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Sir Gilbert Walker and a Connection between El Niño and Statistics

Richard W. Katz

Abstract. The eponym "Walker Circulation" refers to a concept used by atmospheric scientists and oceanographers in providing a physical explanation for the El Niño–Southern Oscillation phenomenon, whereas the eponym "Yule–Walker equations" refers to properties satisfied by the autocorrelations of an autoregressive process. But how many statisticians (or, for that matter, atmospheric scientists) are aware that the "Walker" in both terms refers to the same individual, Sir Gilbert Thomas Walker, and that these two appellations arose in conjunction with the same research on the statistical prediction of climate? Like George Udny Yule (the "Yule" in Yule–Walker), Walker's motivation was to devise a statistical model that exhibited quasiperiodic behavior. The original assessments of Walker's work, both in the meteorology and in statistics, were somewhat negative. With hindsight, it is argued that his research should be viewed as quite successful.

Key words and phrases: Autoregressive process, quasiperiodic behavior, Southern Oscillation, teleconnections, Yule–Walker equations.

1. INTRODUCTION

> It is a natural supposition that there should be in weather free oscillations with fixed natural periods, and that these oscillations should persist except when some external disturbance produces discontinuous changes in phase or amplitude.—*Sir Gilbert T. Walker* (Walker, 1925, pages 340–341)

Much recent attention in the popular press and in the scientific literature has been devoted to the 1997–1998 El Niño event, with all sorts of anomalous weather conditions and consequent societal impacts having been blamed on it (e.g., Changnon, 2000). Figure 1 shows the field of anomalies of sea surface temperature (i.e., deviations from the long-term sample mean) at the peak in intensity of this event, with the magnitude of the anomalies in the equatorial Pacific (shaded in dark red) being among the largest observed during the entire twentieth century.

Richard W. Katz is Senior Scientist, Environmental and Societal Impacts Group, National Center for Atmospheric Research, Box 3000, Boulder, Colorado 80307 (e-mail: rwk@ucar.edu).

At first only recognized as a local phenomenon, the term "El Niño" (or "Christ Child" in Spanish) apparently originated in the nineteenth century as a name fishermen applied to an anomalously warm current that appears off the Peruvian coast around Christmas (Glantz, 2001, page 15). Such El Niño events, or anomalously warm sea surface temperatures in the equatorial Pacific, now are understood as part of a more general atmosphere–ocean phenomenon known as the El Niño–Southern Oscillation (ENSO) (e.g., Glantz, 2001; Allan, Lindesay and Parker, 1996). This phenomenon is the largest single source of climate variations globally on an annual time scale, with its links to distant anomalous weather and climate events (such as droughts or heavy rains) being termed "teleconnections" (e.g., Glantz, Katz and Nicholls, 1991).

The Southern Oscillation (SO) is the atmospheric component of ENSO, loosely speaking a tendency of the atmospheric pressure to "seesaw" between two "centers of action," one in the general vicinity of Indonesia, the other in the tropical–subtropical southeastern Pacific Ocean. A physical explanation for the existence of the SO is provided at least in part by the "Walker Circulation," a large-scale atmospheric circulation consisting of sinking air in the eastern Pacific

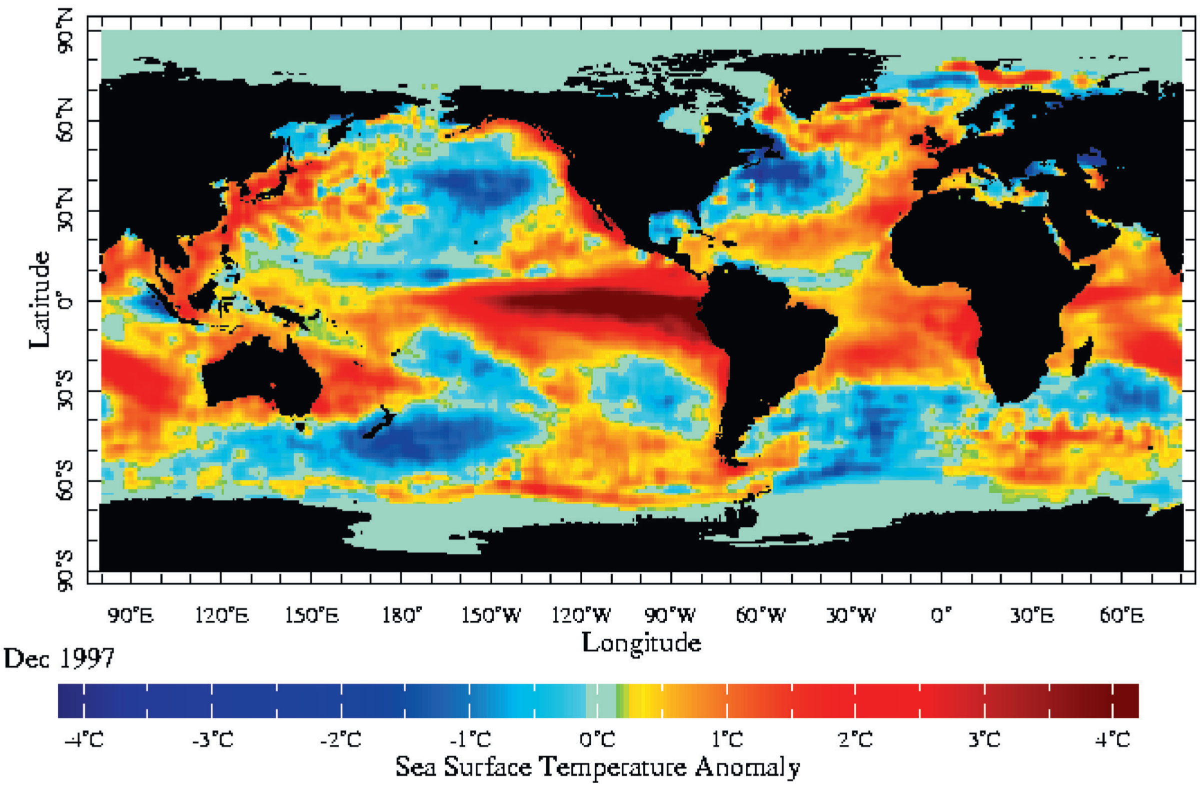

FIG. 1. *Field of sea surface temperature anomalies* (°*C*) (*i.e., deviations from sample mean over time period* 1950–1979) *for December* 1997 [*source*: *Columbia University International Research Institute, NOAA National Centers for Environmental Prediction* (*NCEP*)].

and rising air in the western Pacific and caused by feedback between trade winds and ocean temperatures. It is an eponym familiar to any present-day atmospheric scientist or physical oceanographer and was coined by Bjerknes (1969), who was the first to recognize the physical mechanism by which the SO, the Walker Circulation, and the El Niño phenomenon are linked. In its normal mode (Figure 2), trade winds along the surface flow toward the west, creating a pool of warm water near Indonesia and Australia. This warm water heats the atmosphere, resulting in conditions favorable for convection and precipitation to occur. Higher up in the atmosphere, the winds blow toward the east completing the circulation loop. During an ENSO event, an anomalous Walker Circulation occurs. Weakened trade winds, in conjunction with weakened upwelling of cold water along the equatorial coast of South America, shift the warm pool farther east along with the convection and precipitation (for further details about the Walker circulation, see Trenberth, 1991).

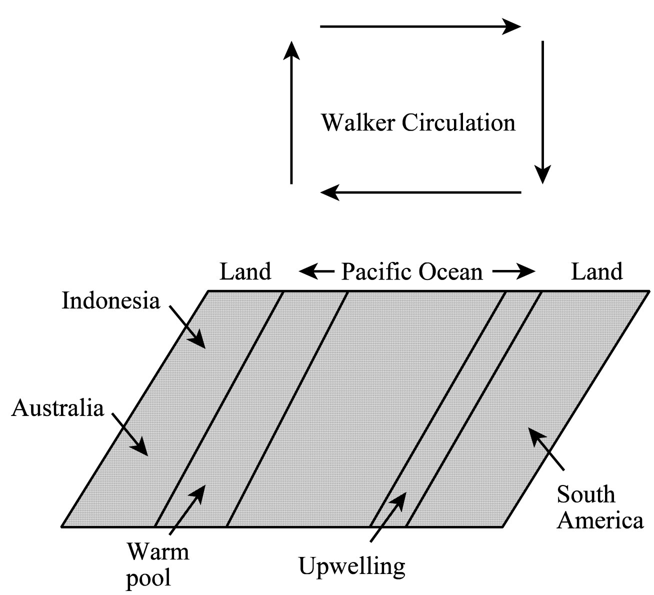

FIG. 2. *Schematic diagram of Walker Circulation.*

Most present-day statisticians would be familiar with the eponym "Yule–Walker equations," relating the parameters of an autoregressive (AR) process to its autocorrelations. For many years, the Yule–Walker equations (or the closely related normal equations for least squares) were the basis of a common method for fitting AR processes to time series, until computational advances made the method of maximum likelihood readily available. These equations still are popular (e.g., used in S-PLUS) for estimating partial autocorrelations and, through a generalization (Whittle, 1963, page 101), for fitting multiple AR processes.

But how many statisticians (or, for that matter, atmospheric scientists) are aware that the "Walker" in both terms refers to the same individual and, moreover, that these two appellations arose in conjunction with the same research? The "Walker" in question is none other than Sir Gilbert Thomas Walker (Figure 3). While stationed in India as Director General of Observatories of that country's meteorological department, Walker became preoccupied with attempts to forecast the monsoon rains, whose failure could result in widespread famine (Davis, 2001). It was in the course of this search for monsoon precursors that he identified and named the "Southern Oscillation" (Walker, 1924).

At that time, the approach most prevalent in the statistical analysis of weather variables was to search for deterministic cycles through reliance on harmonic analysis. Such cycles included those putatively associated with sunspots, the hope being to provide a method for long-range weather or climate forecasting. Walker was quite skeptical of these attempts, especially given the lack of statistical rigor in identifying any such periodicities. Eventually, he suggested the alternative model of quasiperiodic behavior (Walker, 1925). Meanwhile, the prominent British statistician George Udny Yule devised a second-order autoregressive [AR(2)] process to demonstrate that the sunspot time series was better modeled as a quasiperiodic phenomenon than by deterministic cycles (Yule, 1927). To determine whether the SO exhibits quasiperiodic behavior, Walker was compelled to extend Yule's work to a general pth-order autoregressive [AR(p)] process (Walker, 1931).

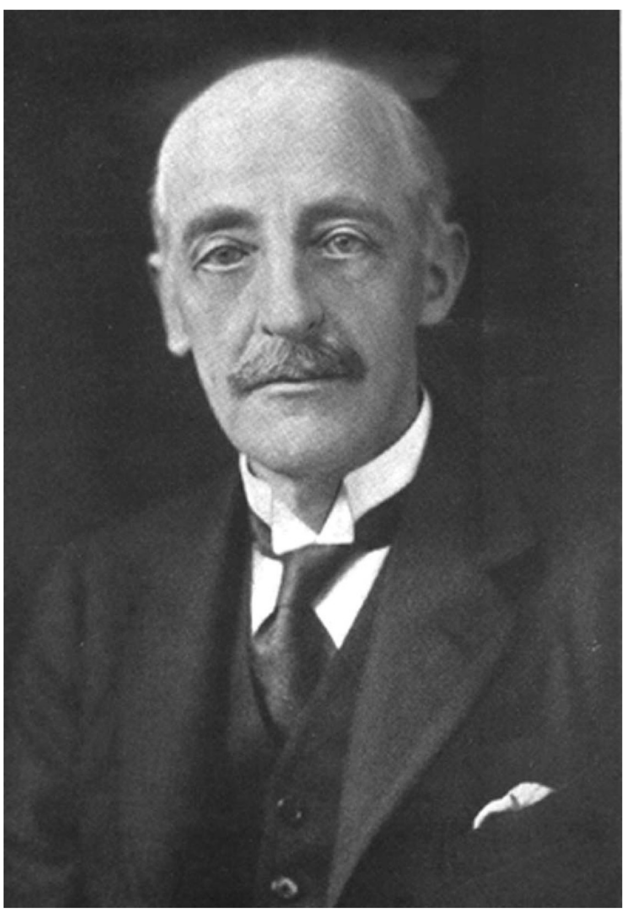

FIG. 3. *Photograph of Sir Gilbert T. Walker* (*source*: *Royal Society*; *Taylor*, 1962).

The focus of the present paper is on the connection between the meteorological and statistical aspects of Walker's research. First some background about Walker's research on what he called "world weather" is provided. Then the development of the Yule–Walker equations is treated, including a reanalysis of the index of the SO originally modeled by Walker. Reaction to his research, contemporaneously and in subsequent years and both in meteorology and in statistics, is characterized. For historical perspective, the present state of stochastic and dynamic modeling of the SO is briefly reviewed, examining the extent to which his work has stood the test of time. Finally, the question of why his work was so successful is considered in the discussion section. For a more formal, scholarly treatment of Walker's work, in particular, or of the ENSO phenomenon, in general, see Diaz and Markgraf (1992, 2000) and Philander (1990) (in addition to the references on ENSO already cited in this section).

2. WALKER'S RESEARCH ON WORLD WEATHER

2.1 Training and Career

In grammar school, Sir Gilbert Thomas Walker, who lived from 1868 to 1958, "showed an early interest in arithmetic and mechanics" (Taylor, 1962, page 167). After being educated under a mathematical scholarship at Trinity College, University of Cambridge, he remained there, assuming an academic career as Fellow of Trinity and Lecturer. Walker was a "mathematician to his finger-tips" (Simpson, 1959, page 67) and was elected Fellow of the Royal Society in 1904 on the strength of his research in pure and applied mathematics, including "original work in dynamics and electromagnetism before ever he turned his thoughts to meteorology" (Normand, 1958). Among his first papers (published in 1895) was one that dealt with the purely mathematical subject of the properties of Bessel functions.

In 1903 Walker left academia, taking charge of the Indian Meteorological Department the next year. This career change seems quite surprising given the fact that he was not a meteorologist, but a "typical Cambridge don and had never read a word of meteorology" (Simpson, 1959, page 67). In fact, it came about through the actions of the previous Director of the Indian Meteorological Department, John Eliot. His rationale for choosing Walker was that he saw the need for his successor to be someone with strong mathematical abilities (Normand, 1953). Walker soon became "engrossed in the problem of monsoon forecasting" (Sheppard, 1959) and spent the next 21 years in India working on what evolved into the broader topic of world weather.

Upon his return to England in 1924, the King of England conferred knighthood upon Walker, primarily for his accomplishments in directing the Indian Meteorological Department. He then became Professor of Meteorology at Imperial College of Science and Technology, University of London, continuing to devote much of his time to the topic of world weather, including forecasting the Indian monsoon. In making the presentation to him in 1934 of the Symons Memorial Medal for "distinguished work in connection with meteorological science," the famous British astrophysicist and geophysicist Sydney Chapman remarked that: "Sir Gilbert has had a long and distinguished career, first as mathematician and then as meteorologist" (Chapman, 1934, pages 184–185). Despite formal retirement at about this time, he remained an active researcher for many years, publishing his last research paper in 1950.

Walker was something of a "Renaissance man," working on diverse topics seemingly unrelated to his primary research focus. For example, he published several papers on the flight of birds, having made observations with a telescope in India (Taylor, 1962). He also worked on the mathematics of the flight of the boomerang (publishing a paper in 1897 that became well known), in the course of which he acquired such expertise in throwing them that he earned the sobriquet "Boomerang Walker" at Cambridge (Chapman, 1934). In India, he retained his interest in the boomerang, as even the Viceroy noticed his throwing (Simpson, 1959). Upon retirement, he wanted to become a glider pilot, but "found that at 65 his reactions were too slow" (Taylor, 1962, page 171). A recent description of Walker's life appeared in Walker (1997), and a publication list in Taylor (1962).

2.2 Statistical Methods

Scientific attempts to forecast the monsoon rains had started at least 25 years before Walker's arrival in India, with official forecasts being issued beginning in 1886 (Normand, 1953, page 463). Lacking any rigorous meteorological or statistical basis (i.e., being derived from a combination of apparent connections and unverified theories), these forecasts were of limited, if any, success. Still these efforts led to the tentative identification of predictor variables for the monsoon, including Himalayan snow cover and atmospheric pressure at distant locations (such as Australia and southern Africa). Other prior meteorological research, not focused on the Indian monsoon per se, indicated that connections existed between the variations in atmospheric pressure at distant locations, including some apparent cycles with estimated periods of about $3\frac{1}{2}$ years (e.g., Lockyer and Lockyer, 1904). Making use of these clues, Walker's "investigation begun with the narrow object of improving the Indian monsoon forecasts developed into an examination of worldwide variations of weather" (Normand, 1953, page 468).

Walker faced a situation in which no quantitative theory for forecasting the Indian monsoon was available, with "no agreed explanation of the general circulation, and certainly no quantitative theory whatever about deviations from the normal" (Normand, 1953, page 468). Even a rudimentary understanding of the general circulation of the atmosphere was not developed until the 1920s and 1930s (Crutzen and Ramanathan, 2000). A satisfactory dynamical explanation for the existence of the SO only was postulated well after his death (Bjerknes, 1969).

"It is a sign of the high quality and flexibility of his mind that he could realize that all the skill he had acquired in the past would be of little use to him in his new situation" (Taylor, 1962, page 170). Instead, Walker "at once saw the part the new branch of mathematics—statistics, very active then under Pearson and others—might play in scientific monsoon forecasting" (Simpson, 1959, page 67). The first to apply statistical methods to the problem, he became a "pioneer in the use of correlation in meteorology" (Normand, 1953, page 464). Despite working in the relative isolation of India, Walker was able to apply and extend recent developments in statistics, published primarily by British researchers, in his study of world weather. He made use of the techniques of correlation, regression and harmonic analysis, a consistent theme being the need to impose "criteria for reality" (Walker, 1914). Faced with the daunting task of sorting through a myriad of possible relationships, he was one of the first to develop a formal treatment of the problem of multiple comparisons.

2.2.1 *Correlation and regression.*

Monsoon prediction. Walker set about using the techniques of correlation and regression in an encyclopedic attempt to describe the relationships among climate variables (especially precipitation, temperature and pressure) around the world. As early as 1908, he was performing correlation analysis and developing multiple regression equations to predict Indian monsoon rainfall, as well as precipitation and related variables such as stream flow (e.g., Nile flood) for other regions. These regression equations involved as many as four to five predictor variables, and he used determinants to solve the system of normal equations. For example, Indian monsoon rainfall (totaled over a number of months and averaged over a number of stations) was related to four predictor variables, including pressure at the quite distant locations of Mauritius and Argentina–Chile (Walker, 1910).

Probable errors. Walker employed Yule's modern notation for correlation just introduced in 1907, and routinely attached "probable errors" (roughly two-thirds standard error in the case of statistics whose large-sample distribution is normal) to statistics such as the correlation coefficient (at that time, it was conventional to quote probable, instead of standard, errors). When first starting to perform regression analyses, Walker had available an expression for the approximate standard error of an ordinary correlation coefficient [i.e., of the form $(1-\rho^2)/n^{1/2}$, for correlation coefficient ρ and sample size n, a result derived by Karl Pearson], but lacked a corresponding one for the multiple correlation coefficient (called "joint" correlation by Walker). In one of a series of somewhat obscure monographs published by the Indian Meteorological Department, Walker set out a long, tedious algebraic argument, in an attempt to justify the use of the same approximate expression in this case as well (Walker, 1910). In this way, he could attach at least a crude measure of uncertainty to the strength of any fitted regression relationship.

Centers of action. Despite a network of meteorological observations that was quite sparse, especially over the Pacific Ocean, Walker tried to decompose the variations in large-scale weather into a few dominant centers of action. These centers were primarily defined in terms of atmospheric pressure averaged over a season. Relying on the correlation coefficient because "the relations between weather over the earth are so complex that it seems useless to try to derive them from theoretical considerations" (Walker, 1923, page 75), he compiled extensive tables of sample correlations between the pressure at different locations. These statistics included what we would today term autocorrelation and cross correlation coefficients, with leading or lagging relationships of up to two seasons being considered.

On purely statistical grounds through careful interpretation of these correlation tables, Walker was able to identify three pressure oscillations central to world weather:

> there is a swaying of pressure on a big scale backwards and forwards between the Pacific Ocean and the Indian Ocean, there are swayings, on a much smaller scale, between the Azores and Iceland, and between the areas of high and low pressure in the N. Pacific (Walker, 1923, page 109).

Besides the aforementioned SO (i.e., the swaying between the Pacific and Indian Oceans), he named the seesaw in pressure between the Icelandic Low and Azores High the "North Atlantic Oscillation" (NAO), and the seesaw in the Pacific, the "North Pacific Oscillation" (NPO) (Walker, 1924). The NAO (e.g., Lamb and Peppler, 1987) now is the focal point for much research on variations in climate on annual to decadal time scales in higher latitudes of the Northern Hemisphere, especially in western Europe (Hurrell, 1995). The NPO (e.g., Wallace and Gutzler, 1981) has received some recent attention as well (Hurrell, 1996). Despite his labeling of these phenomena as "oscillations," he did not necessarily view them as being strictly periodic (recall quote at beginning of Section 1).

Walker further asserted that the SO is the predominant oscillation: "the influence of the Pacific Ocean–Indian Ocean swayings upon world weather seems to be much greater than that of either of the other two" (Walker, 1923, page 110). He also noted a tendency of the SO to persist for at least one to two seasons (his correlation tables included the first- and second-order sample autocorrelation coefficients), suggesting the potential for using the SO in forecasting world weather (Walker, 1924). So when Bjerknes later identified the atmospheric circulation tied to the SO (see Section 1 and Figure 2), he named it after Walker, stating that it "must be part of the mechanism of the still

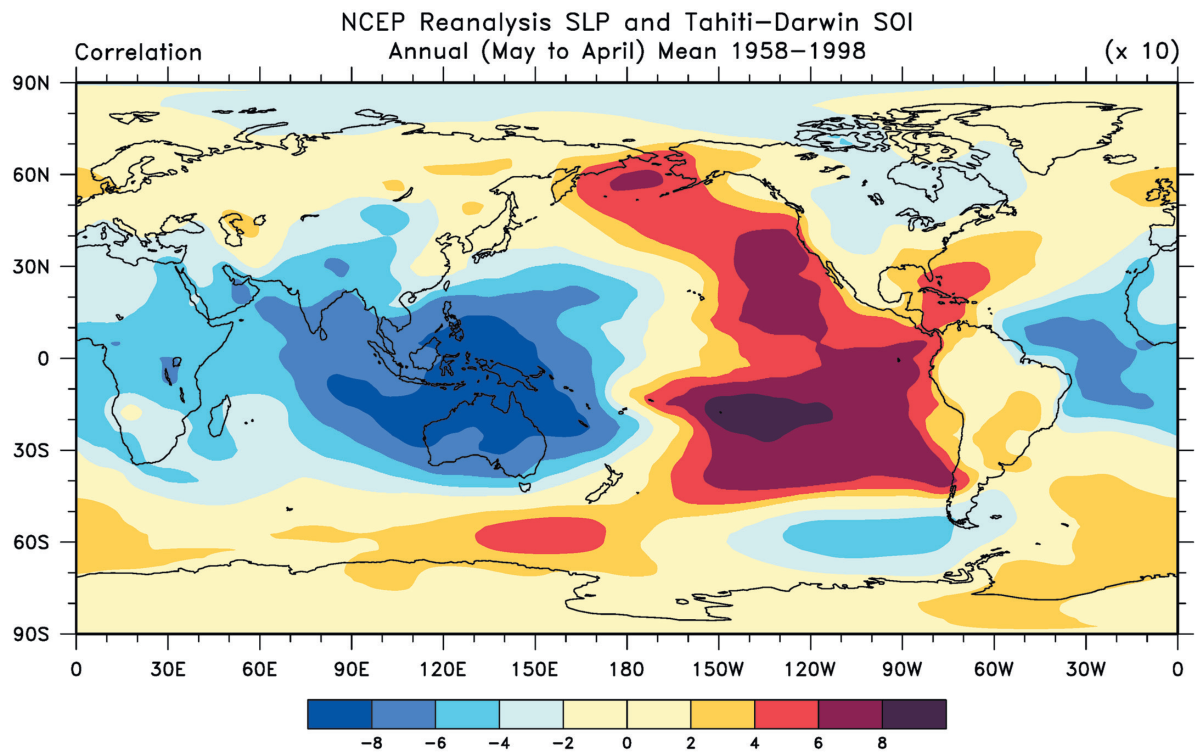

FIG. 4. *Contemporaneous cross correlation (in tenths) between annual (May–April) Tahiti–Darwin SO index (SOI) and sea-level pressure (SLP) at individual grid points (source: NOAA/NCEP; Trenberth and Caron,* 2000).

larger 'Southern Oscillation' statistically defined by Sir Gilbert Walker" (Bjerknes, 1969, page 169).

Taking advantage of more recent meteorological pressure measurements that have both higher spatial density and higher quality than those available to Walker, Figure 4 shows the contemporaneous cross correlation between an annual index of the SO (i.e., difference in pressure between Tahiti and Darwin averaged over 12 months, May–April) and the pressure at individual grid points across the world [i.e., measurements that have been spatially interpolated to a grid (termed an "analysis")]. A large area of positive correlation (shaded in dark red) is evident over the eastern Pacific, from South America to Alaska, with a large area of negative correlation (shaded in dark blue) from India to Australia.

2.2.2 *Multiple comparisons.* The question remains of how Walker was able to disentangle these pressure oscillations from a plethora of apparent relationships, a task that had foiled earlier researchers at least in part because of the difficulties that arise in such "data snooping." As has already been noted, he believed in attaching probable errors to any estimates. However, in practice, he actually adopted a much more stringent procedure to combat the problem of multiplicity. He recognized, in particular, that "it frequently happens that a large number of possible relationships or of periodicities are investigated, and of these the case of the largest 'measure' is examined for reality by the same criterion as that applied to a case that has not been selected for its magnitude" (Walker, 1914, page 13). In fact, he even issued an admonition about "the desirability of publishing the results of all examinations of relationships, not merely those which prove to be close: . . . unless we know that all results are published we do not know how great is the significance of the relationships found" (Walker, 1923, page 77).

Walker test. In another one of the series of monographs previously referred to, Walker (1914) sought to develop a general approach for dealing with the problem of multiple comparisons. Under the assumption that the statistics involved (e.g., correlation coefficients or periodogram ordinates) are independent, he determined the probable value for the largest of the set. In more modern terminology, suppose that m independent tests of significance are performed each at a nominal level α. Then the desired overall level α_0 would be obtained if the individual level is chosen as

$$\alpha = 1 - (1 - \alpha_0)^{1/m}. \quad (1)$$

In practice, Walker would set $\alpha_0 = 0.5$ in (1) to obtain the probable value to attach to the largest of the set of statistics. He made use of a rough large-sample normal approximation for the correlation coefficient and of Schuster's result that a periodogram amplitude is exponentially distributed. For the two cases of correlation and periodogram analysis, a table in Walker (1914) shows how the ratio of the probable value of the largest of the set of m statistics to the probable value of a single one increases as m is increased.

This procedure apparently is the first instance of an "error-rates batchwise" approach to multiple comparisons, using the terminology of John Tukey (Braun, 1994, page 106). In the case of searching for hidden periodicities, Walker's technique constitutes an improvement over the original method of Schuster (1898), which is only appropriate in the case of testing for a periodicity whose frequency is specified a priori. Because it ignores the effect of estimating the variance, Walker's technique still is approximate.

In an early book on economic time series analysis, Davis (1941, pages 188–189) published a table of what are called "Walker probabilities" for the "Walker test" for hidden periodicities (also see Anderson, 1971, page 120). In the case of correlations, Walker routinely relied on his multiple comparisons procedure as a criterion for reality, with typical batch sizes ranging from $m = 15$ to $m = 35$ (e.g., Walker, 1924). A more detailed treatment of his approach to dealing with multiplicity in research on teleconnections is given in Katz and Brown (1991), who showed that in realistic climate applications the Walker test does not differ much in performance from the more modern Bonferroni technique (which does not require independence).

2.2.3 *Harmonic analysis.* Walker's interest in periodogram analysis already has been alluded to in conjunction with the issue of multiple comparisons. In the study of world weather, he actually did not devote much effort to searching for deterministic cycles and, consequently, did not rely much on such techniques. Rather, he felt compelled to comment on harmonic analysis given its popularity within meteorology at the time (e.g., Walker, 1925, 1930). For example, he rederived a proof of Schuster's result on the distribution of a periodogram amplitude, because: "There still are meteorologists and seismologists whose lack of familiarity with mathematical ideas enables them to ignore both Schuster's criterion and that which I have communicated" (Walker, 1930, page 97).

As Editor of the *Quarterly Journal of the Royal Meteorological Society,* Walker advocated the application of criteria for reality (i.e., tests of significance) for periods detected by harmonic analysis as a requirement for publication. He observed that "as is probably true, ninety-five per cent of the periods announced are nonexistent" (Walker, 1936a, page 2). His general attitude is summed up in a statement he made near the end of his research career, commenting on one of the early papers on modern time series analysis by the British statistician Maurice Kendall: "I have spent much time and energy in attempting to diminish faith in very doubtful periodicities" (discussion in Kendall, 1945, page 137).

3. YULE–WALKER EQUATIONS

3.1 AR Process

Much of the early use of autocorrelation was motived by meteorological applications. According to Klein (1997, page 67), the "first published example of serial or autocorrelation on time series data" was a meteorological one, involving the relationship between daily pressure readings at distant locations (Cave-Browne-Cave, 1904). Clayton (1917) also made use of the autocorrelation function for time series of solar radiation and temperature. The notion of an AR(1) process was implicit in even earlier work of Francis Galton and Karl Pearson, involving correlation and regression applied to heredity (Klein, 1997, pages 261–262).

3.1.1 AR(p) *process.* The original impetus for the development of an AR(p) process, with order $p \geq 2$, was to provide a statistical model for quasiperiodic behavior. From a physical perspective, the basic issue is that if cyclic behavior were of deterministic origin, then it could be plausibly attributable to some cause (especially one with a similar deterministic period). The alternative of quasiperiodic behavior arises intrinsically, so no cause need be invoked to explain its origin.

A zero-mean AR(p) process $\{X_t\}$ satisfies the difference equation

$$(2)\quad X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + a_t,$$

where ϕ_k denotes the kth-order autoregression parameter (the ϕ_k must satisfy certain constraints for the process to be stationary and causal) and a_t denotes the innovation (or error) term (zero mean, uncorrelated). Using (2) and taking expectations, the Yule–Walker equations for an AR(p) process are of the form

$$(3)\quad \rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p}, \quad k \neq 0,$$

$$(4)\quad \sigma_a^2 = (1 - \phi_1 \rho_1 - \phi_2 \rho_2 - \cdots - \phi_p \rho_p)\sigma^2,$$

where $\rho_k = \text{corr}(X_t, X_{t-k})$, $\sigma^2 = \text{var}(X_t)$ and $\sigma_a^2 = \text{var}(a_t)$. Equation (3) reflects the fact that the autocorrelation coefficients satisfy the same form of difference equation as does the AR process itself [i.e., (2) neglecting the error term], whereas (4) shows how the innovation variance is reduced relative to the process variance.

3.1.2 *Quasiperiodic behavior.* For simplicity, the case of an AR(2) process is considered. Properties of its autocorrelation function and spectrum can be derived directly from the Yule–Walker equations (3)–(4) with $p = 2$. In particular, its autocorrelation function is a damped sine wave with frequency f,

$$\cos(2\pi f) = |\phi_1|/[2(-\phi_2)^{1/2}] \quad \text{if } \phi_1^2 + 4\phi_2 < 0 \tag{5}$$

(Box and Jenkins, 1976, page 59).

For the spectrum of an AR(2) process to be peaked, the amplitude of the damped sine wave for its autocorrelation function must be large enough. In particular, its spectrum has a peak at frequency f,

$$\begin{aligned} &\cos(2\pi f) = -[\phi_1(1 - \phi_2)]/(4\phi_2) \\ &\qquad \text{if } \phi_2 < 0 \text{ and } |\phi_1|(1 - \phi_2) + 4\phi_2 < 0 \end{aligned} \tag{6}$$

(Jenkins and Watts, 1968, pages 229–230). More generally, the autocorrelation function of an AR(p) process, $p > 2$, can include a mixture of one or more damped sine waves and a spectrum with one or more peaks, depending on the values of the autoregression parameters.

In early practice (e.g., as by Yule and Walker), any "quasiperiod" was identified as having the frequency of the autocorrelation function (5), as opposed to that corresponding to the actual peak in the spectrum (6). Still the frequency values obtained from (5) and (6) do not necessarily differ very much, plus the peak in the spectrum associated with quasiperiodic behavior will be a broad band, as opposed to a thin line. To avoid confusion, a period based on (5) is referred to as a *correlation quasiperiod*, based on (6) as a *spectral quasiperiod*.

3.2 Yule's Model for Sunspot Numbers

Yule's pioneering 1927 paper on a time series model for sunspots has been cited many times, so only a brief treatment is given here. Before this work, the sunspot time series had been subjected to numerous searches for hidden periodicities via harmonic analysis, with an approximately 11-year period being claimed to exist. Yule's genius was to suggest an alternative model in which a deterministic cycle (specifically, a sine wave) is randomly shifted in phase and amplitude. This chance mechanism led directly to the formulation of an AR(2) process.

Given Yule's earlier innovative work in correlation and regression, especially on the concepts of partial correlation and regression, it should not be surprising that he was the first to formulate such a model. Using the method of least squares, he fit this model to the annual time series of sunspot numbers, 1749–1924, obtaining a correlation quasiperiod of about 11 years, or roughly consistent with the results of previous harmonic analyses. In effect, he used the solution to the Yule–Walker equations for the autoregression parameters (3) (with $p = 2$) in interpreting the properties of the fitted model. Making use of the regression analogue to the expression for the innovation variance (4), he concluded that an AR(1) model was unsatisfactory, but that higher than second-order terms need not be added. Yule's AR(2) model explains much more of the variance of the sunspot time series than does a strictly periodic model with one harmonic component.

3.3 Walker's Model for Darwin Pressure

Before the appearance of Yule's work on sunspots, Walker was already thinking about the issue of quasiperiodic behavior as an alternative to deterministic cycles. The context still was world weather and the prediction of the Indian monsoon. For an index based on one of the centers of action involved in the SO, the seasonal mean pressure at Darwin, Australia, 1882–1923, he observed that "though there is no definite 'line' in the periodogram, there appears to be a 'band' with its centre between 3 and $3\frac{1}{4}$ years" (Walker, 1925, page 342). Figure 5 shows the same time series of seasonal pressure at Darwin that Walker analyzed, except that it has been standardized somewhat differently and extended to the present (see the Appendix).

Walker also urged that a distinction be drawn between periods and quasiperiods: "The word 'period' has hitherto had a definite meaning in physical mathematics and it will tend to confusion if it has to bear also a second meaning" (Walker, 1925, page 343). This paper was read before the Royal Meteorological Society on 20 May 1925, with one of the discussants being Harold Jeffreys. Besides being well known for his work in Bayesian statistics, Jeffreys made major contributions to many areas of earth science, including several papers during 1915–1925 on theories of atmospheric circulation. He commented:

> with reference to the possibility of a real periodicity of variable period, that such a phenomenon could be discovered, if it were present, by Clayton's method in which correlation coefficients were worked out between the value of the data and its values one, two, three, etc., days afterwards (Walker, 1925, page 346).

Thus as early as 1925, Walker was given advice on how he might formally model quasiperiodic behavior.

A subsequent paper, read before the Royal Meteorological Society by Walker on 18 November 1925, focuses on uses and abuses of correlation coefficients (Walker and Bliss, 1926). The British statistician Reginald Hooker, known for his ideas about detrending time series (Klein, 1997, Chapter 3), was one of the discussants and commented:

> He had listened on the previous day to an extraordinarily interesting address at the Statistical Society by Mr. Udny Yule, who took as his subject much the same problem as Sir Gilbert Walker had taken, and he hoped that he and Sir Gilbert Walker might collaborate in the investigation of this subject (Walker and Bliss, 1926, page 81).

The paper to which Hooker was referring had the provocative title of "Why do we sometimes get nonsense-correlations between time-series?" (Yule, 1926), in some respects a precursor to Yule (1927). Evidently, no such collaboration between Yule and Walker ever took place. Yule's 1927 paper turned out to be his last significant research, because he retired in 1930 and was a semiinvalid from 1931 onward (Kendall, 1952). Nevertheless, Hooker was certainly prescient with respect to the appearance of the term Yule–Walker equations.

3.3.1 *Derivation of Yule–Walker equations.* As soon will be explained, Walker's application to Darwin pressure required a more complex model than an AR(2) process. So he was compelled to extend Yule's approach, considering an AR process of arbitrary order p and deriving the general form of Yule–Walker equations, (3)–(4) (Walker, 1931). Because the autocorrelation coefficients satisfy a relationship (3) identical to that for the original process (2), Walker argued that the autocorrelation function "may be used to read off the character of the natural periods" of the process (Walker, 1931, page 532). Wold (1938, pages 143–144) claims that Walker derived (3) only for lags $k > p$, but Walker actually stated that (3) holds "in general" (Walker, 1931, page 519) without specifying any conditions on k, so this issue is not completely clear.

3.3.2 *Analysis of Darwin pressure.* In the same paper, Walker applied the general autoregressive model to the seasonal time series of Darwin pressure, now extended through winter 1926 (see Figure 5). His motivation was that Darwin is

> one of the most important centres of action of "world weather," which ... displays surges of varying amplitude and period with irregularities superposed, suggesting that pressure in this region has a natural period

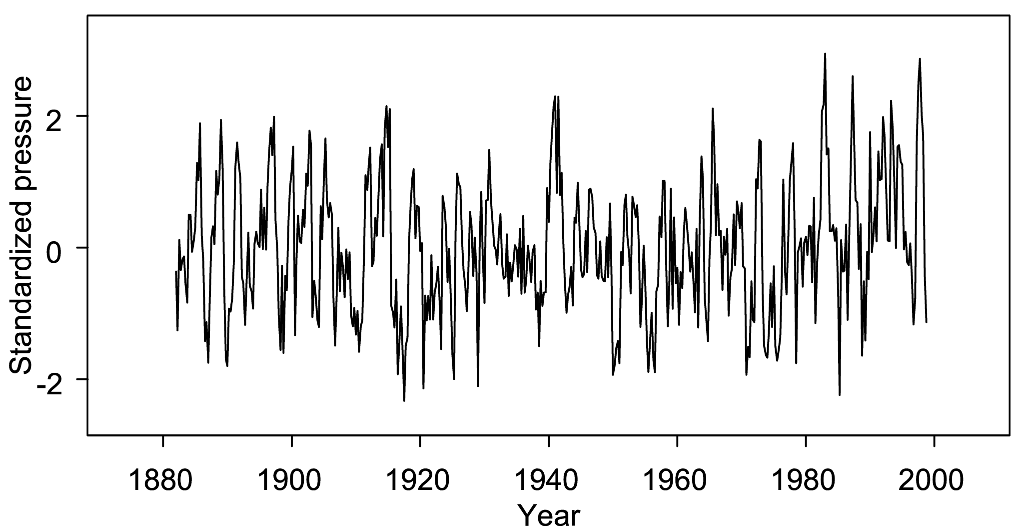

FIG. 5. *Standardized time series of seasonal mean sea level pressure at Darwin, Australia, for winter* 1882–*fall* 1998.

> of its own, based presumably on the physical relationships of world-weather, but that the oscillations are modified by external disturbances (Walker, 1931, page 525).

Unlike the sunspot time series, the sample autocorrelation function for the Darwin time series appeared to him to have a more complex form than that of a damped sine wave. For this reason, he considered a higher-order AR(p) process as a candidate model, observing that the Yule–Walker equations (3) imply that the autocorrelation function is, in general, a sum of damped exponential and damped sine waves (see Section 3.1).

However, Walker did not adopt Yule's estimation method of fitting an AR process directly to the data by regression. Being preoccupied with the issue of quasiperiodicity, he fit a mixture of one damped sine wave and two damped exponentials to the sample autocorrelation function by trial and error. Next this fitted autocorrelation function was converted into the corresponding difference equation, which assumes the form of an AR(4) model. Still, he was cautious about drawing any firm conclusions on the basis of the fitted AR(4) model alone: "There appears to be a periodicity of about $11\frac{1}{2}$ quarters ... but the evidence that it is damped is not conclusive" (Walker, 1931, page 531).

It turns out that this rather indirect approach to model fitting does not necessarily result in sensible parameter estimates. Although Walker's fitted model for the autocorrelation function matches reasonably well the first three or four sample autocorrelation coefficients, his indirectly derived AR(4) model does not. As later pointed out by Wold (1938, pages 145–146), the difficulty is that Walker's fitted autocorrelation function does not actually correspond to an AR process. Before returning to Wold's assessment of Walker's application in Section 4.1, it will be useful to reanalyze the Darwin pressure time series.

3.3.3 *Reanalysis of Darwin pressure.* The time series of mean seasonal pressure at Darwin analyzed by Walker ranges from winter 1882 to winter 1926 (i.e., a length of 177 seasons), or nearly the same number of observations as the sunspot data as analyzed by Yule (Walker, 1931). To eliminate the annual cycle in mean pressure, Walker converted the data into deviations from the corresponding seasonal mean. Here Walker's analysis is repeated, with a few minor modifications that include making use of data corrected for instrumental bias and removing the annual cycle in the pressure standard deviation. The Appendix gives more details on these changes, the rationale being that their nature is such that Walker would have been readily able to implement them.

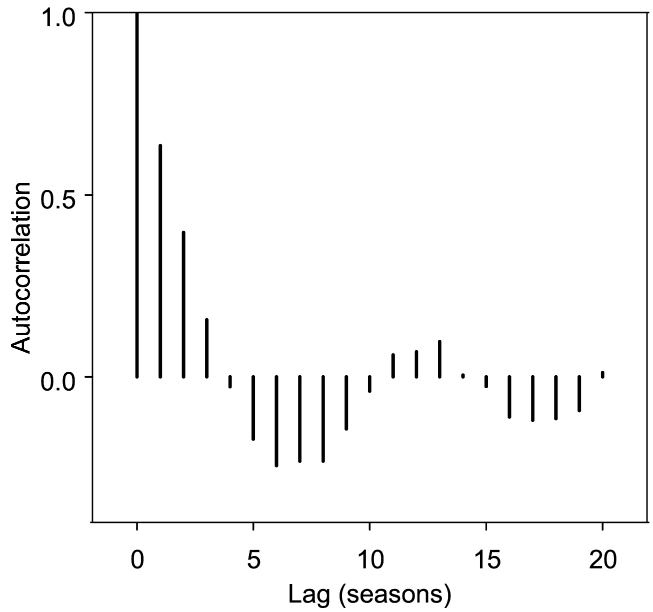

FIG. 6. *Sample autocorrelation function of Darwin pressure, winter* 1882–*winter* 1926.

The reanalyzed data appear as the first portion of the time series in Figure 5, except for slight differences introduced by the standardization being based on a shorter time period. The sample autocorrelation function (Figure 6) indicates at least weak evidence of a damped oscillation. Walker obtained a similar pattern except for a tendency for the autocorrelation to remain positive at higher lags, apparently due to an artificial trend in the data introduced by instrumental bias (see the Appendix). Figure 7 gives the raw periodogram, a nonparametric smoothed spectrum and the spectrum for an AR(4) model fitted by the Yule–Walker equations (3)–(4). A broad band is evident at about 3 to $3\frac{1}{2}$ yr (i.e., at about 0.07 to 0.08 cycles per season in Figure 7), but it is rather weak. Rather than determine the statistical significance of these estimates directly, their reality will be examined in Section 5 in the light of more recent observational evidence and theoretical developments.

3.4 Etymology

The eponym Yule–Walker equations (or "relations") did not originate until long after the work of Yule and Walker was published, with the earliest appearance I have found being in Kendall (1949). Wold (1938, page 178) recognized the equivalence of the Yule–Walker equations to the normal equations that arise in least squares. He solved (3), for $1 \le k \le p$, to

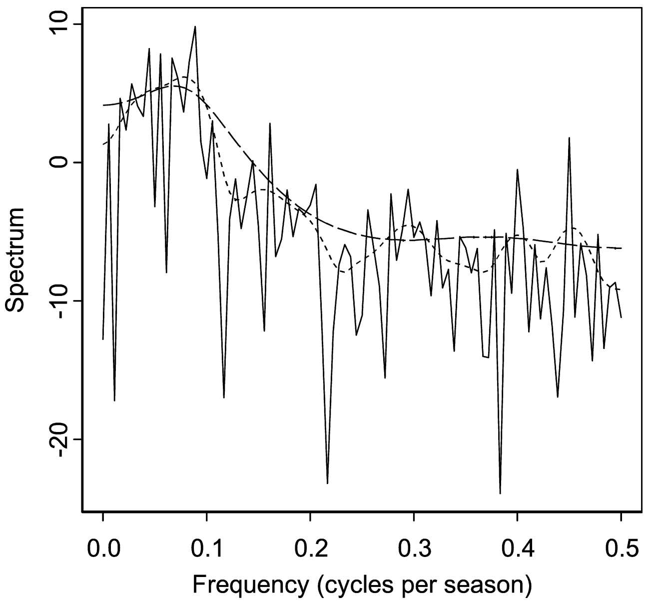

FIG. 7. *Log spectrum of Darwin pressure, winter* 1882–*winter* 1926: *periodogram* (*solid line*), *smoothed using Daniell lag windows* (*short-dashed line*), *and for fitted AR model* (*long-dashed line*).

obtain estimates of the autoregression parameters, observing that this approach reproduces the first p sample autocorrelation coefficients.

At first, the solutions to (3) for the autoregression parameters were referred to as least squares estimates, no matter how the autocorrelation coefficients were estimated (e.g., Whittle, 1954a, b). With the development of the Levinson–Durbin recursion (Durbin, 1960; see Morettin, 1984, for a review), (3) could be solved for the autoregression coefficients in a rapid iterative fashion, convenient when fitting many trial orders of an AR model to the same data. This advance made parameter estimation via the Yule–Walker equations an attractive alternative to regression. Some time afterward, the distinction between least squares and Yule–Walker estimators became prevalent; with the two estimators not being identical because of differences in how the autocorrelation coefficients are estimated (the nonnegative definite version being preferred in the case of Yule–Walker estimators; e.g., Box and Jenkins, 1976, pages 277–279).

4. REACTION TO WALKER'S RESEARCH

4.1 Reaction in Statistics

4.1.1 *Correlation and regression.* It is evident that some of Walker's research on correlation and regression stimulated interest within statistics. For example, a monograph by the British statistician John Wishart, well known for his work in multivariate statistics, begins with: "The use of the multiple correlation coefficient in weather forecasting was suggested by Sir Gilbert Walker in 1910" (Wishart, 1928, page 29). Wishart went on to show that the expression for the standard error of the multiple correlation coefficient used by Walker is overly conservative if no real correlation is present (see Section 2.2.1).

4.1.2 *Multiple comparisons.* Walker's work on multiple comparisons clearly contributed to subsequent developments in statistics on testing for hidden periodicities. In particular, R. A. Fisher extended the Walker test for hidden periodicities by developing a Studentized version (Fisher, 1929). This improvement effectively superceded the Walker test, with "Fisher's test" still being used today.

4.1.3 *Time series analysis.* The appearance of the book by Swedish statistician Herman Wold in 1938 marked the emergence of modern time series analysis (Klein, 1997, page 20). Exploiting recent advances in the theory of stationary stochastic processes, the topic began to be viewed as a bona fide branch of statistics. Yet in the section of Wold's book "On earlier applications of the scheme of linear autoregression" (Wold, 1938, pages 140–146), he was only able to cite the two applications described in the present paper, Yule's to sunspots and Walker's to atmospheric pressure. He critiqued both of these applications, with his assessment of Yule's work being quite laudatory.

On the other hand, Wold's assessment of Walker's application to Darwin pressure is somewhat negative (although to be fair, he qualified his remarks: "a more detailed analysis of the air pressure data is beyond the scope of the present survey"; Wold, 1938, page 146). He does give full credit to Walker for his derivation of the Yule–Walker equations (3)–(4). As mentioned in Section 3.3, Wold raised legitimate concerns about the indirect approach by which Walker fit an AR(4) process to the Darwin pressure, but Wold did not actually attempt to improve upon Walker's parameter estimates by fitting an AR(4) process directly to the data. Rather, Wold observed that a simpler AR(1) process provided "a fairly good fit to the few serial coefficients" (Wold, 1938, page 145) and would be an adequate model for Darwin pressure. He saw no need for a higher order AR model: "it is doubtful whether it would be possible to improve sensibly the approach ... by taking into account more distant elements" (Wold, 1938, page 145), not being convinced of any need for a model with quasiperiodic behavior: "this periodogram does not like that of the sunspots suggest a scheme of linear autoregression with a tendency to periodicity" (Wold, 1938, page 146).

4.2 Reaction in Meteorology

4.2.1 *General.* During the time span of Walker's research career, the general reaction within the meteorological community to statistical methods, especially correlation and regression, was extreme skepticism. For example, a rather technical paper, making extensive use of correlation in modeling rainfall, was read by Fisher before the Royal Meteorological Society on 19 April 1922 (Fisher and Mackenzie, 1922). In the discussion of this paper, it was mentioned that "no new meteorological fact had been discovered by means of correlation coefficients; certainly up to the present no practical forecasts had been obtained from correlation coefficients" (Fisher and Mackenzie, 1922, page 242).

So it should not be surprising that Walker's research discoveries were met with much resistance within meteorology. For example, concerning his pioneering use of correlation in meteorology, Normand (1953, page 464) noted that "meteorologists have not all accorded him whole-hearted thanks."

4.2.2 *Forecasting.* Because the potential value of long-range weather forecasts was perceived to be high, Walker's research did draw much attention, but doubts remained about reliance on statistical methods. For instance, in a review of the series of memoirs of the Indian Meteorological Department produced by Walker, the British meteorologist William Dines (1916, page 130) argued that "correlation is of very little use for the purpose of forecasting unless the coefficients concerned are very high." Dines did admit that "for the purpose of investigating relationships between various elements and their physical causes correlation coefficients, and more especially partial ones, are of very great importance." Walker's response to Dines was to observe that "the object of weather forecasting being practical, we cannot wait until we are absolutely certain of our results" (Walker, 1918, page 223).

Over the years, there were several attempts to "verify" Walker's correlations and multiple regression equations for seasonal forecasting, generally by recalculating the same statistics as the number of observations increased (e.g., Montgomery, 1937). Grant (1956, page 10) questioned Walker's multiple regression equations for predicting rainfall in India, concluding that: "the observed correlations do not prove or even render likely the existence of correlations of useful magnitude between past and future weather." In several instances, the correlations appear to weaken with more data, but not necessarily greater changes than could be reasonably attributable to sampling variations (e.g., Gershunov, Schneider and Barnett, 2001).

4.2.3 *Physical explanation.* Despite his reliance on statistics, Walker always sought physical explanations: "The number of satisfactorily established relationships between weather in different parts of the world is steadily growing ... and I cannot help believing that we shall gradually find out the physical mechanism by which these are maintained" (Walker, 1918, page 223). Despite his lack of formal training in meteorology, he tried to lay out possible research avenues. In an address to the Royal Meteorological Society, he suggested that "variations in activity of the general oceanic circulation will be much more far reaching and important" (Walker, 1927a, page 113) in explaining pressure oscillations such as the SO, and he later recommended searching "for an explanation in terms of slowly changing features, such as ocean temperatures" (Walker, 1936b, page 136).

For reasons that are not completely clear, Walker's appeals for physical explanations went largely unheeded. At the time of his death in 1958, his work was characterized in terms of unrealized expectations:

> Walker's hope was presumably not only to unearth relations useful for forecasting but to discover sufficient and sufficiently important relations to provide a productive starting point for a theory of world weather. It hardly appears to be working out like that (Sheppard, 1959, page 186).

With the recent popularity of the ENSO phenomenon, it is difficult to appreciate that: "As recently as the 1960s, the SO was still largely dismissed as a climate curiosity" (Rasmusson, 1991, page 310). The conventional explanation in meteorology is that only with the breakthrough by Bjerknes (1969) (see the Introduction and Section 2.2.1) was the physical theory available to take advantage of Walker's work. Surely, more rapid progress would have been achieved if researchers in meteorology had taken seriously Walker's discoveries, rather than tending to dismiss them (for additional discussion of the reaction to Walker's work within meteorology, see Brown and Katz, 1991).

5. PRESENT STATE OF MODELING THE SOUTHERN OSCILLATION

Not only has the sunspot time series that Yule modeled become one of the most analyzed data sets in statistics, but on occasion it still is used in meteorology as well to search for sun–weather connections. On the other hand, the SO time series, originally modeled by Walker, received very little attention within meteorology until recent decades. It is therefore somewhat ironic that, at least for long-range weather or climate forecasting, sunspots have proved of so little value (e.g., Pittock, 1978), whereas today the ENSO phenomenon is the primary focus of attention. At least in this respect, it appears that Walker's research discoveries have stood the test of time well.

Since Walker's era, the observational information about the SO has improved. Longer and more reliable records exist as well as refined indices, such as the difference between Tahiti and Darwin pressure (see Figure 4), designed to reflect explicitly the two centers of action involved in the SO pressure seesaw (Walker did not make use of Tahiti pressure measurements, and some doubts have been raised about their reliability before 1935; e.g., Trenberth and Hoar, 1996). Progress also has been made on the dynamical underpinnings of the more general ENSO phenomenon.

5.1 Quasiperiodic Behavior

Recalling the discussion in Section 3.3 about the SO being a quasiperiodic phenomenon (see Figure 7), little doubt remains about this issue within the meteorology community. For example, Chu and Katz (1989) used an AR(3) process for the seasonal Tahiti–Darwin pressure difference since 1935, obtaining an estimated spectral quasiperiod of about 3.6 years; and Trenberth and Hoar (1996) modeled the seasonal Darwin time series for 100 years from 1882 to 1981 (i.e., much of the same data shown in Figure 5) with an ARMA(3, 1) process yielding an estimated spectral quasiperiod of about 4.2 years. The SO time series even has appeared recently in the statistics literature, with Huerta and West (1999) performing a fully Bayesian analysis (involving the use of Markov chain Monte Carlo methods) of the Tahiti–Darwin pressure difference aggregated to a monthly time scale starting in 1950. Their posterior distribution on the order p of an AR process attaches nonnegligible probability between 8 and 17 [roughly consistent with the findings of Chu and Katz (1989), who selected an ARMA(7, 1) model for monthly data over a slightly different time period], and they obtained an estimated spectral quasiperiod of 4 to 5 years (Chu and Katz, 3.3 years). So, notwithstanding the limitations in Walkers's original analysis of the Darwin data, his insight about the SO phenomenon appears to be essentially correct.

5.2 Nonlinear Dynamics

The quasiperiodic feature of the ENSO phenomenon now is viewed as fundamental (Graham and White, 1988), but with its source remaining unclear (e.g., Wang, 2001). The prevailing dynamical explanation is the "delayed oscillator hypothesis" originally formulated by Suarez and Schopf (1988), with alternative versions subsequently being proposed (Wang, 2001). All of these conceptual models involve deterministic nonlinear equations, making use of both positive and negative feedbacks between the atmosphere and ocean in the equatorial Pacific to produce an oscillation on the interannual time scale of ENSO. To obtain the quasiperiodic behavior of the actual ENSO phenomenon, a stochastic forcing term sometimes has been added to the model (Graham and White, 1988). In a sense, this approach is reminiscent of Yule's original idea of introducing randomness into a strictly periodic oscillation.

Finally, it should be noted that general circulation models (GCM's), very complex deterministic numerical models of the atmosphere–ocean system, likewise are starting to be capable of generating "ENSO-like" behavior. For example, Meehl and Arblaster (1998) found that one particular GCM, the NCAR Climate System Model, produces a spectral quasiperiod of about 4 yr or quite close to that observed for ENSO. For reasons that are not yet understood, many GCM's exhibit a peak at shorter periods than that observed (AchutaRao et al., 2000).

6. DISCUSSION

Sir Gilbert Walker's contribution to the Yule–Walker equations arose in conjunction with an attempt to develop a quasiperiodic model for the Southern Oscillation, a pressure seesaw closely related to the El Niño phenomenon. The question remains of why has his work concerning the SO, other pressure oscillations and related teleconnections proved so successful. Within meteorology, the somewhat parochial explanation is that it must have been Walker's climatological expertise. In my judgment, however, it was his expertise in mathematics and statistics, coupled with a dedicated effort to solve a particular scientific problem (namely, long-range weather or climate forecasting) that best explains this success.

A heightened level of interest in collaborative research between statistics and the environmental sciences and geosciences now exists (e.g., Chelton, 1994; Nychka, 2000; Piegorsch, Smith, Edwards and Smith, 1998). However, the lessons gleaned from the earlier history of collaboration in such areas ought to be appreciated as well. In this vein, one last quote from Walker is germane:

> There is, to-day, always a risk that specialists in two subjects, using languages full of words that are unintelligible without study, will grow up not only, without knowledge of each other's work, but also will ignore the problems which require mutual assistance.—*Sir Gilbert T. Walker* (Walker, 1927b, page 321)

APPENDIX

The time series of monthly mean Darwin pressure, for the period January 1882–present, is available from a Web site maintained by the Climate Prediction Center of the National Oceanic and Atmospheric Administration (NOAA): http://www.cpc.ncep.noaa.gov/data/indices/index.html. The seasonal time series, originally analyzed by Walker (1931), was reconstructed from this NOAA data set. Because it is not possible to specify the initial state of the time series (i.e., winter 1882), the required value for December 1881 was obtained from another source, a Web site maintained by the Australian Bureau of Meteorology Research Centre: ftp://ftp.bom.gov.au/anon/home/ncc/www/sco/soi/darwinmslp.html.

Since the time Walker analyzed this data, a correction has been applied to remove an artificial trend introduced by instrumental drift. This trend actually was noticed by Walker (1931, page 529), and he suspected a source such as "some change of barometric correction." Yet he did not remove this trend and argued that "its effect on periodicity will be insignificant." Incidentally, Walker was a contributor to the collection in which these corrected pressures were published (Clayton, 1934, page 575). In fact, the impetus for this series of publications of weather records was research on world weather in which Walker was one of the initiators.

In constructing atmospheric or oceanic circulation indices, today it is common to adjust for annual cycles in the mean and standard deviation simply by standardizing each month or season separately (i.e., subtracting the corresponding sample mean and then dividing by the sample standard deviation) (e.g., Trenberth and Hoar, 1996). Although Walker only removed the seasonal mean in his analysis of Darwin pressure, he did make much use of standardized variables in other work. In particular, he usually expressed multiple regression equations in this form, both for computational convenience and for ease in interpretation (e.g., Walker, 1910). In addition, his correlation or regression analyses generally were conducted separately for each season (i.e., stratifying the data by the season to be predicted), because he was aware of the possibility that variability and covariability might differ depending on the time of year (e.g., Walker, 1924).

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