STAT 479 - Sample Exam III

Note: *The purpose of posting a sample exam is only to indicate the level of the real exam. I encourage you to look at these questions only after you think you have completed your review. If they appear to be beyond your abilities, then that should be taken as an indication that more review is needed, not that you should learn how to do these questions while ignoring the rest of the course material. These questions will* <u>not</u> *be on the exam.*

1. Suppose $\{x_t\}_{t=1}^n$ is a sequence of observed values from a time series $\{X_t\}_{t=-\infty}^{\infty}$.

   (a) Define the Discrete Fourier Transform $X(k)$.

   (b) Show that $X(k)$ is the complex conjugate of $X(n-k)$.

2. Let $Y_t = \sum_{s=-\infty}^{\infty} a_s X_{t-s} + u_t$, where $\{X_t\}$ and $\{u_t\}$ are uncorrelated time series. Show that the spectra satisfy $f_Y(\nu) = |A(\nu)|^2 f_X(\nu) + f_u(\nu)$, where $A(\nu)$ is the IFT of $\{a_s\}$.

3. (a) Define the *power spectrum* $f_X(\nu)$.

   (b) Explain why the raw periodogram is not a satisfactory estimate of the spectral density function. Define the *smoothed* periodogram with smoothing parameter L, and explain why it might improve on the raw periodogram.

4. When estimating the coherence, the periodograms <u>must</u> be smoothed. Reason: If no smoothing is done (i.e. $L = 1$), then the estimated coherence at any frequency ν will be $\rho_{YX}^2(\nu) = 1$, regardless of what the original series were. Explain why this is so.

5. Consider the following simplified way to model a leading economic indicator. We shall assume that the quantity Y_t, measured in the t^{th} time period, is a multiple A (independent of t) of the quantity X in the previous time period, plus white noise w independent of X.

   (a) Write out this model in mathematical terms. Show that the cross spectrum $f_{YX}(\nu)$ and the spectrum $f_X(\nu)$ satisfy $f_{YX}(\nu) = Ae^{-2\pi i\nu}\ f_X(\nu)$.

   (b) How would you estimate the frequency at which the modulus of the cross spectrum reaches a maximum, if all you had available were data from the series $\{X_t\}$?