STAT 368 - Ch.1 - 4 review

<u>One factor at two levels</u>
(e.g. modified/unmodified mortars)

- 2 independent samples: use 2-sample t-test and associated CIs if the 2 variances appear equal. This generalizes to the CRD.
  - What exactly is this t?
  - Welch's test otherwise

- Paired samples, within each pair ($=$ block) the observations are in random order: use paired t-test ($=$ one sample t-test on the differences). This generalizes to the RCBD.
  - What exactly is this t?

- General theory - what is a CI? p-value? standard error? χ^2 r.v.? F r.v.? How does one calculate means and variances of linear functions like $\sum a_i Y_i$? What is plotted on a qqplot?

One factor at several levels
(e.g. levels of cotton in shirts)

- a levels, n observations (replicates) at each level, all an runs in random order; CRD

- The randomization will, we hope, cancel out any systematic effects besides those of the treatments (levels)

- The replication will reduce the random error, and allow us to estimate it.

- General theory - take a model explaining the data, do the (least squares) estimation, make inferences.

  - $y_{ij} = \mu \ + \tau_i + \epsilon_{ij}$

  - $S(\mu, \boldsymbol{\tau}) = \sum_{i=1}^{a} \sum_{j=1}^{n} \left(y_{ij} - E\left[y_{ij} \right] \right)^2$ minimized by $\hat{\mu} = ???$ $\hat{\tau}_i = ???$

  - Minimum value is $S(\hat{\mu}, \hat{\boldsymbol{\tau}}) = SS_E$ (in this 'Full' model) $= \sum_{i=1}^{a} \sum_{j=1}^{n} \left(y_{ij} - \bar{y}_{i.} \right)^2$.

  - Are all $\tau_i = 0$? If so, the min SS is

$$\begin{aligned} SS_{Red} &= S(\hat{\mu}, \mathbf{0}) \\ &= \sum_{i=1}^{a} \sum_{j=1}^{n} \left(y_{ij} - \bar{y}_{..} \right)^2 = SS_T \\ &\overset{how?}{=} SS_E + SS_{Tr}, \end{aligned}$$

    so the increase in SS 'caused by H_0' is

$$SS_{Red} - SS_{Full} = SS_{Tr}$$

    and the appropriate F is

$$F_0 = MS_{Tr}/MS_E.$$

- Model checking - plot residuals against whatever is available, looking for trends, evidence of variance heterogeneity, evidence of non-normality.

- Nonparametric alternatives: Kruskal-Wallis test, Levene's test. Rationale for each?

- Multiple comparisons - what is wrong with producing numerous CIs, all using "estimate $\pm\, t_{\alpha/2} \cdot$ se(estimate)"? Alternatives:

  - Scheffé - for <u>all</u> contrasts

  - Tukey - for all differences among the treatment effects

<u>Several factors</u>, one of which (the 'treatment') is of primary interest, the rest of which are 'nuisance' factors

- Treatment (at a levels) and one nuisance factor at b levels. At each of these b levels (the 'blocks') we carry out a CRD experiment (i.e. we randomize only within blocks). E.g. tips (treatments) pressed into metal coupons (blocks). For this to be useful, responses should be less highly varied within blocks than between blocks.

- Our model allows for 2 systematic sources of variation - treatment and blocks. We looked only at the case of $n = 1$ obs./cell, so only an additive model can be fitted.

  - $\mu_{ij} = \mu + \tau_i + \beta_j$

  - LSEs = ?

- How to get SS_{Full} and SS_{Red} under various hypotheses? (e.g. additional review question #3)

- Treatment + two nuisance factors? If all at p levels, then a $p \times p$ Latin square design will work.
  - Three systematic sources of variation: $\mu_{ij} = \mu + \alpha_i + \tau_j + \gamma_k$
  - LSEs = ?

- Treatment + three nuisance factors? Graeco-Latin squares.

- BIBD - used when a block is not large enough to take all treatments. ‘Balanced’ means ??
  - Algebraic treatment is difficult; numerically we only have to remember to enter the nuisance factor first; SS_{Tr} is then the amount of the remaining variation which is accounted for by the changing levels of the treatment.