

STAT 479 - Sample Exam II

Note: The purpose of posting a sample exam is only to indicate the level of the real exam. I encourage you to look at these questions only after you think you have completed your review. If they appear to be beyond your abilities, then that should be taken as an indication that more review is needed, not that you should learn how to do these questions while ignoring the rest of the course material. These questions will not be on the exam.

- (a) Define what it means for a series to be *invertible*.
(b) Is the zero-mean series defined by

$$X_t = X_{t-1} - .5X_{t-2} + w_t$$

stationary? Why or why not?

- (a) Define the *partial autocorrelation function* of a time series.
(b) Derive the lag-1 partial autocorrelation ϕ_{11} .
- Suppose that you observe data $\{x_t\}_{t=1}^n$ from an invertible MA(1) series. You wish to estimate the parameter θ .
 - Write down, or explain clearly how you would calculate, the function $S(\theta)$ which is minimized by the Maximum Likelihood estimate $\hat{\theta}$.
 - If $\hat{\theta}$ was obtained from R, then the residuals $\hat{w}_1, \dots, \hat{w}_n$ would also be computed. Given these numbers, how would you compute the forecast of X_{n+1} ?
- (a) Identify the following as a particular $ARIMA(p, d, q) \times (P, D, Q)_s$ model:

$$X_t = X_{t-1} + X_{t-12} - X_{t-13} + w_t + \theta w_{t-1} + \Theta w_{t-12} + \theta\Theta w_{t-13}.$$

- Write out an expression, similar to that given in part (a), for an $ARIMA(0, 1, 1) \times (1, 0, 1)_4$ model.
- Consider the problem of predicting a random variable Y , on the basis of data represented by a random variable X . Show that the function of X which minimizes the mean squared error

$$MSE = E[\{Y - g(X)\}^2]$$

is $g(X) = E[Y|X]$, the conditional expectation of Y , given X .