STAT 568 – Assignment 3 – due date is on course outline

For each of the questions which are carried out on R – please e-mail me your R programs. Question numbers from the text refer to the current version; check with me if you are using an earlier version

1. Text, #10.10. In (b) you should (i) analyze only the data from the first order design (the first 17 runs) and make a recommendation. Then (ii) assume that your advice was ignored and that the design was augmented by the axial points with the same centre as was used in (i) Describe the fitted response surface and identify factor settings that maximize the response, assuming that they must remain within the given ranges $[-\alpha, \alpha]$.

2. Text, #11.20 (b) – (d). (Note that there are two analyses which fall under the heading of 'response modelling'.)

3. Minimum mse prediction: Given training data $\{\mathbf{x}_i^t, y_i^t\}_{i=1}^n$ and an untried new input $\mathbf{x}_0$, we wish to predict a r.v. $Y(\mathbf{x}_0)$ by a function $\hat{Y}$ of the data $\mathbf{Y}^n$.

   (a) Show that the minimum MSE predictor of $Y(\mathbf{x}_0)$ is $\hat{y}(\mathbf{x}_0) = E[Y(\mathbf{x}_0)|\mathbf{Y}^n]$.

   (b) Show that the minimum MSE predictor under the GaSP model assumptions is also the BLUP, using only the moment assumptions of the GaSP model. Show also that the minimum MSPE is

   $$\text{MSPE}[\hat{y}(\mathbf{x}_0)] = \sigma_Z^2 \left\{ 1 - \left((\mathbf{f}'(\mathbf{x}_0), \mathbf{r}'(\mathbf{x}_0)) \begin{pmatrix} \mathbf{0}_{k\times k} & \mathbf{F}' \\ \mathbf{F} & \mathbf{R}_{n\times n} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{f}(\mathbf{x}_0) \\ \mathbf{r}(\mathbf{x}_0) \end{pmatrix} \right) \right\}.$$

   (c) Verify that this predictor interpolates the data: $\hat{y}(\mathbf{x}_i^t) = y(\mathbf{x}_i^t)$.

4. Show that on $\chi = [0, 1]$ the n-point design given by $x_i = \frac{i-.5}{n}, i = 1, ..., n$, is minimax in the sense of Lecture 22.

5. Show that for quadratic regression on $[-1, 1]$ the design with mass $1/3$ at each of $0, \pm 1$ is D- and G-optimal.

6. Verify (24.2) and (24.3) in the notes.

7. Verify (24.5) in the notes. [*Hint*: You will repeatedly need the result, established for instance in STAT 312 or 512, that matrix products **AB** and **BA** have the same non-zero eigenvalues.]