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The Economics of Participation and Time Spent in Physical Activity

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Abstract

This paper examines the economics of participation in physical activity by developing a consumer choice model of participation and estimating it using data drawn from the Behavioral Risk Factor Surveillance Survey (BRFSS). Both emphasize that individuals face two distinct decisions: (1) should I participate; and (2) how much time should I spend participating? The results indicate that economic factors like income and opportunity cost of time are important determinants of physical activity and that physical activity is a normal good. Individual characteristics also play an important role in determining the amount of time spent in physical activity. Participation and time spent decline with age. Females, married people, households with children, blacks and hispanics all spend less time engaged in physical activity than males, single people, childless households and whites. Public policy interventions aimed at improving physical activity of Americans targeted to specific sub-populations are likely to be more effective than broad-based policies.

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Introduction

There is a growing perception among policymakers and public health researchers that individual's decisions about participating in physical activity, including sport, has an important economic component. Relatively little attention has been paid to this topic by economists. In this paper, we develop an economic model of the decision to participate in physical activity and estimate the model using a nationally representative data set containing detailed information on participation in sport and other physical activities.

Much of the interest in the economic determinants of physical activity stems from the growing literature on the economic causes and consequences of obesity. Poor nutrition and physical inactivity are discretionary activities that can have a major impact on chronic diseases such as obesity (Cawley, 2004). Many plausible explanations for the rise in obesity have been advanced and a variety of policy interventions have been proposed to reduce the rate of obesity. However, the prevalence of meeting nutrition and and physical activity guidelines is low in the United States (Hill et al., 2004). Despite the important policy and public health aspects of participation in physical activity, little economic research has focused on the topic. There are a few notable exceptions to this. One is recent research by Downward and Riordan (2007) and Downward (forthcoming) on the demand for sports participation. Cawley et al. (2007, 2005) examine the effect of physical education and state policies on physical activity among youths in the United States. Another is research on participation in physical activity from a leisure demand perspective; Davies (2002) is a recent example of research in this area. Still another is statistical analysis by Farrell and Shields (2002) on the economic and demographic determinants of sporting participation in England. A final exception is the tangentially related research on the economic returns to participating in intercollegiate athletics in the United States spawned by interest in Title IX. (Long and Caudill, 1991).

A possible explanation for the failure in meeting physical activity guidelines is a poor understanding of the influence of economic factors on participation in physical activity and sport. Economics is useful for furthering our understanding because it provides a framework for studying how people allocate their time to competing activities and what economic, environmental and demographic factors affect the decision to be physically active. Once the decision to participate is made, the next decisions involve what activity, how often, how intense and how long. This paper begins to bridge this gap in the literature by combining and adapting components of the SLOTH framework developed by Cawley (2004) and commonly used recreation and leisure demand models to investigate the economic determinants of participation in physical activity. The SLOTH framework is based on Becker's (1964) model of labor and leisure choice. This model assumes that individuals derive satisfaction from the consumption of "basic commodities" like meals. These basic commodities are produced by households using time and market commodities according to a production function. The consumption of basic goods has labor market implications because any time used to produce and enjoy basic commodities represents time spent not working. Participation in physical activity and sport is a basic commodity in this context.

The model developed in this paper incorporates the idea that individuals make two separate but related decisions: (1) should I participate in sport?; and (2) how much time should I spend participating in sport? The model developed here generates empirically testable hypotheses about the role of economic factors like income and the opportunity cost of time in determining individual choices about being physically active. For example, the model shows that a change in the opportunity cost of time has both an income and substitution effect on the physical activity participation and duration decisions. These effects work in opposite directions and are empirically testable. We find that the income effect dominates the substitution effect through the analysis of a large, nationally representative data set containing a wealth of information about individual's participation in physical activity.

A Model of Participation in Physical Activity

Our model of participation in physical activity is an extension of the SLOTH framework of time allocation. (Cawley 2004) The framework is based on Becker's (1964) model of labor and leisure choice. The basic idea is that people are involved in the production of their own health. People combine time with market goods to produce health. The SLOTH framework assumes that individuals choose how to allocate their time to maximize their utility subject to budget, time and biological constraints where SLOTH is an acronym for the activities to which individuals allocate their time. Specifically, S represents time spent sleeping; L represents time spent at leisure, O represents time spent at paid work; T represents time spent in transportation to work; and H represents time spent in home production, or unpaid work. Participation in physical activity and sport is captured in L, as is time spent in sedentary leisure activities such as watching television or playing computer games.[1]

We combine the key temporal elements of the SLOTH framework with a recreation demand model (McConnell, 1992) to analyze both time allocation decisions and decisions about the purchase of good and services related to active and passive leisure. The key behavioral decisions behind our

[1]In a recent paper, Aguiar and Hurst (2007) documented trends in the allocation of time in the United States over the last forty years. They find that time allocated to leisure activities increased significantly over this time period. In addition, the adjustment process of reallocating time to leisure differed for men and women.

model are the separate but related decisions to participate in physical activity and how long to participate per episode of physical activity. We extend the SLOTH framework by recognizing that some of the activities in the SLOTH framework require time and goods and services purchased in the marketplace. At the same time, we reformulate and simplify the underlying utility function from the SLOTH framework to emphasize the physical activity participation and duration decisions. Suppose individuals choose how to allocate their time to various activities according to the following utility function:

$$\max U(a, t, z) \tag{1}$$

where a represents the individual's decision to participate in physical activity; t is the amount of time spent per episode of physical activity; and z represents the individual's decision to engage in the other activities in the SLOTH framework.

Individuals choose how to best allocate their time and what bundle of goods and services to purchase subject to time and budget constraints. The budget constraint is

$$Y = F_a + c_a at + c_z z \tag{2}$$

where F_a is the fixed cost of engaging in physical activity; c_a is the variable cost associated with engaging in physical activity; and c_z is the cost all other goods and services. The fixed costs of physical activity are one-time costs or flat recurring costs that individuals incur to participate in physical activity but do not depend on how many times the individuals participates. An example of a fixed cost is the monthly membership dues at a health club. An individual pays this flat, fixed amount regardless of how many times he uses the gym during the month. Variable costs of physical activity are costs that depend on the amount of time or the number of times the individual engages in physical activity. Examples of variable costs are equipment maintenance costs, coaches fees and personal trainer fees.

The time constraint is

$$T^* = at + \theta z \tag{3}$$

where T^* is the time available for consumption activities such as physical activity and θ is time spent consuming z. Assume that T^*, t and θ are measured in the same units such as hours. Let T be the total time available for work and all other activities. Hence, $T^* = T - h$ where h is time spent working. If individuals can choose the amount of hours they work, then wage earnings w can be expressed in terms of total time available and time spent not working

$$wh = w(T - at - \theta z). \tag{4}$$

Equation (4) captures the notion that any time spent in physical activity and other activities is time not available for work and reduces earnings. Thus, the wage is the opportunity cost of engaging in activities other than work. The full budget (or income) constraint includes the opportunity cost of time

$$y_0 + w(T - T^*) = F_a + p_a at + p_z z \tag{5}$$

where $p_a = c_a + w$ and $p_z = c_z + \theta w$ are the full costs of participating in physical activity and other activities.

Comparative Static Analysis

Consumers choose a, t and z to maximize utility subject to the full budget constraint. The lagrangian for this problem is

$$V = U(a, t, z) - \lambda(F_a + p_a \cdot a \cdot t + p_z z - y) \tag{6}$$

We derive the first order conditions characterizing the optimal choices of a, t, and z and conduct a comparative static analysis of the consumer's choice problem. The detailed derivation of the comparative static results are contained in the technical appendix. We analyze the effects of changes in income and the opportunity cost of time on the decisions to participate in physical activity and the amount of time spent participating in physical activity. In the comparative static analysis we treat the decision to participate in physical activity as a continuous, but discrete, count variable rather than a dichotomous variable restricted to take on the values of zero or one. This approach is consistent with the time dimension of the participation in physical activity data used in our empirical analysis, the month prior to the survey. Each episode of physical activity requires a separate participation decision, so the participation decision is made repeatedly over time. As a result, observed episodes of physical activity are not limited to zero or one over the relevant time period.

The sign of the comparative static derivative da/dy describing the effect of a change in income on participation depends on the sign of the cross-partial derivative U_{az}. This cross partial describes the relationship between the marginal utility from participating in physical activity and the marginal utility from other activities like meals or watching television. Participating in physical activity may lead to increased enjoyment of other non-active leisure activities. For example, if an individual decides to go to the gym and work out, the marginal utility from a meal in a restaurant later in the evening could be greater than the marginal utility received by a non-participant. On the other hand, the marginal utility from a meal in a restaurant could be lower because the individual does not want to offset the benefits from physical activity with a high calorie meal in a restaurant. Similarly, the sign of the comparative static derivative dt/dy examining the effect of a change in income on the optimal amount of time spent in physical activity holding da constant depends on the sign of the cross-partial derivative U_{tz} that cannot be signed *a priori*. We hold da constant because the decision about the amount of time an individual participates in physical activity is only relevant if the individual chooses to participate.

The direction and magnitude of the effect of a change in income on the participation decision and the amount of time spent in physical activity are empirically estimated later in this paper. These estimates shed light on the signs of the comparative static derivatives, and the nature of the relationship between the utility derived from time spent in physical activity and the utility derived from consuming other goods and spending time in other leisure activities.

The opportunity cost of time affects the decision to participate in physical activity and the amount of time devoted to physical activity. Recall, $p_a = c_a + w$ and $p_z = c_z + \theta w$. The opportunity cost of time is the wage rate w. Expanding the lagrangian to explicitly show the full cost of time spent in physical activity and all other activities yields

$$V = U(a, t, z) - \lambda(F_a + (c_a + w) \cdot a \cdot t + (c_z + \theta w)z - y). \tag{7}$$

The individual's choices are the same in this expanded model; namely, to choose a, t and z to maximize utility.

We examine the effect of a change in the opportunity cost of time, (dw), on the participation decision, a and the amount of time spent in physical activity, t. The comparative static expressions for both variables contain two terms that can be interpreted as income and substitution effects from standard consumer theory.

The first term of the comparative static expressions resemble the income effect of a change in the price of a market good. In the standard consumer theory model, an increase in the price of a good effectively decreases the consumer's real income, and the income effect is greater as the importance of the good in the consumer's budget increases. The income effect of a change in the opportunity cost of time has the opposite effect on the consumer's real income than the income effect of a change in the price of some market good. This occurs because an increase in the opportunity cost of time effectively means a higher wage and an increase in real income. If physical activity is a normal good, then we would expect the income effect of an increase in the opportunity cost of time to be positive. The income effect is weighted by the amount of time available for all activities other than work. Thus, the total amount of time spent in all non-work activities, rather than only the time in physical activity, determines the effect of a change in the opportunity cost of time on participation. This occurs because the opportunity cost of time is the same for physical activity and all other activities and individuals are constrained by the total amount of time available.

The second term is the substitution effect of a change in the opportunity cost of time. The substitution effect is negative which means that the likelihood of participating in physical activity decreases as the opportunity cost of time increases. The signs of both comparative static expressions, $\partial w/\partial a$ and $\partial w/\partial t$ depend on the signs of the income and substitution effects. We cannot sign the entire expressions because the substitution and income effect move in opposite directions. Our empirical estimates will shed some light on which effect dominates this comparative static result.

In summary, the model developed in this section describes consumer's decisions about participating in physical activities, time spent participating in physical activities and time spent all other activities. We conduct a comparative statics analysis to examine the effect of changes in income and the opportunity cost of time on participation and time spent in physical activity. To our knowledge, no previous research has developed and solved a formal consumer choice model of physical activity participation and time decisions.

Because of the lack of research in this area and the ambiguity of the comparative static analysis, constructing and estimating an empirical model is an important step in research into the economic determinants of physical activity. In the following section we describe a large, nationally representative data set that contains a rich amount of data on participation in physical activity and other economic and demographic factors. We then use these data to estimate an empirical specification of our consumer choice model to estimate the effect of economic factors and individual characteristics on physical activity.

Data Description and Sample Statistics

Since little previous research has focused on the economic determinants of participation in sport and physical activity, we empirically test the comparative static results from our consumer choice model. We use data from the Behavioral Risk Factor Surveillance System (BRFSS). The survey is conducted annually by telephone to a random representative sample of the population over the age of 18 in each U.S. state by the Center for Disease Control and Prevention in conjunction with U.S. states. The survey collects uniform state-specific data on preventative health factors, behavioral risk factors, and other economic and demographic characteristics and includes a rotating selection of modules one of which is on exercise and physical activity.

The BRFSS physical activity data is a rich source of information on participation in physical activities in the United States and has been used in some previous economic research. For example, Chou, et al. (2002a,2002b) used this data set to examine the link between obesity and physical activity. The survey asks about both frequency and duration of participation, which provides a relatively complete picture of self reported physical activities. The survey also asks questions about demographic factors like age, gender, race, ethnicity, and marital status, and questions about economic factors like income and labor market participation. This makes the BRFSS data an ideal setting for examining the economic determinants of physical activity. The physical activity module is not included in every year. We use data from the 2000 BRFSS survey, which included a module about physical activity and exercise.

184,450 persons were surveyed in the 2000 BRFSS survey. The 2000 survey included residents of Puerto Rico, and the exercise module was not administered to residents of Illinois that year. After excluding these observations, and some observations for individuals with a reported age under 18, a sample of 175,246 individuals remained. Table 1 shows some basic summary statistics for this sample of 175,246 individuals.

The average age of an individual in the sample was just under 47 years. 59% of the individuals sampled were female. In terms of minority representation, the sample was 8% black and 7% Hispanic. These categories are mutually exclusive in terms of race and ethnicity in the BRFSS survey methodology, which divides the sample into four categories ("white non-Hispanic," "black non-Hispanic," "black," and "other"). Over half of the respondents were married, and the average number of children present in the household was 0.75. The average number of children present in households that have children is 1.96 and the average number of children in households with a married couple and at least one child is 2.01.

64% of those surveyed were employed. Those who were not in the labor force were identified as short and long term unemployed persons, students, homemakers, people unable to work because of disabilities, and retired persons. 19% of the respondents were retired, 12% dropped out of high school, 32% were high school graduates, 28% had attended one or more years of college without graduating, and 29% were college graduates.

The BRFSS survey asks respondents about income from all sources. This is somewhat limiting because it potentially includes income from sources like pensions, capital, and government assistance programs in addition to income earned from work. Time allocation decisions depend heavily on the opportunity cost of time, which is related to the hourly wage. To the extent that the income variable reported in the BRFSS survey includes unearned income, this variable will be a poor proxy

Table 1: Summary Statistics

Variable	Mean	Standard Deviation
Age	46.88	17.21
Female	0.59	0.49
Black	0.08	0.27
Hispanic	0.07	0.25
Married	0.54	0.50
Number of Children	0.75	1.17
Employed	0.64	0.48
Retired	0.18	0.38
Income	$46,524	34,003
High School Dropout	0.12	0.32
High School Graduate	0.32	0.47
Some College	0.28	0.45
College Graduate	0.29	0.45
Physically Active	0.72	0.45
Average Times per Week Participating	2.36	2.49
Average Minutes Per Week Participating	197	330

for the hourly wage. The BRFSS survey reports income in ranges. The ranges in the survey are less than $10,000, between $10,000 and $15,000, between $15,000 and $20,000, between $20,000 and $25,000, between $25,000 and $35,000, between $35,000 and $50,000, between $50,000 and $75,000, and greater than $75,000. Following Ruhm (2005), the level of income for each individual is coded as the midpoint of the range reported, or 150% of the unbounded top range. Only 150,648 people responded to the income question in the 2000 BRFSS survey. This sub-sample forms the basis for the empirical work in the following sections. From Table 1, the average level of income in the sample was $46,524.

Physical Activity Measures

The 2000 BRFSS survey contained a module of questions on physical activity. These questions were asked to the entire sample except residents of Illinois. The basic physical activity question in the BRFSS survey is

During the past month, did you participate in any physical activities or exercises such as running, calisthenics, golf, gardening, or walking for exercise?

We initially define participation in physical activity using this survey question. From Table 1, 72% of the sample answered yes to this question. This is a relatively high participation rate for physical activity, but the question is not qualified by any statement about the duration of the activity, and many people may answer yes even when they spend very little time participating in physical activity. Fortunately, BRFSS contains more detailed questions about physical activity and exercise than a simple participation question. The survey also elicits detailed information about the type of activity, the frequency of participation, and the duration of participation in physical activity. The first question that elicits additional detail about the type of physical activity undertaken is

What type of physical activity or exercise did you spend the most time doing during the past month?

The responses to this question were a long list of activities from running to raking the lawn. Table 2 contains the entire list of physical activities reported and their frequency in the sample. The responses on Table 2 show a wide variation in reported physical activities in the sample. The activities also differ in a number of important ways. They require different amounts of travel, equipment, time and effort on the part of the participant. Some, like walking or running, can be done alone while others, like soccer or volleyball, require additional participants; still others can be done alone or in groups. In a related paper (Humphreys and Ruseski, 2007), we grouped the activities into five discrete categories roughly based on the underlying characteristics of the activities. We found considerable variation in participation and time spent in physical activity across different groups of physical activity. Because of the heterogeneity of the activities and variation in participation, future research will consider more carefully grouping the activities by common characteristics like equipment required, level of exertion, calories burned and other identifiable factors.

Walking is by far the most common physical activity. Just over 50% of the sample, 65,620 individuals, reported walking as their primary or secondary type of physical activity over the past month. No other type of physical activity was even close. The high reported frequency of walking as the primary form of physical activity in the sample probably reflects the relatively low cost of this activity. Unlike many of the other activities listed on Table 2, walking does not require any specialized equipment or facilities. It can be done in almost any setting under almost any conditions. In addition, walking is relatively undemanding physically so even people who are not in the best physical shape can participate. Recall, the question about physical activity specifically asks about exercising for leisure. In our model, people who walk to work are allocating time to transportation rather than to active leisure.

The survey asks about a first and a second activity participated in over the last month. 31% of those surveyed reported participating in two activities over the previous month. In this paper, we simply add up time spent in the primary and secondary physical activity to get a total measure of time spent in physical activity per week.

BRFSS also contains detailed information about how frequently individuals in the survey participated in physical activities in the past month, and how much time the individuals spent in each activity on average. These data provide enough detail to construct an estimate of the number of times per week and minutes per week that each individual in the survey spent participating in some physical activity. In estimating the duration of participation, we included the reported amount of time spent in both the primary and secondary physical activity.

The last two lines of Table 1 show the average frequency of participation and time spent participating in the sample. The average person in the sample participated in physical activity one and a third times per week. The average amount of time per week spent participating in physical activity was 197 minutes, about three and one quarter hours. A forty hour work week contains 2400 minutes.

Note that the survey asks about physical activities undertaken in the past month. The survey is administered throughout the year. Note that the sample does not constitute a panel. Each individual is contacted only once, but because of the large sample size, the survey takes a year to complete. Many physical activities take place outdoors, leading to some seasonal variation in participation. Table 3 shows the rate of participation in physical activities in the sample by month. The average participation rate of 72% across all months in the sample clearly masks considerable seasonal variation in participation. This seasonal variation is probably due to changes in climate. Runners and joggers may only exercise during warm months in northern states. We account for this seasonal variation in the participation rate in the empirical work described below and its effect on time spent in physical activity.

Econometric Analysis of Participation in Physical Activity

Table 1 shows that 72% of the individuals in the sample report participating in physical activity in the previous month. Of the 175,246 individuals in the 2000 BRFSS, 126,868 were physically active in the previous month and 48,378 were not. Both the indicator variable for time spent in physical activity and the variable for the amount of time spent in physical activity in the previous month

Table 2: Distribution of Physical Activities

Activity	Frequency	Percent
Walking	65620	50.20
Gardening	9213	7.05
Running	8172	6.25
Weightlifting	5437	4.16
Other	4581	3.50
Aerobics class	4357	3.33
Bicycling for pleasure	4032	3.08
Golf	3948	3.02
Home exercise	3250	2.49
Basketball	2424	1.85
Health club exercise	2319	1.77
Swimming laps	2148	1.64
Jogging	1719	1.32
Calisthenics	1608	1.23
Bicycling machine exercise	1272	0.97
Hiking cross country	1080	0.83
Tennis	947	0.72
Softball	791	0.61
Dancing-aerobics/Ballet	784	0.60
Mowing lawn	585	0.45
Bowling	544	0.42
Soccer	505	0.39
Judo/Karate	479	0.37
Snow skiing	465	0.36
Volleyball	476	0.36
Horseback riding	438	0.34
Hunting large game - deer, elk	436	0.33
Skating - ice or roller	426	0.33
Fishing from riverbank or boat	312	0.24
Racquetball	299	0.23
Stair climbing	301	0.23
Boxing	178	0.14
Surfing	159	0.12
Snow shoveling by hand	136	0.10
Carpentry	113	0.09
Waterskiing	115	0.09
Boating (canoeing, rowing, sailing)	100	0.08
Raking lawn	110	0.08
Touch football	104	0.08
Canoeing, rowing in competition	84	0.06
Rowing machine exercise	74	0.06
Mountain climbing	65	0.05
Badminton	35	0.03
Painting/Papering house	43	0.03
Snow shoeing	37	0.03
Backpacking	27	0.02
Rope skipping	26	0.02
Scuba diving	32	0.02
Sledding, tobogganing	26	0.02
Table tennis	21	0.02
Handball	14	0.01
Squash	13	0.01
Stream fishing in waders	12	0.01
Snorkeling	10	0.01
Snow blowing	9	0.01
Paddleball	4	0.00

Table 3: Participation by Month of Survey

Month	% Reporting Participation
January	0.67
February	0.67
March	0.71
April	0.72
May	0.76
June	0.77
July	0.78
August	0.76
September	0.75
October	0.73
November	0.71
December	0.67

contain a large number of zeros. Any econometric analysis of participation in physical activity must account for the large number of zeros observed in the data.

We assume that the zeros present in the data represent "genuine zeros" in the taxonomy developed by Jones (2000), meaning that the observed non-participation in physical activity in this sample is the result of the utility maximizing choices, as described in our theoretical model, of sampled individuals. The alternative explanation for the zeros is some sort of censoring that would take place if the time period mentioned in the survey instrument was so short that some individuals would not participate in the economic activity. Censoring in survey data has been associated with infrequent activities like the purchase of consumer durable goods (vehicles, televisions, etc.). However, we assume that the one month time period referred to in the survey instrument is long enough to avoid censoring.

Both Jones (2000) and Amemiya (1984) discuss the appropriate econometric techniques for dealing with zeros that are the result of utility maximizing decisions in survey data. In the terminology used by Jones, "genuine zeros" call for either a two-part model or a hurdle model. In the terminology used by Amemiya (1984) these are called Tobit Type I and Tobit Type II models. In both cases, the models can be motivated by a latent variable interpretation of the utility maximization model. Define two latent variables y^*_{1i} representing (unobserved) utility from participation in physical activity and y^*_{2i} representing (unobserved) utility from time spent in physical activity.

These latent variables can be interpreted as capturing the utility generated by participating in physical activity and time spent in physical activity defined by the utility function, equation (1) above. Formally, the expressions for these latent variables are

$$\begin{aligned} y_{1i}^* &= x_{1i}'\beta_1 + u_1 \\ y_{2i}^* &= x_{2i}'\beta_2 + u_2 \\ y_{2i} &= y_{2i}^* \qquad \text{if } y_{1i}^* > 0 \\ y_{2i} &= 0 \qquad \text{if } y_{1i}^* \leq 0 \end{aligned} \tag{8}$$

where (u_{1i}, u_{2i}) are realizations from an independent and identically distributed, mean zero, constant variance bivariate normal distribution. The variance of these two variables are σ_1^2 and σ_2^2 and the covariance between them is σ_{12}. By assumption, only the sign of y_{1i}^* is observed. y_{2i}^* is only observed when y_{1i}^* is positive. Also, variables in x_{1i} are observed for all i but variables in x_{2i} may not be observed when the utility of participation is negative ($y_{1i}^* \leq 0$.) Based on this latent variable representation, define an indicator variable (w_{1i}) for participation in physical activity

$$\begin{aligned} w_{1i} &= 1 \quad \text{if } y_{1i}^* > 0 \\ w_{1i} &= 0 \quad \text{if } y_{1i}^* \leq 0. \end{aligned} \tag{9}$$

In this case, the key observed variables in the sample are pairs (w_{1i}, y_{2i}) of the indicator variable for participation in physical activity and the time spent in physical activity, along with the vectors of covariates x_{1i}' and x_{2i}' The likelihood function for these two latent variables can be written

$$L = \prod_0 P(y_{1i}^* \leq 0) \prod_1 f(y_{2i}|y_{1i}^* > 0) P(y_{1i}^* > 0) \tag{10}$$

where $f(\cdot|y_{1i}^* > 0)$ is the conditional density of y_{1i}^* given that the utility from participation is positive $(y_{1i}^* > 0.)$ The subscript 0 on the product operator refers to non-participants and the subscript 1 refers to participants. Note that this likelihood function can also be written

$$L = \prod_0 P(y_{1i}^* \leq 0) \prod_1 P(y_{1i}^* > 0) \times \prod_1 f(y_{2i}|y_{1i}^* > 0). \tag{11}$$

The first term on the right hand side of equation (11) is the Probit model for participation. The second term on the right hand side of equation (11) is the conditional density of the utility from consumption, given that the utility from participation is positive for the observations where positive consumption is observed. Amemiya (1984) shows that, using the definition of a conditional density function, this likelihood function can be re-written

$$L = \prod_0 \left[1 - \Phi\left(\frac{x_i'\beta_1}{\sigma_1}\right)\right] \times \prod_1 \Phi\left[\frac{\left(\frac{x_{1i}'\beta_1}{\sigma_1} + \frac{\sigma_{12}}{\sigma_1\sigma_2}(y_{2i} - x_{2i}'\beta_2)\right)}{\left(\sqrt{1 - \frac{\sigma_{12}^2}{\sigma_1^2\sigma_2^2}}\right)}\right] \frac{1}{\sigma_2}\phi\left(\frac{y_{2i} - x_{2i}'\beta_2}{\sigma_2}\right). \qquad (12)$$

This is the "full double hurdle model" derived by Jones (1992) that includes correlation between the equation error terms (u_{1i}, u_{2i}). Assuming no correlation between u_{1i} and u_{2i} so that $(\sigma_{12} = 0)$, this likelihood function can be written

$$L = \prod_0 \left[1 - \Phi\left(\frac{x_i'\beta_1}{\sigma_1}\right)\right] \prod_1 \Phi\left[\frac{x_{1i}'\beta_1}{\sigma_1}\right] \frac{1}{\sigma_2}\phi\left(\frac{y_{2i} - x_{2i}'\beta_2}{\sigma_2}\right) \qquad (13)$$

where the first two terms on the right hand side are the Probit model for participation and the third term is a truncated normal regression model. This is commonly called the Cragg model in the literature and was first developed by Cragg (1971). Finally, assuming that all observations represent participants $[P(y_{1i}^* = 1)$ and $y_{2i} = y_{2i}^*]$ and that y_{2i}^* is distributed normally with mean $x_{1i}'\beta_1$ and variance σ_1^2 the Cragg Model can be rewritten as

$$L = \prod_0 \left[1 - \Phi\left(\frac{x_i'\beta_1}{\sigma_1}\right)\right] \prod_1 \frac{1}{\sigma_1}\phi\left(\frac{y_{1i} - x_{1i}'\beta_1}{\sigma_1}\right) \qquad (14)$$

which is the familiar Tobit model, which Amemiya calls the Tobit Type I model.

All three of these models can be applied to data that contain zeros generated by corner solutions in a utility maximizing model, and their parameters estimated by maximum likelihood. The "full double hurdle model," equation (12), allows for factors that affect participation and factors that affect time spent to have different signs and for correlation in the equation error terms, meaning that the unobservable factors affecting participation and time spent in physical activity can be correlated. The Cragg model allows for factors that affect participation and factors that affect time spent to have different signs but assumes independence of the error terms. The Tobit model forces factors that affect participation and time spent to have the same sign.

In this setting, the full double-hurdle model is not possible to estimate for the full sample. We lack computing power to numerically evaluate equation (12) across more than roughly 10,000 observations. In practice, most empirical research that treats zeros in survey data as corner solutions uses either the Cragg model or the Tobit model. We estimate the parameters of both models using maximum likelihood.

Results and Discussion

We estimate the parameters of both the Cragg model, equation (13), and the Tobit model, equation (14), using a restricted sample of 74,653 individuals who were employed and were between the age of 25 and 54.[2] The theoretical model developed above points out that the opportunity cost of time is an important determinant of participation in physical activity. The BRFSS data contain only information about total income, not earned income, so we do not have a good measure of an individual's wage, the most common proxy for the opportunity cost of time. We restrict the sample to employed people who are in their prime earning years to reduce the systematic effect of unmeasured variation in the opportunity cost of time on the empirical results. In addition, restricting the sample in this way allows a cleaner interpretation of the effect of the full budget constraint described in equation (5) on decisions to be physically active.

Table 4: Parameter Estimates and z-statistics

Variable	Tobit model	Cragg model time spent	Cragg model participation
Age	-1.63	-1.60	-0.053
	(-8.58)	(-8.33)	(-2.66)
Married	-36.57	-35.39	-4.14
	(-11.22)	(-10.74)	(-0.33)
Number of Children	-4.47	-4.44	0.043
	(-3.68)	(-3.590)	(0.22)
Income (in 000)	0.926	0.905	0.079
	(19.70)	(18.97)	(4.71)
Attended College	60.56	58.81	0.975
	(18.78)	(17.99)	(1.22)
Female	-66.74	-66.86	1.769
	(-23.23)	(-22.98)	(2.05)
Black	-46.50	-45.21	0.463
	(-8.80)	(-8.51)	(0.55)
Hispanic	-41.07	-35.80	-1.226
	(-7.07)	(-5.93)	(-2.51)
Constant	237.39	183.72	6.770
	(27.12)	(21.67)	(0.55)

Time spent: exercise minutes per week

Participation: Indicator variable for participation in physical activity

[2] The summary statistics for the restricted sample are provided in Table 5 in technical appendix.

Table 4 shows the parameter estimates and asymptotic z-statistics (in parentheses) from both the Tobit model, equation (14), and the time spent and participation equations for the Cragg model, equation (13). Both models contained indicator variables for the season during which the physical activity took place, to control for the monthly variation in reported physical activity. The parameters on these variables were significant in the time equations and indicated that more time was spent in physical activity during the summer months. The results for the two alternative econometric approaches are very similar. Clearly the restrictions placed on the data by the Tobit model do not affect the results.

Effect of Income and Opportunity Cost of Time on Physical Activity and Sport

Individuals with higher income are more likely to participate in physical activity. This effect probably reflects the cost of equipment and facilities associated with participation in many of the activities. Higher income allows individuals to afford gym memberships, tennis rackets, home exercise equipment and other goods and services needed to participate in these physical activities. The effect of income on time spent in physical activity is positive but small. An increase in income of $1,000 per year is associated with about a minute more time spent in physical activity per week.

Although the magnitude of the income effect time spent is not large, the fact that the signs of the income effect on both participation and time spent are positive and significant are economically important for two reasons. First, the empirical results indicate that the comparative static derivatives from the model developed above describing the effect of a change in income on participation ($\partial a/\partial y$) and the optimal time spent ($\partial t/\partial y$) are positive, implying that physical activity is a normal good. To our knowledge, this is the first evidence that physical activity and participation in sport are normal goods. This is important for informing policy because it suggests that consumers will respond to economic incentives to become physically active or to increase the amount of time devoted to physical activity in conventional ways. Second, recall that in order for $\partial a/\partial y > 0$ and $\partial t/\partial y > 0$ to be positive, the cross partial derivatives U_{az} and U_{tz} must be positive. These cross partial derivatives describe the relationship between the marginal utility of physical activity and the marginal utility from other activities. This suggests that being physically active leads to increased enjoyment of other activities, including non-active leisure activities.

The results on Table 4 indicate that education is positively associated with time spent in physical activity. We interpret this relationship between educational attainment and time spent in physical activity as the opportunity cost of time effect. A positive relationship between income and education has been widely documented in the economics literature. Evidence shows that more educated people tend to have higher paying jobs and higher hourly wages and, therefore, higher opportunity costs of time. Recall from the model that the effect of a change in the opportunity cost of time on both participation in and duration of physical activity has an income and substitution effect. A higher opportunity cost of time makes non-work related activities more costly and reduces the amount of time spent participating in those activities; therefore, decreasing time spent participating in physical activities as education increases indicates that the substitution effect dominates the income effect. Conversely, a positive relationship between education and time spent in physical activity is suggestive of a dominating income effect. Participation in physical activities entails monetary costs and people with higher income have greater financial means to participate. Our education variable is an indicator variable that is equal to one if the individual attended college and zero otherwise. Attending college does not affect the decision to participate in physical activity and sport but there is a strong positive effect of education on duration. College attendees spend an hour longer, on average, participating in physical activity than those who did not attend college. Our results are suggestive of a dominating income effect.

Effect of Individual Characteristics on Physical Activity and Sport

We examine the influence of age, marital status, number of children, gender and race/ethnicity on physical activity. Age is important in both the participation and duration decisions, even in our sample of 25 to 54 year olds. Older people are less likely to participate in physical activity. In addition, the time spent engaging in physical activity and sports declines with age. The decrease in time spent is about 1 1/2 minutes per week for each one year increase in age. Although the magnitude of the age effect appears small, the cumulative effect is not and it highlights the importance of encouraging working adults to become and stay physically active as they age.

Being married and having children present in the household do not affect the decision to participate in physical activity and sport but these factors negatively affect the time spent. Time spent decreases by about 4.5 minutes per week as the number of children in the household increases. Married people spend over a half-hour less time in physical activity than single people. Both of these results indicate that married couples and households with children have different demands on their time and different opportunity costs of time than unmarried and childless people.

Even after controlling for differences in income, the number of children and marital status, females are more likely to participate in physical activity than males. Recall that this sample excludes men and women who are not in the labor force. However, of those men and women who do choose to be physically active, women spend less time participating than men. Taken together, these results suggest that the opportunity of time for men in the labor force is different from that of women. These difference could be due to occupational choices, if females tend to sort themselves into occupations that require more hours of work, or offer less job flexibility than males. Examples of such occupations include nursing, primary and secondary education, and secretarial work. It could also reflect differences in the underlying types of physical activities preferred by males and females. There is evidence that males are more likely to participate in activities that take more time like group sports and outdoor recreation activities whereas females are more likely to walk for exercise (Humphreys and Ruseski 2007). The decisions of women to spend less time in physical activity is indicative of the differential challenges working women face in allocating time to leisure relative to men. This finding is consistent with the results in Aguiar and Hurst (2007). They find a larger increase in time spent in leisure for men than women over the past 40 years.

Blacks are not more or less likely to participate in physical activity than whites but spend about 45 minutes per week less time in physical activity than whites. Hispanics are both less likely to participate and spend fewer minutes per week exercising. These differences may simply indicate that employed minorities have poorer access to the goods and services needed to participate in physical than whites. One implication of this result is that interventions aimed at increasing the participation these groups might be very effective but focusing attention increasing the amount of time spent in physical activity should be a priority.

Conclusions and Suggestions for Future Research

This research examines participation and time spent in physical activity by developing a consumer's choice model containing these elements and by empirically estimating the model using a large nationally representative data set. A number of interesting conclusions emerge from the analysis. With respect to economic variables, we find that income has a positive and significant effect on both participation in and time spent in physical activity indicating that physical activity is a normal good. We also find that the income effect dominates the substitution effect as the opportunity cost of time increases.

The results have important implications for those designing policy interventions aimed at increasing participation in physical activity. Married people, females, blacks, hispanics and households with children all spend considerably less time engaged in physical activity each week than unmarried, childless, male and white people. Policy interventions designed to target these subpopulations are likely to be more effective than a "one size fits all" approach to policy design. For example, policy interventions somehow linked to daycare for children of employed people are likely to be more effective in increasing the amount of time they spend participating in physical activity. Participation and time spent both decline with age, even in a sample of young adults to middle-aged people. These results suggest that programs aimed at increasing participation in older populations and encouraging continued participation over the life cycle might be particularly effective. Both participation and time spent in physical activity display seasonal variation; both decline during cold weather months and increase during warm weather months. These results suggest that policy interventions aimed at increasing physical activity should take into account this seasonal variation. Finally, the model and empirical results suggest that the opportunity cost of time plays a key role in both the participation and time decision. Any policy interventions that ignore this dimension of the decision to participate in physical activity may not be very effective.

While the model provides new insight into role of economic determinants of participation and time spent in physical activity, it has the potential to be extended in many ways. One clear extension of the model is to include physical activity as an input to the production of health. This extension should allow us to examine the economic links between physical activity and obesity, and also explicitly link physical activity to the consumption of health goods and services. Grossman's (1972) model of health production provides one possible way to expand this model. Empirical studies of health production indicate that more highly educated people are more efficient at producing health. Our finding that the time spent exercising increases with education provides further evidence of the link between education and health production.

The decision to participate in physical activity needs to be explicitly linked to economic outcomes like employment and earnings. Previous research by Long and Caudill (1991), Barron, et al. (2000), and Eide and Ronan (2001) show a clear link between participation in physical activity and labor market outcomes and lieftime earnings. This suggests an important link between participation in physical activity and human capital and labor productivity. Much of the previous literature focused on participation in team sports in secondary schools and college. The importance of age in explaining observed participation and time spent in the broad measures of physical activity examined here suggest that a closer look at the relationship between this type of activity and labor market outcomes warrants additional attention.

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Technical Appendix

Consumers choose a, t and z to maximize utility subject to the full income constraint. The lagrangian for this problem is

$$V = U(a,t,z) - \lambda(F_a + p_a \cdot a \cdot t + p_z z - y) \tag{15}$$

The first order conditions characterizing the optimal choices of a, t and z are found by partially differentiating V with respect to the choice variables and the Lagrange multiplier

$$\begin{aligned}
\frac{\partial V}{\partial a} &= \frac{\partial U}{\partial a} - \lambda p_a t &= 0\\
\frac{\partial V}{\partial t} &= \frac{\partial U}{\partial t} - \lambda p_a t &= 0\\
\frac{\partial V}{\partial z} &= \frac{\partial U}{\partial z} - \lambda p_z &= 0\\
\frac{\partial V}{\partial \lambda} &= -(F_a + p_a \cdot a \cdot t + p_z z - y) &= 0.
\end{aligned}$$

Totally differentiate each first order conditions and set this total differential equal to zero to form the system of equations needed for a comparative static analysis of the consumer's choice problem

$$\begin{aligned}
U_{aa}da + U_{at}dt + U_{az}dz - \lambda t dp_a - \lambda p_a dt - p_a t d\lambda &\equiv 0\\
U_{ta}da + U_{tt}dt + U_{tz}dz - \lambda a dp_a - \lambda da p_a - p_a a d\lambda &\equiv 0\\
U_{za}da + U_{zt}dt + U_{zz}dz - \lambda dp_z - p_z d\lambda &\equiv 0\\
-dF_a - p_a t da - p_a a dt - at dp_a - z dp_z - p_z dz + dy &\equiv 0.
\end{aligned}$$

The system of total differential equations can be expressed compactly in matrix form

$$\begin{bmatrix} U_{aa} & U_{at} - \lambda p_a & U_{az} & -p_a t \\ U_{at} - \lambda p_a & U_{tt} & U_{tz} & -p_a a \\ U_{za} & U_{zt} & U_{zz} & -p_z \\ -p_a t & -p_a a & -p_z & 0 \end{bmatrix} \begin{bmatrix} da \\ dt \\ dz \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda dp_a t \\ \lambda dp_a a \\ \lambda dp_z \\ at dp_a + z dp_z + dF_a - dy \end{bmatrix} \tag{16}$$

and this coefficient matrix is the familiar Jacobian, $|J|$, from standard consumer theory. The Jacobian forms the basis of a comparative static analysis of the effect of changes in exogenous variables on the choice variables. We analyze the effects of changes in income and changes in the opportunity cost of time on the decision to participate in physical activity and on the decision about how much time to spend participating in physical activity.

We first derive comparative static expressions for the the effect of a change in income (dy) on both the participation decision a and the optimal amount of time spent in physical activity t. To find the effect of change in income (dy) on a and t holding dp_a, dp_z and dF_a, divide the system of total differential equations through by dy

$$\begin{bmatrix} U_{aa} & U_{at}-\lambda p_a & U_{az} & -p_a t \\ U_{at}-\lambda p_a & U_{tt} & U_{tz} & -p_a a \\ U_{za} & U_{zt} & U_{zz} & -p_z \\ -p_a t & -p_a a & -p_z & 0 \end{bmatrix} \begin{bmatrix} \frac{da}{dy} \\ \frac{dt}{dy} \\ \frac{dz}{dy} \\ \frac{d\lambda}{dy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}.$$

Since the participation and time decisions are separate decisions, we first solve for the comparative static derivative da/dy by holding dt constant and setting $dt/dy = 0$. In matrix form, the restricted model becomes

$$\begin{bmatrix} U_{aa} & U_{az} & -p_a t \\ U_{za} & U_{zz} & -p_z \\ -p_a t & -p_z & 0 \end{bmatrix} \begin{bmatrix} \frac{da}{dy} \\ \frac{dz}{dy} \\ \frac{d\lambda}{dy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

This matrix is the Jacobian matrix for the participation decision and is denoted $|J_p|$. The comparative static derivative in matrix form is

$$\frac{\partial a}{\partial y} = \frac{|J_a|}{|J_p|} = \frac{\begin{vmatrix} 0 & U_{az} & -p_a t \\ 0 & U_{zz} & -p_z \\ -1 & -p_z & 0 \end{vmatrix}}{\begin{vmatrix} U_{aa} & U_{az} & -p_a t \\ U_{za} & U_{zz} & -p_z \\ -p_a t & -p_z & 0 \end{vmatrix}}.$$

Finding the determinants $|J_a|$ and $|J_p|$ and substituting yields

$$\frac{\partial a}{\partial y} = \frac{|J_a|}{|J_p|} = \frac{U_{az}p_z - p_a t U_{zz}}{p_z(-U_{aa}p_z + U_{za}p_a t) - p_a t(-U_{az}p_z + U_{zz}p_a t)} \tag{17}$$

Next, we solve for the comparative static derivative dt/dy to examine the effect of changes in income on the optimal amount of time spent in physical activity by holding da constant and setting $da/dy = 0$. We hold da constant because the decision about the amount of time an individual participates in physical activity is only relevant if the individual chooses to participate. The restricted model in matrix form becomes

$$\begin{bmatrix} U_{tt} & U_{tz} & -p_a a \\ U_{zt} & U_{zz} & -pz \\ -p_a a & -p_z & 0 \end{bmatrix} \begin{bmatrix} \frac{dt}{dy} \\ \frac{dt}{dy} \\ \frac{d\lambda}{dy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

This is the Jacobian matrix for the time decision and is denoted $|J_d|$. The comparative static result is

$$\frac{\partial t}{\partial y} = \frac{|J_t|}{|J_d|} = \frac{\begin{vmatrix} 0 & U_{tz} & -p_a a \\ 0 & U_{zz} & -pz \\ -1 & -p_z & 0 \end{vmatrix}}{\begin{vmatrix} U_{tt} & U_{tz} & -p_a a \\ U_{zt} & U_{zz} & -p_z \\ -p_a a & -p_z & 0 \end{vmatrix}}.$$

Solving for the determinants $|J_t|$ and $|J_d|$ and substituting yields

$$\frac{\partial t}{\partial y} = \frac{|J_t|}{|J_d|} = \frac{U_{tz}p_z - p_a a U_{zz}}{p_z(-U_{tt}p_z + U_{zt}p_a a) - p_a a(-U_{tz}p_z + U_{zz}p_a a)} \tag{18}$$

The opportunity cost of time affects the decision to participate in physical activity and the amount of time devoted to physical activity. Recall, $p_a = c_a + w$ and $p_z = c_z + \theta w$. The opportunity cost of time is the wage rate w. Expanding the lagrangian to explicitly show the full cost of time spent in physical activity and all other activities is

$$V = U(a, t, z) - \lambda(F_a + (c_a + w) \cdot a \cdot t + (c_z + \theta w)z - y). \tag{19}$$

The system of total differential equations expressed compactly in matrix form is

$$\begin{bmatrix} U_{aa} & U_{at} - \lambda(c_a + w) & U_{az} & -(c_a t + wt) \\ U_{ta} - \lambda(c_a + w) & U_{tt} & U_{tz} & -(c_a a + wa) \\ U_{za} & U_{zt} & U_{zz} & -(c_z + \theta w) \\ -t(c_a + w) & -a(c_a + w) & -(c_z + \theta w) & 0 \end{bmatrix} \begin{bmatrix} da \\ dt \\ dz \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda(dc_a t + dwt) \\ \lambda(dc_a a + dwa) \\ \lambda(dc_z + dw\theta + d\theta w) \\ I \end{bmatrix}$$

where

$$I = atdc_a + (at + \theta z)dw + zdc_z + wzd\theta + dF_a - dy.$$

In this expression the coefficient matrix is the Jacobian $|J|$ for the system of equations based on the expanded or full income constraint. This matrix is denoted $|J_{FI}|$.

We examine the effect of a change in the opportunity cost of time, (dw), on the participation decision, a and the amount of time spent in physical activity, t. Divide the system of total differential equations through by dw, holding dc_a, dc_z, dF_a, and $d\theta$ constant. The system in matrix form becomes

$$\begin{bmatrix} U_{aa} & U_{at}-\lambda(c_a+w) & U_{az} & -(c_at+wt) \\ U_{ta}-\lambda(c_a+w) & U_{tt} & U_{tz} & -(c_aa+wa) \\ U_{za} & U_{zt} & U_{zz} & -(c_z+\theta w) \\ -t(c_a+w) & -a(c_a+w) & -(c_z+\theta w) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial a}{\partial w} \\ \frac{\partial t}{\partial w} \\ \frac{\partial z}{\partial w} \\ \frac{\partial \lambda}{\partial w} \end{bmatrix} = \begin{bmatrix} \lambda t \\ \lambda a \\ \lambda\theta \\ ta+\theta z \end{bmatrix}$$

Since the participation decision and the time decision are sequential, we solve for the comparative static result, da/dw by holding dt constant and setting $\frac{dt}{dy}=0$. The model in matrix form becomes

$$\begin{bmatrix} U_{aa} & U_{az} & -(c_at+wt) \\ U_{za} & U_{zz} & -(c_z+\theta w) \\ -t(c_a+w) & -(c_z+\theta w) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial a}{\partial w} \\ \frac{\partial z}{\partial w} \\ \frac{\partial \lambda}{\partial w} \end{bmatrix} = \begin{bmatrix} \lambda t \\ \lambda\theta \\ ta+\theta z \end{bmatrix}$$

The coefficient matrix is the Jacobian matrix for the participation decision in the model with expanded income constraint and is denoted $|J_{FI_p}|$. The comparative static derivative is

$$\frac{\partial a}{\partial w} = \frac{|J_{FI_a}|}{|J_{FI_p}|} = \frac{\begin{vmatrix} \lambda t & U_{az} & -(c_at+wt) \\ \lambda\theta & U_{zz} & -(c_z+\theta w) \\ ta+\theta z & -(c_z+\theta w) & 0 \end{vmatrix}}{\begin{vmatrix} U_{aa} & U_{az} & -(c_at+wt) \\ U_{za} & U_{zz} & -(c_z+\theta w) \\ -t(c_a+w) & -(c_z+\theta w) & 0 \end{vmatrix}}$$

Finding the determinants of $|J_{FI_a}|$ and $|J_{FI_p}|$, substituting and rearranging yields

$$\frac{\partial a}{\partial w} = \frac{-(ta+\theta z)}{|J_{FI_p}|}\cdot(U_{zz}t(c_a+w)-U_{az}(c_z+\theta w)-\frac{\lambda t}{|J_{FI_p}|}\cdot((-c_z-\theta w)^2-\theta(-c_z-w)(-c_a-w)) \quad (20)$$

where

$$|J_{FI_p}| = (c_z+\theta w)(-U_{aa}(c_z+\theta w)) + U_{za}(t(c_a+w)) - (c_a+w)t(-U_{az}(c_z+\theta w) + U_{zz}t(c_a+w)).$$

Next we examine (dt/dw) by holding da constant and setting $\frac{da}{dw} = 0$. The matrix equation becomes

$$\begin{bmatrix} U_{tt} & U_{tz} & -a(c_a+w) \\ U_{zt} & U_{zz} & -(c_z+\theta w) \\ -a(c_a+w) & -(c_z+\theta w) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial t}{\partial w} \\ \frac{\partial z}{\partial w} \\ \frac{\partial \lambda}{\partial w} \end{bmatrix} = \begin{bmatrix} \lambda a \\ \lambda\theta \\ ta+\theta z \end{bmatrix}$$

The coefficient matrix is the Jacobian for the time decision in the model with the expanded income constraint and is denoted $|J_{FI_d}|$. The comparative static derivative is

$$\frac{\partial t}{\partial w} = \frac{|J_{FI_t}|}{|J_{FI_d}|} = \frac{\begin{vmatrix} \lambda a & U_{tz} & -a(c_a+w) \\ \lambda\theta & U_{zz} & -(c_z+\theta w) \\ ta+\theta z & -(c_z+\theta w) & 0 \end{vmatrix}}{\begin{vmatrix} U_{tt} & U_{tz} & -a(c_a+w) \\ U_{zt} & U_{zz} & -(c_z+\theta w) \\ -a(c_a+w) & -(c_z+\theta w) & 0 \end{vmatrix}}$$

Finding the determinant of $|J_{FI_t}|$ and $|J_{FI_d}|$, substituting and rearranging yields

$$\frac{\partial t}{\partial w} = \frac{-(ta+\theta z)}{|J_{FI_d}|} \cdot (U_{zz}a(c_a+w) - U_{tz}(c_z+\theta w) - \frac{\lambda a}{|J_{FI_d}|} \cdot ((-c_z-\theta w)^2 - \theta((-c_z-\theta w)(-c_a-w)) \quad (21)$$

where

$$|J_{FI_d}| = (c_z+\theta w)(-U_{tt}(c_z+\theta w)) + U_{tz}(a(c_a+w)) - (c_a+w)a(-U_{tz}(c_z+\theta w) + U_{zz}a(c_a+w)).$$

Table 5: Summary Statistics for Sample of Employed Individuals aged 25-54

Variable	Mean	Standard Deviation
Age	39.66	7.69
Married	0.60	0.49
Number of Children	1.10	1.25
Income (in 000)	55.12	34.25
Attended College	0.66	0.48
Female	0.54	0.50
Black	0.08	0.28
Hispanic	0.07	0.26
Physically Active	0.76	0.43
Average Times per Week Participating	2.31	2.24
Average Minutes per Week Participating	199.31	312.30
# of observations	74,653	

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