DEPARTMENT OF ECONOMICS
UNIVERSITY OF ALBERTA

Working Paper No. 2009-13

A New Look at Copper Markets: A Regime-Switching Jump Model

W. Hong Chan
Wilfred Laurier University

D. Young
University of Alberta

March, 2009

A New Look at Copper Markets: A Regime-Switching Jump Model

Wing Hong Chan[†]
School of Business & Economics
Wilfrid Laurier University
Waterloo, Canada, N2L 3C5
Tel: 519-884-0710 (ext. 2773)
wchan@wlu.ca

Denise Young
Department of Economics
University of Alberta
Edmonton, Alberta, T6G 2H4
Tel: 780 - 4927626
denise.young@ualberta.ca

Abstract

GARCH-jump models of metal price returns, while allowing for sudden movements (jumps), apply the same specification of the jump component in both 'bear' and 'bull' markets. As a result, the more frequent but relatively small jumps that occur in both bear and bull markets dominate the characterization of the jump process. Given that large jumps, although less frequent, are still quite common in copper (and other metal) markets, this is a potential shortcoming of current models. More flexibility can be added to the modeling process by allowing for regime-switching. In this paper we specify a model that allows for switching across two separate regimes, with the possibility of different jump sizes and frequencies under each regime, along with a regime-specific GARCH process for the conditional variance. This model is applied to daily copper futures prices over the period of January 2 1980 through the end of July 2007. The model is estimated both with and without factors such as interest and exchange rate movements entering into the specification of the state-dependent mean of the conditional jump size. In some respects, a Regime Switching GARCH-Jump Model performs well when applied to the copper returns data. The results are mixed in terms of whether or not variations of the model that allow jump sizes to be a function of interest or exchange rates offer much of an advantage over a pure time series approach to the modeling of copper returns over the past three decades.

Key Words: *regime switching, Poisson jump, GARCH volatility, copper futures*

JEL Classification: C32, C53, G13

(Current Version: March, 2009)

[†] We thank the Social Science and Humanities Research Council of Canada and Hong Kong University Grant Committee for financial support.

1 Introduction

Movements in copper returns continue to puzzle observers. Early empirical studies that attempted to capture observed price behaviour met with only limited success, while more recent studies incorporating GARCH effects, with and without jump dynamics, have shown promise. A common approach in the early empirical studies was to attempt to characterize the movements of copper (or other metal) spot prices over time, possibly as a function of other characteristics of the economic environment such as interest rates (Heal and Barrow 1980, Slade 1982, Deaton and Laroque 1992). To a large extent this strand of the literature is rooted in economic theory. That is, the time series specifications either implicitly or explicitly draw on economic theories of optimal resource extraction and/or speculative commodity storage, and in many cases, for example, either use interest rates as a control variable or estimate an implied discount rate within the model. For copper, when found to be significant, the role of interest rates does not match up with the predictions from standard models (Heal and Barrow 1980, Deaton and Laroque 1992).

More recent studies have turned to GARCH or GARCH-jump models to examine prices and returns in copper markets. Bracker and Smith (1999) and Smith and Bracker (2003) apply GARCH and EGARCH models to daily copper futures price data and find that these specifications are better able to capture volatility than other models. Chan and Young (2006) apply an Autoregressive Jump Intensity GARCH (ARJI-GARCH) model to the joint distribution of cash and futures returns in copper markets and find that allowing for a common jump across the cash and futures series contributes significantly to the ability to empirically capture the observed dynamics in these markets, at least in the more recent 'post-producer pricing' era.

In ARJI-GARCH models, a single specification of the jump component is applied throughout the entire time period under consideration. For assets that experience periods of both ‘bear’ and ‘bull’ market conditions, observed jumps in returns may be very different, however, depending on the prevailing market sentiment. A standard

ARJI-GARCH specification, therefore, may not offer sufficient flexibility to adequately capture these differences. The characterization of a jump process that is common across all market conditions will end up being dominated by the more frequent relatively small jumps that occur in both 'bear' and 'bull' markets. As a result, a single regime ARJI-GARCH specification will make it difficult to capture the larger jumps that are often observed in a 'bear' market.

Given that large jumps akin to those observed in 'bear' markets for stocks are common in copper (and other metal) markets, this is a criticism that should be examined in the context of copper markets. One way of addressing this problem is to add flexibility to the modeling process by allowing for regime-switching. Regime-switching models have already been applied in a variety of contexts. Much of this work has been done in the context of stock markets (Maheu and McCurdy 2000), exchange rates (Maheu and McCurdy 2002) and interest rates (Gray 1996, Mills and Wang 2006, Chua and Suardi 2007). Power (2007) applies a regime-switching model to agricultural commodity futures. In particular, Power finds that this type of model performs well for Chicago Board of Trade corn futures.

In this paper, we specify a model for returns on copper futures that allows for switching across two separate regimes, with the possibility of different intensities and sizes of jumps in each of the regimes, along with a regime-specific GARCH process for the conditional variance. This model is applied to daily futures prices over the period of January 1980 through the end of July 2007. Furthermore, in the spirit of the earlier literature where other characteristics of the economic environment are allowed to play a role in the process governing copper returns, we allow the jump process to be affected by interest and exchange rates. The consideration of the former is implied by standard economic theories of exhaustible resource extraction, dating back to the seminal work of Hotelling (1931). The latter is based on the fact that copper sales generally involve international trade and will be affected by the cost of obtaining foreign currency. In particular, we consider the 91-day US Treasury Bill rate, the US federal funds rate and the Pound/Dollar and Yen Dollar exchange rates. This provides a simple framework that can be used to examine the roles of possible underlying sources of the observed jumps in various regimes (such as 'bear' and 'bull' markets).

The remainder of the paper is structured as follows. In Section 2 we present a discrete time jump model for copper returns with a state-dependent jump intensity and jump size distribution where jump size is allowed to be affected by exogenous factors (such as interest or exchange rates). Section 3 presents and discusses the results of the model as applied to copper market data. Section 4 concludes.

2 A Regime-Switching GARCH Jump Model for Copper Returns

Copper markets are affected by a variety of factors. Given its thermal and electrical conductivity properties and its use in the creation of many metal alloys, copper is used in a variety of products and applications ranging from manufacturing to the transmission of electricity to telecommunications to residential construction. Copper also has close substitutes in many of these applications. For example, pipes can be made of copper or plastics. Fiber optics can replace copper in telecommunications applications. Therefore shocks that hit the overall economy or particular subsectors will have an influence on the derived demand for copper, and therefore on spot and futures prices. Furthermore, as with markets for financial assets, there is a speculative component to demand in copper markets (Slade *et al* 1993). On the supply side of the market, the opening of new mines, labour disputes, and attempts at collusive behaviour will affect prices and returns.[1] With such a variety of potential sources of shocks to copper returns, it may well be the case that the predominance of certain types or combinations of shocks will lead to different observed patterns of volatility.

An examination of trends in daily copper NYMEX futures over our January 1980 to July 2007sample period, presented in Figure 1, supports this conjecture. We see, as is often observed in stock markets, that there are several periods of stable prices or

[1] There have been many episodes of collusive behaviour in copper markets. One of the most recent overt attempts at collusion was through Conseil Intergouvernemental des Pays Exportateurs de Cuivre (CIPEC) in the early 1970s (Zorn 1978). This organization disbanded in 1988.

sustained general price increases. But there are also periods where returns follow a much different pattern of sharp drops followed by sustained decreases in prices. These two general types of observed market conditions for copper prices can be thought of as being analogous to 'bull' and 'bear' markets. One might also note that Deaton and Laroque (1992), who apply a rational expectations competitive storage model to the study of price behavior for thirteen commodities including copper, show that the long-run invariant stationary distribution of prices oscillates between two regimes. In summary, both the observed behaviour of prices and theoretical models support the case for the existence of switches back and forth across two separate regimes in copper markets.

We model this behaviour empirically through a regime switching GARCH-jump model. This approach, which can be viewed as a direct extension of switching-regime ARCH models, provides a reasonably parsimonious approach to the study of a two-regime state of affairs in copper (or other) markets (Cai 1994, Gray 1996, Hamilton and Susmel 1994). Daily copper futures returns at time t, R_t, are defined as $100\times\ln(FP_t/FP_{t-1})$ where FP is the futures price defined by rolling over expiration (Moschini and Meyers 2002). The information set at time $t, \Phi_t = \{R_t, \cdots, R_1\}$, consists of the history of these copper futures returns which, in our model, are assumed to evolve according to:

$$
\begin{aligned}
&R_t = \mu_{S_t} + \sum_{i=1}^{p} \phi_i R_{t-i} + \sqrt{h_{S_t,t}}\, z_t + J_{S_t,t} \\
&J_{S_t,t} = \sum_{k=1}^{n_{St,t}} Y_{St,T,k} - E\left(\sum_{k=1}^{n_{St,t}} Y_{St,t,k} \right) \\
&z_t \sim NID(0,1), \quad Y_{St,t,k} \sim \mathrm{N}\left(\theta_{\mathrm{S}_t}, \delta_{S_t}^2\right) \qquad (2.1)
\end{aligned}
$$

Equation (2.1) is made up of a returns equation, a jump equation, and a set of distributional assumptions. In this model, the market will be in one of two (unobserved) states or regimes, indexed by $S_t = \{1,2\}$ on any given day. The way in which returns, R_t, evolve in each state or regime is assumed to consist of several components. The first of these is a state-dependent mean, μ_{S_t}. Second, there is an autoregressive component in the returns process, $\sum_{i=1}^{p}\phi_i R_{t-i}$, (wherein each of the previous returns will itself have evolved as a function of the state/regime on that day). The third component, $\sqrt{h_{S_t,t}}z_t$, is a GARCH disturbance with state-dependent volatility that captures smooth movements due to gradual adjustments in copper markets. The exact specification of this regime-dependent volatility is discussed below, following our discussion of the final component in the returns equation, $J_{S_t,t}$, which is a regime-dependent jump with a zero mean. This jump component captures sudden market movements, as one or more 'unusual' events or shocks, $Y_{S_t,t,k}$, hit the market.

The size of the composite jump component at time t, $J_{S_t,t}$, depends on both the size of individual jump shocks ($Y_{S_t,t,k}$) and on the number (n_{S_t}) of these 'shocks' that arrive on that day. Therefore, assumptions must be made regarding the distributions of both of these factors. In our model, the size of $Y_{S_t,t,k}$, conditional on the history of returns contained in the information set, is presumed to be independent, and normally distributed with a regime-dependent mean θ_{S_t} and variance $\delta_{S_t}^2$, where the mean can be specified as a function of one or more exogenous variables (such as interest or exchange rates). That is, both the average size and the variability of jump sizes are allowed to vary over the two states or regimes. The subtraction of the expected value in the jump equation allows $J_{S_t,t}$ to enter the returns equation with a zero mean.

The discrete counting process governing the number of jumps that arrive between periods *t-1* and *t*, n_{S_t} , is assumed to be distributed as a Poisson random variable with regime-dependent parameter $\lambda_{S_t} > 0$ and density:

$$P\left(n_{S_t,t} = j \mid \Phi_{t-1}\right) = \frac{\exp\left(-\lambda_{S_t}\right)\lambda_{S_t}}{j!} \qquad j = 0,1,2,... \quad (2.2)$$

The mean (of the number of individual jumps on day t) and variance (of the number of jumps on day t) for the Poisson random variable are both λ_{S_t}. This parameter is referred to in the literature as the jump 'intensity'.

To complete the specification of the conditional volatility dynamics for the returns, it is also necessary to parameterize the GARCH component of our model that allows us to capture smooth volatility movements in the market. To simplify the construction of the likelihood, we assume that z_t, the N(0,1) GARCH disturbance, and the jump size $Y_{S_t,t,k}$, are independently distributed normal random variables. Furthermore, we assume that $h_{S_t,t}$ is measurable with respect to the information set Φ_{t-1} and follows a GARCH(p,q) (Bollerslev 1986) process. That is,

$$h_{S_t,t} = \omega_{S_t} + \sum_{i=1}^{q} \alpha_{S_{t,i}} \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_{S_{t,i}} h_{t-i} \qquad (2.3)$$

Following Gray (1996)'s generalized regime switching model, we can obtain the regime-independent variance as

$$\begin{aligned} h_{t-1} &= E\left[R_{t-1}^2 \mid \Phi_{t-2}\right] - E\left[R_{t-1} \mid \Phi_{t-2}\right]^2 \qquad (2.4) \\ &= p_{1,t-1}\left[\mu_{1,t-1}^2 + h_{1,t-1} + \lambda_{1,t-1}\left(\theta_1^2 + \delta_1^2\right)\right] \\ &\quad + (1 - p_{1,t-1})\left[\mu_{2,t-1}^2 + h_{2,t-1} + \lambda_{2,t-1}\left(\theta_2^2 + \delta_2^2\right)\right] \\ &\quad - \left[p_{1,t-1}\mu_{1,t-1} + \left(1 - p_{1,t-1}\right)\mu_{2,t-1}\right]^2 \qquad (2.5) \end{aligned}$$

where $p_{1,t-1}$ is the probability of being in state 1 given all information up to *t-1*, and the corresponding regime-independent GARCH disturbance is defined by

$$\begin{aligned} \varepsilon_{t-1} &= R_{t-1} - E\left[R_{t-1} \mid \Phi_{t-2}\right] \qquad (2.6) \\ &= R_{t-1} - \left[p_{1,t-1}\mu_{1,t-1} + \left(1 - p_{1,t-1}\right)\mu_{2,t-1}\right] \qquad (2.7) \end{aligned}$$

This specification of ε_{t-1} contains the expected jump component and therefore allows it to propagate and affect future volatility through the GARCH variance factor.

We specify the transition probabilities for the unobserved regime or state variable, s_t, as a first order Markov process:

$$\begin{aligned}
\Pr[s_t = 1 \mid s_{t-1} = 1] &= P \\
\Pr[s_t = 2 \mid s_{t-1} = 1] &= 1 - P \\
\Pr[s_t = 2 \mid s_{t-1} = 2] &= Q \\
\Pr[s_t = 1 \mid s_{t-1} = 2] &= 1 - Q
\end{aligned} \qquad (2.8)$$

In principle, the transition probabilities can be generalized to be a function of regime duration (Maheu and McCurdy 2000, Maheu and McCurdy 2002) or one or more exogenous variables, such as an interest rate (Gray 1996). For simplicity and to provide a benchmark for future comparison, we assume that these transition probabilities are constant in order to focus on the regime-dependent jump specification. It follows that the probability of being in regime 1 given the information available at *t-1* can be expressed as

$$\begin{aligned}
p_{1,t} &= (1-Q)\left[\frac{g_{2,t-1}(1-p_{1,t-1})}{g_{1,t-1}p_{1,t-1} + g_{2,t-1}(1-p_{1,t-1})}\right] \\
&\quad + P\left[\frac{g_{1,t-1}p_{1,t-1}}{g_{1,t-1}p_{1,t-1} + g_{2,t-1}(1-p_{1,t-1})}\right]
\end{aligned} \qquad (2.9)$$

where $g_{i,t-1} = f(R_{t-1} \mid s_{t-1} = i) = \sum_{j=0}^{\infty} f(R_{t-1} \mid n_{i,t-1} = j, \Phi_{t-1})$ is the conditional density of returns given state i at time t-1.

The conditional density of the copper returns is obtained by integrating out the two discrete valued variables: (i) $n_{S_t,t}$, governing the number of jumps; and (ii) the state variable, s_t, governing the regime transition. This conditional density of returns is:

$$P(R_t \mid \Phi_{t-1}) = \sum_{j=0}^{\infty}[p_{1,t} f(R_t \mid n_{1,t} = j, \Phi_{t-1}) P(n_{1,t} = j \mid \Phi_{t-1}) + (1 - p_{1,t}) f(R_t \mid n_{2,t} = j, \Phi_{t-1}) P(n_{2,t} = j \mid \Phi_{t-1})]. \quad (2.10)$$

Equation 2.10 shows that this model can be expressed as a discrete mixture of distributions, where the mixing is driven by a time-varying Poisson distribution.

The assumptions presented in Equation 2.1 imply that the distribution of returns conditional on the most recent information set, *i* state, and *j* jumps is normally distributed as,

$$f(R_t \mid n_{i,t} = j, \Phi_{t-1}) = \frac{1}{\sqrt{2\pi(h_{i,t} + j\delta_i^2)}} \exp\left(-\frac{(R_t - \mu_i - \theta_i j)^2}{2(h_{i,t} + j\delta_i^2)}\right).$$

Equation 2.10 involves an infinite sum over the possible number of jumps, $n_{i,t}$. In practice, we truncate the maximum number of jumps to a sufficiently large value τ, so that the probability of τ or more jumps is for all practical purposes equal to zero. We empirically check for our particular dataset to machine precision. Equation 2.2 yields a value of 0 for $j \geq \tau$. A second check on the choice of τ is to investigate $\tilde{\tau} > \tau$ to ensure that the likelihood and parameter estimates do not change.

In this model it is possible to allow for time varying jump intensity by considering the conditional jump intensity specification used in Chan and Maheu (2002):

$$\lambda_{S_t,t} = \kappa_{S_t,0} + \kappa_{S_t,1}\lambda_{t-1} + \kappa_{S_t,2}\xi_{t-1}. \quad (2.11)$$

With the restriction of $\kappa_{S_t,0}$ and $\kappa_{S_t,1} > \kappa_{S_t,2}$, the autoregressive jump intensities (ARJI) $\lambda_{S_t,t}$ will be positive at all times. The ex-post error, ξ_{t-1}, captures the unexpected number of jumps in the previous period. Given the state s_t, $\xi_{S_t,t-1}$ is the innovation or unpredictable component of the counting process, measured ex-post as:

$$
\begin{aligned}
\xi_{t-1} &= E\left(n_{s_t,t-1} \mid \Phi_{t-1}\right) - \lambda_{S_t,t-1} \\
&= \sum_{j=0}^{\infty} jP\left(n_{s_t,t-1} = j \mid \Phi_{t-1}\right) - \lambda_{S_t,t-1}.
\end{aligned} \qquad (2.12)
$$

λ_{t-1} are defined by

$$\xi_{t-1} = p_{1,t-1}\xi_{1,t-1} + (1 - p_{1,t-1})\xi_{2,t-1} \qquad (2.13)$$

and

$$\lambda_{t-1} = p_{1,t-1}\lambda_{1,t-1} + (1 - p_{1,t-1})\lambda_{2,t-1} \qquad (2.14)\,.$$

An examination of the autocorrelation properties of the jump intensity residuals, ξ_{t-1}, can be used to determine whether or not time-varying jump intensities are empirically relevant in a given application of the model.

Finally, let $f\left(R_t \mid n_{i,t} = j, \Phi_{t-1}\right)$ denote the conditional density of returns given *i* state and *j* jumps occur, and the information set Φ_{t-1}. Having observed R_t and using Bayes' rule we can infer ex post the probability that a jump of size *j* occurred at time *t*, with the filter defined as

$$P\left(n_{i,t} = j \mid \Phi_t\right) = \frac{f\left(R_t \mid n_{i,t} = j \mid \Phi_{t-1}\right) P\left(n_{i,t} = j \mid \Phi_{t-1}\right)}{P\left(R_t \mid \Phi_t\right)}, \quad \text{j} = 0,1,2,... \quad (2.15)$$

where $P\left(n_{i,t} = j \mid \Phi_{t-1}\right)$ is from Equation 2.2. The filter in Equation 2.15 is an important component of the model in the presence of time-varying jump dynamics, since it enters Equation 2.12 but it also can be constructed and used for inference purposes. For example, the probability that at least one jump occurred could be assessed using $1 - P\left(n_{i,t} = 0 \mid \Phi_t\right)$.

Once the model has been estimated (via maximum likelihood methods), one way to evaluate its performance is via a Pearson goodness of fit test based on the integral transformation of Rosenblatt (1952). For a candidate model with conditional density $\tilde{f}(R_t \mid \Phi_{t-1}, \Theta)$ where Θ is the known parameter vector, Rosenblatt (1952) shows that if the data are drawn from the density $\tilde{f}(\cdot)$ then the series $\{\tilde{u}_t\}_{t=1}^{T}$ defined from

$$\tilde{u}_t = \int_{-\infty}^{R_t} f(v \mid \Phi_{t-1}, \Theta) dv \qquad (2.16)$$

will be *i.i.d. U(0,1)*. Therefore testing whether or not $\tilde{u}_t \sim i.i.d.\ \mathrm{U}(0,1)$ provides a test for correct model specification. The Pearson goodness-of-fit test for $\tilde{u}_t$ is suggested by Vlaar-Palm (1993). Under the null hypothesis of a correctly specified model,

$$\sum_{i=1}^{g} \frac{(n_i - E(n_i))^2}{E(n_i)} \sim \chi^2(g-1), \qquad (2.17)$$

where g is the number of equally spaced groups, and n_i is the number of observation of $\tilde{u}_t$ that occur in group *i*. After g is chosen, $n_i = \sum_{t=1}^{T} I_{it}$ where,

$$I_{it} = \begin{cases} 1 & \text{if } (\mathrm{i}-1)/\mathrm{g} < \mathrm{u}_\mathrm{t} < i/g \\ 0 & \text{otherwise} \end{cases} \qquad (2.18)$$

for $1 \le i \le g$.

3 Results

The regime-switching GARCH jump model from Section 2, using a lag length of one in the autoregressive component of our returns specification, is estimated via Maximum Likelihood methods with a truncation point (see the discussion immediately following Equation 2.10) of 10. We use daily observations on NYMEX futures for copper from January 2 1980 to July 31 2007 where the futures price is defined by rolling over expiration (Moschini and Myers 2002). Returns are calculated as (100×) the log difference in this futures price. The model is estimated both with and without an exogenous variable (change in an interest rate or rate of return for an exchange rate) entering into the specification of the regime-dependent mean of the conditional jump size.[2] Earlier data, although available, were not used as copper markets witnessed significant changes in the late 1970s as North America was in the process of moving away from its producer-price system and the presence of jumps were not found to be a statistically significant characteristic of copper returns in this earlier era (Slade 1988, Chan and Young 2006).

Summary statistics for our data are presented in Table 1. The average return on copper futures prices was close to zero (0.023%), but exhibited substantial volatility with returns ranging from -13% to 11% across the 8058 days in our sample. In comparison, the US 91-day Treasury Bill rate over this period averaged about 6%, ranging from 1% to 17%.[3]. Compared to the Treasury Bill rate, the Federal Funds rate exhibits both a higher mean (around 6.5%) and a larger range (from 1% to 22%). The two exchange rates considered are the Pound/Dollar and Yen/Dollar rates. These correspond to economies with different sectoral compositions in terms of activities that involve the use of copper, with Japan having substantial automotive and high-tech manufacturing sectors. Over our sample period, the average rate of return (calculated as 100 × the log difference) in each exchange rate is close to zero, at -0.002 for the Pound and 0.011 for the Yen. The

---

[2] Specifications where the interest rate is entered into the regime-dependent mean of the returns (μ) led to convergence problems in the solution algorithm.

[3] Although the choice of interest rate in previous studies has not been unique, generally a 30-day or 91-day Treasury Bill rate has been used. For example, Heal and Barrow use a (British) 91-day Treasury Bill rate. In order to check for robustness, we consider both the US 91-day rate and the US Federal Funds rate.

range and standard deviation of these changes are of similar magnitudes for both exchange rates, although slightly larger for the Yen.

As with other time periods considered in the previous literature, the copper returns exhibit strong negative skewness and excess kurtosis. And while there is no evidence of autocorrelation in the copper return series, the squared returns are serially correlated. As expected, these copper returns data indicate that a GARCH model with jumps to capture the leptokurtosis is appropriate.

The results from our Regime-Switching GARCH-Jump Model are presented in Tables 2 (basic model and 91-day Treasury Bill version) and 3 (Fed Fund, Pound/Dollar and Yen/Dollar versions). Two distinct regimes for copper returns are evident. As can be seen from these tables, most of the major features of the two regimes are not affected by whether or not we allow the average jump size to be a function of an interest or exchange rate.

Under Regime 1, the mean return, μ, is statistically different from zero with estimated values ranging from 0.073 to 0.078. By contrast, the mean return in Regime 2 is never significantly different from zero. The first regime could possibly be referred to as a "bull market" with apparently most investors enjoying a decent size of return and the second regime a "bear market" with average returns of zero. These names are only roughly appropriate, given that, as is described below, the first regime is also the one in which large negative jumps are most likely to occur. Note that in Regime 2 returns always exhibit statistically significant negative autocorrelation, (φ_1) implying that a large drop in the futures return is likely to be followed by a rally on the next day. This phenomenon does not appear under Regime 1.

Both the frequency (intensity) and sizes of jumps vary over the two regimes. The Ljung-Box Portmanteau tests applied to the jump intensity residuals, Q_{ξ}, indicate the absence of any autocorrelation. This implies that a time-varying parameterization of these intensities, as presented in Equation 2.11, is not needed. Instead, regime-specific constant intensities suffice. These jump intensities of about 0.17 in Regime 1 and larger at about 0.25 in Regime 2, when entered into Equation 2.2, imply about one jump per week under Regime 1 and one about every 4 to 5 days under Regime 2. These intensities are comparable to an average value of approximately 0.2 found in Chan and Young (2006) (who, as opposed to the constant within-regime intensities here, find that time-varying intensities were needed in a single regime model).

The sizes of the jumps, θ, that occur within the two regimes are very different. Although less frequent, jumps in Regime 1 are relatively large. For the basic model, the average jump size is -0.617 and the variance of the jump sizes is 2.4485 which is over 50% larger than under Regime 2. With the GARCH parameter estimates ($\alpha + \beta$) of 0.86 in Regime 1, the large negative jumps in this period are associated with a persistent volatility process. This implies that it is more likely that one will observe large negative jumps in this regime and the jump impacts transmitted through the lagged residual in the volatility next period will be long lasting. Among the large jumps captured under Regime 1 are the tight supply period of the late 1980s and the copper trading scandal of June 1996 (Edelstein 2001). Thus, although Regime 1 sees higher returns on average, as measured through μ, this regime is also characterized by the risk of occasional large negative jumps.

When we allow the jump size to be a function of the (change of the) rate of interest, we see that under Regime 1, for both the 91-day Treasury Bill and Fed Fund rates, the size of the jump is related positively to changes in the rate of interest. When interest rates increase, for example, jumps are more likely to be positive. This is loosely consistent with theories in the economics of exhaustible resources that posit higher returns during periods of high interest rates.

When we allow the jump size to be a function of the (percentage change of an) exchange rate, the results are quite sensitive to the choice of exchange rate. While jumps in Regime 2 are not affected by either of the exchange rates considered, Regime 1 jumps increase with changes in the Yen/Dollar rate. Given that copper is used in many manufacturing applications in the automotive and high-tech sectors, it is not surprising that under conditions of tight supply, shocks to the Japanese currency are related to jumps in copper returns. Furthermore, it might be noted that the 1996 copper trading scandal mentioned above involved a trader at Tokyo's Sumitomo Corp (Edelstein 2001). In spite of the significant impact of the Yen under Regime 2, inclusion of the exchange rate information does not lead to an appreciable increase in the value of the log-likelihood.

Average durations in each regime are calculated based on an evaluation of the conditional probability $P(S_1 \mid t\text{-}1)$ for each t. Days for which the estimated probability exceeds 0.5 are classified as belonging to Regime 1. In the basic model, the durations for Regime 1 range from 1 to 343 days, with a sample average of about 20 days and a standard deviation of 45, while the observed. For Regime 2, durations range from 1 to 578, with a sample mean of 45 and a standard deviation of 103.[4] Copper markets at any given point in time are more likely to be characterized by Regime 2. In fact, Regime 2 is in place for about 70% of our observations. Results are similar for the models that include interest or exchange rates. Given that Regime 1 offers higher returns on average but also witnesses the largest sudden negative movements via the jump process, it is not obvious whether producers of copper, for example, would prefer to be in the more stable low return Regime 2 or the riskier but higher return Regime 1. An answer to this would require an investigation into the returns from available hedging strategies, and such an investigation would require information regarding the joint behaviour of spot and futures returns.

The probabilities of being in Regime 1 for the first two specifications (Basic and 91-day Treasury Bill) of our model are depicted in Figure 2. The returns under the two regimes for the basic model are depicted in Figure 3. Figure 3 sorts copper returns into two separate groups based on the conditional filter of being in a given state. From Figure 3, we see that Regime 1 captures the (infrequent) relatively more volatile and extreme portions of the return process. Regime 2, on the other hand, picks up the (frequent)

---

[4] The reported durations based on the calculated conditional probabilities are shorter than those implied by the transition probabilities for the individual regimes.

relatively less volatile periods. Note that in both regimes, there is substantial variation around the means for these returns due to both the GARCH and jump components of the model. Our results for copper markets are somewhat similar to than those found by Power (2007) for agricultural commodities. He finds, in a regime-switching model without jumps, evidence of a highly persistent low-mean low-volatility regime and a much less persistent high-mean high-volatility state in corn prices.

The Ljung Box statistics presented at the bottom of Tables 2 and 3 show that the regime switching jump models capture all the serial correlations in both the mean and variance of the futures return process. As mentioned above, the same test on the measurable shock (Chan and Maheu 2002) indicates that there is no need to add a time-varying component to the jump intensity for the regime switching model.

Given that our major results are quite similar across the specifications in Tables 2 and 3, we restrict our reported diagnostics to those for the basic and 91-day Treasury Bill models. These are presented in Table 4.[5] We see that the Regime Switching GARCH-Jump Model performs well in some respects. The Pearson Goodness of Fit values all indicate that the models, with and without the interest rate included, perform well. For the bin size of 50, the Pearson statistic corresponding to the regime-switching jump model has a value of 46.15. This is low compared to the values of 242.52 and 60.22 for a standard GARCH and a single regime GARCH-jump model, respectively. The statistics for the standard GARCH model, having p-values closed to zero for all bin sizes, signal misspecification of the unconditional distribution of futures returns. All jump models pass this particular test of misspecification in the unconditional distribution, providing support for the usage of a Poisson jump structure in modeling copper futures returns. As a robustness check for overfitting, we evaluate the forecasting performance of the regime switching GARCH-jump model using the data from the beginning of year

[5] We also check for parameter stability by estimating the basic and 91-day Treasury Bill specifications separately for 3 time periods: 1975-1985, 1986-1996, and 1997-2007. As might be expected, the earliest sample, which includes data from the tail end of the North American producer-pricing era (which has been excluded from the sample used in Table 2), yields results that are much different from those found for the full sample and for the other two subsamples. The general patterns of signs and relative parameter sizes are quite stable across the latter two subsamples. These results are available from the authors upon request.

2000 to July 31, 2007 as out-of-sample data. The root mean square errors and mean absolute errors are reported in the right panel of Table 4. The results show that the regime switching GARCH jump model generally provides marginally superior out-of-sample forecasts for copper futures return in comparison to the plain GARCH and GARCH-jump models. The regime-switching jump model does not perform as well simpler specifications, however, with respect to an Akaike Information Criterion (AIC) or Schwarz's Bayesian criterion (BC).

In spite of the somewhat disappointing performance of the regime-switching model in terms of a subset of the model diagnostics, the results may still be of economic importance for agents who deal in copper markets. Recall that in Regime 1, average returns are relatively high, but there is a much greater risk of large negative jumps than in the more stable but lower return Regime 2. From a macroeconomic perspective, copper-exporting countries such as Chile or Zambia that may rely on copper sales to generate much of their foreign exchange and fuel growth, may have a strong preference for the less risky Regime 2, as this regime is less likely to generate temporary crises with respect to the management of the economy. One available strategy for managers of these economies is to attempt to influence copper prices through cartelization. Previous attempts to do so, however, have not been successful (Zorn, 1978). Risk preferences may be different, however, for higher income copper exporters such as Canada and Australia, whose economies are more diversified, and for whom the higher average returns of Regime 1 may be attractive.

The results are also of interest in that they may have implications for optimal hedging strategies for firms who are sellers and buyers of copper. The cost of risk management via hedging will be a function of the jump structures, which differ across the two observed regimes. Therefore, knowledge of which regime is in effect and how likely it is that this regime will remain in effect will affect agents' optimal hedging strategies. An examination of whether or not there are significant gains from a hedging strategy that takes regime-switching into account would require an extension of the model where spot price jumps are modeled along with those for futures returns. Finally, the results related to copper are also of general interest as they provide a point of comparison for other metal markets where regime-switching models may better capture observed behaviour than models that are currently in use.

4 Conclusions

In this paper, we examine the ability of a discrete time jump model with a state-dependent jump intensity and jump size distribution to capture the observed dynamics in the returns of copper futures prices. We find two distinct regimes with markedly different behaviour in terms of autocorrelation in the returns and jump size. In the spirit of the early empirical literature on metal prices, we allow the jump size to be a function of interest or exchange rates and find the relationship to sometimes be statistically significant, while leaving the main features of the two regimes unchanged. Diagnostic tests are mixed in terms of indicating whether or not controlling for these in our Regime-Switching GARCH-Jump model contribute much to our ability to capture copper return dynamics in recent years. Possible fruitful directions for future research may include (i) an examination of the implications of regime switching jumps for issues such as the hedging of risk in commodity markets (through the application of a bivariate regime-switching jump model for both spot and futures prices); and (ii) the application of regime-switching models to other non-fuel and fuel mineral markets.

References

Bracker, K. and K. Smith (1999), "Detecting and Modeling Changing Volatility in the Copper Futures Market," *Journal of Futures Markets* 19, 79-100.

Bollerslev, (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics* 31, 307-327.

Cai, J., (1994), "Markov Model of Switching-Regime ARCH," *Journal of Business & Economic Statistics*, 12, 309--316.

Chan, W. H., and J. M. Maheu, (2002), "Conditional Jump Dynamics in Stock Market Returns," *Journal of Business & Economic Statistics*, 20, 377--389.

Chan, W. and D. Young (2006), "Jumping Hedges: An Examination of Movements in Copper Spot and Futures Markets," *Journal of Futures Markets* 26, 169-188.

Chua, L.C. and S. Suardi (2007), "Markov-Switching Mean Reversion in Short-Term Interest Rates: Evidence from East Asian Economies," *The Economic Record*, 83, 383-397.

Deaton, A. and G. Laroque (1992), "On the Behaviour of Commodity Prices," *Review of Economic Studies* 59, 1-24.

Edelstein, D. (2001). Copper Minerals Yearbook, U.S. Geological Survey.

Gray, S. F., (1996), "Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process," *Journal of Financial Economics,* 42, 27--62.

Hamilton, J. D., and R. Susmel, (1994), "Autoregressive Conditional Heteroskedasticity and Changes in Regime,"' *Journal of Econometrics,* 64, 307--333.

Heal, G. and M. Barrow (1980), "The Relationship between Interest Rates and Metal Price Movements," *Review of Economic Studies* 47, 161-181.

Hotelling, H. (1931), "The Economics of Exhaustible Resources," *The Journal of Political Economy*, 39, 137-175.

Maheu, J. M., and T. H. McCurdy, (2000), "Identifying Bull and Bear Markets in Stock Returns," *Journal of Business & Economic Statistics*, 18, 100--112.

Maheu, J. M., and T. H. McCurdy, (2002), "Nonlinear Features of Realized FX Volatility," *Review of Economics and Statistics,* 84, 668--681.

Mills, T. and P. Wang (2006), "Modelling Regime Shift Behaviour in Asian Real Interest Rates," *Economic Modelling*, 23, 952-966.

Moschini, G.C. and R.J. Myers (2002), "Testing for Constant Hedge Ratios in Commodity Markets: A multivariate garch approach," *Journal of Empirical Finance* 59, 589-603.

Power, G., (2007), "A Wavelet-Based Analysis of Commodity Futures Markets," unpublished PhD dissertation, Cornell University.

Rosenblatt, M., (1952), ``Remarks on a Multivariate Transformation,'' *Annals of Mathematical Statistics*, 23, 470--472.

Slade, M. (1982), "Trends in Natural Resource Commodity Prices: An Analysis of the Time Domain," *Journal of Environmental Economics and Management* 9, 122-137.

Slade, M. (1988), Pricing of metals. Kingston: Centre for Resource Studies.

Slade, M., C. Kolstad and R. Weiner (1993), "Buying Energy and Nonfuel Minerals: Final, Derived and Speculative Demand," in A. Kneese and J Sweeney (eds) *Handbook of Natural Resource and Energy Economics*, Volume 3, Amsterdam: Elsevier, 935-1009.

Smith, K. and K. Bracker (2003), “Forecasting changes in copper futures volatility with GARCH models using an iterated algorithm,” *Review of Quantitative Finance and Accounting* 20, 245-265.

Vlaar, P. J. G., and F. C. Palm, (1993), “The Message in Weekly Exchange Rates in the European Monetary System: Mean Reversion, Conditional Heteroskedasticity, and Jumps,” *Journal of Business & Economic Statistics,* 11, 351--360.

Zorn, S.A. (1978) “Producers’ Associations and Commodity Markets: The Case of CIPEC,” in Adams, F.G.and S.A. Klein (eds) *Stabilizing World Commodity Markets*, Lexington, MA: Lexington Books, 215-234.

Table 1: Summary Statistics

	COPPER	TB3M	FEDFUND	POUND	YEN
Mean	0.023	5.955	6.528	-0.002	-0.011
Std.Dev.	1.659	3.014	3.551	0.602	0.655
Min	-13.092	0.800	0.860	-3.866	-5.630
Max	11.194	17.140	22.360	4.667	3.557
Skew	-0.350	0.924	1.157	-0.049	-0.501
Kurtosis	7.075	4.232	4.990	6.465	-0.501
Q_5	6.70	2551.57	8181.70	27.41	4.79
	[0.24]	[0.00]	[0.00]	[0.00]	[0.44]
Q_5^2	150.40	1657.12	1798.91	134.46	103.65
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Jarque-Bera	5739.32	1657.13	3128.06	4034.21	6683.38
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

The data contain daily observations on copper futures returns from January 2, 1980 to July 31, 2007. TB3M and FEDFUND refer to the 90-day US treasury bill rate and the federal fund rate. POUND and YEN refer to the rate of return for the British Pound and the Japanese Yen. P values are in square brackets. Q_i is the modified Ljung-Box portmanteau test, robust to heteroskedasticity, for serial correlation in the standardized residuals with i lag for the respective models. Q^2 is the modified Ljung-Box portmanteau test, robust to heteroskedasticity, for serial correlation in the squared standardized residuals.

Table 2: Estimates of the Regime Switching GARCH-Jump Model

$$R_t = \mu_{s_t} + \phi_{s_t,1} R_{t-1} + \sqrt{h_{s_t,t}} z_t + \left(\sum_{k=1}^{n_t} Y_{s_t,t,k} - E\left(\sum_{k=1}^{n_{s_t,t}} Y_{s_t,t,k}\right)\right);$$
$$h_{s_t,t} = \omega_{s_t} + \alpha_{s_t}\epsilon_{t-1}^2 + \beta_{s_t} h_{t-1}; \quad z_t \sim N(0,1); \quad P(n_t = j) = \frac{\exp^{-\lambda_{s_t}} \lambda_{s_t}^j}{j!};$$
$$Y_{s_t,t,k} \sim N(\theta_{s_t}, \delta_{s_t}^2);$$
$$\theta_{it} = \theta_i + \tilde{\theta}_i \Delta int_t$$

	Basic Model		Interest Rate	
Parameter	R1	R2	R1	R2
μ	0.0753	-.0061	0.0784	-.0049
	(0.0335)	(0.0168)	(0.0326)	(0.0167)
ϕ_1	-.0122	-.0576	-.0132	-.0559
	(0.0184)	(0.0130)	(0.0182)	(0.0134)
ω	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_i	0.0383	0.0113	0.0368	0.0112
	(0.0115)	(0.0093)	(0.0108)	(0.0095)
β_i	0.8217	0.5232	0.8032	0.5255
	(0.0333)	(0.0490)	(0.0367)	(0.0515)
λ_0	0.1653	0.2558	0.1943	0.2492
	(0.0459)	(0.0793)	(0.0484)	(0.0806)
δ_0	2.4485	1.5519	2.2920	1.5430
	(0.2648)	(0.1615)	(0.2277)	(0.1644)
θ_i	-0.617	-0.058	-0.430	-0.058
	(0.2319)	(0.0763)	(0.1929)	(0.0773)
$\tilde{\theta}$			4.8990	1.2286
			(1.2029)	(1.4603)
$P(S_i\|S_i)$	0.9936	0.9959	0.9931	0.9956
	(0.0705)	(0.0538)	(0.0258)	(0.0492)
% of Obs in Regime One		0.30		0.32
Avg Durations in Regime One		19.65		18.59
Avg Durations in regime Two		45.18		40.09
Q_5	7.42		7.76	
	[0.1911]		[0.1700]	
Q_5^2	0.14		0.55	
	[0.9996]		[0.9902]	
Q_ξ	4.89		4.34	
	[0.4290]		[0.5011]	
lnL	-14616.48		-14608.04	

Standard errors are in parentheses. p-values are in square brackets. Q^2 is the modified Ljung-Box portmanteau test, robust to heteroskedasticity, for i^{th} order serial correlation in the squared standardized residuals. $Q(\xi)$ is the same test for the i^{th} order autocorrelations of the jump intensity residuals.

Table 3: The Regime Switching GARCH-Jump Model with alternative jump size functions

$$R_t = \mu_{s_t} + \phi_{s_t,1}R_{t-1} + \sqrt{h_{s_t,t}}z_t + \left(\sum_{k=1}^{n_t} Y_{s_t,t,k} - E\left(\sum_{k=1}^{n_{s_t,t}} Y_{s_t,t,k}\right)\right);$$
$$h_{s_t,t} = \omega_{s_t} + \alpha_{s_t}\epsilon_{t-1}^2 + \beta_{s_t}h_{t-1}; \quad z_t \sim N(0,1); \quad P(n_t = j) = \frac{\exp^{-\lambda_{s_t}}\lambda_{s_t}^j}{j!};$$
$$Y_{s_t,t,k} \sim N(\theta_{s_t}, \delta_{s_t}^2); \theta_{s_t} = \theta_0 + \tilde{\theta}X_t$$

	Fed Fund		Pound		Yen	
Param	R1	R2	R1	R2	R1	R2
μ	0.0780	-.0070	0.0755	-.0057	0.0729	-.0036
	(0.0335)	(0.0169)	(0.0337)	(0.0169)	(0.0333)	(0.0168)
ϕ_1	-.0116	-.0584	-.0111	-.0581	-.0104	-.0568
	(0.0184)	(0.0132)	(0.0185)	(0.0131)	(0.0183)	(0.0130)
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_i	0.0379	0.0118	0.0381	0.0108	0.0370	0.0095
	(0.0115)	(0.0096)	(0.0118)	(0.0093)	(0.0114)	(0.0092)
β_i	0.8171	0.5087	0.8127	0.5080	0.8185	0.5305
	(0.0348)	(0.0535)	(0.0353)	(0.0488)	(0.0333)	(0.0494)
λ_0	0.1662	0.2786	0.1638	0.2730	0.1664	0.2394
	(0.0457)	(0.0918)	(0.0462)	(0.0816)	(0.0443)	(0.0762)
δ_0	2.4570	1.5116	2.5232	1.5351	2.4526	1.5840
	(0.2615)	(0.1650)	(0.2723)	(0.1546)	(0.2598)	(0.1677)
θ_0	-0.5860	-0.0540	-0.5730	-0.0510	-0.5520	-0.0570
	(0.2292)	(0.0730)	(0.2368)	(0.0725)	(0.2241)	(0.0777)
$\tilde{\theta}$	0.7984	0.1472	-0.4697	-0.0897	0.4977	0.0425
	(0.3707)	(0.1324)	(0.2451)	(0.0678)	(0.2039)	(0.0770)
$P(S_i \vert S_i)$	0.9934	0.9958	0.9935	0.9959	0.9933	0.9958
	(0.0689)	(0.0541)	(0.0698)	(0.0535)	(0.0707)	(0.0535)
% of Obs. in R1		0.31		0.30		0.31
Avg Dur in R1		19.50		21.09		19.32
Avg Dur in R2		44.29		48.80		43.98
Q_5		10.41		10.33		10.02
		[0.0645]		[0.0663]		[0.0747]
Q_5^2		6.50		6.31		5.76
		[0.2604]		[0.2774]		[0.3305]
Q_ξ		4.81		5.47		4.98
		[0.4390]		[0.3617]		[0.4182]
lnL		-14613.66		-14613.93		-14613.62

X_t refers to one of the three choices affecting the mean jump size including the federal fund rate, the Japanese Yen, and the British Pound. Standard errors are in parentheses. p-values are in square brackets. Q^2 is the modified Ljung-Box portmanteau test, robust to heteroskedasticity, for i^{th} order serial correlation in the squared standardized residuals. $Q(\xi)$ is the same test for the i^{th} order autocorrelations of the jump intensity residuals.

Table 4: Diagnostic Statistics and Forecasting Performance

	Pearson Goodness of Fit Test No. of Cells				Out-of-sample forecasts			
	50	100	150	200	RMSE	MAE	AIC	BC
Basic Model	46.15	94.57	134.08	186.45	1.6315	1.1631	0.9963	1.0442
	[0.59]	[0.61]	[0.80]	[0.73]				
Interest Rate	40.39	105.84	155.53	208.00	1.6312	1.1638	0.9981	1.0520
	[0.80]	[0.30]	[0.34]	[032.]				
GARCH	242.52	349.12	407.17	501.16	1.6332	1.1670	0.9897	1.0015
	[0.00]	[0.00]	[0.00]	[0.00]				
GARCH-jump	60.22	110.63	163.36	206.99	1.6314	1.1643	0.9875	1.0115
	[0.13]	[0.20]	[0.20]	[0.33]				

The Pearson goodness-of-fit test statistics are based on the integral transformation of the observed data using the respective model. Under the null hypothesis of a correctly specified density, the test statistic is distributed as $\chi^2(g-1)$, where g is the number of bins. P values are in square brackets. The out-of-sample forecasts include data from the beginning of year 2000 to the end of the entire sample. RMSE, MAE, AIC, BC refer to root mean square errors, mean absolute errors, Akaike Information Criterion and Schwarz' BC , respectively.

Figure 1: Copper Futures Prices

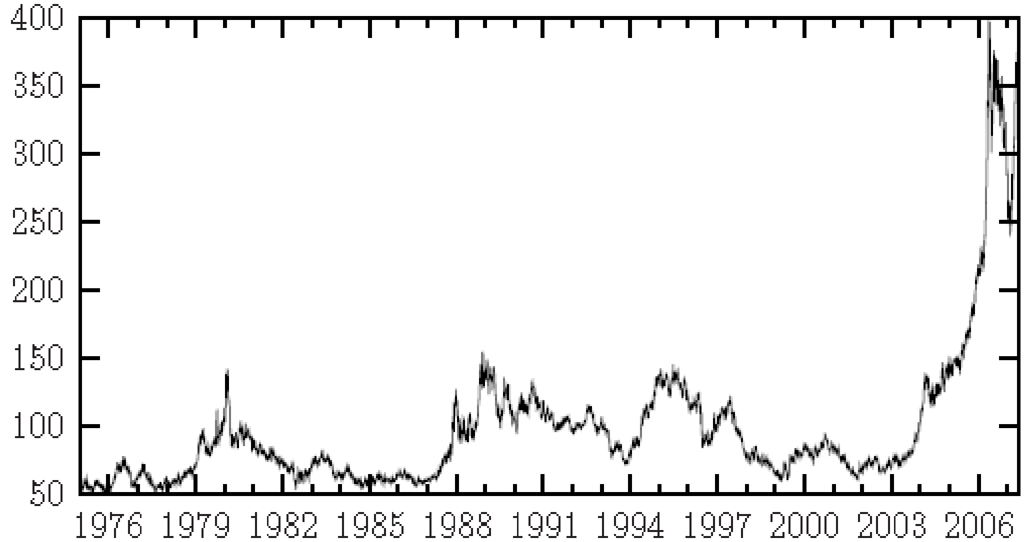
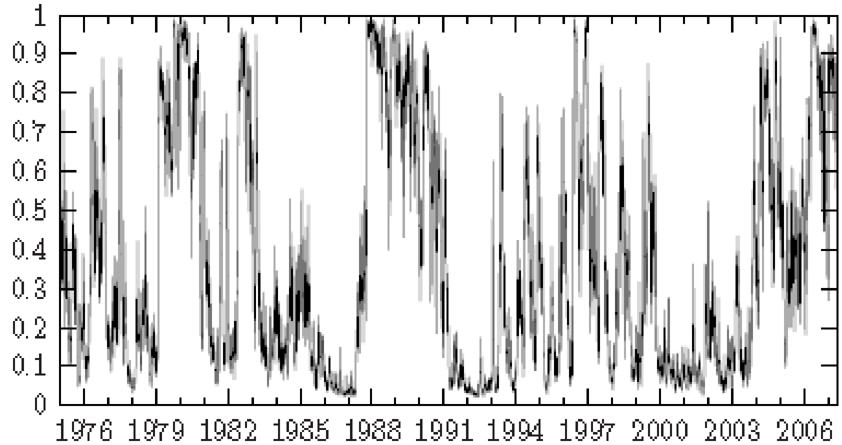


Figure 2: Probabilities of Regime One (Copper Futures)

Original Model



Interest Rate Model

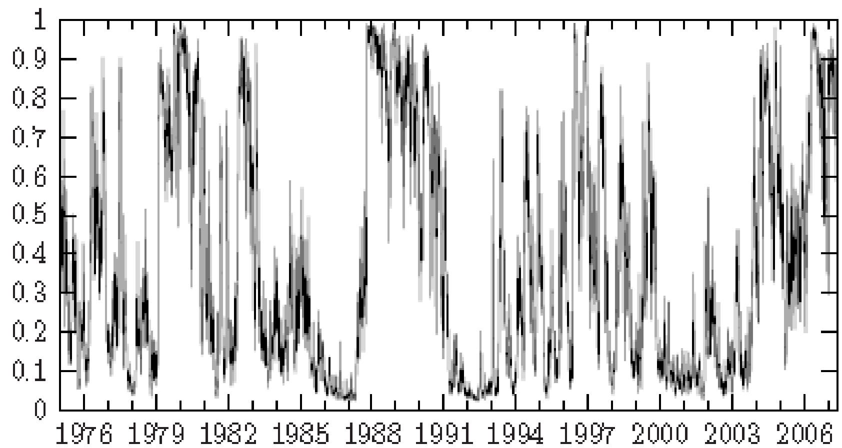
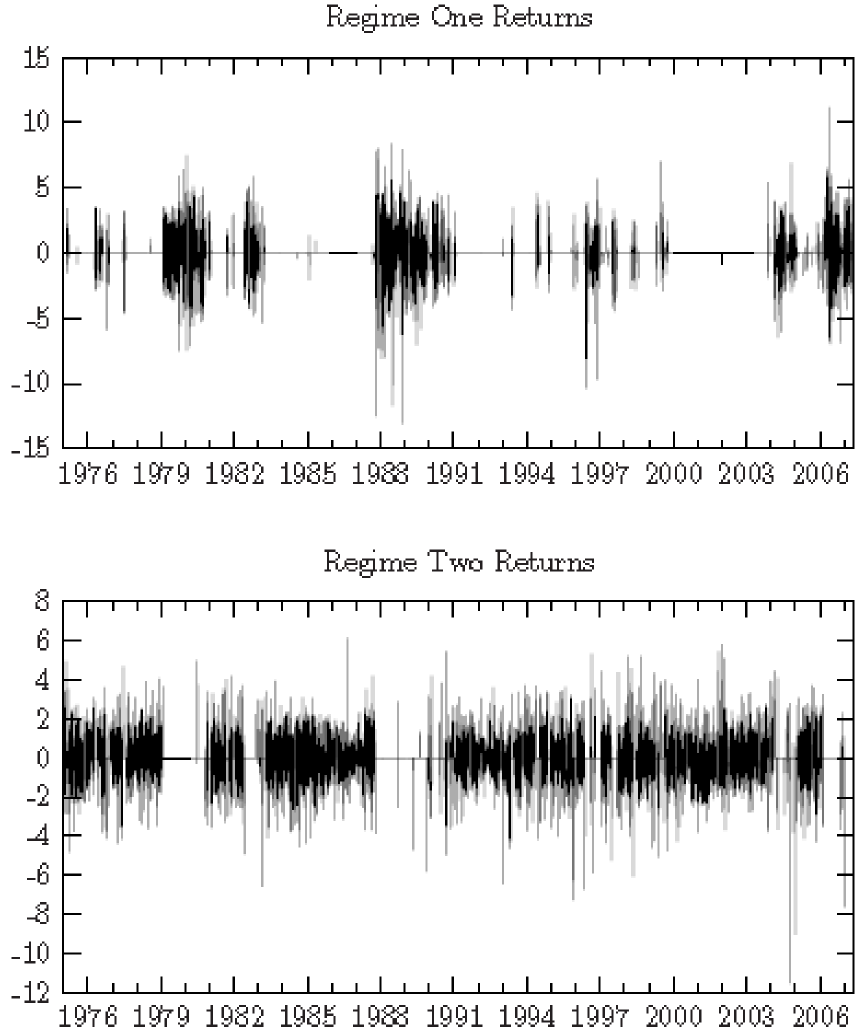


Figure 3: State Dependent Returns (Original Model)



Department of Economics, University of Alberta
Working Paper Series

http://www.economics.ualberta.ca/working_papers.cfm

Recent Working Papers

2009-12 Tort Reform, Disputes and Belief Formation – **Landeo**
2009-11 The Role of the Real Exchange Rate Adjustment in Expanding Service Employment in China – **Xu**, Xiaoyi
2009-10 "Twin Peaks" in Energy Prices: A Hotelling Model with Pollution and Learning – **Chakravorty**, Leach, Moreaux
2009-09 The Economics of Participation and Time Spent in Physical Activity – **Humphreys, Ruseski**
2009-08 Water Allocation Under Distribution Losses: Comparing Alternative Institutions – **Chakravorty**, Hochman, Umetsu, Zilberman
2009-07 A Comparative Analysis of the Returns on Provincial and Federal Canadian Bonds – **Galvani, Behnamian**
2009-06 Portfolio Diversification in Energy Markets – **Galvani**, **Plourde**
2009-05 Spanning with Zero-Price Investment Assets – **Galvani**, **Plourde**
2009-04 Options and Efficiency in Spaces of Bounded Claims – **Galvani**, Troitsky
2009-03 Identifying the Poorest Older Americans – Fisher, Johnson, **Marchand**, Smeeding, Boyle Torrey
2009-02 Time-Saving Innovations, Time Allocation, and Energy Use: Evidence from Canadian Households – **Brenčič, Young**
2009-01 Trigger Happy or Gun Shy? Dissolving Common-Value Partnerships with Texas Shootouts – Brooks, **Landeo**, Spier

Department of Economics
8-14 HM Tory
University of Alberta
Edmonton, AB
T6G 2H4
Canada

Telephone (780) 492-3406
Fax (780) 492-3300
www.ualberta.ca/economics

FACULTY OF ARTS