STAT 479 - Sample Exam I

Note: *The purpose of posting a sample exam is only to indicate the level of the real exam. I encourage you to look at these questions only after you think you have completed your review. If they appear to be beyond your abilities, then that should be taken as an indication that more review is needed, not that you should learn how to do these questions while ignoring the rest of the course material. These questions will* <u>not</u> *be on the exam.*

1. Let $\{X_t\}_{t=1}^n$ be a weakly stationary time series, with autocovariance function $\gamma_X(m)$.

   (a) In terms of $\gamma_X(m)$, what is the *correlation* between X_8 and X_2?

   (b) If $n = 3$, $x_1 = 4$, $x_2 = 2$, $x_3 = 3$, compute the estimate of $\gamma_X(1)$.

2. Suppose that $Y_t = AX_{t-D} + w_t$, where $\{w_t\}$ is white noise independent of $\{X_t\}$ and $D > 0$. The series $\{X_t\}$ and $\{Y_t\}$ are jointly stationary.

   (a) What are the 'best' values A^* and D^* of A and D, for optimal forecasting of Y_t by $\hat{A}X_{t-\hat{D}}$? Give a detailed justification for your answer.

   (b) In the previous question, if data $\{x_t, y_t\}_{t=1}^n$ were available, how would you estimate A and D?

3. Suppose that $\{X_t\}$ and $\{Y_t\}$ are jointly stationary time series *whose means are known to be zero*. Given data $\{x_t, y_t\}_{t=1}^n$, determine an *unbiased* estimate of $\gamma_{XY}(m)$ for $m = 0, ..., n-1$. Explain why your estimate is unbiased.

4. Consider the following model for the movement of a certain type of particle. In the t^{th} unit of time, the particle travels a distance w_t, where $w_1, w_2, \ldots$ are independent random variables with a mean of zero and a variance of σ_w^2. A positive value of w represents a forward movement, a negative value a backwards movement. Let X_t represent the position of the particle after the t^{th} time period ($X_0 = 0$). Is $\{X_t\}$ stationary? Why or why not?

5. A stationary time series $\{Y_t\}_{t=1}^n$, with 3600 data points, was regressed on its own lag-1 values, i.e. the model

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t \qquad (*)$$

   was fitted. Some summary statistics were:

$$\hat{\beta}_1 = .5,\ s_{\hat{\beta}_1} = .25,\ s_e = 1.5.$$

(a) Consider the null hypothesis that $\beta_1 = 0$, versus the alternate hypothesis $\beta_1 \neq 0$. If you had the appropriate tables (and what would these be?) how would you assess the p-value, i.e. exactly what would you look up in the tables?

(b) Recall that in a straight line regression of one series Y_t on another series X_t, the slope estimate is

$$\hat{\beta}_1 = \frac{\sum(y_t - \bar{y})(x_t - \bar{x})}{\sum(x_t - \bar{x})^2}.$$

In (∗) then, what is $\hat{\beta}_1$ really estimating, i.e. what common parameter of time series models is estimated by $\hat{\beta}_1$?