STATISTICS 568 SAMPLE FINAL EXAM

Instructions: Answer any six of these eight equally weighted questions. If you answer more than six then the best six will be counted. If necessary you may use a calculator.

Time: 3 hours.

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1. Describe 'Lenth's method' for the testing of significance of estimated effects $\left\{\hat{\theta}_i\right\}_{i=1}^{I}$. You should describe as many of the details of the computation as you can, but particular attention should be paid to the qualitative aspects – what are the underlying ideas, and when might this method be preferred to others?

2. Consider a 2^{6-2} fractional factorial design with treatment defining contrast subgroup generated by
$$1 = 235 = 246,$$
and block defining contrast subgroup generated by
$$B_1 = 134, \ B_2 = 234.$$
   (a) What are the 16 runs, and which runs are in which blocks?
   (b) What does it mean for an effect to be 'clear'? Show that in this design all main effects are clear.
   (c) Under what assumption(s) about the interactions would you be able to estimate the block×block interaction effect? Assuming that this holds, how exactly would you estimate this effect?

3. Write down, with a verification, the defining contrast subgroup for a 2^{6-2} design where the main effect for factor A and all two-factor interactions involving A are clear. Define 'resolution'. What is the resolution of your design?

...*over*

4. Two choices of generators for a 2^{6-2} design are being considered:

$$\begin{aligned}(i)\ 5 &= 1234,\ 6 = 123,\\ (ii)\ 5 &= 123,\ 6 = 234.\end{aligned}$$

   (a) Which design do you prefer? Justify your answer.

   (b) Show that it is impossible to have a 2_V^{6-2}design.

   (c) Show that in a resolution R design, no effect involving i factors is aliased with effects involving fewer than $R - i$ factors.

5. Consider the 2_{III}^{7-4} design with defining relations

$$4 = 12, 5 = 13, 6 = 23, 7 = 123.$$

   (a) Write down the defining contrast subgroup.

   (b) Which main effects are clear in this design? Justify your answer.

   (c) Explain how you would make a further 8 runs, in such a way that the combined design of 16 runs would have all main effects clear.

6. In an experiment to determine the minimum value on a certain four-dimensional response surface, with independent variables $x_1, x_2, x_3 \in [-1, 1]$, a central composite design was employed and a second order model, parameterized as

$$E[y|\mathbf{x}] = \beta_0 + \mathbf{x}'\mathbf{b} + \mathbf{x}'\mathbf{B}\mathbf{x},$$

was fitted. The estimates (which are being reported in a simplified manner which makes for easy calculations), were

$$\hat{\beta}_0 = 1,\ \hat{\mathbf{b}} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix},\ \hat{\mathbf{B}} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

   (a) Estimate the stationary point of the surface.

   (b) Describe the nature of the stationary point - do you think it represents a minimum, or something else?

   (c) If the experimenter is willing to make further observations in order to better locate the minimum, where would you advise him to look? Justify your answer.

...over

7. (a) Briefly describe the motivation for 'robust parameter design'. What are control and noise factors? Describe two methods of analysis, based on the 'response modelling' approach.

   (b) What is the defining characteristic of a Latin Hypercube design? Construct such a design, for the three-dimensional design space $[0, 1] \times [0, 1] \times [0, 1]$, with 5 design points.

8. Suppose that an experimenter plans to take n observations at points $\{x_i\}$ chosen from $\chi = [-1, 1]$, after which he will fit, by least squares, a straight line response

$$E[y|x] = \theta_0 + \theta_1 x.$$

Denote by $\hat{y}(x) = \hat{\theta}_0 + \hat{\theta}_1 x$ the predicted values and by ξ the design measure, according to which the experimenter will make $n_i = n\xi_i$ of the observations at x_i.

   (a) Express the covariance matrix of $\hat{\boldsymbol{\theta}} = \left(\hat{\theta}_0, \hat{\theta}_1\right)'$ in terms of the moments of ξ.

   (b) A design is 'I-optimal' if it minimizes $\int_\chi \text{VAR}[\hat{y}(x)]\, dx$. Determine the I-optimal design for this situation. (You may assume that n is odd or that n is even, if that is convenient for you.)