STAT 665 - Sample Midterm Exam

Instructor: Professor D. Wiens

Time: 80 minutes
Instructions: Answer all questions in the answer booklets supplied. There are 5 questions; each is worth 20% of the total mark on this exam.

1. (a) Define what it means for a sequence of r.v.s X_n to converge *in probability* to a constant c.

   (b) Prove: If A_n, B_n are sequences of r.v.s such that, as $n \to \infty$, $P(|A_n| \leq B_n) \to 1$ and $B_n \xrightarrow{pr} b$, where b is a finite constant, then $A_n = O_P(1)$.

2. (a) Define what we mean by the *cumulants* of a distribution.

   (b) Show that all cumulants of the Poisson $\mathcal{P}(\lambda)$ distribution equal λ.

3. Suppose that X is a (binomial) $bin(n, p)$ r.v. with $0 < p < 1$. Consider estimating p^2 by $\delta = (X/n)^2$.

   (a) Is this estimator consistent?

   (b) Show that δ (properly normalized) is asymptotically normal. Exhibit the limiting distribution.

   Prove your statements, making reference to any results from class which you use.

4. (a) Define what it means for two sequences of tests to be *asymptotically equivalent.*

   (b) Suppose that V_n and V_n' are two sequences of test statistics such that $W_n = V_n - V_n' \xrightarrow{pr} 0$, and V_n has a continuous limiting distribution function F. Show that, for a fixed constant c, the tests with rejection regions $V_n \geq c$ and $V_n' \geq c$ are asymptotically equivalent.

5. Let $X_{(n)}$ be the largest of n i.i.d. Uniform$(0, \theta)$ random variables. Suppose that we test $H : \theta = \theta_0$ versus $K : \theta > \theta_0$ by rejecting if $n\left(1 - \frac{X_{(n)}}{\theta_0}\right)$ does not exceed $l_\alpha = -\ln(1 - \alpha)$. Determine the limiting power against alternatives of the form $\theta = \theta_0 + \Delta/n$.