**STAT 479 - Assignment 3 - due date is on course outline**

1. Consider the series $X_t = w_t - \theta w_{t-1}$.
   (a) Obtain the cross-spectrum $f_{Xw}(\nu)$.
   (b) Obtain the coherence $\rho^2_{Xw}(\nu)$.

2. 4.4 in the text. Note that in (b) the method to be used is prescribed.

3. 4.8 (a) and (b) only. Hint: Assume $0 < \nu_k < .5$. After formulating this as a linear regression problem and writing down the normal equations - see (2.4) in Chapter 2 - you will encounter terms like $\sum_t \cos^2(2\pi\nu_k t)$, $\sum_t \sin^2(2\pi\nu_k t)$ and $\sum_t \cos(2\pi\nu_k t)\sin(2\pi\nu_k t)$. You can look up the values of these sums, but they are easy to evaluate as follows. Consider $\sum_t \left(e^{2\pi i\nu_k t}\right)^2$. Using the identity studied in class - and used in the inversion of the DFT to recover the data - you can show that this sum equals 0. Now write out its real and imaginary parts - they will involve the trigonometric sums above, and must both equal 0; from this you get the required three sums.

4. Suppose that an impulse response analysis, with $\{X_t\}$ as input and $\{Y_t\}$ as output, led to seemingly significant coefficients $a_{-k}, a_{-k+1}, ..., a_l$ with $k, l > 0$. Show that one can forecast $\{X_t\}$ from $\{Y_t\}$ (and the past history of $\{X_t\}$), i.e. that there are coefficients $\{\beta_0, ..., \beta_{k+l}\}$ so that
$$X_t = \beta_0 Y_{t-k} + \sum_{s=1}^{k+l} \beta_s X_{t-s}.$$
(This is just one example showing that one need not use only the coefficients at lags with one particular sign.)

5. 4.16 in the text. Use smoothing parameters of your choice. Answer (b) by plotting the squared modulus of the IFT relating the powers of $\{u_t\}$ and $\{v_t\}$ to that of $\{x_t\}$. In (c) use the series `prodn`.

6. Suppose we are given a stationary zero-mean series $x_t$ with spectrum $f_X(\nu)$, and we then construct the derived series $y_t = a y_{t-1} + x_t$.
   (a) Show how the theoretical $f_Y(\nu)$ is related to $f_X(\nu)$.
   (b) Plot the function that multiplies $f_X(\nu)$ in (a) for $a = .1$ and for $a = .8$. (This is a recursive filter.)

7. 4.18 in the text. Use the R data series `sunspots`, which is longer than `sunspotz`. For the parametric estimate the 'method of your choice' can be AIC, which is the default in the R command `spec.ar(series)`. Using the periodogram, estimate the main periodicity. Also filter the series in a manner designed to highlight the frequencies corresponding to this periodicity.

8. 4.20 in the text.

9. The dataset `climhyd` in `astsa` is a matrix consisting of six columns, labelled `airtemp`, ..., `inflow`; they are climatic variables related to Shasta Lake in California. We would like to look at possible relations among the five weather factors as they relate to the inflow into the lake. Use the transformed inflow $I = \sqrt{\text{inflow}}$ and transformed precipitation $P = \sqrt{\text{precipitation}}$.
   (a) Argue, on the basis of the coherences, that the strongest determinant of the inflow series is (transformed) precipitation. For this you can plot all the coherences in one command: put the six series (two of which are now transformed) into a matrix ("mat", say), then use the commands `qwe=spec.pgram(mat, ...)` and `plot.spec(qwe, plot.type="coh",... )`. See `help(plot.spec)` for further details.
   (b) Carry out a backward lagged regression with input = precipitation and output = inflow; use a threshold large enough that only two of the lags are significant. Then invert the result so as to obtain a model from which $I_t$ can be predicted $I_{t-1}$ and $P_t$. Estimate the coefficients of this model in a manner of your choosing.