = -\frac{\partial G}{\partial \gamma} / \frac{\partial G}{\partial p}.$$

Therefore, $dp/d\gamma < 0$, as long as $sign(\partial G/\partial \gamma) = sign(\partial G/\partial p)$.

So we have defined the function $p(\gamma)$ such that $E_1^* = E_i^* = E$. Let us depart from this point. Assume that $E_1^* = E + \varepsilon$, where ε is small. We have $(1-E)^2 + \varepsilon = \phi_1(E, p, \gamma)$ and $(1-E)^2 = \phi_i(E, p, \gamma)$, that gives $\phi_1(E(p, \gamma), p, \gamma) + \varepsilon - \phi_i(E(p, \gamma), p, \gamma) = 0$. Hence, $E_1^* > E_i^*$ for $p > p(\gamma)$.

Lastly we must check that $p < \overline{p}_\theta(\gamma)$, in order to get an equilibrium (i.e., $\theta^* > 0$). Thus, p must be larger than $p(\gamma)$, as well as smaller than $\overline{p}_\theta(\gamma)$, so $(\gamma, p) \in \Psi = \{\gamma, p/p > p(\gamma)$ and $p < \overline{p}_\theta(\gamma)\}$.

Therefore, there exists a unique pair of solutions $(E_1^*, E_i^*) \in [0,1] \times [0,1]$ and $\theta^* \in [0,1]$, such that $E_1^* > E_i^*$ for $(\gamma, p) \in \Upsilon \cap \Psi \equiv \Omega$.

Comparative statics in the general case

In the absence of an analytical formulation for efforts, we need to study how the functions $E_1^*(E_i)$ and $E_i^*(E_1)$ are affected by a change in one parameter in order to determine the impact of the change of this parameter on the equilibrium values. Let us call this parameter y, which can be ΔW (with $\overline{W}_G - \underline{W}_R$=constant), p, x_i, γ and K. Let us rewrite $E_1^*(E_i) = 1 - \sqrt{\phi_1(y)}$ and $E_i^*(E_1) = 1 - \sqrt{\phi_i(y)}$, where $\phi_1(y)$ and $\phi_i(y)$ are defined in equation (16) and (17). Thus, $\frac{dE_j^*}{dy} = -\frac{1}{2}(\phi_j(y))^{-\frac{1}{2}}\phi_j'(y)$, where $j = 1, i$. We show that $\phi_1'(y) < 0$ for $y = \Delta W, p$ (for $p < 1/2\gamma$), $\phi_1'(y) = 0$ for $y = x_i$, and $\phi_1'(y) > 0$ for $y = K, \gamma$ (for $\gamma > 1/2p$). The function $E_1^*(E_i)$ is decreasing (respectively, increasing) with K, γ (respectively, $\Delta W, p$) and is constant with x_i. By the same token, we find that $\phi_i'(y) < 0$ (respectively, positive) for $y = \Delta W, x_i, p$ (for $p < 1/2$), and γ (for $\gamma < (2p-1)/2p$) (respectively, $y = K$). The function $E_i^*(E_1)$ is increasing with $\Delta W, x_i, p$, γ (respectively, decreasing with K).

Proof of Corollary 1. Case where $\overline{W}_G = \underline{W}_R$: Necessary condition for interior solutions

Showing that $E_1^* > E_i^*$, where E_1^* is defined by (18) and E_i^* by (19), is equivalent to showing that

$$p(1-2\gamma) > \tfrac{x_i}{1+x_i} - \gamma.$$

If $1 - 2\gamma > 0$, then

$$p < \frac{\gamma - \frac{x_i}{1+x_i}}{2\gamma - 1} \equiv h_1(\gamma),$$

and if $1 - 2\gamma < 0$, then

$$p > \frac{\frac{x_i}{1+x_i} - \gamma}{1 - 2\gamma} \equiv h_2(\gamma).$$

Therefore, $E_1^* > E_i^*$ for $(p, \gamma) \in \Gamma = \{(p, \gamma) \mid (1 + x_i)(p(1-2\gamma) + \gamma) - x_i > 0\}$.

Proof of Corollary 2

The proof consists in the calculation of the derivatives of E_1^* and E_i^* with respect to p, γ.

Proof of Corollary 3

The result of Corollary 3 is trivial for $\alpha \leq \alpha_1$ from the calculation of the derivatives $\partial E_{1t}^*/\partial \underline{W}_G < 0$ and $\partial E_{it}^*/\partial \underline{W}_G < 0$.

Proof of Lemma 3

$\underline{\text{Convexity of } e^*(\alpha) = e_1^*(\alpha) - \sigma(\alpha)}$: We first prove that

$$\frac{d^2 e_1(\alpha)}{d\alpha^2} = \gamma \frac{\partial^2 E_1^*}{\partial \alpha^2} + (1-\gamma)\frac{\partial^2 E_i^*}{\partial \alpha^2} > 0.$$

Simple calculations show that

$$\frac{\partial^2 E_1^*}{\partial \alpha^2} = \frac{(1-\gamma)(1+p\gamma-p)\left(1-E_1^*\right)Kpx_i}{\left[Kx_i(1-p+p\gamma)-p(1-\gamma)\alpha^2\right]^2} > 0,$$

and

$$\frac{\partial^2 E_i^*}{\partial \alpha^2} = \frac{-K\gamma\left[\alpha\frac{\partial E_i}{\partial \alpha}-3(1-E_i)\right]}{\alpha^4(1-\gamma)(1-p)\Delta(1-E_i)^2 W} > 0 \text{ as } \frac{\partial E_i^*}{\partial \alpha} = -\frac{K\gamma}{\alpha^3(1-\gamma)(1-p)\left(1-E_i^*\right)\Delta W} < 0.$$

Then, we compute

$$\frac{d^2\sigma(\alpha)}{d\alpha^2} = p\gamma(1-\gamma)(p\Delta V + \underline{V}_G)[\frac{-2(1-E_1)-2\alpha\frac{\partial E_1}{\partial \alpha}+2\alpha^3\frac{\partial E_i}{\partial \alpha}+\alpha^2(1-\alpha)\left[\frac{\partial^2 E_1}{\partial \alpha^2}-\alpha\frac{\partial^2 E_i}{\partial \alpha^2}\right]}{\alpha^3}] < 0,$$

This expression is negative since the numerator's terms are all negative. Therefore,

$$\frac{d^2 e^*(\alpha)}{d\alpha^2} = \frac{d^2 e_1(\alpha)}{d\alpha^2} - \frac{d^2\sigma(\alpha)}{d\alpha^2} > 0.$$

$\underline{\frac{\partial e(\alpha)}{\partial \alpha} > 0 \text{ when } \alpha \text{ close to } 1}$:

The derivative of $e(\alpha)$ is written as

$$\frac{\partial e(\alpha)}{\partial \alpha} = \frac{\partial e_1(\alpha)}{\partial \alpha} - \frac{d\sigma(\alpha)}{d\alpha},$$

with

$$\frac{de_1(\alpha)}{d\alpha} = \left(\gamma\frac{\partial E_1^*(\alpha)}{\partial \alpha} + (1-\gamma)\frac{\partial E_i^*(\alpha)}{\partial \alpha}\right)p\Delta V,$$

and

$$\frac{d\sigma(\alpha)}{d\alpha} = p\gamma(1-\gamma)\left[\frac{1-\alpha}{\alpha^2}\left(1-E_1(\alpha)+\alpha\frac{\partial E_1(\alpha)}{\partial \alpha}-\alpha^2\frac{\partial E_i(\alpha)}{\partial \alpha}\right)-(1-E_i^*-\frac{1-E_1^*}{\alpha})\right](p\Delta V+\underline{V}_G).$$

We first study the derivative of $e_1(\alpha)$, which is

$$\frac{\partial e_1(\alpha)}{\partial \alpha} = \left(\frac{\alpha(1-\gamma)K}{x_i(1-p)\left(1-E_1^*\right)\Delta W} - \frac{K\gamma}{\alpha^3(1-p)\left(1-E_i^*\right)\Delta W}\right)p\Delta V,$$

using the randomization condition (11), we obtain that

$$\frac{\partial e_1(\alpha)}{\partial \alpha} = \frac{K\alpha(1-\gamma)}{(1-p)\left(1-E_1^*\right)\Delta W \alpha^3}\left(\frac{\alpha^3}{x_i} - \frac{\gamma(p+\theta-p\theta)}{p\theta\gamma+1-p\gamma-\theta\gamma}\right)p\Delta V.$$

Thus for any $0 \leq \theta \leq 1$,

$$\frac{\partial e_1(\alpha)}{\partial \alpha} > 0 \Leftrightarrow \gamma < \frac{\alpha^3}{(\alpha^3+x_i)(p+\theta-p\theta)}.$$

Therefore, a sufficient condition for $\partial e_1(\alpha)/\partial\alpha > 0$ for any θ is

$$\gamma \leq \gamma_1 \equiv \frac{\alpha^3}{\alpha^3+x_i}.$$

Finally, note that when $\alpha \to 1$, then

$$\lim_{\alpha \to 1} \frac{d\sigma(\alpha)}{d\alpha} = -p\gamma(1-\gamma)\left[1 - E_i^* - \frac{1-E_1^*}{\alpha}\right](p\Delta V + \underline{V}_G) < 0 \Rightarrow \frac{\partial e(\alpha)}{\partial \alpha} > 0.$$

Proof of Lemma 4: comparative statics for the optimal effort of the applicant

We study how the optimal level of applicant effort e^* varies with y, where $y = x_i, p, \gamma$,

$$\frac{de^*}{dy} = \frac{\partial e^*}{\partial y} + \frac{\partial e^*}{\partial E_1}\frac{\partial E_1^*}{\partial y} + \frac{\partial e^*}{\partial E_i}\frac{\partial E_i^*}{\partial y}.$$

Proof of Lemma 5: Prior art search

Using (21) and (23), the applicant search more when $\alpha = 1$ than when he is truthful if

$$\gamma\left[E_1^*(1) - E_{1t}^*\right] + (1-\gamma)\left[E_i^*(1) - E_{it}^*\right] > 0$$

or if

$$E_1^*(1) > E_{1t}^* \text{ and } E_i^*(1) > E_{it}^*.$$

Simple calculations shows that for any $\gamma \in (0,1)$, we have $E_1^*(1) > E_{1t}^*$ if

$$p > \frac{x_i}{1+x_i}.$$

Similarly, we have $E_i^*(1) > E_{it}^*$ for any $\gamma \in (0,1)$ if

$$p > \frac{1}{1+x_i}.$$

Thus, a sufficient condition is

$$p > \max\left(\frac{1}{1+x_i}, \frac{x_i}{1+x_i}\right).$$

Proof of Lemma 6

The efforts (24) and (25) can be written as

$$\begin{aligned} E_1^{SPER} &= 1 - \phi_{S1}(\beta)^{\frac{1}{2}}, \\ E_i^{SPER} &= 1 - \phi_{Si}(\beta)^{\frac{1}{2}}, \end{aligned}$$

and we need to study the functions $\phi_{S1}(\beta)$ and $\phi_{Si}(\beta)$. First, we determine that

$$\tfrac{\partial \phi_{S1}(\beta)}{\partial \beta} = \tfrac{K(1-\gamma)p\Delta W(1-\beta)}{x_i \Delta W \Delta W^{SPER}(1-p)p\gamma}(\overline{W}_G - \overline{W}_R) > 0,$$

and, therefore, we can conclude that

$$\frac{\partial E_1^{SPER}}{\partial \beta} < 0.$$

Similarly, we compute $\partial \phi_{Si}(\beta)/\partial \beta$ and show it is negative as long as a pair of solutions exists. Therefore,

$$\tfrac{\partial E_i^{SPER}}{\partial \beta} < 0.$$

Further, there exists a value of $\beta \in (0,1)$ such that $\phi_{S1}(\beta) = \phi_{Si}(\beta)$. Denote β_s such a value. Hence, for any $\beta < \beta_s$, $\phi_{S1}(\beta) < \phi_{Si}(\beta)$ and, therefore, $E_1^{SPER} > E_i^{SPER}$.

Proof of Proposition 3

We first show that for any $\beta > 0$, the introduction of the SPER initiative increases (respectively, reduces) the number of patent applications submitted with $\widetilde{x} = 1$ (respectively, $\widetilde{x} = x_i$). In the emerging field, it is easy to show that the the number of patent applications in absence of the SPER initiative regime is higher than in its presence:

$$\begin{aligned} (1-\gamma) + \gamma(1-p)(1-\theta^*) &> p(1-\gamma) + (1-p)(1-\gamma) + \gamma(1-p)(1-\theta^*_{SPER}), \\ \text{as } \theta^*_{SPER} &> \theta^*. \end{aligned}$$

In the emerging field, the number of applications is higher under the SPER regime, as

$$p\gamma + (1-p)\gamma + (1-p)\gamma\theta^*_{SPER} > p\gamma + (1-p)\gamma + (1-p)\gamma\theta^*.$$

Second, we show that the total number of patents granted is lower in the mature field as $\mu_1^{SPER} < \mu_1$, where

$$\mu_1^{SPER} = p\tfrac{(1-\gamma)(1-\beta)(1-E_i)+\gamma(1-E_1)}{(1-E_1)}.$$

To see that, we differentiate μ_1^{SPER} with respect to β,

$$\frac{d\mu_1^{SPER}}{d\beta} = \frac{\partial\mu_1^{SPER}}{\partial\beta} + \frac{\partial\mu_1^{SPER}}{\partial E_1}\frac{\partial E_1}{\partial\beta} + \frac{\partial\mu_1^{SPER}}{\partial E_i}\frac{\partial E_i}{\partial\beta} < 0,$$

as $\partial\mu_1^{SPER}/\partial\beta < 0$, $\partial\mu_1^{SPER}/\partial E_1 > 0$, $\partial E_1/\partial\beta < 0$, $\partial\mu_1^{SPER}/\partial E_i < 0$ and $\partial E_i/\partial\beta > 0$.

Third, to show that the quality of patents granted in the mature field decreases with β, we first rewrite the quality as

$$\begin{aligned} Q_1^{SPER} &= E_1^{SPER} + (1 - E_1^{SPER})\mu_1^{SPER}, \\ &= p\gamma + E_1^{SPER}(1 - p\gamma) + p(1-\gamma)(1-\beta)(1 - E_i^{SPER}). \end{aligned}$$

The total derivative of the quality is

$$\frac{dQ_1^{SPER}}{d\beta} = \frac{\partial Q_1^{SPER}}{\partial\beta} + \frac{\partial Q_1^{SPER}}{\partial E_1^{SPER}}\frac{dE_1^{SPER}}{d\beta} + \frac{\partial Q_1^{SPER}}{\partial E_i^{SPER}}\frac{dE_i^{SPER}}{d\beta} < 0,$$

as $\partial Q_1^{SPER}/\partial\beta < 0$,$\partial Q_1^{SPER}/\partial E_1^{SPER} > 0$, $\frac{dE_1^{SPER}}{d\beta} < 0$, $\frac{\partial Q_1^{SPER}}{\partial E_i^{SPER}} < 0$ and $\frac{dE_i^{SPER}}{d\beta} > 0$. The quality of patents issued in the emerging field can be written as

$$Q_i^{SPER} = \frac{E_i^{SPER} + (1 - E_i^{SPER})\mu_i^{SPER},}{E_i^{SPER} + (1 - E_i^{SPER})(1 - \beta(1 - \mu_i^{SPER}))}.$$

The total derivative of Q_i^{SPER} is

$$\frac{dQ_i^{SPER}}{d\beta} = \frac{\partial Q_i^{SPER}}{\partial\beta} + \frac{\partial Q_i^{SPER}}{\partial\mu_i^{SPER}}\frac{d\mu_i^{SPER}}{d\beta} + \frac{\partial Q_i^{SPER}}{\partial E_i^{SPER}}\frac{dE_i^{SPER}}{d\beta} > 0,$$

as all of the derivatives are positive with respect to β.

Proof of Proposition 4

Let $A = (E_{1c}^A, E_{ic}^A)$ be a policy such that $E_{ic}^A < E_{1c}^A$. In such a policy, the equilibrium beliefs of the examiner are

$$\begin{aligned} \mu_{1c}^A &= 1, \\ \mu_{ic}^A &= p\frac{1-\gamma}{1-\gamma p} = \mu_i < p. \end{aligned}$$

Now consider a policy $A' = (E_{1c}^{A'}, E_{ic}^{A'})$, such that $E_{ic}^{A'} = E_{ic}^A$ and $E_{1c}^{A'} = E_{1c}^A - \varepsilon$, with $\varepsilon > 0$ but with $E_{1c}^{A'} > E_{ic}^{A'}$. With this new policy, upon receiving $\widetilde{x} = 1$, the examiner will check with accuracy $E_{1c}^{A\prime} > E_{ic}^{A\prime}$. Thus, a bad applicant will never choose to report $\widetilde{x} = 1$, even if he can. Thus, $\mu_1^{A'} = 1$. A bad applicant will report $\widetilde{x} = x_i$, so that

$$\mu_i^{A'} = p\frac{1-\gamma}{1-\gamma p} < p.$$

The examiner exerts effort $E_{1c}^{A'}$, pays a cost $C(E_{1c}^{A'}) = 1/(1-E_{1c}^{A'})$, and follows her judgement. It is obvious that policy A' dominates policy A, since they have the same screening efficiency. But A is more costly than A', as $C(E_{1c}^{A'}) < C(E_{1c}^{A})$, even though $C(E_{ic}^{A'}) = C(E_{ic}^{A})$.

Ex Ante **Commitment: Comparison of the levels of effort**

In the case of commitment, E_c is the solution of

$$\frac{de(E)}{dE}[B_c(E_c) - \frac{\gamma K}{(1-E_c)} - \frac{(1-\gamma)K}{x_i(1-E_c)}] + e(E)\,[p(1-p)\Delta W - \frac{(\gamma x_i+(1-\gamma))K}{x_i(1-E)^2}] = 0.$$

If we define $\underline{E}$ as the solution of

$$p(1-p)\Delta W - (\gamma x_i + (1-\gamma))\frac{K}{x_i(1-E)^2} = 0,$$

it is easy to prove that $\underline{E} < E_c$.

Let us rewrite the condition to obtain E_1^* as

$$p(1-p)\Delta W - \frac{x_i(1-p(1-\gamma))-p(1-\gamma)}{\gamma}\frac{K}{x_i(1-E)^2} = 0,$$

and the condition to obtain E_i^* as

$$p(1-p)\Delta W - \frac{\underline{V}_G(F)^2(1-p\gamma)-\underline{V}_G^2 p\gamma x_i}{(1-\gamma)\underline{V}_G(F)^2}\frac{K}{x_i(1-E)^2} = 0.$$

A comparison of these conditions leads to $E_i^* < \underline{E} < E_c$ for $p < d_1$, and that $E_1^* < \underline{E} < E_c$ for $p < d_2$, where

$$d_1 \equiv \frac{\underline{V}_G(F)^2[1-(1-\gamma)(\gamma x_i+1-\gamma)]}{\gamma(\underline{V}_G(F)^2+x_i\underline{V}_G^2)} \quad \text{and} \quad d_2 \equiv \frac{\underline{V}_G[x_i-\gamma(\gamma x_i+1-\gamma)]}{(1-\gamma)(\underline{V}_G(F)^2+x_i\underline{V}_G^2)},$$

and $d_2 < d_1$. The constellation of parameters (p, γ) for which $p < d_1$ is much larger than for which $p < d_2$. In fact, we can also show that there exists a function $H > d_2$ such that $E_1^* = E_c$ and, therefore, for values of $(p, \gamma) \in \{p, \gamma / d_1 > p > H\}$, $E_i^* < E_c < E_1^*$.

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Canada
Telephone (780) 492-3406 Fax (780) 492-3300
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